This is to propose collaboration between me (Ashwin Ramaswamy) and Professor František Kalvas after our brief meetings at Complexity Weekend. Through our discussions we discovered a mutual interest in studying opinion dynamics and political polarization, and I had the pleasure to go through Dr. Kalvas's model on Reinforcing Spirals Model and Spiral of Silence based on the Hegselmann-Krause model. I am writing down a few examples of where my interests lie, and what I suggest could be directions in which we could take, should we choose to take the collaboration forward. I am also open to other ideas.

1. **Studying the effects of "new issues"**

   A phenomenon I wish to model is how a society might react to the sudden addition of a new issue/subject on which they may have little prior understanding but must quickly form opinions on. Recent history has examples where the sudden addition of a new issue has led to increased polarization across previously established fault lines. This was seen recently in the United States for example, where following the emergence of the Covid-19 situation there were mass demonstrations, and formation and empowerment of fringe groups. The intuition is that the addition of a new issue to which a lot of attention is suddenly being paid can exaggerate existing biases and differences, and thus reinforce polarization even if the issue itself is poorly understood. This also might give insights into why divisive regimes (that benefit from polarization) tend to be known for strong decisions – even faction-neutral decisions (such as cutting down a forest).

   I believe the HK model is inadequate to capture this phenomenon – the influencing forces in HK are all attractive, while exaggeration of differences would require dissimilar agents to repel each other. A model that assumes weighted averages within a completely connected network and allows edge weights to go negative might be a good starting point. A discrete opinion variant of this can be seen in Macy et al. 2003.

2. **A bounded confidence learning model based on Hegselmann-Krause**

   One of the key features of the HK model is that individuals assimilate into their neighbourhoods instantly and perfectly – the opinion of the individual does not matter beyond determining their neighbourhood.

   In our discussions we spoke of using a bias term on top of the regular Hegselmann-Krause updating rule – and the issue that arose is that this would have the effect of agents always gravitating to one of the two extremes. This violates the intuition of bias in my opinion – a bias should not make one constantly gravitate to one of the extremes, but instead should help resist the neighbourhood's influences over the agent's current beliefs to some extent.

   For this reason, I am proposing an updating rule that deviates significantly from the classical HK model. This rule is more similar to what is seen in learning models in cognitive science. Here the agent's default next state (in the absence of influences) is its current state. At every time step, however, the agent assimilates more into its neighbours' opinions by learning the difference between its own opinion and the neighbourhood average. A learning parameter α controls how well an agent assimilates, and can be modelled as either a constant or something that varies across individuals.

$$o_i^d(t) = o_i^d(t-1) + \alpha\left[\frac{1}{N}\sum_{j=1}^{N} o_j^d(t-1) - o_i^d(t-1)\right]$$

If $o_i^d(t) > 1$, set $o_i^d(t) = 1$

If $o_i^d(t) < 0$, set $o_i^d(t) = 0$

Where,

1. $0 < o_i^d(t) < 1$ represents the opinion of the i^{th} agent on the d^{th} dimension.
2. N > 0 is the size of the neighbourhood.
3. α is the learning parameter

The neighbourhood can be determined using the HK rule for n-dimensions by selecting all neighbours falling within a tolerance parameter ε of Euclidean distance in the opinion space.

On the learning rate α

α determines the extent to which an agent assimilates to its neighbourhood. One could assume α to be a constant value for all agents or assume a Gaussian distribution for it. Variable α is an interesting case because it could be taken to represent diversity in the agents' tendency to be influenced by neighbourhood opinions. An important caveat is that high values of α can destabilize the agent's opinion.

References

1. Macy MW, Kitts JA, Flache A, Benard S. Polarization in Dynamic Networks: A Hopfield Model of Emergent Structure. Dyn Soc Netw Model Anal. 2003.