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An agent-based model for emotion contagion and competition in online social media



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HIGHLIGHTS

- An agent-based model that considers newest findings for emotion spread is developed.
- The model's effectiveness is thoroughly validated by reproducing multiple patterns.
- The model reveals that anger is more competitive in negative events.
- A critical gap, which can be a warning signal, is also identified.
- The model can be helpful in study of emotion issues from the view of simulations.

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ABSTRACT

Recent studies suggest that human emotions diffuse in not only real-world communities but also online social media. However, a comprehensive model that considers up-to-date findings and multiple online social media mechanisms is still missing. To bridge this vital gap, an agent-based model, which concurrently considers emotion influence and tie strength preferences, is presented to simulate the emotion contagion and competition. Our model well reproduces patterns observed in the empirical data, like anger's preference on weak ties, anger-dominated users' high vitalities and angry tweets' short retweet intervals, and anger's competitiveness in negative events. The comparison with a previously presented baseline model further demonstrates its effectiveness in modeling online emotion contagion. It is also surprisingly revealed by our model that as the ratio of anger approaches joy with a gap less than 12%, anger will eventually dominate the online social media and arrives the collective outrage in the cyber space. The critical gap disclosed here can be indeed warning signals at early stages for outrage control. Our model would shed lights on the study of multiple issues regarding emotion contagion and competition in terms of computer simulations.

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1. Introduction

Recent years have witnessed the tremendous development of online social media. For example, Twitter attracts more than 310 million monthly active users and produces 500 million tweets every day. In online social media, the high-dimensional user-contributed contents, which offer a "big data" window [1], are easily to be collected and analyzed. Because of this, the

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growth of online social media provides an unprecedented opportunity to study human behavior with unparalleled richness and granularity [2-8].

Emotion contagion on real-world social networks is a traditional research area that has been studied by psychologists and sociologists for many years [9-12]. After the development of Internet, especially the explosion of online social media, researchers found that users' emotions can also transfer to others via virtual online connections [13-18]. Kramer et al. found that sentimental states can be transferred to others via emotion contagion through a massive experiment with controls on Facebook [19], Ferrara et al. analyzed the emotion valence of messages that Twitter users receive before they post positive or negative tweets to highlight the effect of emotion contagion [20]. Several emotion contagion models, from the perspective of computer simulations, have also been developed in previous literatures [21–24]. Hill et al. proposed a SISa model, which is a variant of classical susceptible-infected-susceptible model and included the possibility for spontaneous infection [22]. Wang et al. developed an emotion-based spreader-ignorant-stifler (ESIS) model to simulate information diffusion by considering emotion weights when calculating the spreading probabilities of edges [23]. However, an emotion contagion model that considers multiple online social media characteristics and emotion features is still missing. Aiming at filling this gap, in this paper, we propose an agent-based model to study the mechanisms of online emotion diffusion. In previous studies, users' emotions are simply classified to positive and negative or just a score of happiness, neglecting the detailed aspects of human sentiments. However, fine-grained emotion states such as anger or sadness play essential roles in the bursts of social events like earthquake or terrorist attack. In this paper, we categorize human emotions into four categories which are anger, disgust, joy and sadness [25,26] to systematically investigate and model the detailed mechanisms of different emotion contagion patterns.

On social networks, emotions flow with the diffusion of information, while information diffusion procedure is also influenced by user emotions behind the contents. Previous studies found that emotional tweets are easier to be retweeted and thus spread more quickly than neutral ones [27] and both positive and negative emotions might diffuse in the subsequent comments to corresponding blog entries [28]. In psychology view, each emotion has three dimensions which are valence (positive–negative), arousal (passive/calm–active/excited), and tension (tense–relaxed) [29], among which both valence and arousal impact the diffusion procedure. High arousal emotions such as anger or anxiety boost diffusion more than low arousal emotions such as sadness or contentment [30,31]. For valence, researchers also found that bad is stronger than good, which implies that negative emotions possess stronger impact than positive ones [32]. Kim and Salehan also suggested that negative emotions have higher retweet possibilities than positive ones [33]. Fan et al. found that anger, which is negative and of high arousal, is more influential than joy while sadness has extremely low correlation between connected users [26], which are consistent with the findings on the influence of valence and arousal in diffusion procedure.

Meanwhile, the dynamics beyond information diffusion can be influenced by the structure of underlying network such as the characteristics of nodes and ties. Since emotion spread is tightly coupled with the information flow in terms of retweets or comments, its contagion patterns are accordingly affected by the network features. Robert et al. found that nearly all transmission occurred between close friends, suggesting that strong ties are instrumental for spreading both online and real-world behavior in human social networks [34]. Beyond that, the theory of weak ties, which has been long established, suggests that weak ties enhance diffusion by exchanging novel information across groups [35]. Onnela et al. analyzed mobile communication networks and found that removal of the weak ties results in network collapse [36]. In online social media, weak ties still play a crucial role in the dissemination of information [37]. However, Zhao et al. suggested that weak ties can be categorized into positive ones and negative ones which can enhance and prevent the diffusion respectively [38,39]. For the coupling of emotion and tie strength, different emotions would have different preferences on propagation ties, e.g., Fan et al. suggested that anger emotion prefers weaker ties compared to joy [40]. In other words, joyful messages tend to transfer between close friends while angry ones are more likely to evoke resonances of strangers. Meanwhile, the disclosed difference in different emotions' spread implies the existing of emotion competition, which still remains an open question and is rarely investigated. The existing findings indeed offer insightful mechanisms to establish competent contagion and competition models for emotion in social networks.

