Computer Simulations of Opinions and their Reactions to Extreme Events

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Computer Simulations of Opinions

and Their Reactions to Extreme Events

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We review the opinion dynamics in the computer models of Deffuant et al. (D), of Krause and Hegselmann (KH), and of Sznajd (S). All these models allow for consensus (one final opinion), polarization (two final opinions), and fragmentation (more than two final opinions), depending on how tolerant people are to different opinions. We then simulate the reactions of people to extreme events, in that we modify the opinion of an individual and investigate how the dynamics of a consensus model diffuses this perturbation among the other members of a community. It often happens that the original shock induced by the extreme event influences the opinion of a big part of the society.

1   Introduction

Predicting extreme events is very important when we want to avoid the losses due to earthquakes, floods, stock market crashes, etc. But it is not easy, as reading the newspapers shows. It is much easier to claim afterwards that one has an explanation for this event. A more scientific question is the investigation of the opinions people have after an extreme event: Do they now take objective risks more seriously than before? Do people tend to exaggerate the risks and prefer to drive long distances by car instead of airplane, shortly after a plane crash happened? How do these opinion changes depend on the time which has elapsed since the event, or the geographical distance? It is plausible that the more time has elapsed since the last catastrophe, the less serious is the risk taken by most people. Less clear is the influence of

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geographical distance, e.g. if the probability to die of a terror attack in a far away country is compared with the "customary" risk to die there from a traffic accident.

Geipel, Härta and Pohl [1] looked at the geography question in a region of Germany where 10^4 years ago a vulcano erupted and left the Laach lake. The closer the residents were to that lake, the more seriously they took the risk. But also their general political orientation was correlated with their risk judgment. On the other hand, scientific annoucements led to some newspaper reactions within Germany, independent of the distance, but died down after a few months. Other examples are the reactions to nuclear power plants and their accidents. Volker Jentsch (private communication) suggested to simulate such reactions to extreme events on computers; such simulations are only possible with a reasonable model of opinion dynamics.

A very recent application would be the influence of deadly tsunamis after an earthquake on the opinion of people. Those who live on the affected coasts after that extreme event of December 2004 will remember it as a clear danger. Those who life further inwards on the land, away from the coast, know that tsunamis do not reach them, but they still have learned from the news about the thousands of people killed. Will they judge the danger as higher or as lower than those on the affected coast line? And what about those who live on the coast of a different ocean, where such events are also possible but happened long ago? This example shows how the influence of an extreme event on the opinion of the people can depend on the distances in time and space. This is the question we want to simulate here in generic models.

It would not be desirable to invent a new opinion dynamic model just for the purpose to study reactions to extreme events. Instead, it would be nice if one would have one generally accepted and well tested model, which then could be applied to extreme events. No such consensus is evident from the literature. We thus concentrate here on three models, D, KH and S (Deffuant et al. [2], Krause and Hegselmann [3] and Sznajd [4]) which are currently used a lot to simulate opinion dynamics; we ignore the older voter models [5] or those of Axelrod [6], of Galam [7] and of Wu and Huberman [8], to mention just some examples. We will not claim that as a result of these simulations one may predict public reaction; we merely claim that simulations like these could be a useful starting point in this research field.

Of course, one may question in general whether human beings can be simulated on computers where only a few numbers describe the whole per-

son. More than two millenia ago, the Greek philosopher Empedokles already paved the way to these computer simulation by stating (according to J. Mimkes), that some people are like wine and water, mixing easily, while others are like oil and water, refusing to mix. Thus he reduced the complexity of human opinions to two choices, like hydrophilic or hydrophobic in chemistry, spin up or spin down in physics, 0 or 1 in computer science. And in today's developed countries, we take regular polls on whether people like their government, allowing only a few choices like: very much, yes, neutral, no, or not at all. Simplifying mother Nature thus was not started by us, is common also in sociology, and has been quite successful in physics.

2 General Opinion Dynamics

In this section we review the dynamics of models D of Deffuant et al, KH of Krause and Hegselmann, and S of Sznajd [2, 3, 4]. Their results are quite similar but they differ in their rules on how the opinions are changed. An earlier review of these models was given in [9] with emphasis on the Sznajd model. In that model two people who agree in their opinions convince suitable neighbours to adopt this opinion. In model D, each person selects a suitable partner and the two opinions get closer to each other. For KH, each person looks at all suitable partners and takes their average opinion. "Suitable" means that the original opinions are not too far from each other.

2.1 Deffuant et al

In model D [2] all N agents have an opinion O which can vary continuously between zero and one. Each agent selects randomly one of the other agents and checks first if an exchange of opinions makes sense. If the two opinions differ by more than ϵ ($0 < \epsilon < 1$), the two refuse to discuss and no opinion is changed; otherwise each opinion moves partly in the direction of the other, by an amount $\mu \Delta O$, where ΔO is the opinion difference and μ the convergence parameter ($0 < \mu < 1/2$). The parameter ϵ is called confidence bound or confidence interval. For $\epsilon > 1/2$ all opinions converge towards a centrist one, while for $\epsilon < 1/2$ separate opinions survive; the number of surviving opinions in the latter case varies as $1/\epsilon$. Besides simulations, also analytical approximations were made [10] which agree well with the simulations.

Fig. 1 shows a consensus formation with the number of simulated people
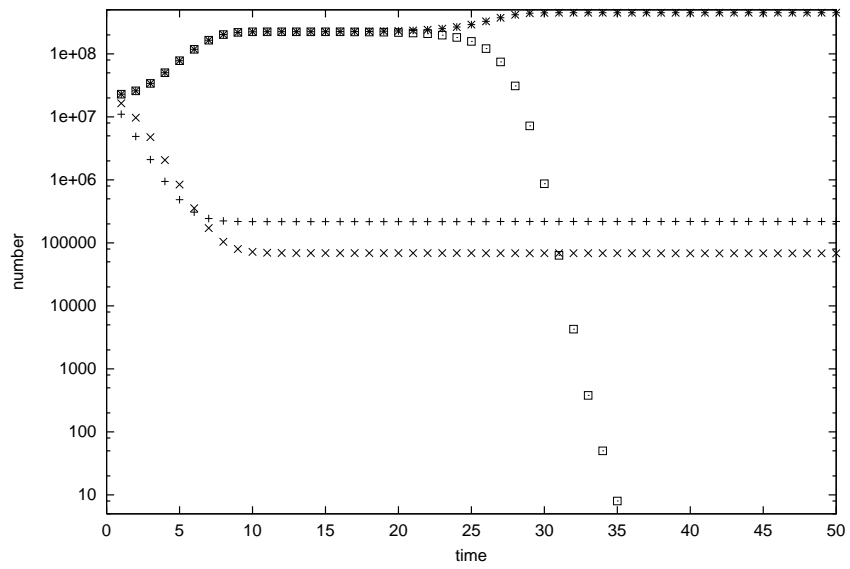
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Figure 1: Standard D model, 450 millions agents, $\epsilon = 0.4$, $\mu = 0.3$, opinions divided in 20 intervals. Shown are intervals 1 (+), 2 (x), 10 (stars), and 11 (squares).

close to that in the European Union, 450 millions, and $\epsilon = 0.4$, $\mu = 0.3$. To plot the results, the opinions were binned into 20 intervals. We show intervals 1, 2, 10 and 11 only. Initially, the numbers of opinions were the same in all intervals; soon two centrist opinions dominate until finally one of them eats up the other. Independent of this power struggle, some extremist opinions survive in the intervals close to zero and close to one. These extremist wings [11] are a general property for $\epsilon < 1/2$ but are not the theme of this "extreme" book.

Various variants of this standard version were published. It is numerically easier to look at integer opinions $O = 1, 2, 3 \ldots, Q$ instead of continuously varying O; a precursor of such work is Galam and Moscovici [12] where both discrete opinions $(0, 1)$ and opinions in between were allowed. If the opinions $O = 1, 2, \ldots Q$ are integers, one can determine unambiguously if two opinions agree or differ. The above expression for $\mu \Delta O$ then needs to be rounded to an integer. If two opinions differ only by one unit, one randomly selected opinion is replaced by the other one, whereas this other opinion remains unchanged.

