EE 243 HW #2

QUESTION 1

(a) False. When an action potential is fired, there is first a large current attributable to Na^+ channels opening up before K^+ channels.

(b) False. During an action potential, Na^+ currents serve to depolarize the cell and K^+ currents serve to hyperpolarize the cell.

(c) True

(d) False. It is possible to record action potentials with a voltage clamp.

(e) False. The poisson process is memoryless

(f) False. If the Fano factor of a neuron is greater than 1, the firing rate variance is greater than the firing rate mean

(g) True

(h) False. An exponential interspike interval distribution does not model the refactory period well.

(i) False. Measurement of action potentials at microsecond resolution

(j) True

(k) True.

(l) True

(m) False. Convolving spike trains w/ a Gaussian kernel to approximate a spike rate $r(t)$ is a type of low-pass filtering

(n) True

(o) False. During the relative refractory period, it is possible for a spike to be generated

QUESTION 2 $\quad f(\theta) = c_0 + c_1 \cos(\theta - \theta_0)$

a. Show θ_0 is preferred direction

Solution: Cos of $\emptyset$ is 1 and cosine values can only be between $\emptyset$ and 1. Assuming that c_0 and c_1 are positive, the max value for $f(\theta) = c_0 + c_1$. To get $c_1(1)$, $\cos(\theta - \theta_0)$ should equal 1 meaning that $\theta - \theta_0 = 0$ leading to $\theta = \theta_0$. This is why θ_0 is the preferred direction.

b. $f(\theta) = -11 + 8\cos(\theta - 125)$

I would tell him "You've made a mistake". This is because $f(\theta)$ returns a negative value at all values of θ but we want distributions with values greater than $\emptyset$.

c. $\cos(\theta - \theta_0)$; $\quad e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$\rightarrow \cos(\theta - \theta_0) + j\sin(\theta - \theta_0) = e^{j(\theta - \theta_0)}$

$\rightarrow e^{j\theta - j\theta_0} = e^{j\theta} e^{-j\theta_0} = [\cos(\theta) + j\sin(\theta)][\cos(\theta_0) - j\sin(\theta_0)]$

$= \cos\theta\cos\theta_0 - j\cos\theta\sin\theta_0 + j\sin\theta\cos\theta_0 + \sin\theta\sin\theta_0$

$\cos(\theta - \theta_0) + j\sin(\theta - \theta_0) = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0 + j(\sin\theta\cos\theta_0 - \cos\theta\sin\theta_0)$

so, $\cos(\theta - \theta_0) = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0$

d. $f(\theta) = c_0 + c_1\cos(\theta - \theta_0)$

$f(\theta) = k_0 + k_1\sin(\theta) + k_2\cos(\theta)$

$f(\theta) = c_0 + c_1\cos(\theta)\cos(\theta_0) + c_1\sin(\theta)\sin(\theta_0)$

$k_0 = c_0$

$k_1 = c_1\sin(\theta_0) \qquad k_2 = c_1\cos(\theta_0)$

e. $y_0 = k_0 + k_1\sin(0) + k_2\cos(0) = k_0 + k_2$

$y_{120} = k_0 + k_1\sin(120) + k_2\cos(120) = k_0 + \frac{\sqrt{3}}{2}k_1 - \frac{1}{2}k_2$

$y_{240} = k_0 + k_1\sin(240) + k_2\cos(240) = k_0 - \frac{\sqrt{3}}{2}k_1 - \frac{1}{2}k_2$

$y_{120} + y_{240} = 2k_0 - k_2$

$y_{120} + y_{240} + y_0 = 3k_0$

$\boxed{\frac{y_{120} + y_{240} + y_0}{3} = k_0}$

$\boxed{k_2 = y_0 - \frac{y_{120} + y_{240} + y_0}{3}}$

$k_1 = \frac{2\left\{y_{240} - \frac{y_{120} - y_{240} + y_0}{3} + \frac{1}{2}\left(y_0 - \frac{y_{120} - y_{240} + y_0}{3}\right)\right\}}{\sqrt{3}}$

$$Y_{240} + \frac{\sqrt{3}}{2}k_1 = k_0 - \frac{1}{2}k_2$$

$$Y_{240} + \frac{\sqrt{3}}{2}k_1 = \frac{Y_{120}+Y_{240}+Y_0}{3} - \frac{1}{2}\left[Y_0 - \frac{Y_{120}+Y_{240}+Y_0}{3}\right]$$

$$Y_{240} + \frac{\sqrt{3}}{2}k_1 = \frac{3}{2}\,\frac{Y_{120}+Y_{240}+Y_0}{3} - \frac{1}{2}Y_0$$

$$= \frac{1}{2}(Y_{120}+Y_{240}+Y_0) - \frac{1}{2}Y_0$$

$$= \frac{1}{2}Y_{120} + \frac{1}{2}Y_{240} \; ; \; \frac{\sqrt{3}}{2}k_1 = \frac{1}{2}Y_{120} - \frac{1}{2}Y_{240}$$

$$\sqrt{3}\,k_1 = Y_{120} - Y_{240}$$

$$k_1 = \boxed{\frac{Y_{120} - Y_{240}}{\sqrt{3}}}$$

fig: Jupyter

QUESTION 3

a. The exponential distribution <u>does not</u> incorporate the refractory period. This is because real data has few or no short ISI's due to the refactory period and the firing rates vary over time.

