Design and Analysis of Experiments for Attenuation Optimization in an Optical Fiber

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Executive Summary

The manufacturing company has decided to introduce a new design for its product, an optical fiber with minimum optical signal attenuation. For the new design, it was essential to complete the experimental runs within $50,000 ($1000 per run) with a limit of 16 runs per day. The design was developed in an iterative process, the initial fractional factorial design of 2^{8-4}_{IV} gave an idea for the follow- up experimental runs. The initial design was followed by a fold-over design and then, a response surface model was fit with the centered dataset. Using the JMP software we were able to identify the significant main effects and interactions on attenuation from the list of identified factors.

These are, **interaction of Furnace Temperature and Germanium Concentration** (X2*X4), **Furnace Temperature^2** (X2*X2), **Germanium Concentration** (X4), **Interaction of Germanium Concentration and Fiber Design** (X4*X5), **Fiber Design** (X5) and **Furnace Temperature** (X2). A model was built to predict attenuation from the significant factors and generate a list of optimal settings/characteristics based on that model. The recognized optimal settings are:

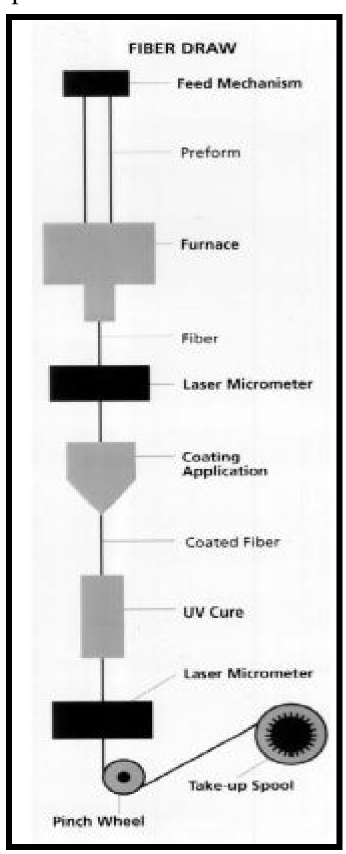
- Furnace Temperature (X2) of 2053 C
- Germanium Concentration (X4) of 0.01
- Fiber Design (X5) of 1.

The lowest attenuation achieved through this design and model is **0.224094**.

The results of the final model indicated a low attenuation value, but there were some limitations of the model like the assumption of block as a nuisance factor and no axial range expansion, which could be accommodated in the future models.

1. Introduction

This project focuses on determining the best combination of settings for the fiber drawing process as well as a few other characteristics to minimize attenuation. A flow process chart of the fiber drawing process is given below:



The manufacturing company has decided to introduce a new design for its product, an optical fiber with minimum optical signal attenuation. For the new design, it was essential to complete the experimental runs within $50,000 ($1000 per run) with a limit of 16 runs per day.

Two main objectives of this project are:

1. Identifying the main and interaction effects that significantly affect the attenuation.
2. To recommend optimal setting by using selected main & interaction effects to predict attenuation.

To analyze what factors impact the most, first, we selected 2^{8-4} fractional factorial design with resolution IV. By using an iterative process and from the output of effect test & normal probability plot, the most significant main & interaction effects are decided and the model is rerun by using significant effects only. Folding has been done to de-alias the significant interaction effects that were aliased with other interactions. Also blocking has been done to eliminate the effect of variable day, so that day will not be considered as a separate variable & model can be rerun from the last data point the next day.

In this analysis, we fit a quadratic model (Response Surface Model) with main & interaction effects in JMP PRO software. The Prediction profiler has been used to predict the optimality of the model, along with interpreting the response surface to minimize attenuation.

