PROGRESSIONS

INTRODUCTION

A movement forward, especially one that advances toward some achievement, is called a progression. A progression is a series that advances in a logical and predictable pattern. Progression has different types. Here we are going to discuss Arithmetic and Geometric progression.

ARITHMETIC PROGRESSION

An arithmetic progression (AP) or arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant. For instance, the sequence 5, 7, 9, 11, 13, 15 … is an arithmetic progression with common difference of 2.

By an arithmetic progression of m terms, we mean a finite sequence of the form

$a, a + d, a + 2d, a + 3d, . . . , a + (m - 1)d.$

The real number a is called the first term of the arithmetic progression, and the real number d is called the difference of the arithmetic progression.

Example 1:
Consider the sequence of numbers 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23.
This sequence has the property that the difference between successive terms is constant and equal to 2.

Here we have: $a = 1$; $d = 2$.

GENERAL TERM OF ARITHMETIC PROGRESSION

The general term of an arithmetic progression with first term a1 and common difference d is:

$$a_k = a_1 + (k-1)d$$

Example: Find the general term for the arithmetic sequence -1, 3, 7, 11 . . . Then find a12.

IMPORTANT

Solution:

Here a1 = 1. To find d subtract any two adjacent terms: d = 7 - 3 = 4. The general term is:

$$a_k = a_1 + (k-1)d$$

a_k = -1 + (k-1) 4
= -1+4k – 4
= 4k -5

To find a_{12} let k = 12.

a_{12} = 4 *12 – 5 = 43

SUM OF AN ARITHMETIC PROGRESSION

The sum of the n terms of an arithmetic progression with first term a1 and common difference d is:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

Also, the sum of an arithmetic progression is equal to

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Example: Find the sum of the 10 terms of the arithmetic progression if a_1 = 5 and d = 4.

IMPORTANT

Solution:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2 \cdot 5 + (10-1) \cdot 4\right]$$

$$S_{10} = 5 \cdot 46$$

$$S_{10} = 230$$

Example: Find 1 + 2 + 3 + . . . + 100

Solution:

In this example we have: a_1 = 1, d = 1, n = 100, a_{100} = 100. The sum is:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{100} = \frac{100}{2}(1 + 100)$$

$$S_{100} = 5050$$

GEOMETRIC PROGRESSION

A geometric progression, also known as a geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio.

For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2.

By a geometric progression of m terms, we mean a finite sequence of the form

$a, ar, ar^2, \ldots, ar^{m-1}$.

The real number a is called the first term of the geometric progression, and the real number r is called the ratio of the geometric progression.

Example:
Consider the finite sequence of numbers 4, 8, 16, 32, 64, 128, 256, 512, 1024
This sequence has the property that the ratio between successive terms is constant and equal to 2.
Here we have: a = 4; r = 2.

N-TH TERM OF THE GEOMETRIC PROGRESSION

The n-th term of the geometric progression is equal to $T_n = ar^{n-1}$

SUM OF A GEOMETRIC PROGRESSION

The sum of the n terms of a geometric progression is equal to

$$S_n = a\frac{\left(r^n - 1\right)}{r - 1} \text{ if } r > 1$$

or

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} \text{ if } r < 1$$

Sum of a Geometric series continued to infinity:
Let us consider the series a, ar, ar^2, ar^3.......

$$S = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a}{1 - r} - \frac{ar^n}{1 - r}$$

If $|r| < 1$, $r^n \to 0$ as $n \to \infty$

$$\therefore S_\infty = \frac{a}{1 - r}$$

Note: Sum to infinity exists only when r is numerically less than 1, i.e. $|r| < 1$.
Example: Find the sum of the series 1+5+25 .... to 10 terms

$a = 1, \quad r = 5$

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

$$\therefore S_{10} = 1\left(\frac{5^{10} - 1}{5 - 1}\right) = \frac{1}{4}\left(5^{10} - 1\right)$$

PRACTICE PROBLEMS (WITH EXPLANATORY ANSWERS)

1. The sums of 'n' terms of two Arithmetic Progressions are in the ratio of (7n + 21) : (3n + 9). What is the ratio of their 8th terms?
A. 7 : 4
B. 4 : 7
C. 7 : 3
D. 3 : 7

2. Sum of three numbers in Arithmetic Progression is 24 and their product is 440. What are the terms?
A. 5, 8, 11
B. 4, 8, 12
C. 6, 8, 10
D. 6, 7, 11

