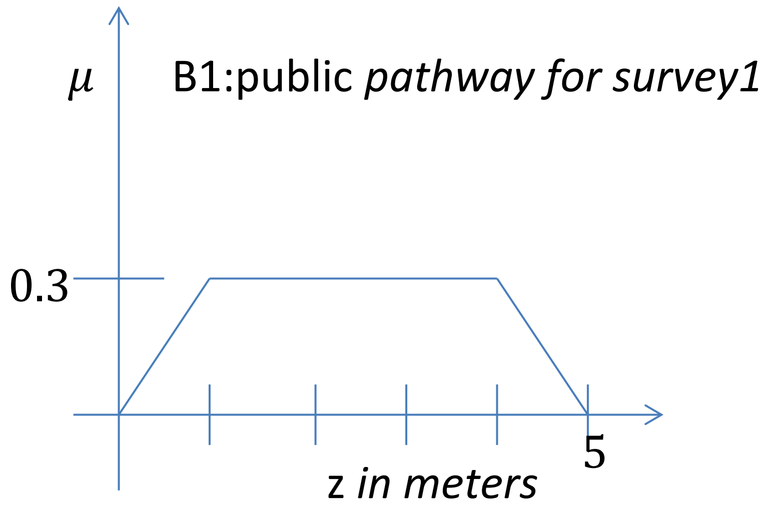
EXAMPLE-1

- A railroad company intends to lay a new rail line in a particular part of a county.
- The whole area through which the new line is passing must be purchased for right-of-way considerations
- It is surveyed in 3 stretches and the data are collected for analysis
- The surveyed data for the road are given by three fuzzy sets B1, B2 and B3
- For the railroad to purchase the land, it must have an assessment of the amount of land to be purchased

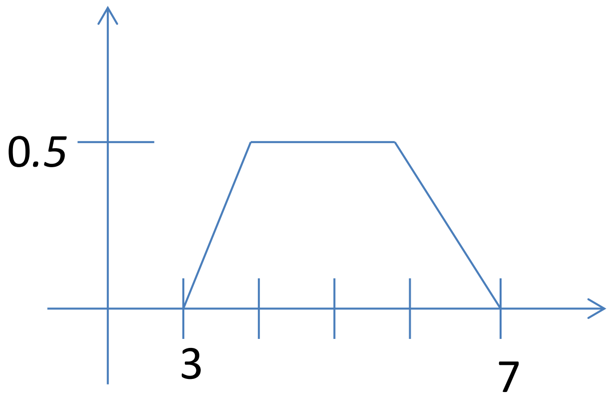
EXAMPLE-1 CONTD…

- The three surveys on right-of-way width are ambiguous
- However, because some of the land along the proposed railway route is already public domain and will not need to be purchased
- The three fuzzy sets are shown in the figures next

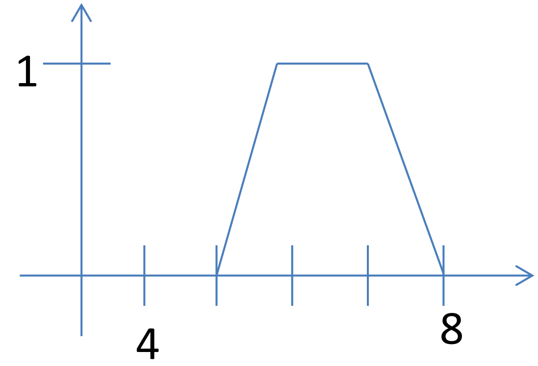


EXAMPLE-1 CONTD…

- Fuzzy set B2: Public right-of-way for survey 2:

