Backward Differences (BD)

Backward difference operator: ∇

$$\left.\begin{aligned} \nabla y_1 &= y_1 - y_0 \\ \nabla y_2 &= y_2 - y_1 \\ \nabla y_3 &= y_3 - y_2 \\ &\vdots \\ \nabla y_n &= y_n - y_{n-1} \end{aligned}\right\}$$ First backward differences.

Higher order backward differences:

$$\nabla^2 y_i = \nabla y_i - \nabla y_{i-1}$$

$$\nabla^3 y_i = \nabla^2 y_i - \nabla^2 y_{i-1}$$

Backward Difference Table

value of 'x'	value of 'y'	1st Difference	2nd Diff.	3rd Diff.	4th. Diff.	5th. Diff
x_0	y_0					
		∇y_1				
x_0+h	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
x_0+2h	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$	$\nabla^5 y_5$
		∇y_3		$\nabla^3 y_4$		
x_0+3h	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$	
		∇y_4		$\nabla^3 y_5$		
x_0+4h	y_4		$\nabla^2 y_5$			
		∇y_5				
x_0+5h	y_5					

Central Differences (CD).

$$\left.\begin{aligned} \delta y_{1/2} &= y_1 - y_0 \\ \delta y_{3/2} &= y_2 - y_1 \\ &\vdots \\ \delta y_{n-1/2} &= y_n - y_{n-1} \end{aligned}\right\}$$ First order central differences

$\delta \leftarrow$ central difference operator

Higher order central differences

$$\delta^2 y_1 = \delta y_{3/2} - \delta y_{1/2}$$

$$\delta^2 y_2 = \delta y_{5/2} - \delta y_{3/2}$$

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Central Difference Table

value of 'x'	value of 'y'	1st Diff.	2nd Differences	3rd Diff.	4th Diff.	5th Diff.
x_0	y_0					
		$\delta y_{1/2}$				
x_0+h	y_1		$\delta^2 y_1$			
		$\delta y_{3/2}$		$\delta^3 y_{3/2}$		
x_0+2h	y_2		$\delta^2 y_2$		$\delta^4 y_2$	
		$\delta y_{5/2}$		$\delta^3 y_{5/2}$		$\delta^5 y_{5/2}$
x_0+3h	y_3		$\delta^2 y_3$		$\delta^3 y_3$	
		$\delta y_{7/2}$		$\delta^3 y_{7/2}$		
x_0+4h	y_4		$\delta^2 y_4$			
		$\delta y_{9/2}$				
x_0+5h	y_5					

Example Find out the missing values in the following data.

x	45	50	55	60	65
y	3.0	?	2.0	?	-2.4

Sol^n

The forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
45	$y_0 = 3.0$			
		$y_1 - 3.0$	$5 - 2y_1$	$3y_1 + y_3 - 9$
50	y_1			
		$2 - y_1$	$y_1 + y_3 - 4$	$-y_1 - 3y_3 + 3.6$
55	$y_2 = 2.0$			
		$y_3 - 2$	$-4.4 - 2y_3$	
60	y_3			
		$-2.4 - y_3$		
65	$y_4 = -2.4$			

$$y = a_0 + a_1 x + a_2 x^2$$
$$y' = a_1 + 2a_2 x$$
$$y'' = 2a_2$$
$$y''' = 0$$

As only three data points are given the function can be represented by a quadratic polynomial.

Hence, $\Delta^3 y_0 = 0$ $\qquad$ $\Delta^3 y_1 = 0$

$\Rightarrow 3y_1 + y_3 - 9 = 0$ — (1) $\qquad$ $\Rightarrow -y_1 - 3y_3 + 3.6 = 0$ — (2)

solving (1) & (2) we have,

$\boxed{y_1 = 2.92}$ $\qquad$ $\boxed{y_3 = 0.22}$