

RATIO, PROPORTION, AND VARIATION

- Introduction
- Basic Rules

INTRODUCTION

RATIOS

A “ratio” is just a comparison between two quantities of same kind. For Example: The ratio between 15 liters of milk to 10 liters of milk is 3:2. 2 kg iron and 1 liter milk cannot be compared. We represent ratios in numbers. The ratio is dimension less.

The **first term** in ratio is called **Antecedent** and **the second term** is called **Consequent**.

If somebody says, the ratio of the two numbers is $X : Y$, then that doesn't mean that first number is X and the second number is Y .

The numbers may be $CX : CY$ (here C is a non-zero constant which is a multiple of CX and CY). The ratio remains the same when divided or multiplied with a constant.

The **ratio will change** if you **add or subtract** a number from both numerator and denominator.

Two ratios $a : b$ and $c : d$ (or a/b and c/d) are said to be equal if $a \times d = b \times c$.

If two ratios are $X : Y$ and $Y : Z$, then you can simply write them as $X : Y : Z$.

PROPORTION

The equality of two ratios is called a proportion. If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion. In a proportion, the first and fourth terms are known as extremes, while the second and third are known as means.

Product of means = Product of extremes.

Mean proportion between a and b is \sqrt{ab}

The compounded ratio of the ratios:

(a : b), (c : d), (e : f) is (ace : bdf).

$a^2 : b^2$ is a duplicate ratio of a : b.

a and b is a sub-duplicate ratio of a : b.

$a^3 : b^3$ is a triplicate ratio of a : b.

$\sqrt[3]{a} : \sqrt[3]{b}$ is a sub-triplicate ratio of a : b.

VARIATION

Two quantities A and B may be such that as one quantity changes in value, the other quantity also changes in value bearing certain relationship to the change in the value of the first quantity.

Direct Variation:

$X \propto Y$ (X varies as Y) $\Rightarrow X = KY$.

Indirect Variation:

$X \propto \frac{1}{Z}$ (X varies as $\frac{1}{Z}$) $\Rightarrow X = \frac{K}{Z}$.

Joint Variation:

$X \propto \frac{Y}{Z}$ (X varies as $\frac{Y}{Z}$) $\Rightarrow X = K \frac{Y}{Z}$.

Where K is a constant.

KEY POINTS

FEW IMPORTANT POINTS TO REMEMBER

1. The equality of two ratios is called a proportion. If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

2. In a proportion, the *first and fourth* terms are known as *extremes*, while the *second and third* are known as *means*.

3. Product of extremes = Product of means

4. Mean proportion between a and b is \sqrt{ab} .



5. The compounded ratio of the ratios:

(a : b), (c : d), (e : f) is (ace : bdf)

6. If $\frac{a}{b} = \frac{c}{d}$, then, $\frac{a+b}{b} = \frac{c+d}{d}$, which is called the **component**.

7. If $\frac{a}{b} = \frac{c}{d}$, then, $\frac{a-b}{b} = \frac{c-d}{d}$, which is called the **dividend**.

8. If $\frac{a}{b} = \frac{c}{d}$, then, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$, which is called the **component and dividend**.

PRACTICE PROBLEMS (EXPLANATORY ANSWERS AT THE END)

1. Arun and Badri together have Rs. 1210. If $\frac{4}{15}$

of Arun's amount is equal to $\frac{2}{5}$ of Badri's amount, how much amount does Badri have?

- A. Rs. 460 B. Rs. 484
C. Rs. 550 D. Rs. 664

2. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

- A. 2 : 5 B. 3 : 5
C. 4 : 5 D. 6 : 7

3. A sum of money is to be distributed among Arun, Varun, Karan, Arjun in the proportion of 5 : 2 : 4 : 3. If Karan gets Rs. 1000 more than Arjun, what is Varun's share?

- A. Rs. 500 B. Rs. 1500
C. Rs. 2000 D. None

4. Seats for Physics, Chemistry and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50%



IMPORTANT

RATIO, PROPORTION, AND VARIATION

and 75% respectively. What will be the ratio of increased seats?

