PERCENTAGES

INTRODUCTION

Calculating percentages is a basic skill. Every day we deal with data which becomes meaningful if we convert the figures into percentage.

By per cent, we mean "out of 100".
If a student gets 27 marks out of 40, and another gets 35 out of 60, can we say that the second student is smarter? We need to know how many marks they have got if the base was 100, so that the figures are comparable.

CONCEPTS OF PERCENTAGES

Concept 1:
Conversion from fraction to percent and vice versa.
1. Fraction to percent: Multiply the fraction by 100
0.2 = 0.2 * 100 = 20%
3/8 = 3/8 *100 = 37.5%
2. Percent to Fraction: Divide the percent by 100
40% = 40/ 100 = 0.4
55% = 55/100 = 0.55

Concept 2:
Increase or Decrease of a quantity
% increase/ decrease = (Quantity increase or decrease / original quantity) * 100

Example: The salary of a man goes up from Rs. 100 to Rs. 125. What is the percentage increase in the salary?
Solution:
Increase = 125-100 = Rs. 25
Therefore, % increase = 25/100 * 100% = 25%

Concept 3: Increase and Decrease by the same Percentage
Example: A Shopkeeper marks up his goods by 20% but then to encourage sales, reduces the

price by 20%. By doing so, he makes a profit and loss of:
1. 0%
2. 4% profit
3. 4% loss

Solution: Start with 100, after the first increase his price goes up to 120. Reducing 20% of 120, he will have to reduce Rs. 24 and new price is 120 – 24 = 96. So after increasing the price by 20% and reducing it by same percentage, he will make a 4% loss.

Some students make the mistake of marking 0%, but increasing and decreasing by the same percentage will not result in the original figure.

Concept 4: Successive Discounts

If successive discounts are made, then each successive discount must be calculated on the discounted price. Do not make the mistake of adding the discounts.

Example: A retail chain gives a discount of 50% and then to increase sales offers another 40% off. By doing this, it has effectively reduced prices by:
1. 90%
2. 10%
3. 20%
4. 70%

Solution: Start with 100, we arrive at 50 after first discount. Then another 40% discount is given, so we discount 50 by 40% and that gives us 20.

So the effective price is 20 – 20 = 30, so the shopkeeper has effectively reduced prices by 70%.

Concept 5: Compound Growth

Typically compound growths are used in investment growth analysis or in population growth (things like cattle population, production output, and growth).

If **P** is the population of a country and if it grows at **r**% per annum, then the population after **n** years will be:

$A = P \left(\frac{100+r}{100} \right)^n$

Example: A bank offers fixed deposit at 10% per annum. What will be the value of Rs. 10, 000 after 3 years?

Solution:
The value will be:
10, 000 * $\left(\frac{110}{100}\right)^3$ = 10, 000 * 1.1 * 1.1* 1.1 = 13, 310

KEY POINTS

Points to remember:

1. If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is 100*R/ (100+R) %.

2. If the price of a commodity decreases by R%, then the increases n consumption so as not to decreases the expenditure is 100*R/ (100-R) %.

3. Value of machine after n years = p (1-R/100) power n.

4. If A is R% more than B, then b is less than A by 100*R/ (100+R) %.

5. If A is R% less than B, then b is more than A by 100*R/ (100-R) %.

PRACTICE PROBLEMS (EXPLANATORY ANSWERS AT THE END)

1. What is 15% of 700?
A. 105 B. 70 C. 115 D. 75

2. A book-seller sold 30% of his books and left with 420 books. How many books did he have initially in his stock?
A. 1400 B. 1000 C. 800 D. 600

3. A county cricket team has won 10 matches and lost 4. If the matches played represent 70% of the total matches in the tournament, then how many more matches should the team win so as to have a record of exactly 75% wins?
A. 5 B. 6 C. 4 D. 3

4. In an examination, Gautam's score is 25% more than Arun's score. By what percentage, Arun's score is less than Gautam's score?
A. 25% B. 20% C. 16.67% D. 15%

5. A 90 liters solution has 10% salt. How many liters of water must be evaporated to leave the solution with 20% salt? (Note: Salt cannot be evaporated)
A. 50 B. 45 C. 30 D. 36

6. In 2008, the price of a mobile phone increases by 20% with respect to that in 2007. By what percentage is its price in 2007 less than that in 2008?
A. 15% B. 16.67% C. 13.33% D. 20%

7. Abhishek secures 42% of the total marks in an exam and gets 29 marks more than the pass mark. Anirban secures 29% of the total marks in the same exam and fails by 23 marks. What is the pass mark as a percentage of the total marks?
A. 28% B. 32.25% C. 34.75% D. 35%

8. The production of rice in the year 1998 was 1500 tonnes, which was 25% of the total food grain production in that year. In the next year, if the production of rice decreased by 4% and production of rice as a percentage of total food grain production increased by 5 percentage points, what is the total food grain production in1999 (in tonnes)?
A. 4800 B. 4200 C. 4000 D. 4440

