

2011

# STATISTICS AND NUMERICAL TECHNIQUES

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP - A

### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i)  $E^{-1}$  is equivalent to

a)  $1 - \nabla$

b)  $1 + \Delta$

c)  $1 - \nabla$

d) none of these.

ii) if  $\text{Var} ( aX + bY ) = a^2 \text{Var} ( X ) + b^2 \text{Var} ( Y )$ , the  $X$  and  $Y$  are

a) mutually exclusive

b) uncorrelated

c) impossible events

d) none of these.



iii) If  $E^c$  is the complement of the event  $E$  then

a)  $P(E^c) = 1 - P(E)$

b)  $P(E^c) = P(E)$

c)  $P(E^c) = 1 + P(E)$

d) none of these.

iv) For a binomial distribution

a) Mean  $>$  Variance

b) Mean = Variance

c) Mean  $<$  Variance

d) none of these.

v) If  $f(x)$  is a polynomial of degree  $n$ , then  $\Delta^n f(x)$  is

a) 0

b) constant

c) 1

d) none of these.

vi) Romberg's method is based on the error in

a) Trapezoidal rule

b) Simpson's 1/3rd rule

c) Weddle's rule

d) none of these.

vii) Order of convergence of Newton-Raphson method is

a) 0

b) 2

c) 1

d) none of these.

viii) Normal distribution is

- a) unimodal
- b) bimodal
- c) trimodal
- d) none of these.

ix) Order of error in Simpson's 1/3rd rule is

- a)  $h^2$
- b)  $h^4$
- c) 1
- d) none of these.

x) Condition for convergence of Fixed-point iteration method to solve the equation  $f(x) = 0$  in  $[a, b]$  is that

- a)  $\phi'(x) < 1$
- b)  $\phi'(x) > 1$
- c)  $\phi'(x) = 1$
- d)  $\phi'(x) \leq 1$

in  $[a, b]$  where  $f(x) = 0$  can be written as  $\phi(x) = x$ .

xi) The formula of the area of a trapezium whose length of the parallel sides are  $a, b$  and the distance between them is  $h$  is

- a)  $\frac{h}{2}(a + b)$
- b)  $\frac{h}{2} + a + b$
- c)  $h(a + b)$
- d)  $h + a + b$ .

xii) The method of bisection for solving equation  $f(x) = 0$  in  $[a, b]$  is based on

- a) Intermediate value theorem
- b) MVT of integral calculus
- c) MVT of differential calculus
- d) Fundamental theorem of Algebra.



xiii)  $\int_a^b f(x) dx$  describe the

- a) area
- b) volume
- c) surface area
- d) volume and surface area both

under the curve  $y = f(x)$  in  $[a, b]$ .

xiv) In Newton's forward and backward interpolation formula the points are

- a) equally spaced
- b) unequally spaced
- c) both of the previous
- d) none of the previous.

### GROUP - B

#### ( Short Answer Type Questions )

Answer any *three* of the following.  $3 \times 5 = 15$

2. Prove that  $P(A \cup B) = P(A) + P(B)$ , if  $A$  and  $B$  are disjoint events.
3. The probabilities of  $X$ ,  $Y$  and  $Z$  being managers are in the ratio  $4 : 2 : 3$  respectively. The probabilities that the bonus scheme will be introduced if  $X$ ,  $Y$ ,  $Z$  become managers are  $\frac{3}{10}$ ,  $\frac{1}{2}$ ,  $\frac{4}{5}$  respectively.
  - i) What is the probability that bonus scheme will be introduced ?
  - ii) If the bonus scheme has been introduced, what is the probability that the manager appointed was  $Y$ ?



4. Evaluate  $\int_0^1 x/(1+x) dx$  using Trapezoidal rule using 5 intervals.
5. Evaluate  $\int_1^2 \log x dx$  using Simpson's 1/3rd rule using 5 intervals.
6. Distinguish between absolute error and relative error with example.

### GROUP - C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

7. a) Use Newton-Raphson method to find a positive root of  $e^x = 3x$  correct to four decimal places.
- b) What are the advantages and disadvantages of Newton-Raphson method ?
- c) State and prove Bayes' theorem.  $6 + 4 + 5$
8. a) Find  $\Delta^2 f(x)$  where  $f(x) = 3x^4 + 8x^2 + 5x + 7$  by taking  $h = 1$ .
- b) Apply Simpson's 1/3rd rule to find  $\int_0^{\pi/2} \cos x dx$  by dividing the range on integration into 6 subintervals.
- c) Prove that if  $E_1$  and  $E_2$  are statistically independent, then  $P(E_1 \cap E_2) = P(E_1)P(E_2)$ .  $6 + 6 + 3$



9. a) Discuss the convergence of fixed point iteration.
- b) Prove that if  $\rho_{xy}$  is the Pearson correlation coefficient between the random variables  $X$  and  $Y$ , then  $-1 \leq \rho_{xy} \leq 1$ .
- c) Apply Newton's forward interpolation to find  $f(x)$  at  $x = 2.5$  from the following table :

$x$	2	3	4	5	6
$f(x)$	1.456	1.689	1.992	2.010	2.225

5 + 5 + 5

10. a) Find  $\sqrt{45}$  using Newton-Raphson method.

- b) Use Gauss-Jordan method to solve

$$p + 2q + r - s = -2$$

$$2p + 3q - r + 2s = 7$$

$$p + q + 3r - 2s = -6$$

$$p + q + r + s = 2.$$

- c) Prove that if  $X \sim \text{Binomial}(n, p)$  then  $E(X) = np$ .

5 + 5 + 5

11. a) Derive the expression of error in the composite trapezoidal rule.

- b) Apply Runge-Kutta method of order 4 to solve  $\frac{dy}{dx} = x + y$ ,

where  $y(0) = 1$  at  $x = 0.1$  and  $0.2$ .

7 + 8



12. a) Apply LU factorization to find the inverse of

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

b) Prove that for a normal distribution :

$$\text{Mean} = \text{Median} = \text{Mode}$$

c) Fit an approximating polynomial to the following data :

$x$	0	3	4
$f(x)$	2.12	4.34	3.19

$$8 + 3 + 4$$