b. $\int_0^{0.001} \lambda e^{-\lambda t} = \int_0^{0.001} 50 e^{-50t}\,dt$

$= +\frac{50}{50} e^{-50t}\Big|_0^{0.001} = e^{-50t}\Big|_0^{0.001} = 0.0487$

$= \boxed{4.87\%}$

$\lambda = \frac{60 \text{ spikes}}{\text{second}}$; $ISI = \frac{1}{60}$

QUESTION 4 Poisson process with rate λ

a. Mean of ISI $= \frac{1}{\lambda}$

b. $P(T > \frac{1}{\lambda}) = e^{-\lambda t}$ where $t = \frac{1}{\lambda}$

$P(T=t) = \lambda e^{-\lambda t}$

$= e^{-\lambda \frac{1}{\lambda}}$

$= e^{-1}$

$= \frac{1}{e}$

$T=t) = \lambda e^{-\lambda t}$

c. $E[T \mid T > \frac{1}{\lambda}] = \int_0^\infty t\, P(T=t \mid T > \frac{1}{\lambda})\, dt$

step function w/ val 1 @ $T > \frac{1}{\lambda}$

$$P(T=t \mid T > \tfrac{1}{\lambda}) = \frac{P(T=t)\, P(T > \frac{1}{\lambda} \mid T=t)}{P(T > \frac{1}{\lambda})} = \frac{P(T=t)}{P(T > \frac{1}{\lambda})} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda \frac{1}{\lambda}}} = \frac{\lambda e^{-\lambda t}}{e^{-1}}$$

$$P(T=t \mid T > \tfrac{1}{\lambda}) = t e^{1-\lambda t}$$

$$\int_{\frac{1}{\lambda}}^\infty t \lambda e^{1-\lambda t}\, dt = \lambda e \int_{\frac{1}{\lambda}}^\infty t e^{-\lambda t}\, dt = \lambda e \cdot \frac{2}{e\lambda^2} = \boxed{\frac{2}{\lambda}}$$

d. $E[T \mid T < \frac{1}{\lambda}] = \int_0^\infty t\, P(T=t \mid T < \frac{1}{\lambda})$

step @ 1 from 0 to $\frac{1}{\lambda}$

$$P(T=t \mid T < \tfrac{1}{\lambda}) = \frac{P(T=t)\, P(T < \frac{1}{\lambda} \mid T=t)}{P(T < \frac{1}{\lambda})} = \frac{\lambda e^{-\lambda t}}{1 - \frac{1}{e}}$$

$$\frac{\lambda}{1-e^{-1}} \int_0^{\frac{1}{\lambda}} t e^{-\lambda t}\, dt = \frac{e-2}{e\lambda^2} \cdot \frac{\lambda}{1-\frac{1}{e}} = \frac{e-2}{e\lambda(1-e^{-1})} = \frac{e-2}{e\lambda - \lambda} = \frac{e-2}{(e-1)} \cdot \frac{1}{\lambda}$$

e. $P(T > \frac{1}{\lambda}) = \frac{1}{e}$ $\quad P(T < \frac{1}{\lambda}) = 1 - \frac{1}{e}$

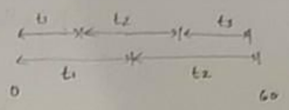
Expected # of spikes $= \sum_{n=1}^\infty n \left(\frac{1}{e}\right)\left(1-\frac{1}{e}\right)^{n-1}$

This is a geometric series that converges to e.

$\therefore$ Expected # of spikes $= e$

f. $e-1$ spikes with $T < \frac{1}{\lambda}$ and 1 spike at $T > \frac{1}{\lambda}$

$$\text{Waiting time} = (e-1) E[T \mid T < \tfrac{1}{\lambda}] + E[T \mid T > \tfrac{1}{\lambda}]$$
$$= (e-1)\left(\frac{1}{\lambda}\right)\left(\frac{e-2}{e-1}\right) + \frac{2}{\lambda}$$
$$= \frac{e-2}{\lambda} + \frac{2}{\lambda} = \boxed{\frac{e}{\lambda}}$$



⑤ Electrode 1 - ISI of 20 ms

Electrod 2 - ISI of 30 ms

a. No neurons are detected in first 60 ms?

$N(s) = 1$ @ time > 60 ms for both electrodes

$\underline{0}$ spikes in 60 ms $= e^{-\lambda(60)}$

$$= e^{-(\frac{1}{20})(60)} = e^{-3} \quad \text{for } 20\text{ ms}$$

$$= e^{-(\frac{1}{30})(60)} = e^{-2} \quad \text{for } 30\text{ ms}$$

$P(0 \text{ spikes in } 60 \text{ from both}) = e^{-5} = \underline{\underline{0.67\%}}$

b. $Pr(T \geq t+s \mid T \geq s) = Pr(T \geq t) = e^{-\lambda t}$ ← this we know.

Now, accounting for λ_1 & λ_2: $\boxed{e^{-(\lambda_1+\lambda_2)t}}$

c. Probability of first spike at time t being from e1

1 spike in $\underline{t}$ ms: $e^{-(\frac{1}{20})t \cdot (\frac{1}{20})t}$

$e^{-(\frac{1}{30})t\,(\frac{1}{30})t}$

$N(t) = N_1(t) + N_2(t)$

$\int_0^\infty P(N_1(t) = 1, N_2(t) = 0)$ $\quad P(N_1(t) = 1, N_2(t) = 0)$

$Pr(N_1(t) = 1, N_2(t) = 0 \mid N(t) = 1)$

Poisson

$$= \int_0^\infty \lambda_1 e^{-(\lambda_1+\lambda_2)t}\, dt$$

$$= \int_0^\infty (0.05)\, e^{-(0.0833)t}\, dt$$

$$= 0.60 = \boxed{\frac{3}{5}}$$