Fuzzy Logic with engineering application

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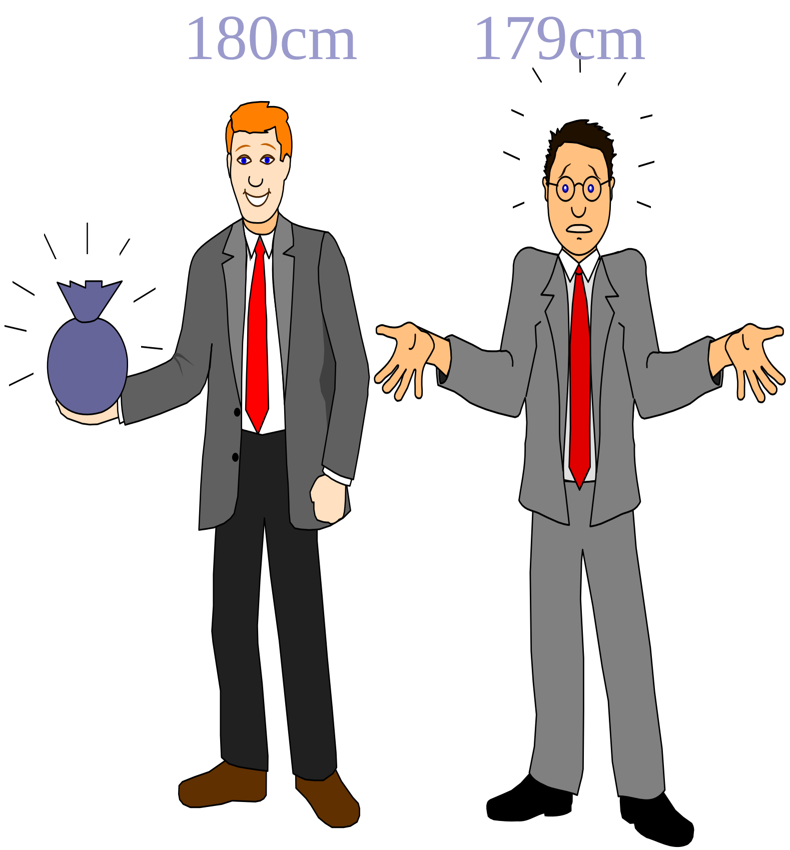
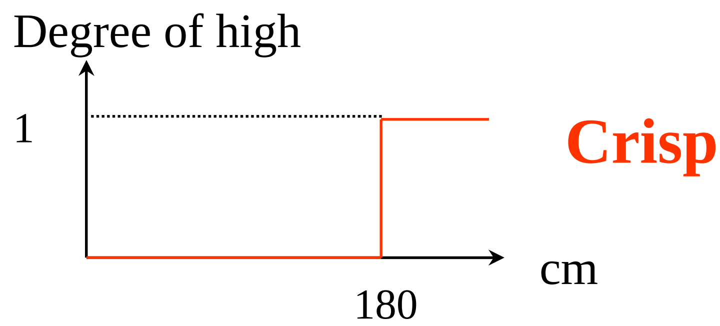
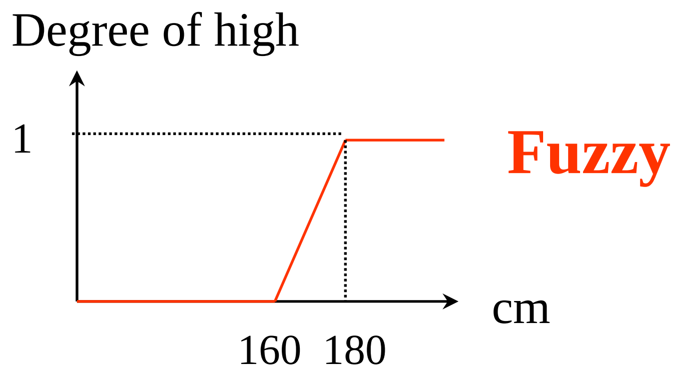
Introduction

Chapter 1

Fuzzy and crisp







THE CASE FOR IMPRECISION

- Our understanding of most physical processes is based largely on imprecise human reasoning.
  - *E.g.*, parking a car, backing up a trailer, navigating a car among others on a freeway, washing clothes, controlling traffic at intersections, and a preliminary understanding of a complex system.
- Requiring precision in engineering models and products translates to requiring high cost and long lead times in production and development.
- When considering the use of fuzzy logic for a given problem, an engineer or scientist should ponder the need for exploiting the tolerance for imprecision.