2. Experimental Design

To reduce the attenuation in the optical fiber, eight factors were recognized as the significant ones. Continuous factors include:

- Fiber draw speed (ranges from 20-30 m/sec)
- Furnace temperature (ranges from 1800-2200 C^{0})
- Draw tension (ranges from 0.5 to 1.0N)
- Germanium concentration (ranges from 0.01 to 0.05)

Categorical factors include:

- Fiber design 1 or 2 (index of refraction profile)
- Draw tower 1 or 2 (one of two manufacturing lines able to make the fiber)
- Raw material supplier 1 or 2 (one of two potential glass material suppliers)
- Coating type 1 or 2 (one of two coatings compatible with the designs)

Overall flow of the design process is given below:

Initial Design: 2^{8-4}_{IV} Fractional Factorial Design	De-aliasing: One factor Fold Over	Accommodating Curvature: Center Points & RSM	Final Model
• Main Factors not aliased with any other effects. • Two- factor interactions aliased with other two- factor interactions. • Resolution IV design with 16 runs.	• The significant main factors and interactions recognized from initial design. • Performed fold-over to de-alias significant interactions.	• Lack of Fit significant after performing fold-over. • Need for accommodation of curvature. • Center Points introduced. • Quadratic Model Fit through Response Surface Modeling.	• Insignificant factors eliminated. • Six significant factors recognized. • Model analyzed for the Fit and Residuals. • Optimal values recognized through Response surface and Prediction profiler.

2.1. Initial Design:

On the Basis of the Instructions provided by the Manufacturing Firm that a maximum of 16 runs can be conducted on a single day, We selected the 2^{8-4} Fractional Factorial Design with Resolution 4, in which 16 runs could be performed, with the combinations of the Four Continuous as well as Four Categorical Variables which were found out to be significant for the fiber drawing process.

Aliasing of Effects

Effects	Aliases
X1*X2	= X3*X8 = X4*X7 = X5*X6
X1*X3	= X2*X8 = X4*X6 = X5*X7
X1*X4	= X2*X7 = X3*X6 = X5*X8
X1*X5	= X2*X6 = X3*X7 = X4*X8
X1*X6	= X2*X5 = X3*X4 = X7*X8
X1*X7	= X2*X4 = X3*X5 = X6*X8
X1*X8	= X2*X3 = X4*X5 = X6*X7

2.2. Iterative Design:

The aliased Effects can be found out to be as follows in the above table and after looking at the effect summary for the model we conducted fold over on the factor which was. In order to break the alias between the main effects and two factor interactions we performed a foldover on the X3 (Draw Tension) factor which came up to be the most insignificant with the highest p-value.

X3(0.5,1)	0.023	0.94914

Blocks were added after 16 runs as suggested by the manufacturing firm to eliminate the effects such as time at continuation of runs as the same level which were conducted on the previous day.

After Foldover the desirability was around 0.5. The addition of center points in our model presented us with proof of curvature in our model, not a linear relationship between factors.

The centered variables were chosen to be the average values of all the four variables:

- 25 for Fiber Draw Speed
- 2000 for Furnace Temperature Range
- 0.75 for draw Tension
- 0.03 for Germanium Concentrations.

The Final sequence of runs for the most optimal Model with the lowest attenuation rate is given below:

