HQASTS402_1117

SESSION – 1

1. Ans: [b]
Each letter can be printed in 26 different ways, the first digit in 9 ways and each of the other two digits in 10 ways. Hence, 26x26x9x10x10= 608400 different plates can be printed.

2. Ans: [a]
Clearly, the first person can occupy any of the 5 seats.
So, there are 5 ways in which the first person can seat himself.
Now, the second person can occupy any of the remaining 4 seats.
So, he can be seated in 4 ways. Similarly, the third can occupy a seat in 3 ways. Hence, by the fundamental principle of counting, the required number of ways
$=5 \times 4 \times 3 = 60$.

3. Ans: [c]
Between 0 to one million we can have either 1 digit numbers, 2 digit numbers, 3 digit numbers, 4 digit numbers or 5 digit numbers (0 to one million excluded) total numbers that can be formed.
(3x3x3x3x3x3)-1(excluded 0) = 729 – 1 = 728.

4. Ans: [d]
Since we need 6 digits palindrome, we need to find the 3 digits only. ___/___.
ut of these three digits unit's place will have only
4 ways (since it should be even 0 cannot be added) other 2 digits can be selected in 10 ways each. 10x10x4=400.

5. Ans: [d]
Number of squares = 8 x 8 + 7 x 7 + 6 x 6 + …… + 1 x 1.= 204
= 8 x 9 x 17/6 = 204

6. Ans: [b]
Number of ways a black square can be chosen=32 Number of ways white square can be chosen which is not in the same row or columns as a black square=24.
The total number of ways=32x24=768.

7. Ans: [b]
If only one of the boxes has a green ball, it can be in any of the 6 boxes. So, this can be achieved in 6 ways.
If two of the boxes have green balls and then there are 5 arrangement possible. i.e., the two boxes can be one of 1-2 or 2-3 or 3-4 or 4-5 or 5-6.
If 3 of the boxes have green balls, there will be 4 options in which the 3 boxes are in consecutive positions. (that is, 1–2–3 or 2–3–4 or 3–4–5 or 4–5–6)
If 4 boxes have green balls, there will be 3 options.
(that is 1–2–3–4 or 2–3–4–5 or3–4–5–6)
If 5 boxes have green balls, then there will be 2 options.
(that is, 1–2–3–4–5 or2–3–4–5–6)
If all 6 boxes have green balls, then there will be just 1 options.
Total number of options = 6 + 5 + 4 + 3 + 2 + 1 = 21

8. Ans: [c]
This is the typical Pigeon-hole principle problem. We have 128 boxes that can have 120, 121,….144 oranges. If we start filling the boxes with 120, 121,…..141 oranges, we will be able to fill 25 boxes with distinct number of oranges. The remaining boxes will have a non-distinct number of oranges. Hence, we can fill 25×5=125 boxes with the number of oranges from 120 to 144(both inclusive), i.e,. 5 boxes with same number of oranges. There are however 3 more boxes. If we start from120 oranges again, we will have 6 boxes containing 120,121 and 122 oranges each. The minimum number of boxes is 6.

9. Ans: [b]
If 'n' particular items need to be arranged in a row in such a way that none of them acquires its correct place
$$= n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.....+(-1)^n\frac{1}{n!}\right) = n!\sum_{r=0}^{n}(-1)^r\frac{1}{r!}$$
So, number of ways of derangement of 5 letters
$$= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$
$$= 120\times\left(\frac{120-120+60-20+5-1}{120}\right) = 44$$

10. Ans: [a]
Number of 5 digit numbers, that can be formed by 1, 3 5, 7 and 9 without repetition = 5 × 4 × 3 × 2 × 1= 120
So, each digit of given number occurs 24 times.
Hence, the sum of digits in each position = 24 (1 + 3 + 5 + 7 + 9) = 600
So, the sum of all numbers
= 600 (1 + 10 + 100 + 1000 + 100000)
= 6666600

11. Ans: [b]
Number of way the specified two not sitting together
$= (n-1)! - (n-2)! \times 2 = (n-2)!(n-1-2)$
$= (n-2)!(n-3)$
So, the required result = $\frac{(n-2)!(n-3)}{(n-2)!\times 2} = \frac{(n-3)}{2}$

12. Ans: [d]
Consider that there are 24 dots.
Then the number of ways of putting 3 '+' sign between 23 places = $23C_3$
From each of its combinations, on subtracting 1 from each addend we get the required result.
Example:
4 + 10 + 6 + 4 can be changed to 3 + 9 + 5 + 3 = 20
So, the no. of non negative integral solution=$^{23}C_3$ = (23 x 22 x 21)/(3 x 2 x 1) = 1771.

13. Ans: [d]

To construct 2 roads, three towns can be selected out of 4 = 4 x 3 x 2 = 24.

Now, if third road goes from the third town to the first town, a triangle is formed, and if it goes to the fourth town, a triangle is not formed.

So there are 24 ways to form a triangle and 24 ways of avoiding triangle.

14. Ans: [c]

Let us consider * represents ladies and M represents men. Then, the seating arrangements be =*M*M*M*M*M*

Number of ways the 5 men to be seated=5!

Number of ways the 3 ladies to be seated= $^6P_3$ Total number of ways=5! ×$^6P_3$ =14,400.

15. Ans: [c]

The number of ways to arrange the six members = 6!/(4!×2!) = 15.

Number of ways to arrange the four members = 4! = 24

Total number of ways=24 ×15=360.

Aliter:

Total number of possible combinations is 6! = 720.

Since there are no specific conditions there would exactly half number of ways Joey would be ahead of Frankie and exactly half number of ways where Frankie would be ahead of Joey. Therefore, in order to satisfy Frankie's requirement, the six mobsters could be arranged in 360 different ways $\left(\frac{720}{2}=360\right)$.

16. Ans: [c]

Let the number of men participating in the tournament be n.

Since every participant played two games with every other participant.

Therefore the total number of games played between men and women = 2 x $^nc_2$ = n(n – 1). And the number of games played with each woman = 2n.

But there are two women,

Hence the total number of games played with 2 women = 2 x 2n = 4n.

{n(n – 1)} – 4n = 66. n^2 – 5n – 66 = 0

(n – 11)(n + 6) = 0.

n = 11(n < 0, is not possible).

Number of participants = 11men + 2women = 13.

(The total number of games played by them is 2($^{13}C_2$) = 156.)

17. Ans: [d]

The digits which can be recognized as digits on the screen of a calculator when they are read inverted i. e., upside down are 0, 1, 2, 5, 6, 8 and 9. Since a number cannot begin with zero hence leftmost digits can never be 0 as when an 'n' digit number read upside down it will become a number of less than n digits.

Hence,

Number of digits	Total number of numbers
1	7
2	6x 6=6^2
3	6x7 x6=6^2 x7
4	6 x7 x7 x6= 6^2x 7^2
5	6 x7 x7x7 x6= 6^2 x7^3
6	6 x7x7x7x7 x6= 6^2 x7^4

Thus, the number of required numbers

= 7 + 6 + 6^2 + 6^2.7 + …… + $6^2.7^4$ 7 + $6^2(7^5–1)/(7–1)$

=7+6(7^5-1)

=6x7^5+1 = 100843.

18. Ans: [d]

Part A can be attempted in 3 ways

(i)The student does not solve the question. (ii)The student attempts the first part of the question.

(iii)The student attempts the second (i.e., alternative) part of the question.

The total number of choices in the first part = 3^5.

Hence, the required number of ways in which a student must attempt at least one question =3^5-1=242.

Similarly we can show that there are

2^4 –1 i.e., 15 choices for part B.

Hence the required number of ways = 242x15=3630.

19. Ans: [b]

Ball 2 can be distributed b/w 2 boxes (viz., box 2 and box 4) in 2 ways.

Now, the remaining 6 balls can be distributed in any of the 5 boxes in 5^6 ways.

Hence, the required number of ways =2 x 5^6 ways=31250 ways.

20. Ans: [a]

The number of ways of choosing 26 cards out of 52 cards is $^{52}C_{26}$. There are two ways in which each card can be dealt because a card can be either from a first pack or from the second pack.

Hence, the number of ways in which the card can be dealt is $^{52}C_{26}$ x2^{26}.

SESSION – 2

1. Ans: [c]

3 boys can be seated in 1st, 3rd and 5th places. So, they can be seated in 3! ways.

3 girls can be seated in 2nd, 4th and 6th places. So, they can be seated in 3! ways.

So, required number of arrangements = 3! ×3!

2. Ans: [a]

Number of ways of filling the first place = 1 (only 2)

Number of ways of filling the second place = 7

(leaving the first digit)

Number of ways of filling the third place = 6
(leaving the first 2 digits)
Number of ways of filling the fourth place = 5
(leaving the first 3 digits)
$\therefore$ Required number of ways $=1\times7\times6\times5 \ =210$

3. Ans: [c]
Number of rings = 3
The 3 rings have to be worn in 4 fingers.
$\therefore$ Required number of ways $=4P_3=\frac{4!}{1!}=4\times3\times2\times1$
$=24$

4. Ans: [b]
Total number of letters in the word 'HEXAGON' = 7
These 7 letters can be arranged themselves in $7!$ ways
$\therefore$ Required number of ways $=7!$
$=7\times6\times5\times4\times3\times2=5040$

5. Ans: [a]
Total number of flags = 3 + 2 + 2 = 7
Out of these 7 flags, 3 are red, 2 are yellow and 2 are green.
$\therefore$ No. of different signals which can be transmitted
$=\frac{7!}{3!2!2!}=\frac{7\times6\times5\times4\times3!}{3!\times4}=\frac{7\times6\times5\times4}{4} \ =7\times6\times5=210$

6. Ans: [b]
A straight line is formed with 2 points
No of available points = 8
$\therefore$ Required number of ways $={}^8C_2=\frac{8\times7}{2}=28$

7. Ans: [b]
10 pearls of one colour can be arranged in $\frac{(10-1)!}{2}$ ways
$=\frac{9!}{2}$
The number of arrangements of 10 pearls of the other colour in the 10 places between the first 10 pearls = $10!$
$\therefore$ The required number of ways $=\frac{1}{2}\times9!\times10!=5(9!)^2$

8. Ans: [b]
Total number of balls in the box = 2 + 3 + 4 = 9
Total number of selections of 3 balls out of 9 balls = $9C_3$
Number of selections in which no any green ball is selected = $6C_3$
Hence the required number of selections = $9C_3 - 6C_3$ = 64

9. Ans: [b]
Let us consider the possibilities as number with one six or number with no six or 2 sixes.
Number with 2 sixes.
The digits are 0, 1, 2, 3, 4, 5, 6

Number ending in zero $=\frac{{}^5C_1\times3!}{2!}=\frac{5\times3\times2}{2}=15$
(a) Numbers ending in 5 and starting with 6
$=5C_1\times2!=10$
(b) Numbers ending in 5 and not starting with 6
$=4C_1=4$
Numbers with one six or no sixes
(a) ending in 0 $={}^6C_3\times3!=\frac{6\times5\times4\times3\times2}{1\times2\times3}=120$
(b) ending in 5 $={}^5C_1\times{}^5C_2\times2!=5\times\frac{5\times4}{1\times2}\times2=100$
$\therefore$ Required number of ways $=15+10+4+120+100$
$=249$
$\therefore$ There are a total of 249 numbers.

10. Ans: [b]
The word 'MANAGEMENT' contains 10 letters,
2M, 2A, 2E, 2N, 1G and 1T
The new words should end with G
$\therefore$ Only 9 letters to be considered.
$\therefore$ Required number of ways $=\frac{9!}{2!2!2!2!}=\frac{9!}{(2!)^4}$

11. Ans: [a]
number of even places = 4
number of even digits = 5
number of odd places = 5
number of odd digits = 4
Odd digits can be arranged in $\frac{4!}{2!\times2!}$ = 6 ways
Even digits can be arranged in $\frac{5!}{2!\times3!}$ = 10 ways
Hence the required number of ways = 6 x 10 = 60 ways.

12. Ans: [c]

	B_1		B_2		B_3	
The possibilities are	3		1		1	
	$^5C_3$	x	$^2C_1$	x	$^1C_1$ x 3	= 60
	2		2		1	
	$^5C_2$	x	$^3C_2$	x	$^1C_1$ x 3	= 90

Total no of ways = 60 + 90 = 150.

13. Ans: [d]
Total number of choices $=4^4$
No of correct choice = 1
$\therefore$ No of wrong choices $=4^4-1$

14. Ans: [d]
There are 8 males and 8 females 4 persons have to be selected with atleast one male.
The possibilities are (i) 1 male 3 female (ii) 2 male 2 female (iii) 3 male 1 female (iv) 4 male

No of ways of selecting 1 male and 3 female $= {}^8C_1 \times {}^8C_3$

$$= 8 \times \frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 448$$

No. of ways of selecting 2 male and 2 female

$$= {}^8C_2 \times {}^8C_2 = \frac{8 \times 7}{1 \times 2} \times \frac{8 \times 7}{1 \times 2} = 784$$

No. of ways of selecting 3 male and 1 female

$$= {}^8C_3 \times {}^8C_1 = 448$$

No. of ways of selecting 4 male $= {}^8C_4 = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$

$\therefore$ Required number of ways $= 448 + 784 + 448 + 70$

$$= 1750$$

15. Ans: [b]

Number of ways of arranging the 7 boys around the table $= 6!$

In between the boys, there are 7 places

$\therefore$ No of ways of arranging the 7 girls in the 7 places $= 7!$

$\therefore$ Required number of ways $= 6! \times 7!$

16. Ans: [a]

When the books have to be made into four equal bundles, the 4 groups are not distinct. When the 4 groups do not have a distinct identify, the number of ways in which the books can be divided into 4 equal groups of 3 each

$$= \frac{12!}{4!(3!)^4}$$

17. Ans: [a]

Let x, y, z be the number of balls received by the three persons, then x > 5, y>5, z>5 and x + y + z = 21

Let u > 0, v > 0, w > 0 then x + y + z = 21

u + 5 + v + 5 + w + 5 = 21

u + v + w = 6

Total number of solutions $= {}^{(6+3-1)}C_{3-1} = {}^8C_2 = 28$.

