# Workshop:
# **Measures of Polarization**

Aug 16, 2022
Lorentz Workshop Othering and Polarisation
Leiden

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# Polarization: Several conceptions

1. (Group) Polarization is the degree of **extremeness** of a group (social psychology)
   Polarization is not the opposite of consensus?

2. Polarization is strong **correlation of attitudes** on various topics
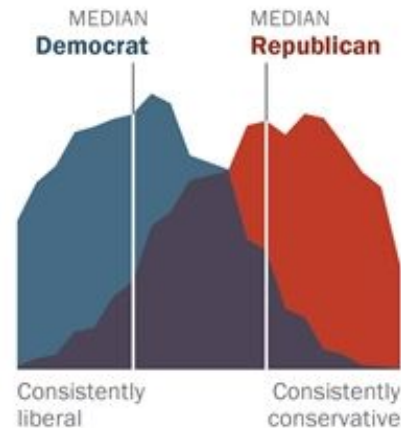   There cannot be polarization on one topic?

3. Polarization is large **difference** in me(di)an **opinions of two groups**
   What if groups are not clearly given exogenously?

4. Polarization on one topic is **large antagonisms** (opinion difference) between individuals
   But how to measure this exactly?

Isn't polarization a **process** instead of a **state**?

Practically, it doesn't matter much. To measure change we need to measure state.

# **Polarization: Definition**

In Physics/Chemistry (among others)

*A process that brings about electric or magnetic poles.*

Polarization in social sciences

1. *A process of accentuation of differences*
2. *A state of accentuated differences*

# Let us look at potential data sets of individuals

*E.g., liberal-conservative*  *E.g., liberal-conservative*

| ID | Issue1 | Issue2 | Issue… | Group1 | Group2 | Group... | AffectA | AffectB | Affect… |
|----|--------|--------|--------|--------|--------|----------|---------|---------|---------|
| 1 | 10 | 0 | … | A | c | … | 5 | - 5 | … |
| 2 | 6 | 8 | … | B | b | … | 1 | 4 | … |
| 3 | 9 | 2 | … | B | a | … | - 1 | 3 | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

Basic assumption:

We are interest in **polarization in this society** of this individuals as

an **aggregate macroscopic property**.

→ **How accentuated are differences?**

# Concept 1: Issue polarization

| ID | Issue1 | Issue2 | Issue… | Group1 | Group2 | Group... | AffectA | AffectB | Affect… |
|----|--------|--------|--------|--------|--------|----------|---------|---------|---------|
| 1 | 10 | 0 | … | A | c | … | 5 | - 5 | … |
| 2 | 6 | 8 | … | B | b | … | 1 | 4 | … |
| 3 | 9 | 2 | … | B | a | … | - 1 | 3 | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

How accentuated are the differences among individuals with respect to **one issue**?

# Concept 2: Issue alignment

| ID | Issue1 | Issue2 | Issue… | Group1 | Group2 | Group... | AffectA | AffectB | Affect… |
|----|--------|--------|--------|--------|--------|----------|---------|---------|---------|
| 1 | 10 | 0 | … | A | c | … | 5 | - 5 | … |
| 2 | 6 | 8 | … | B | b | … | 1 | 4 | … |
| 3 | 9 | 2 | … | B | a | … | - 1 | 3 | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

How aligned are **several issues**?

How much do they boil down to **one dimension**?

# Concept 3: Issue partisanship

| ID | Issue1 | Issue2 | Issue… | Group1 | Group2 | Group... | AffectA | AffectB | Affect… |
|----|--------|--------|--------|--------|--------|----------|---------|---------|---------|
| 1 | 10 | 0 | … | A | c | … | 5 | - 5 | … |
| 2 | 6 | 8 | … | B | b | … | 1 | 4 | … |
| 3 | 9 | 2 | … | B | a | … | - 1 | 3 | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

How accentuated are differences on an issue (Issue1) between existing groups (in Group1 labels)?