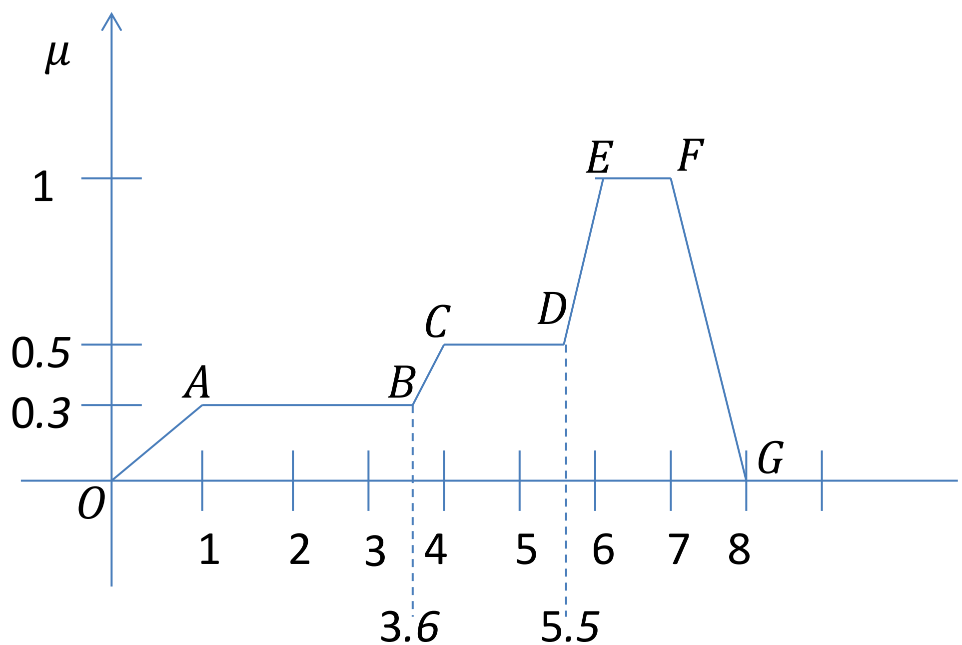


- Fuzzy set B3: Public right-of-way for survey 3:



EXAMPLE-1 CONTD…

- We want to aggregate these three survey results to find the single most nearly representative right-of-way width to allow the railroad to make its initial estimate of the right-of-way purchasing cost.
- The union of the three surveys is given by



COMPUTATION OF THE BOUNDARY EQUATIONS

- We use the 2 point formula to find the equations of the boundary lines
- Equation of the line 'OA' (It joins (0, 0) to (1, 0.3))
- $y - 0 = [(0 - 3\,)/(\,1 - 0)].\,(x - 0)$ i.e. y = (0.3).x
- Equation of the line 'AB' (It is parallel to X-axis)
- Y = 0.3
- Equation of the line 'BC' (It joins (3.6,0.3) to (4, 0.5))

$$\mathrm{y} - 0.3 = \left(\frac{0.5 - 0.3}{4 - 3.6}\right) \times \left(\mathrm{x} - 3.6\right)$$

- That is y = (x-3)/2

COMPUTATION OF THE BOUNDARY EQUATIONS

- Equation of the line ‘CD’ (It is parallel to the X-axis)
- y = 0.5
- Equation of the line ‘DE’ (It joins the points (5.5,0.5) to (6, 1))
- $(y-0.5)=\left(\frac{1-0.5}{6-5.5}\right)\times\left(x-5.5\right)$ i.e. y = x – 5
- Equation of the line ‘EF’ (It is parallel to the X-axis)
- y = 1
- Equation of the line ‘FG’ (It joins the points (7, 1) to (8, 0)
- $(y-1)=\left(\frac{0-1}{8-7}\right)\times(x-7)$ i.e. y = 8 - x

EXAMPLE-1 CONTD…

- We shall use the centroid method to find z*
- z*using the centroid method is given by

$$z* = \frac{\int \mu_B(z).zdz}{\int \mu_B(z)dz}$$

- So, z* =

$$\frac{\int_0^1 \left(0.3z)zdz + \int_1^{3.6}(0.3)zdz + \int_{3.6}^{4} \left((z-3)/2\right)zdz + \int_4^{5.5}(0.5)zdz + \int_{5.5}^{6}(z-5)zdz + \int_6^7 zdz + \int_7^8 (8-z)zdz\right.}{\int_0^1 \left(0.3z)dz + \int_1^{3.6}(0.3)dz + \int_{3.6}^{4} \left((z-3)/2\right)dz + \int_4^{5.5}(0.5)dz + \int_{5.5}^{6}(z-5)dz + \int_6^7 dz + \int_7^8 (8-z)dz\right.}$$

