# Project 4

MGMT 237G
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code clarity and accuracy will determine the grades.

**<u>Due date: by Noon on next Wednesday</u>**

**1.** Compare the convergence rates of the four methods below by doing the following:
Use the Binomial Method to price a 6-month European Call option with the following information: the risk-free interest rate is 5% per annum and the volatility is 24%/annum, the current stock price is \$32 and the strike price is \$30. Divide the time interval into $n$ parts to estimate the price of this option. Use $n = 10,20,40,80,100,200,$ and $500$ to estimate the price and draw them all in one graph, where the horizontal axis measures $n$, and the vertical one the price of the option.

(a) Use the binomial method in which

$$u = \frac{1}{d}, \qquad d = c - \sqrt{c^2 - 1}, \qquad c = \frac{1}{2}\left(e^{-r\Delta} + e^{(r+\sigma^2)\Delta}\right), \qquad p = \frac{e^{r\Delta} - d}{u - d}$$

(b) Use the binomial method in which

$$u = e^{r\Delta}\left(1 + \sqrt{e^{\sigma^2\Delta} - 1}\right), \qquad d = e^{r\Delta}\left(1 - \sqrt{e^{\sigma^2\Delta} - 1}\right), \qquad p = 1/2$$

(c) Use the binomial method in which

$$u = e^{\left(r - \frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}}, \qquad d = e^{\left(r - \frac{\sigma^2}{2}\right)\Delta - \sigma\sqrt{\Delta}}, \qquad p = 1/2$$

(d) Use the binomial method in which

$$u = e^{\sigma\sqrt{\Delta}}, \qquad d = e^{-\sigma\sqrt{\Delta}}, \qquad p = \frac{1}{2} + \frac{1}{2}\left(\frac{\left(r - \frac{\sigma^2}{2}\right)\sqrt{\Delta}}{\sigma}\right)$$

**Outputs**:

i. Graphs: 4 plots in one .png file

**2.** Take the current price of GOOG. Use risk-free rate of 2% per annum, and strike price that is the closest integer (divisible by 10) to 110% of the current price. Estimate the price of the call option that expires in January of next year, using the Binomial Method. GOOG does not pay dividends. To estimate the historical volatility, use 60 months of historical stock price data on the company. You may use *Bloomberg* or *finance.yahoo.com* to obtain historical prices and the current stock price of GOOG.

(a) Compare your estimated option price with the one you can get from *Bloomberg* or *finance.yahoo.com* and comment.

(b) If the two are different in part (a), find the volatility that would make your estimated price equal to the market price and comment.

**Outputs**:

i. Writeup: comments in a .pdf file for parts (a) and (b)

**3.** Consider the following information on the stock of a company and options on it:

$S_0 = \$49, K = \$50, r = 0.03, \sigma = 0.2, T = 0.3846$(20 weeks)$, \mu = 0.14.$

Using the Binomial Method (any one of them) estimate the following and draw the graphs:

(i) Delta of the call option as a function of $S_0$, for $S_0$ranging from \$20 to \$80, in increments of \$2.

(ii) Delta of the call option, as a function of T (time to expiration), from 0 to 0.3846 in increments of 0.01.

(iii) Theta of the call option, as a function of $S_0$, for $S_0$ranging from \$20 to \$80 in increments of \$2.

(iv) Gamma of the call option, as a function of $S_0$, for $S_0$ranging from \$20 to \$80 in increments of \$2.

(v) Vega of the call option, as a function of $S_0$, for $S_0$ranging from \$20 to \$80 in increments of \$2.

(vi) Rho of the call option, as a function of $S_0$, for $S_0$ranging from \$20 to \$80 in increments of \$2.

**Outputs**:

ii. Graphs: 6 plots all in one .png file

**4.** Consider 12-month put options on a stock of company XYZ. Assume the risk-free rate is 5%/annum and the volatility of the stock price is 30 % /annum and the strike price of the option is \$100. Use a Binomial Method to estimate the prices of European and American Put options with current stock prices varying from \$80 to \$120 in increments of \$4. Draw them all in one graph, compare and comment.

**Outputs**:

i. Graphs: plot in a .png file
ii. Writeup: comments in a .pdf file

**5.** Compare the convergence rates of the two methods below by doing the following:
Use the Trinomial Method to price a 6-month European Call option with the following information: the risk-free interest rate is 5% per annum and the volatility is 24%/annum, the current stock price is \$32 and the strike price is \$30. Divide the time interval into $n$ parts to estimate the price of this option. Use n = 10, 15, 20, 40, 70, 80, 100, 200 and 500 to compute the approximate price and draw them in one graph, where the horizontal axis measures $n$, and the vertical one measures the price of the option. The methods are:

(a) Use the trinomial method applied to the stock price-process ($S_t$) in which

$$u = \frac{1}{d}, \qquad d = e^{-\sigma\sqrt{3\Delta}},$$

$$p_d = \frac{r\Delta(1-u)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(1-d)}, \quad p_u = \frac{r\Delta(1-d)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(u-1)}, \quad p_m = 1 - p_u - p_d$$

(b) Use the trinomial method applied to the Log-stock price-process ($X_t$) in which

$$\Delta X_u = \sigma\sqrt{3\Delta}, \quad \Delta X_d = -\sigma\sqrt{3\Delta}$$

$$p_d = \frac{1}{2}\left(\frac{\sigma^2\Delta + \left(r - \frac{\sigma^2}{2}\right)^2 \Delta^2}{\Delta X_u{}^2} - \frac{\left(r - \frac{\sigma^2}{2}\right)\Delta}{\Delta X_u}\right),\; p_u = \frac{1}{2}\left(\frac{\sigma^2\Delta + \left(r - \frac{\sigma^2}{2}\right)^2 \Delta^2}{\Delta X_u{}^2} + \frac{\left(r - \frac{\sigma^2}{2}\right)\Delta}{\Delta X_u}\right),\; p_m = 1 - p_u - p_d$$

**Outputs**: Graphs: plot in a .png file

**4.** Use Halton's Low-Discrepancy Sequences to price European call options. The code should be generic: it will ask for the user inputs for $S_0$, $K, T, r$, $\sigma$, $N$ (number of points) and $b_1$ (base 1) and $b_2$ (base 2). Use the Box-Muller method to generate Normals such as:

$$\begin{cases} Z_1 = \sqrt{-2ln(H_1)}cos(2\pi H_2) \\ Z_2 = \sqrt{-2ln(H_1)}sin(2\pi H_2) \end{cases}.$$

where $H_1$ and $H_2$will be the Halton's numbers with base $b_1$ and base $b_2$accordingly.

For the price of the call option you may use the following formula:

$$C = Ef(W_T) = e^{-(rT)}E\left(max\left(0, S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma W_T} - K\right)\right)$$

**Inputs:** *S_0, K, T, r, Sigma, N, b1, b2*
**Outputs**:

*i.* Values: *C*

**5.** **[*Optional –NOT for grading*]** How much are you willing to pay to play this game: You toss a fair coin. If it is a Tail then you get \$7 in 18 months. If it is a Head then you lose \$2 immediately. The one and two-year zero-coupon rates are 4% and 6% respectively. Would the amount you are willing to pay to play this game increase or decrease if the payoff (in case of Tails) happens in 36-months?

**6.** **[*Optional –NOT for grading*]** Value an American Put option that has no maturity (perpetual option). What's the delta of the option if it is at-the-money?