Working paper
Conceptualizing and measuring polarization: A review*

Paul C. Bauer
Mannheim Centre for European Social Research (MZES)

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Feedback? Please contact me at mail@paulcbauer.eu.

Abstract

The increasingly popular concept of polarization is used to describe various social phenomena such as political, opinion, health and income polarization. Despite this popularity it is still debated within disciplines how polarization should be conceptualized and how it should be measured. In this study I provide a systematic, interdisciplinary overview of the conceptual and measurement literature on polarization. I first describe the challenge of conceptualizing polarization, a task that requires taking decisions about how to aggregate individual positions on one or more scales. Different distributional aspects may matter and the concept's meaning is related to the measurement level of the underlying scales. Subsequently, I review various polarization measures that have been developed during the last decades.

Corresponding author: mail@paulcbauer.eu; www.paulcbauer.eu. Document generated with R Markdown.

1 Introduction

The concept of polarization is discussed in various social science classics and studied across various disciplines and sub-disciplines in the social sciences (e.g. Ross 1920; Galtung 1966; Blau 1977; Deutsch 1977; Marx 1867). In political science, for instance, the concept is applied to attitudes and the concept's popularity as grown, driven by a more polarized political reality in the United States. A growing number of empirical studies investigates levels and trends of political polarization in the United States both on the elite/party level and the level of the public (e.g. Hetherington 2009, 2009; Layman, Carsey, and Horowitz 2006; Layman and Carsey 2002; Poole and Rosenthal 1984; Stone, Rapoport, and Abramowitz 1990; Baldassarri and Gelman 2008; Fiorina and Abrams 2008; Fischer and Mattson 2009; Lelkes 2016). More recently, European scholars have picked up the concept (e.g. Adams, Green, and Milazzo 2012; Adams, De Vries, and Leiter 2012; Down and Wilson 2010; Munzert and Bauer 2013). Nonetheless, despite the sizable volume of scholarship on the concept, scholars disagree on how much political polarization there actually is among elites and the public. The debate on polarization is itself polarized (Lelkes 2016). Scholars often provide evidence from a series of measures instead of suggesting one ideal measure (cf. DiMaggio, Evans, and Bryson 1996; Fiorina and Abrams 2008; Lelkes 2016). We can find similar debates in related disciplines such as economics.

Broadly speaking, the level of polarization describes to what degree individuals' positions on one or several scales are separated.[1] A process or trend of (de-)polarization describes how the level of polarization changes over time. However, the nature of this separation, i.e. to what extent different characteristics of a distribution reflect the idea of polarization is still subject to debate, a debate that is linked to the scale(s) that underlie one's concept of polarization. Scholars arrive at different conclusions regarding the levels and trends of polarization of different phenomena and polarization's impact on other phenomena (e.g. violence) and are also critical of certain measures (e.g. Downey and Huffman 2001). In part, such differing conclusions are caused by the use of different conceptualizations and related measures of polarization.

In the present study I pursue the following research question: *How can we conceptualize and measure polarization?* In seeking an answer to this question our study contributes to current scholarship in several ways.

First, I describe the choices necessary when conceptualizing polarization. In doing so I emphasize that this process is ultimately an abstract mathematical task. This task entails choosing what kind of and how many scales underlie our potential polarization concept and which distributional characteristics reflect higher/lower levels of polarization. Such characteristics,

[1] I speak of individuals throughout the paper, however, generically we could think of other types of actors such as firms, parties, countries etc.

e.g. the extremeness or clustering of scale positions, have been discussed in more or less explicit manner in the past. Because it's an abstract task it is not surprising that measures were coined with similar characteristics for closely related concepts such as "agreement". Importantly, as we study the polarization of different phenomena we may find that different distributional characteristics matter from a conceptual point of view. For instance, both income and opinion polarization may lead to conflicts in a group of individuals. However, the income distribution to cause maximum conflict may differ from the opinion distribution that causes maximum conflict, requiring different measures of polarization for the two phenomena.
Second, I review current polarization measures. In doing so I extend upon previous reviews in Sociology, Political Science or Economics (e.g. Fiorina and Abrams 2008; Hetherington 2009) both by classifying them in a more systematic way and by reviewing measures across disciplines as well as across subareas such as political, social or health polarization.[2] I try to provide a more systematic perspective by classifying subconcepts and measures of polarization through the set of choices on which they are based. One such choice, for instance, is whether to incorporate a single scale or several scales. Thereby, I link formerly separated literatures and review measures of both unidimensional and multidimensional polarization. *Generally*, the idea is to provide scholars with a guide to the concept of polarization and, in doing so, emphasize that the study of polarization of different phenomena requires a careful consideration of the conceptual aspects that underlie different measures which in turn are linked to the actual phenomenon we study. Our review is necessarily selective in its discussion of measures because the literature turned out to be quite vast.

In Section 2 I start by outlining how polarization is conceptualized and discuss the choices made in this process as well as the role of measurement levels. In Section 3 I provide an overview of various polarization subconcepts. In Section 4 I review measures of polarization, both for unidimensional polarization and for multidimensional polarization. Section 5 provides some concluding thoughts.