18. Ans: [c]

A and B occupy 1st and 9th places, 2nd and 10th places, 3rd and 11th places, 4th and 12th places and so on.

Number of ways of doing this = 18

Number of arrangements of A and B = 2

The other 24 alphabets can be arranged themselves in $24!$ ways

Hence the required number of ways $= 18 \times 2 \times 24!$

$$= 36 \times 24!$$

19. Ans: [b]

For drawing a circle 3 non collinear points are needed.

Total number of points = 10

Number of collinear points = 7

$\therefore$ Required number of ways $= {}^3C_3 + {}^3C_2 \times {}^7C_1 + {}^3C_1 \times {}^7C_2$

$$= 1 + 3 \times 7 + 3 \times \frac{7 \times 6}{1 \times 2} = 1 + 21 + 63 = 85.$$

$\therefore$ Number of circles drawn = 85

20. Ans: [b]

Total number of people 8 (4 men and 4 women)

A committee of 5 is to be formed with maximum 2 women and 1 woman holding one of the 2 posts of the committee

The number of ways = Number of committees with 4 men and 1 woman + Number of committees with 3 men and 2 women – Number of committees with 3 men and 2 women where both the women are occupying the 2 posts

$$= \frac{{}^4C_4 \times {}^4C_1 \times 5!}{3!} + \frac{\left({}^4C_3 \times {}^4C_2 \times 5!\right)}{3!} - {}^4C_3 \times {}^4C_2 \times {}^2C_2 \times 2!$$

$$= \frac{1 \times 4 \times 120}{6} + \frac{4 \times 6 \times 120}{6} - \frac{4 \times 6 \times 1 \times 2}{1}$$

$$= 80 + 480 - 48$$

$$= 512$$

$\therefore$ Required number of ways = 512

21. Ans: [a]

Select any 3 places for A, B and C. They need no arrangement among themselves as A would always come before B and B would always come before C.

The number of ways this is done $= {}^8C_3$

The remaining 5 people have to be arranged in 5 places in $5!$ ways.

$\therefore$ Required number of ways $= {}^8C_3 \times 5!$

$$= \frac{8!}{5!\,3!} \times 5! = \frac{8!}{3!}$$

22. Ans: [c]

No of ways of choosing one person between the 2 brothers $= 10\,P_1 = 10$ ways

The remaining 9 persons can be arranged in $9!$ ways

The 2 brothers can be arranged in $2!$ ways

$\therefore$ Required number of ways $= 10 \times 9! \times 2!$

$$= 10!2!$$

SESSION – 3

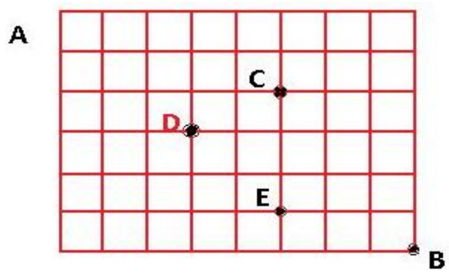
PERMUTATION AND COMBINATION - CASE STUDY

CASE STUDY DESCRIPTION

- **In the city of Honolulu, all roads are either in**
- **North-South direction or East-West direction**
- **Forming a grid like structure**

Route Description

- Distance between two adjacent roads in both North-South and East-West directions are equidistant
- It implies that each cell shown in the figure is a square
- Length of each side of square is 500 m.



Questions

1. **What is the minimum distance covered to travel from A to B?**

   - The minimum distance to be covered from A to B will cover all together 14 sides (8 horizontal and 6 vertical) of different squares.
   - Length of each side of square is 500 m
   - So the distance = 14 × 500 = 7000 m

2. **How many ways are there to get from A to B by covering shortest possible distance?**

   - Going from A to B, there are 14 total sides of different squares to be covered which can be covered either in $^{14}C_6$ or $^{14}C_8$ ways = 3003

3. **If a delivery van has to start from A, deliver a parcel to C and reach B, covering the minimum possible distance along the way in how many ways can this be managed?**

   - As the previous problem, covers the minimum distance
   - The number of ways to go from A to C = $^6C_4$
   - The number of ways to go from C to B = $^8C_4$
   - Total number of ways = 6C4 × 8C4 = 15 × 70 =1050

4. **If there is an underground subway joining D and E, and a man walks from A to B via road, takes the subway to E and then walks to B, how many minimum length routes can he take?**

   - Number of ways of reaching D from A covering the minimum distance = 6!/(3! × 3!) = 20
   - Number of ways of reaching E from D via subway = 1
   - Number of ways of reaching B from E covering the minimum distance = 5!/(3! × 2!) = 10
   - Therefore number of minimum length routes= 20 × 1 × 10 = 200

5. **How many shortest path's routes can a person take if he desires to go from A to B visiting both C and D along the way?**

   - The shortest route can be A-C-D-B OR A-D-C-B.
   - Number of shortest paths along A-C-D-B = {6!/(4!2!)} × 2 × {8!/(5!3!)} = 15 × 2 × 56 = 1680
   - Number of shortest paths along A-D-C-B = {6!/(3!3!)} × 2 × {8!/(4!4!)} = 20 × 2 × 70 = 2800
   - Therefore minimum number of shortest paths = 1680

6. **How many shortest paths/routes are possible between A and B if point C is to be avoided?**

   - From previous answers, we can see that total number of ways to
   - Reach B from A taking shortest path = 3003
   - Reach B from A via C taking shortest path = 1050
   - Therefore, total number of ways to reach B from A without passing through C = 3003 – 1050 = 1953

SESSION – 4

1. Ans: [a]

$$AB^2 + BC^2 = 26^2 \quad \left(\text{Diagonal of a rectangle} = \sqrt{\ell^2 + b^2}\right)$$

$$2(AB + BC) = 68 \quad \left(\text{Perimeter} = 2(\ell b + bh)\right)$$

$$\Rightarrow AB + BC = 34$$

$$(AB + BC)^2 = AB^2 + BC^2 + 2AB \times AC$$

$$\Rightarrow 34^2 = 26^2 + 2AB \times AC$$

$$\Rightarrow 2\,AB \times AC = 34^2 - 26^2 = (34+26)(34-26) = 480$$

$$\Rightarrow AB \times AC = 240$$

Area of the rectangle $= AB \times AC$

$= 240$ sq.cm

2. Ans: [b]

Radius of the hemisphere = 3.5 cm

$\Rightarrow$ Radius of the base of the cone $= 3.5$ cm

Let h be the height of the cone.

$\therefore h + 3.5 = 15.5 \quad \Rightarrow \quad h = 12$ cm

Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 + \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 154 + \frac{539}{6}$$

$$= \frac{924 + 539}{6} = \frac{1463}{6} = 243.82 \text{ cubic cm}$$

Slant height of the cone $= \ell = \sqrt{h^2 + r^2}$

$\Rightarrow$ Slant height $= \sqrt{144 + 12.25} = \sqrt{156.25} = 12.5\,\text{cm}$

Total surface of the toy = curved surface of the cone + curved surface of the sphere

$= \pi r\ell + 2\pi r^2$

$\Rightarrow$ Total surface $= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

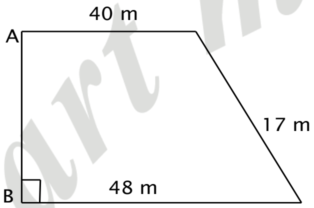
$= \frac{275}{2} + 77 = 137.5 + 77 \quad = 214.5\,\text{cm}^2$

3. Ans: [b]

Length of the fence $ABCD = 120\text{ m}$

$\Rightarrow AB + BC + CD + DA = 120\text{ m}$

$\Rightarrow AB + 48 + 17 + 40 = 120\text{ m}$



$\Rightarrow AB = 120 - 105$

$\Rightarrow AB = 15\text{ m}$

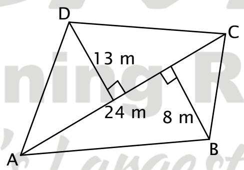
Area of the trapezium $= \frac{1}{2}(BC + AD) \times AB$

$= \frac{1}{2}(48 + 40) \times 15$

$= \frac{1}{2} \times 88 \times 15 = 660\text{ m}^2$

4. Ans: [c]

Area of ABCD = Area of $\triangle ABC$ + Area of $\triangle ADC$



$= \frac{1}{2} \times 24 \times 8 + \frac{1}{2} \times 24 \times 13$

$= 96\text{ sq.m} + 156\text{ sq.m} = 252\text{ sq.m}$

5. Ans: [c]

Let the edge of the cube be x cm.

$\therefore$ Lateral surface area $= 4x^2$

New lateral surface area $= 4x^2 \times \frac{144}{100}$

$= 4x^2 \times 1.44$

$= 4 \times (1.2\,x)^2$

Volume of the cube $= x^3$

Volume of the new cube $= (1.2\,x)^3 = 1.728\,x^3$

$\therefore$ Change in volume $= 0.728 = 72.8\%$

6. Ans: [b]

Volume of the metallic sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3^3 = 36\pi$

Let 'h' be the height of the cone

$\therefore \frac{1}{3}\pi r^2 h = 36\pi$

$\Rightarrow \frac{1}{3}\pi \times 12^2 \times h = 36\pi$

$\Rightarrow h = \frac{3 \times 36\pi}{144\pi} = 3\text{ cm}$

7. Ans: [c]

Let the edge of the cube be 'a' units.

$\therefore$ Surface area of the cube $= 6a^2 = 1734\text{ cm}^2$

$\Rightarrow a^2 = 289 \Rightarrow a = 17\text{ cm}$

Volume of the cube $= a^3 = (17)^3 = 4913\text{ cm}^3$

8. Ans: [a]

Curved surface area of the tent $= \pi r\ell$

$= \frac{22}{7} \times 7 \times \sqrt{24^2 + 7^2}$

$= \left(\frac{22}{7} \times 7 \times 25\right)\text{m}^2 = 550\text{ m}^2$

Width of the Canvas $= 1.25\text{ m}$

$\therefore$ Length of the Canvas $= \frac{\text{Area}}{\text{width}} = \frac{550}{1.25} = 440\text{ m}$

9. Ans: [a]

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 5^2 \times 15 = \frac{8250}{7}$

$= 1178.59\text{ cm}^3$

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 6$

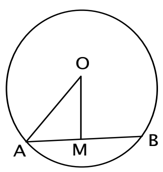
$= \frac{1100}{7} = 157.14\text{ cm}^3$

$\therefore$ Volume of the remaining solid

$= (1178.59 - 157.14)\text{ cm}^3$

$= 1021.45\text{ cm}^3$

10. Ans: [d]



Radius of the circle = OA = 10 cm

Length of the chord AB = 12 cm

OM is perpendicular from O to AB

$\Rightarrow AM = \frac{1}{2} AB = 6 \text{ cm}$

In ΔOAM, $OA^2 = OM^2 + AM^2$

$\Rightarrow 10^2 = OM^2 + 6^2$

$\Rightarrow OM^2 = 10^2 - 6^2 = 100 - 36 = 64$

$= OM = 8 \text{ cm}$

11. Ans: [a]

In $\Delta ABC, \frac{AP}{PB} = \frac{2}{4} = \frac{1}{2}$

$\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$

$\Rightarrow \frac{AP}{AP+PB} = \frac{AQ}{AQ+QC}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

In ΔAPQ and ΔABC,

$\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common

$\therefore \Delta APQ \sim \Delta ABC$ [SAS Criterion]

$\Rightarrow \frac{PQ}{BC} = \frac{AP}{AB} = \frac{1}{3}$

$\Rightarrow 3PQ = BC$

$\Rightarrow PQ = \frac{1}{3} BC = \frac{1}{3} \times 12 \text{ cm} = 4 \text{ cm}$

12. Ans: [b]

The sum of the angles of a polygon of 'n' sides is (2n – 4) right angles.

$\therefore$ The sum of the angles of a pentagon is (2 × 5 – 4) right angles = 6 × 90° = 540

One angle = 140°

$\therefore$ The sum of the remaining angles = 540° – 140°

= 400°

The angles are in the ratio 1:2:3:4.

The size of the greatest angle $= \frac{4}{10} \times 400° = 160°$

13. Ans: [c]

AB, CD are two parallel chords of a circle.

AB = 16 cm, CD = 30 cm

OM and ON are perpendiculars to AB and CD.

$AM = \frac{1}{2} AB = 8 \text{ cm}$

Let r be the radius of the circle.

