Nonlinear regression analysis

$(x_1, y_1), (x_2, y_2) \cdots (x_m, y_m)$

Let, $y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$ [polynomial of n^{th} degree]

$$S = e_1^2 + e_2^2 + \cdots + e_m^2$$

$$= [y_1 - (a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n)]^2 + [y_2 - (a_0 + a_1 x_2 + a_2 x_2^2 + \cdots + a_n x_2^n)]^2 + \cdots + [y_m - (a_0 + a_1 x_m + a_2 x_m^2 + \cdots + a_n x_m^n)]^2$$

$$\frac{\partial S}{\partial a_0} = -2[y_1 - (a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n)] - 2[y_2 - (a_0 + a_1 x_2 + a_2 x_2^2 + \cdots + a_n x_2^n)] \cdots - 2[y_m - (a_0 + a_1 x_m + a_2 x_m^2 + \cdots + a_n x_m^n)] = 0$$

$$\Rightarrow m a_0 + a_1 \sum_{i=1}^{m} x_i + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n = \sum_{i=1}^{m} y_i \quad \text{— (1)}$$

$$\frac{\partial S}{\partial a_1} = 0 = -2x_1[y_1 - (a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n)] - 2x_2[y_2 - (a_0 + a_1 x_2 + \cdots a_n x_2^n)] - 2x_m[y_m - (a_0 + a_1 x_m + a_2 x_m^2 + \cdots + a_n x_m^n)]$$

$$\Rightarrow a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} = \sum_{i=1}^{m} x_i y_i \quad \text{— (2)}$$

$$a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + \cdots + a_n \sum_{i=1}^{m} x_i^{2n} = \sum_{i=1}^{m} x_i^n y_i \quad \text{— (n+1)}$$

Power function fitting.

Let, $y = ax^c$ where, a & c are to be obtained.

$\Rightarrow \log y = \log a + c \log x$.

$\Rightarrow Y = a_0 + a_1 X$

where, $Y = \log y$, $a_0 = \log a$, $a_1 = c$, and $X = \log x$.

Problem ① Fit a polynomial of the second degree to the data points given in the following table:

x	y
0.0	1.0
1.0	6.0
2.0	17.0

Problem ② Determine the constant a and b by the method of least squares such that $y = ae^{bx}$ fit the following data

x	y
2	4.077
4	11.084
6	30.128
8	81.897
10	222.62

Prob: Fit a second order polynomial to the following data:

i	1	2	3	4	5	6
x	0	0.5	1.0	1.5	2.0	2.5
y	0	0.25	1.0	2.25	4.0	6.25

Ans: $a_0 = 0$, $a_1 = 0$, $a_2 = 1$

Hence, $y = x^2$

3) a) An experiment on the life of a cutting tools at different speeds has given the values: (5)

Speed v (ft/min.)	350	400	500	600
Life, t (min)	61	26	7	2.6

Find the best values of 'a' and 'b' in the law $V = ae^{bt}$ by the method of least squares.

Soln.

Given eqn: $V = ae^{bt}$

$\Rightarrow \ln V = \ln a + bt$

$\Rightarrow v = K + bt$

$ma_1 + a_2\Sigma x_i = \Sigma y_i$

$a_1\Sigma x_i + a_2\Sigma x_i^2 = \Sigma x_i y_i$

V	350	400	500	600	
$v = \ln V$	5.858	5.991	6.215	6.397	$\Sigma v_i = 24.461$
t	61	26	7	2.6	$\Sigma t_i = 96.6$
t^2	3721	676	49	6.76	$\Sigma t_i^2 = 4452.76$
vt	357.338	155.766	43.505	16.6322	$\Sigma v_i t_i = 573.2412$

Hence, the eqn: $4K + 96.6b = 24.461$ — (1)

$96.6K + 4452.76b = 573.2412$ —

$\Rightarrow K = 6.3145$ $\quad b = -8.25 \times 10^{-3}$

$\Rightarrow \ln a = 6.3145$

$\Rightarrow a = e^{6.3145} = 552.5257$

9