	X1	X2	X3	X4	X5	X6	X7	X8	Block	Y
1	30	2200	1	0.01	1	1	1	2	1	0.89153...
2	30	2200	1	0.05	2	2	2	2	1	2.15756...
3	20	2200	1	0.01	1	2	2	1	1	0.95672...
4	20	1800	0.5	0.01	1	1	1	1	1	2.23195...
5	20	1800	1	0.01	2	2	1	2	1	2.29139...
6	20	2200	0.5	0.01	2	1	2	2	1	0.96964...
7	20	1800	0.5	0.05	2	2	2	1	1	1.18072...
8	20	2200	0.5	0.05	1	2	1	2	1	2.93012...
9	20	1800	1	0.05	1	1	2	2	1	1.76358...
10	30	2200	0.5	0.01	2	2	1	1	1	1.02115...
11	30	1800	0.5	0.05	2	1	1	2	1	1.08767...
12	30	1800	1	0.05	1	2	1	1	1	1.91626...
13	30	1800	0.5	0.01	1	2	2	2	1	2.27594...
14	30	1800	1	0.01	2	1	2	1	1	2.34667...
15	20	2200	1	0.05	2	1	1	1	1	2.26067...
16	30	2200	0.5	0.05	1	1	2	1	1	2.88020...
17	30	2200	0.5	0.01	1	1	1	2	2	0.91884...
18	30	2200	0.5	0.05	2	2	2	2	2	2.19293...
19	20	2200	0.5	0.01	1	2	2	1	2	0.93876...
20	20	1800	1	0.01	1	1	1	1	2	2.26693...
21	20	1800	0.5	0.01	2	2	1	2	2	2.31028...
22	20	2200	1	0.01	2	1	2	2	2	0.97291...
23	20	1800	1	0.05	2	2	2	1	2	1.16157...
24	20	2200	1	0.05	1	2	1	2	2	2.84696...
25	20	1800	0.5	0.05	1	1	2	2	2	1.76811...
26	30	2200	1	0.01	2	2	1	1	2	1.03050...
27	30	1800	1	0.05	2	1	1	2	2	1.13981...
28	30	1800	0.5	0.05	1	2	1	1	2	1.86972...
29	30	1800	1	0.01	1	2	2	2	2	2.35851...
30	30	1800	0.5	0.01	2	1	2	1	2	2.23055...
31	20	2200	0.5	0.05	2	1	1	1	2	2.25776...
32	30	2200	1	0.05	1	1	2	1	2	2.93041...
33	25	2000	0.75	0.03	1	2	1	2	3	0.71882...
34	25	2000	0.75	0.03	1	1	2	2	3	0.68621...
35	25	2000	0.75	0.03	1	1	1	1	3	0.64984...
36	25	2000	0.75	0.03	1	2	2	1	3	0.69677...

The nuisance Factors which were added were Blocks for limiting runs which were not computed while considering factors for the response surface runs. Finally, a model was selected with 36 runs which turned out to be the most optimal for this experiment. While considering the Response Surface Design the nuisance factors were not included to increase the overall optimality.

3. Analysis and Results

The initial design for this process was a 2^{8-4}_{IV} fractional factorial design with 16 runs. After fitting a model, this design gave an initial idea of the significant main effects and interactions. The final result came out to be slightly different from the initial findings, but there were some effects that remained consistently significant in the whole process. These were the Furnace Temperature (X2) and the Germanium Concentration (X4). The effect summary of the initial analysis on jmp is given below (factors having an insignificant effect on variance have been removed using the normal probability plot):

Effect Summary

Source	LogWorth	PValue
X1*X7	9.701	0.00000
X4(0.01,0.05)	7.314	0.00000
X1*X8	7.234	0.00000
X5	6.811	0.00000
X2(1800,2200)	4.866	0.00001
X8	3.035	0.00092 ^
X3*X8	2.507	0.00311
X6	2.341	0.00456
X1*X4	1.487	0.03261
X3*X7	1.420	0.03805

Then, a fold-over has been performed on the variable Draw Tension (X3) to de-alias the corresponding interaction effects that came out to be significant in the initial analysis. The fold-over results gave more insight into the selection of significant factors. The effect summary is given below:

Effect Summary

Source	LogWorth	PValue
X1*X7	30.550	0.00000
X4(0.01,0.05)	18.260	0.00000
X6*X7	17.318	0.00000
X5	16.017	0.00000
X2(1800,2200)	7.283	0.00000

But, the Lack of Fit was also significant in this model.

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	10	0.04006294	0.004006	3.2860
Pure Error	16	0.01950741	0.001219	Prob > F
Total Error	26	0.05957034		0.0168*
				Max RSq
				0.9987

This indicated that there might be a curvature in the model that hasn't been recognized. A response surface model was then fit to accommodate this curvature. The model has then been analyzed and fitted using the significant factors and interactions. These were recognized by a Normal Probability plot and the effect summary. These significant factors and interactions are listed below (with significance in decreasing order):