- A. 2 : 3 : 4 B. 6 : 7 : 8
C. 6 : 8 : 9 D. None

5. In a mixture 60 litres, the ratio of milk and water 2 : 1. If the this ratio is to be 1 : 2, then the quantity of water to be further added is:

- A. 20 litres B. 30 litres
C. 40 litres D. 60 litres

6. The ratio of the number of boys and girls in a school is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

- A. 8 : 9 B. 17 : 18
C. 21 : 22 D. Cannot be determined

7. Salaries of Badri and Vinayak are in the ratio 2 : 3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40 : 57. What is Vinayak's new salary?

- A. Rs. 17,000 B. Rs. 20,000
C. Rs. 25,500 D. Rs. 38,000

8. If $0.75 : x :: 5 : 8$, then x is equal to:

- A. 1.12 B. 1.2
C. 1.25 D. 1.30

9. The sum of three numbers is 98. If the ratio of the first to second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is:

- A. 20 B. 30 C. 48
D. 58

10. If Rs. 782 be divided into three parts, proportional to, $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ then the first part is:

- A. Rs. 182 B. Rs. 190
C. Rs. 196 D. Rs. 204

11. The salaries Arjun, Bheem, Chirag are in the ratio 2 : 3 : 5. If the increments of 15%, 10%

RATIO, PROPORTION, AND VARIATION

and 20% are allowed respectively in their salaries, then what will be new ratio of their salaries?

- A. 3 : 3 : 10 B. 10 : 11 : 20
C. 23 : 33 : 60 D. Cannot be determined

12. If 40% of a number is equal to two-third of another number, what is the ratio of first number to the second number?

- A. 2 : 5 B. 3 : 7
C. 5 : 3 D. 7 : 3

13. The fourth proportional to 5, 8, and 15 is:

- A. 18 B. 24
C. 19 D. 20

14. Two number are in the ratio 3: 5. If 9 is subtracted from each, the new numbers are in the ratio 12: 23. The smaller no. is:

- A. 27 B. 33
C. 49 D. 55

15. In a bag, there are coins of 25 p, 10 p and 5 p in the ratio of 1: 2: 3. If there is Rs. 30 in all, how many 5 p coins are there?

- A. 50 B. 100
C. 150 D. 200

16. 60 litres of diesel is required to travel 600 km using a 800 cc engine. If the volume of diesel required to cover a distance varies directly as the capacity of the engine, then how many litres of diesel is required to travel 800 km using 1200 cc engine?

- A. 80 litres B. 90 litres
C. 120 litres D. 170 litres

17. Rs.432 is divided amongst three workers A, B and C such that 8 times A's share is equal to 12 times B's share which is equal to 6 times C's share. How much did A get?

- A. Rs.192 B. Rs.133
C. Rs.144 D. Rs.128

18. If 20 men or 24 women or 40 boys can do a task in 12 days working for 8 hours a day, how many men working with 6 women and 2 boys take to do a task four times as big working for 5 hours a day for 12 days?

- A. 8 men B. 2 men
C. 122 men D. 24 men

19. Two cogged wheels of which one has 54 cogs and other 32 cogs, work into each other. If the former turns 80 times in three quarters of a minute, how often does the other turn in 8 seconds?

- A. 48 B. 135
C. 24 D. None

20. The monthly incomes of A and B are in the ratio 4: 5, their expenses are in the ratio 5: 6. If 'A' saves Rs.25 per month and 'B' saves Rs.50 per month, what are their respective incomes?

- A. Rs.400 and Rs.500
B. Rs.240 and Rs.300
C. Rs.320 and Rs.400
D. Rs.440 and Rs.550

21. The mean proportional between two numbers is 9 and the third proportional of the two numbers is 243. What is the larger of the two numbers?

- A. 27 B. 81
C. 9 D. None

22. The distance (in m) to which a boy can throw a stone is inversely proportional to its weight (in kg). He breaks the stone into 3 pieces whose weights (in kg) are in the ratio 2: 4: 3. He then throws the stones one by one. The sum of the distances they cover is 39 meters. To what distance (in m) can he throw the unbroken stone?

