An Optimization Based Inverse Kinematics of Redundant Robots Avoiding Obstacles and Singularities

V. V. M. J. Satish Chembuly
Department of Mechanical Engineering
National Institute of Technology, Warangal
Telangana, India-506004
satishchv@gmail.com

Hari Kumar Voruganti
Department of Mechanical Engineering
National Institute of Technology, Warangal
Telangana, India-506004
harikumar@gmail.com

ABSTRACT

Redundant manipulators are characterized by a high number of degrees of freedom (DOF) than the required number to perform a given task. This additional DOF of the robot enhances it to work in the cluttered environment by avoiding obstacles and provides improved dexterity while performing a given task. Inverse Kinematics (IK) of redundant manipulators has infinite solutions. Among these infinite solutions, only those solutions are preferred which fulfill the criteria such as joint distance minimization, singularity avoidance, and joint torque minimization. This paper focuses on the IK of redundant manipulators for a given path with secondary objectives as performance criteria. The IK problem is formulated as an optimization problem by choosing the joint distance and singularity avoidance as objectives and collision with obstacles in the workspace as constraints. Simulations have been performed on serial redundant manipulators by varying different types of obstacles and their positions in the workspace. Results are also reported on redundancy resolution of serial manipulators based on singularity avoidance criterion.

CCS CONCEPTS

Robotics →Redundant robots→ Inverse kinematics

KEYWORDS

Redundant Robot, Redundancy Resolution, Inverse Kinematics, Obstacle Avoidance, Singularity Avoidance

1 INTRODUCTION

Kinematic redundancy of the robot occurs when a robot possesses more degrees of freedom than required to perform the given task [1].

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In such cases, the kinematic equations relating joint angles and end-effector position/orientation in task space often leads to multiple/infinite solutions.
The choice of picking best among available solutions is based on criteria such as minimization of joint torque [2], energy minimization [3], obstacle avoidance [4], singularity avoidance [5] and joint limit avoidance [6] of the robot while end-effector is performing a given task in the workspace. The problem of choosing a unique solution from the solution set is called redundancy resolution.

Many researchers have proposed several methods in IK of redundant manipulators. These are broadly classified as algebraic [7], geometric [8] and iterative approaches [9]. The complexity of IK problem increases with the increase in kinematic redundancy. This problem has been solved as the distance minimization between the end-effector location and task space by using evolutionary algorithms. Koker [10] used neural networks and genetic algorithm together as a hybrid approach in order to get the solutions to the IK problems for a robotic manipulator. But the approach gave the solutions with a decline in the precision level. Bingul and Ertunc [11] also used neural-network approach using back propagation algorithm to get the IK solution. But, it failed to give multiple solutions to the IK equations and results show considerable large errors in the joint variables.

These methods suffer from certain shortcomings. In algebraic approach closed form solutions are not guaranteed; iterative approach converges to only one solution based on the initial guess and for geometric approach closed form solutions of first three joints must exist geometrically.

A lot of research has been carried out in redundancy resolution methods, mostly redundancy resolution considered at velocity and acceleration level in local and global optimal control. Local control determines the joint trajectory at each point along the end-effector path in which the performance criterion is locally optimized. Global approach determines the joint trajectory by optimization of performance criteria over the entire end-effector path.

The most popularly adopted local optimization control problem is by the use of pseudo-inverse of Jacobian [12], the pseudo-inverse method has been applied by including null space term to optimize secondary objective such as singularity avoidance, obstacle avoidance, joint limit avoidance, minimization of joint torque and maximization of manipulability measure. Obstacle avoidance is to choose a configuration such that collision of robot with obstacles is avoided. Baillieul [13] proposed an extended Jacobian method, which supplemented the velocity equations by an additional set of equations equal to the number of unknown joint rates. Nakamura [14] proposed an algorithm to avoid obstacles by placing a restriction on joint angles indirectly while using the pseudo-inverse of the manipulator Jacobian to resolve the redundancy. Kircanski and Petrovic [15] proposed a combination of the analytical and the pseudoinverse solution. An extensively used method for obstacle avoidance proposed by Khatib [16] is artificial potential field method, where the manipulator moves in the field of forces that attract the end-effector towards the goal and repel the manipulator parts from the obstacles. Maciejewski and Klein [17] proposed obstacle avoidance by introducing a secondary task which controls the point on the robot closest to the obstacle, by maximizing its distance from the surface of the obstacle. There are some drawbacks with these methods like potential field methods suffer with local minima and pseudo-inverse approach is computationally expensive. In contrast to above resolution schemes Singla et al. [18] proposed a high index norm minimization approach to resolve redundancy for serial manipulators. They have explored the norms of intermediate indices and observed that 8-norm solutions are better than other minimum norm solutions. Zhang et al. [18] proposed an equivalence in position and velocity level resolution schemes.