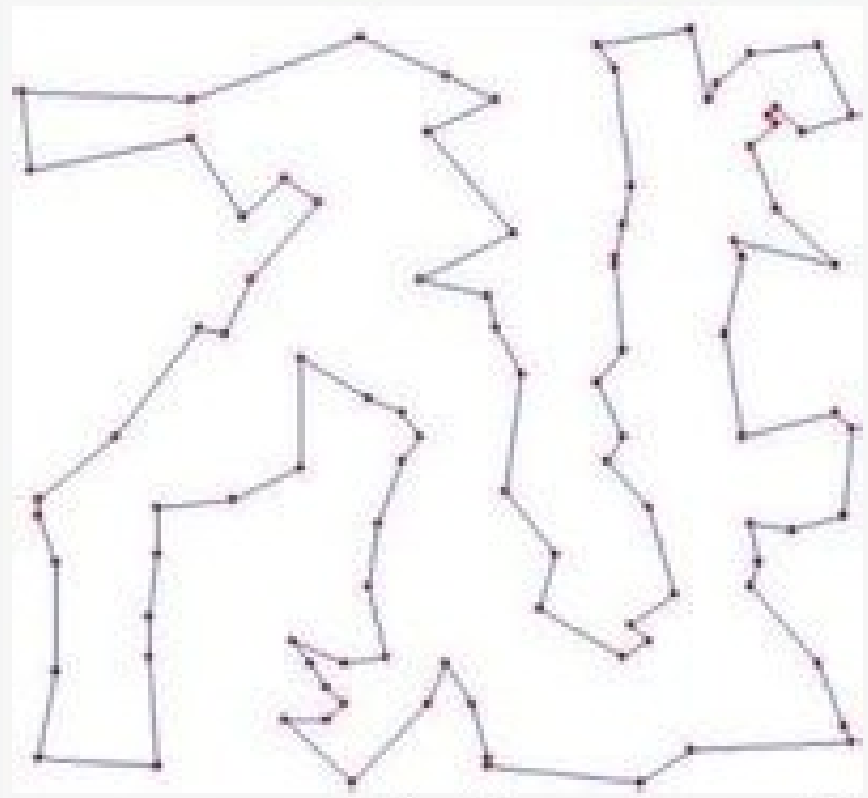
Eample:“traveling salesrep” problem

- A sales representative

wants to minimize total distance traveled by considering various itineraries and schedules between a series of cities on a particular trip.

For example, for 100 cities there are

100 × 99 × 98 × 97×· · ·×2 × 1, or about 10200, possible routes to consider!



TSP问题

Eample:“traveling salesrep” problem

- algorithms have been developed to solve the traveling salesrep problem in an optimal sense; that is, the exact answer is not guaranteed but an optimum answer is achievable.
- remarkable reduction in cost is due solely to the acceptance of a lesser degree of precision in the optimum solution.

THE UTILITY OF FUZZY SYSTEMS

- fuzzy systems more and more popular as solution schemes, and it will make fuzzy systems theory a routine offering in the classroom as opposed to its previous status as a “new, but curious technology.”
- It contains all of what algebra has to offer, plus more, because it can handle all kinds of information not just numerical quantities.

The primary benefit of fuzzy systems theory is

- To approximate system behavior where analytic functions or numerical relations do not exist. Fuzzy systems have high potential to understand the very systems that are devoid of analytic formulations: complex systems.
- Complex systems can be new systems that have not been tested, they can be systems involved with the human condition.

such as biological or medical systems, social, economic, or political systems, (where the vast arrays of in and outputs could not all possibly be captured analytically or controlled in any conventional sense.)

Fuzzy systems are very useful in two general contexts

1. in situations involving highly complex systems whose behaviors are not well understood.
2. in situations where an approximate, but fast, solution is warranted.

UNCERTAINTY AND INFORMATION

Only a small portion of the knowledge (information) for a typical problem might be regarded as certain, or deterministic.

Unfortunately, the vast majority of the material taught in engineering classes is based on the presumption that the knowledge involved is deterministic.

Types of Uncertainty

- Stochastic uncertainty
  - E.g., rolling a dice
- Linguistic uncertainty
  - E.g., low price, tall people, young age
- Informational uncertainty
  - E.g., credit worthiness, honesty

Where uncertainty arises from?