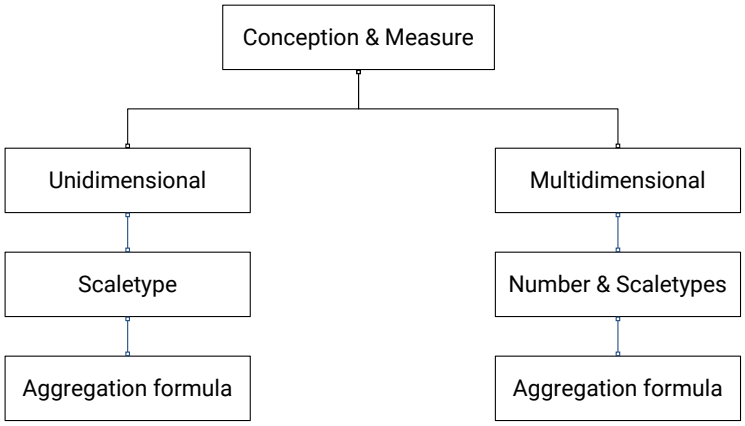
2 Conceptualizing polarization: A set of choices

The concept of polarization was discussed at various stages throughout the last century. Definitions are rare but helpful. Within peace & conflict research Galtung (1996, 14:90) [1966] characterizes the concept as follows: "Polarization means reducing [...] conflict formation to

[2]While several helpful reviews exist these are often discipline specific and topic specific (see e.g. DiMaggio, Evans, and Bryson 1996).

the most simplistic level, assigning all m parties to one or the other of two camps, wrapping all n conflict themes together in one super-theme. All positive, cooperative relations are within the camps and all negative relations between them [...]". In economics, Esteban and Ray (2012, 2) provide a more mathematical and abstract characterization of polarization as "separation or distance across clustered groups in a distribution". Galtung (1996)'s definition already emphasizes the element of clustering into (two) groups and the type of relations that should exist within those groups. Esteban and Ray (2012)'s definition also emphasizes the role of clustering, adds that there may be more than two clusters and alludes to separation or distance between groups. The ideas of separation vs. distance come into play when we deal with different, qualitative or quantitative scale types. Below we'll present the task of conceptualizing polarization as a set of choices, discuss the significant role of measurement levels and dimensionality and try to formulate some general principles.

Figure 1: Choices in conceptualizing/measuring polarization



Fundamentally, conceptualizing and measuring polarization entails several choices as depicted in Figure 1. First, we have to choose whether we investigate polarization on a single scale or several scales. Or put differently, whether the type of polarization we are interested is uni- or multidimensional in nature.[3] For instance, *income polarization* is unidimensional as the underlying data are positions on a single scale measuring income. In contrast, *ethnic income polarization* is twodimensional or multidimensional respectively, as we need data on both a scale differentiating individuals' incomes and a scale measuring individuals' belonging to ethnic groups.

Second, we have to choose what types of scales flow into our polarization measure, i.e. what

[3]Mostly, I use "dimension" and "scale" as synonyms, however at times their meaning slightly differs.

measurement levels we are dealing with. A commonly used classification of measurement levels distinguishes *nominal, ordinal, interval and ration* scales as depicted in Figure 2 (Stevens 1946).[4] For instance, income is often measured on a ratio level, but may also be measured on an ordinal scale that assigns individuals to categories. Ethnicity is usually measured on a nominal scale comprising different ethnic categories. Scholars have developed measures for different measurement levels.

Third, we have to choose which *distributional characteristics* reflect polarization. Some may argue that polarization is represented by a distribution in which many individuals are located at the extremes of a scale or several scales. Others may suggest that the main aspect is not extremeness but clustering. When individuals are located in clearly distinguishable clusters, even if not extreme, we may speak of polarization. If we deem several aspects relevant we have to make a decision on how to weight them. Naturally, such distributional aspects are tied to the measurement level of the underlying scales. Subsequently, we have to decide on how to aggregate individuals' positions on the scale(s) as to reflect the distributional characteristics we want to pick up both conceptually and in terms of measurement. Different ways of aggregation emphasize different distributional characteristics. For instance, the variance is strongly influenced by individuals that are distant from the distributional mean. Ultimately, a polarization measure will assign uni- or multidimensional distributions values that should reflect our conceptual idea of polarization.

Fourth, we have too choose which type of actors underlie our concept of polarization. In abstract terms we can simply speak about units that are distributed across one or several scales. However, the concept's meaning may change as we investigate different types of units, such as individuals, firms, parties etc. The type of unit probably also determines what overall population size we investigate. The population size in turn may affect what measure we use. For instance, we may want a polarization measure for small groups ($N = 2$) to have different characteristics than a polarization measure for larger groups ($N >= 500$). In other words, depending on whether we investigate small group or large group polarization, we may want to adapt any measure we are using. Generally it is argued that polarization is a matter of groups so that the contribution of isolated individuals should be negligible (Esteban and Ray 2012, 3).[5] However, there are "small group situations" where a single actor that is distant from others may be responsible for conflicts or a break down of cooperative relations.