- A. 3 B. 4
C. 6 D. 2

23. What is the value of $\frac{2a+2b}{2a-2b}$, given that $b \neq 0$ and $\frac{a^2 - ab + b^2}{a^2 + ab - 4b^2} = \frac{1}{3}$?

- A. 4 B. $\frac{4}{3}$
C. $\frac{3}{4}$ D. 3

SOLUTION WITH EXPLNATORY ANSWER

1. $\frac{4}{15}$ Arun = $\frac{2}{5}$ Bhadri

Arun = $(\frac{2}{5} * \frac{15}{4})$ Bhadri

Arun = $\frac{3}{2}$ Bhadri

$\frac{\text{Arun}}{\text{Bhadri}} = \frac{3}{2}$

Arun : Bhadri = 3 : 2

\therefore Bhadri's share = Rs. 1210 * $\frac{2}{5}$ = 484.

Answer: Option B

2. Let the third number be x

Then, first number = 120% of x = $\frac{120x}{100} = \frac{6x}{5}$

Second number = 150% of x = $\frac{150x}{100} = \frac{3x}{5}$

\therefore Ratio of first two numbers = $\frac{6x}{5} : \frac{3x}{5}$
= 12x : 15x
= 4 : 5

Answer: Option C

3. Let the shares of Arun, Varun, Karan and Arjun be Rs. 5x, Rs. 2x, Rs. 4x and Rs. 3x respectively.



Then, $4x - 3x = 1000$

$x = 1000$.

∴ B's share = Rs. $2x$
= Rs. (2×1000)
= Rs. 2000.

Answer: Option C

4. Originally, let the number of seats for, Physics, Chemistry and Biology be $5x$, $7x$ and $8x$ respectively.

Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$).

$$\left(\frac{140}{100} \times 5x\right), \left(\frac{150}{100} \times 7x\right) \text{ and } \left(\frac{175}{100} \times 8x\right)$$

$$7x, \frac{21x}{2} \text{ and } 14x$$

∴ The required ratio = $7x : \frac{21x}{2} : 14x$
 $14x : 21x : 28x$
 $2 : 3 : 4$.

Answer: Option A

5. Quantity of milk = $60 \times \frac{2}{3}$ litres
= 40 litres

Quantity of water in it = $(60 - 40)$ litres
= 20 litres.

New ratio = 1 : 2

Let quantity of water to be added further be x litres.

Then, milk : water = $40 : 20 + x$

$$\text{Now } \frac{40}{20+x} = 12$$

$$20 + x = 80$$

$$x = 60.$$

Quantity of water to be added = 60 litres.

Answer: Option D

6. Originally, let the number of boys and girls in the college be $7x$ and $8x$ respectively. Their increased number is (120% of $7x$) and (110% of $8x$).

$$\left(\frac{120}{100} \times 7x\right) \text{ and } \left(\frac{110}{100} \times 8x\right)$$

$$C \text{ and } \frac{44x}{5} = 21 : 22.$$

Answer: Option C

7. Let the original salaries of Badri and Vinayak be Rs. $2x$ and Rs. $3x$ respectively.

$$\text{Then, } \left(\frac{2x+4000}{3x+4000}\right) = \frac{40}{57}$$

$$57(2x + 4000) = 40(3x + 4000)$$

$$6x = 68,000$$

$$3x = 34,000$$

$$\begin{aligned}\text{Vinayak's present salary} &= (3x + 4000) \\ &= \text{Rs.}(34000 + 4000) \\ &= \text{Rs. } 38,000\end{aligned}$$

Answer: Option D

8. $(X \times 5) = (0.75 \times 8)$

$$x = (X \times 5) = (0.75 \times 8)$$

$$\Rightarrow x = \frac{6}{5} = 1.2$$

Answer: Option B

9. Let the three parts be A, B, C.

$$\text{Then, } A : B = 2 : 3 \text{ and } B : C = 5 : 8$$

$$= \left(5 \times \frac{3}{5}\right) : \left(8 \times \frac{3}{5}\right) = 3 : \frac{24}{5}$$

$$A : B : C = 2 : 3 : \frac{24}{5}$$

$$= 10 : 15 : 24$$

$$B = 98 \times \frac{15}{49} = 30.$$

Answer: Option B

10. Given ratio = $\frac{1}{4} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9.$

$$\text{1st part} = \text{Rs.} \left(782 \times \frac{6}{23} \right) = \text{Rs. } 204$$