In fact, traditional controlled experiments on emotion diffusion are extremely difficult because of the high costs and spatio-temporal limitations. While with a reliable diffusion model, researchers could study various emotion-related issues such as emotion competition and emotion dynamics in hot-event through computer simulation. However, few existing diffusion models take the influence of emotion and tie-strength preferences into accounts concurrently. In this paper, based on previous works, we propose an agent-based model which combines the two critical features that function profoundly in the contagion process. Based on the model, we study the tie-strength preferences of different emotions, emotion diffusion and how emotions compete in social network. Our model perfectly reproduces the vitality difference, the tie-strength preference and the retweet interval difference of different emotions. The comparison results with previous ESIS model [23] further demonstrate the effectiveness of our model. Moreover, our model offers an comprehensive understanding of emotion contagion and competition, and the emergence of dominated emotion that widely disseminates in the network. Our model would shed lights on study of emotion issues in terms of computer simulations.

2. Background

In this section, we introduce our dataset and several up-to-date technologies and findings, which are the foundations of the present work. Our data set is publicly available and can be downloaded freely through https://doi.org/10.6084/m9.figshare.4311920.v2.

Table 1Statistical results of emotion proportions and emotion correlations of anger, joy, disgust and sadness. The number of joyful messages are larger than that of others. Anger and Joy have large correlations which indicate that they are easy to influence other users. Statistics of anger and joy are emphasized by bold fonts.

	Anger	Joy	Disgust	Sadness
Proportion	19.2%	39.1%	13.7%	28.0%
Correlation	0.41	0.35	0.04	0.03

2.1. Dataset

Although we concentrate on the emotion contagion model, several empirical results are still necessary to define and estimate the parameters of the model. We collect over 11 million tweets posted by approximately 100,000 users over half a year in Weibo (a Twitter's variant in China) to perform empirical experiments. Following edges link these users and these connected users compose a directed network denoted as G(V, E, W), where V and E represent all the users and following edges respectively. E is the set of edges' strengths derived from common friends definition (which can be seen in 2.3). To investigate the contagion mechanisms of fine-grained emotions, we employ a Bayesian classifier introduced in [25] to categorize tweets into four emotions which are anger, disgust, joy and sadness. The proportion of the four emotions in these tweets are illustrated in Table 1, which are important parameters in our model.

2.2. Emotion correlation

From the view of conventional social theory, homophily is widely disclosed in social networks [41]. After the explosion of online social media, researchers found that users' physiological states are also assortative in online social networks [1,13]. Moreover, anger and joy are revealed to be significantly correlated and anger's correlation is even higher than joy [40]. However, disgust and sadness possess extremely low correlations. The emotion with higher correlation indicates that it is easier to affect others, which makes it an important feature of the diffusion process. In this paper, we apply the correlation calculation method on the connected users in our data and achieve similar results as in [40], as can be seen in Table 1. In our model, we apply the correlation results of four emotions to construct the model. While in the simulations, we only consider anger and joy and treat the diffusion of disgust and sadness as random reposts due to their extremely low influence.

2.3. Tie-strength preference

Besides the emotion correlation, tie strengths also have notably influence on diffusion. For example, users tend to communicate via strong ties [36], but weak ties are also irreplaceable in diffusion due to its "bridge" function between different groups. In this paper, we use the following three tie-strength definitions as in [40]:

- Common friends. More common friends of two users indicates that their relationship is stronger. Because of this, the first metric we apply is the proportion of *common friends* [36,38], which is defined as $c_{ij}/(k_i 1 + k_j 1 c_{ij})$ for the tie between user i and j, where c_{ij} denotes the number of common friends of i and j. And k_i and k_j represent the degrees of i and j respectively.
- Reciprocity. A higher ratio of reciprocity indicates more trust social connections and more significant homophily [42]. Therefore, we define the proportion of reciprocal links, through which messages of a specific emotion disseminate, as a measurement of tie-strength of this emotion. Note that the result of reciprocity strength is a proportion rather than an averaged strength as that of the other two definitions.
- Retweets. The idea of retweet strength is that larger number of retweets represents more frequent interactions. Note that, different from the previous two metrics, retweet strength evolves over time. Therefore, we count only the retweets that occurred before the relevant emotional retweet. Moreover, to smooth the comparison between anger and joy, the retweet strength (denoted as S) is normalized by $(S S_{min})/(S_{max} S_{min})$, in which S_{min} and S_{max} separately represent the minimum and maximum values of all observations.

Based on these three metrics, researchers found that different emotions have different edge preferences in dissemination procedure [40], i.e., anger prefers weaker ties than joy. People tend to share joyful messages with their close friends, but can be easily enraged by strangers' anger in online social media. In this paper, we study the tie-strength preference by the simulation of our model and achieve similar results. Moreover, we reveal the underlying reason of different tie-strength preferences of the two emotions.

2.4. Information diffusion and emotion competition

Information diffusion is a traditional research area and several classical information diffusion models, such as linear threshold model (LT) and independent cascade model (IC) [43–46] are well-developed in previous literatures. LT model considers the weight of connection w_{yv} between y and v and generates threshold θ_v for node v. v becomes active if $\Sigma_y w_{yv} \ge \theta_v$, which means the sum of weights between v and its active neighbors exceeds a threshold. With the development of online social media, researchers found that information diffusion presented quite different mechanisms on this platform because of its unique pushing and republishing features [47,38]. Meanwhile, network structure such as tie strength exerts significant impact on the diffusion procedure [36]. Zhao. et al. proposed an information diffusion model which considers the pushing and republishing features and the tie-strength to investigate the relationship between the tie strength and information propagation in online social networks [39]. Our model absorbs the advantages of previous literatures such as the pushing and republishing mechanisms, the preference of tie strength in diffusion procedure, and thresholds which are defined to determine the condition of diffusion.