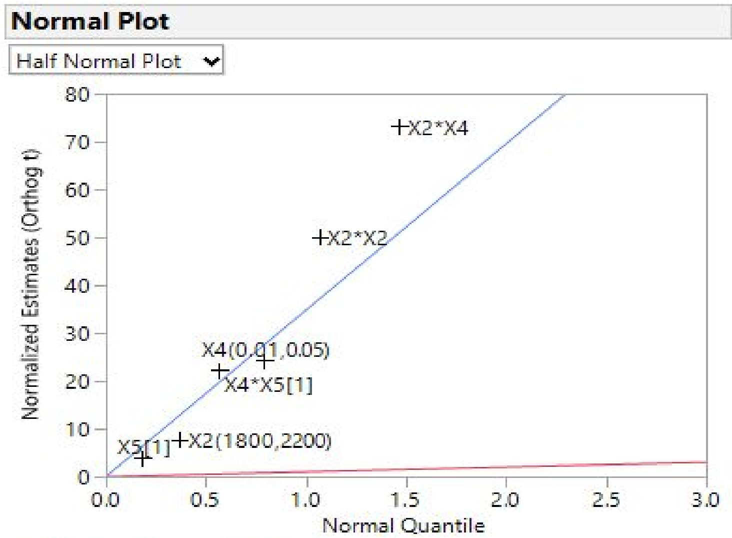
- Interaction of Furnace Temperature and Germanium Concentration. (X2*X4)
- Furnace Temperature^2 (X2*X2)
- Germanium Concentration (X4)
- Interaction of Germanium Concentration and Fiber Design (X4*X5)
- Fiber Design (X5)
- Furnace Temperature (X2)

The effect summary from the jmp report is shown below:

Effect Summary

Source	LogWorth	PValue	
X2*X4	33.749	0.00000	
X2*X2	28.999	0.00000	
X4(0.01,0.05)	20.052	0.00000	^
X4*X5	19.003	0.00000	
X5	17.556	0.00000	^
X2(1800,2200)	7.883	0.00000	^

The Half- Normal Probability plot:



Blue line has slope equal to Lenth's PSE.
Red line has slope 1.

The interactions involve the same factors as main effects and hence this provides additional support to their significance, according to the principle of “hierarchy”.
The resulting model showed an insignificant Lack of Fit as shown below (high p-value):

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	2	0.00373774	0.001869	0.8652
Pure Error	27	0.05831878	0.002160	Prob > F
Total Error	29	0.06205652		0.4323
				Max RSq
				0.9970

The overall analysis of the final model is given below:

3.1. Analysis of Model Adequacy:

The Summary of Fit and Analysis of Variance for the final model are given below:

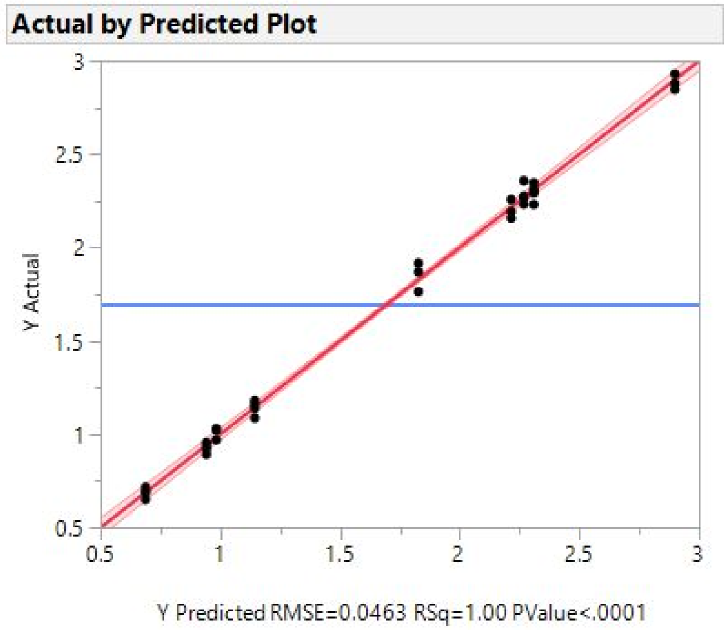
Summary of Fit

RSquare	0.996802
RSquare Adj	0.99614
Root Mean Square Error	0.046259
Mean of Response	1.697448
Observations (or Sum Wgts)	36