The idea of everybody talking to everybody with the same probability is perhaps realistic for scientific exchanges via the internet, but, in politics, discussions on city affairs are usually restricted to the residents of that city and do not extend over the whole world. Putting agents onto a square lattice [2] with interactions only between lattice neighbours [13] is one possibility. In recent years, small-world networks and scale-free networks [14] were simulated intensively as models for social networks. In the standard version of the Barabási-Albert model, the most popular model of scale-free networks, one starts with a small number m of agents all connected to each other. Then, one by one, more members are added to the population. Each new member selects randomly m previous members as neighbours such that the probability of selecting one specific agent is proportional to the number of neighbours this agent had before. In this way, the well connected people get even more connections, and the probability of one agent to have been selected as neighbour by k later members is proportional to $1/k^3$. (In contrast, on the square lattice and on the Bethe lattice, every agent has the same number of neighbours, and for random graphs the number of neighbours fluctuates slightly but its distribution has a narrow peak.) In opinion dynamics, only network neighbours can influence each other.

Putting Deffuant agents [15] onto this Barabási-Albert network, with continuous opinions, again for large confidence intervals ϵ a complete consensus

is found whereas for small ϵ the number of different surviving opinions varies roughly as $1/\epsilon$. An opinion cluster is a set of agents sharing in final equilibrium the same opinion, independent of whether these agents are connected as neighbours or separated. Varying the total number N of agents one finds that the number of small opinion clusters with 1, 2, 3, ... agents is proportional to N, while the number of large opinion clusters comprising an appreciable fraction of the whole network is of order unity and independent of N. This result reminds us of the cluster size distribution for percolation [16] above the threshold: There is one infinite cluster covering a finite fraction of the whole lattice, coexisting with many finite clusters whose number is proportional to the lattice size. One may compare this distribution of opinions with a dictatorship: The imposed official opinion coexists with a clandestine opposition fragmented into many groups.

This scale-free network can be studied in a complicated and a simple way: In the complicated way, if a new agent Alice selects a previous agent Bob as neighbour of Alice, then Alice is also neighbour of Bob, like in mutual friendships. This is the undirected case. The directed case is the simpler way: Bob is a neighbour for Alice but Alice is not a neighbour for Bob; this situation corresponds more to political leadership: the party head does not even know all party members, but all party members know the head. Apart from simplifying the programming, the directed case seems to have the same properties as the undirected one [15].

Also changing from continuous to discrete opinions $O = 1, 2, \ldots, Q$ does not change the results much but it simplifies the simulation [17], particularly when only people differing by one opinion unit discuss with each other (corresponding to $\epsilon \sim 1/Q$). Again the number of opinion clusters varies proportional to N for $N \to \infty$ at fixed N/Q. A consensus is reached for $Q = 2$, but not for $Q > 2$. A scaling law gives the total number of final opinions as being equal to N multiplied by a scaling function of N/Q. This law has two simple limits: For $Q \gg N$ there are so many opinions per person that each agent has its own opinion, separate from the opinions of other agents by more than one unit: no discussion, nobody changes opinion, N clusters of size unity. In the opposite case $Q \ll N$, all opinions have lots of followers and thus most of them survive up to the end. These simple limits remain valid also if people differing by up to ℓ opinion units (instead of $\ell = 1$ only) influence each other; a consensus is then formed if ℓ/Q (which now plays the role of the above ϵ) is larger than $1/2$. (The more general scaling law for arbitrary N/Q now becomes invalid). This threshold of $\epsilon = 1/2$, which

has emerged so often in the previous examples, is supposed to be a universal feature of the Deffuant dynamics, as long as the symmetry of the opinion spectrum with respect to the inversion right $\leftrightarrow$ left is not violated [18]. The symmetry means that the opinions O and $1 - O$ ($Q - O$ for integer opinions) are equivalent and can be exchanged at any stage of the dynamics without changing the corresponding configuration. In this way, the histogram of the opinions is at any time symmetric with respect to the central opinion $1/2$ ($Q/2$ for integer opinions). If we instead let O and $1 - O$ ($Q - O$) play different roles the threshold will in general be different. As a matter of fact, in [19] one introduced such an asymmetry in that the "convincing power", expressed by the parameter μ, is no longer the same for all agents but it depends on the opinion of the agent. More precisely, μ increases with the opinion of the individual, and this implies that those agents with low values of O are less convincing than those with high values of O. In this case the opinion distribution is no longer symmetric with respect to $O = 1/2$ ($Q/2$) and the consensus threshold is larger than $1/2$.

In all this work, first the scale-free network was constructed, and then the opinion dynamics studied on the fixed network. Not much is changed if opinion dynamics takes place simultaneously with network growth [20], in agreement with Ising and Sznajd models [21].

2.2   Krause-Hegselmann

The KH model [3] was simulated less since only small systems seemed possible to be studied. Only recently, for discrete opinions, an efficient algorithm was found to study millions of agents [22], compared with at most 300,000 for continuous opinions [23]. Again we have opinions O continuous between zero and one, or discrete $O = 1, 2, \ldots Q$. At every iteration, every agent looks at all other agents, and averages over the opinions of those which differ by not more than ϵ (continuous opinions) or ℓ (discrete opinions) from its own opinion. Then it adopts that average opinion as its own. As in the D model, also the KH model shows a complete consensus above some threshold and many different opinions in the final configuration if ϵ is very small. However, in this case, there are two possible values for the threshold [24], depending on how many neighbours an agent has on average: if this number of neighbours, or average degree, grows with the number of agents of the community, there is consensus for $\epsilon > \epsilon_0$, where $\epsilon_0 \sim 0.2$; if instead the average degree remains finite when the population diverges, the consensus threshold is $1/2$ as in the D
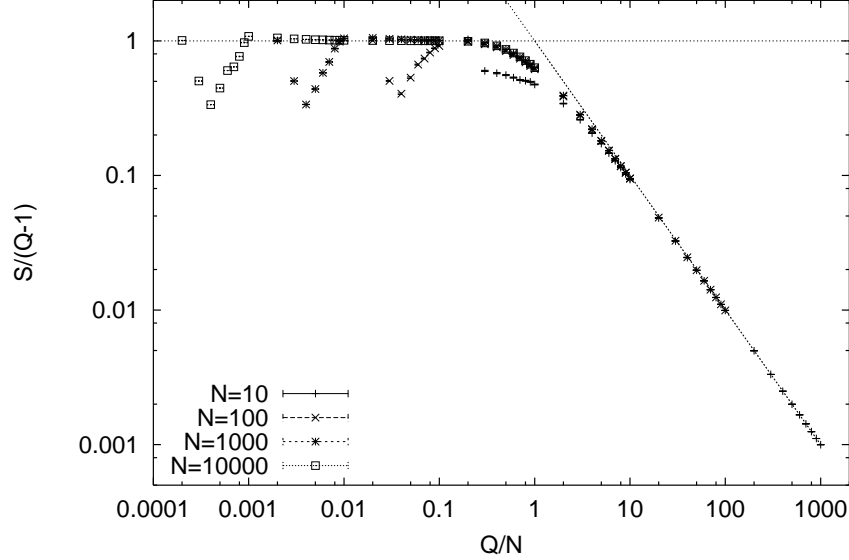
Figure 2: Scaling law for the number S of surviving opinions in the discrete KH model, from [22]. For the D model the figure looks similar [17] except that the downward deviations at the left end of the data sets are weaker.

model. Various ways of opinion averaging were investigated [25]. Hegselmann and Krause [3] also simulated asymmetric ϵ choices, which may depend on the currently held opinion.

Fig. 2 shows that the same scaling law as for the discrete D model also holds for the discrete KH model [22] on a scale-free Barabási-Albert network. For the usual version of the model, in which all individuals talk to each other, but with discrete opinions and discussions only between agents differing by one opinion unit, up to $Q = 7$ a consensus is reached while for $Q > 7$ several opinions remain. (The role of well-connected leaders in a similar opinion model on a Barabási-Albert network was studied in [26].)

