NOLTA, IEICE

Paper

Public opinion formation with the spiral of silence on complex social networks

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Received April 11, 2014; Revised July 23, 2014; Published January 1, 2015

Abstract: Public opinion is formed through social interactions of individuals. A mechanism behind the formation of a highly dominant public opinion is a sociological theory called *the spiral of silence*. Here we study opinion dynamics resulting from the spiral of silence, using an agent-based model with complex interaction networks. We show that an extremely dominant public opinion arises in the presence of a moderate proportion of neutrals and its dominance level is enhanced by social interactions. Furthermore, we demonstrate that a correlation between characteristics and social interactions of the individuals has a large influence on the opinion formation dynamics.

Key Words: public opinion formation, the spiral of silence, complex networks, social media, agent-based models

1. Introduction

In modern society today, people are exposed to a great deal of information from mass and social media. The mass media mainly provide major information in a global society, whereas the social media enable to transmit individual-level and community-level information within local groups. The online social media have contributed to rapid spreading of information and played a crucial role in group behavior such as protest movements [1] and riots [2]. The new communication technologies with anonymity on the Internet have also brought about the social issues such as cyber-bullying [3] and flaming [4]. Moreover, an increasing concern has been given to political polarization in the Internet-based online social media [5–8]. One of the mechanisms behind these group behavior is *the spiral of silence* propounded by Noelle-Neumann [9, 10], which is a sociological theory describing a dynamic

Nonlinear Theory and Its Applications, IEICE, vol. 6, no. 1, pp. 15–25 DOI: 10.1587/nolta.6.15

process by which those who perceive their opinion to be minor tend to remain silent and a seemingly majority opinion becomes more prominent. The processes for the formation of a highly dominant opinion or attitude in groups have long been studied in social psychology and political science, but they are still the subject of much debate due to the diversification of information sources and the complexity of human interactions via social media.

In the present paper, we focus on public opinion formation resulting from the spiral of silence [9, 10]. In spite of the variety of individuals' beliefs and preferences, a single opinion often becomes extremely dominant for political matters or social issues in both real and virtual communities. The spiral of silence is a possible factor to facilitate the dominance of a majority opinion. If people perceive their opinion to be in the minority, they would hesitate to express their opinion in public for fear of isolation. Some people, who are strongly conform to the social norm, may express the majority opinion even if it is opposite to their opinion in mind. In contrast, other people who perceive their opinion to be in the majority are willing to express their opinion in public. As a result, mass and social media mostly spread the majority opinion and do not transmit other minor opinions. A repeat of the above processes accelerates the expansion of the majority opinion, eventually leading to an extremely dominant public opinion with potential to cause totalitarianism and authoritarianism. Although the assumptions and conditions for the spiral of silence in the age of mass media have been considerably discussed [10, 11], it is controversial how they are affected by individual differences and complex social communications in the age of new media [12].

Our aim is to investigate the conditions for the emergence of a highly dominant public opinion by using an agent-based model with complex interaction networks. Each agent makes a decision on whether to express either of two opposing opinions or keep silence, depending on the individual characteristics and the climate of opinion in the reference group. The individual characteristics include the belief (i.e. the opinion in mind) and the cooperativeness. These two factors are represented by fixed parameters for each agent. Formulating the payoffs for speaking out an opinion and keeping silent, we assume that each agent makes a decision such that the expected payoff is maximized [13]. Therefore, it is possible that the expressed opinion is different from the opinion in mind. The decision making model that we use is regarded as a threshold model, which has been widely used to study collective behavior including opinion formation [14–18].

We introduce two factors that are missing in the previous mathematical models for the spiral of silence. One is the neutrals who are not inclined to either of the two opposing opinions. The other is the complex topology of social interaction networks, which is often found in Twitter [19], Facebook [20], social networking sites [7], and other online social networks [21, 22]. Using complex networks, we can take into account the difference in the size of the reference group among the agents [23]. We also investigate the interplay between individual characteristics and social interactions by changing the arrangements of specific types of agents: the neutrals who do not have a specific opinion and the hardcore non-conformists who express their opinion in mind independently of the opinions of others. The outcome of the opinion dynamics is evaluated by the prevalence of opinion expression and the degree of the dominance of the majority opinion.

The remainder of this paper is organized as follows. In Sec. 2, we introduce an agent-based model for opinion formation on information transmission networks. In Sec. 3, we specify the conditions for the occurrence of the spiral of silence phenomenon and the formation of a highly dominant public opinion. We mainly emphasize the important role of neutrals in the opinion formation dynamics. Simulations are performed with random networks, scale-free networks, and a real online social network. Finally, we give a concluding remark in Sec. 4.