# Political Polarization in the American Public



**Democrats and Republicans More Ideologically Divided than in the Past**

*Distribution of Democrats and Republicans on a 10-item scale of political values*
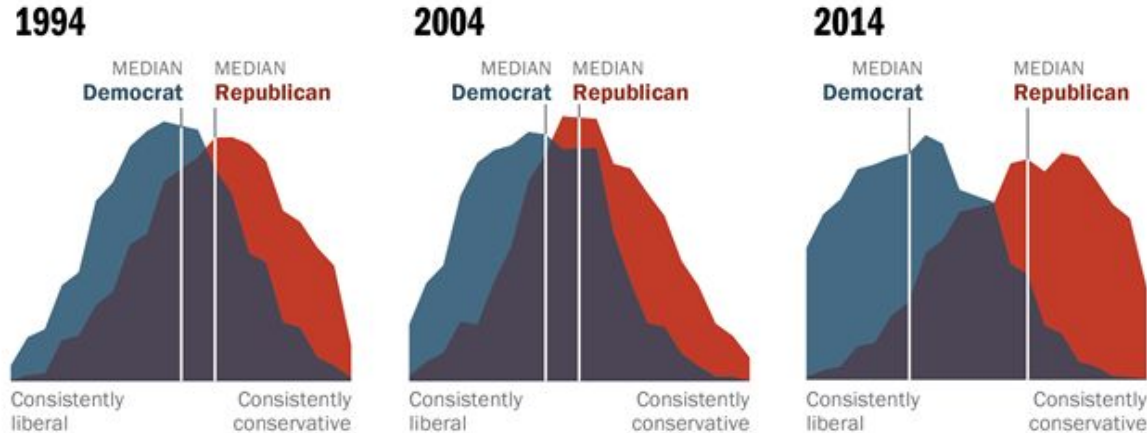
Source: 2014 Political Polarization in the American Public
Notes: Ideological consistency based on a scale of 10 political values questions (see Appendix A). The blue area in this chart represents the ideological distribution of Democrats; the red area of Republicans. The overlap of these two distributions is shaded purple. Republicans include Republican-leaning independents; Democrats include Democratic-leaning independents (see Appendix B).

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**Mix of issue alignment and issue partisanship**

Pew Research Center Report 2014, **Political Polarization in the American Public**
https://www.pewresearch.org/politics/2014/06/12/political-polarization-in-the-american-public/

# Concept 4: Affective polarization

| ID | Issue1 | Issue2 | Issue… | Group1 | Group2 | Group... | AffectA | AffectB | Affect… |
|----|--------|--------|--------|--------|--------|----------|---------|---------|---------|
| 1 | 10 | 0 | … | A | c | … | 5 | - 5 | … |
| 2 | 6 | 8 | … | B | b | … | 1 | 4 | … |
| 3 | 9 | 2 | … | B | a | … | - 1 | 3 | … |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

How negative are the **feelings of individuals about groups** (in Group1 labels)?