Finally, all of the above choices are linked to the phenomenon we study. For instance, if we investigate income polarization there are only so many scales types (probably ordinal or higher)

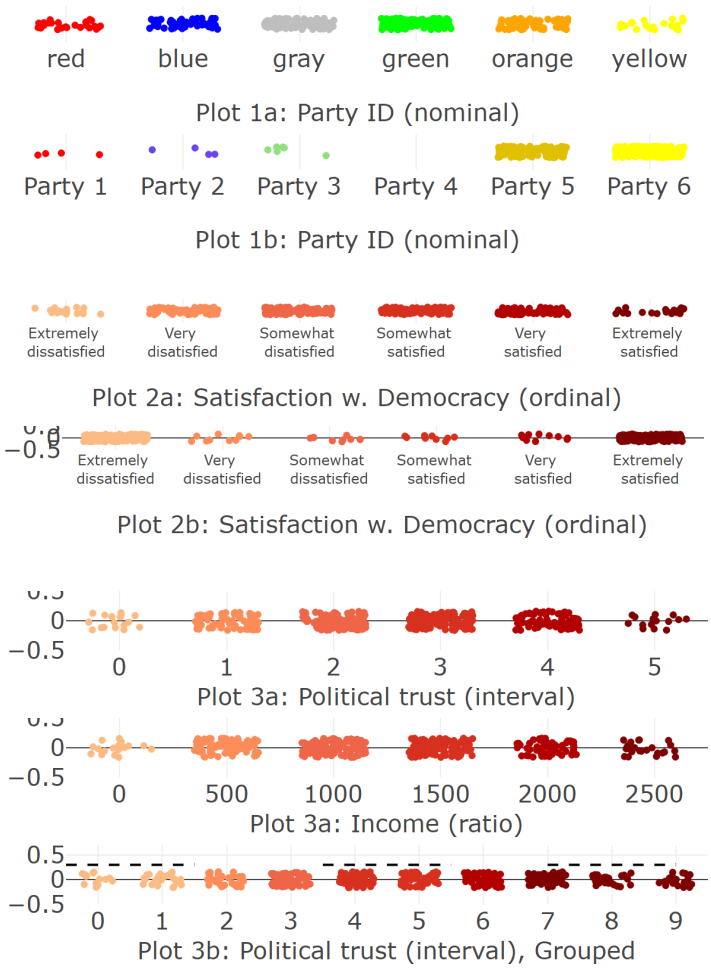
[4] Note that there are other classifications of measurement levels.

[5] The literature on party system polarization discusses how the number of parties may affect their incentive to converge or spread along an ideological continuum (Downs 1957; Dalton 2008).

that make sense when we operationalize and measure polarization. This may change when we investigate ethnic or opinion polarization.

Once we construct a measure we can use it to assign *values/levels of polarization* to different groups of units based on data collected at a particular moment in time t. Given the we have data across time it may allow us to observe *trends/processes of polarization* (e.g. DiMaggio, Evans, and Bryson 1996, 693), i.e. whether the level of polarization of a group of individuals increases or decreases over time.

Figure 2: Nominal, ordinal, interval and ratio scales



Note: Simulated data.

Let's turn to the notion of measurement levels. When we differentiate characteristics of individuals we assign those individuals to values on a scale. Scales differ in terms of measurement

level and a scale's measurement level determines what conceptualization of polarization we can have. As we will see below moving up the ladder of measurement levels allows us to add depth to an eventual conceptualization of polarization. We start our discussion with *unidimensional* or *single scale polarization*.

On *nominal scales*, see Figure 2 Plot 1a and 1b, the only informative characteristic is category membership, i.e. we can count the number of individuals in different categories (frequency). Early scholarly references to polarization seem to be rooted in a *nominal* view.[6] Take for instance how Ross (1920, 415) refers to polarization: "*Sometimes the spirit of faction gains mysteriously the upper hand and society polarizes into opposing groups which may have no more substantial basis than the parties of Greens and Blues [. . .] Generally, however, it is antagonism of economic interests or ideas which rives society in twain.*". In this view, polarization occurs when people are separated into categories, along dividing lines. Such dividing lines are also referred to as cleavages (Lipset and Rokkan 1967; Zuckerman 1975) and Section 6.3 (appendix) discusses the link between the concepts of polarization and cleavage.[7] On a single scale it is often argued that polarization is highest when individuals are concentrated in two equally sized groups or categories as in Figure 2 Plot 1b rather than spread across all categories as in Figure 2 Plot 1a (e.g. Reynal-Querol 2002). Without additional information it would not matter in which two categories they concentrate. Pervasive spread is also called 'heterogeneity' in the respective literature. The question of how individuals are distributed across categories matters. For instance, with ethnic polarization we could either argue that it is the situation with two equally sized ethnic groups that may increase the probability of conflict or the situation in which a large group dominates and suppresses one or several small groups In other words, it's not that simple.

Ordinal scales enhance thinking about polarization. On a ranked scale, categories towards the opposing endpoints "lie further apart". Here it matters in which categories individuals are located. A distributions with many individuals located in the opposite extreme categories, e.g. extremely dissatisfied and extremely satisfied, as in Figure 2 Plot 2b could be seen as a higher level of polarization than a concentration in the categories towards the middle as in Figure 2, Plot 2a. In other words, even without a quantifiable distance between categories, the idea of dispersion becomes meaningful and it matters in which categories individuals concentrate.

On *interval* scales, distances between scale values are meaningful. Hence, a potential conception of polarization may incorporate the distance of individuals towards the mean, the scale endpoints, within and between sub clusters etc. This opens the venue for a whole new set of

[6] I am grateful to Andreas Diekmann for hinting to the early work of Ross (1920).

