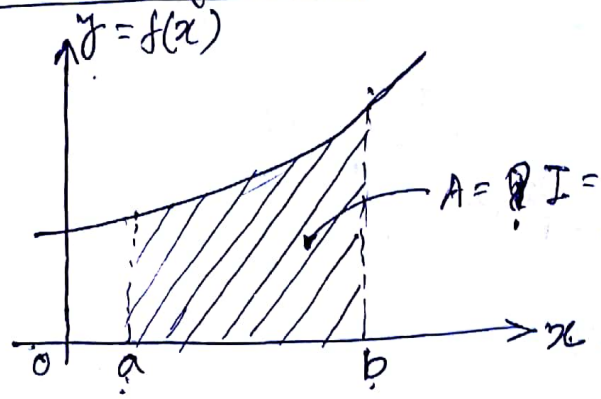
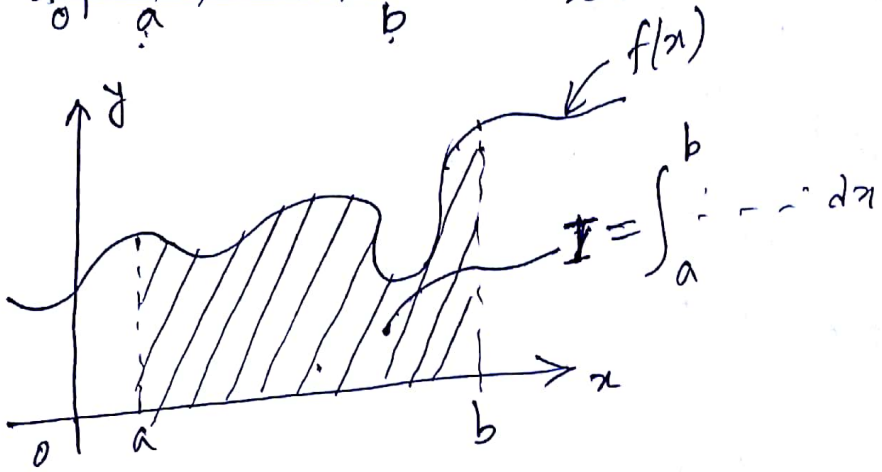
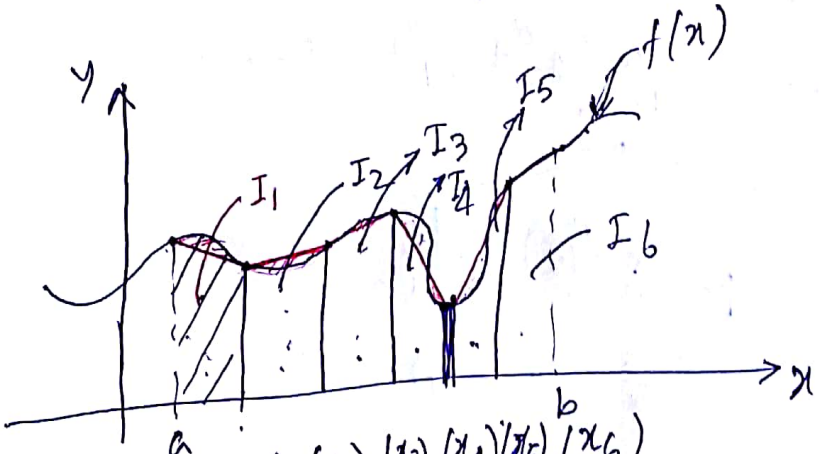
DSNM (17/03/2021)