# Issue polarization in the European Social Survey?
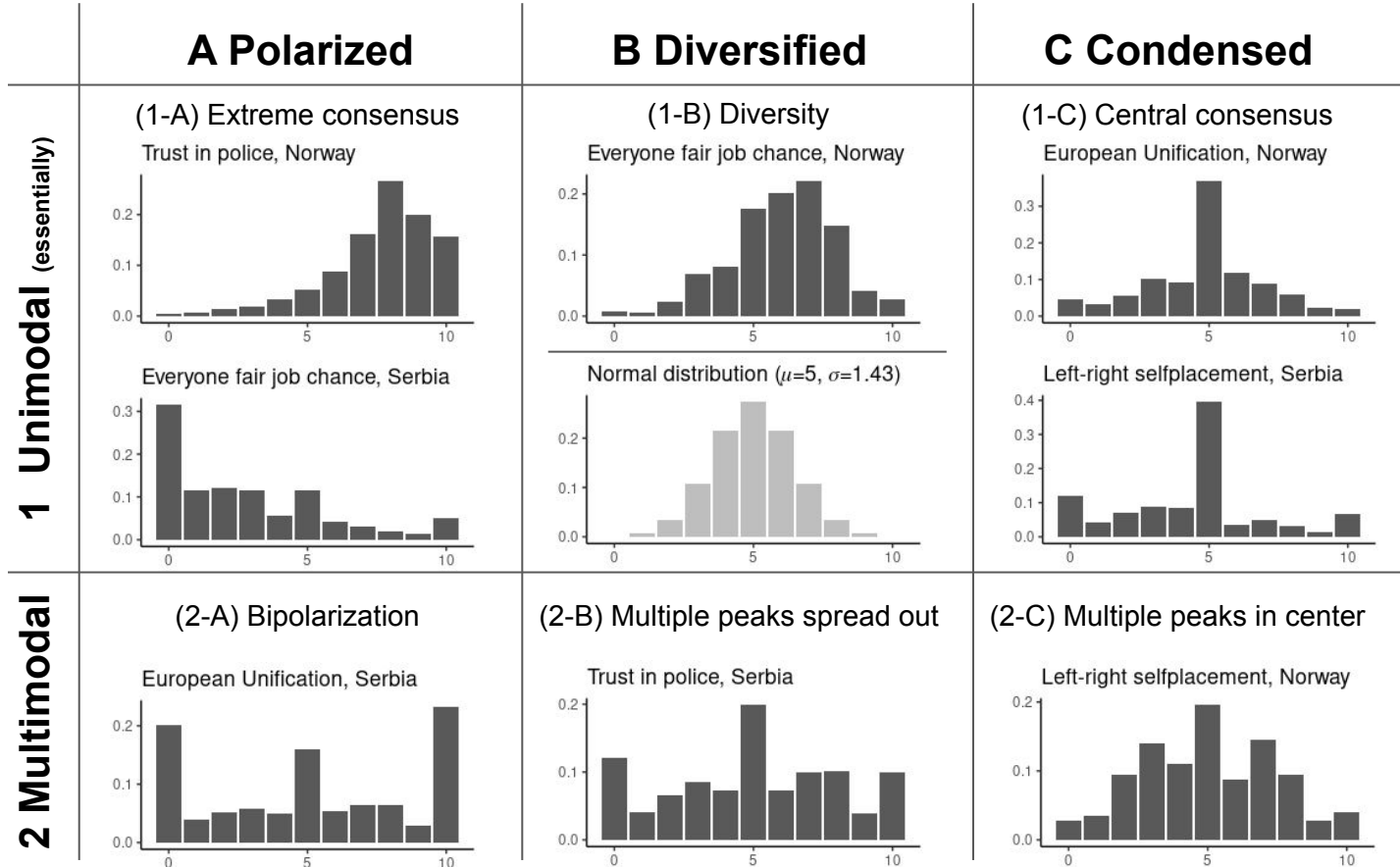


Lorenz, J., Neumann, M., & Schröder, T. (2021). Individual attitude change and societal dynamics: Computational experiments with psychological theories. Psychological Review, 128(4), 623. https://doi.org/10.1007/s43545-022-00342-7

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# **Polarization:** Many aspects even in 1-d distribution

Bramson, A., Grim, P., Singer, D. J., Fisher, S., Berger, W., Sack, G., & Flocken, C. (2016). Disambiguation of social polarization concepts and measures. The Journal of Mathematical Sociology, 40(2), 80-111.

→ List 9 different polarization aspects:

Spread                          Distinctness

Dispersion                      Group divergence

Coverage                        Group consensus

Regionalization                 Size parity

Fragmentation

# **Polarization as antagonisms between individuals**

The conceptual measurement problem:
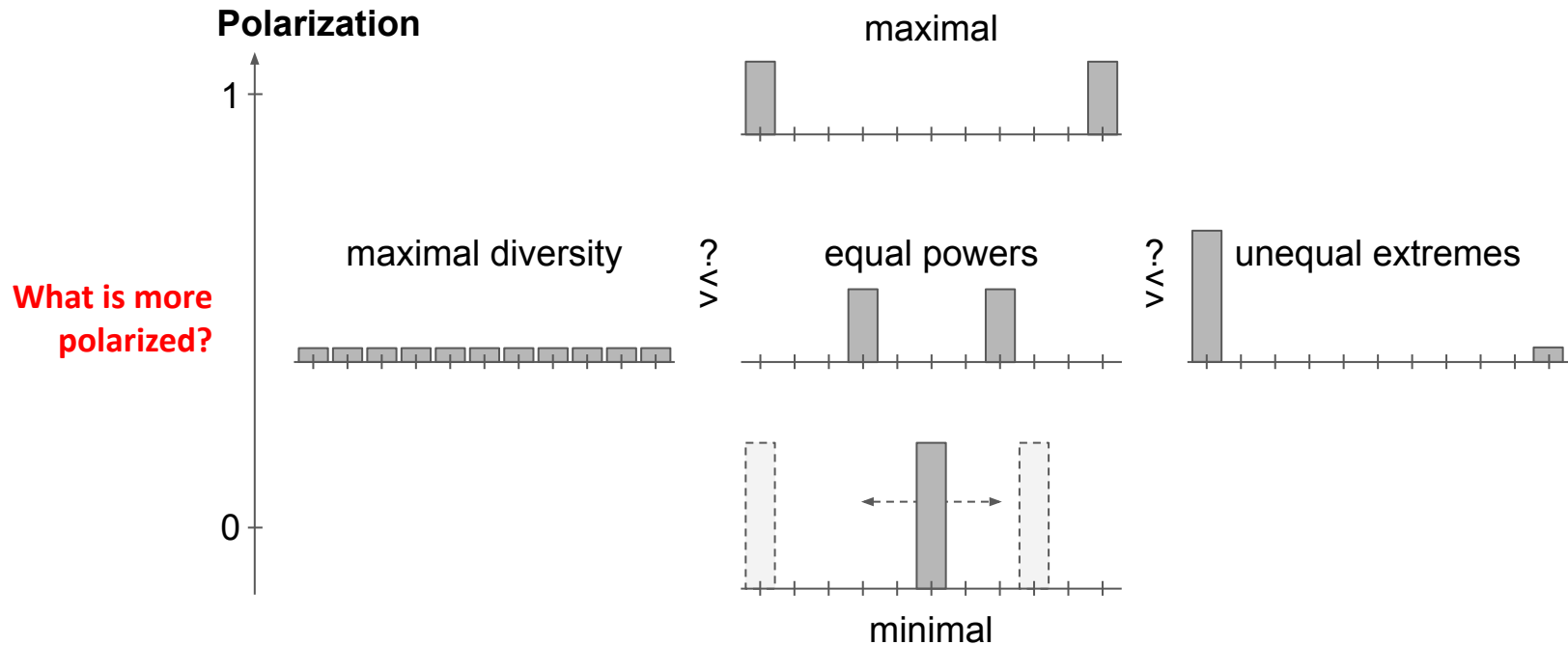
# Polarization