This paper demonstrates the redundancy resolution scheme of the serial redundant manipulator at position level by considering obstacle avoidance and singularity avoidance as a secondary criterion. As the proposed approach is developed as resolution scheme at position level, the forward kinematic model itself is used for constraint formulation of robot, while travelling in task space. Hence, finding inverse of Jacobian is not required. Thus, the approach is computationally efficient and effective for planar redundant manipulators with obstacles in the workspace. Effectiveness of method lies in simple and effective modelling of obstacles and classical optimization methods adopted to solve the problem. Both convex and non-convex obstacles can be handled using this approach. Restart procedure with different initial guesses is incorporated in the algorithm to avoid the problem of getting IK solution at local minima. This avoids the use of evolutionary optimization techniques, which are computationally expensive. The proposed approach can be easily incorporated to real-time applications because of its efficiency. Effectiveness of the method is demonstrated by simulating various case studies of redundant manipulators, shown in Section 6.

2 KINEMATICS OF REDUNDANT MANIPULATOR

Consider a $n-DOF$ planar manipulator shown in the fig 1. If the length of i^{th}link is l_i, θ_i represents the angle between the i^{th} link and the $x-axis$. The end-effector position $E(x_{te}, y_{te})$ is given by

$$x_{te} = l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 + \dots + l_n \cos\theta_n \qquad (1)$$

$$y_{te} = l_1 \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 + \dots + l_n \sin\theta_n \qquad (2)$$

Forward kinematic model of the end effector is related as

$$E(x_{te}, y_{te}) = f(\theta),\ E \in R^m, \theta \epsilon R^n \qquad (3)$$

where m is the task space dimension and n is the joint space dimension.

Redundant manipulators have joint space dimension (n) greater than task space dimension (m). IK solution of these manipulators have infinite number of configurations. As the kinematic equations involves nonlinear and transcendental terms, closed form solutions are not possible always. In such a case, iterative techniques have been employed.

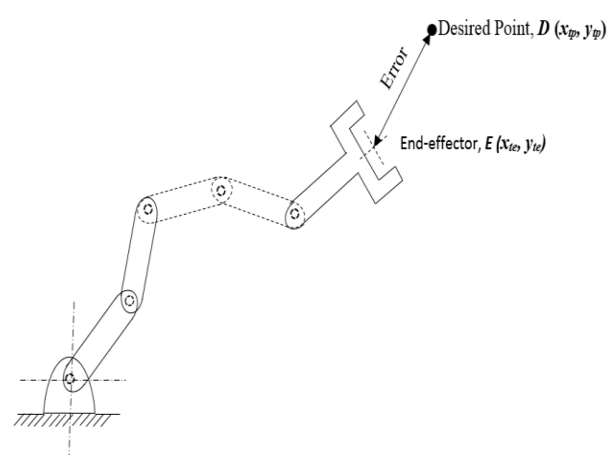

Figure 1: Kinematic model of 2-dimensonal robot

Here, the problem of determining IK solution and redundancy resolution has formulated as nonlinear constrained optimization problem. The objective function is to minimize the joint movement with respect to initial configuration and the constraint is to reach the task coordinates. The optimization problem is stated as

minimize

$$f = (\theta_{i1} - \theta_{f1})^2 + (\theta_{i2} - \theta_{f2})^2 + \cdots (\theta_{in} - \theta_{fn})^2 \qquad (4)$$

subject to

$$g = (x_{te} - x_{tp})^2 + (y_{te} - y_{tp})^2 = 0 \qquad (5)$$

where,

θ_{in} is the initial joint position of the n^{th} joint.

θ_{fn} is the final joint position of the n^{th} joint.

x_{tp}, y_{tp} are the coordinates of the desired point.

The optimization problem is solved by using gradient based classical optimization algorithm, handles the equality constraints by sequential quadratic programming (SQP) technique.