Let OM = x

MN = 23

$\therefore ON = 23 - x$

In ΔOMA,

$OA^2 = OM^2 + MO^2$

$\Rightarrow r^2 = x^2 + 64$ ... (i)

In ΔONC,

$OC^2 = ON^2 + NC^2$

$\Rightarrow r^2 = (23-x)^2 + 15^2 \Rightarrow r^2 = x^2 - 46x + 754$ ... (ii)

Equating (i) and (ii) we get,

$x^2 + 64 = x^2 - 46x + 754$

$\Rightarrow 46x = 690$

$\Rightarrow x = \frac{690}{46} = 15 \text{ cm}$

$r^2 = x^2 + 64$

$\Rightarrow r^2 = 15^2 + 64$

$\Rightarrow r^2 = 225 + 64 = 289$

$\Rightarrow r = 17 \text{ cm}$

14. Ans: [d]

Since ABCD is a cyclic quadrilateral,

$\angle B + \angle D = 180°$

$\angle D = 130°$

$\Rightarrow \angle B = 50°$

$\angle ACB = 90°$ [Angle in a semicircle]

In ΔBAC,

$\angle BAC + \angle ABC + \angle BCA = 180°$

[sum of the angles of a triangle]

$\Rightarrow \angle BAC + 50° + 90° = 180°$

$\Rightarrow \angle BAC = 180° - (50° + 90°)$

$= 180° - 140° = 40°$

15. Ans: [b]

$\angle BAD = \frac{1}{2} \angle BOD$ [Angle at the centre of a circle is equal to twice the angle on the circle]

$\Rightarrow \angle BAD = \frac{1}{2} \times 160° = 80°$

$\angle BCD = 80° - \angle BAD$

[Opposite angles of a cyclic quadrilateral]

$= 180° - 80°$

$= 100°$

$\angle BPD = \angle BCD$ [Angles in the same segment]

$\Rightarrow \angle BPD = 100°$

16. Ans: [c]

In ΔOPQ,

$\angle POQ + \angle OQP + \angle OPQ = 180°$ [Angle sum property]

$\Rightarrow 110° + 2\angle OPQ = 180°$

$\Rightarrow 2\angle OPQ = 70°$

$\Rightarrow \angle OPQ = 35°$

In quadrilateral OPTQ,

$\angle O + \angle P + \angle T + \angle Q = 360°$

$\Rightarrow 110° + 90° + \angle T + 90° = 360°$

$\Rightarrow \angle T = 360° - 290°$

$\Rightarrow \angle PTQ = 70°$

17. Ans: [c]

Let $\angle BAC = y$

$x + y + 90° = 180° \qquad \Rightarrow x + y = 90°$

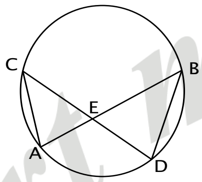
If ℓ_1 and ℓ_2 are parallel,

$78° = y + 35° \qquad \Rightarrow y = 43°$

$\therefore x = 90° - y = 90° - 43° = 47°$

18. Ans: [b]

AB and CD, the chords of the circle intersect at O.



In ΔAEC and ΔDEB,

$\angle AEC = \angle DEB$ (Vertically opposite angles)

$\angle ACE = \angle DBE$ (Angles in the same segment of the circle are equal)

$\angle EAC = \angle EDB$ (Angles in the same segment)

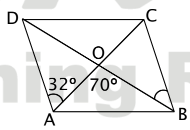
$\therefore \Delta AEC \sim \Delta DEB$ (A.A.A. similarity criteria)

$\Rightarrow \frac{AE}{DE} = \frac{EC}{EB}$

(Corresponding angles of similar triangles are proportional)

$\Rightarrow \frac{3}{DE} = \frac{5}{4} \Rightarrow DE = \frac{4 \times 3}{5} = 2.4\text{ cm}$

19. Ans: [c]



AD || BC, AC is a transversal

$\therefore \angle BCO = \angle DAO$ [Interior alternate angles]

$\Rightarrow \angle BCO = 32°$

$\angle BOC = 180° - 70° = 110°$ (linear pair).

$\angle DBC = 180° - (\angle BOC + \angle BCO)$

$= 180° - (110° + 32°) = 180° - 142° = 38°$

20. Ans: [a]

Let n be the number of sides

$\therefore$ sum of the interior angles = (2n – 4) × 90°

sum of the exterior angles = 180°

Here (2n – 4) × 90° = 3 × 180°

$\Rightarrow 2n - 4 = 6$

$\Rightarrow 2n = 10$

$\Rightarrow n = 5$

The number of sides = 5

SESSION – 5

1. Ans: [a]

$\pi r^2 = 154 \times 10000$ sq.m

$\Rightarrow r^2 = \frac{154 \times 10000}{22} \times 7 = r^2 = 490000$

$= r = 700$ m

Perimeter of the garden

$= 2\pi r = 2 \times \frac{22}{7} \times 700\text{ m} = 4400\text{ m}$

$\therefore$ The required length

$= 25 \times 4400 = 110000\text{ m} = 110\text{ km}$

2. Ans: [d]

Volume of the sphere $= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)\text{cm}^3$

Volume of the cone $= \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)\text{cm}^3$

$\therefore$ Volume of wood wasted

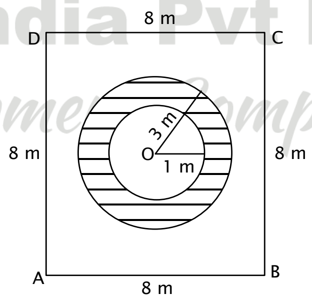
$= \frac{4}{3}\pi \times 9 \times 9 \times 9 - \frac{1}{3}\pi \times 9 \times 9 \times 9\text{ cm}^3$

$= \pi \times 9 \times 9 \times 9\text{ cm}^3$

$\therefore$ Required percentage

$= \frac{\pi \times 9 \times 9 \times 9}{\frac{4}{3}\pi \times 9 \times 9 \times 9} \times 100\% = \frac{3}{4} \times 100\% = 75\%$

3. Ans: [b]



The radius of the pond is 1m

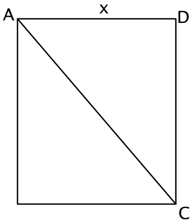
The length of the rope = 3m

$\therefore$ Required area = Area of the circle with centre O and radius 3m – Area of the circle with centre O and radius 1m

$= \pi(3^2) - \pi(1^2) = 9\pi - \pi = 8\pi$ sq. m.

4. Ans: [c]

Let the side of the square be 'x' m.



Then $AB + BC = x + x = 2x$ m

$AC = \sqrt{x^2 + x^2} = \sqrt{2}x\ m = 1.41x\ m$

Saving on $2x\ m = 0.59x\ m$

$\therefore$ Saving percent

$$= \frac{0.59x}{2x} \times 100\% = \frac{59}{2}\% = 29.5\% \text{ approximately}$$

5. Ans: [b]

Let 'r' be the radius of the hemisphere.

$\therefore$ Volume $= \frac{2}{3}\pi r^3$

The biggest sphere inscribed will have diameter 'r'.

$\therefore$ Radius $= \frac{1}{2}$

Volume of the sphere $= \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{1}{6}\pi r^3$

$\therefore$ Ratio of volume of hemisphere to volume of sphere

$$= \frac{2}{3}\pi r^3 : \frac{1}{6}\pi r^3 = 12 : 3 = 4 : 1$$

6. Ans: [d]

Let the back wheel make n revolutions

At this time the front wheel would have made n + 5 revolutions

The distance covered by both the wheels is the same

$\therefore n \times 36 = (n + 5) \times 30$

$\Rightarrow 36\,n = 30\,n + 150$

$\Rightarrow 6n = 150$

$\Rightarrow n = 25$

$\therefore$ Distance covered $= 25 \times 36 = 900$ ft

7. Ans: [c]

The sides of the triangle happen to be a Pythagorean triplet.

$\therefore$ The triangle is a right triangle.

For a right triangle,

The measure of in radius $= \dfrac{\text{product of the } \perp^{r} \text{ sides}}{\text{perimeter of the triangles}}$

$$= \frac{72 \times 21}{72 + 21 + 75} = \frac{1512}{168}\ 9\ \text{cm}$$

8. Ans: [a]

Let the radius of the base of the cylinder be R and the height be H.

$\therefore \frac{H}{2R} = \frac{1}{2} = H = R$

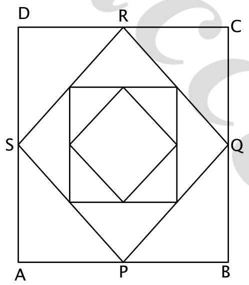
Area of its surface $= 2\pi RH = 2\pi R^2$

Sum of the areas of its ends $= 2\pi R^2$

$\therefore \dfrac{\text{Area of its surface}}{\text{Sum of the areas of its ends}} = \dfrac{2\pi R^2}{2\pi R^2} = \dfrac{1}{1}$

$\therefore$ The required ratio is 1:1.

9. Ans: [c]



The side of the first square = 16 cm

The side of the 2nd square $= \sqrt{8^2 + 8^2} = 8\sqrt{2}$ cm

The side of the 3rd square $= \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = 8$ cm

The side of the 4th square $= \sqrt{4^2 + 4^2} = 4\sqrt{2}$ cm

and so on.

$\therefore$ Sum of the areas of all the squares

$= (16)^2 + (8\sqrt{2})^2 + 8^2 + (4\sqrt{2})^2 + \ldots$ infinity

This sum is an infinite Geometric Series with first term 16^2 and common ratio $\frac{1}{2}$.

$\therefore$ Required sum $= \dfrac{16^2}{1 - \frac{1}{2}} = 256 \times 2 = 512$ sq.cm.

10. Ans: [d]

Radius of the cylinder $= \frac{1.5}{2}\ m = 0.75\ m$

Height of the cylinder = 1 m

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \pi \times \left(\frac{3}{4}\right)^2 \times 1 = \frac{9}{16}\pi$$

Let r be the radius of the sphere.

$\therefore \frac{4}{3}\pi r^3 = \frac{9}{16}\pi \Rightarrow r^3 = \frac{27}{64} \Rightarrow r = \frac{3}{4} = 0.75$

$\therefore$ Diameter of the sphere $= 2 \times 0.75 = 1.5$ m

11. Ans: [d]

Area of triangle $= r \times s$

Where r is the inradius of the triangle and s, the semiperimeter.

$\therefore$ Area $= 2.5 \times \frac{28}{2} = 35\ \text{cm}^2$

12. Ans: [d]

The quantity of milk in the cylinder $= \frac{22}{7} \times 14 \times 14 \times 20$ mℓ

The quantity of milk in 1 bottle $= 770$ mℓ

$\therefore$ Required number of bottles $= \frac{22}{7} \times \frac{14 \times 14 \times 20}{770} = 16$

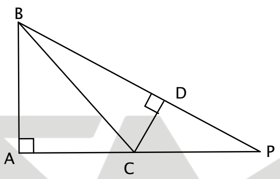
13. Ans: [c]

Sin A/a = SinB/b = Sinc/c

a + b = 90 cm. Cosine rule gives $\cos 60° = \frac{a^2 + 80^2 - b^2}{2 \times a \times 80}$

Solving a = 17 and b = 73

14. Ans: [c]



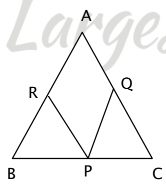
Δ CDP ~ Δ ABP

as $\angle$CDP = $\angle$BAP

$\angle$DPC = $\angle$BPA

$\therefore$ DP/ AP = CP/ BP $\Rightarrow$ DP × BP / AP × PC = 1

15. Ans: [c]



ABC is a Δ and P, Q, R are the midpoints of sides BC, CA and AB respectively.

$\therefore$ PQ || AB

And PQ = $\frac{1}{2}AB = \frac{1}{2}(30) = 15cm$

Similarly, RP || AC

and RP = $\frac{1}{2}$AC = $\frac{1}{2}(21) = 10.5\ cm$

$\therefore$ Perimeter of ARPQ = (AR + RP +PQ + QA) cm.

= (15.0 + 10.5 + 15.0 + 10.5) cm. = 51 cm

16. Ans: [c]

$AC = BC \Rightarrow \angle CAB = \angle ABC$ [base angles of an isosceles triangle]

In ΔABD, $\angle ADB + \angle ABD + \angle BAD = 180°$ [Angle sum properly)

$\Rightarrow 91° + 47° + \angle BAD = 180°$

$\Rightarrow \angle BAD = 180° - (91° + 47°) = 42°$

17. Ans: [d]

Let R_1 be the original radii and R_2 the new radii.

$2\pi R_1 = 4\pi,\ 2\pi R_2 = 8\pi$

$\Rightarrow R_1 = 2,\ R_2 = 4$

Original area $= \pi R_1^2 = 4\pi$

New area $= \pi R_2^2 = 16\pi$

$\therefore$ The new area quadruples.

18. Ans: [a]

Let the side of the cube be 'x' cm.

The diameter of each sphere $= \frac{x}{3}$ cm

$\therefore$ Sum of the volumes of the spheres $= 27 \times \frac{4\pi}{3} \times \left(\frac{x}{6}\right)^3 \text{cm}^3$

Volume of the cube $= x^3$ cu. cm. $= \frac{\pi x^3}{6}$ cu. cm.

$\therefore$ The required part $= \frac{\frac{\pi}{6}x^3}{x^3} = \frac{\pi}{6}$

19. Ans: [b]

The sum of the interior angles of a polygon $= \frac{2n-4}{n} \times 90°$

$\Rightarrow$ An interior angle of an octagon $= \frac{16-4}{8} \times 90° = 135°$

An exterior angle of an octogon $= \frac{4 \times 90}{8} = 45°$

$\therefore$ Required ratio $= 135° : 45° = 3 : 1$

20. Ans: [d]

Area of plot ABCD = Area of ΔADE + Area of ΔAFB + Area of BCEF

$= \frac{1}{2} \times 50 \times 120 + \frac{1}{2} \times 40 \times 30 + 40 \times 90$

$= (3000 + 600 + 3600)$ sq.m $= 7200$ sq. m.