Answer: Option D

11. Let Arjun = 2k, Bheem = 3k and Chirag = 5k.

$$\begin{aligned}\text{Arjun's new salary} &= \left(\frac{115}{100} \right) \text{ of } 2k \\ &= \left(\frac{115}{100} \right) \times 2k \\ &= \frac{23k}{10}\end{aligned}$$

$$\begin{aligned}\text{Bheem's new salary} &= \left(\frac{110}{100} \right) \text{ of } 3k \\ &= \left(\frac{110}{100} \right) \times 3k \\ &= \frac{33k}{10}\end{aligned}$$

$$\begin{aligned}\text{Chirag's new salary} &= \left(\frac{120}{100} \right) \text{ of } 5k \\ &= \left(\frac{120}{100} \right) \times 5k = 6k\end{aligned}$$

$$\text{New ratio} = 23 : 33 : 60$$

Answer: Option C

12. Let 40% of A = $\frac{2}{3}$ B

$$\text{Then, } \left(\frac{40}{100} \right) A = \frac{2}{3} B$$

$$\frac{2A}{5} = \frac{2B}{3}$$

$$\frac{A}{B} = \frac{2}{3} \times \frac{5}{2} = \frac{5}{3}$$

$$A : B = 5 : 3.$$

Answer: Option C

- 13.** Let the fourth proportional to 5, 8, 15 be x.

$$\text{Then, } 5 : 8 :: 15 : x$$

$$5x = (8 \times 15)$$

$$\frac{8 \times 15}{5} = 24$$

Answer: Option B

- 14.** Let the numbers be 3x and 5x.

$$\text{Then, } \left(\frac{3x-9}{5x-9} \right) = \frac{12}{23}$$

$$23(3x - 9) = 12(5x - 9)$$

$$9x = 99$$

$$x = 11.$$

$$\therefore \text{The smaller number} = (3 \times 11) = 33.$$

Answer: Option B

- 15.** Let the number of 25 p, 10 p and 5 p coins be x, 2x, 3x respectively.

Then, sum of their values

$$= \text{Rs.} \left(\frac{25x}{100} + 10 \frac{2x}{100} + 5 \frac{3x}{100} \right) = \text{Rs.} \frac{60x}{100}$$

$$\therefore \frac{60x}{100} = 30$$

$$\therefore \frac{30 \times 100}{60} = 50$$

$$\text{Hence, the number of 5 p coins} = (3 \times 50)$$

$$x = 150.$$

Answer: Option C

- 16.** To cover a distance of 800 kms using a 800 cc engine, the amount of diesel required

$$= \frac{800}{600} \times 60 = 80 \text{ litres.}$$

However, the vehicle uses a 1200 cc engine and the question states that the amount of diesel required varies directly as the engine capacity.

i.e., for instance, if the capacity of engine doubles, the diesel requirement will double too.

Therefore, with a 1200 cc engine, quantity of diesel required

$$= \frac{1200}{800} \times 80 = 120 \text{ litres.}$$

Answer: Option C

- 17.** 8 times A's share = 12 times B's share = 6 times C's share.

Note that this is not the same as the ratio of their wages being 8 : 12 : 6

In this case, find out the L.C.M of 8, 12 and 6 and divide the L.C.M by each of the above numbers to get the ratio of their respective shares.

The L.C.M of 8, 12 and 6 is 24.

$$\therefore \text{the ratio } A : B : C :: \frac{24}{8} : \frac{24}{12} : \frac{24}{6}$$

$$\Rightarrow A : B : C :: 3 : 2 : 4$$

$$\text{The sum of the total wages} = 3x + 2x + 4x = 432$$

$$\Rightarrow 9x = 432 \text{ or } x = 48.$$

Hence, A who gets 3x will get $3 \times 48 = \text{Rs.144}$

Answer: Option C

- 18.** Amount of work done by 20 men
= 24 women
= 40 boys or amount of work done by 1 man
= 1.2 women
= 2 boys.