## Minimum (pol = 0)

- All have the same opinion
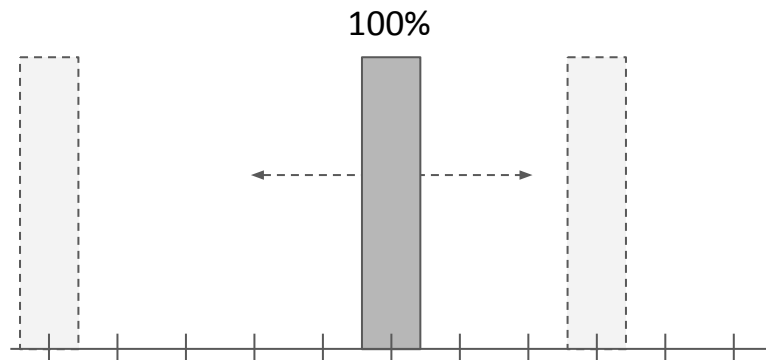
## Maximum (pol = 1)

- One half maximally positive
- Other half maximally negative



100%



50%                                    50%

# Average distance between pairs

$$M = c\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}d(x_i, x_j)$$

For c=1: Mean absolute difference

https://en.wikipedia.org/wiki/Mean_absolute_difference

Probabilistic:

Expected value of the distance between a random pair.

# The unified view: Average distance between pairs

When the characteristics (Ethnicity, Attitude, Wealth) are from a discrete set of n labels or numbers we can also compute

$$M = c \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j d(x_i, x_j)$$

where $p_i$ is the fraction of the population with characteristic $x_i$.

# For Polarization:

$$M = c \sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j d(x_i, x_j)$$

$x_i$ are discrete number, e.g., from a $x_1 = 0$ to $x_{11} = 10$

$d(x_i, x_j) = |x_i - x_j|$

We set $c = 2/|x_{11} - x_1|$.



Assume the maximal case:

M = 2/|10-0|*(0.5*0.5|10-0| + 0.5*0.5|0-10|) = 0.2*(2.5+2.5) = 1

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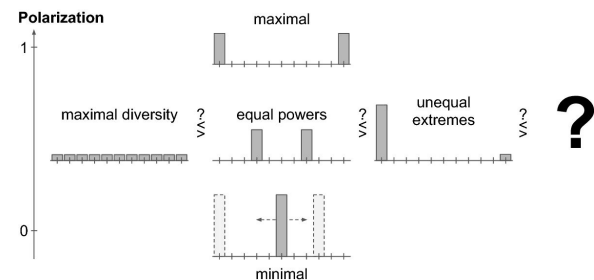
# For Polarization: Extension of Estaban and Rey

$$\text{Pol}_\alpha = \frac{2^{1+\alpha}}{x_n - x_1} \sum_{i=1}^{n} \sum_{j=1}^{n} p_i^{1+\alpha} p_j |x_i - x_j|$$

$\alpha = 0$ delivers our old measure

$\alpha > 0$ weights antagonism more when the first selected person is from a larger group with the same attitude

This can be called **"identificiation-weighted"**.

For $\alpha \gtrless 1.6$ the measure "degenerates"

(can deliver values larger than one).

Esteban, J. M., & Ray, D. (1994). On the measurement of polarization. Econometrica: Journal of the Econometric Society, 819-851.

