Fourier Transforms and its Applications in Image Enhancement

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Abstract— Images are used in varieties of fields in today's world to help procure insightful information such as in medical diagnostics, object detection, fingerprint evaluation, satellite monitoring etc. Image processing techniques are often used as an optimised method to enable the efficient processing of these tasks. Fourier transform is one such important image processing method that can mathematically transform the image to frequency domain from the spatial domain. Throughout the course of this paper, we apply a number of frequency domain filters for reducing the noise as well as to detect the edges. The quality of reconstructed images is analysed by calculating different parameters like mean squared error and peak signal noise ratio. The best filter for a particular application is then determined through the tabulation and analysis of these results

I. INTRODUCTION

Image processing [1] includes the different techniques of manipulating the default nature of an image so as to either substantially improve its pictorial information for human perception and interpretation or in order to render it in a way that is more suitable for processing, storage and representation on machines. Most present day image processing techniques involve treating the image as a two dimensional signal represented in a numerical matrix and then applying signal processing techniques.

Image processing consists of all the methods that take in an image (an array of pixels, each with data indicating its brightness or "grey scale" value or perhaps any colour information such as rgb values or 1-bit monochrome) and output a processed and substantially improved image. [2] The resulting image in most cases is of similar specifications (number of pixels and number of grey levels) to the original image. Other aspects of the resulting pixels such as brightness will have been modified according to the process that takes into reason the original value of the pixel and its neighbour's value in most cases, or for instance its position or spatial relationship to the other pixels in its proximity . Sometimes, a new image is produced from the combination of several pictures. The spatial, time or frequency domains are where these principal methods of processing are described to work in.

II. LITERATURE REVIEW

D Maheshwari and V Radha conducted a study in which a number of filters including Median filter, relaxed median filter, Wiener, Centre weighted median, averaging filters are applied to remove noises present in a compound image. They sought to assess the performance of the proposed filters against different types of noises such as salt-and pepper noise, Gaussian noise and speckle noise. The PSNR value of each filter was calculated and used to obtain the comparative performance. From these study it was concluded that the relaxed median, with highest PSNR value was more efficient in reducing all the 3 filters from compound images.

The authors Ajay Kumar Boyata and Birendra Kumar Joshi proposed an algorithm based on log energy distribution approach using an integrating wavelet transformation and wiener filter to remove Poisson – Gaussian noise present in an image. The algorithm was successful in giving a constant level of accuracy along with valid results. Propose method was more efficient in a high noise environment. The paper by Mandeep Kaur and Dinesh Kumar presents a two dimensional FFT removal algorithm to decrease the effect of the periodic noise in natural and strain images. For the periodic pattern of the artefacts, the authors extracted and removed the peaks which are corresponding to periodic noise in the frequency domain by applying the two dimensional FFT on the strain and natural images. The mean filter is then applied to get further more effective results. The performance of the proposed method is tested on both natural and strain images. The results of this proposed method was compared with the mean filter based periodic noise removal from which it was found that the proposed method significantly improved the noise removal.

The authors Mukesh Chandra Arya and Ashish Semwal haveperformed a comparison on the results of applyinganaverage/mean filter, a median filter and a wiener filter on lung image so as to remove noise while preserving edges.The cumulative performance of these filters were judged on the basis ofthe parameters peak signal to noise ratio(PSNR), meansquare error(MSE) and root mean square error(RMSE). The results were simulated on MATLAB. The paper proves thatthe Wiener filter indeed provides the optimal results for noise reductionfrom a lung image in comparison to Average/mean filter andmedian filter.In this paper, Lakhwinder Kaur, Savita Gupta and R.C. Chauhan propose to implement an adaptive threshold estimation methodcalled NormalShrink. It aims to perform image denoising in the waveletdomain based on the generalized Gaussian distributionmodeling of subband coefficients.Since the parameters requiredfor estimating the threshold depending on subband data, this method is computationallymore efficient and adaptive. Thispaper comes to the conclusion that when compared to theother denoising techniques such as Wiener filtering,BayesShrink and SureShrink, the Normal sink outperformsthe competition most of the time. In this paper, Devanand Bhonsle, Vivek Chandra and G.R.Sinha propose to implement a bilateral filtering to denoise medicalimagery which are often corrupted by Gaussian white noise of random values of variances. They conclude by deducing from the result that this method removes additive white Gaussian noise effectively but offers poor performances in the removal of salt and pepper noise. They also find that it provides an significantly improved performance in the removal of noise in high frequency area but seems to often fail in the removal of noise in low frequency area