As we mentioned above, by using discrete opinions it is possible to speed up the algorithm compared to the continuous case. The implementation of an algorithm for KH with discrete opinions must be probabilistic, because the value of the average opinion of compatible neighbours of an agent must necessarily be rounded to an integer and this would make the dynamics trivial, as in most cases the agent would keep its own opinion. We start with a community where everybody talks to everybody else, opinions $O =$

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$1, 2, .., Q$ and a confidence bound ℓ. After assigning at random opinions to the agents in the initial configuration, we calculate the histogram n_O of the opinion distribution, by counting how many agents have opinion O, for any $O = 1, 2, ..., Q$. Suppose we want to update the status of agent i, which has opinion k. The agents which are compatible with i are all agents with opinion $\overline{k} = k - \ell, k - \ell + 1, ..., k, k + 1, ..., k + \ell - 1, k + \ell$. Let $n_{k\ell} = n_{k-\ell} + n_{k-\ell+1} + ... + n_{k+\ell-1} + n_{k+\ell}$ be the total number of compatible agents. Then we say that agent i takes opinion $\overline{k}$ with the probability $p_{\overline{k}} = n_{\overline{k}}/n_{k\ell}$, which just amounts to choosing at random one of the agents compatible with i and taking its opinion. Let k_f be the new opinion of agent i. We simply need to withdraw one agent from the original channel k and add it to the channel k_f to have the new opinion histogram of the system, and we can pass to the next update. Notice that in this way the time required for a sweep over the whole population goes like $(2\ell+1)N$, where N is as usual the total number of agents and $2\ell + 1$ the number of compatible opinions. In the original algorithm with continuous opinions, instead, the time to complete an iteration goes as N^2, because to update the state of any agent one needs to make a sweep over the whole population to look for compatible individuals and calculate the average of their opinions. The gain in speed of the algorithm with discrete opinions is then remarkable, especially when $\ell \ll N$.

We have seen that the presence of the second factor N in the expression of the iteration time for the continuous model is exclusively due to the fact that we consider a community where every agent communicates with all others. If one instead considers social topologies where each agent interacts on average with just a few individuals, like a lattice, the iteration time will grow only linearly with N, and the algorithm will compete in speed with that of D. As a matter of fact, in many such cases the KH algorithm is much faster than the D algorithm.

2.3 Sznajd

The S model [4] is the most often studied model, and the literature up to mid-2003 was reviewed in [9]. Thus we concentrate here on the more recent literature.

The most widespread version uses a square lattice with two opinions, $O = \pm 1$. If the two opinions in a randomly selected neighbour pair agree, then these two agents convince their six lattice neighbours of this opinion; otherwise none of the eight opinions changes. If initially less than half of
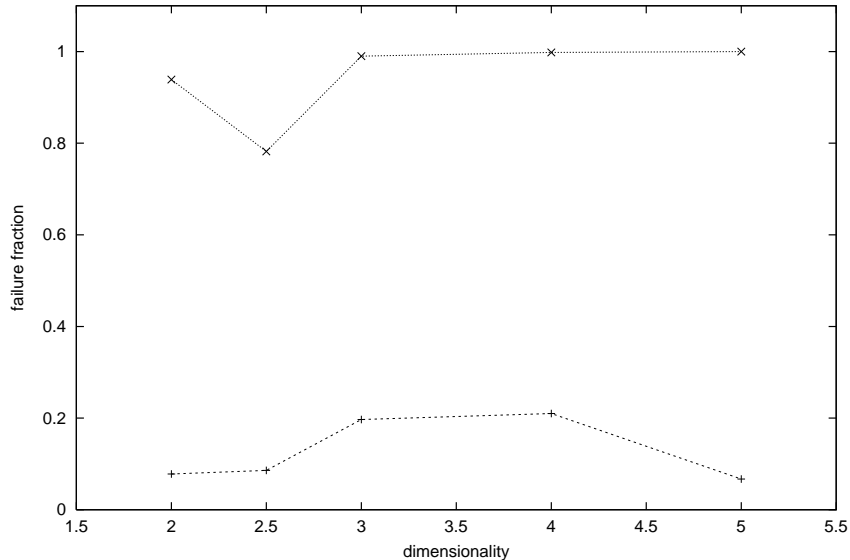
Figure 3: Variation with dimensionality of the probability not to reach a complete consensus. $d = 2.5$ represents the triangular lattice. The upper data refer to four opinions, the lower ones to three opinions, in small lattices: 19^2, 7^3, 5^4, 5^5. For larger lattices, the failures for three opinions vanish. Opinion O can only convince opinions $O \pm 1$.

the opinions have the value 1, at the end a consensus is reached with no agent having opinion 1; if initially the 1's have the majority, at the end everybody follows their opinion. Thus a phase transition is observed, which is the sharper the larger the lattice is. The growth of nearly homogeneous domains of -1's and 1's is very similar to spinodal decomposition of spin $1/2$ Ising magnets.

With $Q > 2$ possible opinions ($O = 1, 2, ..., Q$), always a consensus is found except if only people with a neighboring opinion $O \pm 1$ can be convinced by the central pair of opinion O; then a consensus is usually possible for $Q \leq 3$ but not for $Q \geq 4$ in a variety of lattice types and dimensions, see Fig. 3 (from [9]).

The greatest success of the S model is the simulation of political election results: The number of candidates receiving v votes each varies roughly as $1/v$ with systematic downward deviations for large and small v. This was obtained on both a Barabási-Albert [27] and a pseudo-fractal model [28]. Of

course, such simulations only give averages, not the winner in one specific election, just like physics gives the air pressure as a function of density and temperature, but not the position of one specific air atom one minute from now.

Schulze [29] simulated a multilayer S model, where the layer number corresponds to the biological age of the people; the results were similar as for the single-layer S model. More interesting was his combination of global and local interactions on the square lattice: two people of arbitrary distance who agree in their opinions convince their nearest neighbours of this opinion. Similarly to the mean field theory of Slanina and Lavicka [30], the times needed to reach consensus are distributed exponentially and are quite small. Therefore up to 10^9 agents could be simulated. The width of the phase transition (for $Q = 2$, as a function of initial concentration) vanishes reciprocally to the linear lattice dimension [29].

If the neighbours do not always follow the opinion of the central pair, but do so only with some probability [4], one may describe this probability through some social temperature T: The higher the temperature is, the higher is the probability to change opinion [31]. Then $T = 0$ means nobody changes opinion, and $T = \infty$ means everybody follows the S rule. Alternatively, one may also assume that some people permanently stick with their opinion [31, 32]. In this way, a more democratic society is modeled even for $Q = 2$ such that not everybody ends up with the same opinion.

In an S model with continuous opinions and confidence bound ϵ similar to the D and KH models, always a consensus was found independently of ϵ [33].

3 Damage Spreading

How is it possible to describe the reaction of people to extreme events in quantitative terms? From the previous discussion we have learnt that opinions can be treated as numbers, integer or real. A change of opinion of an arbitrary agent i is thus simply the difference between the new opinion and the old one. During the dynamical evolution, as we have seen above, opinions change, due to the influence of the people on their acquaintances. This is, however, the "normal" dynamics within a community. What we would like to investigate is instead how much a sudden perturbation ("extreme event") would alter the opinion variables of the agents of the system. The concept

of perturbation need not be exactly defined: for us it is whatever causes opinion changes in one or a few[1] agents of the system. We have in mind localized events, like strikes, accidents, decisions involving small areas, etc. We assume that people shape their own opinions only through the interactions with their acquaintances, without considering the influence of external opinion-affecting sources like the mass media, which act at once on the whole population.

In order to evaluate the effect of a perturbation on the public opinion it is necessary to know the opinion distribution of the agents when nothing anomalous takes place ("normal state"), and compare it with the distribution determined after the occurrence of an extreme event. From the comparison between these two replicas of the system we can evaluate, among other things, the so-called Hamming distance, i. e. how many agents have changed their mind, and how the influence of the perturbation spread as a function of time and distance from the place where the extreme event occurred.