# Polarization measures

- **Polarization**
  (Esteban & Ray, 1994)
- **Mean Absolute Deviation**
- **Standard Deviation**
- **Disagreement**
  (Van der Eijk, 2001)
- ...

$$\text{Pol}_0(p) = \frac{2}{n} \sum_{i,j=0}^{n} p_i p_j |i - j|$$

$$\text{Pol}_\alpha(p) = \frac{2^{1+\alpha}}{i - j} \sum_{i,j=0}^{n} p_i^{1+\alpha} p_j |i - j|$$

$$\text{Pol}_1(p) = \frac{4}{n} \sum_{i,j=0}^{n} p_i^2 p_j |i - j|$$

$$\text{MAD}(p) = \frac{2}{n} \sum_{i=0}^{n} p_i |i - \bar{x}|$$

$$\text{SD}(p) = \frac{2}{n} \sqrt{\sum_{i=0}^{n} p_i (i - \bar{x})^2}$$

| | | Pearson Correlation |
|---|---|---|
| $\text{Pol}_0$ | MAD | 0.99 |
| $\text{Pol}_0$ | SD | 0.99 |
| $\text{Pol}_0$ | Dis | 0.96 |
| MAD | SD | 0.98 |
| MAD | Dis | 0.93 |
| SD | Dis | 0.93 |
| $\text{Pol}_0$ | $\text{Pol}_{0.4}$ | 0.97 |
| $\text{Pol}_0$ | $\text{Pol}_1$ | 0.39 |
| $\text{Pol}_0$ | $\text{Pol}_{1.6}$ | -0.23 |

Empirically the same

These measure two aspects

Based on 4,004 attitude landscapes from ESS

Esteban, J. M., & Ray, D. (1994). On the measurement of polarization. Econometrica: Journal of the Econometric Society, 819-851.
Van der Eijk, C. (2001). Measuring agreement in ordered rating scales. Quality and Quantity, 35(3), 325-341.
Gestefeld, M., Lorenz, J., Henschel, N.T. et al. Decomposing attitude distributions to characterize attitude polarization in Europe. SN Soc Sci 2, 110 (2022). https://doi.org/10.1007/s43545-022-00342-7