3. What is the sum of the series, 2 + 4 + 7 + 5 + 12 + 6 +………up to 200 terms?
A. 60600
B. 30300
C. 15150
D. 90900

4. What is the sum to 'n' terms of the series 1 + 11 + 111 + 111 +…….?
A. $\frac{10^{n+1}-9n}{81}$
B. $\frac{10^{n+1}-10-9n}{81}$
C. 8(10n – 1 – 10)
D. 8(10n – 1 + 10)

5. What is the value of 'x', if x = 1 – 2 + 3 + 2 – 3 + 4 +…………..up to 100 terms?
A. – 627
B. 629
C. 628
D. 627

6. Divide 104 into 4 parts which are in Arithmetic Progression such that the product of the first and the fourth part is 32 less than the product of the second and the third parts.
A. 20, 32, 24, 28
B. 19, 21, 23, 25
C. 20, 24, 28, 32
D. 20, 24, 32, 28

7. The first and last terms of an Arithmetic Progression are 8 and 64 respectively. What is the number of terms?
A. 10
B. 9
C. 8
D. Indeterminate

8. What are the three numbers in Geometric Progression whose sum is 21 and whose product is 216?
A. 4, 6, 9
B. 3, 6, 12

C. 6, 6, 9
D. 7, 8, 6

9. The sum of the terms of an infinite Geometric Progression is 20 and the sum the squares is 100. The first term of the series is,
A. 6 B. 8 C. 12 D. 16

Fast forward 1: *What comes next in the following sequence? 1, 4, 5, 6, 7, 9, 11,... (Hint: spelling)*

10. If x < 1, then what is the sum of the series: $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$to infinity is,
A. $\frac{1}{1-x}$
B. $\frac{1}{(1-x)^2}$
C. $\frac{1}{1-x^2}$
D. $\frac{1}{1+x^2}$

11. Sum of three numbers in Arithmetic Progression is 18 and the sum of their squares is 116. What are the terms?
A. 3, 6, 9
B. 4, 6, 8
C. 2, 6, 10
D. 2, 4, 12

12. What is the 12th term of an Arithmetic Progression whose first term is 262 and common difference is – 5?
A. 207
B. 212
C. 217
D. 214

13. What is the first term and the common difference of an Arithmetic Progression if its 5th term is 14 and its 11th term is 29.
A. 4, 2.5
B. 6, 1.5
C. 0, 3
D. −3, 4.5

14. An elastic ball is dropped from a height of 150 m. It is known that the ball rebounds to half the height of the last drop. What is the distance traversed by it by the time it comes to rest?
A. 300 m

B. 450 m
C. 600 m
D. 400.99 m

15. A ball thrown up 12 feet into the air bounces back to half its previous height every time it hits the ground. What is the total distance travelled by the ball (approximately, in feet)?

A. 24
B. 48
C. 12
D. 18

Fast forward 2: *There is a three digit number. The second digit is four times as big as the third digit,*
while the first digit is three less than the second digit. What is the number?

16. The greatest value of positive integer n, so that the sum to n terms of the series $1 + \frac{1}{2} + \frac{1}{2^2} + \ldots.$ is less than $\left(2 - \frac{1}{500}\right)$ is
A. 9
B. 10
C. 11
D. 12

17. What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?
A. 897
B. 164,850
C. 164,749
D. 149,700

18. Given A = 2^{65} and B = (2^{64}+2^{63}+2^{62}+...+2^0)
A. B is 2^{64} larger than A
B. A and B are equal
C. B is larger than A by 1
D. A is larger than B by 1

19. Find the sum of all terms of following series. 81, 27, 9, 3, 1, 1/3, 1/9, 1/27,....... Infinite terms.
A. 120
B. 121.5
C. 122.5
D. 242.5

20. Find the 28th term of the sequence 2, 5, 8, 11 ……299 from last?
A. 83
B. 198
C. 218
D. 243

21. Find the last term of the series 5, 7, 9,....... to 91 terms
A. 180
B. 185
C. 190
D. None

22. Find the arithmetic mean of 256 and 576?
A. 464
B. 444
C. 416
D. 832

23. Find the arithmetic mean of 23, 47, 80 and 150?
A. 64
B. 154
C. 75
D. 150

24. Find the geometric mean of 256 and 576?
A. 364
B. 344
C. 384
D. None

25. Find the geometric mean of 1234321, 1234321 and 1234321?
A. 1111
B. 1234321
C. 123.4321
D. None

SOLUTION AND EXPLANATORY ANSWERS

1. Let a & d be first term and common difference of first AP and a’ and d’ be first term and common difference for second AP.
So by putting n=1 we get ratio a/a’=28/12 or 7/3.