This kind of comparative analysis is by no means new in science, and it is commonly adopted to investigate a large class of phenomena, the so-called *damage spreading* processes. Damage spreading (DS) was originally introduced in biology by Stuart Kauffman [34], who wanted to estimate quantitatively the reaction of gene regulatory networks to external disturbances ("catastrophic mutations"). In physics, the first investigations focused on the Ising model [35]. Here one starts from some arbitrary configuration of spins and creates a replica by flipping one or more spins; after that one lets both configurations evolve towards equilibrium according to the chosen dynamics under the same thermal noise (i.e. identical sequences of random numbers). It turns out that there is a temperature T_d, near the Curie point, which separates a phase where the damage heals from a phase in which the perturbation extends to a finite fraction of the spins of the system.

The simplest thing one can do is just to follow the same procedure for opinion dynamics models. The perturbation consists in changing the opinion variable of an arbitrarily selected agent in the initial configuration. After that, the chosen opinion dynamics applies for the two replicas. Preliminary studies in this direction already exist, and they deal with the Sznajd model on the square lattice. In [36] one adopted a modified version of Sznajd where the four agents of a plaquette convince all their neighbours if they happen

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[1]Here "a few" means that the agents represent a negligible fraction of the total population, which vanishes in the limit of infinitely many agents.

to share the same opinion; here the perturbed configuration is obtained by changing the opinion of all agents which lie on a line of the lattice. In [37] the shock consists in the sudden change of opinions of some finite fraction g of the whole population and the time evolution of the number of perturbed agents is studied as a function of g. More importantly, the authors of the latter paper show that in several cases critical shocks in social sciences can be used as probes to test the cohesion of society. This recalls the strategy of natural sciences: if we hit an iron bar with a hammer, from the velocity of the sound in the bar we are able to derive its density. In section 3.2 we will present new results on damage spreading for the Sznajd opinion dynamics [38]. Here we focus on the other two consensus models, D and KH. We shall first analyze the models for real-valued opinions, then we will pass to integer opinions. In all our simulations we defined the amount of damage as the number of agents differing in their opinions in an agent-to-agent comparison of the two replicas; we ignored the amount by which they differ.

An important issue is the choice of a suitable social topology. A bidimensional lattice lends itself to a geographical description of the damage spreading process: we can assume that the sites represent the position in space of the agents, and that the "acquaintances" of an agent be its spatial neighbours. In this way the lattice would map the distribution of people in some geographic area and the distances between pairs of agents on the lattice can be associated to physical distances between individuals. On the other hand, the regular structure of the lattice and the prescription of nearest-neighbour friendship endow the system with features which never occur in real communities. In fact, on the lattice each agent has the same number of friends and people who are geographically far from each other are never friends. These unrealistic features can be removed by adopting a different kind of graph to describe the social relationships between the agents. A Barabási-Albert (BA) network [39] could be a good candidate: it is a non-regular graph where the number of acquaintances of an agent varies within a wide spectrum of values, with a few individuals having many friends whereas most people have just a few. On the other hand the BA network is a structure with a high degree of randomness and can hardly be embedded in an Euclidean bidimensional surface, so a geographical characterization of the damage propagation would be impossible. In our opinion the ideal solution would be a graph which includes both the regular structure of the lattice and the disorder of a random graph. A possibility could be a lattice topology where the connection probability between the agents decays with some

negative power of the Euclidean distance, being unity for nearest neighbours. In what follows we shall however consider only the square lattice and the BA network.

3.1 Continuous Opinions

If opinions are real numbers, we need a criterion to state when the opinion of an agent is the same in both replicas or different due to the initial perturbation. Since we used 64-bit real numbers, we decided that two opinions are the same if they differ by less than 10^{-9}. In order to determine with some precision the fraction of agents which changed their opinions, it is necessary to repeat the damage spreading analysis many times, by starting every time from a new initial configuration without changing the set of parameters which constrain the action of the dynamics: the final result is then calculated by averaging over all samples. In most our simulations we have collected 1000 samples, in a few cases we enhanced the statistics up to 10000.

For KH with continuous opinions a detailed damage spreading analysis has recently been performed [40], for the case in which the agents sit on the sites of a BA network. The dynamics of the KH model is fixed by a single parameter, the confidence bound ϵ, which plays the role of temperature in the Ising model. Like in the Ising model, it is interesting to analyze the damage propagation as a function of the control parameter ϵ; it turns out that there are three phases in the ϵ-space, corresponding to zero, partial and total damage, respectively. The existence of a phase in which the initial perturbation manages to affect the state (here the opinions) of all agents is new for damage spreading processes, and is essentially due to the fact that opinions are real-valued. In this case, in fact, the probability for a "damaged" opinion to recover its value in the unperturbed configuration is zero; on the other hand, to perturb the opinion of an agent it suffices that one of its compatible neighbours be affected, and the probability of having a compatible "disturbed" neighbour increases with the confidence bound ϵ. The only circumstance which can stop the propagation of the damage is when the perturbed agents are not compatible with any of their neighbours. The considerations above allow us to understand why the critical threshold $\epsilon_s = 1/2$ found in [40], above which damage spreads to all agents of the system, coincides with the threshold for complete consensus of the model, as in this case all agents share the same opinion and so they are all compatible with each other, which means that any agent was affected by each of its

neighbours at some stage. Another interesting result of [40] is the fact that the two critical thresholds which separate the "damage" phases in the ϵ-space do not seem to depend on the degree d_0 of the first node affected by the shock, although the Hamming distance at a given ϵ increases with d_0. This means that it is irrelevant whether the shock initially affected somebody who has many social contacts or somebody who is instead poorly connected: if damage spreads in one case, it will do in the other too.

It is important to study as well how damage spreads under the D opinion dynamics. The hope is to be able to identify common features which would allow to characterize the spreading process independently of the specific consensus model adopted. In section 2.2 we stressed the analogies between the KH and the D model, so we expected to find similar results. For the D model we need to fix one more parameter to determine the dynamics, the convergence parameter μ. The value of μ affects exclusively the time needed to reach the final configuration, so it has no influence on our results: we set $\mu = 0.3$. Fig. 4 shows how the Hamming distance varies with the confidence bound ϵ for the D model on a BA network. The total number of agents is 1000. We remark that the damage is here calculated when the two replicas of the system attained their final stable configurations. We have also plotted the corresponding curve for the KH model, as obtained in [40]. The two curves are quite similar, as we expected, and the thresholds for the damage spreading transition are very close to each other. Again, for $\epsilon > \epsilon_s = 1/2$, all agents will be affected by the original perturbation.

As we explained in the introduction, our main aim is to attempt a spatial characterization of the damage spreading process, which would be impossible on a BA network. This is why from now on we shall focus on the lattice topology. Here we start by changing the opinion variable of the agent lying on the center site of the lattice; if the lattice side L is even, as in our case, the center of the lattice is not a site, but the center of a plaquette, so we "shocked" one of the four agents of the central plaquette. We refer to the initially shocked agent as to the origin. We will address the following issues:

- How far from the origin can the perturbation go?

- What is the probability for an agent at some distance from the origin to be itself affected?

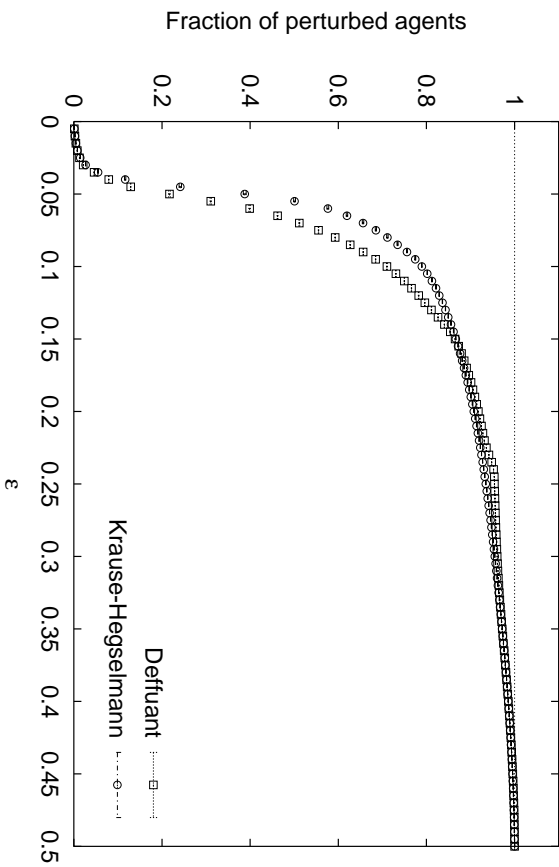- How does this probability $p(d, t)$ vary with the distance d and with the time t?

