# Project 5 by
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Q1. Least Square Monte Carlo :

a) **Laguerre polynomials**

```
Q1.a [method = Laguerre]
 S0 = 36.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 4.2076689  0.       ]                    [ 3.98954488  5.18396363]  [ 4.05437298  3.64325059]  [ 4.19203344  0.3715943 ]
[  1.  nan]     [ 4.4871167  0.       ]                    [  3.98008112  11.29980822]  [ 4.07565702  9.16980116]  [ 4.26751805  4.89398134]
[  2.  nan]     [ 4.84680593  0.       ]                   [  3.95796233  18.3387495 ]  [  4.08532585  15.71096712]  [  4.31456721  10.98122623]

 S0 = 40.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 1.80072793  0.       ]                    [  1.37009677  23.91428194]  [ 1.677248    6.85722321]  [ 1.78574476  0.83206186]
[  1.  nan]     [ 2.31581127  0.       ]                    [  1.434298   38.0649877]   [  1.80172162  22.19911701]  [ 2.14704694  7.28748223]
[  2.  nan]     [ 2.89227396  0.       ]                    [  1.54455538  46.59719641]  [  1.93427687  33.12262614]  [  2.29167158  20.76575017]

 S0 = 44.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 0.62878238  0.       ]                    [  0.49442231  21.3682937 ]  [ 0.61167739  2.72033554]  [ 0.62586107  0.46459796]
[  1.  nan]     [ 1.11576557  0.       ]                    [  0.60746465  45.55624689]  [  0.84259215  24.48304815]  [ 1.06547145  4.50758893]
[  2.  nan]     [ 1.68865437  0.       ]                    [  0.71370993  57.73499036]  [  1.00598812  40.42664154]  [  1.29974216  23.03089448]
```

b) **Hermite polynomials**

```
 Q1.b [method = Hermite]
  S0 = 36.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 4.2076689  0.       ]                    [ 4.17242696  0.83756447]  [ 4.19449418  0.31311222]  [ 4.21081297  0.0747224 ]
[  1.  nan]     [ 4.4871167  0.       ]                    [ 4.41853379  1.5284406 ]  [ 4.48053943  0.14658121]  [ 4.47546235  0.25972908]
[  2.  nan]     [ 4.84680593  0.       ]                   [ 4.74330532  2.13543961]  [ 4.81734088  0.6079273 ]  [ 4.82748757  0.39857922]

 S0 = 40.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 1.80072793  0.       ]                    [ 1.7683478   1.79816904]  [ 1.79596391  0.26456063]  [ 1.7920265   0.48321703]
[  1.  nan]     [ 2.31581127  0.       ]                    [ 2.27766775  1.64709108]  [ 2.32075586  0.2135145 ]  [ 2.31473794  0.04634795]
[  2.  nan]     [ 2.89227396  0.       ]                    [ 2.81140731  2.79595416]  [ 2.87140415  0.72157107]  [ 2.8758163   0.56902154]

 S0 = 44.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 0.62878238  0.       ]                    [ 0.61148475  2.75097137]  [ 0.63188487  0.49341282]  [ 0.63483142  0.96202489]
[  1.  nan]     [ 1.11576557  0.       ]                    [ 1.08795898  2.49215428]  [ 1.11104275  0.42328099]  [ 1.11059689  0.46324149]
[  2.  nan]     [ 1.68865437  0.       ]                    [ 1.65948327  1.72747619]  [ 1.68895203  0.01762716]  [ 1.69385572  0.30801756]
```

c) **Simple Monomials**

```
Q1.c [method = Monomials]
 S0 = 36.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 4.2076689  0.       ]                    [ 4.16275494  1.06743091]  [ 4.19247715  0.36104906]  [ 4.20066273  0.16650964]
[  1.  nan]     [ 4.4871167  0.       ]                    [ 4.43745505  1.10676072]  [ 4.46125672  0.5763162 ]  [ 4.4897774   0.05929645]
[  2.  nan]     [ 4.84680593  0.       ]                   [ 4.73865692  2.23134603]  [ 4.82523086  0.44514003]  [ 4.84435452  0.05057787]

 S0 = 40.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 1.80072793  0.       ]                    [ 1.78138534  1.07415374]  [ 1.79605255  0.25963851]  [ 1.79296383  0.43116437]
[  1.  nan]     [ 2.31581127  0.       ]                    [ 2.26208332  2.32004898]  [ 2.30632908  0.4094546 ]  [ 2.31659751  0.03395079]
[  2.  nan]     [ 2.89227396  0.       ]                    [ 2.81042583  2.82988879]  [ 2.85426714  1.31408085]  [ 2.89300097  0.02513607]

 S0 = 44.000000
['T/K', 'nan']  ['Binomial-using J-R Model', '% deviation']  ['K=2', '% deviation']  ['K=3', '% deviation']  ['K=4', '% deviation']
[ 0.5  nan]     [ 0.62878238  0.       ]                    [ 0.61548456  2.11485216]  [ 0.6282096  0.0910937]   [ 0.63042619  0.26142843]
[  1.  nan]     [ 1.11576557  0.       ]                    [ 1.09682622  1.69743155]  [ 1.10079381  1.34183806]  [ 1.12146413  0.51073028]
[  2.  nan]     [ 1.68865437  0.       ]                    [ 1.64673582  2.48236373]  [ 1.68225187  0.37914801]  [ 1.68968444  0.06099951]
```

The results are compared against the Binomial Model (J-R with 100000 simulations).

Based on the above results, we can see that the Monomials with K=4 gives the best results (close to the Binomial-using J-R model).
General trend is increasing K the % error (Deviation from actual) decreases. But we also see the error increases (drastically) when increase the stock price ( out of the money) or increase time to expiration. Laguerre method gives least performance for higher T/Stock Price.
Roughly on performance basis,
Monomial > Hermite> Laguerre

Q2.

```
Forward-start European Put Price = 3.160828  with var = 0.000725   ****[1.007263 sec]
Forward-start American Put Price = 3.476219  with var = 0.000995   ****[25.511993 sec]

Process finished with exit code 0
```