PE 7033-Well Test Analysis I
HW# 2
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Spring 2017

Given Date: January 19, 2017
Due Data: January 25, 2017
Subject: Pressure-Derivative, Smoothing, and Log-Log Diagnostic Plots

Problem (100 pts). The pressure-derivative is used for as a diagnostic tool for model identification, when it is plotted on a log-log plot against the elapsed time. It is generated by numerical differentiation of pressure change data (Δp, defined as $\Delta p = p_i - p(t)$, for a drawdown test, where pi is the initial pressure at the beginning of the test, and $p(t)$ is the flowing bottom-hole pressure) or $p(t)$ with respect to natural logarithm of time. Recall that the pressure-derivative is defined by

$$\frac{d\Delta p}{d\ln t} = t\frac{d\Delta p}{dt} = t\frac{d\left[p_i - p(t)\right]}{dt} = -t\frac{dp(t)}{dt} = -\frac{dp(t)}{d\ln t} \tag{1}$$

One of the numerical methods is to fit a Lagrangian second degree polynomial based on ln$t$ or t that passes through successive time points and then differentiate this polynomial to generate the derivative data given below. A Lagrange interpolating polynomial of degree 2 passing exactly through the points (lnt_k,Δp_k), (lnt_{k+1}, Δp_{k+1}), and (lnt_{k+2}, Δp_{k+2}) for each k ($1<k<N-2$, where N represents the total number of sampled or measured pressure data, Δp_i, $i=1,2,\ldots,N$):

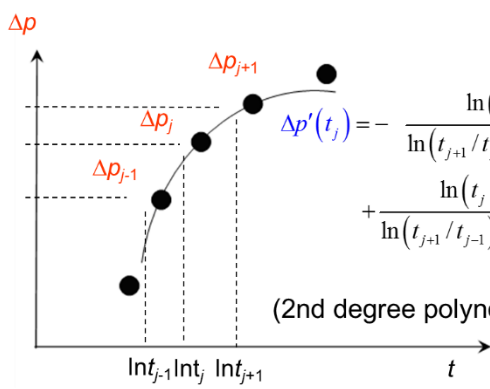
$$\Delta p_k(\ln t) = \sum_{i=k}^{k+2} \Delta p_i L_i(\ln t), \tag{2}$$

where L_i is called the Lagrange coefficient polynomial and is defined by

$$L_i(\ln t) = \prod_{\substack{j=k \\ j\neq i}}^{k+2} \frac{\left(\ln t - \ln t_j\right)}{\left(\ln t_i - \ln t_j\right)}, \tag{3}$$

Note that $L_i(\ln t_j) = 0$ when $i \neq j$ and $L_i(\ln t_i) = 1$.

(a) (**25 pts**) Derive the Bourdet et al. three-point derivative formula is given by:



$$\Delta p'(t_j) = -\frac{\ln\left(t_{j+1}/t_j\right)}{\ln\left(t_{j+1}/t_{j-1}\right)\ln\left(t_j/t_{j-1}\right)}\Delta p_{j-1} + \frac{\ln\left(t_{j+1}t_{j-1}/t_j^2\right)}{\ln\left(t_{j+1}/t_j\right)\ln\left(t_j/t_{j-1}\right)}\Delta p_j + \frac{\ln\left(t_j/t_{j-1}\right)}{\ln\left(t_{j+1}/t_{j-1}\right)\ln\left(t_{j+1}/t_j\right)}\Delta p_{j+1}$$

Bourdet method
(2nd degree polynomial through successive three points)

(b) **(25 pts)** Consider drawdown pressure change data given in an Excel file entitled as **Table_HW_No_2_Data** in Harvey. Generate derivative data by using the Bourdet three-point formula based on Eq. 2.

(c) **(15 pts)** Make a log-log plot of Δp and $\Delta p'$ (generated in step b) versus t on the same graph paper, identify ½ slope line, zero-slope line, and unit-slope line if any exists based on Bourdet derivative data and their time intervals. You are free to use Excel, Grapher, Matlab, or any other graphics software to make your log-log plot, and you should show the slope lines on your plot by the appropriate lines.

(d) **(35 pts)** As you will notice in Parts b and c, derivative data computed in part b are quite noisy. Suppose we want to smooth the data by using a differentiation interval length based on a criterion defined by:

$$L = \min\left\{ \ln\left(\frac{t_{j+1}}{t_j}\right), \ln\left(\frac{t_j}{t_{j-1}}\right) \right\}, \tag{4}$$

where L defines a minimum distance based on the logarithmic (natural log) time scale between the left- and right-hand points of the central time points where the derivative is computed and is a dimensionless parameter. You may try L values in such that $0 \le L \le 0.3$, if $L = 0$, i.e., no smoothing is applied. Show how smoothing improves flow regime identification. This part may require an implementation of Eq.4 into your algorithm that you will write in Part b.