2. Methods

2.1 Decision making of each agent

Let us describe an opinion formation model consisting of N interacting agents, where each agent is a proponent, an opponent, or a neutral, for a social issue. Namely, the ith agent ($i = 1, \ldots, N$) has an opinion in mind, represented by the parameter $\alpha_i \in \{-1, 0, 1\}$: $\alpha_i = 1$ for a proponent; $\alpha_i = -1$ for an opponent; $\alpha_i = 0$ for a neutral. The ith agent also has a parameter β_i representing the cooperativeness for group synchrony, which is assumed to be uniformly distributed in the range $[1, \beta_{\max}]$. The agents with a larger value of β_i are more likely to express the majority opinion in the reference group. Each agent has three options to choose: expressing an opinion of agreement (state A), expressing an opinion of disagreement (state D), and keeping silent (state S). The decision making of each agent depends on the interplay between the characteristics (i.e. α_i and β_i) of the agents and the expressed opinions in the reference group. Note that the expressed opinion and the opinion in mind can be different.

Here we define the payoffs for the three options. The expected payoff for expressing the opinion of agreement and that for expressing the opinion of disagreement are, respectively, given as follows [13]:

$$U_i^{\mathrm{A}} = \alpha_i + p_i\beta_i - (1-p_i)\beta_i, \quad (1)$$
$$U_i^{\mathrm{D}} = -\alpha_i + (1-p_i)\beta_i - p_i\beta_i, \quad (2)$$

where $p_i \in [0,1]$ denotes the dominance level of the opinion of agreement in the reference group as defined later. The first terms of the above equations correspond to the reward for speaking out the opinion in mind. The second and third terms correspond to the reward for expressing an opinion same as the neighbor's one and the penalty for expressing an opinion opposite to the neighbor's one, respectively. The payoff functions for the two opinions are essentially equivalent because $U_i^{\mathrm{A}} = -U_i^{\mathrm{D}}$. We newly introduce the expected payoff for the silent state in order to consider the spiral of silence. The payoff for remaining silent is given by

$$U_i^{\mathrm{S}} = \beta_i'(1-q_i), \quad (3)$$

where $q_i \in [0,1]$ is the proportion of the neighboring agents who have expressed either of the two opinions as defined later and $\beta_i' = \delta\beta_i$ represents the cooperativeness to a neighboring agent who is still silent. The parameter δ is used to adjust the balance between the payoff for expressing an opinion and that for keeping silent.

We define the proportion q_i of the neighboring agents who have expressed either of the two opinions and the dominance level p_i of the opinion of agreement. The reference group of an agent consists of its neighboring agents in the interaction network. The number of the neighbors of the ith agent, or the size of the reference group, is denoted by k_i. Each agent can perceive that there are k_i^{A} agents who have expressed the opinion of agreement, k_i^{D} agents who have expressed the opinion of disagreement, and k_i^{S} agents who are still silent. These numbers can change with time while the total number is fixed at $k_i = k_i^{\mathrm{A}} + k_i^{\mathrm{D}} + k_i^{\mathrm{S}}$. The proportion of the agents who have expressed an opinion is given by

$$q_i = \frac{k_i^{\mathrm{A}} + k_i^{\mathrm{D}}}{k_i}. \quad (4)$$

The proportion of the silent agents is given by $1 - q_i$. We evaluate the dominance level of the opinion of agreement in the reference group by

$$p_i = \frac{k_i^{\mathrm{A}}/k_i + (w/2)(1-q_i)}{q_i + w(1-q_i)}, \quad (5)$$

where $w \in [0,1]$ is a constant parameter representing the influence of silent neighbors on the evaluation of the dominance level. Each silent neighbor is regarded to have a weight $w/2$ evenly for the two opinions. If w is very small, then $p_i \sim k_i^{\mathrm{A}}/(k_i^{\mathrm{A}} + k_i^{\mathrm{D}})$. In this case, once a neighboring agent expresses an opinion, the value of p_i becomes 0 or 1 and the ith agent highly tends to immediately express the same opinion. A succession of these processes means that an initially expressed opinion determines the majority opinion. To avoid this situation, we set the w value to be not too small.

The ith agent chooses the option corresponding to the largest payoff among U_i^{A}, U_i^{D}, and U_i^{S}. By comparing these payoffs, we can obtain the decision boundaries. Figure 1 shows which option is chosen depending on k_i^{A}/k_i and k_i^{D}/k_i. When $w = 1$, expressing the opinion of agreement is chosen if

$$\beta_i(1+\delta)\frac{k_i^{\mathrm{A}}}{k_i} - \beta_i(1-\delta)\frac{k_i^{\mathrm{D}}}{k_i} - \beta_i\delta + \alpha_i > 0, \quad (6)$$