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	19.341868	3.22364	1506.460
Error	29	0.062057	0.00214	Prob > F
C. Total	35	19.403924		<.0001*

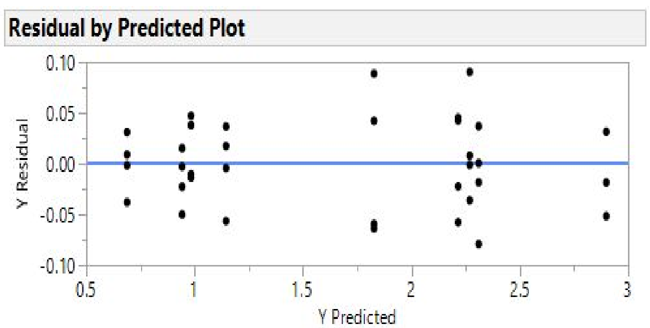
From the above results, we can conclude that the model has an excellent fit with 99.68% of the variance in the data explained. Also, this model is statistically significant, as indicated by the low p-value (<.0001) from the Analysis of Variance analysis.
The actual v/s predicted plot is given below:

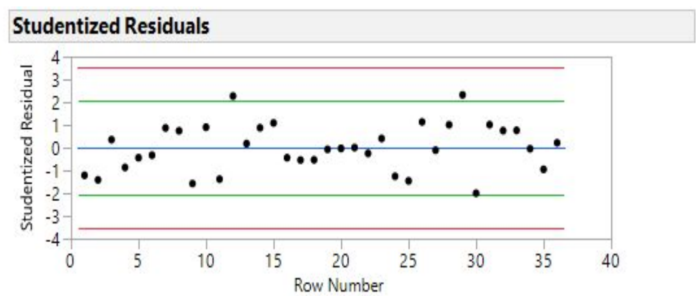


It shows that the data points are closely fitted to the line.

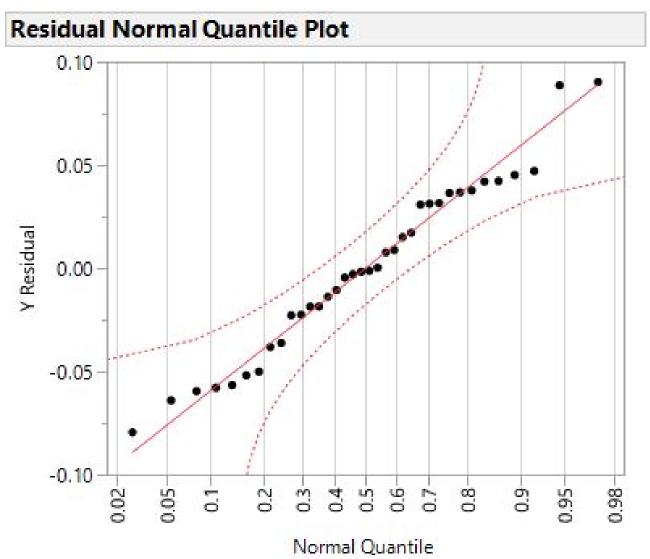
3.2. Residual Analysis

The residual plots are given below:





Externally studentized residuals with 95% simultaneous limits (Bonferroni) in red, individual limits in green.



From the residual analysis, we can see that:

- The residual plots look good except for two potential outliers.
- The Q-Q plot also looks good, with the residuals almost fitting the line.

