MGMT 237E - Empirical Methods
Homework 1

Yi Chi Chan, Ashwin Kumar Ashok Kumar, Zahofang Shi, George Bonebright

January 12, 2016

1. 1. $m(t) = E[e^{tj}] = \sum_{j=0}^{\infty} e^{tj} Pr(J_t = j)$
$= \sum_{j=0}^{\infty} e^{tj} (\frac{e^{-\lambda}\lambda^j}{j!})$
$= e^{-\lambda} \sum_{j=0}^{\infty} \frac{(\lambda e^t)^j}{j!}$
$= e^{-\lambda}(e^{\lambda e^t})$
$= e^{\lambda(e^t - 1)}$

1. 2. $k(t) = ln(m(t)) = \lambda(e^t - 1)$
$Mean : \mu = \kappa_1 = k'(0) = \lambda e^t \mid_{t=0} = \lambda$
$Variance : \sigma^2 = \kappa_2 = k''(0) = \lambda e^t \mid_{t=0} = \lambda$
$\kappa_3 = \kappa_4 = \lambda e^t \mid_{t=0} = \lambda$
$Skewness = S(j) = \frac{\kappa_3}{\kappa_2^{\frac{3}{2}}} = \lambda^{\frac{-1}{2}}$
$ExcessKurtosis = K(j) - 3 = \frac{\kappa_4}{\kappa_2^2} = \lambda^{-1}$

1. 3. $ConditionalDistribution : Y_t \mid J_t \ N(j\alpha, j\delta^2) \mid j = J_t$
$m(t; Y_t \mid J_t) = E[e^{tY_t} \mid J_t] = \int_{-\infty}^{\infty} e^{tY_t} \frac{1}{\sqrt{2\pi j\delta^2}} e^{-\frac{1}{2j\delta^2}(Y_t - j\alpha)^2} dY_t$
$= \frac{1}{\sqrt{2\pi j\delta^2}} \int_{-\infty}^{\infty} e^{tY_t - \frac{1}{2j\delta^2}(Y_t^2 - 2j\alpha Y_t + j^2\alpha^2)} dY_t$
$= \frac{1}{\sqrt{2\pi j\delta^2}} \int_{-\infty}^{\infty} e^{j\alpha t + \frac{1}{2}j\delta^2 t^2 - (j\alpha t + \frac{1}{2}j\delta^2 t^2)} e^{-\frac{1}{2j\delta^2}(Y_t^2 - 2(j\alpha + j\delta^2 t)Y_t + j^2\alpha^2)} dY_t$
$= e^{j\alpha t + \frac{1}{2}j\delta^2 t^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi j\delta^2}} e^{-\frac{1}{2j\delta^2}(Y_t - [j\alpha + j\delta^2 t])^2} dY_t$
$= e^{j\alpha t + \frac{1}{2}j\delta^2 t^2} \mid j = J_t$
$= e^{J_t\alpha t + \frac{1}{2}J_t\delta^2 t^2}$

1. 4. $m(t) = E_j[m(t; Y_t] \mid J_t) = E_j[E[e^{tY_t} \mid J_t]]$
$= \sum_{j=0}^{\infty} e^{(j\alpha t + \frac{1}{2}j\delta^2 t^2)} \frac{e^{-\lambda}\lambda^j}{j!}$
$= e^{-\lambda} \sum_{j=0}^{\infty} \frac{[\lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)}]^j}{j!}$