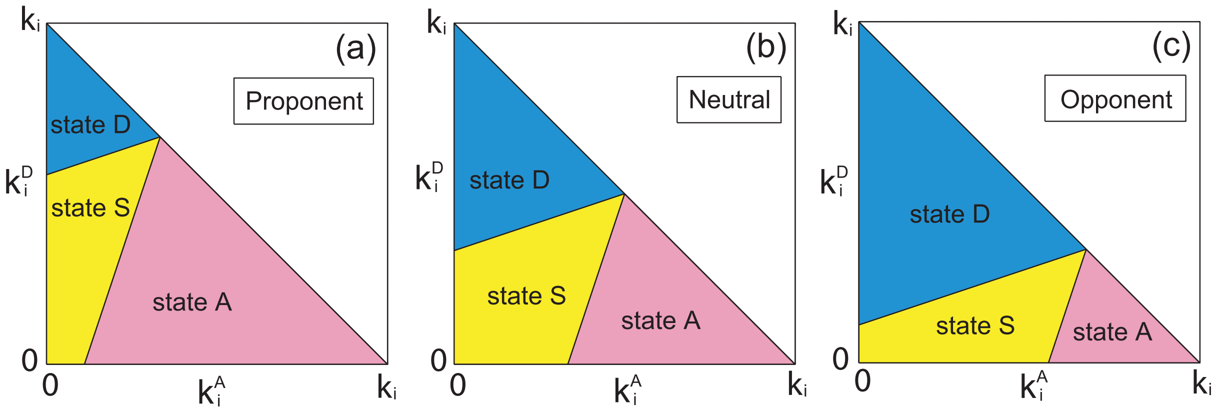

Fig. 1. Decision making of each agent when $w = 1$ in Eq. (5). The ith agent with state S, who has not yet expressed an opinion, chooses to express the opinion of agreement (red), express the opinion of disagreement (blue), or keep silent (yellow), according to the largest payoff among U_i^{A}, U_i^{D}, and U_i^{S}. The option to be chosen depends on k_i^{A}/k_i and k_i^{D}/k_i, which are the proportion of the agents with state A and that with state D in the reference group, respectively. The cooperativeness parameter is set at $\beta_i = 3$ and the balance parameter at $\delta = 1/2$. (a) A proponent agent with $\alpha_i = 1$. (b) A neutral agent with $\alpha_i = 0$. (c) An opponent agent with $\alpha_i = -1$.

expressing the opinion of disagreement is chosen if

$$-\beta_i(1-\delta)\frac{k_i^{\mathrm{A}}}{k_i} + \beta_i(1+\delta)\frac{k_i^{\mathrm{D}}}{k_i} - \beta_i\delta - \alpha_i > 0, \tag{7}$$

and keeping silent is chosen otherwise. The thresholds for decision making are dependent on the belief parameter α_i, the cooperativeness parameter β_i, and the balance parameter δ.

2.2 Simulation methods

The social interactions of the agents are characterized by the connection matrix $C = (C_{ij})$ for $i, j = 1, 2, \ldots, N$: $C_{ij} = C_{ji} = 1$ if the ith and jth agents have social interactions; $C_{ij} = C_{ji} = 0$ otherwise. To avoid self-connections, the diagonal terms are set at $C_{ii} = 0$ for all $i = 1, \ldots, N$. The size of the reference group of the ith agent is given by the degree $k_i = \sum_{j=1}^{N} C_{ij}$. The average size of the reference groups is then given by the mean degree $\langle k \rangle = (1/N)\sum_{i=1}^{N} k_i$. We adopt random networks generated by the Erdős-Rényi model [24] in Sec. 3.1. We also consider Barabási-Albert scale-free networks generated by the preferential attachment rule [25] and an online social network obtained from real data [26] in Sec. 3.2.

We denote the proportion of neutrals by r. To eliminate the initial advantage of one opinion over the other, the proportion of the proponents and that of the opponents are evenly set at $(1-r)/2$. The value of parameter α_i is randomly generated according to the above proportions. The value of parameter β_i is randomly generated from a uniform distribution in the range $[1, \beta_{\max}]$. The parameter values are set at $N = 1000$, $\beta_{\max} = 6$, $\delta = 1/2$, and $w = 1$ unless otherwise noted.

In the initial state, all the agents are silent. The agents with $\beta_i < 1/\delta$, corresponding to the hardcore non-conformists, first express their opinions without being affected by the opinions of the other agents, because Eq. (6) or Eq. (7) holds in the initial state with $k_i^{\mathrm{A}} = k_i^{\mathrm{D}} = 0$. For the given parameter values, 20% of the total population are the non-conformists. They trigger the spread of opinion expression. Once an agent expresses an opinion, the expressed opinion is unchanged. In each time step t, only the agents with state S evaluate the payoffs and choose an option with the largest payoff. The states of the agents are updated synchronously. The above procedure is iterated until no agent updates its state and a convergence state is reached.

3. Results

3.1 Opinion formation on random networks

We show the results of opinion formation on random networks where the degrees are distributed around the mean degree $\langle k \rangle$. The spread of opinion expression is demonstrated in Fig. 2. The network state is given by $N_{\mathrm{A}}(t)$, $N_{\mathrm{D}}(t)$, and $N_{\mathrm{S}}(t)$, representing the number of agents with state A, that with state D, and that with state S, respectively. Since the total population is constant, $N_{\mathrm{A}}(t)+N_{\mathrm{D}}(t)+N_{\mathrm{S}}(t)=N$. Figure 2(a) shows that the opinion expression is fully prevalent and a highly dominant public opinion is formed due to the spiral of silence effect. Figure 2(b) demonstrates that the opinion expression is almost prevalent but neither of the opinions is highly dominant. These two cases imply that an emergence of the spiral of silence phenomenon depends on the proportion of the neutrals. Even if one opinion seems to be a majority in the beginning, the opposing opinion can be globally dominant in the final state, as seen from Fig. 2(b). Figure 2(c) shows that the opinion expression only locally spreads and most agents remain silent in the final state. To evaluate the outcome of the spread of opinion expression, we introduce the prevalence of opinion expression and the dominance level of the majority opinion.