3.3. Regression Analysis and Prediction Expression:

The Parameter Estimates of the reduced final model are given below:

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.5275185	0.024532	21.50	<.0001*
X2(1800,2200)	-0.063843	0.008177	-7.81	<.0001*
X4(0.01,0.05)	0.1978684	0.008177	24.20	<.0001*
X5[1]	0.1603988	0.008177	19.61	<.0001*
X2*X2	1.2961209	0.025859	50.12	<.0001*
X2*X4	0.5994172	0.008177	73.30	<.0001*
X4*X5[1]	0.181269	0.008177	22.17	<.0001*

The prediction expression is given below:

Prediction Expression

$$0.5275184948$$

$$+ -0.063843397 \cdot \left(\frac{(X2 - 2000)}{200} \right)$$

$$+ 0.1978684101 \cdot \left(\frac{(X4 - 0.03)}{0.02} \right)$$

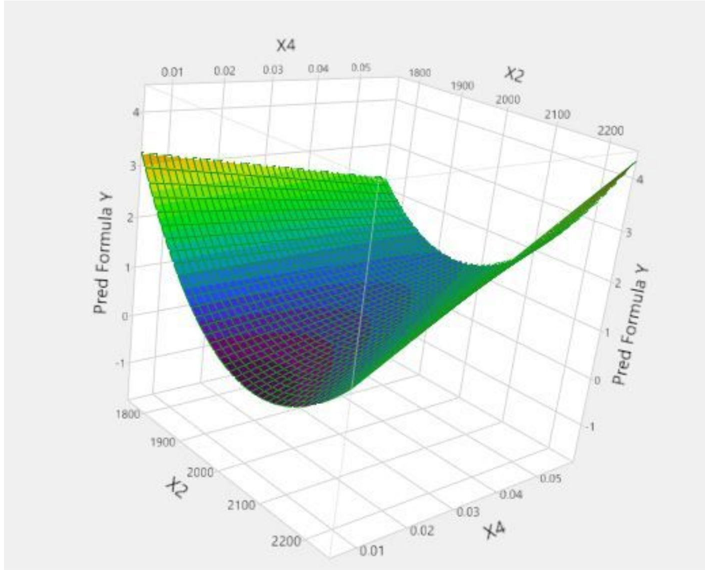
$$+ \mathrm{Match}(X5) \begin{pmatrix} \text{"1"} \Rightarrow 0.1603987509 \\ \text{"2"} \Rightarrow -0.160398751 \\ \text{else} \Rightarrow . \end{pmatrix}$$

$$+ \left(\frac{(X2 - 2000)}{200} \right) \cdot \left(\left(\frac{(X2 - 2000)}{200} \right) \cdot 1.2961209217 \right)$$

$$+ \left(\frac{(X2 - 2000)}{200} \right) \cdot \left(\left(\frac{(X4 - 0.03)}{0.02} \right) \cdot 0.59941718 \right)$$

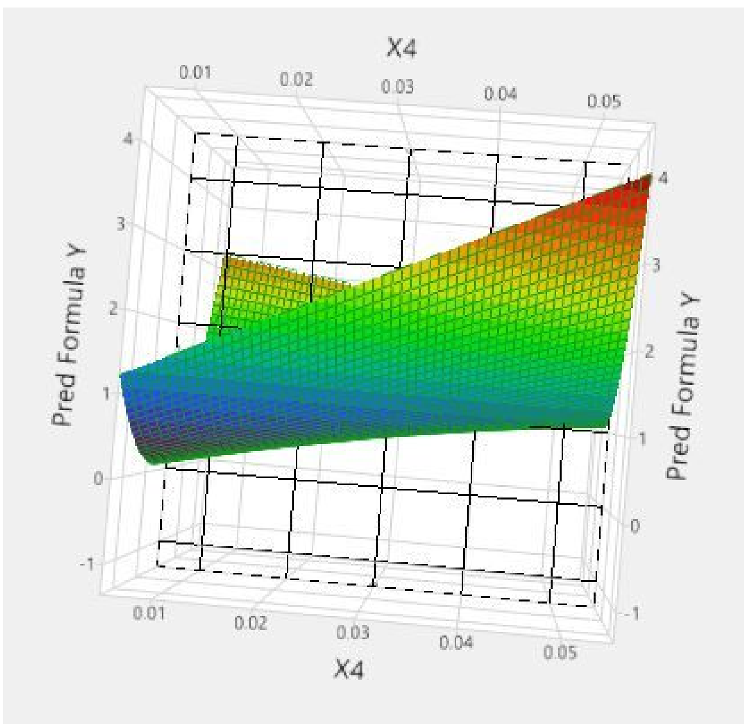
$$+ \left(\frac{(X4 - 0.03)}{0.02} \right) \cdot \mathrm{Match}(X5) \begin{pmatrix} \text{"1"} \Rightarrow 0.1812689877 \\ \text{"2"} \Rightarrow -0.181268988 \\ \text{else} \Rightarrow . \end{pmatrix}$$