SESSION – 6

1. Ans: [c]

Probability of selecting the first bag = $\frac{1}{2}$

Probability of selecting a white ball from the first bag = $\frac{4}{7}$

Probability of selecting the second bag = $\frac{1}{2}$

Probability of selecting a white ball from the second bag = $\frac{3}{7}$

∴ Required Probability = $\frac{1}{2} * \frac{4}{7} + \frac{1}{2} * \frac{3}{7} = \frac{1}{2}$

2. Ans: [c]

A leap year has 366 days (52 weeks and 2 days)
The extra 2 days can be
(i) Sunday and Monday or
(ii) Monday and Tuesday or
(iii) **Tuesday and Wednesday or**
(iv) **Wednesday and Thursday or**
(v) Thursday and Friday or
(vi) Friday and Saturday or
(vii) Saturday and Sunday.
The favourable case is 53 Wednesdays and 52 Mondays.
So there are two possibilities i.e. (iii) and (iv).

∴ Required probability = $\frac{2}{7}$

3. Ans: [b]

Given that events A and B are independent,
$P(A \cap B) = P(A) \times P(B)$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A) \times P(B)$
$0.45 = 0.15 + P(B) - 0.15 \times P(B)$
$0.3 = 0.85 \times P(B)$
∴ $P(B) = \frac{30}{85} = \frac{6}{17}$

4. Ans: [d]

Case 1: One is spade king and other spade card.

Probability = $\frac{1C_1 * 12C_1}{52C_2}$

Case 2: One is non-spade king and other spade card

Probability = $\frac{3C_1 * 13C_1}{52C_2}$

∴ Required Probability = $\frac{1C_1 * 12C_1}{52C_2} + \frac{3C_1 * 13C_1}{52C_2} = \frac{1}{26}$

5. Ans: [c]

A number divisible by 4 and formed using the digits 2, 4, 6 and 8 should have the last two digits 24 or 28 or 48 or 64 or 68 or 84 and thus can be formed in six ways.

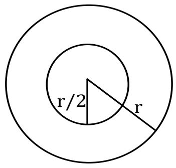
In each of these cases, the four digit number can be formed using the remaining 2 digits in 2! = 2 ways. So the number divisible by 4 can be formed in 12 ways.

Total four-digit numbers that can be formed using the digits 2, 4, 6 and 8 without repetition and any condition = 4! = 24.

Required probability = $\frac{1}{2}$

6. Ans: [b]

Given a circle of radius r, take a smaller circle with the same centre and radius $\frac{r}{2}$ is shown.



If we choose a point inside the smaller circle, it will be closer to the centre rather than to the circumference.

∴ Required probability

$\frac{\textit{area of the smaller circle}}{\textit{area of the bigger circle}} = \frac{\pi (\frac{r}{2})^2}{\pi r^2} = \frac{1}{4}$

7. Ans: [c]

Eight persons are to speak one after the other. The total number of arrangements possible is 8!

Favourable case is A speaks before B, B speaks before C and C speaks before D.

So the number of arrangements possible to our favourable case is $\frac{8!}{4!}$

∴ Required Probability = $\frac{1}{4!} = \frac{1}{24}$

8. Ans: [a]

The total number of outcomes in 4 throws of a die = 6^4

To get a total of 10, there are 2 possibilities.

$3 + 3 + 3 + 1 = \frac{4!}{3!} = 4$ ways

$3 + 3 + 2 + 2 = \frac{4!}{2!2!} = 6$ ways.

Required Probability = $\frac{10}{1296} = \frac{5}{648}$

9. Ans: [b]

First a ball is transferred from Bag A to Bag B, then a ball from B to A. And finally a ball drawn from Bag A and that is red.

So the possibilities are RRR, RBR, BRR and BBR

Required Probability = $\frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} + \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} + \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} + \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} = \frac{640}{1100} = \frac{32}{55}$

10. Ans: [b]

The number of ways that ten balls can be arranged is 10!.

Favourable case is no two black balls should be placed adjacent.

That can be done in $6! \times 7P_4$ ways.

∴ Required Probability = $\frac{6! \times 7P_4}{10!} = \frac{1}{6}$

11. Ans: [b]

Let the number on the ball picked first = a, second = b, third = c and fourth = d.

The four numbers a, b, c and d are distinct.

Four distinct balls can be picked in (10 × 9 × 8 x 7) ways

Thus the number of ways in which a > b > c > d $= \frac{1}{24} \times 10 \times 9 \times 8 \times 7 = 210$

Required Probability $= \frac{210}{10^4} = \frac{21}{10^3}$

12. Ans: [c]

Let P(A) be the probability of throwing total 4 and P(B) the probability of throwing total 6.

$\therefore P(A) = \frac{3}{36} = \frac{1}{12}, P(B) = \frac{5}{36}$

$P(A \text{ or } B) = \frac{8}{36} = \frac{2}{9}$

$P(\text{neither A nor B}) = \frac{7}{9}$

$\therefore P(4 \text{ before } 6) = \frac{1}{12} + \frac{7}{9}.\frac{1}{12} + \frac{7}{9}.\frac{7}{9}.\frac{1}{12} + ... = \frac{1}{12}\left[\frac{1}{1-(7/9)}\right] = \frac{3}{8}$

13. Ans: [a]

x	4m + 1	4m + 3	4m + 2	4m
y	4n + 3	4n + 1	4n	4n + 2
last digit of $3^x + 3^y$	0	0	0	0
Number of possibilities	25 × 25	25 × 25	25 × 25	25 × 25

If a number ends in 0 then the number must be divisible by 5. Also $3^x + 3^y$ will never end with 5.

Required Probability $= \frac{625 \times 4}{100^2} = \frac{1}{4}$

14. Ans: [c]

The number of ways that three persons can be arranged in 7 seats is $7P_3$ ways.

Let the seat numbers be 1, 2, 3, 4, 5, 6 and 7. Given that 4 should be always occupied and 3 and 5 should be always empty.

So one person would be in 1 or 2, one will be in 4 and the one will be in 6 or 7 and there are totally 4 different ways. In each possibility, 3 can be arranged in 6 ways.

∴ Total number of possible ways = 24.

Required Probability $= \frac{24}{7 * 6 * 5} = \frac{4}{35}$

15. Ans: [a]

There are totally 90 numbers between 10 and 99.

The product of digits should be 12, so the possibilities are 26, 34, 43 and 62.

The probability of selecting three numbers with replacement and the product of the digits not 12 $= \frac{86}{90}\frac{86}{90}\frac{86}{90} = \frac{43}{45} * \frac{43}{45} * \frac{43}{45}$

$\therefore$ Required Probability $= 1 - \left(\frac{43}{45}\right)^3$

SESSION – 7

1. Ans: [b]

First we calculate the probability of selecting 4 non paired sock.

The first sock can be chosen in 20 ways and the probability is 1.

The probability of selecting the next sock is $\frac{18}{19}$ as we cannot take the paired sock. Similarly for the next sock it is $\frac{16}{18}$ and next is $\frac{14}{17}$. Thus the probability that non paired sock is taken is $\frac{18 * 16 * 14}{19 * 18 * 17} = \frac{224}{323}$

Required Probability $= 1 - \frac{224}{323} = \frac{99}{323}$

2. Ans: [c]

Let two digit number that A thought be 'xy', where x and y are single digit numbers.

Therefore, 10x + y = p(x + y) + 3, where p is a natural number

Also x + y > 3

Possible values of x and y for which p is a natural number are:

(x = 1, y = 5), (x = 2, y = 3), (x = 3, y = 1, 3, 5, 9), (x = 4, y = 7), (x = 5, y = 1, 2, 9), (x = 7, y = 3, 5, 8) and (x = 9, y = 4)

There are 14 such two digit numbers that give a remainder of 3 when divided by the sum of the digits.

Probability that B thought of the same number as A $= \frac{1}{14}$

3. Ans: [b]

The total number of possibilities = 5! = 120 ways

Favourable case is only one ball occupies the place corresponding to its number.

If first ball placed in first box, then all the other balls should be wrongly placed.

And that can be done in $4!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}) = 9$ ways.

Required Probability $= \frac{9 * 5}{120} = \frac{3}{8}$

4. Ans: [b]

Two persons out of of 7 can be selected in $7C_2$ ways. These two persons born on the same day of the week and there are 7 possibilities (Sun – Sat).

The remaining 5 persons have possibilities to have birthday on remaining 6 days and the total possibility is $6P_5$.

$\therefore$ Required Probability $= \frac{7C_2 * 7 * 6P_5}{7^7} = \frac{2160}{7^5}$

5. Ans: [d]

Probability of heads showing on 50 coins $= 100C_{50} \times p^{50} (1 - p)^{50}$

Probability of heads showing on 51 coins $= 100C_{51} \times p^{51} (1 - p)^{49}$

Given that both are equal.

$100C_{50} \times p^{50} (1 - p)^{50} = 100C_{51} \times p^{51} (1 - p)^{49}$

$\frac{51}{50}(1-p) = p$

$51(1-p) = 50p$

$\therefore p = \frac{51}{101}$

6. Ans: [b]
On throwing an unbiased die, the possible outcomes are 1, 2, 3, 4, 5 and 6.
Favourable case is that the outcome should be more than 2. So there are 4 favourable cases.
Required probability = $\frac{4}{6} = \frac{2}{3}$

7. Ans: [d]
A number to be divisible by 3, the sum of its digits should be divisible by 3.
To make a 5 digit number using the digits 1, 2, 3, 4 and 5 without repetition, all the digits have to be used.
Sum = 1 + 2 + 3 + 4 + 5 = 15.
Here the sum is 15 and it is divisible by 3. It is clear that, all the 5 digit numbers formed using the digits 1, 2, 3, 4 and 5 without repetition are divisible by 3.
So the required probability is 1.

8. Ans: [b]
On throwing an unbiased die, the possible outcomes are 1, 2, 3, 4, 5 and 6.
Total number of outcomes on throwing three dice= 6^3
Favourable case is that all three dice should show the same number.
It can be any one of the six numbers 1, 2, 3, 4, 5 and 6 and can be selected in 6 ways.
Required probability = $\frac{6}{6^3} = \frac{1}{36}$

9. Ans: [a]
The total number of outcomes on tossing 4 coins is $2^4 = 16$.
Favourable cases are HTHT and THTH.
Required probability = $\frac{2}{16} = \frac{1}{8}$

10. Ans: [d]
A normal year has 365 days i.e., 52 weeks and a day.
So it is obvious that normal year has 52 Thursdays.
Required probability is 1.

11. Ans: [d]
It is given that no two cubes have the same colour. So all the cubes are of different colours. Favourable case is to draw a red coloured cube.
As we don't know the colours of the cube, the bag may or may not contain the red coloured cube. So we cannot determine the probability.

12. Ans: [b]
Given that the tie is impossible, so the events are mutually exclusive.
P(A or B or C will win) = P(A win) + P(B win) + P(C win) = 0.25 + 0.1 + 0.3 = 0.65

13. Ans: [a]
P(drawing a black ball) = $\frac{11}{20}$
P(drawing a white ball) = $\frac{9}{19}$ (since drawing ball without replacement)
P(first black and second white) = $\frac{11}{20} * \frac{9}{19} = \frac{99}{380}$

14. Ans: [b]
Three cards are to be drawn from a pack of 52 cards. The total number of outcomes possible is $52C_3$.
Selecting 1 suit out of 4 suits can be done in $4C_1$ ways.
Selecting 3 cards out of 13 cards can be done in $13C_3$ ways.
Required Probability = $\frac{4C_1 * 13C_3}{52C_3} = \frac{22}{425}$

15. Ans: [c]
Total number of outcomes = $10C_2$
Given that none of the balls is blue,
the two balls have to be selected from 3 red and 5 green balls only.
$\therefore$ Required Probability = $\frac{8C_2}{10C_2} = \frac{28}{45}$

16. Ans: [a]
i) 1 – defective

$$\frac{{}^{16}C_1 \cdot {}^{4}C_1}{{}^{20}C_2} = \frac{16 \times 4}{\frac{20 \times 19}{1 \times 2}} = \frac{16 \times 8}{20 \times 19} = \frac{32}{95}$$

ii) 2 – defective

$$\frac{{}^{4}C_2}{{}^{20}C_2} = \frac{4 \times 3}{20 \times 19} = \frac{3}{95}$$

$$\text{P (Atleast 1 defective)} = \frac{32}{95} + \frac{3}{95} = \frac{35}{95} = \frac{7}{19}$$

17. Ans: [c]
Let A = Event that A speaks the truth
B = Event that B speaks the truth

Then $P(A) = \frac{75}{100} = \frac{3}{4}$

$P(B) = \frac{80}{100} = \frac{4}{5}$

$P(A - \text{lie}) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(B - \text{lie}) = 1 - \frac{4}{5} = \frac{1}{5}$

Now, A and B contradict each other = [A lies and B true] or [B true and B lies]

$= P(A) \cdot P(B - \text{lie}) + P(A - \text{lie}) \cdot P(B)$

$= \left(\frac{3}{5} \times \frac{1}{5}\right) + \left(\frac{1}{4} \times \frac{4}{5}\right) = \frac{7}{20}$

$= \left(\frac{7}{20} \times 100\right) = 35\%$

18. Ans: [b]
Here, S = {1, 2, 3, 4, ..., 19, 20}.
Let E = event of getting a multiple of 3 or 5 = {3, 6 , 9, 12, 15, 18, 5, 10, 20}.
$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$.