The prevalence of opinion expression is given by

$$\rho = \frac{N_{\mathrm{A}}+N_{\mathrm{D}}}{N}, \tag{8}$$

where N_{A} and N_{D} represent $N_{\mathrm{A}}(t)$ and $N_{\mathrm{D}}(t)$ at the converged state, respectively. Figure 3(a) shows the prevalence of opinion expression with variation of the proportion r of the neutrals. For a relatively small value of r (r smaller than around 0.4), almost all the agents eventually express the opinions. When r is moderate (r between around 0.4 and 0.7), the opinion expression globally spreads or locally terminates. These two final states are coexisting. The frequency of the global prevalence of opinion expression decreases with an increase in r. For a sufficiently large value of r (r larger than around 0.7), the opinion expression only locally spreads and a global prevalence is not achieved. This is because Eqs. (6) and (7) are not satisfied in the presence of many neutrals.

The ratio between the prevalences of the two opinions is evaluated by

$$\sigma = \frac{N_{\mathrm{D}}}{N_{\mathrm{A}}}. \tag{9}$$

Since a public opinion is formed when a global prevalence of opinion expression is achieved, we obtain the σ value only from the simulations with $\rho > 0.8$. Figure 3(b) shows the cumulative distribution of σ, i.e. the probability that the value of $N_{\mathrm{D}}/N_{\mathrm{A}}$ is less than σ. The approximate point symmetry of the distribution with respect to $(1, 1/2)$ originates from the essential equivalence of the two opinions. In

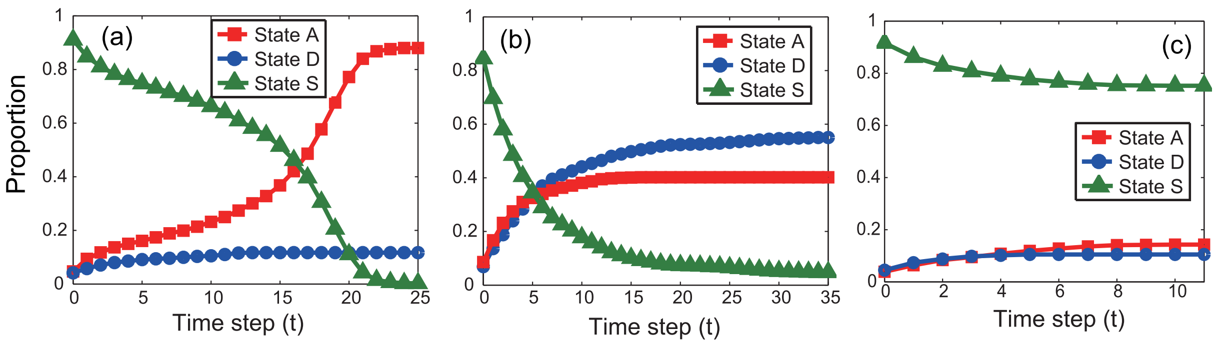

Fig. 2. The spread of opinion expression in the agent-based model on random networks with $\langle k \rangle = 6$. The squares, circles, and triangles represent N_{A}, N_{D}, and N_{S}, respectively. The proportion of the neutrals in the three cases are set at (a) $r = 0.5$, (b) $r = 0.2$, and (c) $r = 0.5$, respectively. (a) The opinion expression is fully prevalent and the highly dominant opinion is formed, due to the spiral of silence. (b) The opinion expression is almost prevalent, but the disparity between the two opinions is small. (c) The opinion expression is not prevalent.