As the theory of economics of attention [48–52], user's attention is a scarce resource that multiple information compete for. With the development of Internet, especially the explosion of online social media, netizens receive redundant messages from various online channels such as Facebook, Twitter and online forums, resulting in extremely severe information overload problem. Because of this, information competes for users' finite attention and the winners go vital in cyber space [53,54]. To investigate the competition mechanisms, researchers also proposed competition models [55–58], e.g., Weng et al. proposed an agent-based model to study memes competition in Twitter. However, except for hashtags, topics and memes, emotions also compete for users' limited attention and the competitive emotion would pervasively diffuse in the network. Based on this idea, we introduce a structure in our model to represent users' limited attention and analyze the diffusion results from the perspective of emotion competition.

3. Agent-based model

Agent-based model [59–62,56,63,64] is widely employed in previous works to study network community, discussion process, emotion expression patterns and meme competition, etc. Inspired by the model introduced in [56], we develop an emotion contagion model that combines emotion influence and tie strengths, to study emotion contagion mechanisms in online social networks. We simulate the diffusion process on the real-world following network G(V, E, W) and diffusion behaviors are reflected in terms of retweets which occur on connected edges. Each user has a screen which contains N time-ordered messages that he/she has received. Messages posted by his/her followees will be pushed into the screen. In one circulation, a randomly selected user posts new messages with probability P_n , or reposts several ones that in the screen. If the number of messages in the screen exceeds N, the old messages will be removed. This process stems from the pushing and republishing mechanisms in online social media that users either post new messages or read messages he/she received and then repost some of them. The critical principle of the dissemination is that if one message with emotion of significant influence and the tie strength between the two users are strong, the message will be preferentially reposted. The model is defined as follows:

- Step 1: Randomly select a user u from V.
- Step 2: *u* posts a new message *m* with the probability P_n . The emotion of *m* is decided by the statistical results of empirical data on which the probabilities (denoted as P_{anger} , $P_{disgust}$, P_{joy} and $P_{sadness}$) of anger, disgust, joy and sadness are 19.2%, 13.7%, 39.1% and 28.0%, respectively, as can be seen in Table 1. At last, *m* is pushed to the screens of all followers of *u*. If the screen of each follower of *u* contains more than *N* tweets, oldest tweets will be removed to narrow the screen until its size is *N*. The procedure of pushing new messages is illustrated in Fig. 1(a).
- Step 3: If u does not post new message, he/she will read all the messages in his/her screen. When he/she reads a message that has emotion i, and the tie strength of u and his/her followee v who posts or reposts this message is w_{uv} , the retweet tendency is defined as $t = w_{uv} * e^{c_i 1}$, where c_i is the correlation of emotion i. If the tendency t exceeds a threshold τ , the message will be reposted and pushed to the screens of all followers of u. We also remove oldest tweets that in screens of u's followers if the screens overloads. We call the above procedure republishing, which is shown in Fig. 1(b).
- Repeat the above three steps for *M* times.

Table 2 illustrates important parameters in our model. P_n is the probability of posting new message, the value of which (0.379) is calculated based on our empirical Weibo data. $P_{emotion}$ such as P_{anger} and P_{joy} is the probability of four newly posted emotions, which can be adjusted to reflect the variation of new messages with one dominant emotion in social events. w_{uv} is calculated by the common friends strength definition, representing the strength of the edge between users u and v. It is worthy noting that tie strengths calculated in terms of reciprocity definition only have binary values (reciprocal or not) and cannot be employed in our model, and retweet strength definition requires empirical data that not available to the present study and the value also evolves quickly with time, hence it is neither not considered in the simulation. Because of above limitations, common friends definition, which reflects the inherent closeness of two users in a continuous manner, is chosen to calculate w_{uv} in the model. N represents a user's screen size, which is set to 20 in this paper because Sina Weibo presents 10 newest messages per page in its mobile App of smart devices, which dominates the traffic. After removing isolated nodes,

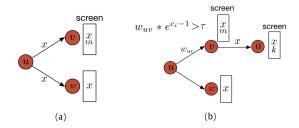


Fig. 1. Pushing and republishing procedures in our model. In (a), v and w follow u. When u posts a new tweet x with probability P_n , x is put into the top of v and w's screens. (b) illustrates the republishing process. v reads x that in his/her screen, if the retweet tendency $w_{uv} * e^{\epsilon_i - 1}$ (c_i is the correlation of the emotion i delivered by x) is higher than τ , x will be reposted by v and enter the screen of v, who is the follower of v, implying the emotion v initially posted by v transfers from v to v.

Table 2The description of parameters of the model.

Parameter	Meaning	Value
P_n	The probability of posting a new tweet	0.379
$P_{emotion}$	The probability of posting a tweet with one emotion, such as P_{anger}	See Table 1
Cemotion	The correlations of the four emotions	See Table 1
w_{uv}	The tie-strength between u and v	-
N	The size of the screen	20
M	The total number of steps	8,853,200
τ	The retweet threshold	0.00~0.10

we achieve 88,532 connected users which are used to construct G. Because of this, the simulation steps M is arbitrarily set to 8,853,200, which is 100 times of the network size and supposed to be sufficient in simulating the diffusion. We tune the threshold τ in the following simulation and further estimate the threshold in real online social media.

Our model considers two important features: emotion correlation and the tie-strength. Emotion correlation represents emotions' influence. Hence emotion with high correlation such as anger is easier to be reposted. Meanwhile, as described in [36], users with stronger connections have more communications. Our model simulates the diffusion based on these two findings as described in Step 3: if the product of e^{c_i-1} and tie strength w_{uv} and is larger than a threshold τ , the message will be reposted. This definition combines the influence of content that is presented by emotion and the network structure which is presented by tie-strength. The code of the model is also publicly available and it can be freely downloaded through https://doi.org/10.6084/m9.figshare.5092522.v1.