Let us therefore, find out the amount of men required, if only men were working on the task, to complete the new job under the new conditions.

And then make adjustments for the women and boys who are already employed on the task.

The man hours required to complete the new task = 4 times the man hours required to complete the old task. (As the new task is 4 times as big as the old task)

So, the new task is $20 \times 12 \times 8 \times 4$ man hours task.

Let 'n' be the number of men required to complete the new task.

Equating the man hours of task,
We get $20 \times 12 \times 8 \times 4 = n \times 5 \times 12$.
 $n = 128$.

The new task will require 128 men, if only men worked on the task to complete it.

However, the problem states that 6 women and 2 boys are working on the job.

6 women are the equivalent of $\frac{6}{1.2} = 5$ men
and 2 boys = 1 man.

i.e., the equivalent of $5 + 1 = 6$ men are already working on the project.

Therefore, 122 more men are required to work with 6 women and 2 boys to complete the task.

Answer: Option C

- 19.** Less Cogs => more turns and less time
=> less turns

| | Cogs | Time | Turns |
|----------|------|------|-------|
| A | 54 | 45 | 80 |
| B | 32 | 8 | ? |

$$\text{Number of turns required} = 80 \times \frac{54}{32} \times \frac{8}{45}$$

∴ Number of turns required is 24 times.

Answer: Option C

- 20.** Let A's income be = $4x$
A's expenses, therefore = $4x - 25$

Let B's income be = $5x$
B's expenses, therefore = $5x - 50$

We know that the ratio of their expenses =
 $5 : 6$

$$\Rightarrow 24x - 150 = 25x - 250$$

$$\Rightarrow \text{Therefore, } x = 100.$$

$$\Rightarrow \text{A's income} = 4x = 400 \text{ and B's income} \\ = 5x = 500.$$

Answer: Option A

- 21.** Let the two numbers be a and b .

$$\text{Mean proportional} = \sqrt{a \times b} = 9$$

$$\Rightarrow a \times b = 81 \quad \text{--(1)}$$

$$\text{Third Proportional} = 243$$

$$\Rightarrow \frac{a}{b} = \frac{b}{243}$$

$$\text{From (1) we have, } a = \frac{81}{b}$$

$$\Rightarrow \frac{81}{b^2} = \frac{b}{243}$$

$$\Rightarrow 19683 = b^3$$

$$\Rightarrow b = 27$$

$$\Rightarrow a = \frac{81}{b} = 3$$

The bigger of the two numbers is 27.

Answer: Option A

- 22.** Let x be the distance in meters to which a 1 kg stone can be thrown

Let the big stone weigh 9 kg and the pieces weigh 2 kg, 4 kg and 3 kg respectively.

$$\text{Distance covered by 2 kg stone} = \frac{x}{2}$$

[Inverse Proportionality]

$$\text{Distance covered by 4 kg stone} = \frac{x}{4}$$

$$\text{Distance covered by 3 kg stone} = \frac{x}{3}$$

$$\text{Sum of the distances} = \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 39$$

$$\Rightarrow \frac{13x}{12} = 39$$

$$\Rightarrow x = 36$$

Distance covered by a 9 kg stone =

$$\frac{x}{9} = \frac{36}{9} = 4 \text{ m}$$

Answer: Option B

- 23.**

$$\frac{a^2 - ab + 2b^2}{a^2 + ab + 4b^2} = \frac{1}{3}$$

$$3a^2 - 3ab + 6b^2 = a^2 + ab + 4b^2$$

$$2a^2 - 4ab + 2b^2 = 0$$

$$2(a^2 - 2ab + b^2) = 0$$

$$(a - b)^2 = 0$$

$$\Rightarrow (a - b) = 0$$

$$\Rightarrow a = b$$

$$\frac{2a+2b}{2a-2b} = \frac{4a}{a} = 4$$

Answer: Option A



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