Numerical Integration



$y = f(x) = a + bx^2$





$$I = I_1 + I_2 + I_3 + I_4 + I_5 + I_6$$

$$I_1 = \frac{x_1 - x_0}{2}\left[f(x_0) + f(x_1)\right] = \frac{h}{2}\left[y_0 + y_1\right] = \int_{x_0}^{x_1} y\,dx$$

$$I_2 = \frac{x_2 - x_1}{2}\left[f(x_1) + f(x_2)\right] = \frac{h}{2}\left[y_1 + y_2\right] = \int_{x_1}^{x_2} y\,dx$$

$$I_3 = \frac{x_3 - x_2}{2}\left[f(x_2) + f(x_3)\right] = \frac{h}{2}\left[y_2 + y_3\right] = \int_{x_2}^{x_3} y\,dx$$

$$I_4 = \frac{x_4 - x_3}{2}\left[f(x_3) + f(x_4)\right] = \frac{h}{2}\left[y_3 + y_4\right] = \int_{x_3}^{x_4} y\,dx$$

$$I_5 = \frac{x_5 - x_4}{2}\left[f(x_4) + f(x_5)\right] = \frac{h}{2}\left[y_4 + y_5\right] = \int_{x_4}^{x_5} y\,dx$$

$$I_6 = \frac{x_6 - x_5}{2}\left[f(x_5) + f(x_6)\right] = \frac{h}{2}\left[y_5 + y_6\right] = \int_{x_5}^{x_6} y\,dx$$

← Trapezoidal rule of Integration
Numerical

$$I = \sum_{i=1}^{n} I_i = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_6\right]$$

$$I = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

$$\int_{x_0}^{x_1} y\,dx = \int_{x_0}^{x_1} \left[y_0 + h y_0' + \frac{h^2}{2} y_0'' + \cdots\right] dx$$

$$= h y_0 + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \cdots \quad \text{——— (1)}$$

$$\frac{h}{2}[y_0 + y_1] = \frac{h}{2}\left[y_0 + y_0 + h y_0' + \frac{h^2}{2} y_0'' + \frac{h^3}{6} y_0''' + \cdots\right]$$

$$= h y_0 + \frac{h^2}{2} y_0' + \frac{h^3}{4} y_0'' + \frac{h^4}{12} y_0''' + \cdots \quad \text{——— (2)}$$

$$E_1 = (1) - (2) = \left(\frac{h^3}{6} - \frac{h^3}{4}\right) y_0'' + \cdots$$

$$\Rightarrow E_1 = -\frac{1}{12} h^3 y_0'' + \cdots$$

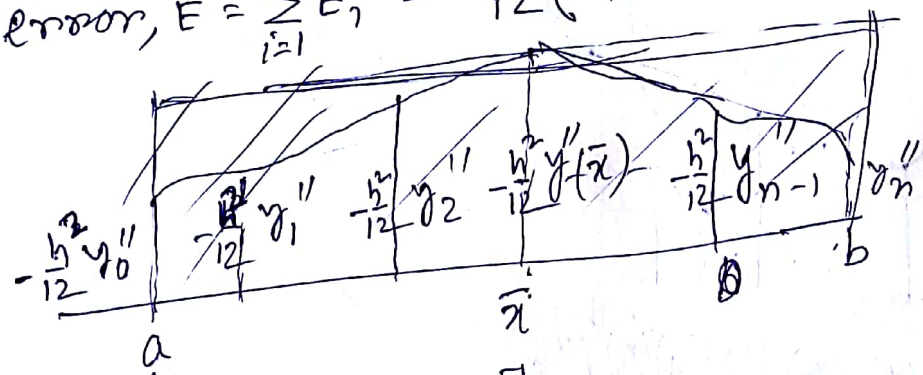
$$E_2 = -\frac{1}{12} h^3 y_1'' + \cdots$$

$$E_3 = -\frac{1}{12} h^3 y_2'' + \cdots$$

$$\vdots$$

$$E_n = -\frac{1}{12} h^3 y_{n-1}'' + \cdots$$

Total error, $E = \sum_{i=1}^{n} E_i = -\frac{h^3}{12}\left(y_0'' + y_1'' + \cdots + y_{n-1}''\right) + \cdots$



$$(a-b)\left[-\frac{h^2}{12} y''(\bar{x})\right]$$

$$E_T \leq -\frac{(a-b)}{12} h^2 y''(\bar{x})$$

$$|E_T| \leq \frac{a-b}{12} h^2 y''(\bar{x})$$