19. Ans: [d]

$12C_3 \rightarrow \text{total} = \frac{12\times11\times10}{1\times2\times3} \Rightarrow 220$

$5C_3 + 4C_3 + 3C_3 \rightarrow 15 \rightarrow \text{favourable}$

$\frac{15}{220} \Rightarrow \frac{3}{44}$

p (not same colour) $\rightarrow 1-\frac{3}{44} \rightarrow \frac{41}{44}$

20. Ans: [d]
Let A = Amit stands first ; B = Vikas stands first ; C = Vivek stands first;
P(A) = 1/(2 + 1) = 1/3
P(B) = 2/(2 + 5) = 2/7
P(C) = 1/(1 +7) = 1/8
∴ Required probability = (1/3) + (2/7) + (1/8) = 125/168

21. Ans: [a]
Probability to select red ball
$\Rightarrow \frac{6}{18} = \frac{1}{3}$

Probability to select the smallest red ball $\Rightarrow \frac{1}{6}$

Total probability $= \frac{1}{3}\times\frac{1}{6} = \frac{1}{18}$

22. Ans: [a]
No. of ways to select 2 members from a group of 13 = $^{13}C_2$
No. of ways to select 1 mathematician from a group of 5 = $^5C_1$
No. of ways to select 1 physicist from a group of 8 = $^8C_1$
The probability that one will be mathematician and the other a physicist
$= \frac{^5C_1\times {}^8C_1}{^{13}C_2}$ = 20/39

23. Ans: [a]
Total = 600
70% at 600 (only suitable)
$\rightarrow 420$

$\Rightarrow \frac{420}{600} = \frac{7}{10}$

24. Ans: [b]
P (not should) $= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{1}{4}$

P (solved) = 1 – P (not solved) = $1-\frac{1}{4} = \frac{3}{4}$

25. Ans: []
Two child family
1^{st} one $\rightarrow$ Boy $-$ 1

2^{nd} one $\rightarrow$ Boy, Girl $\rightarrow \frac{1}{2}$

Total probability $= 1\times\frac{1}{2} = \frac{1}{2}$

SESSION – 8

PROBABILITY – CASE STUDY

Introduction

- Probability is a numeric value of the likelihood, or chance, of something happening.
- Do we have a better chance of this event occurring or do we have a better chance of it not occurring?
- Generally, we discuss probability as a fraction, a decimal, or a percent (%).

Probability Rules

$0 \le P \le 1$

- Where 0 is an event that has no chance of occurring
- Where 1 is an event that is sure to occur
- In between, the possibility that an event will occur
- The sum of the probabilities of all possible outcomes of an event is one (1).

3 Approaches to Probability

1. Classical – probability of success is based on prior knowledge of the process involved.
2. Empirical – the outcomes are based on observed data, not on prior knowledge of a process.
3. Subjective – based on the decision maker's opinion regarding the chances that an event will occur.

Case Study Description

Grand Prize

- Dream Cruise is a famous Cruise line.
- Provides commuter service between Hawaiian islands – three round trips daily (total of six cruises per day).
- Promotional contest awarding a large prize to be run one day per month on each cruise.

Contest Description

- The day each month for contest to be run will be selected randomly on the first of each month.

- On each cruise that day, all passengers (excluding crew) will write down their birthday (month and date).
- If any two people on the cruise have the same birthday, they will place their names in a hat and one name will be selected to receive the grand prize.

Other Case Information

- Capacity of each cruise is a maximum of 40 passengers.
- The Marketing Manager believes there is a very low chance of a birthday match, so only a small chance of giving away the large prize.
- Marketing Manager states that the probability for a match will be 40/365 (10.95%) for a full cruise and less than that when there are fewer than 40 passengers on board.

Probability Question

The owner wants to know:

- What is the probability of one or more birthday matches on cruise of 20, 30, or 40 passengers?

Is the Marketing Manager accurate in her assessment? Let's test

Multiplication Rule for Independent Events

- Independent events are events in which the occurrence of the events will not affect the probability of the occurrence of any of the other events.
- Example: The date of one's birth has no bearing on others birth date.
- For independent events, the probability of all of the events occurring is equal to a product of the probabilities of each event occurring P(A and B) = P(A) * P(B) * ...

Calculating the Probability

- Out of 20 passengers, what is the probability of two or more passengers having the same birthday?
- Looked at another way, what is the probability of 20 passengers <u>not</u> sharing a birthday?
- 20 independent events with each event as a corresponding person <u>not</u> sharing their birthday with any of the previously analyzed people.

Assumptions

DISREGARD:

- Leap Year
- Twins

*There are 365 birthdays equally likely

(Now, you will be given 15 minutes to complete the task.)

Calculating the Probability

<u>Classical Probability</u>:

P(A) = the probability of at least two passengers having the same birthday

$$P(A) = \frac{\text{number of ways for event to occur}}{\text{Total number of possible outcomes}}$$

The complement of event A

P(A') = the probability of there <u>not</u> being any two passengers with the same birthday.

Then, because P(A) and P(A') are the only two possibilities and are also mutually exclusive,

P(A') = 1 – P(A)

20 Passenger Probability Calculation

P(1) = for one person, there are 365 distinct birthdays

$$P(1) = \frac{365}{365}$$

P(2) = for two people, there are 364 different ways that the second could have a birthday without matching the first:

$$P(2) = \frac{364}{365}$$

P(3) = if person 3 is born on any of the 363 days of the year other than the birthdays of people 1 and 2, person 3 will not share their birthday.

$$P(3) = \frac{363}{365}$$

Calculating for all 20 passengers:

$$P(A') = \frac{365}{365} * \frac{364}{365} * \frac{363}{365} * \frac{362}{365} * \frac{346}{365} = 0.588528$$

Therefore, the probability of at least two passengers having the same birthday is:

P(A) = 1 – 0.588528 or 41.15%

30 Passenger Calculation

30 Passengers:

$$P(A') = \frac{365}{365} * \frac{364}{365} * \frac{363}{365} * \frac{362}{365} * \frac{336}{365} = 0.293665$$

P(A) = 1 – 0.293665 **= 70.63%**

40 Passenger Calculation

40 Passengers:

$$P(A) = \frac{365}{365} * \frac{364}{365} * \frac{363}{365} * \frac{362}{365} * \frac{326}{365} = 0.108760$$

P(A) = 1 - 0.108760

= 89.12%

Birthday Probability Summary

20 Passengers = 41.15%

30 Passengers = 70.63%

40 Passengers = 89.12 %

Seems implausible, doesn't it?

Understanding Probability

- Some of the terms people use for probability are:
  - Chance
  - Likelihood
- Our probability calculation reflects that it is likely that there are some birthday matches among the other people in the group, not matching just to me.

Tests of this probability:

- Try to find the names of the celebrities who are born on the same date and month as of yours.
- Example:
- Kamal Hasan and Anushka Shetty share the same birthday.
- Rajini Kanth and Yuvraj Singh share their birthday too.

Awarding Prizes

- With six cruises daily (carrying the same number of passengers: 20, 30, and 40), what are the chances that Dream Cruise will end up awarding two or more major prizes during a given month?

Awarding Prizes

1. Consider the number of cruises on the one day per month the contest is run (3 round trips, 6 cruises total)
2. Multiply by the probability of a birthday match assuming group size (20, 30, and 40 passenger possibilities).

   6×0.4115 = 2.47 prizes/month for 20 passenger cruises

   6×0.7063 = 4.24 prizes/month for 30 passenger cruises

   6×0.8912 = 5.35 prizes/month for 40 passenger cruises

Conclusion:

- The Marketing Manager statement was: "probability for a match will be 40/365 (10.95%) for a full cruise and less than that when there are fewer than 40 passengers on board."

  But we can see that his **predictions are incorrect.**

Final Thought on Probability

- *If I make everything predictable, human beings will have no motive to do anything since the future is totally determined.*
- *If I make everything unpredictable, human beings will have no motive to do anything as there is no rational basis for any decision.*
- *I must therefore create a mixture of two.*

[from E. F. Schumacher]

SESSION – 9

1. Ans: [c]
$\log_2(\log_2(\log_3(\log_3 27^3)))$
$= \log_2(\log_2(\log_3(\log_3 3^9)))$
$= \log_2(\log_2(\log_3(9\log_3 3)))$
$= \log_2(\log_2(2\log_3 3))$
$= \log_2 1 = 0$

2. Ans: [c]
$[1/\log XYZ_{xy} + 1/\ logXYZ_{yz} + 1/\log XYZ_{xz}]$
$\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} xz$ $\quad (\log_b a = 1/\log_a b)$
$= \log_{xyz} xy * yz * zx$
$= \log_{xyz}(xyz)$^2
$= 2\log_{xyz} xyz = 2$

3. Ans: [a]
$\log_5 x^2 = 6$
$x^2 = 5^6$
$x^2 - 5^6 = 0$
$(x + 5^3)(x - 5^3) = 0$
Roots are 5^3 and -5^3
Product of roots = – 15625

4. Ans: [c]
$\log_2 3 * \log_3 4 * \log_4 5 * \ldots\ldots\ldots * \log_{n-1} n * \log_n n + 1 = 10$
$\log_2 n + 1 = 10$
$n + 1 = 2^{10}$
n = 1024 – 1 = 1023

5. Ans: [a]
$\frac{\log_3(3^5(3^4 * 3^{\frac{1}{3}})\text{^}1/2)\text{^}1/2}{\log_2(2^6)\text{^}1/4 \ * \ \log_e e^{-10}}$
$= \frac{\log_3(3^5 * 3^{13/6})^{1/2}}{\frac{3}{2}\log_2 2 \ * \ -10}$
$= - \frac{43}{102}$

6. Ans: [b]
If log2 = 0.30103, the number of digits in5^{20}
Let $5^{20} = x$
$\log_{10} x = \log_{10} 5^{20} = 20\log_{10} 10/2 = 20(\log_{10} 10 - \log_{10} 2)$
= 20 (1- 0.3010) =13.980
Characteristic = 13
So, No. of Digits. = 13 + 1 = 14

7. Ans: [b]
$\log_{10}5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$
$\Rightarrow \log_{10}5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + \log_{10}10$
$\Rightarrow \log_{10}[5\ (5x + 1)] = \log_{10}[10(x + 5)]$
$\Rightarrow 5(5x + 1) = 10(x + 5)$
$\Rightarrow 5x + 1 = 2x + 10$
$\Rightarrow 3x = 9$
$\Rightarrow x = 3.$

8. Ans: [a]
$\log_{32} x = 0.8.$

$x = (32)^{0.8}$

$= (2^5)^{4/5}$

$= 2^4 = 16.$

9. Ans: [d]
$\log_{10} 36000 = \log_{10} (36 \text{ x } 1000) = \log_{10} 36 + \log_{10} 1000$
$= \log_{10} 36 + 3 \log_{10}10 = y + 3$

10. Ans: [a]
$\log_8 2 + \log_8(1/2) = \log_8(2 * (1/2)) = \log_8 1 = 0.$

11. Ans: [c]
$\log_{\sqrt{2}}(\frac{1}{8}) = \log_2 1 / 2(2^{-3}) = 2 * (-3)\log_2 2 = -6.$

12. Ans: [b]
$\log_{0.2} 2 = \log 2 / (\log 2 - \log 10) = 0.3/(0.3 - 1) = -3/7$

13. Ans: [c]
$x^3 = 128$
$x = 2^{7/3}$
$\log_x 2 = \log_{2^{7/3}} 2 = (3/7) * \log_2 2 = 3/7.$

14. Ans: [c]
$\log_{(\frac{1}{x})} x\text{^}2 = -\sqrt{2}\, x$
$x^{x\sqrt{2}} = x^2$
$x = \sqrt{2}$

15. Ans: [c]
$a\text{^}(2\log_a x) = 49$
$\log_a 49 = 2 \log_a x$
$49 = x^2$
$x = 7$

16. Ans: [b]
$\log_2 100 + \log_{\sqrt{2}} 100 + \log_{\sqrt[3]{2}} 100 + \log_{\sqrt[4]{2}} 100 + \ldots\ldots$till 20 terms
$\rightarrow \log_2 100 + \log_{2^{1/2}} 100 + \log_{2^{1/3}} 100 + \log_{2^{1/4}} 100 \ldots\ldots \log_{2^{1/20}} 100$
$\rightarrow \log_2 100 + 2\log_2 100 + 3\log_2 100 + \ldots\ldots 20\log_2 1000$
$= \log_2 100\ [1 + 2 + 3 + 4 + 5 + \ldots.. + 20]$
$= 210 * (\log 100)/(\log 2)$
$= 420/(\log 2).$

17. Ans: [b]
$\log_x a + \log_{x^2} a + \log_{x^4} a + \log_{x^8} a \ \ldots\ldots \infty$
$= \log_x a * [1 + (1/2) + (1/4) + (1/8) + \ldots. \infty$
$= \log_x a\left(\frac{1}{1-1/2}\right)\ldots$ (G.P. up to ∞)
$= \log_x a * 2$
$= \log_{\sqrt{x}} a.$

18. Ans: [b]
Let $\log_{\sqrt{2}} 32 = n \Rightarrow \left(\sqrt{2}\right)^n = 32$
$\Rightarrow 2^{n/2} = 2^5$
$\Rightarrow \frac{n}{2} = 5 \Rightarrow n = 10$
$\therefore \log_{\sqrt{2}} 32 = 10$
Let $\log_5\left(\frac{1}{125}\right) = n \Rightarrow 5^n = \frac{1}{125} = 5^{-3}$
$\Rightarrow n = -3$
$\therefore \log_5\left(\frac{1}{125}\right) = -3$
Hence $\log_{\sqrt{2}} 32 + \log_5\left(\frac{1}{125}\right) = 10 - 3 = 7$

19. Ans: [a]
$\log_8 x + \log_8\left(\frac{1}{6}\right) = \frac{1}{3}$
$\Rightarrow \frac{\log x}{\log 8} + \frac{\log \frac{1}{6}}{\log 8} = \frac{1}{3}$
$\Rightarrow \log x + \log\frac{1}{6} - \frac{1}{3}\log 8 = 0$
$\Rightarrow \log x + \log\frac{1}{6} - \frac{1}{3}\log\left(2^3\right) = 0$
$\Rightarrow \log x + \log\frac{1}{6} - \log 2 = 0$
$\Rightarrow \log x = \log 2 - \log\frac{1}{6}$
$\Rightarrow \log x = \log(2 \times 6) = \log 12$
$\Rightarrow x = 12$

20. Ans: [a]
$\log_x \frac{p}{q} + \log_x \frac{q}{r} + \log_x \frac{r}{p}$
$= \log_x\left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log_x 1 = 0$

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1. Ans: [c]

$7a+8b=53$ ... (i)

$9a+5b=47$ ... (ii)

(i) $\times 5 \to 35a+40b=265$ ... (iii)

(ii) $\times 8 \to 72a+40b=376$ ... (iv)

$(iv)-(iii) \to 37a=111$

$\Rightarrow a=\frac{111}{37}=3$

Substituting for a in (i),

$21+8b=53 \Rightarrow b=\frac{32}{8}=4$

$\therefore$ The solution is (3, 4)

2. Ans: [b]

Let the present age be x years.