4. Results

In this section, we first report the results of our emotion contagion model, including tie-strength preferences, user vitality difference and retweet interval difference of different emotions and estimation of parameters τ by comparing the simulation with empirical results. Then we compare our model with a previous emotion contagion model and study emotion diffusion and competition mechanisms using our model. We also launch different parameters for different users, resulting in a heterogeneous model.

4.1. Tie-strength preference

Based on the three strength definitions, we calculate the average strength of ties on which anger and joy emotion transfer. In this experiment, values of parameters are illustrated in Table 2. Note that given the extremely low influence of disgust and sadness, which can be learnt from their poor correlations in the social network as can be seen in Table 1, only anger and joy are explored in the following simulations of emotion contagion. As can be seen in Fig. 2, the average tie-strength grows up with the growth of τ , which is determined by the retweet condition in our model. Moreover, all the results of the three strength definitions indicate that anger prefers transferring through weaker ties than joy. When the diffusion threshold τ is very small, e.g., $\tau=0$, all messages can be retweeted regardless of the emotion and tie-strength. As a result, both anger and joy diffuse on edges that have similar average strengths. With the growth of τ , edges with small strengths prevent some messages spreading on them, especially the ones with small correlations. On some edges, joyful messages cannot spread while angry ones could still pass. As a result, the average tie-strength for contagion of both anger and joy grows up with the growth of τ while their difference increases accordingly. Classical social theory suggests that weak ties play a key role in diffusion process because they always connect different communities and transfer information from one group to another. Because of this, the preference of weaker ties could make anger easier to spread through deep penetration. The preferences of tie strengths unraveled by our model are consistent with the empirical results in [40], which also suggested that anger prefers weaker ties than joy. Moreover, our model reveals where the different preferences come from. The diffusion in the

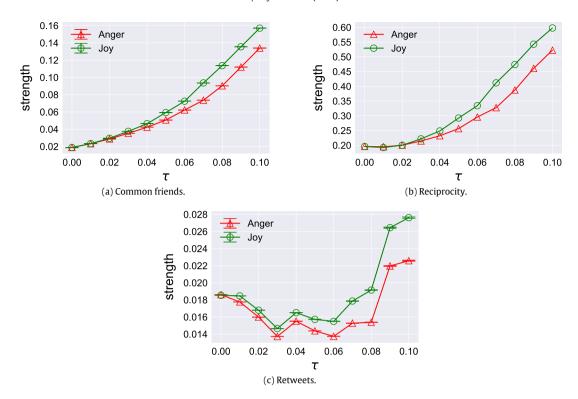


Fig. 2. Tie-strength preferences of anger and joy. In all the three strength definitions, including common friends (a), reciprocity (b) and retweets (c), anger prefers weaker ties than joy, especially when τ is large. Standard errors are calculated for (a) and (c) but the values are extremely small and might be not that obvious in the plots, suggesting that corresponding conclusions from these statistics are significant and solid. (b) does not have standard error since the strengths calculated by reciprocity only produces the percentage of reciprocity links.

network depends on the tie-strength and the influence of the contents' emotion simultaneously. More influential emotions such as anger do not rely on ties with large strengths too much, which makes it easily spread via weaker ties than joy.

Moreover, we estimate the value of τ based on empirical analysis and the results are illustrated in Fig. 3. We calculate the difference of average tie-strength preference between anger and joy, which is defined as $\bar{s}_{joy} - \bar{s}_{anger}$, where \bar{s}_{anger} and \bar{s}_{joy} are the average tie strengths of anger and joy respectively. The empirical results of strength preference comes from [40], which are calculated from six months tweets posted by the network users based on the three strength definitions. As can be seen, we can draw the conclusion that the value of τ in real online social media ranges from 0.05 to 0.08. Our model results are assortative with observations from the real-world data and we can further estimate the threshold in Weibo, which are solid evidences of the reliability of our model.

4.2. User vitality

The user vitality, which reflects the extent of a user's activeness in online social media, can be an excellent indicator of users' intention in participating online activities. Here the user vitality is intuitively defined as the number of messages posted per day by a user after the registration in the empirical data. And for the simulations in the model, since all agents share the same age, the user-activity can be directly valued by the total number of posted messages by an agent. Then besides strength preferences, it is surprisingly found that in both simulation and empirical data, anger-dominated users demonstrate higher vitalities than their joy-dominated counterparts, as can be seen in Fig. 4, in which anger (joy)-dominated users are those posting more angry (joyful) tweets than that of other emotions. The similarly long-tailed distribution of user vitalities reproduced by our model, as shown in Fig. 4, again indicates the model's effectiveness in replicating emotion contagion in social media. And in fact, the consistency between the empirical data and our model further implies that being a sentiment with high arousal, anger can indeed enhance users' activeness in online social media and make itself competent in capturing attention during the contagion.

4.3. Retweet interval

Besides the tie-strength preference and user vitality, we calculate retweet intervals as another measure to validate our model. For a tweet R which is reposted by another tweet T, the retweet interval is defined as the difference between the

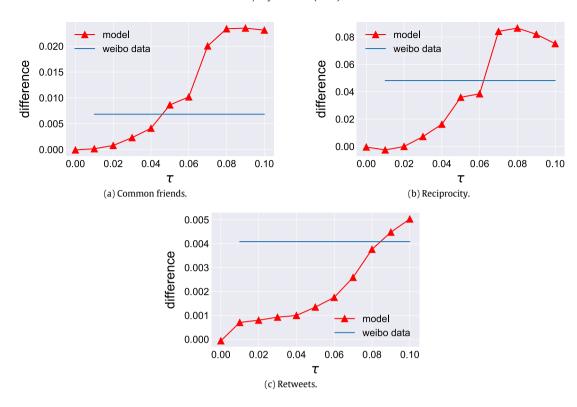


Fig. 3. The tie-strength difference of joy and anger. The horizontal lines represent the difference calculated from realistic Weibo data [40].