EXAMPLE-1 CONTD…

- $$= \frac{\left[0.3\frac{z^3}{3}\right]_0^1 + \left[0.3\frac{z^2}{2}\right]_1^{3.6} + \left[\frac{z^3}{6} - \frac{3z^2}{4}\right]_{3.6}^{4} + \left[0.5\frac{z^2}{2}\right]_4^{5.5} + \left[\frac{z^3}{3} - 5.\frac{z^2}{2}\right]_{5.5}^{6} + \left[\frac{z^2}{2}\right]_6^{7} + \left[8.\frac{z^2}{2} - \frac{z^3}{6}\right]_7^{8}}{\left[0.3\frac{z^2}{2}\right]_0^1 + [0.3.z]_1^{3.6} + \left[\frac{z^2}{4} - \frac{3z}{2}\right]_{3.6}^{4} + [0.5.z]_4^{5.5} + \left[\frac{z^2}{2} - 5.z\right]_{5.5}^{6} + [z]_6^7 + \left[8.z - \frac{z^2}{4}\right]_7^{8}}$$

- = 4.9m

EXAMPLE-1
(MEAN MAX MEMBERSHIP METHOD)

- In the **mean max membership method** for defuzzification z* is given by (a + b)/2, where **a and b are the minimum and the maximum values where the maximum membership occurs**
- This method (also called **middle-of-maxima**)
- Is closely related to the first method (**Max membership principle**),
- Except that the **locations of the maximum membership can be non-unique** (i.e., the maximum membership can be a plateau rather than a single point).
- This method is given by the expression (Sugeno, 1985; Lee, 1990)

EXAMPLE-1
(WEIGHTED AVERAGE METHOD)

- The weighted average method is the most frequently used in fuzzy applications since it is one of the more computationally efficient methods.
- Unfortunately, it is usually restricted to symmetrical output membership functions.
- It is given by the algebraic expression

$$z* = \frac{\sum \mu_C(\bar{z}).\bar{z}}{\sum \mu_C(\bar{z})}$$

- $\bar{z}$ is the **centroid of each symmetric membership function**.

EXAMPLE-1 CONTD…

- In this case there are 3 symmetric regions
- So, the defuzzified value is

$$z^* = \frac{(0.3 \times 2.5) + (0.5 \times 5) + (1 \times 6.5)}{0.3 + 0.5 + 1}$$

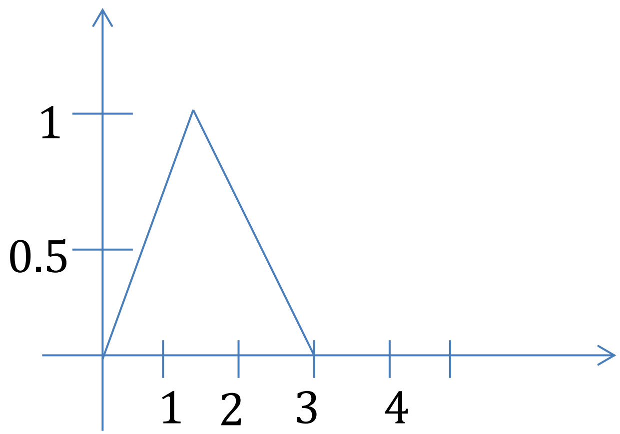
$$= \frac{.75 + .25 + 6.5}{1.8} = 5.41$$

