While adding up several numbers of different absolute accuracy the following ~~rules~~ steps may be adopted:

i) Isolate the number with greatest absolute ~~error~~ accuracy.

ii) Round-off all other numbers retaining in them one digit more than in the isolated number,

iii) Add up, and

iv) Round-off the sum by discarding one digit.

Ex-1.1

Sum of the following numbers:

0.1532, 15.45, 0.000354, 305.1, 8.12, 143.3, 0.0212, 0.643 and 0.1734 where in each of which all the given digits are correct.

Greatest absolute error is in 305.1 & 143.3, which is 0.05.

Hence round-off all the other number to two decimal digits.

These are: 0.15, 15.45, 0.00, 8.12, 0.02, 0.64 and 0.17

Hence, the sum is

$S = 305.1 + 143.3 + 0.15 + 15.45 + 8.12 + 0.02 + 0.64 + 0.17$

$= 472.59$

$= \cancel{472.59}$

$= 472.6$ [round-off to ~~one~~ by decreasing one significant digits]

$\therefore$ Round-off error $= 0.01$

absolute error $E_A = 2(0.05) + 7(0.005)$

$= 0.1 + 0.035$

$= 0.135$.

≈ 0.14

Hence, the total error $= 0.01 + 0.14 = 0.15$.

Thus, $S = 472.6 \pm 0.15$

Solution of nonlinear ~~algebraic~~ equation by numerical methods

Let us consider $f(x) = 0$ where $f(x)$ nonlinear in 'x'.

example: $x^3 + 3x^2 + 7x + 5 = 0$

$x^3 - 2x + 6 = 0$

$x^n + ax^{n-1} + bx^{n-2} + \dots + tx^1 + R = 0$

~~In a non~~ To solve ~~t~~ nonlinear ~~eq~~ equations we need to apply numerical technique.

There are several numerical techniques or methos for solving nonlinear equations.

Solving a nonlinear equation means to find out its roots.

BISECTION METHOD

Let, $f(x)$ is continuous between $x = a$ and $x = b$.