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Figure 4: Fraction of perturbed agents in the final configuration as a function of ϵ for the D and the KH model on a Barabási-Albert network.

To discuss the first issue, we need to calculate the *range r* of the damage, i. e. the maximum of the distances from the origin of the agents reached by the perturbation. The damage probability $p(d, t)$ is the probability that, at time t, a randomly chosen agent at distance d from the origin changed its mind, due to the initial shock. Here the time is represented as usual by the succession of opinion configurations created by the dynamics. The time unit we adopted is one sweep over all agents of the system. To calculate $p(d, t)$ we proceed as follows: after t iterations of the algorithm, we select all sites which are at the distance d from the origin and which lie on-axis with respect to the origin, as in the scheme below
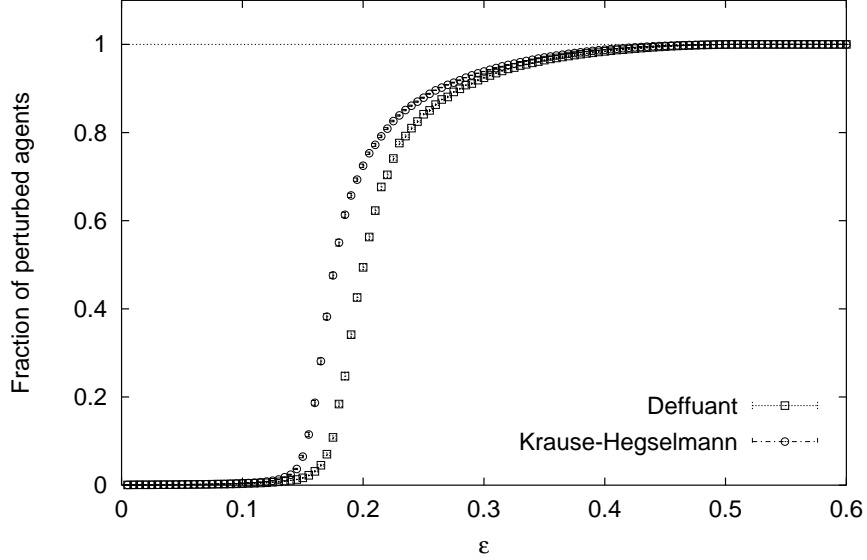

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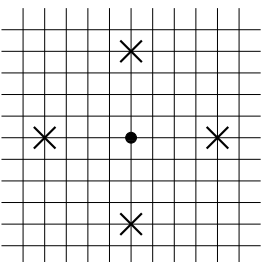
Figure 5: As Fig. 4, but for agents sitting on the sites of a square lattice.

where the black dot in the middle represents the origin and the crosses mark the agents to be monitored. The damage probability is simply given by the fraction of these agents whose opinions differ from those of their counterparts in the unperturbed configuration (e. g. if two of the four agents changed their mind, the probability is $2/4 = 1/2$). Note that by construction d must be a multiple of the lattice spacing (in our illustrated example $d = 4$). At variance with the evaluation of the damage range r, where we review all lattice sites, for the damage probability we neglected the off-axis sites because the lattice is not isotropic and the corresponding data would be affected by strong finite size effects due to the lack of rotational symmetry. To derive $p(d, t)$ only from four sites is of course difficult and we need to average over many samples for the data to have statistical meaning; we found that a number of samples of the order of 10^3 is enough to obtain stable results. We calculated $p(d, t)$ for all distances from the center to the edges of the lattice and for all intermediate states of the system from the initial random configuration to the final stable state.

We will present mostly results relative to the D model. The corresponding analysis for the KH model leads to essentially the same results. For the purpose of comparison with Fig. 4, we plot in Fig. 5 the Hamming distance
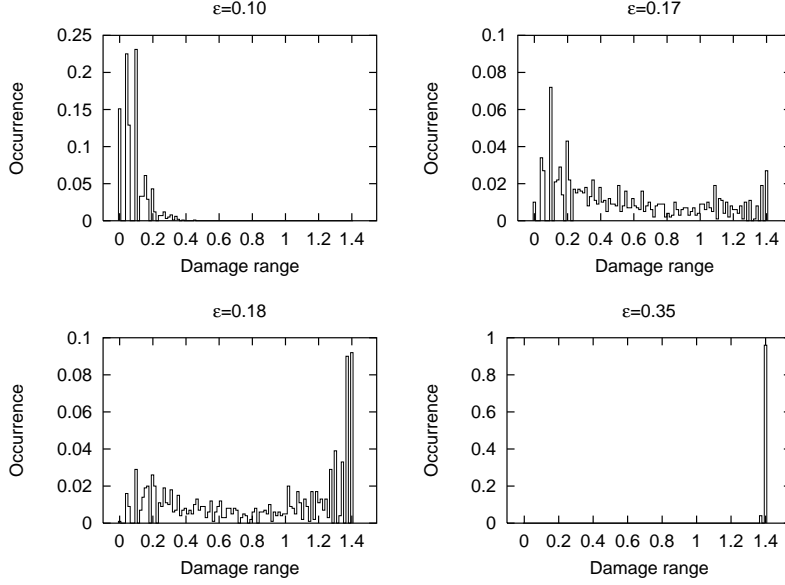
17

Figure 6: D model, continuous opinions. Histograms of the damage range corresponding to four values of ϵ; the lattice size is 40^2.

as a function of ϵ, for the D and the KH model. The curves refer to a lattice with 40^2 agents: the two patterns are again alike. The damage spreading thresholds are close, but they lie quite a bit higher than the corresponding values relative to the BA network. This is basically due to the fact that in a BA network each vertex lies just a few steps away from any other vertex (small world property), and this makes spreading processes much easier and faster. Indeed, in the damage spreading phase, the time needed for the perturbation to invade the system is much longer for the lattice than for the BA network.

Since the amount of the damage is a function of ϵ, the range r of the damage is also a function of ϵ. It is interesting to analyze the histograms of the values of r for different values of the confidence bound. In Fig. 6 we show four such histograms, corresponding to $\epsilon = 0.10, 0.17, 0.18, 0.35$. Note that the values of r reported on the x-axis are expressed in units of $L/2$ (half of the lattice side), which is the distance of the center site from the edges of the lattice; since the farthest points from the origin are the four vertices of the square, the maximal possible value of r is $L\sqrt{2}/2$ (which corresponds to $\sqrt{2} \sim 1.414$ in the figure). In the top left frame ($\epsilon = 0.10$), damage does
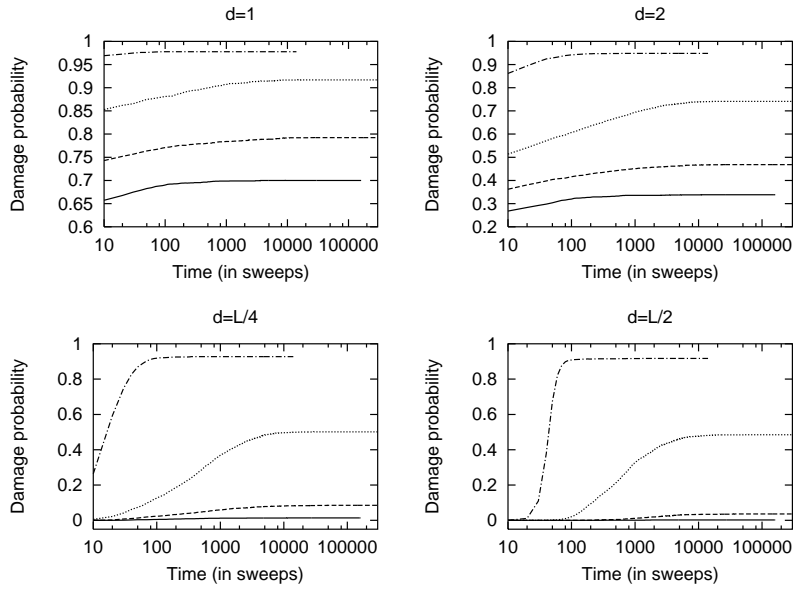
18