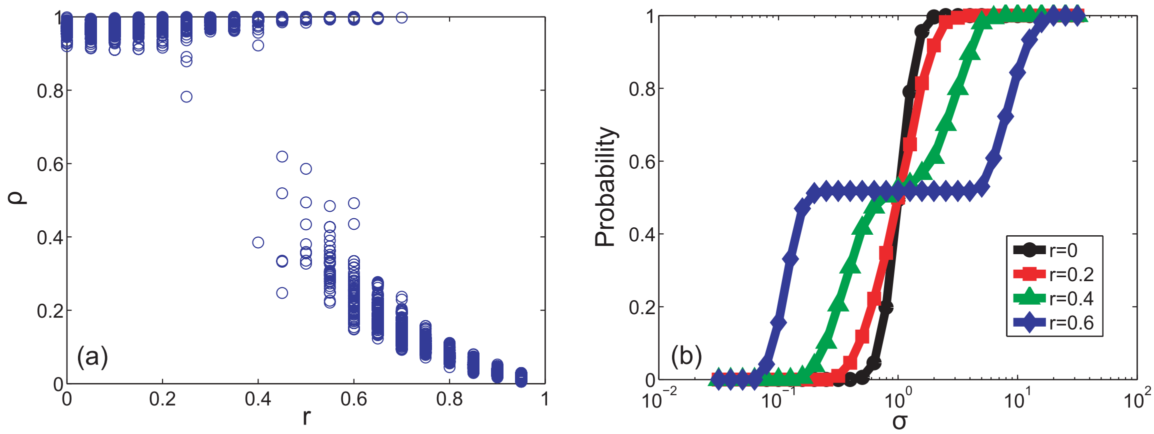

Fig. 3. Effect of the neutrals on opinion formation in random networks with $\langle k \rangle = 6$. (a) The prevalence ρ of opinion expression plotted against the proportion r of the neutrals. For each value of r, the results for 500 simulations are superimposed. (b) The cumulative distribution of σ for the different proportion of neutrals.

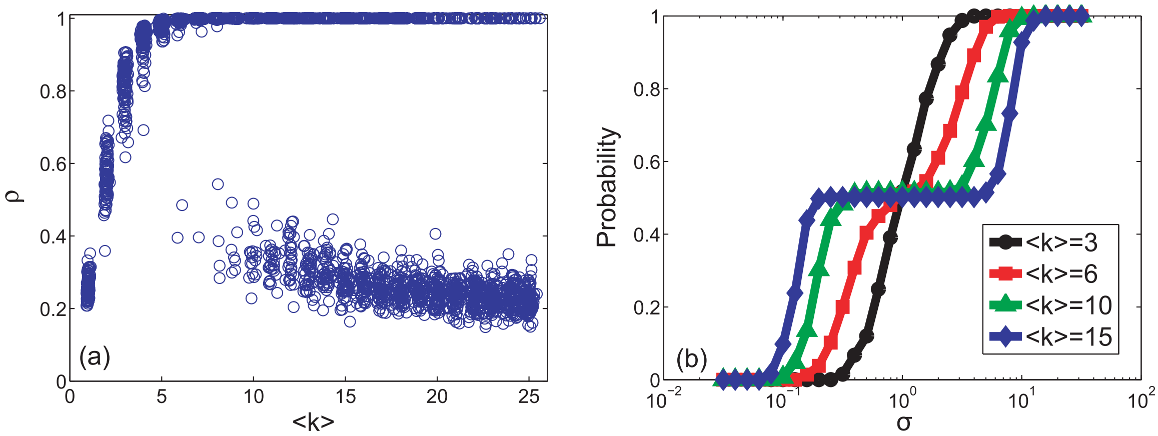

Fig. 4. Effect of the size of reference groups on opinion formation in random networks with $r = 0.4$. (a) The prevalence ρ of opinion expression plotted against the average size $\langle k \rangle$ of the reference groups. For each value of $\langle k \rangle$, the results for 500 simulations are superimposed. (b) The cumulative distribution σ for the different average size of reference groups.

the absence of the neutrals, the two opinions are almost equally expressed and the σ value concentrates around 1. The plateau at the center found in the case of $r = 0.6$ indicates that the proportion of one opinion is about ten times as large as that of the other opinion. An increase in the proportion of the neutrals facilitates the dominance level of the public opinion, although the formation of a public opinion becomes less frequent.

The average size $\langle k \rangle$ of the reference groups also significantly influences the public opinion formation. Figure 4(a) shows that the opinion expression is not globally prevalent for a relatively small value of $\langle k \rangle$. The agents are not fully interacting in such a network and the spread of opinion expression is confined in a local subnetwork. As $\langle k \rangle$ is increased, there appears a coexistence of the cases with global and local prevalence. With a further increase in $\langle k \rangle$, the number of the simulations resulting in a global prevalence gradually decreases. This is because the agents with a larger number of neighboring agents require more opinion expression in the reference group in order to express an opinion. This effect inhibits the spiral of silence phenomenon and prevents a global prevalence. Figure 4(b) shows the cumulative distribution of σ. The dominance level of the public opinion is enhanced by an increase in the social interactions.

The phase diagrams in Fig. 5 reveal the impacts of the neutrals and the social interactions on the public opinion formation. Figure 5(a) shows the prevalence of opinion expression on the parameter plane of $\langle k \rangle$ and r. The upper white region indicates that a global prevalence with $\rho > 0.8$ is not

