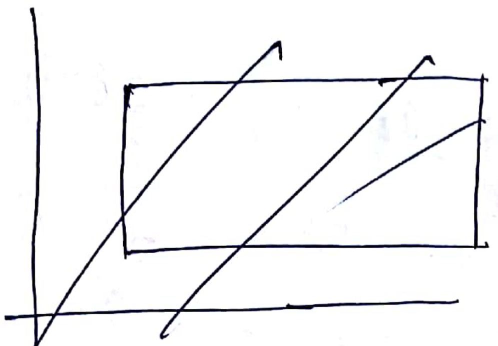
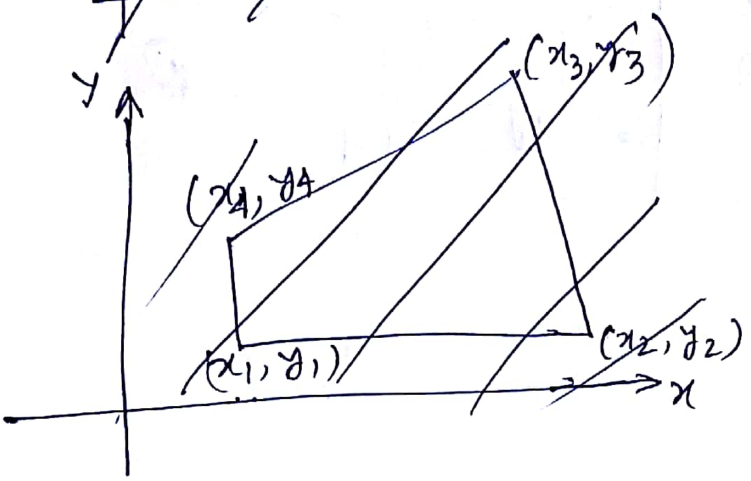
DSNM (31/03/2021)

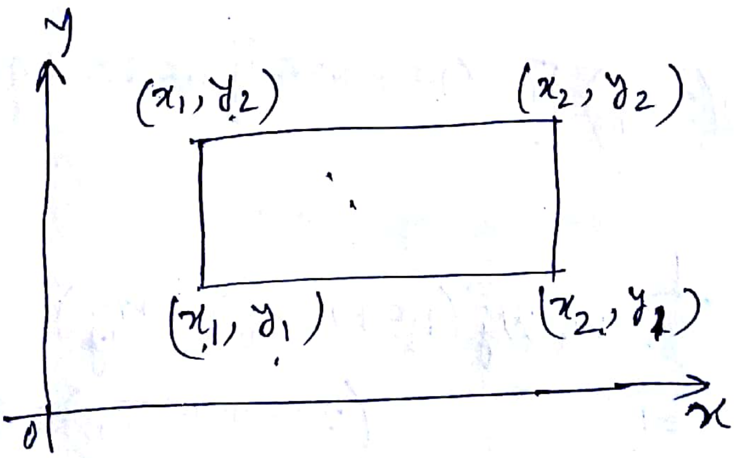
$$I = \int_{y=y_1}^{y=y_2} \int_{x=x_1}^{x=x_2} f(x,y)\, dx\, dy = \int_{-1}^{+1} \int_{-1}^{+1} \phi(\xi,\eta)\, d\xi\, d\eta$$

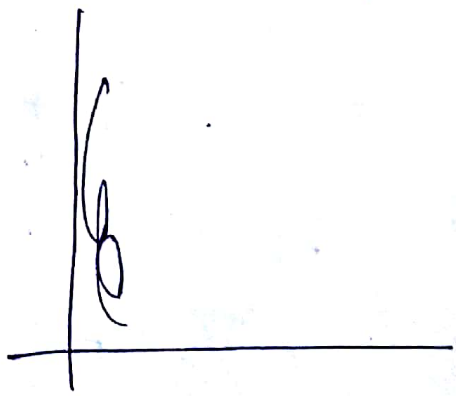
$$= \sum_{i=1}^{m} \sum_{j=1}^{n} w_i\, w_j\, \phi(\xi_i, \eta_j)$$

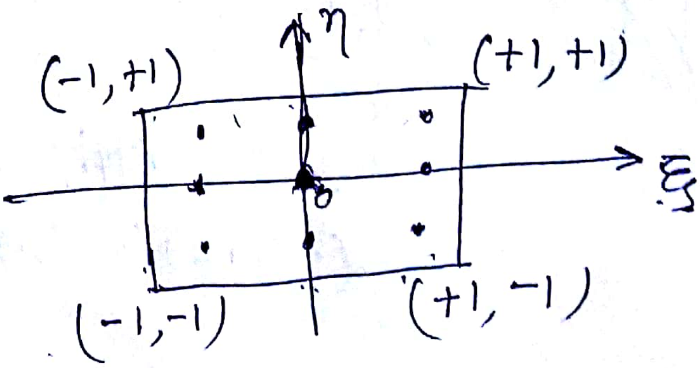
Let us assume $m = n = 3$











Example. Evaluate the following integral I by using

i) One point Gauss quadrature (GQ) technique.

ii) Two point GQ technique

iii) Three " GQ technique

$$I = \int_{y=1}^{y=3}\int_{x=1}^{x=2} xy(1+x)\,dx\,dy. \Rightarrow \text{Analytical value} : 15.33$$