3 OBSTACLE AVOIDANCE

Redundant manipulators are controlled to track the desired end effector trajectory, while simultaneously ensuring that no part of the robot collides with any obstacles in the workspace and also self-collision of links. Collision avoidance requires collision detection, for which links are modeled as line segments and obstacles are modeled as polygons. The problem of detecting collisions between links and the obstacles is performed by finding the intersection of line segment and polygons. Similarly, self-collision of links are identified as the intersection of two line segment. If the configurations lead to the collision are identified, then those configurations are to be avoided. This is handled by penalty approach. The modified constraint equation with penalty is given as

$$g_1 = (x_{te} - x_{tp})^2 + (y_{te} - y_{tp})^2 + \boldsymbol{C} = 0 \quad (6)$$

Penalty is added during the execution of optimization algorithm when the configurations leads to collisions are identified.

$\boldsymbol{C}$ = 0; when there is no collision detection

$\boldsymbol{C}$ = p; (high positive value); when there is a collision.

4 SINGULARITY AVOIDANCE

Singular configurations are defined as the configurations of the robot at which the required joint rates to achieve an end-effector motion along one or more directions are extremely high. At singular configuration, Jacobian loses its full rank. Singularities of serial manipulators are of two types, boundary singularity, and interior singularity. Boundary singularities are observed when the robot is fully stretched out in such a way that end-effector is very near to the boundary of the workspace. Interior singularities occurred by lining up two or more joint axes. Because of this, robot performance is affected. Hence, these singular configurations are to be avoided. Measure of manipulability can be used as the potential function for singularity avoidance, which is given as follows

$$\mu = \sqrt{\det JJ^T} \quad (7)$$

The measure of manipulability is non-negative at non-singular configurations. It becomes zero only at singular points. Higher the manipulability measure, the robot is away from the singular configuration. Here, the problem of singularity avoidance is performed by maximizing the manipulability measure with the same constraint shown in Eq. 6.

5 IMPLEMENTATION

Redundancy resolution is carried out by defining it as an optimization problem, where the objective functions chosen are joint distance and manipulability measure subjected to task constraints and obstacles in the workspace. A set of coordinates in the task space are given as inputs to the defined objective function. By implementing classical optimization algorithm, the objective function is minimized. Collision check is performed by a computational geometry method called ray casting algorithm. Configuration with collisions are avoided by constraint formulation. Detailed scheme of algorithm is explained in the form of flow chart shown in fig. 2

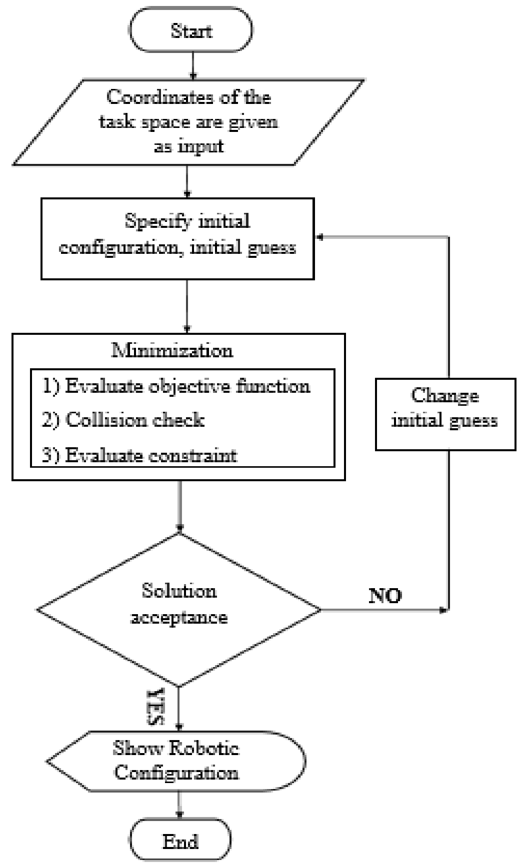

Figure 2: Flow chart for inverse kinematics scheme of redundant manipulators

6 SIMULATION RESULTS AND DISCUSSION

This section demonstrates how the redundancy resolution has been applied on serial redundant planar manipulators for a given path by taking obstacle avoidance and singularity avoidance as secondary criteria of performance.

6.1 Redundancy resolution and obstacle avoidance

In this case, a 5-DOF planar redundant robot of equal link lengths (10mm) has considered. Polygonal obstacles are chosen in the workspace, the robot has to trace the path in the workspace without colliding the obstacles. Here, the problem of redundancy resolution is carried out by solving each point on the path as constrained optimization problem with Eq. 4 as objective function and Eq. 6 as the constraint. Results are reported by varying the position of the obstacles and path to be traced by the end-effector, to show the efficacy of the proposed resolution scheme.