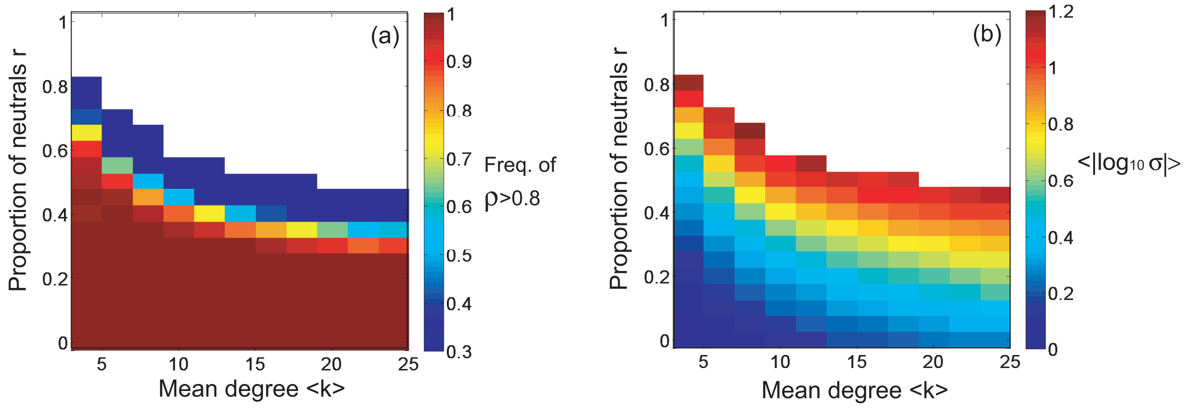

Fig. 5. Phase diagrams showing the outcome of the opinion formation on random networks. For each parameter set, 500 simulations were performed. (a) The frequency of the simulations resulting in a global prevalence with $\rho > 0.8$ in the parameter plane of the proportion r of the neutrals and the average size $\langle k \rangle$ of the reference groups. The upper region (white color) indicates that there were no case with $\rho > 0.8$ within the 500 simulations. (b) The dominance index $\langle |\log_{10} \sigma| \rangle$ in the same parameter plane, which is the average over the cases with $\rho > 0.8$.

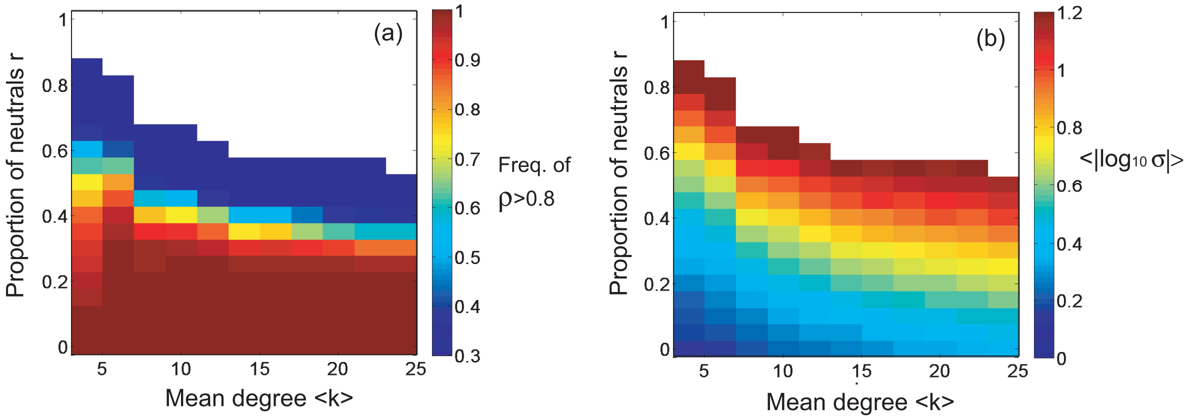

Fig. 6. The same as Fig. 5, but for scale-free networks.

observed in all the simulations, mainly due to the large proportion of the neutrals. In the blue region, a global prevalence rarely occurs. Figure 5(b) shows the averaged dominance level of the majority opinion. First we calculate the values of σ for 500 simulations and then compute $\langle |\log_{10} \sigma| \rangle$ where the brackets mean the average over the simulations with $\rho > 0.8$. It is not distinguished by this measure which opinion is dominant, because $|\log_{10} \sigma| = |\log_{10}(1/\sigma)|$. The comparison between Figs. 5(a) and 5(b) shows that a public opinion becomes extremely dominant when a global prevalence of opinion expression is not likely to be achieved but actually achieved by chance.

3.2 Opinion formation on scale-free and online social networks

In the previous subsection, we have assumed that the social interactions of the agents are almost homogeneous. Here we consider more heterogeneous connectivity often found in online social networks. In such networks, the size of the reference group is different depending on the agents. The hub agents with many social interactions are expected to have a large influence on decision making of the neighboring agents. We examine how the correlation between the characteristics and the size of the reference group of an agent alters the outcome of the opinion formation.