3.4. Response Surface Analysis:



A response surface model is fit over the Predicted values and the curvature of the surface model has been studied. The steepest point of the surface gives the lowest attenuation value. The surface model was plotted between X2, X4 and Predicted Y over two values of X5(Categorical).

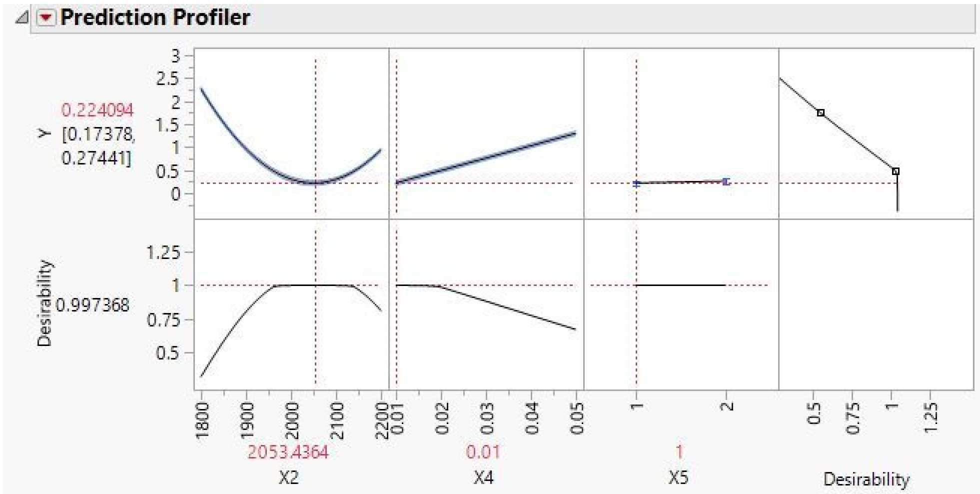
- The above model is selected as the best model among the values of X5(1,2). The value of X5 = 1 gives the lowest attenuation amongst the two values.
- From the above surface plot, it can be seen that the attenuation values decrease as X4 decreases and attains the lowest value at an X4 value of 0.01.
- A steep descent can be observed in the X2 axis which attains the maximum valley point at around 2000.
- The point where these two points merge is found to be the deepest point on the surface, which is dark blue in color and it denotes the lowest attenuation value.



The rotated view of the surface (side view) gives a clear representation of the linear decrease in the attenuation value as the fall in X4 value. It can be seen to attain the **lowest value of 0.22** around the value of 0.01 X4 (for a fixed value of X5 =1).

4. Recommendations:

After analyzing the results from the prediction profiler in jmp, we get the optimum range of values of the significant factors:



The ideal values to attain the highest desirability (Lowest attenuation) can be found by the prediction profiler. The above figure shows the prediction profiler for X5=1. Which gives the maximum desirability of 99.73%.

The lowest attenuation produced by this model is 0.224094 which can be attained by the following settings:

- Furnace Temperature (X2) of 2053 C
- Germanium Concentration (X4) of 0.01
- Fiber Design (X5) of 1.

We recommend that these settings could be used to get a lower attenuation value.

Limitations:

- Block effects were not considered in the models (Assumption: Considered it as nuisance factor).
- Experimentation with an expanded range of significant variables wasn't carried out, the attenuation could be checked in the expanded axial values.

Recommendations for Future Experimentation:

- From the surface model, it is evident that the value of decreases with the decrease of germanium concentration. Hence, a lower germanium concentration may be tested to get even better attenuation results.
- Even though the fiber design factor didn't play a huge factor it reduced the attenuation from .22 from .26. Hence an experiment with a different fiber design (refraction index) might help decrease the attenuation values.