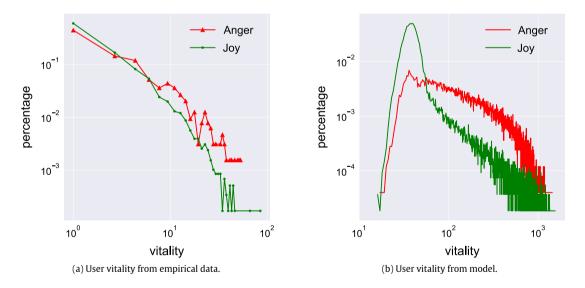


Fig. 4. The distributions of user vitalities for respectively anger and joy-dominated users. (a) User vitality distribution from the empirical data. (b) User vitality distribution in simulations and here we set $\tau=0.06$, which is close to empirical circumstance, as can be seen in Fig. 3. It is worth noting that different from the empirical data, there is an increment in vitality distribution of the model as the user vitality is very low. It comes from the configuration of the model that agents are evenly selected to publish or republish tweets at each step, which results in relatively less low-vitality users as compared to the empirical data. In fact, as discussed in Section 4.6, the heterogeneity of the extended model will fully diminish this phenomenon.

posting time of *T* and *R*. This metric reflects the activity of the tweet being reposted. If one tweet is reposted quickly, i.e., the retweet intervals are small, it could diffuse faster than ones with large retweet intervals. We investigate the distribution of retweet intervals on both empirical Weibo data and simulation data. As can be seen in Fig. 5, the distribution of retweet intervals on simulation data is similar with that on realistic Weibo data, again implying the effectiveness of our model

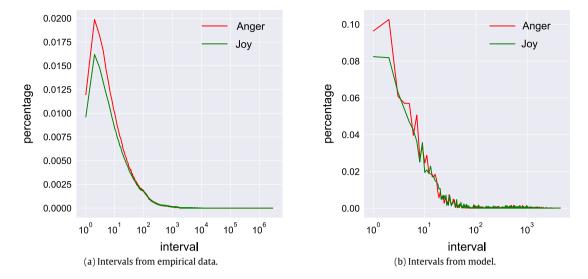


Fig. 5. The distributions of retweet intervals for anger and joy emotions respectively. (a) Retweet interval distribution of the empirical data. (b) Retweet interval distribution of the simulations data and here we set $\tau = 0.06$, which is close to empirical circumstance, as can be seen in Fig. 3.

in simulating emotion contagion. Moreover, both distributions demonstrate that anger emotion presents smaller retweet intervals than joy, indicating that anger, the high arousal emotion, can trigger quickly information contagion in social media.

4.4. Model comparison

In this section, we compare our agent-based emotion contagion model with an existing ESIS model (Emotion-based spreader-ignorant-stifler model) [23]. This model categorizes users to three classes which are spreaders, ignorants and stiflers. Spreaders are users who post or repost tweets and they may spread messages to others via the network. Ignorants are users who do not know the information yet but may receive the message and become spreaders. Stiflers are users who already know the information, but lose interest in it and will not deliver it to others. At first, a user u posts a message which makes u a spreader and all other users are ignorants. In each step, spreaders will spread messages to their neighbors with a probability. A spreader will turn into a stifler with probability σ when the spreader meets another spreader and the spreader will turn into stifler spontaneously with probability δ . The critical mechanism of this model is the message diffusion from spreaders to ignorants. The novel idea in this model is that it considers the emotion distribution of retweets on each edge. The spreading probability is defined as $\lambda w_{i,j,o}$, where λ is the spreading probability when we do not consider emotion, $w_{i,j,o}$ is the strength of emotion o on the edge that connects user i and j. $w_{i,j,o}$ is defined as $\frac{N_{i,j,o}}{N_{i,j}}$, where $N_{i,j,o}$ represents the number of retweets with emotion o that user i reposts from j, $N_{i,j}$ means the total number of retweets that i reposts from j. By performing method, ESIS model considers the emotion preferences for each edge. For example, user i might prefer to receive joyful messages from j, hence if j post or repost a message with joyful emotion, i would repost this message with more likelihoods.

Both the ESIS model and our agent-based model consider the effect of emotion but in different perspectives. ESIS model considers the emotion preference of each edges based on the empirical data; but our model calculates emotion correlation to construct the retweet tendency. However, ESIS model lacks multiple contagion mechanisms in online social media. First, it ignores the pushing and republishing schemes which are widely employed in social media. Then ESIS model considers the diffusion of one message, neglecting the competition of multiple messages. In modern online social media, many messages are posted and reposted simultaneously and compete for users' finite attention, which is reflected by a finite screen in our model. Also, ESIS model calculates the spreading probability by applying the emotion proportion on each edge, ignoring the inherently closeness of two users. But based on previous research, most messages transmissions happen on strong ties [34].

