DSNM Assignment

Q1 Solve, $I = \int_1^2 \frac{dx}{x}$ by using both simpson's $\frac{1}{3}$ and 3/8 rule by dividing the domain into 6 equal divisions.

Soln: Given: $h = \frac{1}{6}$, $f(x) = \frac{1}{x}$

x_n	$f(x) = y_n$
1	$1 = y_0$
7/6	$\frac{1}{7/6} = \frac{6}{7} = y_1$
8/6	$\frac{1}{8/6} = 6/8 = y_2$
9/6	$\frac{1}{9/6} = \frac{6}{9} = y_3$
10/6	$\frac{1}{10/6} = \frac{6}{10} = y_4$
11/6	$\frac{1}{11/6} = \frac{6}{11} = y_5$
2	$\frac{1}{2} = y_6$

By simpson 1/3 rule

$$\int_1^2 \frac{1}{x}dx = \frac{h}{3}\left(y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right)$$

$$= \frac{\frac{1}{6}}{3}\left[\left(1+\frac{1}{2}\right) + 4\left(\frac{6}{7}+\frac{6}{9}+\frac{6}{11}\right) + 2\left(\frac{6}{8}+\frac{6}{10}\right)\right]$$

$$= \frac{1}{18}\left[\frac{3}{2} + 24\left(\frac{1}{7}+\frac{1}{9}+\frac{1}{11}\right) + 12\left(\frac{1}{8}+\frac{1}{10}\right)\right]$$

$$= \frac{1}{18}\left[\frac{3}{2} + \frac{24(99+77+63)}{7 \times 9 \times 11} + 12\left(\frac{18}{80}\right)\right]$$

$$= \frac{1}{6}\left[\frac{1}{2} + \frac{8 \times 239}{7 \times 9 \times 11} + \frac{72}{80}\right]$$

$$= \frac{1}{6}\left[0.5 + 2.7590 + 0.9\right] = \frac{4.159}{6}$$

$$\int_1^2 \frac{1}{x}\,dx = \cancel{0.2908}\ 0.6913$$

Now, <u>By simpson's 3/8 rule</u>

$$\int_1^2 \frac{1}{x}\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$= \frac{3\times 1/6}{8}\left[\left(1+\frac{1}{2}\right)+3\left(\frac{6}{7}+\frac{6}{8}+\frac{6}{10}+\frac{6}{11}\right)+\left(2\times\frac{6}{9}\right)\right]$$

$$= \frac{1}{16}\left[\frac{3}{2}+18\left(\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{11}\right)+\frac{4}{3}\right]$$

$$= \frac{1}{16}\left[1.5+\frac{18(880+770+616+560)}{6160}+1.333\right]$$

$$= \frac{1}{16}\left[2.8333+8.2577\right]$$

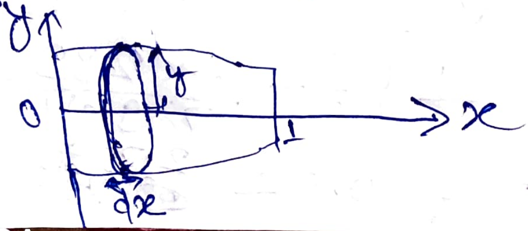
$$= \frac{11.091}{16} = 0.693$$

Q2 A solid revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x=0$ and $x=1$ and a curve through the points with the following coordinates.

x	y
0.00	1.00
0.25	0.9896
0.50	0.9589
0.75	0.9089
1.00	0.8415

Estimate the volume of the solid formed by using the simpson's 1/3 rule.

Soln:



Volume of elemental cylinder, $dV = \pi y^2 dx$

$\therefore$ Volume of solid, $V = \int dV = \int_0^1 \pi y^2 dx$

Now, we have, $h = 0.25$

x	0.00	0.25	0.50	0.75	1.00
$f(x) = y$	1.00	0.9896	0.9589	0.9089	0.8415
$g(x) = y^2$	1.00	0.97930	0.91948	0.82609	0.70812
	y_0	y_1	y_2	y_3	y_4

$\therefore$ Volume, $V = \int_0^1 \pi y^2 dx = \pi \int_0^1 y^2 dx$

Now, By simpson's $\frac{1}{3}$ rule

$$V = \pi \times \int_0^1 y^2 dx$$

$$V = \pi \times \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right]$$

$$= \pi \times \frac{0.25}{3}\left[(1.00 + 0.70812) + 4(0.97930 + 0.82609) + 2(0.91948)\right]$$

$$= \frac{3.142 \times 0.25}{3}\left[1.70812 + 7.22156 + 1.83896\right]$$

$$= 0.26183 \times 10.76864$$

$$= \underline{\underline{2.8195}}$$