9. Rohit's salary is 20% less than Gaurav's salary. If Karan's salary is Rs. 20000 and it is 25% more than Gaurav's salary, then what is Rohit's salary (in Rs.)?
A. 12800 B. 19200 C. 16000 D. 24000

10. In 2000, Abhilash paid a tax of 20% of his salary. In 2001, his salary increased by 33.33%

and the tax scheme changed. Under the new tax scheme, he had to pay a fixed sum of Rs. 1500 and an additional 20% on the amount above Rs. 10000. If he is paying Rs. 100 more in taxes, under the new scheme, what is his salary (in Rs.) in 2001?
A. 12800 B. 19200 C. 16000 D. 24000

11. The success rate of the Indian cricket team in Australia was 25% from 60 matches. If 54 matches were played since then, how many more matches would India have lost if its overall success rate is 50%?
A. 12 B. 14 C. 18 D. 22

12. Due to inflation the total cost of monthly household items has gone up by 25%, but the salary of Gupta had increased by only 20%. Initially, Gupta is used to spend 40% of his salary on household items. What percentage of the present salary would Gupta spend to buy the same quantities of household items?
A. 40% B. 41.33% C. 41.67% D. 42%

13. The cost of production of a Rolex watch which is sold at 35% profit went up by 28%. What should be the percentage increase in the selling price to maintain the profit percentage the same even at the new cost of production?
A. 28% B. 35% C. 63% D. 50%

14. The price of an LED TV has been decreasing every year by a constant percentage over the last 4 years. If the cost of the LED TV was Rs. 150000, 4 years ago and it costs Rs. 98415 now, then find the annual rate of decrease.
A. 8% B. 9% C. 10% D. 15%

15. If the area of a rectangle is increased by 56% and its breadth is increased by 20%, what is the percentage increase in its perimeter?
A. 20% B. 24% C. 30% D. Indeterminate

16. St Mary's Public School has 30% more students than St Joseph's Public School. If 60 students move from St Mary's Public School to St Joseph's Public School, the two schools will have the same number of students. What is the total number of students in both the schools?
A. 600 B. 400 C. 800 D. 920

17. Sharma's tutorial center was started in the year 2004. Every year a certain number of students enroll for class XII tutorials. This number increased by 20% every year. If the number of students in the year 2008 is 576 more than the number of students in the year 2007, how many students enrolled in the year 2005?
A. 2000 B. 2160 C. 2340 D. 2400

18. Rocket Singh is a salesman and has to choose between two schemes of remuneration. Scheme I has a fixed salary of Rs. 3700 and a commission of 2% on sales above Rs. 50000. Scheme II has no fixed salary, but offers 3% commission on the sales up to Rs. 50000 and increases at a rate of 1 percentage point for every increase of Rs. 50000 of sales up to a maximum of 20% of sales. What is the minimum value of the sales (in Rs.) above which he can prefer Scheme II?
A. 50000
B. 89000
C. 140000
D. No such sales value exists.

SOLUTION WITH EXPLNATORY ANSWER

1. 15% of 700 = $\frac{15}{100}$ × 700 = 105.Hence, the correct answer is **option A**.

2. The initial number of books with the book seller is 100%. If 30% of the books are sold he is left with 70%.

Given that 70% of the total books = 420 i.e., $\frac{70}{100}$ × (total books) = 420.So, total number of books = $\frac{420 \times 100}{70}$ = 600.
Alternately,
70% → 420 books
100% → ? books
Through cross multiplication, we have:
The total number of books = $\frac{100}{70}$ × 420 = 600.
Hence, the correct answer is **option D**.

3. 10 wins + 4 losses = 14 matches played.
The 14 matches played represent 70% of the total matches in the tournament.

70% → 14
100% → ?
Through cross multiplication, we have:
The total matches in the tournament are 20.
For the team to have exactly 75% wins, it has to win 75% of (20) = 15 matches.
Since it has already won 10 matches, it still has to win 5 more matches.
Hence, the correct answer is **option A**.

4. Let the score of Arun be 100.
Then the score of Gautam = 100 + 25% (100) = 100 + 25 = 125.
The percentage by which Arun's score is less than that of Gautam is = $\frac{125-100}{125} \times 100 = \frac{25}{125} \times 100 = 20\%$.
Hence, the correct answer is **option B**.

5. It is given that when a salt solution is said to evaporate, only the water content in it evaporates but not the salt.
Amount of salt in the solution = 10% (90) = 9 kg. after 'x' kg of water is evaporated, the amount of solution left behind = (90 – x) kg.
The amount of salt remains unchanged.
The latest percentage of salt in the solution = $\frac{9}{90-x} \times 100$
= 20 (given)
⇔ 900 = (90 - x) 20
⇔ 20x = 1800 – 900 = 900
⇔ x = 45.
The amount of water that is evaporated = 45 kg
Hence, the correct answer is **option B**.