Similarly ratio of (a + d) and (a'+ d') can be found to be 7/3 by putting n = 2.
Solving we get ratio d/d' also as 7/3.
Hence ratio of 8th terms i.e., (a + 7d)/(a'+7d') is also 7/3.
Hence, the correct answer is **option 3.**

2. Let the three numbers in AP be a – d, a and a + d

Sum = 3a = 24 => a = 8

Product = (a – d) a (a + d) = a (a^2 - d^2) = 8(64 – d^2) which is given to be 440.

Solving we get d = 3

Hence numbers are 5,8,11

Hence, the correct answer is **option 1.**

3. S_1 = 2 + 7 + 12 +………up to 100 terms

S_1 = 2+7+12+….497=24950

S_2 = 4 + 5 + 6 +………up to 100 terms

S_2 = 4+5+6+….103=5350

$\Rightarrow S_1+S_2$ = 24950 + 5350 = 30,3000

Hence, the correct answer is **option 2.**

4. S = 1 + 11 +111 +1111 + ……

$\Rightarrow S = \frac{1}{9}$ [9 + 99 + 999+ ….]

$\Rightarrow S = \frac{1}{9}$ [(10 -1) + (100-1) + (1000-1) + ….]

$\Rightarrow S = \frac{1}{9}$ [(10 + 100+ 1000 +……) – n)]

$\Rightarrow S = \frac{1}{9} [(10^n-1)\frac{10}{9} - n]$

$\Rightarrow S = \frac{10^{n+1}-10-9n}{81}$

Hence, the correct answer is **option 2.**

5. x = 1 – 2 + 3 + 2 – 3 + 4 +…………..up to 100 terms

⇒x = (1-2+3) + (2-3+4) + ……. (33-34+35) + 34

⇒x = Sum of 3 series and last number 34 as under

A= 1+2+3….33

B= 3+4+5….35 = A + 66

C = -(2+3+4…35)= -A – 33

A+B+C = A+33 = 33*17 + 33 = 594

x = 594 + 34

x = 628

Hence, the correct answer is **option 3.**

6. Let four number in AP be a – 3d , a – d , a + d and a + 3d

Sum = 4a = 104 which implies a = 26

Product of first & fourth number = (a – 3d) (a + 3d) = $a^2 - 9d^2$

Product of second & third number = (a – d) (a + d) = $a^2 - d^2$

Difference of 2 products = 8 d^2 which is given to be 32.

Hence d = 2 and numbers will be 20, 24, 28, 32

Hence, the correct answer is **option 3.**

7. Let a, d and n be first term, common difference and number of terms of AP.

So, Last Term = a + (n – 1)d = 8 + (n – 1)d which is given to be 64.

Since there are two unknowns 'n' and 'd' we cannot determine them separately.

Hence, the correct answer is **option 4.**

8. From the answer options only pair 3, 6, 12 forms a GP. Further sum and product is 21 and 216 respectively.

Hence, the correct answer is **option 2.**

9. Let a and r be first term and common multiple of GP.

So, $\frac{a}{1-r}$ = 20 (1)

$\frac{a^2}{1-r^2}$ = 100 (2)

Squaring equation (1) we get $\frac{a^2}{(1-r)^2}$ = 400.

Dividing this by equation (2) and solving we get

$5r^2$ – 8r + 3 = 0.

Solving this we get either r = 1 or r = 3/5.