III. FOURIER TRANSFORMS

The Fourier Transformations [3] has a paramount significance in image related enhancement and processing . It gives us the freedom to perform tasks which would be otherwise impossible to perform through any other procedure; its efficiency permits us to perform other tasks faster. It gives us a more powerful alternative to linear spatial filtering [4]; which makes it far more efficient to make use of the Fourier transform rather than a spatial filter for a large filter. We are able to isolate and process particular image “frequencies” using this fourier transform, and as a result we are able to obtain a high degree of perfection when we perform low-pass and high-pass filtering.

A. THE ONE DIMENSIONAL DISCRETE FOURIER TRANSFORM

Discrete Fourier transform (DFT) [5] proceeds to obtain a same-length sequence of equally-spaced samples of the discrete-time Fourier transform (DTFT) from a finite sequence of equally-spaced samples of a function , which is a complex-valued function of frequency. The DTFT is sampled at intervals equal to the reciprocal of the duration of the input sequence.
For a discrete set of successions of length N,

We define its discrete Fourier transform to be the sequence$f = [f0, f1, f2 \dots \dots (N-1)]$

where,$F = [F0, F1, F2, \dots \dots F(N-1)$

$$Fu = \frac{1}{N} \sum_{x=0}^{N-1} exp\left[-2\pi \frac{xu}{N}\right] fx \qquad (1)$$

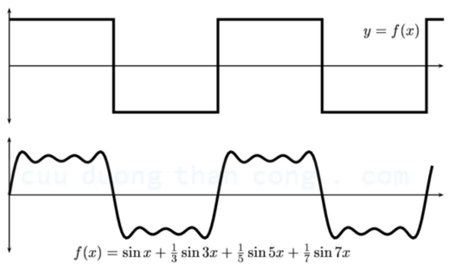


Figure 1: A function and it’s trigonometric approximation

The inverse Fourier transform [6] implements a discrete version of the continuous Fourier synthesis wave by performing the reverse Fourier transform and constructing a waveform from its Fourier coefficients.
For a Fourier synthesis wave of length N

$$x_u = \sum_{x=0}^{N-1} \exp\left[2\pi i \frac{xu}{N}\right] F_u \qquad (2)$$

B. TWO DIMENSIONAL DISCRETE FOURIER TRANSFORMS

In a two dimensional context, The discrete fourier transform takes a matrix as input, and returns another matrix, of the same size, as output in the two dimensional discrete fourier transforms. If the original matrix values are , where and are the indices, then the output matrix values are . The Fourier transform of the matrix f is called matrix F. The original matrix is the inverse Fourier transform of the output matrix.
The two-dimensional discrete Fourier transform definition strikes a resemblance to the one-dimensional Foruier transform. Assuming that the x indices are from 0 to M-1 and the y indices are from 0 to N-1, the forward and inverse transforms for an matrix are:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \exp\left[-2\pi i\left(\frac{xu}{M}+\frac{yv}{N}\right)\right].$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v) \exp\left[2\pi i\left(\frac{xu}{M}+\frac{yv}{N}\right)\right]$$

The original matrix is then rewritten in terms of sums of corrugations [7] by the 2-D DFT. A matrix whose elements give the height of the corrugation for which the horizontal and vertical distances between consecutive troughs are 1/i and 1/j is the magnitude of the Fourier transform.