Then, $7(x+7)+3(x-3)=12x$

$\Rightarrow 7x+49+3x-9=12x$

$\Rightarrow 2x=40 \quad \Rightarrow x=20$ years

$\therefore$ Age after 3 years = 20 + 3 = 23 years

3. Ans: [a]

Let the initial number of chickens be x.

$\frac{x\times 30}{1}=\frac{(x-10)\times 150}{3}$

$\Rightarrow 90x=150x-1500$

$\Rightarrow 60x=1500$

$\Rightarrow \quad x=\frac{1500}{60}=25$

So, the initial number of chickens = 25

4. Ans: [b]

Let tree II grow x feet after 1 year.

$\therefore \left(\frac{3x}{7}+x\right)\times 3=3$

$\Rightarrow \frac{10x}{7}=1 \quad \Rightarrow x=\frac{7}{10}$ ft

Tree II takes $\frac{7}{\frac{7}{10}}$ years to grow 7 ft.

$\therefore$ Time required = 10 years

5. Ans: [d]

$x^2-7x+12=0$

Sum of the roots = 7, product of the roots = 12

$\Rightarrow \alpha+\beta=7, \quad \alpha\beta=12$

Sum of the reciprocals of the roots $=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{7}{12}$

Product of the reciprocals of the roots $=\frac{1}{\alpha\beta}=\frac{1}{12}$

$\therefore$ The required equation is $x^2-\frac{7}{12}x+\frac{1}{12}=0$

$\Rightarrow 12x^2-7x+1=0$

6. Ans: [b]

$\sqrt{4x+9}-\sqrt{11x+1}-\sqrt{7x+4}=0$

$\Rightarrow \sqrt{4x+9}-\sqrt{7x+4}=\sqrt{11x+1}$

$\Rightarrow (4x+9)+(7x+4)-2\sqrt{(4x+9)(7x+4)}=11x+1$

$\Rightarrow 2\sqrt{(4x+9)(7x+4)}=12$

$\Rightarrow (4x+9)(7x+4)=36$

$\Rightarrow 28x^2+79x+36=36 \qquad \Rightarrow 28x^2+79x=0$

$\Rightarrow x=0$ or $-\frac{79}{28}$

As $x\ge -\frac{1}{11}$, $x=-\frac{79}{28}$ is not a root.

$\therefore$ The solution is x = 0

There is 1 solution

7. Ans: [d]

$(\cos p-1)x^2+(\cos p)x+\sin p=0$

If the equation has real roots,

$\cos^2 p-4\sin p(\cos p-1)\ge 0$

$\Rightarrow \cos^2 p-4\sin p\cos p+4\sin p\ge 0$

$\Rightarrow (\cos p-2\sin p)^2-4\sin^2 p+4\sin p\ge 0$

$\Rightarrow (\cos p-2\sin p)^2+4\sin p(1-\sin p)\ge 0$

$(\cos p-2\sin p)^2$ is always ≥ 0

For $1-\sin p$ to be non-negative, $\sin p\le 1$

This is possible in the interval $(0,\pi)$

8. Ans: [a]

α, β are the roots of $ax^2+bx+c=0$

$\alpha+\beta=-\frac{b}{a}, \quad \alpha\beta=\frac{c}{a}$

But $\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$\Rightarrow \alpha+\beta=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$\Rightarrow (\alpha\beta)^2(\alpha+\beta)=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow \frac{c^2}{a^2}\left(-\frac{b}{a}\right)=\frac{b^2}{a^2}-2\frac{c}{a}$

$\Rightarrow -bc^2=ab^2-2a^2c$

$\Rightarrow 2a^2c=ab^2+bc^2$

$\Rightarrow ab^2, ca^2, bc^2$ are in A.P..

9. Ans: [d]

Since α is the root of $a^2x^2 + bx + c = 0$

$a^2\alpha^2 + b\alpha + c = 0$

Since β is the root of $a^2x^2 - bx - c = 0$

$a^2\beta^2 - b\beta + c = 0$

Let $f(x) = a^2x^2 + 2bx + 2c$

$f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$

$= 2(a^2\alpha^2 + bx + c) - a^2\alpha^2$

$= 2\times 0 - a^2\alpha^2 = -a^2\alpha^2 < 0$

$f(\beta) = a^2\beta^2 + 2b\beta + 2c$

$= 3\alpha^2\beta^2 - 2(\alpha^2\beta^2 - b\beta - c)$

$= 3\alpha^2\beta^2 - 0$

$= 3\alpha^2\beta^2 > 0$

$\therefore$ In the interval (α, β), $f(x)$ becomes 0 atleast once

Hence $\alpha < \gamma < \beta$

10. Ans: [c]

$2\sqrt{5} - 1 > \sqrt{3} \Rightarrow \tan^{-1}(2\sqrt{5} - 1) > \tan^{-1}\sqrt{3} = \frac{\pi}{3} > 1$

$\therefore A = \tan^{-1}(2\sqrt{5} - 1) > 1$

Let the other root be B

Then AB = 1 $\Rightarrow B = \frac{1}{A} < 1$

11. Ans: [b]

p and q are the roots of $x^2 + px + q = 0$

$\Rightarrow pq = q, p + q = -p$

$\Rightarrow q(p-1) = 0 \Rightarrow q = 0$ or $p = 1$

If $q = 0$, we get $p = 0$

If $p = 1$, we get $q = -p - p = -2$

Thus $p = 1$ or 0

12. Ans: [c]

$ax^2 + 2bx + c = 0$

Since a, b, c are in G.P., $b^2 = ac \Rightarrow b = \sqrt{ac}$

The equation can be written as

$ax^2 + 2\sqrt{ac}\,x + c = 0$

$\Rightarrow \left(\sqrt{a}x + \sqrt{c}\right)^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}, -\sqrt{\frac{c}{a}}$

Also $ax^2 + 2bx + c = 0$ has equal roots.

So the two given equations have a common root if $-\sqrt{\frac{c}{a}}$ is a root of $dx^2 + 2ex + f = 0$

$\Rightarrow d\left(\frac{c}{a}\right) - 2e\sqrt{\frac{c}{a}} + f = 0$

$\Rightarrow \frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0 \quad \left[\because b = \sqrt{ac}\right]$

$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

$\Rightarrow \frac{a}{d}, \frac{b}{e}, \frac{c}{f}$ are in H.P.

13. Ans: [a] 14. Ans: [d] 15. Ans: [c] 16. Ans: [a]

17. Ans: [a] 18. Ans: [b]

19. Ans: [b]

x = 1 is a root of the quadratic equations

$ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$

$\therefore a + a + 3 = 0$ and $1 + 1 + b = 0$

$\Rightarrow a = -\frac{3}{2}$ and $b = -2$

$\Rightarrow ab = \left(-\frac{3}{2}\right)(-2) = 3$

$\Rightarrow ab = 3$

20. Ans: [b]

$x^2 + 6x = x^2 + 6x + 9 - 9 = (x + 3)^2 - 9$

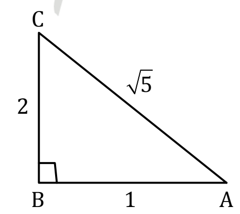
So, $x^2 + 6x$ is least when $x + 3 = 0$ or $x = -3$

$x^2 + 6x - 27 = (x + 3)^2 - 9 - 27$

Therefore, the least value of $x^2 + 6x - 27 = -9 - 27 = -36$

SESSION – 11

1. Ans: [a]



By Pythagoras theorem,

$AC = \sqrt{AB^2 + BC^2} = \sqrt{1+4} = \sqrt{5}$

$\sin A = \frac{2}{\sqrt{5}}; \operatorname{cosec} A = \frac{\sqrt{5}}{2}$

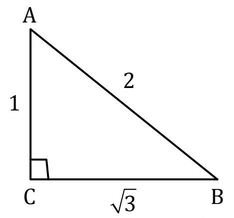
$\cos A=\frac{1}{\sqrt{5}};\sec A=\frac{1}{\cos A}=\sqrt{5}$

$\sec A.\sin A+\tan^2 A-\operatorname{cosec} A$

$=\sqrt{5}.\frac{2}{\sqrt{5}}+4-\frac{\sqrt{5}}{2}$

$=2+4-\frac{\sqrt{5}}{2}=6-\frac{\sqrt{5}}{2}=\frac{12-\sqrt{5}}{2}$

2. Ans: [c]



By Pythagoras theorem, the third side

$BC=\sqrt{4-1}=\sqrt{3}$

$\therefore \cos B=\frac{\sqrt{3}}{2}$

Now $3\cos B-4\cos^3 B$

$=\frac{3\sqrt{3}}{2}-4\left(\frac{\sqrt{3}}{2}\right)^3$

$=\frac{3\sqrt{3}}{2}-\frac{4\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}$

$=\frac{3\sqrt{3}}{2}-\frac{3\sqrt{3}}{2}=0$

3. Ans: [c]

$\tan 30^\circ \sec 45^\circ+\tan 60^\circ \sec 30^\circ$

$=\frac{1}{\sqrt{3}}\times\sqrt{2}+\sqrt{3}\times\frac{2}{\sqrt{3}}$

$=\frac{\sqrt{2}}{\sqrt{3}}+2=\frac{\sqrt{2}+2\sqrt{3}}{\sqrt{3}}$

4. Ans: [c]

$$=\frac{\frac{4}{(\sqrt{3})^2}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}-2\times\left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$=\frac{\frac{4}{3}+\frac{4}{3}-1}{\frac{3}{4}+\frac{1}{2}}=\frac{\frac{4+4-3}{3}}{\frac{3+2}{4}}$$

$$=\frac{\frac{5}{3}}{\frac{5}{4}}=\frac{5}{3}\times\frac{4}{5}=\frac{4}{3}$$

5. Ans: [b]

$\cos^2 60^\circ \tan^2 30^\circ+\sin 30^\circ \cos 0^\circ \sin 60^\circ \tan 45^\circ$

$=\left(\frac{1}{2}\right)^2\left(\frac{1}{\sqrt{3}}\right)^2+\frac{1}{2}\times 1\times\frac{\sqrt{3}}{2}\times 1$

$=\frac{1}{4}\times\frac{1}{3}+\frac{\sqrt{3}}{4}=\frac{1+3\sqrt{3}}{12}$

6. Ans: [c]

$\tan^2 30^\circ+\frac{1}{2}\sin^2 45^\circ+\frac{1}{3}\cos^2 30^\circ+\frac{1}{\tan^2 60^\circ}$

$=\left(\frac{1}{\sqrt{3}}\right)^2+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2+\frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2$

$=\frac{1}{3}+\frac{1}{4}+\frac{1}{4}+\frac{1}{3}=\frac{4+3+3+4}{12}=\frac{14}{12}=\frac{7}{6}=1\frac{1}{6}$

7. Ans: [c]

In one minute the wheel turns through 360 revolutions $=360\times 2\pi$ radians

$\therefore$ In one second it turns through

$=\frac{360\times 2\pi}{60}$ radians $=12\pi$ radians

8. Ans: [b]

$\sin 420^\circ=\sin\left(360^\circ+60^\circ\right)=\sin 60^\circ=\frac{\sqrt{3}}{2}$

$\cos 390^\circ=\cos\left(360^\circ+30^\circ\right)=\cos 30^\circ=\frac{\sqrt{3}}{2}$

$\cos\left(-330^\circ\right)=\cos 300^\circ=\cos\left(360^\circ-60^\circ\right)$

$\sin\left(-330^\circ\right)=-\sin 330^\circ=-\sin\left(360^\circ-30^\circ\right)$

$=\sin 30^\circ=\frac{1}{2}$

$\therefore \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}+\frac{1}{2}.\frac{1}{2}=\frac{3}{4}+\frac{1}{4}=1$

9. Ans: [a]

In a cyclic quadrilateral opposite angles are supplementary

i.e. $A+C=180^\circ$ and $B+D=180^\circ$

$C=180^\circ-A$ and $D=180^\circ-B$

$\therefore \cos C=\cos\left(180^\circ-A\right)=-\cos A$

$\cos D=\cos\left(180^\circ-B\right)=-\cos B$

hence $\cos A+\cos B+\cos C+\cos D$

$=\cos A+\cos B-\cos A-\cos B=0$

10. Ans: [b]

$x^2+y^2+z^2=r^2\sin^2\theta\cos^2\varphi+r^2\sin^2\theta\sin^2\varphi+r^2\cos^2\theta$

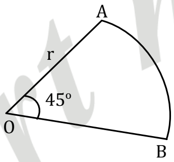
$= r^2\sin^2\theta\left(\cos^2\varphi+\sin^2\varphi\right)+r^2\cos^2\theta$

$= r^2\sin^2\theta+r^2\cos^2\theta$

$= r^2\left(\sin^2\theta+\cos^2\theta\right)=r^2$

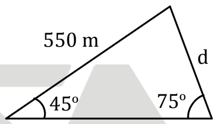
11. Ans: [b]
First, $2\pi/5$ radians $= (2\pi/5)\,x\,(180/\pi) = 72^\circ$ (π radians $= 180^\circ$)
Now, let the two complementary angles be
x° and $x + 72^\circ$
Since $x^\circ + (x^\circ + 72) = 90^\circ$,
$x = 9^\circ$
The angles are 9° and 81°

12. Ans: [b]
r is the length of the rope



$\theta = 45^\circ = [45\,x\,\pi/180]c = (\pi/4)c$
$\pi/4 = 88/r$
$r = (88\,x\,4)/\pi$
$r = 112\,m$
Length of the rope is 112 m.