Figure 6 shows the phase diagrams of the prevalence of opinion expression and the dominance level of the majority opinion in scale-free interaction networks. The results are qualitatively similar to those in the random networks in Fig. 5. Namely, the global prevalence of opinion expression rarely happens but once it happens an extremely dominant public opinion is brought about by the spiral of silence effect. In the scale-free networks, the range of the proportion of neutrals for the rare prevalence

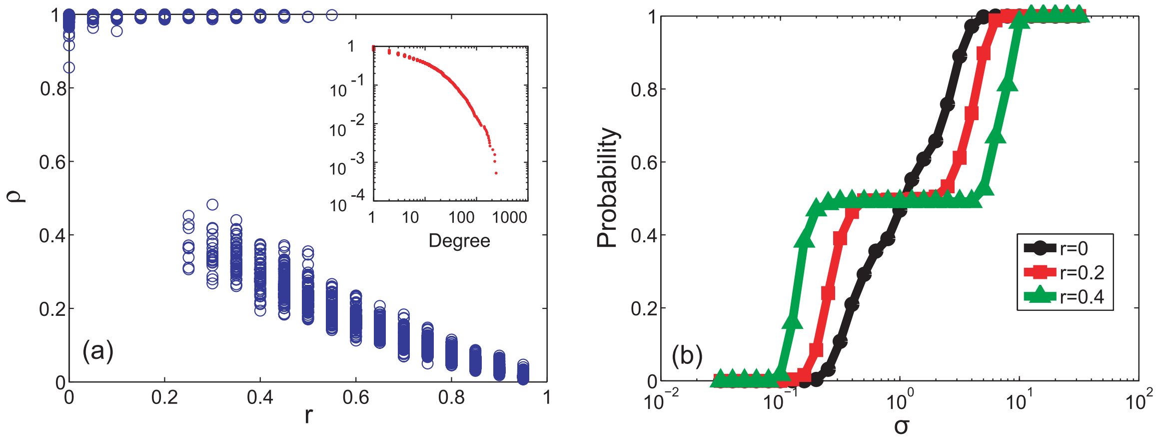

Fig. 7. The same as Fig. 3, but for the online social network. The inset in (a) shows the cumulative degree distribution of this network.

of opinion expression and the emergence of highly dominant public opinion seems to be larger than that in random networks. This is due to the difference of the connectivity between the two types of networks. The scale-free networks contain hub agents who require opinion expression of many neighbor agents in order to express an opinion. These agents are more likely to continue to keep silent and prevent a global prevalence of opinion expression. The result suggests that the online social media having scale-free structure can increase the possibility of the emergence of a highly dominant public opinion.

Finally, we investigate opinion dynamics on a Facebook-like online social network [26]. The nodes correspond to university students and the links correspond to sending of online messages. In the original dataset, a directional link is provided if a student sends one or more messages to another student. We regarded the unidirectional links to be bidirectional ones and obtained a network consisting of 1899 nodes and 13838 links. The mean degree is given by $\langle k \rangle \sim 15$. The cumulative distribution of the degrees is shown in the inset of Fig. 7(a). The distribution shows that the network consists of a small number of hubs and a large number of low-degree nodes. The connectivity is heterogeneous, but the network structure is neither random nor scale-free. The effect of the proportion of neutrals on the prevalence of opinion expression is shown in Fig. 7(a). The result is essentially similar to that for random networks in Fig. 3(a), although the range of r for the global prevalence is smaller. Figure 7(b) shows that a highly dominant public opinion is observed at a moderate value of r as in the other types of networks. These results suggest that the role of the neutrals does not much depend on the network topology.

In the above numerical experiments, the agents with different characteristics have been randomly assigned to the network nodes. Namely, the property for decision making and the number of social interactions have been assumed to be uncorrelated each other. Inspired by the study pointing out the importance of non-hub nodes in dynamical processes on complex networks [27], we consider the correlation between the agents' characteristic and connectivity. We focus on the specific types of agents: the neutrals with $\alpha_i = 0$ and the non-conformists with $\beta_i < 1/\delta$. Figure 8(a) shows the dominance level of the majority opinion for random and targeted assignment of the neutrals. The proportion of the neutrals is set at $r = 0.2$ for all the cases. In the "Random" case, the neutrals are randomly assigned to the nodes independently of the degree. In the "Target (x-y%)" case, the neutrals are assigned to the nodes with the degree in the range from top x% to y%. The proponents and opponents are randomly assigned to the remaining nodes. If the neutrals are located at the top 20% of hub nodes, a global prevalence is not achieved; this case is not shown in the figure. Within the cases where a global prevalence is achieved, the majority opinion is more dominant when the neutrals are concentrated on the higher-degree nodes. Once the neutrals having many social interactions express an opinion, the avalanche of opinion expression is likely to occur. Figure 8(b) shows the results for random and targeted assignment of the non-conformists. The proportion of the non-conformists is set at 0.2 for all the cases. The half of the non-conformists are proponents and

