Iterative Methods for [illegible]
Algebraic Equations

Gauss Elimination method : A direct method, where the amount of computation is known in advance.

In iterative or indirect methods, which start from an approximation to the true solution the amount of computation depends on the accuracy required.

~~In indirect method~~, ~~the~~ ~~with~~
If the indirect method is convergent, with each iterative operation the intial approximation converges towards the exact solution.

In general, the direct method should be preferred over the indirect method, but in the cases of matrices with a large number of zero elements, the iterative method would be advantageous.

$$\left.\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n &= b_n
\end{aligned}\right\} \text{— (1)}$$

where, $a_{ii} \neq 0$
in case $a_{ii} = 0$ the eq^ns should be rearranged.

$$\Rightarrow \left.\begin{aligned}
x_1 &= \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 - \cdots - \frac{a_{1n}}{a_{11}}x_n \\
x_2 &= \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1 - \frac{a_{23}}{a_{22}}x_3 - \cdots - \frac{a_{2n}}{a_{22}}x_n \\
x_3 &= \frac{b_3}{a_{33}} - \frac{a_{31}}{a_{33}}x_1 - \frac{a_{32}}{a_{33}}x_2 - \cdots - \frac{a_{3n}}{a_{33}}x_n \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
x_{n-1} &= \frac{b_{n-1}}{a_{(n-1)(n-1)}} - \frac{a_{(n-1)1}}{a_{(n-1)(n-1)}}x_1 - \cdots - \frac{a_{(n-1)(n-2)}}{a_{(n-1)(n-1)}}x_{n-2} - \frac{a_{(n-1)n}}{a_{(n-1)(n-1)}}x_n
\end{aligned}\right\} \text{— (2)}$$

$$x_n = \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1 - \frac{a_{n2}}{a_{nn}}x_2 - \cdots - \frac{a_{n(n-2)}}{a_{nn}}x_{n-2} - \frac{a_{n(n-1)}}{a_{nn}}x_{n-1}$$

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} - \frac{1}{a_{ii}}\sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij}x_j^{(k)}$$

$[X]^{k+1} = [B][X]^k + C$ Jacobi Method (method of simultaneous displacements)

In the ~~2nd~~ 1st eqn of the ~~second~~ 2nd set of eqn we get $x_1^{(2)}$ by substituting $x_2^{(0)}, x_3^{(0)} \dots x_n^{(0)}$.
In the 2nd eqn of the same set of eqn we get $x_2^{(1)}$ by substituting $x_1^{(1)}, x_3^{(0)} \dots x_n^{(0)}$.

In the 3rd eqn of 2nd set of eqn we get $x_3^{(1)}$ by substituting $x_1^{(1)}, x_2^{(1)}, \dots x_4^{(0)} \dots x_n^{(0)}$.

$$\therefore x_i^{K+1} = \frac{b_i}{a_{ii}} - \frac{1}{a_{ii}} \sum_{j=1}^{i-1} a_{ij} x_j^{K+1} - \frac{1}{a_{ii}} \sum_{j=i+1}^{n} a_{ij} x_j^{K}$$

~~In this case~~ This improvement ~~is obtained~~ in the convergence obtained, which is the Gauss-Seidel method. (Method of successive displacement)

Jacobi and Gauss-Seidel methods converge, for any choice of initial approximation $x_j^{(0)}$ $(j = 1, 2 \dots n)$, if every equation of the system of 2nd set of eqn satisfies the condn that the sum of the absolute values of the coefficients a_{ij}/a_{ii} is almost equal to, or in at least one equation less than unity. i.e. 1st ~~consider~~ ensure that $a_{ii} \neq 0$

$$\sum_{j=1, j\neq i}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| \leq 1$$

Rearranging condition

$$\sum_{j=1}^{n} |a_{ij}| < |a_{ii}|, \text{ when } i \neq j.$$

Gauss- ~~Sie~~ Seidel Method converges twice as fast as the Jacobi method.

Example - Solve the following system of Linear equations by using both the Jacobi Method and Gauss-Seidel Method.

$$16x_1 + 4x_2 + 8x_3 = 4$$
$$4x_1 + 5x_2 - 4x_3 = 2$$
$$8x_1 - 4x_2 + 22x_3 = 5$$

Example:

$$x_1 - 2x_2 + 5x_3 = 12$$
$$5x_1 + 2x_2 - x_3 = 6$$
$$2x_1 + 6x_2 - 3x_3 = 5$$

Ans:

	3	4	5
x_1	1.09	1.61	1.08
x_2	0.17	1.34	1.35
x_3	1.71	2.12	2.61

Assume: $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$.