[7] The cleavage literature often related several scales with each other, e.g. a scale measuring individuals' center vs. periphery belonging and a scale measuring vote choice.

measures. In Figure 2 Plot 3a there are no clusters. Visually this is emphasized as each scale value is represented by a distinct color. However, we could also try to identify sub clusters. In Figure 2 Plot 3a colors and dashed black lines indicate that individuals with values 0-1, 4-5 and 7-9 belong to sub clusters. Accordingly, a polarization concept can be enhanced by the idea that clustering in such groups matters. Naturally, such clusters can be pre-specified or identified in an explorative manner (e.g. by some algorithm or visually).[8] Finally, the *ratio* measurement level comes with a meaningful zero point and allows comparisons between individuals such as X's income is 'one-half or twice as much' as Y's income. To sum up, the measurement level of the scales we work with determines they way in which we can conceptualize polarization. Naturally, we could always assume that certain scales have a higher measurement level as is often done in applied research.

Despite the fact that empirical research primarily focuses on unidimensional polarization, early theoretical accounts have already emphasized the concept's *multidimensional* character. For instance, it is argued that societal conflict is more likely when polarization extends across dimensions/scales (Ross 1920, 164), i.e. conflict is more likely when a society polarizes simultaneously along the lines of economic wealth and religious affiliation rather than only in terms of economic wealth.[9] When we move from uniscale to multiscale polarization, we aggregate positions on scales that may potentially have different measurement levels.
As a quick illustration of the data underlying multidimensional polarization measures, Figure 3 visualizes three examples. Figure 3 Plot 1 visualizes the idea of *residential polarization* or *geographic polarization* (Fiorina and Abrams 2008, 563), namely, that people of a certain category (e.g. party ID or ethnic background) are concentrated in certain districts of a city.[10] It comprises two nominal scales, one differentiating immigrants from non-immigrants and another one differentiating districts.[11] Figure 3, Plot 2 visualizes a two-dimensional distribution comprising a nominal and an ordinal scale, namely race vs. income. This data and distribution could constitute the basis for a measure of *racial health polarization*. Figure 3 Plot 3 visualizes a three-dimensional distribution comprising three ordinal scales, namely attitudes towards redistribution, gender equality and immigration. Such data underlies measures of multidimensional

[8] For instance, Gigliarano and Mosler (2009, 457–58) provide an example with groups that are exogenously defined. See also Duclos, Esteban, and Ray (2004, 1738).

[9] Ross (1920) also recognizes the fact that the probability of conflict may actually decrease with a rising number of cross-cutting dividing lines that separate people from each other: "*A society, therefore, which is riven by a dozen oppositions along lines running in every direction, may actually be in less danger of being torn with violence or falling to pieces than one split along just one line. For each new cleavage contributes to narrow the cross clefts, so that one might say that society is sewn together by its inner conflicts*" (Ross 1920, 165). In other words, if the dividing lines are not somehow linked with each other, the probability is higher that there aren't any stable groups.

[10] In principle, we could replace the categorical scale for the districts with a scale measuring distance.

[11] This example emphasizes that the idea of residential polarization is close to the idea of segregation.

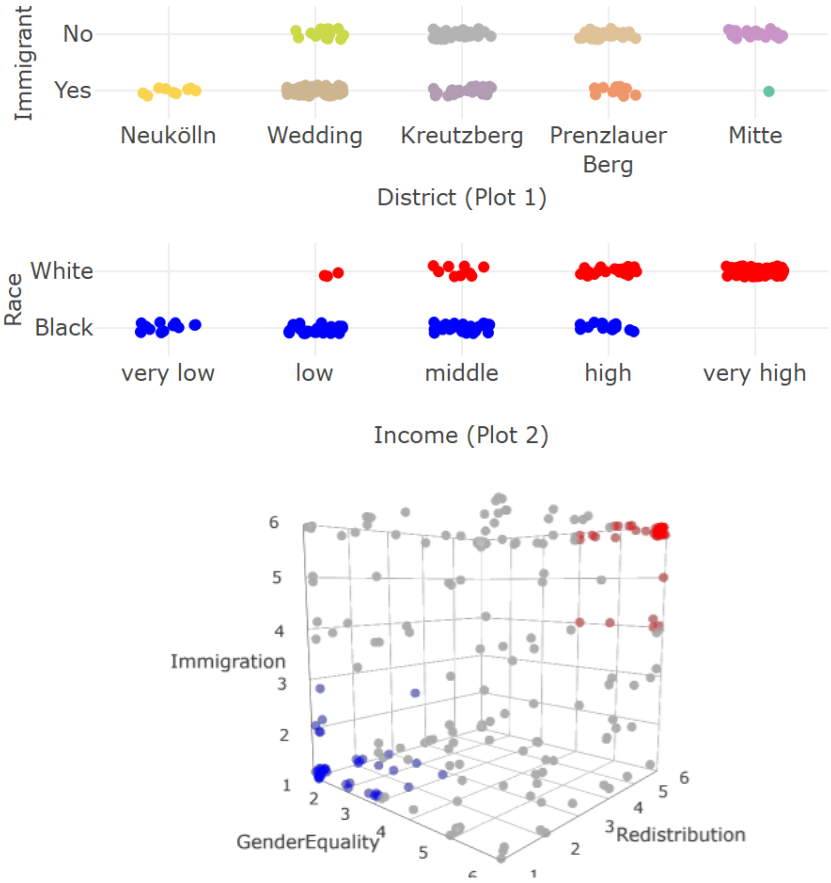
opinion polarization. Importantly, when moving from unidimensional to multidimensional polarization, the fact that the underlying scales may be of different measurement levels adds complexity.