We simulate the ESIS model and calculate the tie-strength preferences of anger and joy, the distribution of user vitalities and the distribution of retweet intervals, as can be seen in Fig. 6. In ESIS model, messages diffuse on the network separately without the order information, but the retweets strengths are changed over time based on definition. Because of this, we only calculate the tie-strength preference results by applying common friends and reciprocity, as can be seen in Fig. 6(a) and (b). When λ is small, anger prefers weaker ties than joy in ESIS model. But with the growth of λ , ESIS model fails to reproduce this pattern. For the user vitality, ESIS model also fails to reproduce the pattern that anger-dominated users have higher vitalities than joy and the vitality distribution is different with the empirical data. However, from Fig. 6(d), ESIS model demonstrates

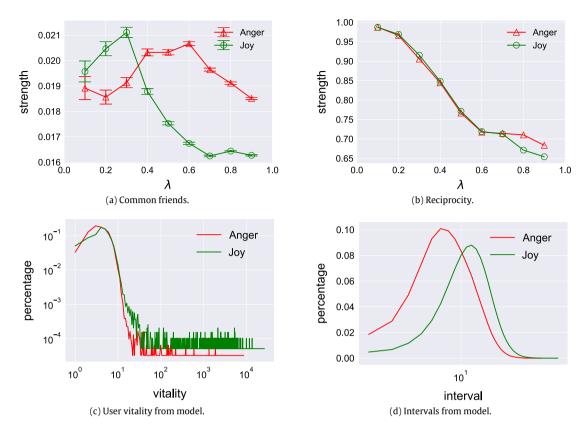


Fig. 6. The results of the ESIS model. (a) and (b) are the tie-strength preferences results. ESIS model fails to reproduce the tie-strength preferences of anger and joy. (c) is the distribution of user vitalities. (d) is the distribution of retweet intervals.

that the retweet intervals of anger is shorter than that of joy and the distribution is similar with that in empirical data. In conclusion, the ESIS model only reproduces the retweet interval pattern but fails in the replication of tie-strength preference and user vitality pattern, indicating that our model has advantages in simulating emotion contagion by considering multiple characteristics such as the pushing–republishing mechanism, closeness of users and emotion correlation.

4.5. Emotion competition

Because of the effectiveness of our model in simulation of emotion contagion, we employ it to thoroughly investigate emotion competition based on the simulation results, which is challenging to be probed in realistic scenarios. Fig. 7(a) illustrates the retweets proportions of anger and joy. When $\tau = 0$, tweets with all the four emotions can be transferred via all edges and accordingly the retweets proportions of anger and joy are assortative with P_{anger} and P_{joy} , respectively. While with the growth of τ , angry tweets can spread on some weak ties through which joyful tweets cannot transfer. As a result, the proportion of angry retweets exceeds that of joy when $\tau > 0.02$ (Fig. 7(a)), and the proportion of all angry tweets also exceeds that of joy when $0.02 < \tau < 0.7$ (Fig. 7(b)). When τ is too large, the percentage of angry tweets is smaller than that of joy because the number of retweets declines, as can be seen in Fig. 7(b). Anger-dominated users also reach a peak when $\tau = 0.03$. Because of the higher influence of anger, its number of retweets increases with the rising of τ and the total angry tweets and anger-dominated users also reach peaks at specific τ . However, as P_{anger} is much lower than P_{joy} , the number of anger-dominated users are smaller than that of joy for all τ . Anger-dominated users have larger vitalities based on the above analysis, making the number of angry tweets exceeds that of joyful tweets for some specific τ , as can be seen in Fig. 7(b). From the view of emotion competition, people's finite attention are filled with messages with different emotions. When the number of messages with one emotion increases, it becomes more competitive. Two factors that influence the amount of messages with one emotion are the probability of newly posted emotional messages and the amount of republishing messages with this emotion because both pushing and republishing tweets will appear in followers' screens. According to Weibo data, P_{joy} is much larger than P_{anger} because of positive-bias [65], while anger is of higher influence and easier to be republished which produces more angry retweets. In conclusion, anger's higher influence can enhance its competition and makes it spread extensively from both the perspectives of messages and users.

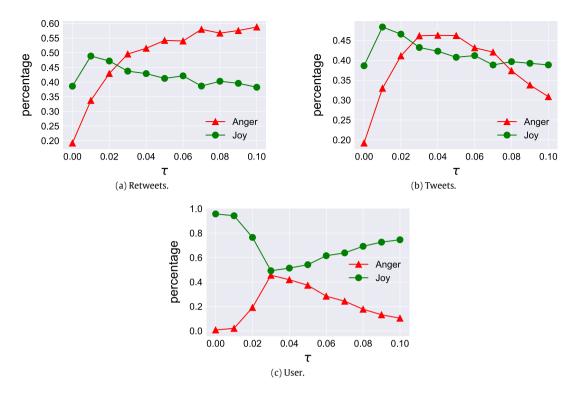


Fig. 7. The simulation results of anger and joy, in which emotion probabilities of new messages are 19.2%, 13.7%, 39.1% and 28.0% for anger, disgust, joy and sadness, respectively. These proportions are calculated from real Weibo data. In all of the three figures, x-axises represent the threshold τ and y-axises are the percents of retweets, tweets and users after the simulation. (a) and (b) separately illustrate the percentages of anger and joy in reposted messages and all messages including new tweets and reposted tweets. (c) is the proportion of anger-and joy-dominated users. Users who post or repost more angry (joyful) tweets than other emotions will be similarly labeled as anger(joy)-dominated users. And the number of a specific emotion dominated users reflects the diffusion scope of this emotion.

Table 3The values of parameters in the emotion competition experiment.