6. Let us assume that price of mobile in 2007 be Rs. 100.
So price of mobile in 2008 = 20% more = 120
To make price in 2008 as 100, we need to multiply by
100/120
So, corresponding price in 2007 will be
100/120 * 100 = 83.33
Hence price is less by 16.67 %
Hence, the correct answer is **option B**.

7. Let x be total marks in exam.
Passing marks = 0.42x – 29
Also, passing marks = 0.29x + 23
So, 0.42x – 29 = 0.29x + 23
⇒ 0.13x = 52

$\Rightarrow x = 400$

Passing marks =
$0.42x - 29 = 0.42(400) - 29 = 139$
Pass marks as a percentage of total marks = 139/400 =34.75%
Hence, the correct answer is **option C**.

8. Let x be food grain production in 1999
Production of rice in 1999 = 0.96(1500)
Production of rice as a percentage of total food grain production = 30%
So, 30% of x = 0.96(1500)
$\Rightarrow x = 0.96\ (1500)/0.3$
$\Rightarrow x = 4800$
Hence, the correct answer is **option A.**

9. Salary of Rohit : Gaurav : Karan =
80 : 100 :125
Now, 125 units correspond to Rs 20,000
So, 80 units will correspond to
20000 * 80/125 = 12800
Hence, the correct answer is **option A.**

10. Let S be salary of Abhilash in 2000.
Tax liability in year 2000 = 0.2 S
Salary in 2001 = 1.33 S
New Tax liability = 1500 + 0.2 (1.33 S – 10000)
So, 100 + 0.2S = 1500 + 0.2 (1.33 S – 10000)
$\Rightarrow 0.066\ S = 600$
$\Rightarrow S = 9000$
$\Rightarrow$ Salary in 2001 = 1.33 S = 12000
Hence, the correct answer is **option D.**

11. From first 60 matches, India won 25% = 15 Wins.
For total 114 matches, India won 50% = 57 Wins
So in 54 matches, India won 57 – 15 = 42 matches.
So, India lost 54 – 42 = 12 matches out of 54 matches.
Hence, the correct answer is **option A.**

12. Assume that the salary of Mr Gupta is Rs 100
Expenditure on house hold items = 40
New Inflated expenditure = 1.25(40) = 50
New Salary = 120
% of Salary into this expenditure =
50/120 = 41.67%
Hence, the correct answer is **option C.**

13. If cost is going up by 28% and profit margin is to be kept the same, the selling price must also go up by same 28%.
Hence, the correct answer is **option A.**

14. Average value for
Actual / Original = $\sqrt[4]{98415/150000}$ = 0.9
Hence the value will be decrease by 10% every year.
Hence, the correct answer is **option C.**

15. Let L, B and A be length, breadth and Area of the rectangle. Also let L', B' and A' be the new length, breadth and area of rectangle.

A' = 1.56 A
B' = 1.2 B
L' = A'/B' = (1.56/ 1.2) * A/B

⇒ L' = 1.3 L
So new perimeter = 2 L' + 2B' = 2.6 L + 2.4 B
And old perimeter = 2L + 2B
Increase in perimeter = 0.6 L + 0.4 B
% Change in perimeter = $\frac{0.6\,L + 0.4\,B}{2L+2B}$
This will vary depending on values of L & B which are both unknown. Hence this is indeterminate.
Hence, the correct answer is **option D.**

16. Let the number of students in St. Joseph Public school be x.
So, the number of students in St. Mary Public school will be
1.3 x 1.3 x – 60 = x +60
⇒ 0.3 x = 120
⇒ x = 120/0.3
⇒ x = 400
Total number of students =
2.3 x = 2.3 * 400 = 920
Hence, the correct answer is **option D.**

17. Let the number of students who enrolled in 2005 be x
So in subsequent years it will be 1.2 x for 2006, 1.44x for 2007 and 1.728x for 2008.
1.728x – 1.44x = 576⇒ 0.288 x = 576
⇒ x = 2000
Hence, the correct answer is **option A.**

18. Let x be minimum salary at which he can start preferring Scheme II.

So, in scheme I his remuneration = 3700 + 0.02(x – 50,000)
Obviously the breakeven will happen in 3rd slab.
So, in scheme II his remuneration = 0.03(50,000) + 0.04(50,000) + 0.05(x – 100,000)
3700 + 0.02(x – 50,000) = 0.03(50,000) + 0.04(50,000) + 0.05(x – 100,000)
⇒ 3700 -1000 = 0.03x + 1500 + 2000 -5000
⇒ 0.03 x = 4200
⇒ x = 4200/0.03
⇒ x = 140,000
Hence, the correct answer is **option C.**