13. Ans: [c]
By using sine formula



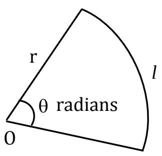
$\frac{d}{\sin 45^\circ} = \frac{550}{\sin 75^\circ}$

$d/(1/\sqrt{2}) = 550/\{(\sqrt{3}+1)/(2\sqrt{2})\} \rightarrow d = \frac{550\times 2}{\sqrt{3}+1}$

$= 402.9$ m

14. Ans: [d]
$\tan\theta = -4/3$
$\sin\theta = 4/5$ [sin is +ve in 2nd quad]
$\operatorname{cosec}\theta = 5/4$

15. Ans: [a]

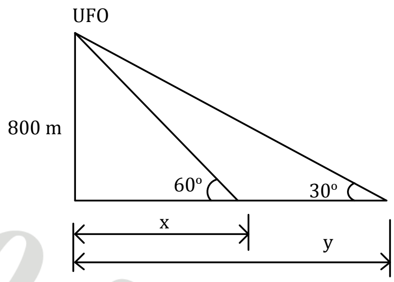


θ = lr

Length of the arc of a semicircle of radius $r = \pi r$
Perimeter of the sector of a circle of radius $r = 2r + \theta r$
Given, $\pi r = 2r + \theta r$

$2 + \theta = \pi$
$\theta = \pi - 2$
Converting θ into degree value
$\theta = [(\pi - 2)x\ 180/\pi]$
$\theta = \frac{1440}{22} = 65.45^\circ$

16. Ans: [a]



Tan 60° = 800/x; tan 30° = 800/y
X = 800/√3; y = 800√3
Boat travelled in 30 sec = y – x
800√3 – 800/√3 = 923.7 m

17. Ans: [b]
$\angle A, \angle B, \angle C$ are in arithmetic progression

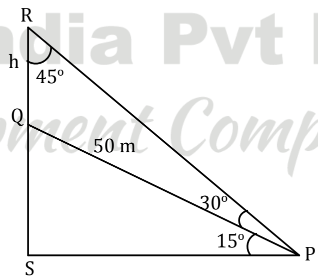
$2\angle B = \angle A + \angle C$

$3\angle B = \angle A + \angle B + \angle C$

$\angle B = 60^\circ$

Now from sine formula, we have
b/Sin B = c/Sin C
b/c = Sin B/Sin C
√3/√2 = sin 60°/sin C
Sin C = 1/√2
$\angle C = 45^\circ$
$\angle A = 75^\circ$

18. Ans: [b]



Let QR represent the tower
Let P be the point 50m down the hill from the base of the tower.
Let h be the height of the tower
In Δ PQR, applying sine formula
PQ/sin 45° = h/ sin 30°
50/(1/√2) = h/ (1/2)
h = 35.36 m = 35 m

19. Ans: [a]

Given sin θ + cos θ = √2 ... [sin θ = cos 90 – θ)]

Applying

Cos(90 – θ) + cos θ = √2

2cos[{(90 – θ) + θ}/2]cos [{(90 – θ) – θ}/2] = √2

Cos A + cosB = 2cos[(A+B)/2]cos[(A – B)/2]

2cos 45°.cos(45 – θ)= √2

2.(1/√2).cos(45 - θ)= √2

cos(45 – θ) = 1

(45 – θ) = 0°

Θ = 45°

20. Ans: [c]

Cosec 5π/6 = cosec(π – π/6) = cosec π/6 = cosec 30° = 2

Cot π/6 = 1/tan π/6 = 1/tan 30° = √3

$(\sqrt{3})^2 + 2 + 3(1/\sqrt{3})^2 = 3 + 2 + 1 = 6$

SESSION – 12

1. Ans: [d]

$F(x) = max\,(2x+1, 3-4x)$ is minimum when $2x + 1 = 3 - 4x$

i.e.$6x = 2$

$x = 2/6 = 1/3$

therefore minimum possible value of f(x) is

$(2x + 1)(at\ x = 1/3) = 2 * 1/3 + 1 = 5/3$

or (3 – 4x) at x = 1/3 = 3 – 4.1/3 = 5/3

2. Ans: [d]

$F(x) = ax2 - b|x|$

$ax^2 > 0$ for $a > 0$ and

$(-b|x|) > 0$ for $b < 0$

$F(x) = ax2 - b|x| > 0\ for\ x \neq 0$

$F(0) = ax2 - b|x| = 0\ for\ x = 0$

If $x = 0$ $f(x)$ is minimised whenever, $a > 0, b < 0$.

3. Ans: [a]

Min $\{f(x2), h(x)\} < 3$

$f(x2) < 3\ or\ h(x) < 3$

$2x^2 - 1 < 3\ or\ x^2 + x + 1 < 3$

$X2 - 2 < 0\ or\ x2 + x - 2 < 0$

$-\sqrt{2} < x < \sqrt{2}\ or\ (x+2)\ (x-1) < 0$

$-\sqrt{2} < x < \sqrt{2}\ or\ -2 < x < 1$

Both the above ranges for x, satisfy the inequality Min $\{f(x2), h(x)\} < 3$

$-2 < x < \sqrt{2}$ (as $-2 < -\sqrt{2}$ and $\sqrt{2} > 1$)

4. Ans: [b]

$F(x) = |x-2| + |2.5-x| + |3.6-x|$

Sub. from the options, the values of x in the function,

when $x = 2.3$

$F(x) = 0.3 + 0.2 + 1.3 = 1.8$

when $x = 2.5$

$F(x) = 0.5 + 0 + 1.1 = 1.6$

When $x = 2.7$

$F(x) = 0.7 + 0.2 + 0.9 = 1.8$

Thus for any value of $x, f(x)$ will be greater than 1.6

→ f(x) is minimum at x = 2.5.

5. Ans: [b]

$F(x) = |3x-2| + |2x-3|$

$= 3|x - 2/3| + 2\,|x - 3/2|$

For $x \geq 3/2, f(x) = 3x - 2 + 2x - 3$

$= 5(x-1)$

So minimum value = $5\,(3/2 - 1) = 5/2$

For $3/2 \geq x \geq 2/3$,

$F(x) = 3x - 2 + 3 - 2x = x + 1$

So minimum value = $2/3 + 1 = 5/3$

For $x \leq 2/3, f(x) = 2 - 3x + 3 - 2x$

$= 5(1-x)$

So, minimum value = $5\,(1 - 2/3) = 5/3$

So, minimum value of $f(x)$ is $5/3$.

6. Ans: [b]

Since $|x| \geq -x$ for any value of x,

$k - x \leq |x| + k$ for any value of x

$f(x) = |x| + k$ for any $x \geq k$ as $|x| \geq 0$

and $f\ (0) = k$

So, minimum value of $f\ (x)$ is k.

7. Ans: [b]

Given $f(x) = ax2 + bx + c\ (a \neq 0)$.

3 is a root of $f\ (x)$

$9a + 3b + c = 0$ ... (1)

Also, $f(5) = -3f(2)$.

$25a + 5b + c = -3(4a + 2b + c)$

$= -12a - 6b - 3c$

$37a + 11b + 4c = 0$ ... (2)

From two equations $a - b = 0\ a = b$

Thus we get $f(x) = ax2 + ax + c$

Dividing $f(x)$ by $x - 3$, we get $c = -12a$

$F(x) = ax2 + ax - 12a$

$F(x) = 0, -4$ is the second root.

8. Ans: [b]

$F(x) = x3 - 4x + p$

$F(0) = +p$ and

$F\ (1) = 1 - 4 + p = -3 + p$

$F(0)$ and $f(1)$ are of opposite signs.

If p is positive, $(p-3)$ has to be negative and p has to take values less than 3 i.e. $0 < p < 3$.

9. Ans: [c]

For D_f, |x| - x > 0,|x| > x i.e., x < |x|

which is true if x < 0 .

D_f = (– ∝, 0).

10. Ans: [d]

Since $f(-x) = \log\frac{1-x}{1+x} = \log\left(\frac{1+x}{1-4}\right)^{-1} = -\log\frac{1+x}{1-x} = -f(x)$

$\therefore$ f(x) is odd.

11. Ans: [a]

Let $y = \frac{2x}{x^2+1}$ ==> $x^2y - 2x + y = 0$

Since x is real, discriminant $4 - 4y^2 \geq 0$

$1 - y^2 \geq 0$

$y^2 \leq 1$

$|y| \leq 1$

$-1 \leq y \leq 1$.

12. Ans: [d]

As f(x) = max (2x + 1, 3 – 4x)

We know that f(x) would be minimum at the point of intersection of these curves.

i.e., 2x + 1 = 3 – 4x

i.e., 6x = 2 ==> $x = \frac{1}{3}$

Hence, minimum value of f(x) is $\frac{5}{3}$.

13. Ans: [b]

Minimum possible value of any expression inside mod is zero. So we will check for x = 3, – 2 and x = 5. At x = 3 we will get minimum value, which is 7.

14. Ans: [a]

Using property (iii) with x = 1,

f(3) = f(1) + 12(1) + 12 = 1 +12 +12 = 25

since f(1) = 1 by property (i).

Using property (ii) with x = 3,

f(6) = 4f(3) + 6 = 4(25) + 6 = 106

Therefore, the value of f(6) is 106.

15. Ans: [b]

f(x) = |x – 2| + |2.5 – x| + |3.6 – x| attains minimum value when any of the terms = 0.

16. Ans: [b]

If a = 2, the function is constant.

17. Ans: [d]

g(x) = max (5 – x, x + 2)

We have to draw graph and the find the point of intersection.

y = 5 – x

y = x + 2

Hence at the point of intersection of two straight line.

Smallest of g(x) = 3.5

18. Ans: [d]

The denominator $x^2 - 3x + 2$ has real roots. Hence the maximum value of the function f(x) will be infinity.

19. Ans: [d]

g(x + 1) + g(x – 1) = g(x)

g(x + 2) + g(x) = g(x + 1)

Adding these two equation, we get

g(x + 2) + g(x – 1) = 0

g(x + 3) + g(x) = 0 … (1)

g(x + 4) + g(x + 1) = 0

g(x + 5) + g(x + 2) = 0

g(x + 6) + g(x + 3) = 0

g(x + 6) – g(x) =0 (From (1))

20. Ans: [d]

Given function = f(1) + f(2) + f(3) + f(4)+ ……. = n^2f(n)

Given f(1) = 3600

For n = 2,

f(1) + f(2) = 2^2f(2)

i.e. 2^2f(2) – f(2) = f(1)

f(2) = f(1)/(2^2–1) … (1)

For n = 3

f(1) + f(2) + f(3) = 3^2f(3)

put the value of f(2) from (1)

→ f(1) + f(1)/(2^2 – 1) = 3^2f(3) – f(3)

→ f(1)+ f(1)/(2^2 – 1) = (3^2 – 1)f(3)

now take f(1) in left side

i.e. f(1) = [1 + 1/(2^2 – 1)] = f(3)(3^2 – 1)

i.e. f(3) = f(1) x 2^2/(2^2 – 1) x 1/(3^2 – 1)

f(3) = 600

Similarly

f(9) = f(1) x (2^2 x 3^2 x 4^2 ………. 8^2)/((2^2 – 1)(3^2 – 1)(4^2 – 1) ……. (9^2 – 1))

f(9)=80

SESSION – 13

1. Ans: [b]

Using the formula n(A ∪ B) = n(A) + n(B) – n(A ∩ B),

then n(A ∩ B) = n(A) + n(B) – n(A ∪ B)

= 20 + 28 – 36

= 48 – 36 = 12

2. Ans: [d]

Let A be the set of people who like cold drinks.

B be the set of people who like hot drinks.

Given

(A ∪ B) = 60 n(A) = 27 n(B) = 42 then;

n(A ∩ B) = n(A) + n(B) – n(A ∪ B)

= 27 + 42 – 60

= 69 – 60 = 9

= 9

Therefore, 9 people like both tea and coffee.