To sum up, scales that underlie a concept of multidimensional polarization may differ. Nontheless, it seems as if we can still formulate a general principle that applies to both singlescale and multiscale polarization: *Polarization is at a maximum when individuals cluster in two cells in the respective distribution. A cell representing a value on a single scale, or a combination of values on several scales. Moreover, when a ranking/distance is meaningful on one or several of the underlying scales, polarization would be at a maximum when the two cells represent the endpoints on those scales. Polarization is at a minium when all individuals cluster in a single cell.*[12] While it seems possible to define states of maximum and minimum polarization it is less clear how to treat and distributional states between those two. For instance, we may wonder how a situation with 3 clusters (with everyone in one of 3 cells) in which two clusters occupy the endpoints, compares to a situation in which everyone is located in one of two clusters but those clusters do not lie at the end points.

And, importantly, if we don't define (maximum) polarization in the abstract distributional sense but rather in relation to a phenomenon, e.g. the distribution that leads to maximum conflict among parties (see above), the above principle wouldn't describe a maximally polarized distribution.

[12] For an argument, why such a distribution is not necessarily the one creating most conflicts see above.

Figure 3: Multidimensional distributions



Note: Simulated data.

3 Polarization subconcepts

By subconcepts I mean polarization concepts that specify polarization for specific social phenomena, e.g. *political polarization*, *health polarization* etc. and I already mentioned a few above. Just as the general concept has been rarely defined (cf. DiMaggio, Evans, and Bryson 1996, 692)[13], there are few definitions of corresponding subconcepts.[14] For some subconcepts several measures have been developed. Table 1 provides a quick, non-exhaustive overview of

[13] See Galtung (1996) and Esteban and Ray (2012) for exceptions.

[14] While there are few non-formal definitions one could in principle regard a mathematical formula that aggregates values on scales in a particular way as a formal definition.

polarization subconcepts and classifies whether corresponding measures are primarily single scale, multiscale or both. Table 1 also provides references that contain definitions or summaries of the corresponding literatures.

I start with *income polarization*, as it is in this literature that one can find one of the first elaborate and explicit discussions of the concept.[15] Wolfson (1994) observed that the quite common notion of the "disappearing middle class" is typically associated with the idea of increasing income inequality. At the same time conventional scalar measures of inequality do not reflect the idea that observations disappear from the middle of a distribution (cf. Levy and Murnane 1992). Consequently, Wolfson (1994, 356) suggests that "summary measures based on concepts like polarization should be given equal space along with Lorenz-consistent inequality measures when describing trends in income distribution" as it may be "closer to the general public's vernacular concept of inequality". As income is measured on different types of scales (ordinal, interval etc.), different measures are used for *income polarization*.

Ethnic polarization was coined to contrast ethnic heterogeneity long assumed to foster the likelihood of conflict between ethnic groups (José G Montalvo and Reynal-Querol 2005). Ethnic polarization is conceptualized on a single nominal scale that differentiates between ethnic groups. Closely linked is the concept of *religious polarization* (Montalvo and Reynal-Querol 2002, 2003), only that the underlying scale comprises religious instead of ethnic categories.

The concept of *political polarization* is used to describe polarization among political actors such as parties, party members, elected party representatives, but also among voters and the public more generally.[16] The concept is broad in its understanding and was operationalized in different ways, i.e. using a variety of measures based on various types of scales. Layman, Carsey, and Horowitz (2006, 87) provides an overview of conceptions/measures that have been used to assess polarization among members of the U.S. Congress. These measures are normally based on at least two scales, one measuring party membership (Democrat vs. Republican) and one measuring voting behavior or opinions. In terms of the public Fiorina and Abrams (2008) suggests that the most direct evidence of political polarization in the public are citizens' positions on public policy issues, i.e. on opinion scales. A corresponding conceptualization may comprise either a single or several of such scales. Again, various conceptualizations and corresponding measures have been suggested in the literature (see Fiorina and Abrams 2008 for an overview), including conceptions that combine a nominal Party ID scale and other types

[15] Importantly, I ignore measures that may have been developed in the natural sciences here.

[16] A common finding seems that party conflict in the U.S. Congress evolves along a single dimension, a single liberal-conservative cleavage (Layman, Carsey, and Horowitz 2006, 87)

of scales.[17]

Polarization of opinions is also termed *(public) opinion polarization* (Baldassarri and Gelman 2008; Munzert and Bauer 2013, Footnote 2) which simply entails looking at 'the distribution of political attitudes across all Americans' (Baldassarri and Gelman 2008, 414) that is their positions on various attitude scales.