- arises because of complexity;(for example, the complexity in the reliability network of a nuclear reactor.)
- ignorance,
- various classes of randomness,
- the inability to perform adequate measurements
- lack of knowledge
- vagueness, like the fuzziness inherent in our natural language

UNCERTAINTY AND INFORMATION

- Fuzzy sets provide a mathematical way to represent vagueness and fuzziness in humanistic systems.
- For example, teaching child to bake cookies

1)take them out when the temperature inside the cookie dough reaches 375◦F,

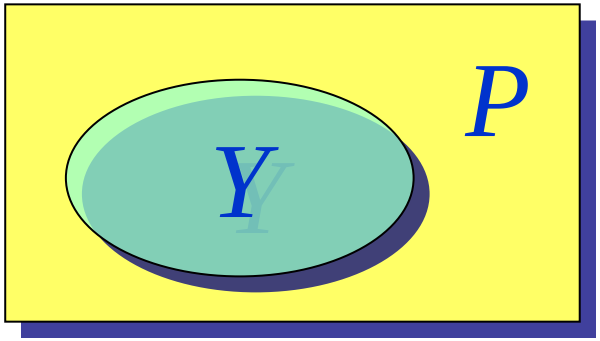
2)take them out when the tops of the cookies turn *light brown*.

Crisp Sets

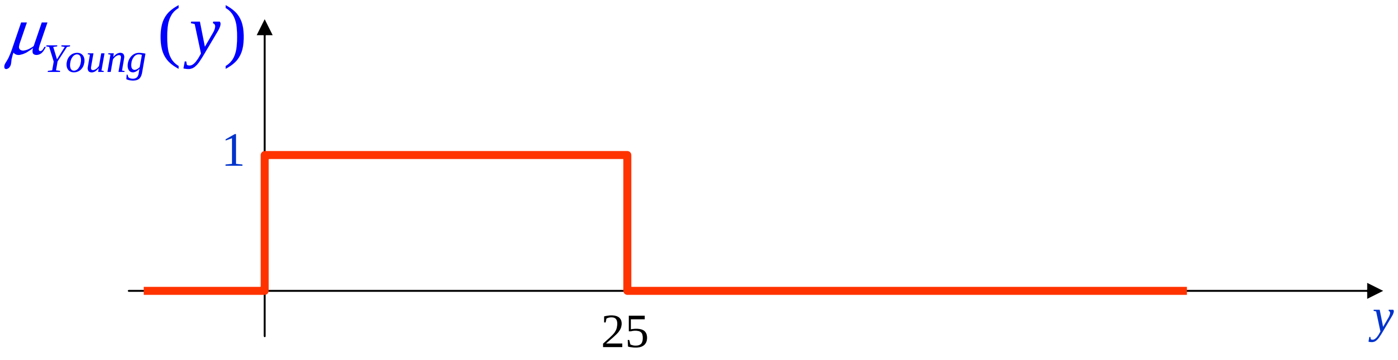
$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

P : the set of all people.

Y : the set of all young people.