Figure 7: D model, continuous opinions. Time evolution of the damage probability. Each frame refers to a fixed distance d from the origin, the curves are relative to different values of ϵ; the lattice size is 40^2.
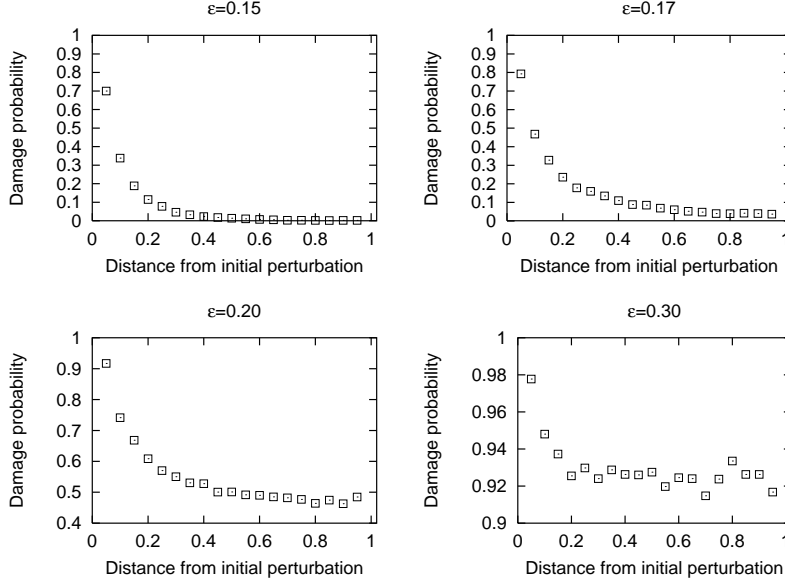
19

Figure 8: D model, continuous opinions. Dependence of the damage probability on the distance d from the origin, when the system has reached the final stable configuration; the lattice size is 40^2.

not spread and in fact the histogram is concentrated only at low values of r. In the other two frames, instead, we are near the threshold for damage spreading, and we see that the damage often reaches the edge of the lattice ($r = 1$ in the plot) and even the farthest vertices ($r = \sqrt{2}$). The step from $\epsilon = 0.17$ to $\epsilon = 0.18$, in spite of the little difference in the value of the confidence bound, is quite dramatic and signals the phase transition: in the first case (top right) it is more likely to have short ranges than long ones, in the other (bottom left) we have exactly the opposite. In the last frame, the range is almost always maximal; looking at Fig. 5, we can see that for $\epsilon = 0.35$ more than 90% of the agents are disturbed, so it is very likely that the perturbation reaches one of the four vertices of the square.

The study of the damage probability $p(d, t)$ is more involved, as it is a function of two variables, the distance d and the time t. A good working strategy is to analyze separately the dependence of $p(d, t)$ on the two variables. We can fix the distance to some value d_0 and study how the damage probability at d_0 varies with time. We can also fix the time to t_0 and study how the probability at time t_0 varies with the distance from the origin. On

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top of that, we should not forget the dependence on ϵ, which determines the "damage" state of the system.

In Fig. 7 we explicitely plot the time dependence of the damage probability at four different distances from the origin, $d = 1, 2, L/4, L/2$. In each frame, we have drawn four curves, corresponding (from bottom to top) to $\epsilon = 0.15, 0.17, 0.20, 0.30$. We remark that the probability is the higher the larger ϵ, since this corresponds to a larger number of affected agents. All curves increase with time, which shows that the damage does not heal, and they reach a plateau long before the system attains the final opinion configuration. Note the rapid rise of the probability at the two largest distances ($L/4$ and $L/2$), for the two values of ϵ which fall in the damage spreading phase ($\epsilon = 0.20, 0.30$).

Fig. 8 shows how the damage probability varies with the distance from the origin, at the end of the time evolution of the system. The values of the distance on the x-axis are renormalized to the maximal distance on-axis from the origin, $L/2$, as in Fig. 6. We have again four frames, one for each of the four values of ϵ we have considered in Fig. 7. We notice that for $\epsilon = 0.15$, which is slightly below the threshold, the damage probability at the edge (top left) is zero, whereas for $\epsilon = 0.17$, which is near the threshold, it is small but nonzero (top right) and it is about $1/2$ for $\epsilon = 0.20$ (bottom left). We tried to fit the curves with simple functions of the exponential type. We found that the decrease with the distance is stronger than exponential: for low ϵ, $p(d,t)$ (at fixed t) is well approximated by $a \exp(-bd)/d$.

We remind that we have chosen to introduce the shock in the system just at the beginning of the evolution. If one instead would perturb the system some time later, the amount of the damage and the corresponding probabilities would decrease; however, the results of the analysis would be qualitatively the same.

3.2   Discrete Opinions

There is essentially one reason which justifies the use of real-valued opinions: the opinions of any two individuals are never exactly the same, although they can be arbitrarily close. This is what commonly happens in society, where no two persons have exactly the same idea or judgement about any issue. In fact, our opinion about somebody or a special event can fall anywhere between the two edges "very bad" and "very good", something like the spectrum of visible light, where one can pass smoothly from red to violet.
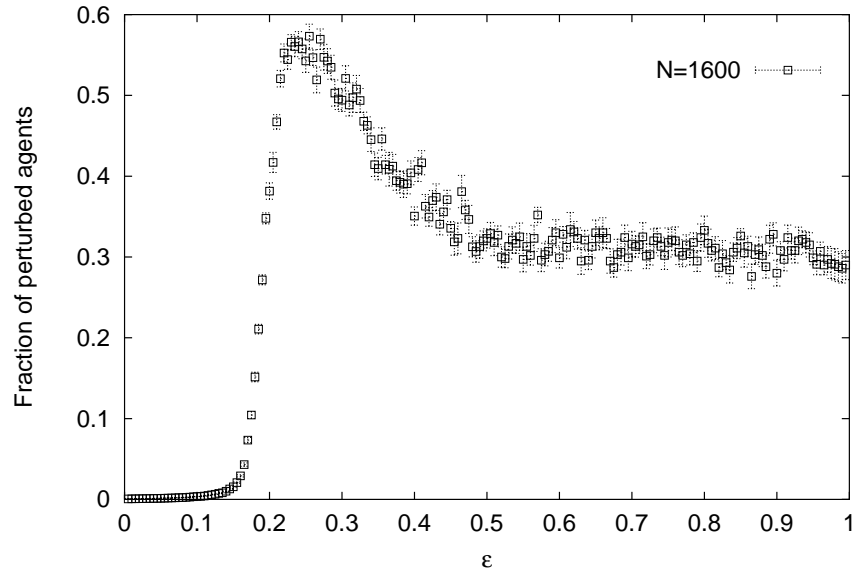
Figure 9: D model, integer opinions. Fraction of perturbed agents in the final configuration as a function of ϵ for agents sitting on the sites of a square lattice.

On the other hand, for all practical purposes, this continuous spectrum of possible choices can be divided in a finite number of "bands" or "channels", where each channel represents groups of close opinions. This is actually what teachers do when they evaluate the essays of their students with marks, which are usually integers. Also electors have to choose among a finite number of parties/candidates. Finally, for the case we are mostly interested in, i. e. the reaction of people to extreme events, the only possible quantitative investigation for sociologists consists in making polls, in which the interviewed persons have to choose between a few options.