EXAMPLE-2

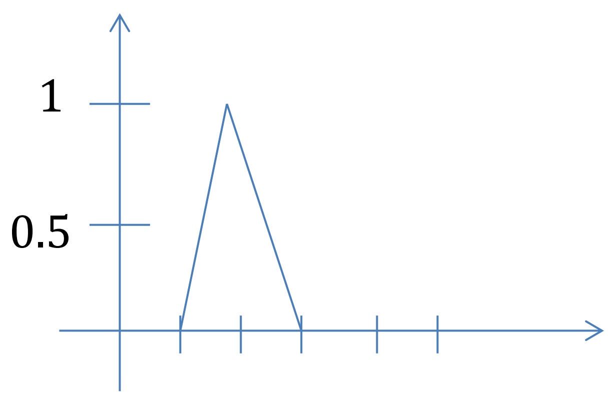
- Many products such as tar, petroleum jelly and petroleum are extracted from crude oil
- In a newly drilled oil well three sets of oil samples are taken and tested for their viscosity
- The results are given in the form of three fuzzy sets B1, B2 and B3
- All defined on a universe of normalised viscosity as shown in the figures below
- We want to find the most nearly representative viscosity value for all three oil samples
- Hence find z* for the three fuzzy viscosity sets

EXAMPLE-2 CONTD…

- The three membership functions are given by

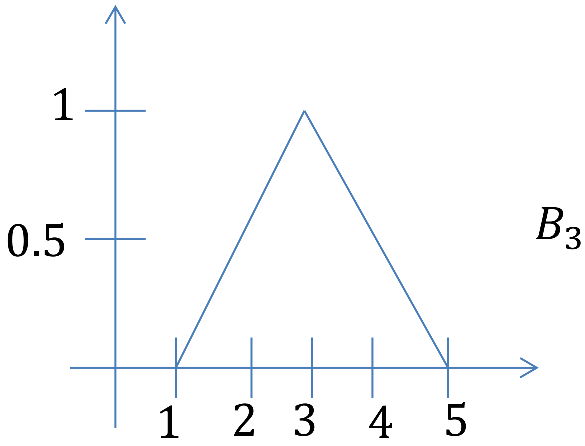


B_1

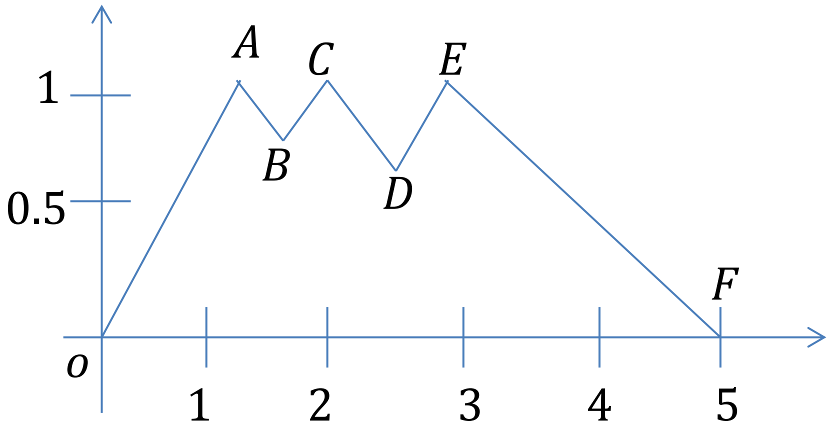


B_2

EXAMPLE-2 CONTD…



- The logical union of the 3 fuzzy sets is given as



EQUATIONS OF BOUNDARY LINES

- The line OA is joining the two points (0, 0) and (1.5, 1)
- Its equation is
- $$(y-0) = \frac{1-0}{1.5-0} \times (x-0)$$ i.e. y = (2/3)x = 0.67 x
- Similarly all other equations can be obtained.
- So, $$z^* = \frac{\int z.\mu_B(z)dz}{\int \mu_B(z)dz}$$

$$\frac{\int_0^{1.5}(0.67z)zdz + \int_{1.5}^{1.8}(2-0,67z)zdz + \int_{1.8}^{2}(z-1)zdz + \int_2^{2.33}(3-z)zdz + \int_{2.33}^{3}(0.5z-0.5)zdz + \int_3^5(2.5-0.5z)zdz}{\int_0^{1.5}(0.67z)dz + \int_{1.5}^{1.8}(2-0,67z)dz + \int_{1.8}^{2}(z-1)dz + \int_2^{2.33}(3-z)dz + \int_{2.33}^{3}(0.5z-0.5)dz + \int_3^5(2.5-0.5z)dz}$$

- =2.5m.

COMPUTATIONS

- The other computations are similar to those in example-1