# Pol$_0$ vs. Pol$_1$ and stylized facts
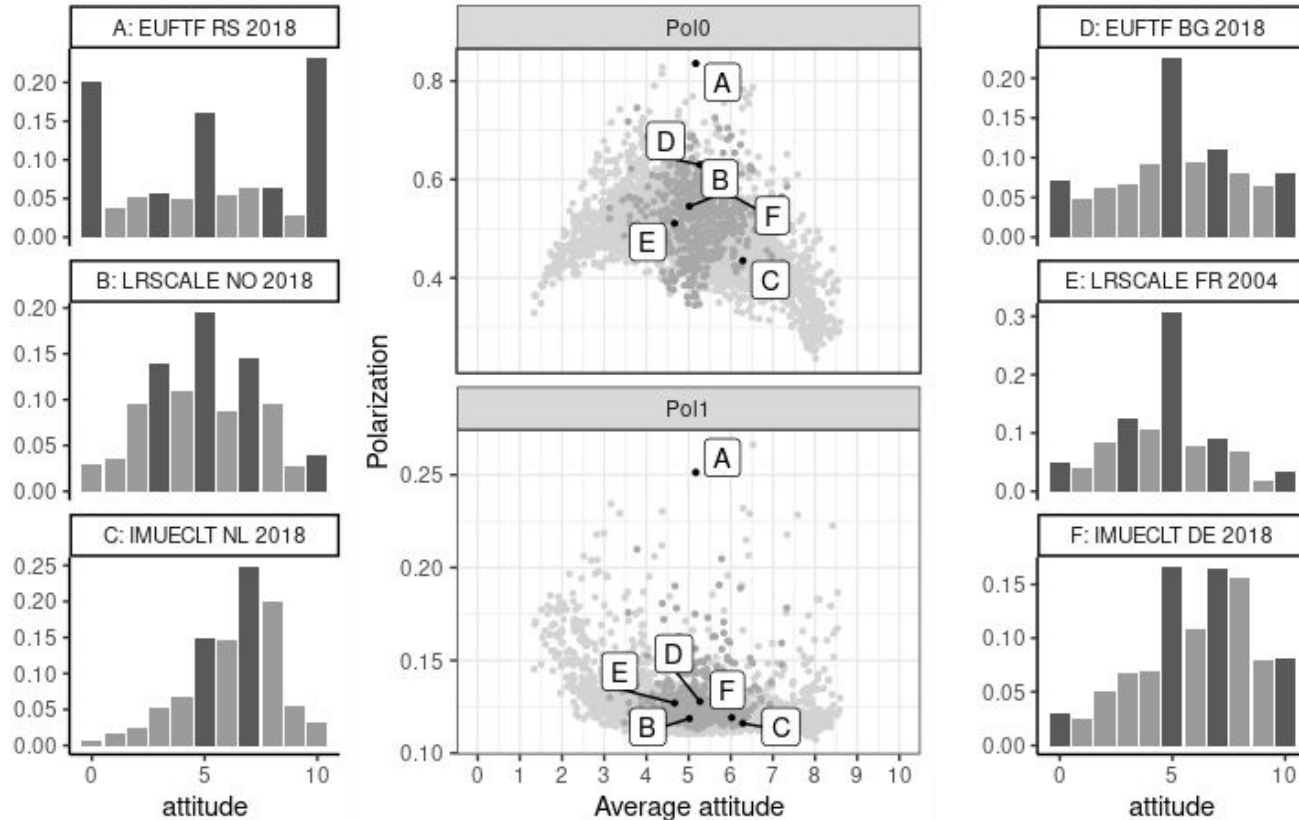


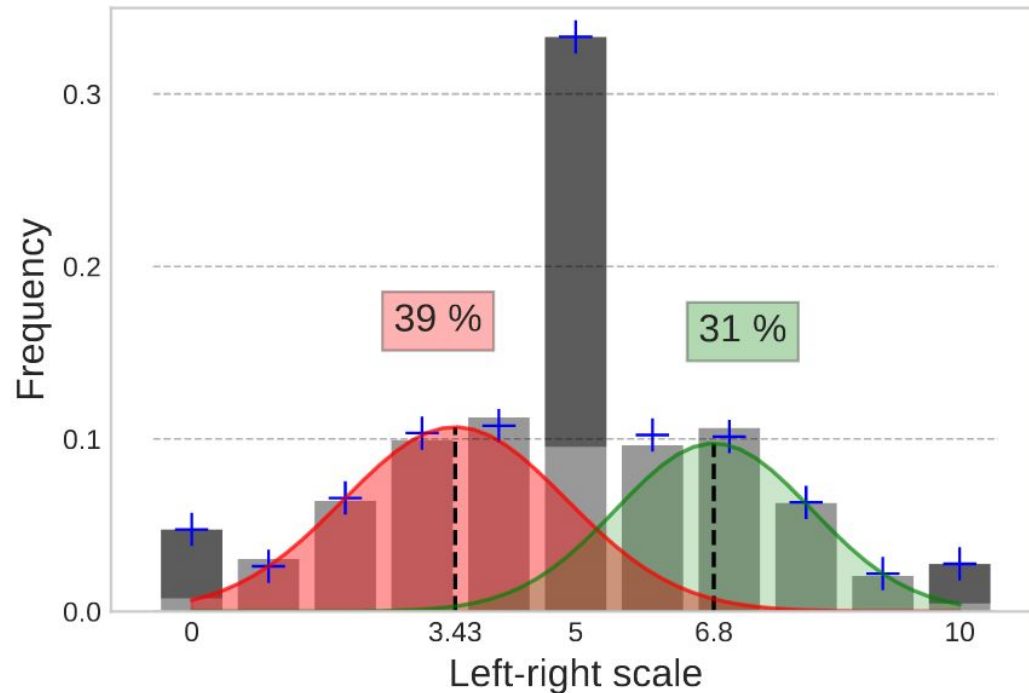**Stylized Facts**

Landscapes tend to have

1. Discrete peaks
   a. Central
   b. Left extremists
   c. Right extremists
2. Smooth peaks
   a. Moderate left
   b. Moderate right

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# Pentamodal model of opinion landscapes

$$\pi = w_{\text{ExL}}\pi^{\text{ExL}} + w_{\text{ExR}}\pi^{\text{ExR}} + w_{\text{C}}\pi^{\text{C}} + w_{\text{ModL}}\pi^{\text{ModL}} + w_{\text{ModR}}\pi^{\text{ModR}}$$



$$\pi^{\text{ExL}} = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$
$$\pi^{\text{ExR}} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$
$$\pi^{\text{C}} = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]$$
$$\pi^{\text{ModL}} = [\pi_0^{\text{ModL}}, \pi_1^{\text{ModL}}, \ldots, \pi_9^{\text{ModL}}, \pi_{10}^{\text{ModL}}]$$
$$\pi^{\text{ModR}} = [\pi_0^{\text{ModR}}, \pi_1^{\text{ModR}}, \ldots, \pi_9^{\text{ModR}}, \pi_{10}^{\text{ModR}}]$$