•	• •
Parameter	Value
P_n	0.379
Panger	0.25
P _{disgust}	0.25
P_{joy}	0.25
P _{sadness}	0.25
Cemotion	See Table 1
w_{uv}	Calculated by common friend definition
N	20
M	8,853,200
τ	0.00~0.10

Moreover, in some circumstances, especially when negative societal events occur, people will post many angry messages in online social media and the possibility of publishing angry tweets will grow significantly [40]. Because of this, we tune the emotion probabilities of newly posted tweets to 25% for all the four emotions to study the emotion competition from a equitable start. Values of parameters in this experiment are illustrated in Table 3. As can be seen in Fig. 8(b) and (c), the proportions of angry tweets and anger-dominated users are much more than that of joy messages. When $\tau=0$, the proportions of angry and joyful tweets and users are similar. With the growth of τ , the proportions of anger surpass joy and their difference in coverage reach a maximum at specific τ , which is about 0.04 for tweets and 0.01 for users respectively. However, when τ is too large, tweets with both anger and joy are hard to spread in the network, leading to a decreasing gap between them. In this simulation, joy has no superiority in the newly posted messages, and anger still has more influence. As a result, anger could dominate users' attention, which makes it more competitive than joy. In conclusion, when negative events occur, people will post many angry tweets which are easily to influence others. As a result, angry users will dominate the whole network. It can well explain the bursty rise of negative events, in which anger dominates the public opinion [40]. Because of anger's high influence, website managers should try to prevent its extensively and unreasonably spread from the very beginning.

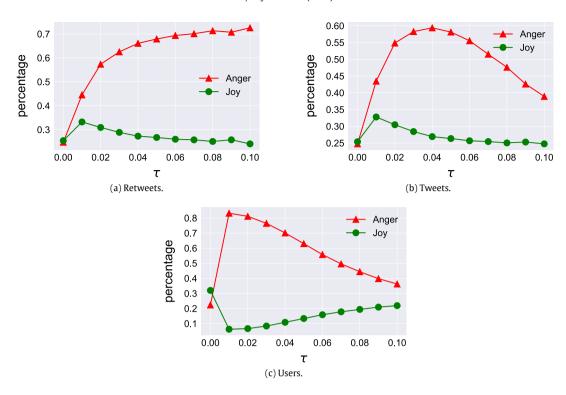
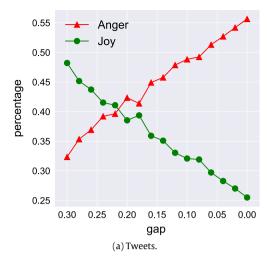


Fig. 8. Simulation results of emotion competition. New tweets have the same probabilities (25%) to be labeled to the four emotions. (a) illustrates the percentages of angry and joyful retweets. The proportion of angry retweets increases with the growth of τ while of joy, the value first increases to a peak and then declines. (b) is the percentages of total tweets including new tweets and reposted tweets. The proportions of both anger and joy increase first and then begin to drop. And the values of anger far exceed that of joy. (c) represents the anger and joy dominated users and the values of anger are still much more than that of joy.

To find the critical point that anger surpasses joy in the network, we tune the probability of newly posted messages of anger and joy, denoted as P_{anger} and P_{joy} , to investigate when anger could dominate the network as P_{anger} approaches P_{joy} . As can be seen in Fig. 9, when the gap is large, e.g., $P_{joy} - P_{anger} = 0.3$, both joyful tweets and joy-dominated users are more than angry ones. With shrinking of the gap, the number of both angry messages and anger-dominated users increases, and when the gap is smaller than 0.22 and 0.12, the proportion of angry messages and anger-dominated users exceeds that of joy messages and joy-dominated users respectively. The different critical gaps for tweets and users comes from the vitality difference, i.e., angry users demonstrate higher activeness in online activities. As a result, the number of angry messages exceeds that of joy although the number of anger-dominated users are still smaller than that of joy-dominated users. This finding demonstrates that when the proportion of newly posted angry messages is 12% less than that of joy, the network could be dominated by angry users. Network managers should be aware of that and adopt corresponding techniques to prevent the collective outrage in cyber space.

4.6. Heterogeneity extension of the model

Our agent-based model sets the probability of publishing new tweets (P_n) and the proportion of four emotions ($P_{emotion}$) based on empirical data, as seen in Table 2. However, we could launch different parameters for different users, which results in a heterogeneous model. For a user u, we assign the probability of posting new messages of u to P_n^u and assign the proportions of four emotions of his/her non-retweeting tweets to P_{anger}^u , $P_{disgust}^u$, P_{joy}^u and $P_{sadness}^u$. When u is selected, it posts new message with the probability P_n^u and the emotion of the new message is determined by $P_{emotion}^u$. Values of parameters in the heterogeneous model are illustrated in Table 4. As can be seen in Fig. 10, which demonstrates the results of the heterogeneous model, the emotion diffusion patterns such as tie-strength preference, user vitality and retweet interval can be well reproduced by the model. Fig. 10(a), (b) and (c) illustrate the results of tie-strength preferences of anger and joy, in which the tie-strength is defined by common friends, reciprocity and retweets that are defined in Section 2.3. Consistent with the previous non-heterogeneous model, results of all the three strength metrics indicate that anger, the emotion of high correlation, prefers weaker ties than joy in emotion contagion. Moreover, we estimate the proper retweet threshold (τ) by calculating the difference of retweeting strengths, as seen in Fig. 10(d), (e) and (f). The differences results indicate that the proper retweet threshold on realistic social media is approximately 0.05 to 0.08. Users vitality distribution and retweet



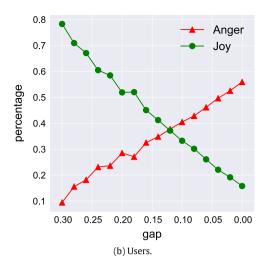


Fig. 9. The proportions of angry and joyful messages and anger and joy-dominated users with the reduction of $P_{joy} - P_{anger}$. In this figure, we fix $P_{disgust}$ and $P_{sadness}$ to 0.25 and adjust $P_{joy} - P_{anger}$. For example, when $P_{joy} = 0.30$ and $P_{anger} = 0.20$, the gap of them is 0.10. (a) illustrates that when the gap is smaller than 0.20, the proportion of angry messages surpasses that of joy messages. (b) illustrates the proportion of anger and joy-dominated users. When the gap is smaller than 0.12, the proportion of anger-dominated users exceeds that of joy-dominated users.

