4) a) Solve the following equations using Gauss-Seidel method:

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

Soln: Here, $|a_{11}| > |a_{12}| + |a_{13}|$, $|a_{22}| > |a_{21}| + |a_{23}|$ & $|a_{33}| > |a_{31}| + |a_{32}|$.

$$\Rightarrow \quad x_{i+1} = \frac{1}{27}\left[85 - 6y_i + z_i\right]$$

$$y_{i+1} = \frac{1}{15}\left[72 - 6x_{i+1} - 2z_i\right]$$

$$z_{i+1} = \frac{1}{54}\left[110 - x_{i+1} - y_{i+1}\right]$$

x	1	2.963	2.232	2.408	2.424
y	1	3.481	3.651	3.580	3.574
z	1	1.918	1.928	1.926	1.926

$$\boxed{\begin{array}{l} 2.425 \\ 3.573 \\ 1.926 \end{array}} \begin{array}{l} \text{by} \\ \text{Gauss} \\ \text{- Elimination} \\ \text{(Exact)} \end{array}$$

（10）

4) b) Solve the following system of equations by Gauss elimination method:

$$5x_1 + x_2 + x_3 + x_4 = 4$$
$$x_1 + 7x_2 + x_3 + x_4 = 12$$
$$x_1 + x_2 + 6x_3 + x_4 = -5$$
$$x_1 + x_2 + x_3 + 4x_4 = -6$$

$$\text{sol} - \begin{Bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{Bmatrix}$$

（10）

5) a) Find a root of the following equation using Bisection method correct to four decimal places: $x^3 - 9x + 1 = 0$ upto 5th iteration which lies between 2 and 4.

Soln:

It no.	a	b	mid ab = (a+b)/2	f(a)	f(b)	f(mid a-b)
0	2	4	3	-9	29	1
1	2	3	2.5	-9	1	-5.875
2	2.5	3	2.75	-5.875	1	-2.95
3	2.75	3	2.875	-2.95	1	-1.11
4	2.875	3	2.9375	-1.11	1	-0.090
5	2.9375	3	2.9687	-0.09	1	0.4467
6	2.9375	2.9687	2.9531	-0.09	0.4467	0.175

（5）

5) b) Using Newton-Raphson method, find a real root of the following ⑧ equation correct to four decimal places: $3x = \cos x + 1$ up to fifth iterations. Take initial value $x_0 = \pi/4$

soln

$f(x) = 3x - \cos x - 1 = 0$

$f'(x) = 3 + \sin x$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{3x_n - \cos x_n - 1}{3 + \sin x_n}$

It. No.	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0	$\pi/4$	0.6491	3.7071	0.6103
1	0.6103	0.1278	3.5731	0.5745
2	0.5745	-0.11586	3.5434	0.6072
3	0.6072	3.43×10^{-4}	3.5706	0.6071
4	(0.6071)	5.926×10^{-6}		

⑤

5) c) Find a root of the equation $x - e^{-x} = 0$ correct to four decimal places by secant method. Given the roots lies between 0 and 1. Solve up to seventh iterations.

soln

$f(x) = x - e^{-x} = 0$

$\therefore f'(x) = 1 + xe^{-x}$

Now, $x_{i+1} = x_i - \dfrac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$

i	x_i	x_{i-1}	$f(x_i)$	$f(x_{i-1})$	x_{i+1}
0	1	0	0.632	-1	$1 - \dfrac{0.632(1-0)}{0.632+1}$ $= 0.613$
1	1	0.613	0.632	0.0713	$1 - \dfrac{0.632(1 - 0.613)}{0.632 - 0.0713}$ $= 0.564$
2	0.613	0.564	0.0713	-4.929	$0.613 - \dfrac{0.0713(0.613 - 0.564)}{(0.0713 + 4.929)}$ $= 0.6123$
3	0.6123	0.564	0.0702	-4.929	$0.6123 - \dfrac{0.0702(0.6123 - 0.564)}{(0.0702 + 4.929)}$ $= 0.6116$
4	0.6123 0.6116	0.6116 0.564	0.0702 0.0692	-4.929	$0.6116 - \dfrac{0.0692(0.6116 - 0.564)}{(0.0692 + 4.929)}$ $= 0.6109$
5	(0.6109)	0.564	0.068		$= 0.6109$

⑦