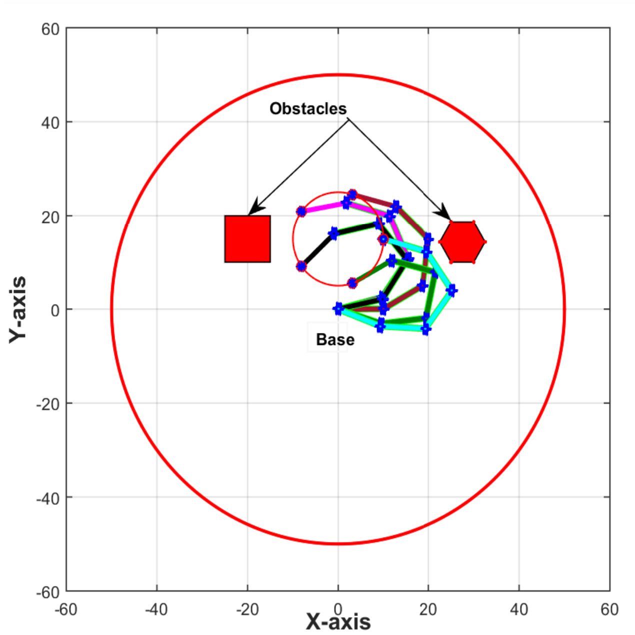

Figure 3: Robot configurations tracking circular Path

In Fig. 3 and 4, the robot is following a circular and straight-line path with polygonal obstacles in the workspace. Fig. 5 shows redundancy resolution scheme without collision avoidance. Fig. 6 and 7 shows robot following the straight line and circular path with collision avoidance. In each of these cases, the computational time with the proposed approach is about 30-60seconds. Fig. 6 illustrates that the optimization problem may not converge easily because the path in which the robot has to travel is very narrow and difficult to reach. This problem can be handled by restarting the optimization algorithm with different initial guesses. Restarting procedure is implemented when the method cannot find configurations that satisfy constraint. It is observed from all these cases that the solutions are resolved by minimizing the objective function while satisfying the constraints of reaching task space and avoiding collisions.

6.2 Manipulability measure and singularity avoidance

Singularity avoided configurations are determined by maximizing manipulability measure. i.e. Eq. 7 as the objective function and Eq. 5 as the constraint. In this case, boundary singularity avoidance for 5-DOF manipulator is considered.

