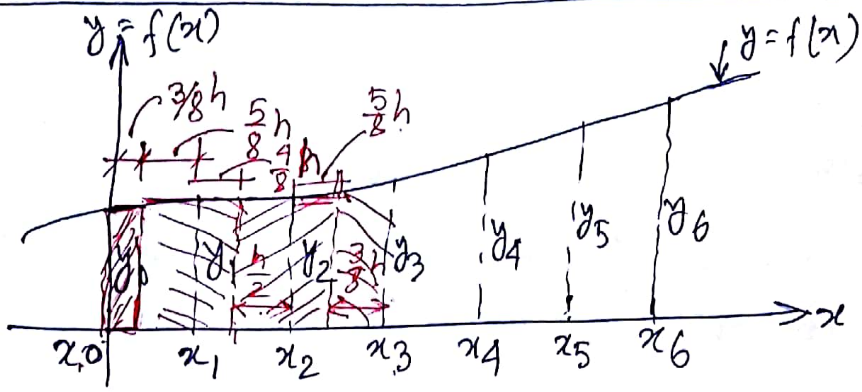
Simpson's 3/8 rule for numerical integration:



$$I = \int_{x_0}^{x_6} f(x)\,dx = \int_{x_0}^{x_3} f(x)\,dx + \int_{x_3}^{x_6} f(x)\,dx$$

$$\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + y_3\right]$$

$$\int_{x_3}^{x_6} f(x)\,dx = \frac{3h}{8}\left[y_3 + 3y_4 + 3y_5 + y_6\right]$$

$$I = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5) + 2y_3 + y_6\right]$$

When we divide the ~~domain~~ domain into 'n' number of ~~IO~~ equal ~~st~~ strips then,

$$\boxed{I = \frac{3h}{8}\Big[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + y_9 + \cdots + y_{n-3}) + y_n\Big]}$$

Assignment: 1) 27/03/21

Prob: 1) Solve, $I = \int_0^1 \frac{dx}{x}$ by using both simpson's 1/3 rule and 3/8 rule by dividing the domain into 6 equal divisions.

Prob: 2) A solid revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x=0$ and $x=1$, and a curve through the points with the following coordinates

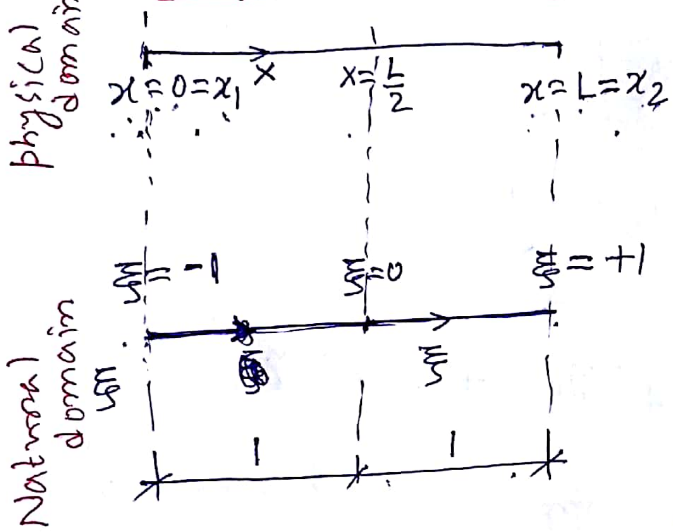
P.T.O.

x	y
0.00	1.00
0.25	0.9896
0.50	0.9589
0.75	0.9089
1.00	0.8415

Estimate the volume of the solid formed by using the simpson's 1/3 rule.

Gauss Quadrature method for numerical integration

Step 1: To change the domain and the integrant from physical to natural.



$$\boxed{x=\frac{x_2+x_1}{2}+\frac{x_2-x_1}{2}\xi}$$

$$x=\frac{x_2+x_1}{2} \quad [\text{when } \xi=0]$$

$$=\frac{L+0}{2}$$

$\therefore x=\frac{L}{2}$ when $\xi=0$

At, $\xi=-1$ $\quad x=\frac{x_2+x_1}{2}+\frac{x_2-x_1}{2}(-1)$

$$\Rightarrow x=x_1$$

At $\xi=+1$ $\quad x=\frac{x_2+x_1}{2}+\frac{x_2-x_1}{2}(+1)$

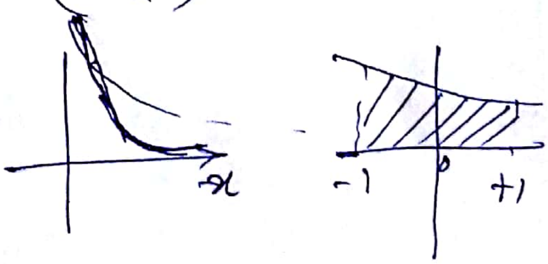
$$\Rightarrow x=x_2$$

$$\int_{x_1}^{x_2} f(x)\,dx \Longrightarrow \int_{-1}^{+1}\phi(\xi)\,d\xi$$

$$x=\frac{x_2+x_1}{2}+\frac{x_2-x_1}{2}\xi$$

$$\Rightarrow \frac{dx}{d\xi}=\frac{x_2-x_1}{2}$$

$$\Rightarrow dx=\left(\frac{x_2-x_2}{2}\right)d\xi$$



$$I=\int_2^5\frac{dx}{x}=\int_{-1}^{+1}\frac{1}{\left(\frac{5+2}{2}\right)+\left(\frac{5-2}{2}\right)\xi}\left(\frac{5-2}{2}\right)d\xi$$

$$=\int_{-1}^{+1}\frac{1.5\,d\xi}{3.5+1.5\xi}$$