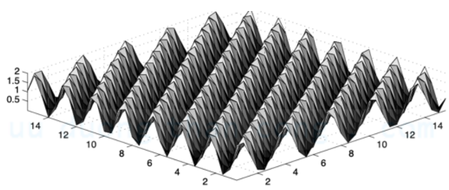


Figure 2: A ‘corrugation’ function formed after 2D DFT

IV. IMAGE ENHANCEMENT USING FREQUENCY DOMAIN TECHNIQUE BY MODIFYING THE FOURIER TRANSFORM

Due to the low efficiency in calculating the DFT, the previous algorithms cannot be used for computing the same. The fast Fourier transform or FFT [8] significantly decreases the amount of time which is needed to compile a DFT. There are various versions of the FFT algorithm, one of the oldest and the most efficient algorithms being the Decimation in time [9] algorithm. This FFT method works recursively by dividing the original vector into two halves, computing the FFT of each half, and then putting the results together. According to the equation, the FFT is at peak efficiency in the case of the value of the power of 2 being the vector length

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n) e^{-\left(i\frac{2\pi nk}{N}\right)} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x(2n) e^{-\left(i\frac{2\pi\times(2n)k}{N}\right)} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) e^{-\left(i\frac{2\pi(2n+1)k}{N}\right)} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x(2n) e^{-\left(i\frac{2\pi nk}{\frac{N}{2}}\right)} + e^{-\left(i\frac{2\pi k}{N}\right)} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) e^{-\left(i\frac{2\pi nk}{\frac{N}{2}}\right)} \\
&= \mathrm{DFT}_{\frac{N}{2}} \left[\left[x(0), x(2), \ldots, x(N-2)\right]\right] + W_N^k \, \mathrm{DFT}_{\frac{N}{2}} \left[\left[x(1), x(3), \ldots, x(N-1)\right]\right]
\end{aligned} \tag{3}
$$

Emphasizing the high-frequency contents of an image to enhance the edge and detail information in it is an efficient method to increase the visual appearance of the image[10]. This process of manipulating or altering digital images in order to obtain results that are more suitable for interpretation, display or further image analysis is called image enhancement. Converting an image to its Fourier transform can help in implementing these adjustments more efficiently. The image processing techniques discussed in this manuscript are noise reduction and edge enhancement.

The basic steps involved in the filtering of an image using the frequency domain technique [11] are:

i. T Transform the input matrix of the image to its corresponding Fourier transform.
ii. Apply filter on the transformed image for enhancement.
iii. Take the inverse Fourier transform of the image to get the resulting enhanced image.

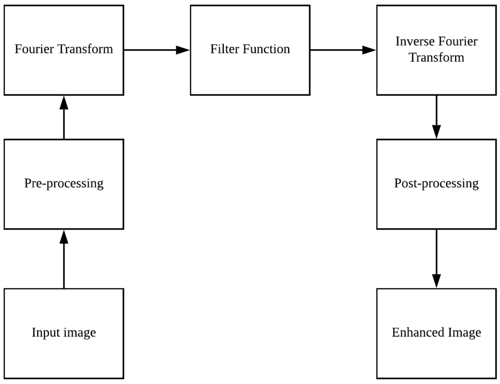


Figure 3: Frequency Domain Filtering Operations

V. NOISE REDUCTION IN IMAGES USING FOURIER TRANSFORM

Noise [12] in an image is any unwanted pixel present. It destroys image quality and results in loss of information. Images can get distorted due to various types of noise such as gaussian noise, Poisson noise, speckle noise, salt and pepper noise and many more. These noises can occur from a noise sources present in the vicinity of image capturing devices, faulty memory location or may be introduced due to faulty image capturing devices like camera.

In this manuscript we'll be focusing on filtering gaussian noise from an image. Gaussian noise [13] is a type of noise, caused by random fluctuations in the input signal. Such noises will be normally distributed. A gaussian filter can be implemented by first creating a low pass filter and the multiplying it with the image transform and finally inverting the result.