Table 4The values of parameters in the heterogeneous model.

Parameter	Value
P _n P _{emotion}	calculated from empirical data calculated from empirical data
C _{emotion}	See Table 1
w_{uv} N	Calculated by common friend definition 20
M	8,853,200
τ	0.00~0.10

interval distribution are respectively shown in Fig. 10(e) and (f). Both distributions are similar with their counterparts in empirical data. And anger-dominated users have higher vitalities and angry tweets show small retweet intervals respectively, which are consistent with the empirical results and the results of previous non-heterogeneous model (Figs. 4 and 5). Particularly, compared to the previous model, there are more low-vitality users, i.e., the heterogeneous model reproduces the vitality distribution better. The reason is that the values of P_n and $P_{emotion}$ vary for different users. As a result, several users post less new tweets because they have low P_n . If the neighbors of user u have low $P_{emotion}$ for high correlation emotions such as anger and joy, u will repost less tweets. Because of the above reasons, the proportion of low vitality users increases. In conclusion, the model that launches different parameters for different users can also reproduce emotion contagion patterns of realistic social media, further implying the effectiveness of our agent-based model. Moreover, the parameters of our model can be adjusted based on realistic social media data and the results can also reproduce emotion contagion patterns, which reflects the flexibility of our model.

5. Conclusion

The explosive development of online social network produces massive data which provides an unprecedented opportunity to investigate human behavior. In this paper, we propose an agent-based model, which outperformed the existing solution, to well simulate the emotion contagion. Our model absorbs advantages of previous findings by considering the pushing and republishing of online social media, the threshold which is the diffusion condition and the competition of users' limited attention. Meanwhile, the most innovative point of our model is that we combine the influence of content which is presented by emotion, and the network feature which is tie-strength. The primary idea of this combination is that higher emotion influence and stronger ties enhance diffusion in social network. By analyzing the simulation result, we find that as compared to joy, angry users possess higher vitalities, angry tweets have smaller retweet intervals and anger can spread on weaker ties which means it could widely diffuse across communities. These findings are consistent with previous empirical observations on Weibo. Our model successfully reproduces these emotion contagion patterns in online social media, which demonstrates its significance. The comparison results with previous ESIS model further indicate its effectiveness, which is caused by considering the above characteristics. Our model can also be used to study the emotion competition, e.g., from

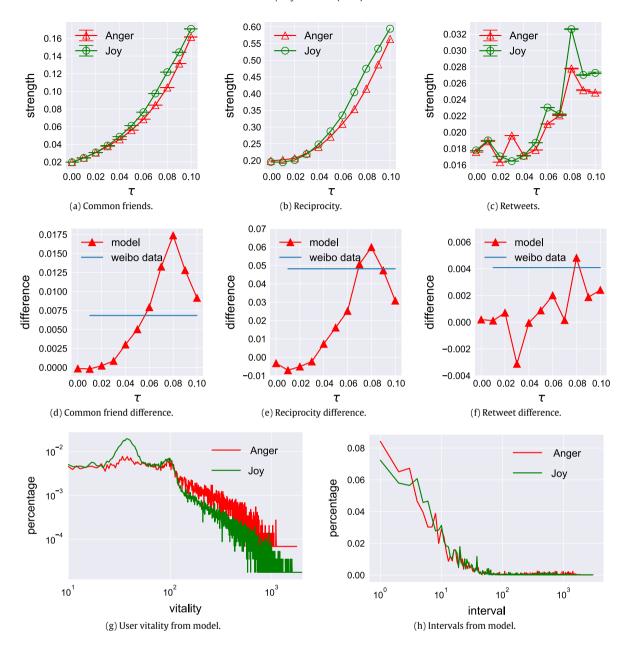


Fig. 10. The results of the heterogeneous model. (a), (b) and (c) are the tie-strength preferences results. All the results of three strength metrics demonstrate that anger prefers weaker ties than joy. (d), (e) and (f) are the strength differences which are used to detect retweet threshold τ in realistic data. (g) and (h) respectively illustrate the distribution of user vitalities and the distribution of retweet intervals when $\tau=0.06$, which is the value that similar to the threshold on realistic Weibo data based on the results of strength differences, as seen in (d), (e) and (f).

the simulation results, we find that higher influence of anger can enhance its competition. Moreover, when the probability of newly posted angry tweet is high, i.e., in circumstances of negative societal events, anger will influence massive users and even dominate the network due to higher influence and preference of weaker ties. We also give a guide value that when the proportion of newly posted anger messages is 12% less than that of joy, the network could be dominated by negative users. This is a warning signal in early stages, i.e., in some societal events, anger could widely propagate and lead to the collective online rage. Finally, the presented model can be easily extended to be heterogeneous by launching different parameters for different agents, which greatly enriches its applications in more realistic scenarios.

Our model demonstrates that anger's high influence and weak-tie preference can boost its dissemination, and in some conditions, it can even dominate the network. Network managers should realize anger's capability of being highly contagious and apply techniques to prevent the unreasonable prevalence of anger. Our emotion contagion model grabs multiple critical

features of online diffusion and can be used to study multiple issues of emotion diffusion and even forecasting by adjusting parameters. For example, we simulate the diffusion of collective social event by changing the probabilities of newly posted emotions. It is thus believed that our model will offer insights for exploring other emotion related issues from the perspective of computer simulations.

In our future work, we will further optimize out agent-based model. For example, we will consider to find the more appropriate retweet tendency definition by mining empirical social media data. The retweet threshold τ might be also agent-dependent and it is worthy to set different τ for different agents. Moreover, we will apply our model to study emotion contagion in different scenarios through computer simulation.

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