Presentation link: https://youtu.be/gB5Bl0b7VBs

1. Executive Summary

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Re: Asset Class Optimization, Volatility Time Series Analysis, and Money Manager Recommendations

The Markowitz's portfolio optimization method was used to optimally partition client's endowment into three asset classes, equities, real estate, and market-neutral hedge funds and as specified by the client the investment in hedge funds was constrained to 20% of the total investment. For 2020, we recommend investing $553.1 million in equities, $244.3 million in real estate and remaining $199.4million in hedge funds. Returns for equity were forecasted for the coming five years using historical data as old as 1928 using GARCH model for time-series forecasting. According to our analysis, we forecast the volatility for next year to be around 14.75%. After evaluating performance of ten money managers, we shortlisted manager number 2, 7, 8, 9 for the interviews.

2. Data Sources & Transformations

For Markowitz's portfolio optimization final project data set was provided by the client and used unchanged. To forecast the volatility for the next five years, stock index data from last 91 years was used. For the current year, returns were provided, but this value does not capture the returns for the entire year, therefore we excluded this value from the calculations. Depending on the events in the rest of the year, the actual value of returns can be significantly different from the value provided in the data. To recommend money managers, we analyzed ten money manager's performance over the last few years. For this data we considered money manager's performance for the current financial year, since we were doing a comparative study instead of absolute. We used Roy's sharpe ratio which uses benchmark performance as the target return[1]. Also, we have excluded managers with experience less than 5 years. If data for money managers past portfolios was used, managers with good performance in equities, real estate and hedge funds would have been selected due to the strong correlation with client's requirements.

3. Methods and Metrics

For asset class allocation we followed standard Markowitz Portfolio Optimization based on assumptions given by the client, including constraint that no more than 20% of portfolio could be allocated to Market Neutral sector. We used the Sharpe ratio as the scoring metric to calculate optimal allocations. The sharpe ratio for a single asset is the ratio of excess return of an investment asset A over the rate of return paid by a risk-free government bond and the standard deviation of the time series of returns of the investment asset. The expected Sharpe ratio for the weighted sum of three assets A, B, and C is:

$$\frac{(w_1 u_A + w_2 u_B + w_3 u_C) - r_f}{\sqrt{{w_1}^2{\sigma_A}^2 + {w_B}^2{\sigma_B}^2 + +{w_3}^2{\sigma_C}^2 + 2w_1 w_2(R_{AB}\sigma_A\sigma_B) + 2w_2 w_3(R_{BC}\sigma_B\sigma_C) + 2w_1 w_3(R_{AC}\sigma_A\sigma_C)}}$$

where w_i, u_i and σ_i are the weight, mean and standard deviation of i_{th} investment, and R_{ij} is the covariance between i^{th} and j_{th} investment.

In Markowitz Portfolio Optimization, the expected asset returns and variances, and the Covariance between each pair of assets, are all assumed fixed, while the weights are varied to maximize the Sharpe Ratio.

We estimate the volatility using the GARCH model. This model forecasts the volatility by taking into consideration auto-correlation between adjacent year's values and the mean reversion property. There are

various versions of the this model, we have used the GARCH (1,1) model which takes into consideration past one year's actual volatility and estimated volatility. The formula for this model is:

$$\sigma_n{}^2 = \gamma V_l + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

where:

V_l= the long-term observed average variance through year n-1

u_{n-1}^2 = the observed variance in year n-1

σ_{n-1}^2 = the previous forecast for variance in year n-1

σ_n^2 = the forecast for variance in year n

We used mean square error as the scoring metric for this model which is given by,

$$\frac{1}{m}\sum_{i=1}^{m}(|u_n| - \sigma_n^2)^2$$

We split the data, into train and test set to validate the performance of the model. With total size of the data 91, we used recent 10 years data for testing and the remaining 81 years data for training the model.

We ranked the money managers performance by calculating the Roy's safety first ratio given by the formula:

$$\frac{r_x - r_t}{\sigma_{r_x}}$$

where, r_x is the average rate of return of x, r_t is target return and σ_{r_x} is standard deviation of r_x. Also, we did not consider managers with experience less than 5 years, as the benchmark perfored well in three out of the last four years and hence the new managers have higher chance of getting selected. But we also included manager 8 instead of manager 4 who had a hgher Roy's ratio due to manager 8's experience of ten years. Though manager 8's performance in starting few years was not the best, she has consistently performed better than manager 4 in the recent years.

4. Findings

Using the Markowitz's portfolio optimization, we maximized the expected sharpe ratio and calculated the optimal weight parameters w1, w2, w3 as 0.55, 0.24 and 0.2 respectively. From these values we recommend the amount to be invested in equities, real estates and hedge funds. For volatility forecasting, using data from 81 years the optimal values for α, β and γ were calculated to be 0.226, 0.772, and 0.001 respectively. The train error and test error for the above-mentioned values is 1.56% and 1.7%. From these values the volatility forecast was calculated to be 14.75%. The optimized value of γ is very low, indicating our forecast does not depend on long term average of the returns, it depends on the volatility and it's estimate from the previous year. Using the Roy's safetyt-first ratio as well as experience we ranked the money managers and recommend moanagers 2,7,8 and 9 to be interviewed.

5. Refereneces:

1. https://en.wikipedia.org/wiki/Efficient_frontier
2. https://www.investopedia.com/articles/08/annualized-returns.asp
3. https://www.investopedia.com/articles/07/sharpe_ratio.asp
4. https://www.investopedia.com/articles/stocks/11/5-ways-to-measure-money-managers.asp
5. https://math.berkeley.edu/~btw/thesis4.pdf
6. https://machinelearningmastery.com/develop-arch-and-garch-models-for-time-series-forecasting-in-python/