ITERATION METHOD

In contrast to the bracketing methods where two values (a & b) are needed in iteration method we require one or more starting value.

Newton raphson method

$$f(x+h) = f(x) + f'(x)\,h + \frac{h^2}{2!} f''(x) + \cdots$$

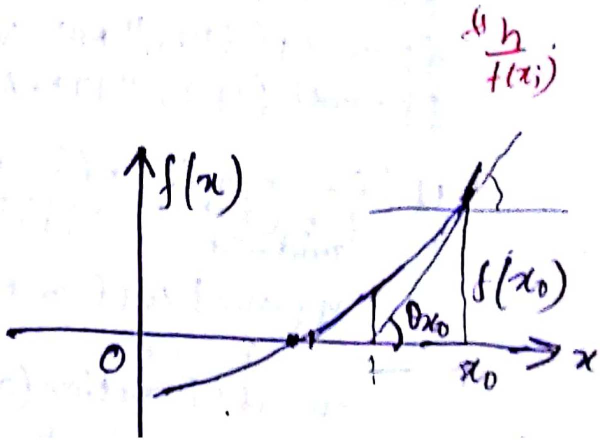
$$\Rightarrow 0 = f(x) + hf'(x) \quad \text{[As we seek } f(x+h) = 0 \text{ i.e. the root]}$$

$$\Rightarrow h = -\frac{f(x)}{f'(x)}$$

$$\therefore f(x_{i+1}) = f(x_i) + hf'(x_i)$$

$$\Rightarrow h = -\frac{f(x_i)}{f'(x_i)}$$

$$\therefore \boxed{x_{i+1} = x_i + h = x_i - \frac{f(x_i)}{f'(x_i)}}$$



Example - 1

$f(x) = x^3 - 2x - 5 = 0$

$x_0 = 2$.

Ex-3

$f(x) = e^{-x} - x$

$x_0 = 0$

Ans $= 0.56714390$

Modified Newton-Raphson

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}$$

Example-2

$f(x) = x\sin x + \cos x = 0$.

$x_0 = \pi = 3.1416$.

Secant Method

Evaluation of the derivatives of the function is not always possible in Newton-Raphson method. In secant method the derivative at x_i is approximated as

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

Hence, $$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

This formula requires two initial approximations to the root.

~~Example.~~

Example $f(x) = x^3 - 2x - 5 = 0$

$x_{-1} = 2$ and $x_0 = 3$.

Rate of convergence of Newton-Raphson method

Let, the root $= r$

Hence, $f(r) = 0$

$\therefore f(r) = f(x_n) + (r - x_n) f'(x_n) + \frac{1}{2}(r - x_n)^2 f''(x_n) = 0$

$\Rightarrow -\frac{f(x_n)}{f'(x_n)} = (r - x_n) + \frac{1}{2}(r - x_n)^2 \frac{f''(x_n)}{f'(x_n)}$

$\Rightarrow x_{n+1} - x_n = (r - x_n) + \frac{1}{2}(r - x_n)^2 \frac{f''(x_n)}{f'(x_n)}$ $\left[x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\right]$

$\Rightarrow x_{n+1} - r = \frac{1}{2}(x_n - r)^2 \frac{f''(x_n)}{f'(x_n)}$

$\Rightarrow \varepsilon_{n+1} = \frac{1}{2}\varepsilon_n^2 \frac{f''(x_n)}{f'(x_n)}$ ← Hence, quadratic convergence