Also, $f(a) * f(b) < 0$

Then atleast one root exists between a & b

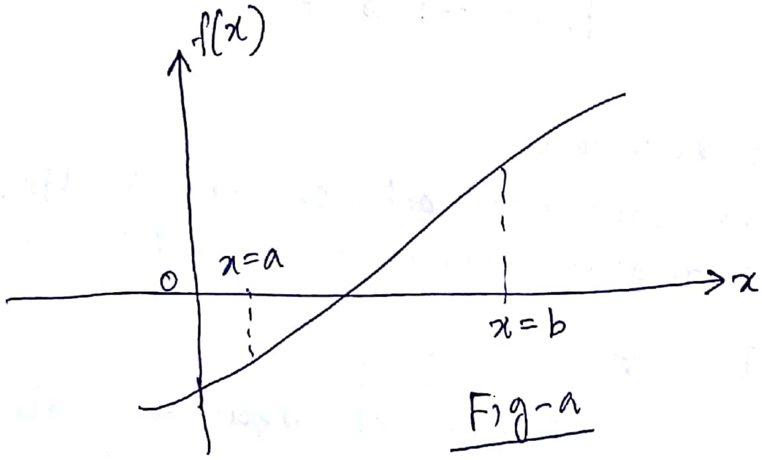


Fig-a

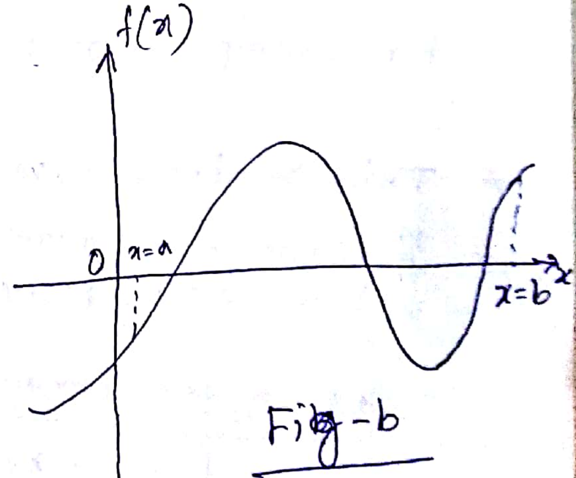


Fig-b

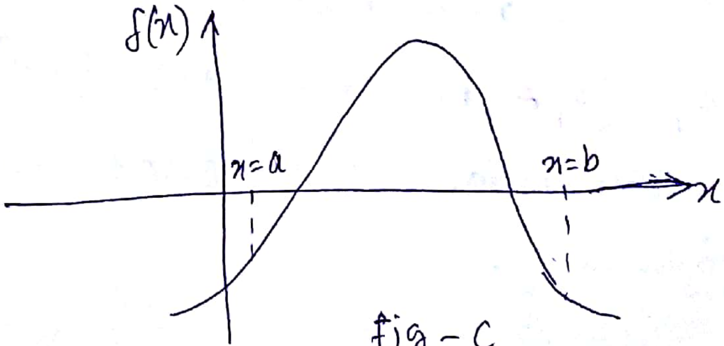


fig-c

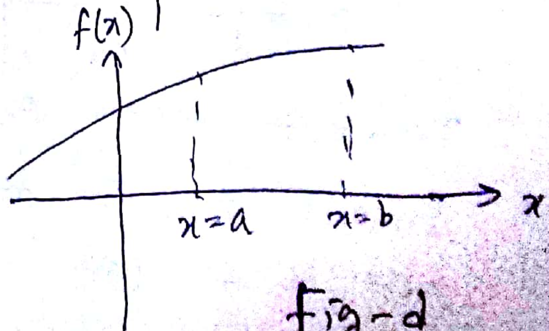
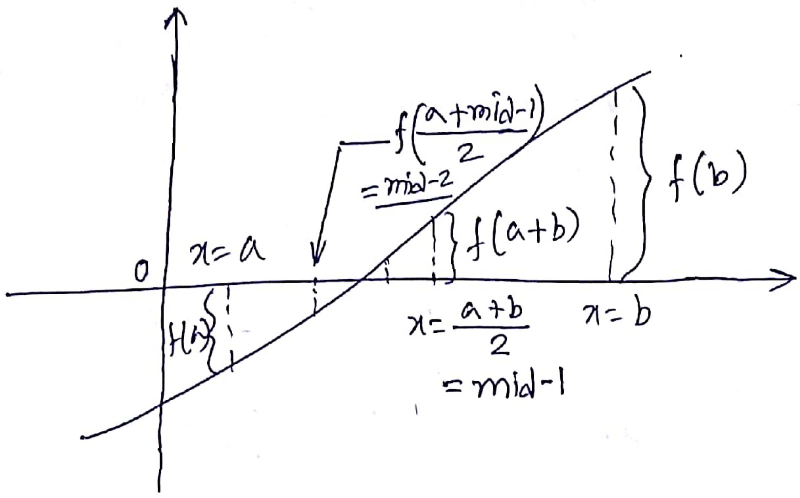


Fig-d



a & b are called brackets, or initial values

After each step interval between the initial values becomes $\frac{|b-a|}{2}$

$\therefore$ " nth " " " " " " $\frac{|b-a|}{2^n}$

$$\frac{|b-a|}{2^n} \leq \varepsilon \quad [\varepsilon \text{ is a small value called tollerance.}]$$

$$\Rightarrow 2^n \geqslant \frac{|b-a|}{\varepsilon}$$

$$\Rightarrow n \log_e 2 \geqslant \log_e\left(\frac{|b-a|}{\varepsilon}\right)$$

$$\Rightarrow n \geqslant \frac{\log_e\left(\frac{|b-a|}{\varepsilon}\right)}{\log_e 2}$$

For example, $n \geqslant 10$ when $|b-a| = 1$ & $\varepsilon = 0.001$

This method always succeeds
If there are more than one root between the initial values a & b, bisection method finds one of the roots.

Criteria for termination of iterations:

$$\varepsilon_r = \left|\frac{x_r' - x_r}{x_r'}\right| \times 100 \quad \left[\begin{array}{l} x_r \leftarrow \text{old } \cancel{\text{value}} \text{ mid value,} \\ x_r' \leftarrow \text{new mid value.} \end{array}\right]$$

Terminate when $\varepsilon_r < \varepsilon_p$ ($\varepsilon_p \leftarrow$ prescribed tollerance)

In addition, the maximum number of iterations may also be satisfied in advance.

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steps in Bisection method

1) Get two 'x' values a & b such that $f(a) * f(b) < 0$

2) Find out $\frac{a+b}{2}$ and $f\left(\frac{a+b}{2}\right)$

3) If $\left| f\left(\frac{a+b}{2}\right) \right| < \varepsilon$ [ε ← is a predefined very small value]

$\left(\frac{a+b}{2}\right)$ is the required root, hence, terminate.

otherwise proceed to step 4

4) If $f\left(\frac{a+b}{2}\right) * f(a) < 0$, take b as $\frac{a+b}{2}$ and go to step 3

else if,

$f\left(\frac{a+b}{2}\right) * f(b) < 0$, take a as $\frac{a+b}{2}$ and go to step 3.

Example:

Given, $f(x) = x^3 - x - 1 = 0$

find out one root of the above equation between $x = 1$ & 2

steps

n	a	$f(a)$	b	$f(b)$	$mid = \frac{a+b}{2}$	$f(mid)$	update
1	1	-1	2	5	1.5	0.875	b = mid
2	1	-1	1.5	0.875	1.25	-0.29688	a = mid
3	1.25	-0.29688	1.5	0.875	1.375	0.22461	b = mid
4	1.25	-0.29688	1.375	0.22461	1.3125	-0.05151	a = mid
5	1.3125	-0.05151	1.375	0.22461	1.34375	0.08261	b = mid
6	1.3125	-0.05151	1.34375	0.08261	1.32812	0.01458	b = mid
7	1.3125	-0.05151	1.32812	0.01458	1.32031	-0.01871	a = mid
8	1.32031	-0.01871	1.32812	0.01458	1.32422	-0.00213	a = mid
9	1.32422	-0.00213	1.32812	0.01458	1.32617	0.00621	b = mid
10	1.32422	-0.00213	1.32617	0.00621	1.3252	0.00204	b = mid
11	1.32422	-0.00213	1.3252	0.00204	1.32471	-0.00005	a = mid

Hence, the approximate root = 1.32471