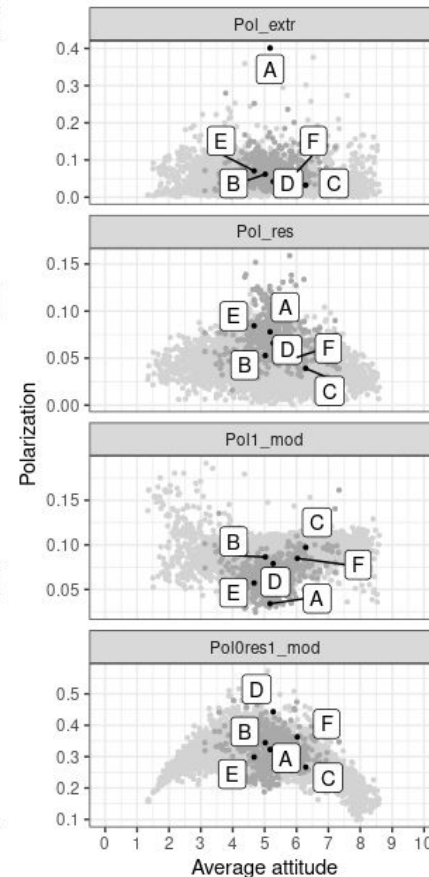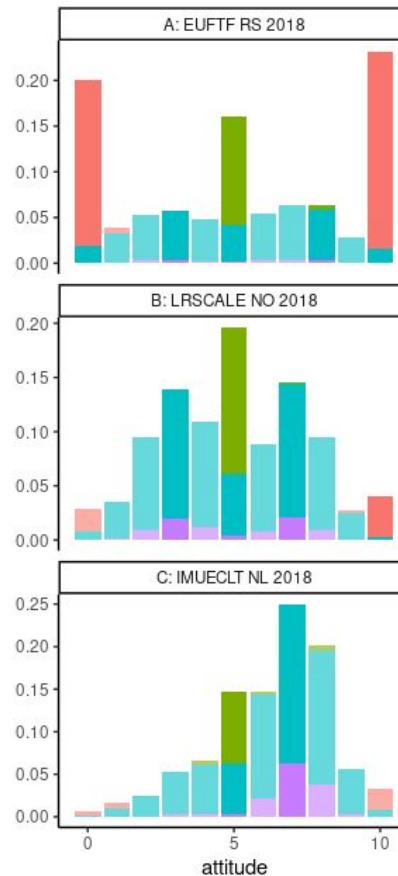# Towards an empirically relevant decomposition of polarization

Partial polarization measure for subpopulation q<p with respect to the total population

$$\text{Pol}_\alpha(q,p) = \frac{2^{1+\alpha}}{n} \sum_{i,j=0}^n q_i^{1+\alpha} p_j |i-j|.$$

$\text{Pol}_0$ is always larger than $\text{Pol}_1$

$$\text{Pol}_0(q,p) \geq \text{Pol}_1(q,p)$$

(Warning: I am not sure if this is general for $\alpha_2 > \alpha_1$.
Unfortunately, I forgot... )



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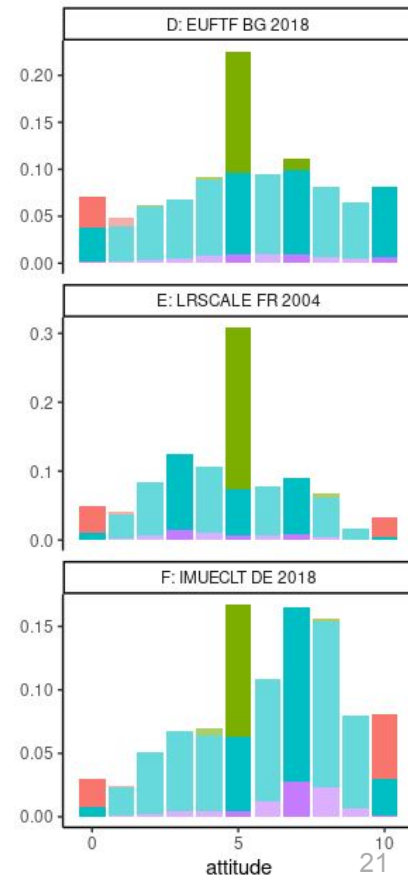
# Conclusion on Decomposition of Polarization

- The decomposition by groups and by measures are interesting to explore, they can tell what drives polarization differences
- It does not help so clear with the conceptual problem
- Is there a decomposition Fitting more directly to this?
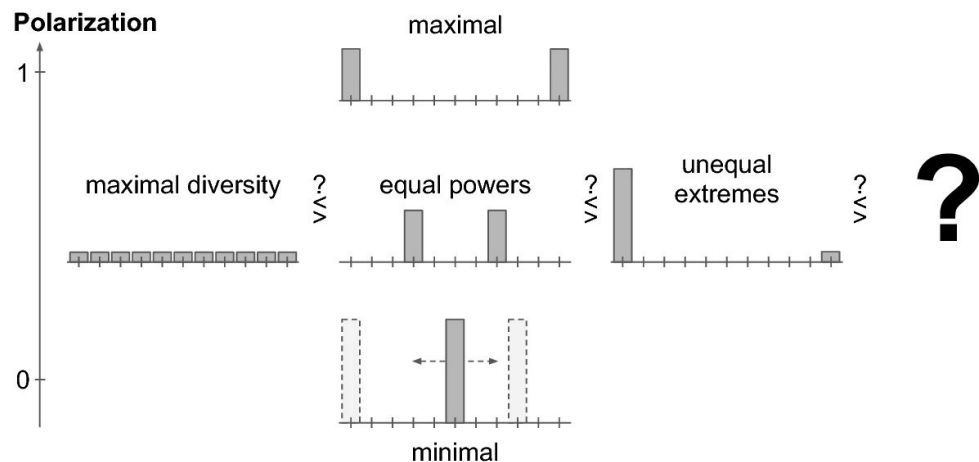
# Backup

