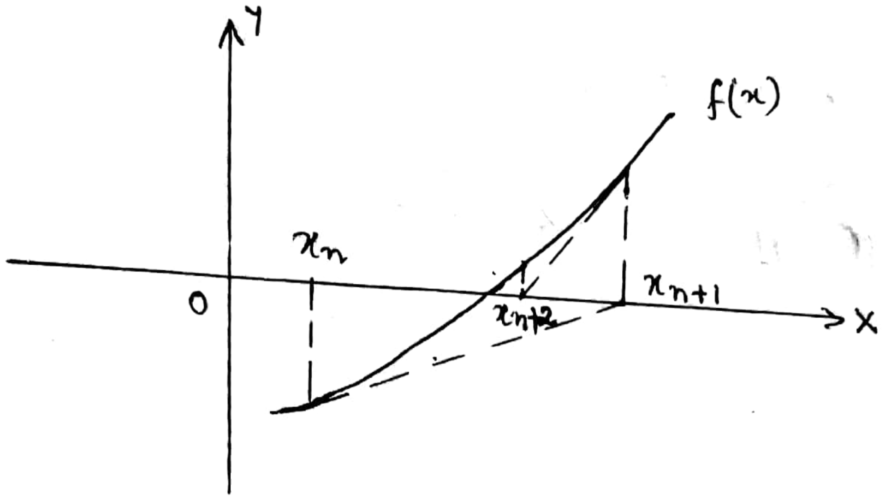
EXTRAPOLATION



Given, $f(x)=0$

Choose x_n as the 1st approximation and suppose x_n+h is its closer approximation.

$\therefore \; x_n + h = x_{n+1}$

Since, $f(x)=0$ is approximately satisfied by x_n+h

$f(x_{n+1}) = f(x_n+h) = 0$ —— (1)

From Taylor's series

$f(x_{n+1}) = f(x_n+h) = f(x_n) + h f'(x_n) + \frac{h^2}{2} f''(x_n) + \cdots$

Since, h is small, neglecting its higher powers we get,

$f(x_n+h) = f(x_n) + h f'(x_n) = f(x_n) + (x_{n+1} - x_n) f'(x_n)$

From eq^n (1) we get,

$f(x_n) + (x_{n+1} - x_n) f'(x_n) = 0.$

or $(x_{n+1} - x_n) f'(x_n) = - f(x_n)$

or $\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$

Prob.-10 Find the root of the equation $f(x) = x^3 - x - 1 = 0$ in the vicinity of $x = 2$ by Newton-Raphson method.

Soln.-

$$f(x) = x^3 - x - 1$$

$$\therefore f'(x) = 3x^2 - 1$$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$$

The iterations are presented in tabular form.

Iteration No.	x_n	x_{n+1}
1	2	1.54545
2	1.54545	1.359609
3	1.359609	1.325801
4	1.325801	1.324719
5	1.324719	1.3247179

The estimated root of the equation $x = 1.3247179$

Prob.-11 Find the root of the equation $f(x) = x^3 - x - 1 = 0$ in the vicinity of $x = 2$ using modified Newton-Raphson method.

Soln:-

$$f(x) = x^3 - x - 1$$

$$\therefore f'(x) = 3x^2 - 1$$

$$x_0 = 2 \quad \therefore f'(x_0) = 3 \times 2^2 - 1 = 11$$

$$\therefore x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{11}$$

Iterations are presented in tabular form:

Iteration No.	x_n	x_{n+1}
1	2	1.54545
2	1.54545	1.441294
3	1.441294	1.391044
4	1.391044	1.363714
5	1.363714	1.34804

The estimated root of the given equation: $x = 1.34804$