$= e^{-\lambda} e^{\lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)}}$
$= e^{\lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)} - 1}$
$Cumulant - GeneratingFunction : k(t) = \lambda(e^{(\alpha t + \frac{1}{2}\delta^2 t^2)} - 1)$
$Mean : \mu = \kappa_1 = k'(0) = \lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)}(\alpha + \delta^2 t) \mid_{t=0} = \lambda\alpha$
$Variance : \sigma^2 = \kappa_2 = k''(0) = \lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)}[(\alpha + \delta^2 t)^2 + \delta^2] \mid_{t=0} = \lambda(\alpha^2 + \delta^2)$
$\kappa_3 = k'''(0) = \lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)}(\alpha + \delta^2 t)[(\alpha + \delta^2 t)^2 + 3\delta^2] \mid_{t=0} = \lambda\alpha(\alpha^2 + 3\delta^2)$
$\kappa_4 = k''''(0) = \lambda e^{(\alpha t + \frac{1}{2}\delta^2 t^2)}(\alpha + \delta^2 t)^2[(\alpha + \delta^2 t)^2 + 6\delta^2] + 3\delta^4 \mid_{t=0} = \lambda[\alpha^2(\alpha^2 + 6\delta^2) + 3\delta^4]$
$Skewness : S(Y_t) = \frac{\kappa_3}{\kappa_2^{\frac{3}{2}}} = \frac{\alpha(\alpha^2+3\delta^2)}{\lambda^{\frac{1}{2}}(\alpha^2+\delta^2)^{\frac{3}{2}}}$
$ExcessKurtosis : K(Y_t) - 3 = \frac{\kappa_4}{\kappa_2^2} = \frac{\alpha^4+6\alpha^2\delta^2+3\delta^4}{\lambda(\alpha^2+\delta^2)^2}$

1. 5. A Poisson-Normal mixture is potentially a more accurate model of returns than a log-normal model for three reasons:

(1) Excess kurtosis accounts for the likelihood that a greater number of returns will fall into the "tails" of the distribution.

(2) Because α is a term included in the skewness equation, it can dictate the polarity of the term and alter whether the distribution is positively or negatively skewed.

(3) Realistically, investment returns are volatile, the value λ represents the "jumps" and affects both the mean and variance of the Poisson-Normal which more accurately models true market behavior.

1. 6. $E[r_t] = E[\epsilon_t + Y_t] = E[\epsilon_t] + E[Y_t] = \mu + \lambda\alpha = 0.0792 + 1.512(-0.0259) = 0.040039$

ϵ_t and Y_t are independent.
$m(t; r_t) = E[e^{t(\epsilon_t + Y_t)}] = E[e^{t\epsilon_t} e^{tY_t}] = m(t; \epsilon_t) * m(t; Y_t)$

$k(t; r_t) = ln(m(t; r_t)) = ln(m(t; \epsilon_t)) + ln(m(t; Y_t)) = k(t; \epsilon_t) + k(t; Y_t)$
$= \mu t + \frac{1}{2}\sigma^2 t^2 + \lambda(e^{\alpha t + \frac{1}{2}\delta^2 t^2} - 1)$

$\kappa_2(r_t) = \sigma^2 + \lambda(\alpha^2 + \delta^2) = 0.0324$
$\kappa_3(r_t) = \lambda(\alpha^3 + 3\alpha\delta^2) = -0.00022$
$\kappa_4(r_t) = \lambda(\alpha^4 + 6\alpha^2\delta^2 + 3\delta^4) = 0.0000232$

$SD(r_t) = \sqrt{\sigma^2 + \lambda(\alpha^2 + \delta^2)} = \sqrt{0.1699^2 + 1.512((-0.0259)^2 + 0.0407^2)} = 0.18$

$Skewness : S(r_t) = \frac{\kappa_3(r_t)}{\kappa_2(r_t)^{\frac{3}{2}}} = -0.0379$
$ExcessKurtosis : K(r_t) - 3 = \frac{\kappa_4(r_t)}{\kappa_2(r_t)^2} = 0.022$

2. Problem 2 R code included as a separate file.

2. 4. In order to provide an expected return of %20 at a constant Sharpe Ratio of 0.50 your methods of diversification will be limited. The Sharpe Ratio is defined as: $\frac{\mu - r_f}{\sigma}$.

The major concerns with the portfolios that fit these criteria are generally attributable to the leverage required to achieve the %20 returns. By exposing yourself to a magnified position in market you are increasing your expected return but also the volatility of your portfolio.

In order to achieve %20 returns the portfolios will have to hold a combination of long and short positions. (The short position could include loaning money from a bank in order to increase capital for investment). By holding a combination of long and short positions, you are increasing the risk faced by your portfolio. Which, in turn, will decrease your portfolio's Sharpe Ratio.