Sol^n

$$x = \frac{x_2+x_1}{2} + \frac{x_2-x_1}{2}\xi$$

$$= \frac{2+1}{2} + \frac{2-1}{2}\xi$$

$$\boxed{x = 1.5 + 0.5\xi} \text{ --- (1)}$$

$$\boxed{dx = 0.5\,d\xi} \text{ --- (2)}$$

$$y = \frac{y_2+y_1}{2} + \frac{y_2-y_1}{2}\eta$$

$$= \frac{3+1}{2} + \frac{3-1}{2}\eta$$

$$\Rightarrow \boxed{y = 2+\eta} \text{ --- (3)}$$

$$\Rightarrow \boxed{dy = d\eta} \text{ --- (4)}$$

$$I = \int_{y=1}^{y=3}\int_{x=1}^{x=3} xy(1+x)\,dx\,dy$$

$$= \int_{-1}^{+1}\int_{-1}^{+1} (1.5+0.5\xi)(2+\eta)\left[1+(1.5+0.5\xi)\right]0.5\,d\xi\,d\eta$$

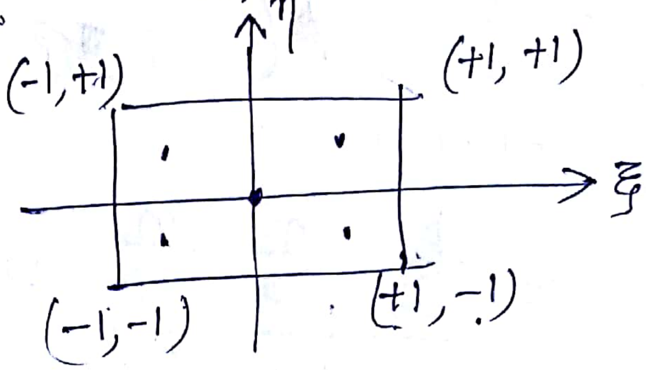
$$= \boxed{\sum_{j=1}^{1}\sum_{i=1}^{1} w_i w_j (1.5+0.5\xi_i)(2+\eta_j)(2.5+0.5\xi_i)\,0.5}$$

Applying 1-poin Gauss rule

$$= (2)(2)(1.5)(2)(2.5)(0.5) = \boxed{15}$$

$$I = \int_{-1}^{+1}\int_{-1}^{+1} (1.5+0.5\xi)(2+\eta)(2.5+0.5\xi)\,0.5\,d\xi\,d\eta$$

$$= \sum_{j=1}^{2}\sum_{i=1}^{2} (1.5+0.5\xi_i)(2+\eta_j)(2.5+0.5\xi_i)\,0.5\,W_i W_j$$



No. of terms
$= (n^m)$
$n \leftarrow$ no. of Gauss points
$m \leftarrow$ ~~no~~ dimension of the domain.

$$= W_1W_1(1.5+0.5\xi_1)(2+\eta_1)(2.5+0.5\xi_1)\,0.5$$
$$+ W_1W_2(1.5+0.5\xi_1)(2+\eta_2)(2.5+0.5\xi_1)\,0.5$$
$$+ W_2W_1(1.5+0.5\xi_2)(2+\eta_1)(2.5+0.5\xi_2)\,0.5$$
$$+ W_2W_2(1.5+0.5\xi_2)(2+\eta_2)(2.5+0.5\xi_2)\,0.5$$

For 2-point Gauss rule $W_1 = W_2 = 1$
$\xi_1 = \eta_1 = -\frac{1}{\sqrt{3}} = -0.57735$; $\xi_2 = \eta_2 = +\frac{1}{\sqrt{3}} = +0.57735.$

$\boxed{I = 15.33}$

Applying 3-point Gauss rule

$$I = \sum_{j=1}^{3}\sum_{i=1}^{3} (1.5+0.5\xi_i)(2+\eta_j)(2.5+0.5\xi_i)\,0.5\,W_iW_j$$

$$= W_1W_1(1.5+0.5\xi_i)(2+\eta_j)(2.5+0.5\xi_i)\,0.5$$
$$+ W_1W_2(1.5+0.5\xi_i)(2+\eta_j)(2.5+0.5\xi_i)\,0.5$$
$$+ W_1W_3(1.5+0.5\xi_i)(2+\eta_j)(2.5+0.5\xi_i)\,0.5$$
$+ W_2W_1$ - - - - - - - -
$+ W_2W_2$ - - - - - - - -
$+ W_2W_3$ - - - - - - - -
$+ W_3W_1$ - - -
$+ W_3W_2$ - - - -
$+ W_3W_3$ - - - -

$\xi_1 = \eta_1 = 0.0$; $\xi_2 = \eta_2 = -0.774596\ldots$; $\xi_3 = \eta_3 = +0.774596\ldots$

$W_1 = 0.88888\ldots$
$W_2 = W_3 = 0.5555\ldots$

$$I = \int_{z=z_1}^{z=z_2} \int_{y=y_1}^{y=y_2} \int_{x=x_1}^{x=x_2} f(x,y,z)\, dx\, dy\, dz$$

$$= \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} \phi(\xi, \eta, \zeta)\, d\xi\, d\eta\, d\zeta$$

$$\simeq \sum_{k=1}^{p} \sum_{j=1}^{n} \sum_{i=1}^{m} w_i w_j w_k\, \phi(\xi_i, \eta_j, \zeta_k)$$

Total number of terms $= p * n * m$

Problem: Evaluate the following integration by using

i) One point GQ technique and,

ii) Two point GQ technique

$$I = \int_{y=-1}^{y=0} \int_{x=0}^{x=2} x^3 y^4\, dx\, dy$$

Also compare the ~~rsl~~ result with the analytical value.