These examples show that it is more realistic to use integers rather than real numbers for the opinion variables of consensus models. Here we will repeat the damage spreading analysis of the previous section for the D model with integer opinions on a square lattice. We will see that the results are quite different from those we found before, due to the phenomenon of *damage healing*.

To start with, we must fix the total number Q of possible opinions/choices. Since we performed simulations for systems with few thousands agents, we decided to allow for a number of choices of the same order of magnitude, therefore we set $Q = 1000$. The confidence bound must be an integer ℓ, but for consistency with the notation we have adopted so far, we will still use a real ϵ, again between 0 and 1, so that ℓ is the closest integer to ϵQ.

In Fig. 9 we show the variation of the Hamming distance with the confidence bound ϵ, for a lattice with 40^2 sites. We immediately notice the difference with the analogous Fig. 5 for continuous opinions: after the rapid variation at threshold, the fraction of damaged sites reaches a peak, then it decreases and finally it forms a plateau at large ϵ. Going from real to integer opinions we have no more total damage, i. e. the perturbation can affect at most some fraction $f < 1$ of the total population (here $f \sim 0.6$), but it has no chance to affect all agents. If we increase the number of agents N but we keep Q fixed to the same value, the height of the final plateau decreases, going to zero when $N/Q \to \infty$.

Why does this happen? Taking a look at Fig. 10 helps to clarify the situation. Here we see the histograms of the damage range for $\epsilon = 0.18, 0.25, 0.35, 0.45$. If we compare the frame relative to $\epsilon = 0.18$ (top left) with its counterpart for continuous opinions (Fig. 6, bottom left), we see that they are basically the same. We are close to the transition so there is some finite probability for the damage to reach the edges and even the vertices of the square. We notice that the histogram is continuous, in the sense that any value of the
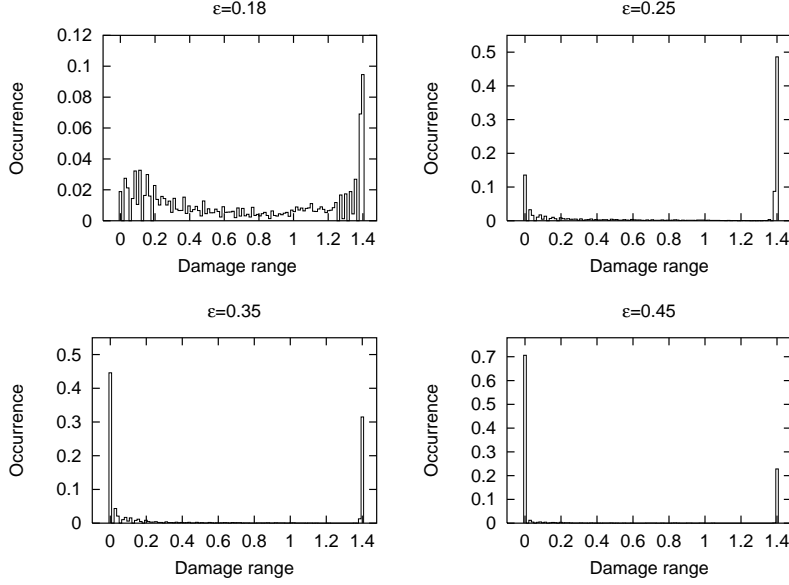
Figure 10: D model, integer opinions. Histograms of the damage range corresponding to four values of ϵ; the lattice size is 50^2.

range between the two extremes is possible. If we now look at the other three frames, the situation is very different: the range can be either very short or very long. In particular, when ϵ is very large (bottom right), the range is zero or maximal. That means that either the damage heals, or it spreads to all agents. In fact, for large ϵ ($> 1/2$), there is complete consensus in the final configuration (see section 2.1), so all agents will end up with the same opinion. The question is then whether the final opinion in the perturbed configuration coincides or not with that of the unperturbed configuration; in the first case we have no damage, in the second total damage.

Now, real-valued opinions can be modified by arbitrarily small amounts, and that would still correspond to damage. On the contrary, the variations of integer opinions are discontinuous steps, and the latter are much more unlikely to occur. In this way, it is virtually impossible for a single agent to trigger a "jump" of the final opinion of all agents of the system to a different value. So, for large ϵ and many agents, the original perturbation will be healed by the dynamics[2] (no damage), whereas for continuous opinions even

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[2]The non-vanishing probability for total damage in Fig. 10 is a finite size effect, as the total number Q of opinions is about the same as the population N.
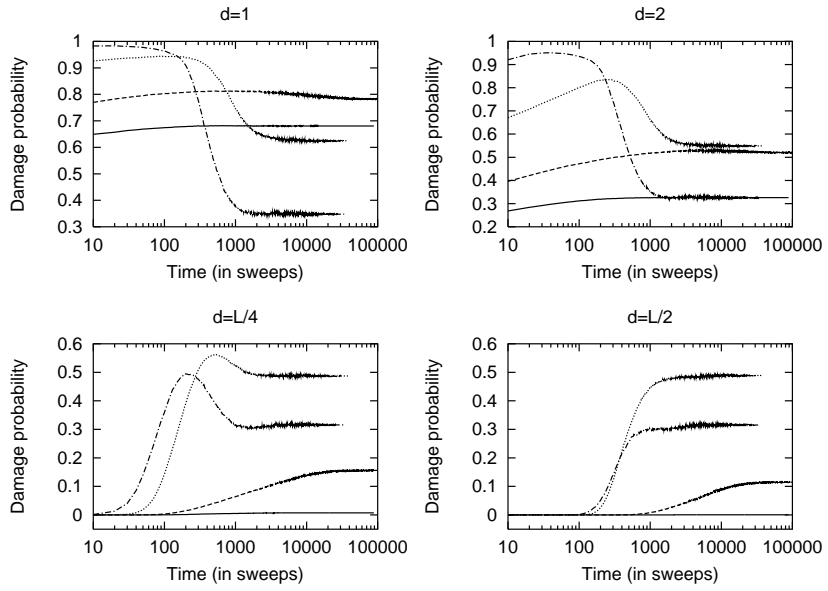
24

Figure 11: D model, integer opinions. Time evolution of the damage probability. Each frame refers to a fixed distance d from the origin, the curves are relative to different values of ϵ; the lattice size is 50^2.
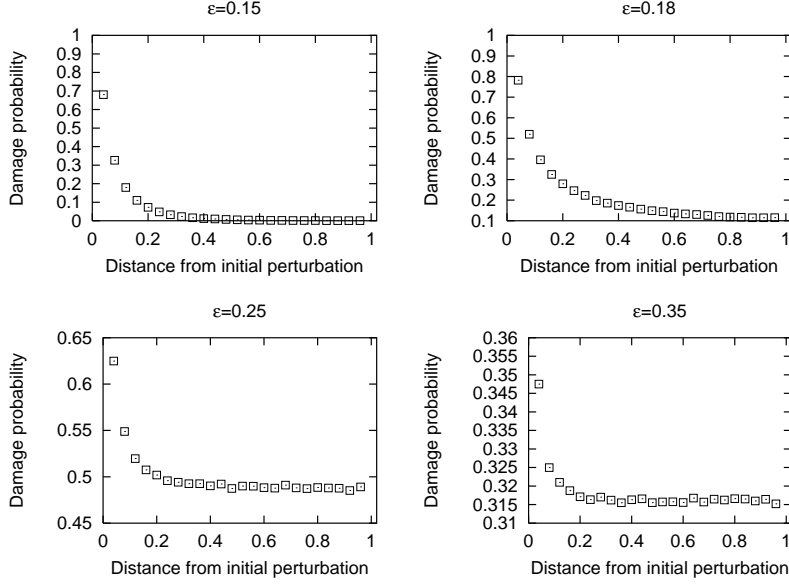
Figure 12: D model, integer opinions. Dependence of the damage probability on the distance d from the origin, when the system has reached the final stable configuration; the lattice size is 50^2.

a small shock manages to shift a little bit the final opinion of the community (total damage).