Somewhat related is *affective polarization* which describes "the tendency of people identifying as Republicans or Democrats to view opposing partisans negatively and co partisans positively" (Iyengar and Westwood 2014, 2). Conceptualizing affective polarization thus requires data on two scales, a Party ID scale, as well as a scale measuring feelings towards other partisans. *Trait polarization* designates the difference Americans perceive between the traits of presidential candidates (Hetherington, Long, and Rudolph 2016). This concept is also based on two scales, a Party ID scale and a scale measuring the evaluation of traits of presidential candidates.

According to Fiorina and Abrams (2008, 563), *geographic (opinion) polarization* or *residential (opinion) polarization*[18] describes the tendency of like minded people to cluster together and live in the same place. This concept requires a scale(s) that locates individuals geographically, e.g. in a district as in the example above or even two scales measuring longitude and latitude and it requires a scale measuring opinions. Naturally, individuals can be geographically polarized not only with regard to opinions, but all sorts of social phenomena. In principle, we should further specify on which scale individuals are geographically polarized, and we could do so by talking about, e.g. geographic income polarization.

Intergroup polarization focuses on tracking "changes between subgroups of the population, distinguishing people along sociodemographic lines" (Baldassarri and Gelman 2008, 414). In other words, this concept distinguishes itself in that one of the underlying scales is sociodemographic in nature (e.g. age, income etc.). In that it is a synonym for *social polarization* which refers to "the measurement of the distance between different social groups, defined on the basis of variables such as race, religion, or ethnicity" (Fusco and Silber 2014, 841). In other words, besides at least one scale separating individuals into different social groups, we need another scale that measures the distance on some phenomenon, e.g. income.

Health polarization refers to the distance between individuals regarding their health. Normally, health polarization is conceptualized on a single scale, e.g. a scale measuring self-assessed health. But for instance, Apouey and Silber (2013) extent the concept health polarization with a scale measuring socio-economic status that one could call *socio-economic health polarization*. Permanyer and D'Ambrosio (2015) enhance the concept with a second scale measuring race,

[17] Party sorting designates a stronger relation between policy positions and party ID (Fiorina and Abrams 2008, 563).

[18] This idea is closely linked to the idea of geographical or residential segregation.

thus conceptualizing the idea of *racial health polarization*.
To sum up, there is a number of subconcepts coined for different social phenomena. These differ in terms of the underlying scale types, their dimensionality (number of scales) and the measures that have been proposed to gauge them.

Table 1: Polarization subconcepts

	Name	Dimensionality (N)	Scaletypes	References
1	Income Polarization	u	o, i, r	Wolfson 1994, Esteban and Ray 1994
2	Ethnic Polarization	u	n	Reynal-Querol 2001; Montalvo and Reynal-Querol 2010
3	Religious Polarization	u	n	Reynal-Querol 2001; Montalvo and Reynal-Querol 2010
4	Political Polarization	various	various	DiMaggio et al. 1996, Layman et al. 2006
5	(Public) Opinion Polarization	various	various	DiMaggio et al. 1996
6	Affective polarization	m	n + i	Iyengar and Westwood 2014
7	Trait polarization	m	n + i	Hetherington, Long, and Rudolph 2016
8	Geographic / residential polarization	m	n + i	Fiorina and Abrams 2008, Others
9	Intergroup polarization	m	various	Baldassarri and Gelman 2008, Others
10	Social polarization	m	various	Fusco and Silber 2014, Others
11	Health polarization	various	n, o	Apouey and Silber 2013, Permanyer and D'Ambrosio 2015

Notes: Table presents a non-exhaustive list of polarization subconcepts;
Dimensionality: Number of dimensions/scales underlying the conception/measure;
Scaletypes: n = nominal, o = ordinal, i = interval, r = ratio; References: exemplary references

4 Measures of polarization

Above I gave a quick overview of various polarization subconcepts. In line with those subconcepts a large number of polarization measures have been proposed in disciplines such as Sociology, Political Science and Economics. Table 2 provides a non-exhaustive list of measures and categorizes them. Section 6.2 in the appendix provides formulas for some of those measures. While the conceptual ideas of polarization have been around for a long time, its empirical measurement started much later.
Measures can be categorized along the choices I presented in Section 2 namely *dimensionality*, *measurement level*, *tradition aka distributional focus* and *formal characterization*. First, *dimensionality* describes the number of scales they integrate. Unidimensional measures aggregate positions on single scales, multidimensional measures aggregate positions on at least two scales. Second, regarding the *measurement level(s)* of the underlying scales. This may be one of nominal, ordinal, interval/ratio or a particular combination thereof for multiscale measures. Third, in terms of the *distributional aspects* they tap into. Esteban and Ray (2012, 3) suggest that there are two different traditions, one could call them the "cluster tradition" and

the "extremeness tradition". The first tradition focuses on capturing the presence of clusters in uni- or multidimensional space. The second focuses on the disappearance of the 'middle class' (middle categories) in uni- or multidimensional space (see also Gigliarano and Mosler 2009).[19] Naturally, following the 'cluster tradition' the question arises of how groups, i.e. clusters are identified. In Figure 2 Plot 3b I arbitrarily marked groups (dashed black lines) as an illustration. On the one hand we could pre-define groups, on the other hand some aggregation clustering algorithm may find those groups in an explorative manner, e.g. by identifying groups that would minimize within group distance and maximize between group distance.
Finally, measures can be classified according to whether they are formally characterized, i.e. derived from a formalized set of rules or not. In the next section I describe a subsample of the measures provided in Table 2.

