Mgmt 237e: Homework 1

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Please use Matlab/R to solve these problems. You can just hand in one set of solutions that has all the names of the contributing students on it in each group. The problem set is due on January 12 by 8.00 AM. Use the electronic drop box to submit your answers. Submit the Matlab or R file and the file with a short write-up of your answers separately.

[The quality of the write-up matters for your grade. Please imagine that you're writing a report for your boss at Goldman when drafting answers these questions. Try to be be clear and precise.]

Problem 1: Modeling heavy-tails with jumps

The distribution of stock returns has fat tails (see lecture 1). As a result, we need models that deliver fat-tailed distributions (e.g., to price options on stocks). One way of doing this is to introduce jumps in the model. This problem describes the building blocks of how to do that.

Consider a Poisson-distributed random variable $J_t \sim \text{Poisson}(\lambda)$ whose mass function is

$$\Pr(J_t = j) \quad = \quad \frac{\exp(-\lambda)\,\lambda^j}{j!} \qquad j = 0, 1, 2, \ldots, \infty$$

From the definition that $\Pr(J_t = j)$ a probability distribution, we know that

$$1 \quad = \quad \sum_{j=0}^{\infty} \frac{\exp(-\lambda)\,\lambda^j}{j!}$$

This is the power series representation of the exponential function given by: $\exp(\lambda) = \sum_{j=0}^{\infty} \frac{\lambda^j}{j!}$.

1. Show that the moment-generating function is given by[1]:

$$m(t) = \exp\left[\lambda(e^t - 1)\right]$$

2. Using the corresponding cumulant-generating function, derive the mean, variance, skewness and excess kurtosis.

3. Next, consider a random variable Y_t that is Poisson-normal mixture. We can write this as:

$$Y_t = \sum_{j=0}^{J_t} \eta_{t,j} \qquad \eta_{t,j} \sim i.i.d.\ \mathrm{N}\left(\alpha, \delta^2\right)$$

   In other words, the random variable Y_t is a random sum of normal random variables. The random variable J_t is interpreted as the number of jumps that occur over an interval of time Δt, e.g. 1 hour, one day, 1 week, etc. Conditional on J_t jumps, Y_t is normal with mean $j\alpha$ and variance $j\delta^2$. Derive the conditional moment generating function of Y_t (conditional on J_t). In other words, assume you know the number of jumps J_t.

4. Derive the moment-generating function of the Poisson-normal mixture random variable Y_t. Hint: you can use the law of total iterated expectations.

$$m(t) = \mathbb{E}\left[\exp\left(tY_t\right)\right]$$

   Report the skewness and excess kurtosis.

5. Explain carefully why this Poisson-normal mixture is potentially a better model of returns than the log-normal model.

6. Suppose that log returns r_t are given by the sum of a normal random variable $\varepsilon_t \sim N(\mu, \sigma^2)$ and a Poisson-normal mixture random variable Y_t:

$$r_t = \varepsilon_t + Y_t$$

   The random variables ε_t and Y_t are independent. The following 5 parameter estimates

$$(\mu, \sigma, \lambda, \alpha, \delta) = (0.0792, 0.1699, 1.5120, -0.0259, 0.0407)$$

[1] For any discrete random variable Z, the moment generating function is defined as

$$\mathbb{E}\left[\exp\left(tZ\right)\right] = \sum_{z=0}^{\infty} \exp\left(tz\right) P(Z = z)$$

This is just the discrete analog of the continuous-valued case discussed in class.

were chosen to match the moments of annual returns on the S&P 500 as best as possible. Using these parameter estimates, evaluate the standard deviation, skewness and kurtosis of log stock returns.

Problem 2: The Currency Carry Trade

The Deutsche Bank G10 Currency Future Harvest Index is, at any one time, composed of long futures contracts on the three G10 currencies associated with the highest interest rates and short futures contracts on the three G10 Currencies associated with the lowest interest rates.* The Index re-evaluates interest rates quarterly and, based on the evaluation, re-weights the futures contracts it holds. Immediately after each re-weighting, the Index will reflect an investment on a 2:1 leveraged basis in the three long and three short futures contracts (unless USD is one of the six currencies associated with the highest or lowest interest rates, in which case the Index will reflect an investment on a 1.66:1 leveraged basis). The PowerShares DB G10 Currency Harvest Fund tracks this index. Its ticker symbol is *DBV*. The price data is provided to you in the spreadsheet. Also provided in a second spreadsheet is the S&P500 index, whose ticker symbol is *GSPC*.

Throughout this problem, use the adjusted closing values of the index. These values have adjusted for dividends and splits.

1. Visualizing the data.
   (a) Create time series plots of the daily log-returns for *DBV* and *GSPC*.
   (b) Create histograms of the daily log-returns for *DBV* and *GSPC*.
2. Shape of Return Distribution:
   (a) Test the null that the skewness of daily log returns is zero at the 5% significance level.
   (b) Test the null that the excess kurtosis of daily log returns is zero at the 5% significance level.
   (c) Test the null that the daily log returns are normally distributed at the 5% significance level using the Jarque-Bera test.
3. Compare all of these numbers in (a) and (b) to the same numbers for daily log returns on the S&P 500 measured over the same sample in one single table.
4. Suppose you are a fund manager with a target return in mind (say 20 % per annum), and suppose that the ratio of expected returns to standard deviation is 0.50 for both investments. Using these numbers, discuss the different nature of the risks you would face if you invested in equity or currency markets to achieve that target return (with the appropriate leverage).