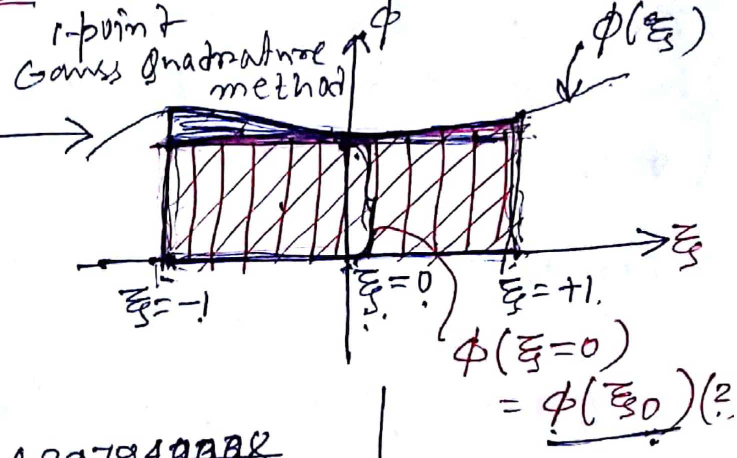
$$\int_{x_1}^{x_2} f(x)\,dx = \int_{-1}^{+1}\phi(\xi)\,d\xi \simeq \phi(\xi_0)(2)$$

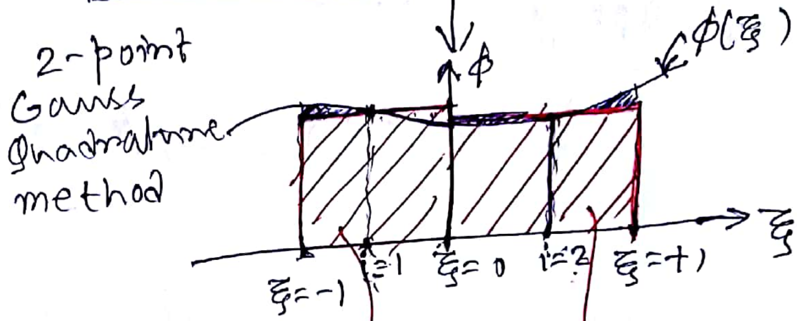


Example:

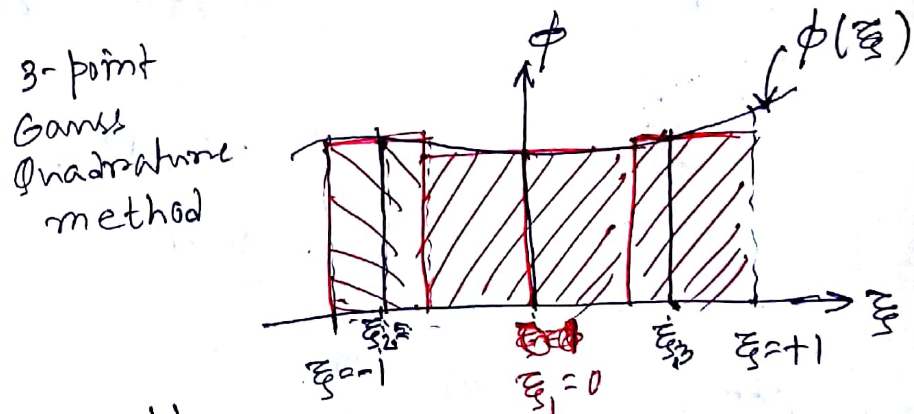
$$\int_2^5 \frac{dx}{x} = \log_e x\Big|_2^5 = \log_e\left(\frac{5}{2}\right) = \cancel{0.397940008} = \boxed{0.916290732}$$

$$\Downarrow$$

$$\int_{-1}^{+1}\frac{1.5}{3.5+1.5\xi}\,d\xi$$



$$\int_{x_1}^{x_2} f(x)\,dx = \int_{-1}^{+1}\phi(\xi)\,d\xi \simeq \phi(\xi_1)(1) + \phi(\xi_2)(1)$$



$$\int_{x_1}^{x_2} f(x)\,dx = \int_{-1}^{+1}\phi(\xi)\,d\xi = \phi(\xi_1)\omega_1 + \phi(\xi_2)W_2 + \phi(\xi_3)W_3$$

$$\sum W_i = 2$$

$$\int_{x_1}^{x_2} f(x)\,dx = \int_{-1}^{+1}\phi(\xi)\,d\xi \simeq \sum_{i=1}^{n} W_i\,\phi(\xi_i)$$

n-point Gauss Quadrature technique/method is sufficient to evaluate integration containing integrant of degree 2n-1 or less.

Example :

$$I = \int_2^5 \frac{dx}{x} = \int_{-1}^{+1} \frac{1.5\, d\xi}{3.5 + 1.5\xi}$$

Analytical value of $I = 0.916290732$

Apply 1-point Gauss rule

$$I = \sum_{i=1}^{1} w_i \phi(\xi_i) = w_1 \phi(\xi_1) = (2)\,\phi(\xi = 0) = (2)\left[\frac{1.5}{3.5 + 1.5(0)}\right]$$

$$= 0.857142$$

Apply 2-point Gauss rule

$$I = \sum_{i=1}^{2} w_i \phi(\xi_i) = w_1 \phi\left(\xi_1 = -\frac{1}{\sqrt{3}}\right) + w_2 \phi\left(\xi_2 = +\frac{1}{\sqrt{3}}\right)$$

$$= (1)\,\phi(\xi = -0.5773) + (1)\,\phi(\xi = +0.5773)$$

$$= (1)\frac{1.5}{3.5 + (1.5)(-0.5773\ldots)} + \frac{1.5}{3.5 + (1.5)(+0.5773\ldots)}$$

$$= 0.912967$$

Apply 3-point Gauss rule

$$I = \sum_{i=1}^{3} w_i \phi(\xi_i) = w_1 \phi(\xi_1) + w_2 \phi(\xi_2) + w_3 \phi(\xi_3)$$