# Segregation, Polarization, and Inequality

From the BIGSSS Summer School in Computational Social Science on Social Cohesion Jul 2022:

Social Division Between Groups

**Segregation**     **Polarization**

**Inequality**
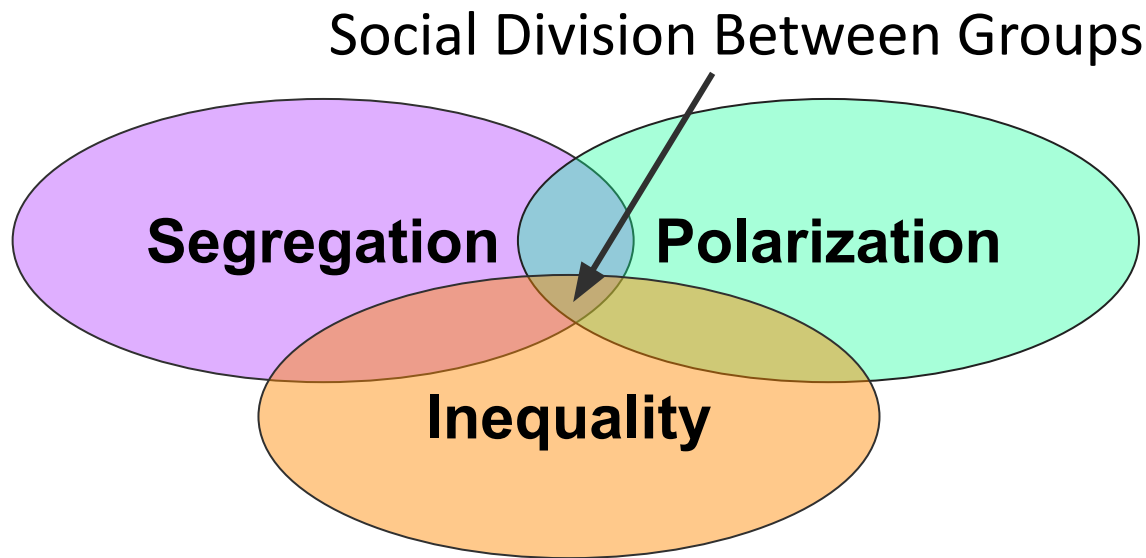
→ Average antagonism between individuals in a society

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# Some differences

On differences in **categorical scales**
(Ethnicity, Social Class, Gender, …)

On differences in **bounded order/metric scales**
(Attitude surveys, Likert scales)

**Segregation**

**Polarization**

**Inequality**
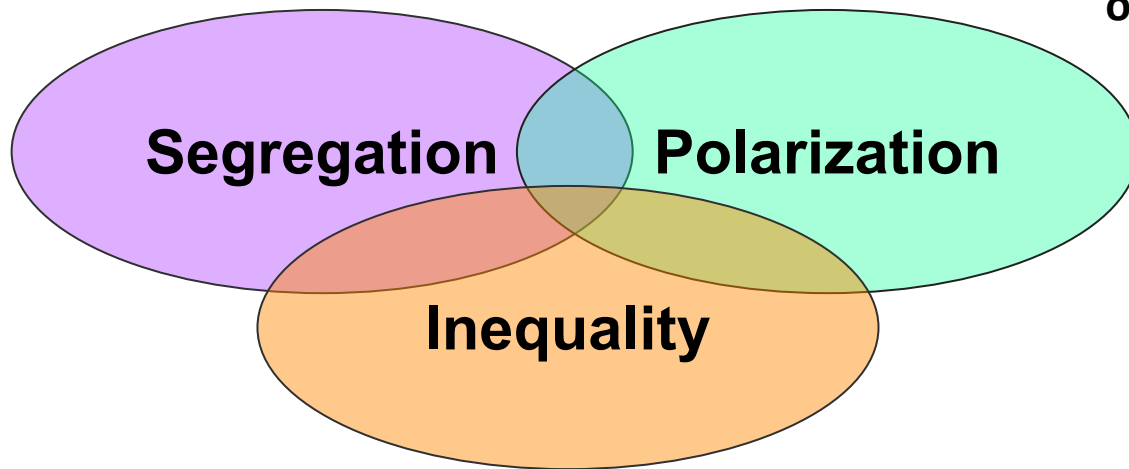
On differences on a **continuous scale**
(Resources quantified in monetary value)