$$Young = \{ y \mid y = age(x) \le 25, x \in P \}$$

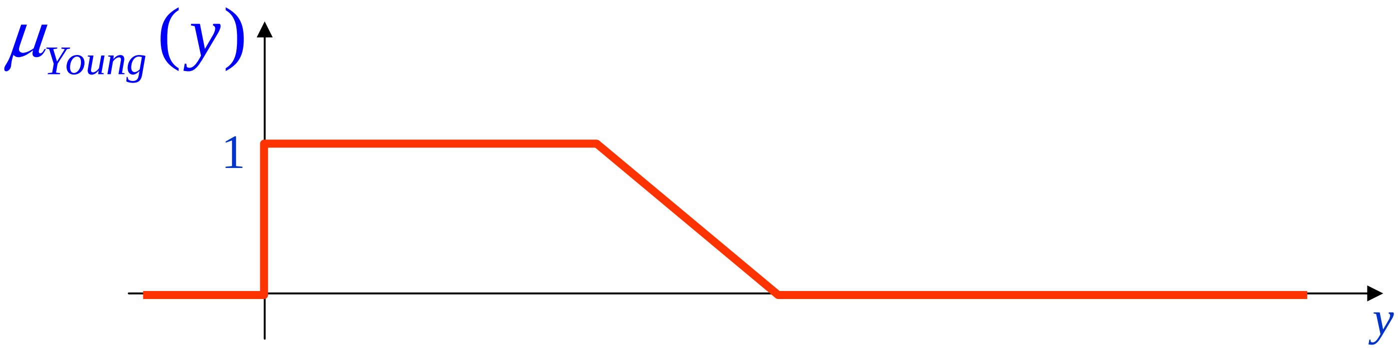


Fuzzy Sets

fuzzy sets $\mu_A(x) \in \{0,1\}$

$$\mu_{\underset{\sim}{A}}(x) \in [0, 1]$$

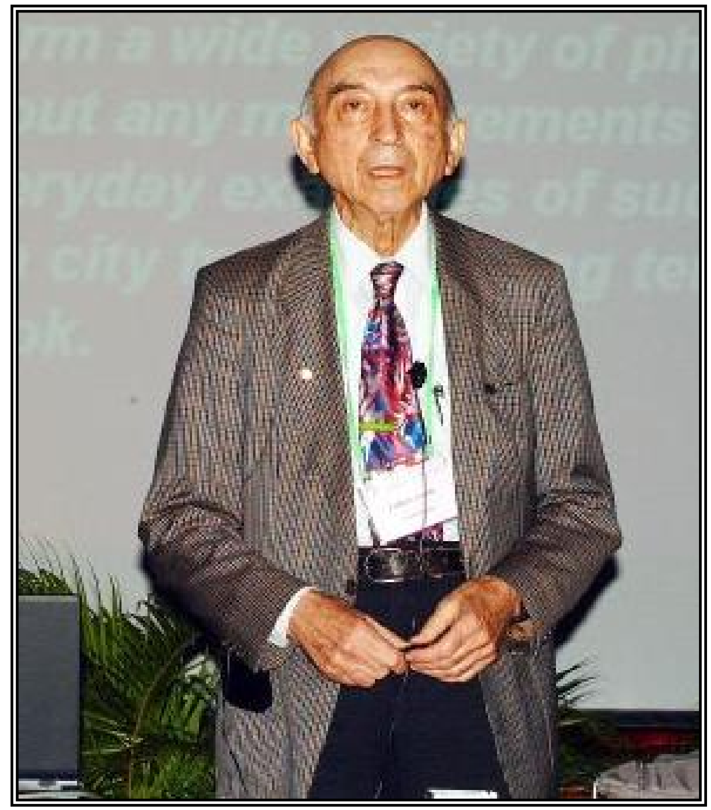
Example



Fuzzy Sets

Lotfi A. Zadeh, The founder of fuzzy logic.

L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, pp. 338-353, 1965.



*L. A. Zadeh，2002-11

CHANCE VERSUS FUZZINESS

- Example involves a personal choice.
- Suppose you can chose two glasses of liquid.
- The liquid in the first glass is described to you as having a 95% chance of being healthful and good.
- The liquid in the second glass is described as having a 0.95 membership in the class of "healthful and good" liquids.
- Which glass would you select, keeping in mind that the first glass has a 5% chance of being filled with nonhealthful liquids, including poisons?

The difference in the information content between chance and fuzziness.

- Suppose we are allowed to test the liquids in the glasses. The prior probability of 0.95 in the case becomes a posterior probability of 1.0 or 0; that is, either the liquid is benign or not.
- However, the membership value of 0.95, which measures the extent to which the drinkability of the liquid is "healthful and good," remains 0.95 after measuring or testing.

Chance versus fuzziness

- *Fuzziness describes the lack of distinction of an event.*

- *whereas chance describes the uncertainty in the occurrence of the event.*

SETS AS POINTS IN HYPERCUBES

The membership function:

- For a universe with only one element, unit interval [0,1];
- For a two-element universe: the unit square;
- For a three-element universe: unit cube.
- For a universe of *n* elements:

  unit hypercube, $I^n = [0, 1]^n$.

