DATA STRUCTURES AND NUMERICAL METHODS

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Note No. 01

APPROXIMATION AND ERRORS

SOURCES OF ERROR:

1. Error in data

   Example: Let radius of a circle= 4.126 unit and that $\pi=\frac{22}{7}$

   Now if we calculate area of the circle as, $3.14\times(4.13)^2$, then error occurs.

2. Inherent Error

   Error in the formula itself.

   Approximation in the formulae used in numerical analysis.

   Example: Approximation in the slope of a function- $\frac{df}{dx}\approx\frac{f(x+\Delta x)-f(x)}{\Delta x}$

3. Residual Error

   While finding the values of an infinite series, we often take up to some finite number of terms and leave the remaining terms.

   Example: $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+....$

   In the above series if we take only first four terms and neglect the rest a residual error is generated.

4. Rounding-off Error

   A decimal number having large number of terms or an infinite number of terms are rounded to some terms after the decimal.

   Example: 4.3256 is rounded as 4.33 causing an error 4.33-4.3256=0.0044.

APPROXIMATION AND ERRORS

5. Truncation Error

In case, the terms of a number are truncated after some terms without rounding-off, the error is called truncation error.

Example: If 4.3256 is truncated after two places of decimal then we get error 4.3256-4.32=0.0056.

This is called truncation error.

SIGNIFICANT DIGITS

A significant digit of an approximate number is

1) Any non-zero digit.
2) In decimal representation, zero can be a significant digit if
   a) it lies between two significant digits as in 0.4302.
   b) it is used as place holder or to indicate retained place as in 0.432200.
      Last 00 are significant.
   c) It has not served as to fix the position of the decimal point. In 0.0023, 00 are not significant digits as these are used to fix decimal.
3) In an integer, zero is significant if it lies between two significant digits. If it lies at the end of an integer then zero may or may not be a significant digit and may depend on expressing the number in exponent form.

Example: Let the number be 784000

Then in $7.84\times10^{5}-$ there are 3 significant digits.

In $7.8400\times10^{5}-$ there are 5 significant digits.

In $7.84000-$ there are 6 significant digits.

APPROXIMATION AND ERRORS

Number	Significant digits/figures
0.2500	4
0.0025	2
4.8032	5
4.8000	5
$4.8X10^{-3}$	2
4.2500	5
4.023	3
4.203	4

ROUNDING OF NUMBERS

A number x is often required to be presented by a number x_1 with smaller number of significant digits. The number x_1 is chosen to keep the rounding-off error $|x_1 - x|$ minimum.

Rules of rounding-off

To round-off a number up to n significant digits, drop all digits on the right of n significant digits with the following rules:

i. If the discarded digit is less than 5, leave the remaining digits unchanged.

ii. If the discarded digit is more than 5, then add '1' to the last retained digit.

iii. If the discarded digit is exactly 5, then add '1' to the last retained digit if 5 is in odd position after the decimal. Do not add '1' if 5 is in even position.

APPROXIMATION AND ERRORS

Example: Round-off the number $\pi = 3.1415926536$ to five, four and three significant digits.

Solution:

$$\pi = 3.1415926536$$

$$= 3.141592654$$

$$= 3.14159265$$

$$= 3.1415926$$

$$= 3.141593$$

$$= 3.14159$$

$$= 3.1416$$ (rounded to '5' significant digits)

$$= 3.142$$ (rounded to '4' significant digits)

$$= 3.14$$ (rounded to '3' significant digits)