Table 2: Measures of polarization

	Name	Dimensionality (N)	Scaletypes	References	Phenomenon
1	RQ Index	u	n	Reynal-Querol 2001	Religion, Ethnic
2	A Index	u	o	Apouey 2007	Health
3	Ordinal consensus	u	o	Leik 1966	none
4	ER index	u	i, d	Esteban and Ray 1994	Income
5	d-squared	u	d	Blair and Lacy 2000	Opinions
6	Average synthetic opinion Score	u	i, d	Baldassarri and Gelman 2008	Opinion
7	Variance	u	c	Fisher?	Various
8	Kurtosis	u	c	?	Various
9	DER Index	u	c	Duclos, Esteban, and Ray 2004	Income
10	Kobus 2012	m	c	Kobus 2012	Gender + Education
11	Silber and Yalonetzky 2011	m	o, n	Silber and Yalonetzky 2011	Life Chances
12	Apouey and Silber 2013	m	o, n	Apouey and Silber 2013	Socioeconomic Status + Health
13	POLOR^{1,2,3,4}, SP^{alpha}, SP^{Gini}	m	o, n	Fusco and Silber 2014	Health, citizenship
14	Average pair-wise correlation	m	i + i	Baldassarri and Gelman 2008	Opinions
15	Cronbach's alpha	m	i + i	DiMaggio, Evans, and Bryson 1996	Opinions
16	P N,alpha	m	n	Permaneyer 2012	Religion, Religiosity
17	P b N,alpha	m	n	Permaneyer 2012	Religion, Religiosity
18	Overlap measure	m	r	Anderson, Linton, and Leo 2012	Lifetime GDP per Capita, Country
19	Trapezoidal measure	m	r	Anderson, Linton, and Leo 2012	Lifetime GDP per Capita, Country
20	GM Index	m	i + o	Gigliarano and Mosler 2009	Income, education
21	Kleiner Index	m	i	Tuuli-Marja Kleiner 2016	Ideology
22	Dalton	m	i	Dalton 2008	Party System/Ideology

Notes: Table presents a non-exhaustive list of polarization measures; Dimensionality: Number of dimensions/scales underlying the conception/measure; Scaletypes: n = nominal, o = ordinal, i = interval, r = ratio, d = discrete; References: exemplary references;

4.1 Measures of unidimensional Polarization

Nominal scales assign individuals to categories of concepts such as ethnicity or religious denomination. On the individual-level we know to which category an individual belongs, aggregation gives us the frequency across categories. Figure 2, Plot 1b illustrates a distribution

[19]The second tradition goes back to Wolfson (1994). Wolfson (1994; see also Levy and Murnane 1992) starts by suggesting that the quite common notion of the "disappearing middle class" is typically associated with the idea of increasing income inequality. At the same time conventional scalar measures of inequality do not reflect the idea that observations disappear from the middle of a distribution (cf. Levy and Murnane 1992).

across 6 categories (N = 400), where most observations are located in two categories in contrast to Plot 1a. It is generally argued that a society in which individuals are evenly distributed across categories is less polarized than a society in which individuals are concentrated in two groups. However, when individuals are concentrated in a single group we would not speak of polarization. Measures such as the Herfindahl-Hirschmann Index (HHI) (and analog the Ethno-Linguistic Fractionalization Index - ELF) reflect heterogeneity - that is to what extent individuals are widely spread across categories - but ignore the aspect of bipolarity (see 6.2.2). Reynal-Querol (2001, see also 2002) develops the so-named RQ index for nominal scales that tries to capture how far a distribution is away from a bipolar distribution, a distribution in which all individuals are located in two categories (see Section 6.2.10). The RQ index can be applied to measure polarization for "nominal phenomena" (Reynal-Querol 2002). Importantly, in using this measure we have to agree with the notion that a distribution in which all individuals concentrate in two categories is maximally polarized.

Various measures where constructed for *ordinal scales*. For instance, Leik (1966) - who gives an insightful introduction to the logic underlying ordinal scales - develops a measure of ordinal consensus (see Section 6.2.4). Similarly, Blair and Lacy (2000) develop a measure of dispersion for discrete data (see Section 6.2.6). Apouey (2007) develops two median-based measures focusing on an ordinal scale of self-rated of health, i.e. self-assessed health status (SAH) (see Section 6.2.9).[20]

On *interval/ratio* scales the magnitude of differences between the scales' values is meaningful. Empirical polarization research in political science and sociology that focuses on single scales mostly relies on classic summaries of distributions such as the variance or the kurtosis, that reflect the idea of dispersion or bi-modality (see Section 6.2.1). DiMaggio, Evans, and Bryson (1996) give an overview of conceptualizations and corresponding measures up until 1996. For instance, Down and Wilson (2010, 69) use the variance as a measure of polarization in their study on opinion polarization and inter-party competition.