The presence of damage healing is also clearly visible in Fig. 11, which is the counterpart of Fig. 7 for integer opinions. The four curves of each frame refer to $\epsilon = 0.15$ (continuous), 0.18 (dashed), 0.25 (dotted) and 0.35 (dot-dashed). The damage probability is no longer monotonically increasing as in Fig. 7, but it displays various patterns, depending on the confidence bound and the distance from the origin. In particular, observe the behaviour of the curve for $\epsilon = 0.35$ and $d = 1$ (top left frame, dot-dashed line): here the probability is initially close to 1, because we are examining a neighbour of the shocked agent, but after few iterations it falls to about 0.3, due to healing. We also note the curious shape of the two upper curves for $d = L/4$, which recalls the pattern of the Hamming distance with ϵ of Fig. 9: the damage probability rapidly rises to a maximum and then it decreases to an approximately constant value.

Fig. 12 shows the dependence of the damage probability on the distance in the final opinion configuration, for $\epsilon = 0.15, 0.18, 0.25, 0.35$. The curves

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look similar as those of Fig. 8. Again, the damage probability decreases faster than exponentially.
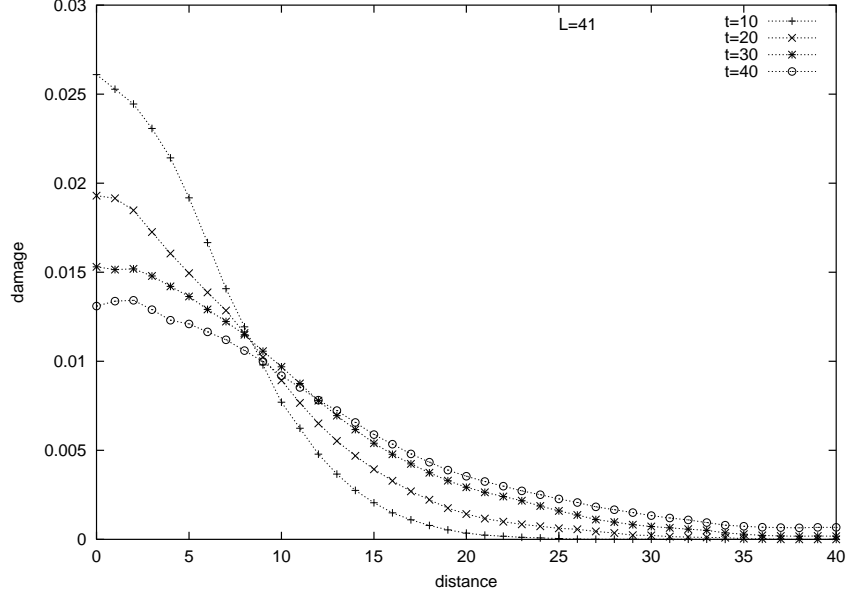

Figure 13: S model, two opinions. Dependence of the damage probability on the distance d from the origin, for various times on a 41×41 lattice.

We conclude with some new results on damage spreading for the S model with two opinions on a square lattice [38], which complement the analyses of [36, 37]. Fig. 13 shows the damage probability as a function of distance at various times. We see that the values of the probability are quite low; in fact, the system always evolves towards consensus, so the damage will heal on the long run, as it happens in the D and KH models (with discrete opinions) when the confidence bound ϵ is above the threshold for complete consensus.

If damage would spread like in a diffusion process, the distance covered by the propagation of the perturbation would scale as the square-root of the time t, and the probability to damage a site at distance d would follow for long times a scaling function $f(d/\sqrt{t})$. Fig. 14 shows that for $t \gg 1$ this seems indeed to be the case, even though damage spreading is not a random diffusion process.

Applications of these techniques to the case of several different themes on which people may have an opinion will be given elsewhere [41].
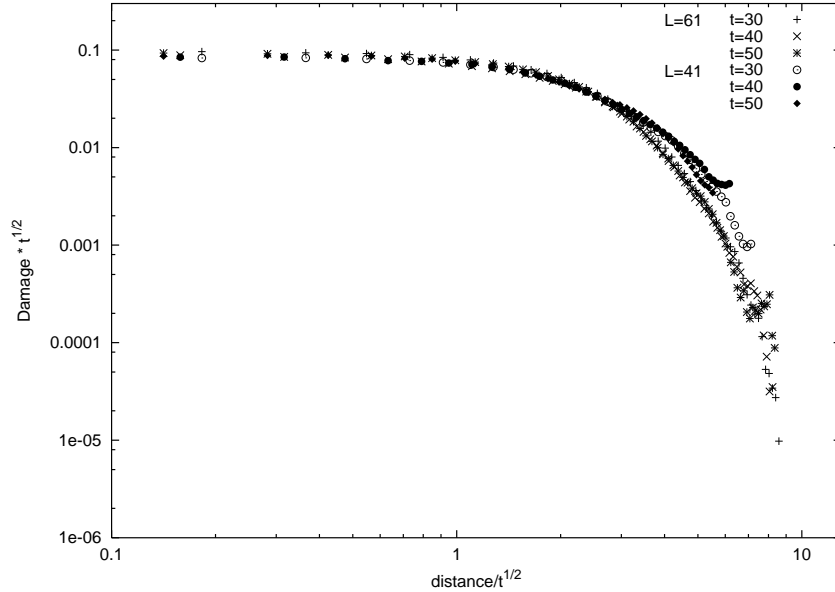
Figure 14: S model, two opinions. Rescaling of the damage probability curves of Fig. 13, also for larger lattices. Here we plot the damage probability times $\sqrt{t}$ versus $d/\sqrt{t}$ (t is the time). For $t > 20$, the curves for different times roughly overlap.

4 Discussion

The three main models D, KH and S discussed follow different rules but give similar results: They end up in a final state where no opinion changes anymore. Depending on the confidence interval ϵ for continuous opinions, or ℓ for discrete opinions, this final state contains one opinion (consensus), two (polarization) or three and more (fragmentation). In the discrete case with Q different opinions, there is a maximum Q (2 for D, 3 for S, 7 for KH) for which a consensus usually is found. These numbers may correspond to the maximum number of political parties which may form a stable coalition government. The three rules differ in that S describes missionaries who don't care about the previous opinions of those whom they want to convince; KH describes opportunists who follow the average opinion of their discussion partners; and D describes negotiators who slowly move closer to the opinion of their discussion partner. Election results were successfully simulated by

the model of S but not by that of D and KH, perhaps simply because nobody tried it yet with D and KH.

The reaction of people to extreme events was investigated by performing a damage spreading analysis on the three consensus models we have introduced. The extreme event induces a change of opinion in one (or a few) agent(s), the dynamics propagates the shock to other agents. We represented the social relationships between people with a square lattice and a scale-free network á la Barabási-Albert. In both cases we found that there is quite a wide range of values of the confidence interval ϵ (or ℓ) for which the original shock influences the opinions of a non-negligible fraction of the community. For very tolerant people and continuous opinions, the whole community will be affected by the event on the long run. By using integer-valued opinions, instead, we found that the perturbation cannot affect more than a maximal fraction of the population (it can be sizeable, though). On the lattice we could as well study how the influence of the extreme event on the opinions varies with the distance in time and space from the event. The damage probability at a fixed distance from the original shock varies very rapidly with time; it increases up to a plateau for continuous opinions, it follows more involved patterns for integer opinions. Our analysis also shows that the effect of the perturbation falls faster than exponentially with the distance from the place where the event took place.

What have we achieved with these simulations? We did *not* find a way to predict earthquakes or floods, nor did we propose a method how to convince people to judge these dangers objectively, instead of being overly influenced by events close in time and space, and of forgetting the lessons from distant catastrophes which happened long ago. Our simulations give quantitative data for these space-time correlations of opinions and extreme events. Once sociology delivered quality data on real people and their opinions [1], one can compare these results with the simulations and modify if needed the simulations until they give a realistic description. Only then can the simulations be used to predict how danger perception will develop in space and time.

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