Now since r =1 is meaningless for an infinite GP therefore r = 3/5

Now putting value of r in equation (1) we get a = 8

Hence, the correct answer is **option 2.**

10. Let S be sum of given series

S = 1 + 2x + 3x^2 + 4x^3+…...to infinity

xS = x + 2x^2 + 3x^3+…...to infinity

Subtracting two equations,

S(1 – x) = 1 + x + x^2 + x^3+…...to infinity

$S(1 - x) = \frac{1}{1-x}$

$S = \frac{1}{(1-x)^2}$

Hence, the correct answer is **option 2.**

11. Let a-d, a and a + d be three numbers in AP.

Sum = 3a = 18 => a = 6

$(a-d)^2 + a^2 + (a+d)^2 = 3a^2 + 2d^2 = 116$

$=> 3(6)^2 + 2d^2 = 116$

$=> 2d^2 = 8$

$=> d^2 = 4$

$=> d = 2$

Hence numbers are 4, 6, 8

Hence, the correct answer is **option 2.**

12. Let a, d, n be first term, common difference and number of terms of AP.

Given a = 262, n = 12, d = - 5

Nth term = a + (n – 1) d = 262 + 11(–5) = 262 – 55 = 207

Hence, the correct answer is **option 1.**

13. Let a, d, n be first term, common difference and number of terms of AP.

Nth term = a + (n – 1) d

For 5^{th} term, a + 4d = 14

For 11^{th} term, a + 10d = 29

Solving 2 equations we get d = 2.5 and a = 4

Hence, the correct answer is **option 1.**

14. Ball is dropped from height of 150 m.
In first rebound, it will rebound to 75m and total distance covered in up and down direction will be 2×75.In second rebound distance covered will be 2×37.5

So Total distance covered

= 150 + 2(75 + 37.5 + 18.75 + ………)

$= 150 + 2\ \frac{75}{\frac{1}{2}}$

= 150 + 300

= 450

Hence, the correct answer is **option 2.**

15. The distance travelled by the ball is given by 2 (12 + 6 + 3 + 1.5 + ….). The series is in infinite geometric progression.

The total distance covered (approximately in m) = 2 {12/(1-1/2)} = 2 (24) = 48

(Since the sum of the series a, ar, ar^2,….. is given by a/(1 – r), where 'a' is the first term of the progression and 'r' is the common ratio).
Hence, the correct answer is **option 2**.

16. $S = 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots$.. $\Rightarrow S_{GP} = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S = 2\,[1 - \frac{1}{2^n}]$
$\Rightarrow S = 2 - \frac{1}{2^{n-1}}$
Now, $S <= 2 - \frac{1}{500}$
$2 - \frac{1}{2^{n-1}} <= 2 - \frac{1}{500}$
$\Rightarrow \frac{1}{2^{n-1}} >= 500$
$\Rightarrow n <= 9$
Hence, the correct answer is **option 1.**

17. The smallest 3 digit number that will leave a remainder of 2 when divided by 3 is 101. The next number that will leave a remainder of 2 when divided by 3 is 104, 107, …. The largest 3 digit number that will leave a remainder of 2 when divided by 3 is 998.

So, it is an AP with the first term being 101 and the last term being 998 and common difference being 3.

Sum of an AP $= \left(\frac{\text{First Term} + \text{Last Term}}{2}\right) * \text{Number of Terms}$

We know that in an A.P., the nth term $a_n = a_1 + (n - 1)*d$

In this case, therefore, 998 = 101 + (n - 1)* 3
i.e., 897 = (n - 1) * 3
Therefore, n - 1 = 299
Or n = 300.

Sum of the AP will therefore, be $\frac{101 + 998}{2} * 300$ = 164,850

Hence, the correct answer is **option 2.**

18. B is in G.P. with $a = 2^0$, r = 2, n = 65

$\therefore$ $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{2^0(2^{65} - 1)}{2 - 1}$

$\therefore$ B = 2^{65} - 1
=> B = A - 1
$\therefore$ A is larger than B by 1
Hence, the correct answer is **option 4.**

19. Given series is infinitely diminishing series with a = 81 & r = 1/3.

Sum of series = a / (1-r) = 81 / (1-1/3) = 81 * 3 /2 = 243 / 2 = 121.5.
Hence, the correct answer is **option 2.**

20. Given series, 2, 5, 8, 11 ….. 299
299, 296, 293 ……2
$Tn = a + (n-1)d$
$\mathrm{T28} = 299 + (28-1)(-3) = 218$
Hence, the correct answer is **option** 3**.**

21. Given series , 5, 7, 9 …..
$\mathrm{T91} = 5 + (91-1)(2) = 185$
Hence, the correct answer is **option 2.**

22. $\mathrm{AM} = \frac{256+576}{2} = 416$
Hence, the correct answer is **option 3.**

23. $\mathrm{AM} = \frac{23+47+80+150}{4} = 75$
Hence, the correct answer is **option 3.**

24. $\mathrm{GM} = \sqrt{256 \times 576} = 384$
Hence, the correct answer is **option 3.**

25. $\mathrm{GM} = \sqrt[3]{1234321 \times 1234321 \times 1234321} = 1234321$
Hence, the correct answer is **option 2.**