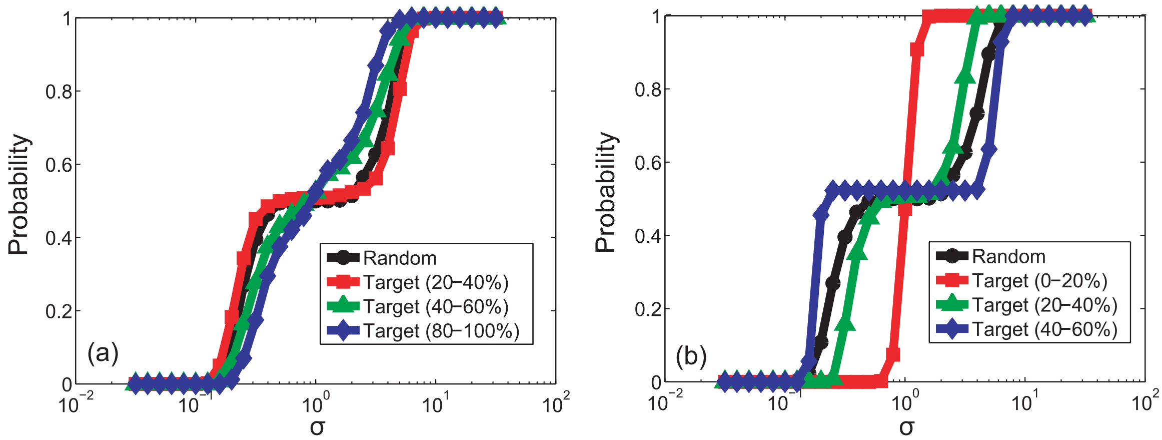

Fig. 8. Effect of the correlation between individual characteristic and the size of the reference group in the online social network. (a) The cumulative distribution of σ for different positions of the neutrals with $a_i = 0$. The proportion of the neutrals is set at $r = 0.2$. In the "Random" case, the neutrals are randomly assigned. In the "Target (x-y%)" case, the neutrals are assigned to the nodes with the degree in the range from top x% to y%. (b) The same as (a), but for different positions of the non-conformists with $\beta_i < 1/\delta$. The proportion of the non-conformists is set at 0.2, i.e. $\delta = 1/2$.

the other half are opponents. When the non-conformists are assigned to the top 20% of hub nodes, neither of the opinions is dominant. If they are assigned to the non-hub nodes with lower degrees in the "Target" case, the global prevalence of opinion expression is less reached and the dominance level becomes larger.

A comparison between Figs. 8(a) and 8(b) implies that the positions of the non-conformists have much greater impact on the dominance level of the majority opinion than those of the neutrals. The neutrals, wherever they are located in the network, promote more or less the bias of the two opinions due to the spiral of silence effect. On the other hand, the non-conformists play different roles depending on their locations in the network. If they are located on the hub nodes, their opinion expressions trigger the opinion expressions of many neighboring non-hub agents. As a result, the expressions of the two opinions are almost even and unlikely to be biased. If they are located on the non-hub nodes, the opinion incidentally expressed by a hub agent is likely to determine the dominant one and the bias rapidly expands. These effects of targeted positioning of the agents with specific characteristics can depend on the degree correlation of the social network and require further investigation.

4. Conclusions

We have investigated public opinion formation with the spiral of silence in an agent-based model on complex interaction networks. The decision making of each agent has been modeled based on the payoffs, which are determined by the agent's characteristics and the expressed opinions of the other agents in the reference group. By iterating the decision making of the agents, we have observed the spread of opinion expression. We have shown that a moderate proportion of neutrals are required for the formation of a highly dominant public opinion. We have demonstrated that there is a trade-off between the dominance level of the public opinion and the frequency of the achievement of the global prevalence. This property has been commonly observed in the random, scale-free, and online social networks. The simulations with variation of the average size of the reference groups have suggested that social interactions enhance the dominance level of the public opinion. Furthermore, we have examined the correlation between the individual characteristic and the size of the reference group of the agents. The results have suggested that the dominant public opinion is more likely to be formed when the neutrals are concentrated on the higher-degree nodes and the non-conformists are on the lower-degree nodes as long as the global prevalence of opinion expression is achieved.

In most mathematical studies on the spiral of silence, the role of specific characteristics of individuals has not been fully focused. We have shown that a highly dominant public opinion is formed when not a few neutrals exist. In other words, the spiral of silence is possible in the situation where many people do not have any evidence to support a specific opinion. Since the neutrals do not have beliefs and tend to express a majority opinion compared with non-neutrals, they are required for intensification of the dominance of the majority opinion. It is also a new finding that the correlation between the characteristics and the social interactions of the agents is important for the formation of a highly dominant public opinion. In particular, the public opinion becomes more dominant when the non-conformists have less social interactions. This means that non-hub people can play a significant role in the formation of public opinion formation.

In the present study, we have assumed for simplicity that the interaction networks of agents are bidirectional. However, many social networks are directed in the real world [28, 29]. Our model can be applied to directed social networks by changing the definition of the reference group. In a directed network, the reference group of each agent should consist of only the neighboring agents who share incoming links to the agent. The numbers of incoming and outgoing links of an agent, which are different among agents in directed networks, characterize how the agent is passive and active. Therefore, simulations of our model using directed social networks could provide a more realistic insight into opinion formation with the spiral of silence.

Acknowledgments

This research is partially supported by JSPS KAKENHI Grant Number 24700222 (G.T.) as well as by the Aihara Innovative Mathematical Modelling Project, the Japan Society for the Promotion of Science (JSPS) through the "Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST Program)," initiated by the Council for Science and Technology Policy (CSTP) (G.T., R.F., and H.S.).

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