Both investments have excess kurtosis which indicates greater "tail risk" (a greater probability that returns will fall farther from the mean in the distribution).

It appears that the ideal portfolio composed of only the two assets, GSPC and DBV, would hold a long position in GSPC and a short position in DBV. The investment in the currency index (DBV) has an increased volatility due to its composition of both long and short positions in different currencies. The Sharpe Ratio for DBV is greater than that of the GSPC.

Besides systematic risk which is inherent in any portfolio and cannot be "diversified out", additional risks include: the likelihood of a large change in relative currency value due to a country defaulting, the (il)liquidity of holding large positions, and any compounding effect that may be present when holding a position in the S&P500 and the USD (through DBV) simultaneously.

```
#Empirical method HW1 Problem 2
#part 1
setwd("C:/Users/SallyShi/Desktop/MGMT237E-Empirical Methods in
    Finance/HW1")
library(xts)
Fund <- read.csv("DBV.csv")
SP <- read.csv("GSPC.csv")

Fund[,1]<-as.Date(Fund[,1],format("%m/%d/%Y"))
#create xts object
Funddata<-as.xts(Fund[,7],order.by=Fund[,1])
Fundret<-diff(log(Funddata),lag=1)
Fundret<-Fundret[-1,]
head(Fundret)

SP[,1]<-as.Date(SP[,1],format("%m/%d/%Y"))
SPdata<-as.xts(SP[,7],order.by=SP[,1])
SPret<-diff(log(SPdata),lag=1)
SPret<-SPret[-1,]

#time-series plot of daily log-returns
plot(Fundret,main="Time series plot of DBV log return")
plot(SPret,main="Time series plot of GSPC log return")
```

```
hist(Fundret,main="DBV log return histogram",breaks=40)
hist(SPret,main="GSPC log return histogram",breaks=40)

#part 2
library(fBasics)
basicStats(Fundret)
t3<-function(ret){
  S3<-skewness(ret)
  T<-length(ret)
  t3<-S3/sqrt(6/T)
  t3
}

t4<-function(ret){
  K4<-kurtosis(ret)
  T<-length(ret)
  t4<-K4/(sqrt(24/T))
  t4
}

t3(Fundret)
t4(Fundret)
normalTest(Fundret,method='jb')

criticalVal=abs(qnorm(0.05/2))
#abs(t3(Fundret))>criticalVal
#reject the null hypothesis, log return of DBV is negatively skewed at 5% significant level
#t4(Fundret)>criticalVal
#reject the null hypothesis, log return of DBV has heavy tails at 5% significant level
#Jarque-Bera test Asymptotic p Value: < 2.2e-16
#reject the null hypothesis at 5% significant level

basicStats(SPret)
t3(SPret)
t4(SPret)
normalTest(SPret,method='jb')
normalTest(SPret,method='jb')@test$p.value

#abs(t3(SPret))>criticalVal
#reject the null hypothesis, log return of GSPC is negatively skewed at 5% significant level
#t4(Fundret)>criticalVal
#reject the null hypothesis, log return of GSPC has heavy tails at 5% significant level
#Jarque-Bera test Asymptotic p Value: < 2.2e-16
#reject the null hypothesis at 5% significant level


#part 3
stats<-data.frame(cbind(basicStats((Fundret)),basicStats((SPret))))
table<-rbind(stats["Mean",],stats["Variance",],stats["Stdev",],
    stats["Skewness",])
table["Excess kurtosis",]<-stats["Kurtosis",]
table["skewness test Tvalue",]<-cbind(t3(Fundret),t3(SPret))
```

```r
table["kurosis test Tvalue",]<-cbind(t4(Fundret),t4(SPret))
table["Jarque-Bera test Pvalue",]<-cbind(normalTest(Fundret,method=
    'jb')@test$p.value,normalTest(SPret,method='jb')@test$p.value)
colnames(table)=c("DBV","GSPC")
table
```