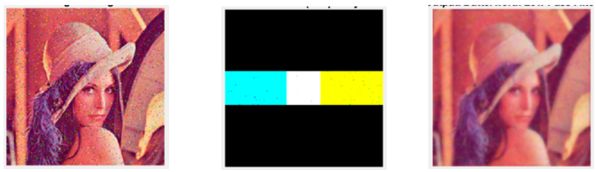


Figure 4: Gaussian Noise reduction of lenna.jpg

VI.. EDGE DETECTION IN IMAGES USING FOURIER TRANSFORM

Edge detection can have many applications in the computer vision field. Once we detect the edges, the output image can be helpful for feature extraction, pattern detection and various other purposes.
The process to detect edges using frequency domain technique are as follows[1]:

i. Transform the input image into the Fourier domain.
ii. Apply the High pass filter.
iii Convert back the image into its original form using Inverse Fourier Transformation.

A filter, in pure mathematical terms, is a matrix with values varying from 0 to 1. If the component is 1, then the frequency is allowed to pass, else the frequency gets blocked [5]. High pass filter suppresses low frequency components without affecting high frequency parts if the image in the frequency domain..
In an image usually the edges are made of higher frequencies and high pass filter attenuates everything other than the edges in the image. So we apply high pass filter on our transformed image for edge detection.

The three main high pass filters are

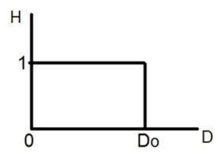
i. An ideal kernel
ii. Gaussian high pass kernel
iii. Butterworth High Pass Filter kernel.

i) Ideal High Pass Filter:

An Ideal High Pass filter cuts all frequencies lower than a specified cut-off frequency.

$$L(u,v) = \begin{cases} 0 \\ 1 \end{cases} 0 \; if \; U(u,v) \le U0, 1 \; if > U0 \tag{4}$$

This is a representation of ideal high pass filter[3]



ii) Gaussian High Pass Filter

I The Gaussian low pass filter attenuates frequency components that are further away from the centre in the frequency domain. [4][5] Hence smoother cut off process takes place resulting in more accurate image.

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$$H(u,v) = 1 - e^{-d^2(n,v)/2D0^2} \quad (5)$$

The standard deviation of the function here is represented by D0 also known as sigma. In this case value of Do is taken greater than 1 [4]. Due to the smoother cut off process ringing effect reduces in the case of Gaussian filter compared to Ideal filter.

From the given output images 2 , It's clear that the Gaussian filter is smoother than ideal filter.

iii) Butterworth High Pass Filter

The third high pass filter is the Butterworth filter. A Butterworth filter of order n and cut off frequency D0 can be represented as [4]

$$H(u,v) = 1/[1 + (\frac{D0}{D(u,v)})]^{2\Pi} \quad (6)$$

The Butterworth high-pass filter can control the sharpness of output image by changing its order. A Butterworth high pass filter keeps frequencies outside radius D0 and discards values inside. We ensure a smooth transition between 0 and 1, that reduce ringing effects in the output image.

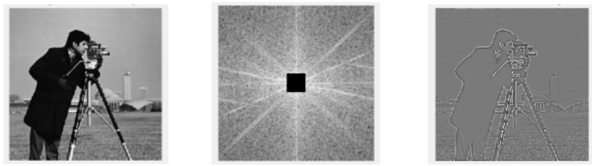

Figure 5: Edge detection using high pass filter

VII. ANALYSIS OF VARIOUS DENOISING TECHNIQUES

Images constitute of different kinds of noises. The method of noise reduction used for a type of noise may not be effective for the other types of noise. Here we apply different methods to reduce the noises present and enhance the image, we also compared this image with the original one. The image consisting of noise will be filtered using three various types of filters. The image is first converted to its center shifted Fourier transform before applying the filter onto the image. The inverse fftshift function is used to reconstruct the image. Analysis of full reference quality metrics functions such as immse,psnr and ssim, enabled us to deduce the most suitable filter for each of the noises