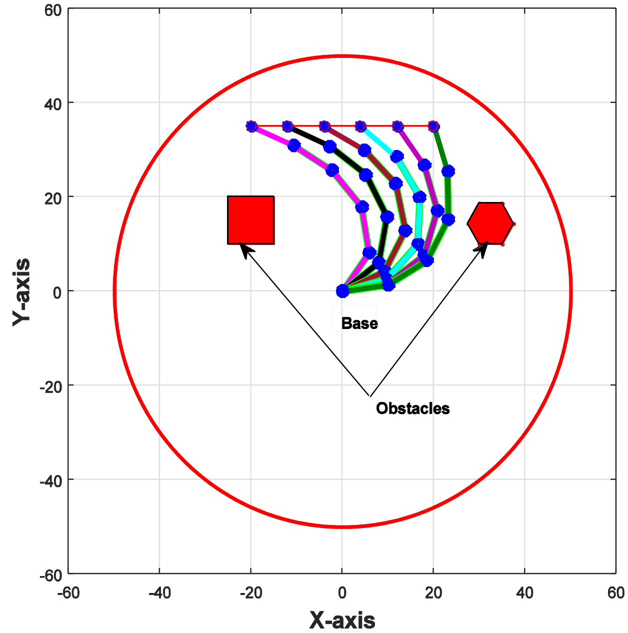

Figure 4: Robot configurations tracking straight line Path

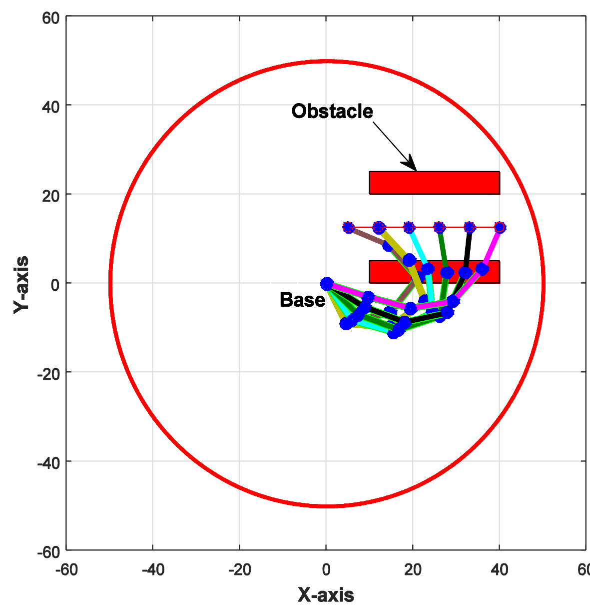

Figure 5: Robot configurations tracking a path without collision avoidance

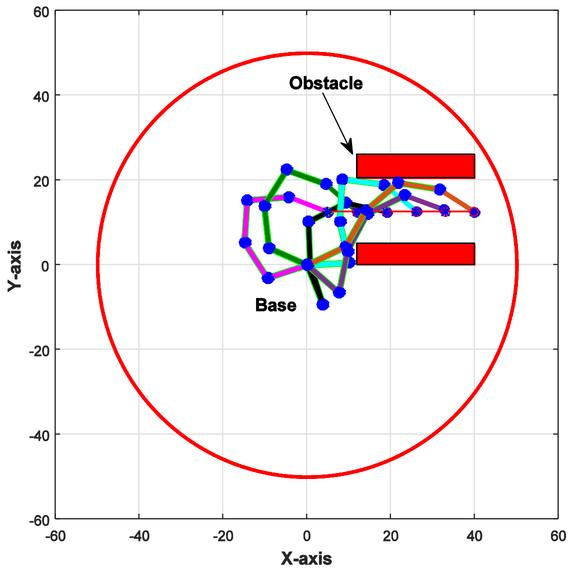

Figure 6: Robot configurations tracking a path with collision avoidance

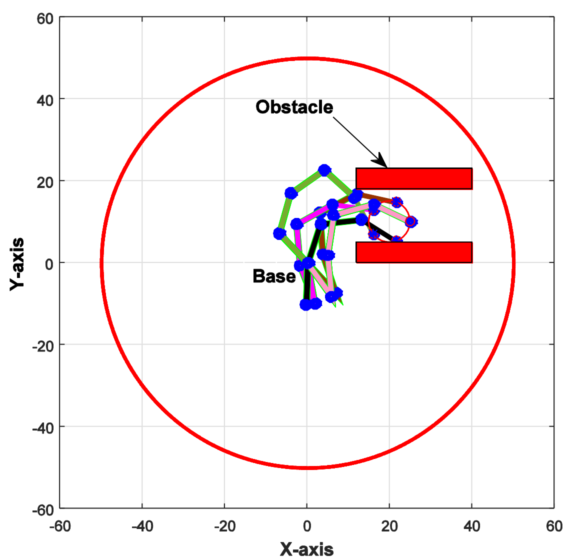

Figure 7: Robot configurations tracking circular path in presence of obstacles

Fig. 8 shows singular configurations while the robot is tracking a given path in the workspace, where the robot configurations are stretched. Fig. 9 shows the configurations that are away from singular points. Manipulability values are also calculated for both the cases to show how far the manipulator is away from singularities. Fig. 10 illustrates manipulability measure along the points of the given path for both singular and non-singular configurations. The value of manipulability measure for singular configuration is varying in the range of 580-860 on the corresponding points of the path. Whereas for non-singular case it is about 738-1020.