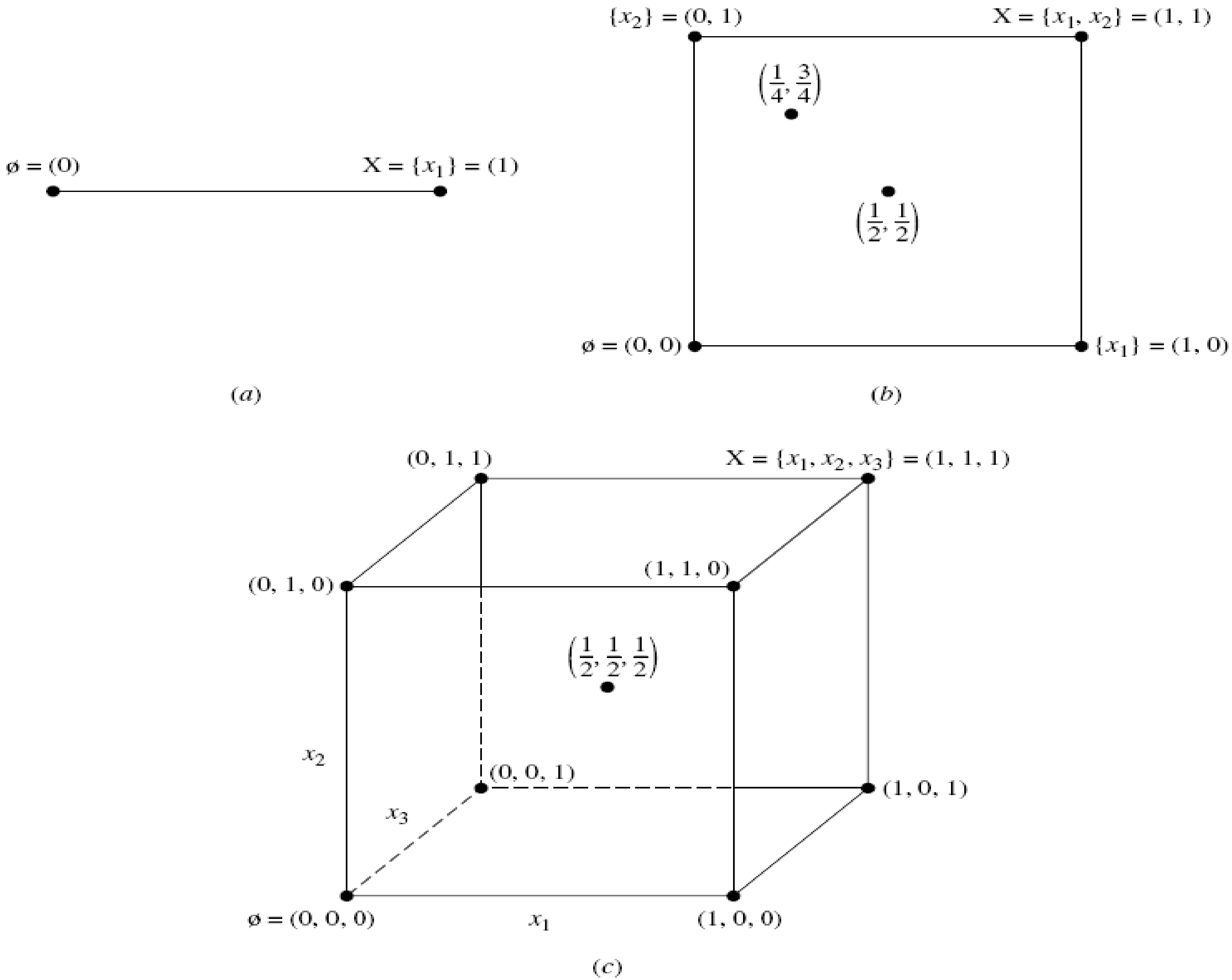


FIGURE 1.4
''Sets as points'' [Kosko, 1992]: (*a*) one-element universe, (*b*) two-element universe, (*c*) three-element universe.

SETS AS POINTS IN HYPERCUBES

- the point (1, 1, 1), is called the whole set, X,

- the point (0, 0, 0), is called the null set, ∅.
- The centroids, represent single points where the membership value for each element in the universe equals1/2.

(A membership value of ½ indicates that the element belongs to the fuzzy set as much as it does not – that is, it holds equal membership in both the fuzzy set and its complement. In a geometric sense, this point is the location in the space that is farthest from any of the vertices and yet equidistant from all of them.)

Applications

- Pattern recognition and clustering
- Fuzzy control
  - Automobiles, air-condition, robotics
- Fuzzy decision
  - Stock market, finance, investment
- Expert system
  - Database, information retrieval, image processing
- Combined with other field
  - Neural network, genetic algorithms