Mgmt 237e: Homework 4
Non-stationary Time Series and GARCH

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Please use Matlab or R to solve these questions (when data is involved). The problem set is due on February 16th. Use the electronic drop box to submit your answers. Submit the Matlab/R file and the file with a short write-up of your answers. The points for each question are allocated equally to each of the sub-questions.

Problem 1: Unit root testing in series with and without time trends

Unit roots (60 pts): This problem is designed to get us to think about whether macroeconomic variables like U.S. log real GDP or financial variables like log exchange rates (log-prices) contain a unit root component, or have only transitory components with a deterministic time trend, or have a bit of both.

For this problem, we will use two data series. First, there is the U.S. exchange rate vs. a broad weighted index available on CCLE in the file '**dollar_broadindex_tradeweighted.xlsx**.' You can transform this series to logarithms and use it from January 1971 through 2008. Secondly, you can download quarterly U.S. real GDP per capita from the Federal Reserve Bank of St. Louis (see the FRED database) or the Bureau of Economic Analysis (BEA). The series should be available from Q1 1947 through Q3 2015.

1. (35 pts) Suppose the series y_t follows one of the two possible stochastic processes: (i) a random walk with drift:

$$y_t = \mu + y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \text{WN}\left(0, \sigma^2\right)$$

or, (ii) an AR(1) process

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \text{WN}\left(0, \sigma^2\right)$$

where $|\phi| < 1$. Notice that we are interested in detecting whether $\phi = 1$ or $\phi < 1$. This type of model is a reasonable specification for the logarithm of asset prices like log exchange rates.[1] There is no time trend in this model because we do not think that a log exchange rate will deterministically increase over time.

(a) For both process (i) and (ii) derive an expression for the following variance ratio:

$$\frac{\mathbb{V}\left(y_t - y_{t-k}\right)}{k\sigma^2}$$

What behavior do we expect to see in each case as $k \to \infty$?

(b) Plot the variance ratio for the log exchange rate data for different values of k. What do you conclude?

(c) In Matlab and R, there are functions to calculate variance ratio tests. The null hypothesis is that the series has a unit root versus the alternative that it does not. However, in practice, the implementation of these tests is more general than what we assumed above. They do not necessarily assume that ε_t is white noise as it can be heteroskedastic and have some autocorrelation. In Matlab's econometrics toolbox, there is the function '**vratiotest.m**' which calculates the test. In R, there is a CRAN package you can download called 'vrtest.'

2. (25 pts) Testing for a unit root in the logarithm of US real GDP per capita.

(a) Plot the logarithm of U.S. real GDP per capita. Plots its first difference. Does it look like you should use a constant and/or a time trend?

(b) Carefully describe the appropriate alternative model and the null hypothesis. Explain your choices.

(c) Report the results of your test.

[1]Note: For the sake of this problem, I have written down both a model under the null hypothesis (the model (i) with $\phi = 1$) and a model under the alternative (the model (ii) with $\phi < 1$). In practice, however, you are not necessarily testing for model (i) vs. (ii). This is because both models could be wrong. A good hypothesis test has power against many alternatives. In other words, if model (i) under our null hypothesis is wrong, we want to be able to detect that it is wrong even when the 'true' model is any one of a large number of possibilities.

Problem 2: Risk Management using Value-at-Risk and GARCH models (40 points)

1. You are in charge of risk management at Deutsche Bank, and you oversee a $2 bn long position in currencies. The daily prices (for one share in the currency fund) have been provided to you in the spreadsheet. The maximum VaR over a 20-day trading period for currency trading was recently set at $ 100 million.

   (a) (20 pts) Specify and estimate a parsimonious model for the conditional volatility of the daily log returns on this currency position. You can abstract from variation in the conditional mean of returns. Carefully explain why you chose this model. Provide some evidence that this model is a good fit for the data.

   (b) (10 pts) Based on these estimates, develop a forecast for the 20-trading-day return volatility on Feb 1, 2016 (end of day). Report the exact number and explain how you arrived at this number. You can assume daily returns (in levels) are independently distributed over time.

   (c) (10 pts) Use this estimate to construct a measure for the 20-trading-day VaR at 5% on the $2 bn long position in currencies on Feb 1, 2016 (end of day). Report the exact number and explain how you arrived at this number.