3. Ans: [c]

4. Ans: [b]

5. Ans: [b]

6. Ans: [a]
There are 25 prime numbers in the given set A.
$\therefore$ Number of sets having exactly 3 elements $= {}^{25}C_3 \rightarrow 2300$

7. Ans: [a]
Power set of A, $A^P = 2^{25}$
Power set of $A^P = 2^{2^{25}}$
No of non-empty sets = $(2^{2^{25}}) - 1$

8. Ans: [b]
Let A be the set of people who speak English.
B be the set of people who speak French.
A – B be the set of people who speak English and not French.
B – A be the set of people who speak French and not English.
A ∩ B be the set of people who speak both French and English.
Given,
n(A) = 72   n(B) = 43   n(A ∪ B) = 100
Now, n(A ∩ B) = n(A) + n(B) – n(A ∪ B)
= 72 + 43 – 100
= 115 – 100 = 15
Therefore, number of persons who speak both French and English = 15
n(A) = n(A – B) + n(A ∩ B)
⇒ n(A – B) = n(A) – n(A ∩ B)
= 72 – 15 = 57
and n(B – A) = n(B) – n(A ∩ B)
= 43 – 15 = 28
Therefore, number of people speaking English only = 57
Number of people speaking French only = 28

9. Ans: [b]
Let A denote the set of students who received medal in athletics, J be the set of students who got medal in Judo and S be the set of students who got medal in swimming.
n(A) = 35, n(J) = 15, n(S) = 18
$n(A \cup J \cup S) = 58$ and $n(A \cap J \cap S) = 3$
Now, $n(A \cup J \cup S) = n(A) + n(J) + n(S) - n(A \cap J) - n(S \cap J) - n(A \cap S) + n(A \cap J \cap S)$
$\Rightarrow n(A \cap J) + n(S \cap J) + n(A \cap S) = 13.$
Number of students who received medals in exactly two of the three = $n(A \cap J) + n(S \cap J) + n(A \cap S) - 3\, n(A \cap J \cap S) = 13 - 9 = 4.$

10. Ans: [c]

11. Ans: [a]

12. Ans: [d]
Let n(A) = 300, n(B) = 250 and $n(A \cap B) = 100$
$\Rightarrow n(A \cup B) = 450$
The number of families which do not subscribe to any of these newspapers is 1000-450 = 550.

13. Ans: [d]
Total number of students = 25
n(A) = Number of students taking economics = 12
n(B) = Number of students taking politics
We have to find $n(A \cap B)$ and $n(B \cap A')$
Now, $n(A) = n(A \cap B') + n(A \cap B)$
$\Rightarrow n(A \cap B) = 12 - 8 = 4$
$\Rightarrow n(A \cup B) = 12 + n(B) - 4$
$\Rightarrow n(B) = 17$
$n(B) = n(A' \cap B) + n(A \cap B)$
$\Rightarrow n(B \cap A') = 17 - 4 = 13.$

14. Ans: [b]
Let n(A) = 450, n(B) = 300 and $n(A \cap B) = 200$
$\Rightarrow n(A \cup B) = 550$
The number of persons who read neither is 840 – 550 = 290

15. Ans: [d]
Let A be the set of persons who prefer fish and B be the set of persons who prefer chicken. Then,
$n(A \cup B) = 100 - 10 = 90$, $n(A) = 100 - 50 = 50$, $n(B) = 60$
$\Rightarrow n(A \cap B) = 20$

16. Ans: [b]

17. Ans: [d]

18. Ans: [b]
$n(\text{Bunglows}) = \frac{1}{4}$
$n(\text{Flats}) = \frac{1}{4}$
Since, one-eight of those who own bunglows do not own flats.
$\therefore$ Remaining $\frac{7}{8}$ own flats as well.
$\therefore n(\text{Both}) = \frac{7}{8} \times \frac{1}{6} = \frac{7}{48}.$

19. Ans: [c]
$$n(\text{Bunglows} \cup \text{Flats}) = \frac{1}{6} + \frac{1}{4} - \frac{7}{48}$$
$$= \frac{8 + 12 - 7}{48} = \frac{13}{48}$$
This means that the total fraction owning either a bunglow or a flat $= \frac{13}{48}$.

20. Ans: [a]
The fraction of society owning neither a bunglow nor a flat $= 1 - \frac{13}{48} = \frac{35}{48}$.

SESSION – 14

CONSOLIDATED LEARNING

1. Ans: [a]

BEAUTY is a 6-letter word. The vowels among the 6 letters are E, A, and U. The question stipulates that the vowels should appear together. i.e., EAU should appear as 1 unit. Let us call this unit X. So, we need to re arrange BXTY. 4 distinct objects can be rearranged in 4! ways. We have one more step to cover before arriving at the final answer. The unit of vowels, EAU (3 distinct letters, which we have taken as X) can be rearranged in 3! ways. Therefore, the total number of ways of reordering BEAUTY such that the vowels appear together is 4! * 3! The correct answer is 4! * 3! ways

2. Ans: [a]

'INSTITUTE' has 9 letters.

I repeats 2 times, T repeats 3 times, others occur once.

$\therefore$ Required number of ways $= \dfrac{9!}{3!\,2!}$

3. Ans: [c]

Since the word starts with the capital consonant, the first place can be selected from the 3 capital consonants.

$\therefore$ Number of ways $= {}^{3}C_{1} = 3$ ways

The three small consonants can be selected from the 5 small consonants in ${}^{5}C_{3} = \dfrac{5\times4}{1\times2} = 10$ ways

The two small vowels can be selected from the 4 vowels in ${}^{4}C_{2} = \dfrac{4\times3}{1\times2} = 6$ ways

Now, the 3 consonants and the 2 vowels can be arranged in 5! ways.

$\therefore$ Required number of ways $= 3\times10\times6\times5!$

$$= 3\times10\times6\times120 = 21600$$

4. Ans: [a]

The digits are 1, 2, 3, 4, 5, 6, 7, 8.

Out of the above 8 digits, 4 are even.

The total possible ways of arranging the 8 digits = 8!

Number of ways of arranging the four even numbers in the first 4 places = 4!

The other four places can be filled with the remaining four numbers in 4! ways.

$\therefore$ The number of ways of filling the first four places with even digits = $4!\times4!$

Required probability

$$= \frac{4!\times4!}{8!} = \frac{4!}{5\times6\times7\times8} = \frac{4\times3\times2}{5\times6\times7\times8}$$

$$= \frac{1}{70}$$

5. Ans: [b]

n(S) = 26

n(consonant) = 21

$n(P) = \dfrac{21}{26}$

6. Ans: [c]

$n(S) = 8\{HHH, HHT, HTH, THH, HTT, TTH, THT, TTT\}$

$n(E) = 1\{HTT\}$

$n(P) = \dfrac{1}{8}$

7. Let the number of sides added be x

If originally the polygon has x sides,

Sum of the interior angles $= (2n - 4)\times90°$

After adding x sides,

Sum of the interior angles $= [2(n + x) - 4]\times90°$

$\therefore [2(n+x) - 4]\times90 = (2n-4)\times90 + 540$

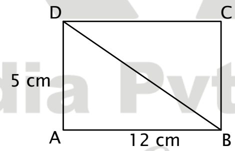
$\Rightarrow 180n + 180x - 360 = 180n - 360 + 540$

$\Rightarrow 180x = 540$

$\Rightarrow x = \dfrac{540}{180} = 3$

$\therefore$ Number of sides to be added = 3.

8. Ans: [c]



The ratio of the adjacent sides is 5:12.

$\therefore$ Let the sides be 5 x and 12x

$\therefore$ Perimeter = 5x +12x + 5x + 12x = 34

$\Rightarrow$ 34x = 34 $\Rightarrow$ x = 1

Hence the sides are 12 cm and 5 cm.

The length of the diagonal = $\sqrt{5^2 + 12^2}$

(Pythagoras theorem)

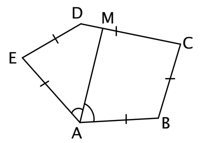
$= \sqrt{25+144}$

$= \sqrt{169} = 13$ cm

9. Ans: [c]

ABCDE is a regular pentagon.

$\therefore \angle A = \angle B = \angle C = \angle D = \angle E$



Sum of the angles of a pentagon = (2n – 4) × 90°

= (2 × 5 – 4) × 90° = 540°

$\therefore \angle A = \angle B = \angle C = \angle D = \angle E = \frac{540°}{5} = 108°$

In quadrilateral ABCM,

$\angle BAM = \frac{1}{2} \times 108° = 54°, \angle B = 108°, \angle C = 108°$

sum of the angles of a quadrilateral = 360°

$\therefore \angle BAM + \angle AMC + \angle C + \angle B = 360°$

$\Rightarrow 540° + \angle AMC + 108° + 108° = 360°$

$\Rightarrow \angle AMC = 360° - 270° = 90°$

10. Ans: [c]

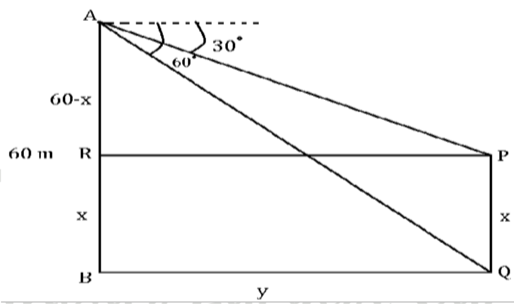
$\frac{\tan 203° + \tan 22°}{1 - \tan 203° \tan 22°} = \tan(203° + 22°)$

$= \tan 225° = \tan(180° + 45°)$

$= \tan 45° = 1$

$\Rightarrow \tan 203° + \tan 22° + \tan 203° \tan 22° = 1$

11. Ans: [c]



Let x be the height of shorter building and y be the distance between the two buildings.

tan 30° = $\frac{AR}{RP} = \frac{60-x}{y} = \frac{1}{\sqrt{3}}$

$\therefore$ y = $\sqrt{3}(60 - x)$ ... (1)

tan 60° = $\frac{AB}{BQ} = \frac{60}{y} = \sqrt{3}$

$\therefore$ y = $\frac{60}{\sqrt{3}}$ ... (2)

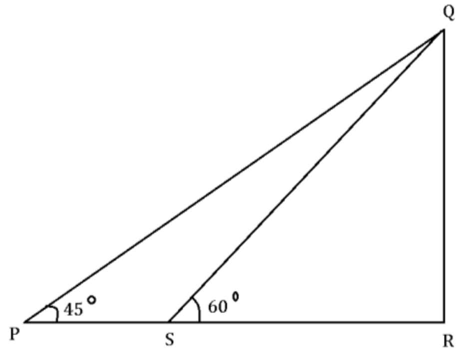
Comparing (1) and (2)

$\sqrt{3}(60 - x) = \frac{60}{\sqrt{3}}$

$\therefore$ 180 – 3x = 60

$\therefore$ x = 40

12. Ans: [a]



tan 45°= $\frac{QR}{PR} = \frac{QR}{PS+SR}$ =1 ⟹ QR=PS+SR ... (1)

tan 60°= $\frac{QR}{SR} = \sqrt{3}$ ⟹ SR= $\frac{QR}{\sqrt{3}}$ ... (2)

From (1) and (2)

PS + SR = $\sqrt{3}$. SR

$\therefore$ PS = ($\sqrt{3}$ – 1)SR

$\therefore$ SR = $\frac{PS}{\sqrt{3}-1} = \frac{PS}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{PS\sqrt{3}+1}{2}$

If it takes car 10 minutes to cover PS, it will take 10. $\frac{\sqrt{3}+1}{2}$ =5($\sqrt{3}$ + 1) minutes to cover SR.

13. Ans: [a]

$\log 1.5 = \log\left(\frac{3}{2}\right) = \log 3 - \log 2$

= 0.4771 – 0.3010 = 0.1761

14. Ans: [c]

If log2 = 0.3010 and log3 = 0.4771, then the value of $\log_5 512$ = 3.876

$= \frac{\log 512}{\log 5}$

= log 2^9/log [10/2]

= 9 log 2_{10}/log 10_{10} – log 2_{10}

= 9(0.3010)/1 – 0.3010

= 2.709/0.699 = 3.876

15. Ans: [b]

F(x) = 3x² – 7x – 5

F(x – 2) = 3(x – 2)² – 7(x – 2) – 5 = 3x² – 12x + 12 – 7x + 14 – 5 = 3x² – 19x + 21

F(x – 2) = 3x² – 19x + 21

16. Ans: [c]

F(x) = √(x + 11)

For domain (x + 11) ≥ 0

⟹ x ≥ – 11

Hence domain of f(x) is [– 11, ∞)

17. Ans: [b]

$5x^2 + 4x + p(p-2) = 0$

The roots are real if discriminant ≥ 0

$\Rightarrow 16 - 20p(p-2) \geq 0$

$\Rightarrow 4 - 5p(p-2) \geq 0$

$\Rightarrow p(p-2) \leq \frac{4}{5}$

The roots will be of opposite sign if $\frac{p(p-2)}{5} < 0$

$\Rightarrow p(p-2) < 0 \Rightarrow p < 0$ and $p > 2$

or

$p > 0$ and $p < 2$

$\Rightarrow 0 < p < 2$ or $(0, 2)$

18. Ans: [c]

Since α, β are the roots of the equation

$(x-a)(x-b) = c$ or $x^2 - (a+b)x + ab - c = 0$

$\alpha + \beta = a + b,$ $\quad\quad \alpha\beta = ab - c$

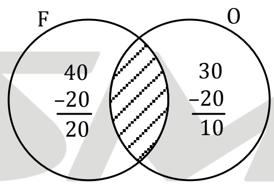
Since $a + b = \alpha + \beta$ and $ab = \alpha\beta + c$, a, b are the roots of

$x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

$\Rightarrow (x-\alpha)(x-\beta) + c = 0$

$\therefore$ The required roots are a and b.

19. Ans: [a]



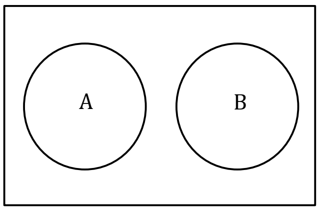
n(F) = 40 n(O) = 30 n(F ∩ O) = 20

The number of workers working in the factory only = n(F) – n(F ∩ O) = 40 – 20 = 20.

The number of workers working in the office only = n(O) – n(F ∩ O) = 30 – 20 = 10

Total number of workers = 20 + 20 + 10 = 50

20. Ans: [c]



$A \subseteq A \cup B$ is true.