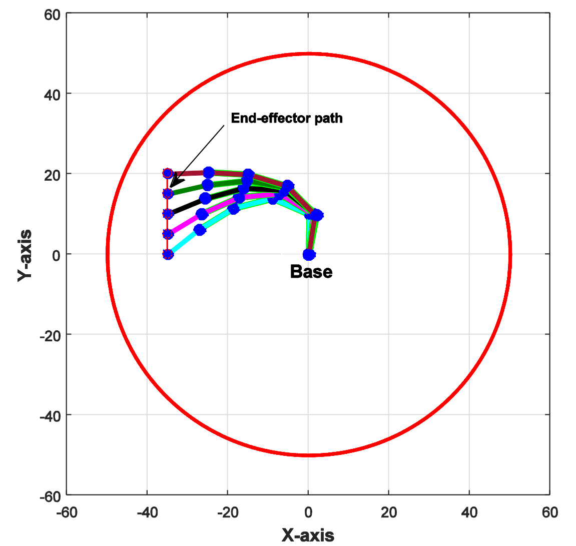

Figure 8: Robot configurations without singularity avoidance

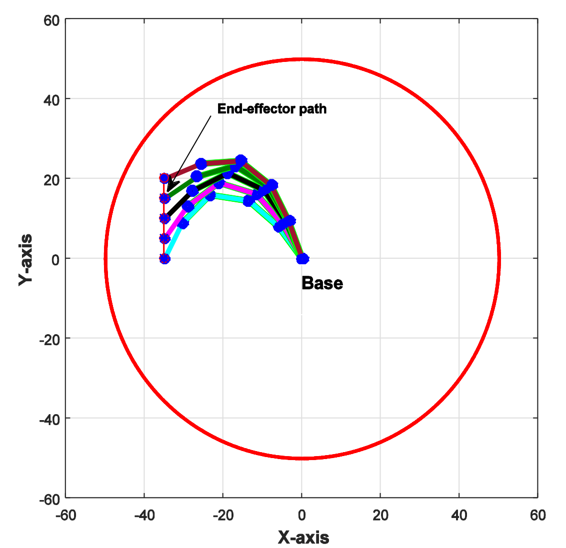

Figure 9: Robot configurations with singularity avoidance

The percentage improvement of manipulability measure is about 27.8% for singularity avoidance case compared to singular configurations. The proposed scheme leads to increase in the manipulability value, which improves the capability of controlling the robot far from its singularity.

The combined implementation of obstacle avoidance and singularity avoidance for a given path is handled by solving it as a constrained optimization problem with Eq.7 as the objective function and Eq. 6 as the constraint

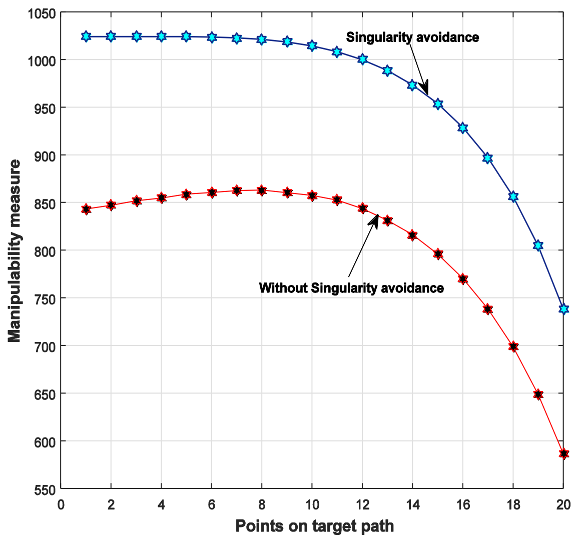

Figure 10: Manipulability measure along desired path

Fig. 11 shows the configurations of the robot while satisfying above stated conditions.

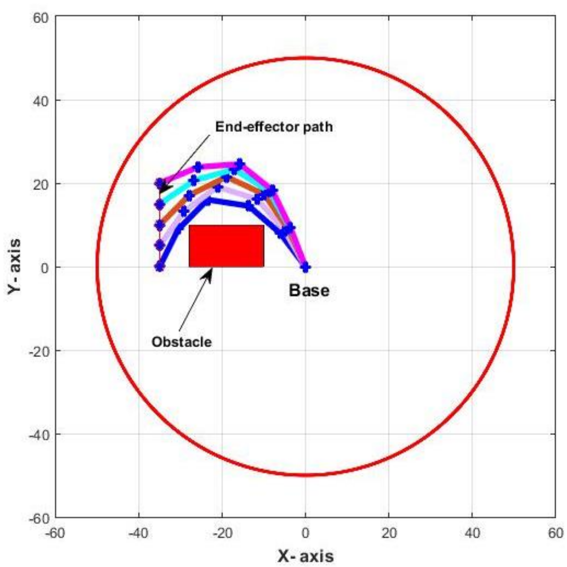

Figure 11: Robot configurations with obstacle and Singularity avoidance

7 CONCLUSIONS

Inverse kinematic solutions that are able to take the advantage of kinematic redundancy is studied in this paper. The problem of the Inverse kinematics of redundant manipulators for a given path is solved as a classical optimization problem. Redundancy resolution of the 5-Dof manipulator is carried out by two performance criterions such as obstacle avoidance and singularity avoidance. Case studies of serial redundant manipulators are shown under different variations in the workspace with respect to obstacles and end-effector path, to demonstrate the efficiency of the proposed approach. This approach is computationally efficient and can be applied to more DOF serial redundant manipulators. This paper focused only on the 2-Dimensional workspace and can be extended to 3D workspace under a dynamic environment.

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