A. Gaussian Noise

These are those noises whose values are normally distributed. The equation represents its probability density function.

$$p_G(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \quad (7)$$

Here z is the grey level value, μ is mean grey value and σ its standard deviation. Gaussian noise arises in images due to factors like poor lighting, high temperature etc. Low pass filters can be used to remove these noises. Among the 3 filters, Gaussian low pass filter had least mean squared error compared to the other filters (Ideal and Butterworth). It also had more structural similarity index value and SNR value. From these results we came to a conclusion that Gaussian low pass filter was more effective in removing the Gaussian noises in the image.

Filter	Mean-Squared error	SSIM Index	Peak-SNR value	SNR Value
Ideal	0.0069	0.7087	21.6114	15.4252
Gaussian	0.0065	0.7208	21.8868	15.7006
Butterworth	0.0075	0.6954	21.2713	15.0851

B. Salt and Pepper Noise

These noises are caused by sharp and sudden disturbances in the input signal. In image it is seen as random black and white pixels. After adding these noises using a function, we applied the 3 low pass filters noise and found that the Gaussian low pass filter had lower mean squared error, higher structural similarity index value and SNR value compared to the other filters. In conclusion, Gaussian low pass filter was more effective in removing salt and pepper noise and restoring image quality.

Filter	Mean-Squared error	SSIM Index	Peak-SNR value	SNR Value
Ideal	0.0052	0.6816	22.7881	16.69
Gaussian	0.0044	0.7078	23.5551	17.36
Butterworth	0.0054	0.6866	22.6791	16.49

C. Poisson Noise

These noises can occur due to the nature of EM waves. These noises obey the Poisson distribution and are given as:

$$P(f_{(pi)}) = k) = \frac{\lambda^{k}{}_{i} e^{-\lambda}}{k!} \quad (8)$$

We applied the 3 filters on the image after adding Poisson noise, we found that Ideal low pass filter had had lower mean squared error, higher structural similarity index value and SNR value compared to the other filters and therefore was more effective in removing Poisson noise and restoring image quality.

Filter	Mean-Squared error	SSIM Index	Peak-SNR value	SNR Value
Ideal	0.0032	0.7779	24.9791	18.79
Gaussian	0.0036	0.7466	24.4166	18.23
Butterworth	0.0048	0.7125	23.2061	17.02

D. Speckle Noise

These noises are inherently present in an image in the form of grains thereby degrading the image quality. For generating this type of noise we multiply each value in the image matrix with a random pixel value. Its probability density function follows gamma distribution and is given by:

$$F(g) = \frac{g^{\alpha-1} e^{\frac{-g}{a}}}{\alpha-1!\,a^{\alpha}} \quad (9)$$

To remove such noises, we applied 3 low pass filters on the image after generating Speckle noise and found that the Gaussian low pass filter had lower mean squared error, higher structural similarity index value and SNR value compared to the other filters. So Gaussian low pass filter was more effective in removing Speckle noise and restoring image quality.

Filter	Mean-Squared error	SSIM Index	Peak-SNR value	SNR Value
Ideal	0.0049	0.6991	23.1235	16.93
Gaussian	0.0041	0.7219	23.8610	17.67
Butterworth	0.0051	0.6992	22.9625	16.77

VIII. CONCLUSION

The pupose of this paper was to apply Image processing techniques on Noisy images using Fourier transformation and to perform edge detection on images using Fourier transforms. After a short review of the various filters used for image processing, we have achieved satisfactory results in denoising images using our proposed implementation of the denoising filters. We have also performed an analysis to figure out which filter is best in denoising a specific noise in an image. We have also implemented filters using the Fourier transforms for detecting the edges in images.

There is considerable amount of limitations in using this technique for denoising in the present world. Therefore, there is window for improvement by adopting the latest methods and algorithms used in the world of computer vision.

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