In 1994 Esteban and Ray (1994) formally derive a measure of polarization for discrete scales/distributions - the so-named ER index (see Section 6.2.5) .[21] While the measure can easly be mistaken as a unidimensional measure it requires exogenously defining groups using

[20]Measures of inequality "are mean based. Thus, cardinality must be imposed on ordinal SAH categories in order to calculate a mean, which can then be used to compute an inequality measure such as the Gini coefficient. This approach is interesting (especially when dealing with reporting bias), but it has a shortcoming: imposing cardinality is a supra-ordinal assumption that changes the original properties of the SAH" (Apouey 2007, 875). "Apouey's polarization measure is an extension of traditional income bi-polarization measures, and, as such, does not include information on any salient social characteristics in the analysis" (Permanyer and D'Ambrosio 2015, 313).

[21]According to the authors polarization measurement starts with Esteban and Ray (1991), Esteban, Gradin and Ray (1998,2007) and Duclos, Esteban and Ray (2004)" (Esteban and Ray 2012)

a second scale. In 2004 Duclos, Esteban, and Ray (2004, 1739) follow up on their previous work and provide a polarization measurement theory that is relevant to describe cases in which distributions can be described by density functions.[22] Naturally, at some point the focus shifted to multidimensional polarization - not only theoretically but also in terms of empirical measurement. We'll shortly discuss some multidimensional measures below.

4.2 Measures of multidimensional Polarization

In principle there are situations in which polarization along a single scale may matter. If it is extreme enough, e.g. think of extreme income or wealth polarization, any other differences between group members may loose their significance and it may lead to conflicts. Often, however, polarization seems to matter more when it is multidimensional in nature. Classic notions of polarization (e.g. Ross 1920 etc.) tend to link at least two scales with each other, for instance class/wealth or religion/wealth and corresponding measures of multidimensional polarization aggregate positions on at least two scales. Different authors have developed measures that aggregate positions on several scales with the same or differing measurement levels.

The *nominal-ordinal 2-dimensional combination* is covered by different measures. Permanyer and D'Ambrosio (2015) axiomatically characterize measures that combine a nominal scale with an ordinal scale. As an example they use the nominal scale race (black vs. white) and the ordinal scale self-assessed health (very poor, poor, fair, good, to very good). In other words, they define a social polarization index that incorporates the fact that a society is partitioned into groups on the basis of salient social characteristics (e.g. race or ethnicity) and that these groups may be clustered in certain areas of another scale's distribution, e.g. have mostly low values of self-assessed health. Their measure can also be used for a combination of nominal scales (see Section 6.2.16).

Likewise, Fusco and Silber (2014, 850) propose a number of measures that allow for measuring multidimensional polarization combining a nominal and an ordinal scale. In their example they combine self-assessed health (Very good, good, fair, bad, to very bad) and citizenship (yes vs. no). Thereby, the authors borrow indices from the literature on segregation and equality of opportunity and distinguish between a segmentation/stratification approach and an antipodal

[22] Duclos, Esteban, and Ray (2004, 1739) emphasize a problem with their original measure (ER index): It is "based on a discrete, finite set of income groupings located in a cardinal ambient space of possible income values". Conceptually this is problematic since the measure presents an "unpleasant discontinuity" because of its foundation in a population distributed over a discrete/distinct number of points. Practically, it assumed that the population has already been sorted into the relevant groups, an exogenous process. Recommendations on how to do that practically are given in Esteban, Gradín, and Ray (2007). But for aggregated sample data (along income intervals) it is unclear what kind of differences across time/entities represent significant ones.

approach. The former focuses on the overlap of ordinal variables between population subgroups. The latter departs from the idea that polarization is maximal when individuals of a nominal subgroup are all in the same - either the lowest or highest - ordinal category (Fusco and Silber 2014, 841) (see Section 6.2.17).

Further measures combine *nominal-interval/ratio scales*. Permanyer (2012) provides an example with data from the World Values Survey in which religious affiliation is the nominal scale and religiosity the interval scale, that is used to construct a distance measure (see 6.2.14). Permanyer (2012) builds on the RQ index criticizing that it's not formally characterized, i.e. derived from a clear set of axioms and that it fails to capture any variation of the level of alienation that exists between groups (which represents the second dimension). For instance, certain religious groups may be more distant from others, e.g. the distance maybe smaller within Christian and Islamic denominations than across those groups. In other words, it's only defined on the basis of the population-weights that these groups represent.[23] Consequently, Permanyer (2012) suggests to add distance as a second dimension. Permanyer (2012) provides two indices that may be used for a combination of a nominal and an interval scale that are generalizations of the RQ index and incorporate an element of alienation (distance) within and between groups.

Anderson, Linton, and Leo (2012) focus on polarization among countries in terms of GDP per capital (life-time over time). The authors develop two statistics - the *Overlap measure* and the *Trapezoidal measure* - that can discern changes in the underlying distributions of groups (e.g. groups of countries). These statistics reflect combinations of increasing (decreasing) subgroup location differences and decreasing (increasing) subgroup spreads, which are the characteristics of polarization (de-polarization) in many dimensions (Anderson, Linton, and Leo 2012, 53). Concretely, the authors use their measure to gauge polarization between countries grouped according to time (1990, 1995, 200, 2005) and continent (Africa vs. The Rest).[24]

Gigliarano and Mosler (2009) construct multivariate indices of polarization. They consider polarization as the presence of groups which are internally homogeneous, externally heterogeneous, and of similar size. In that Gigliarano and Mosler (2009) follow the second strand in the polarization measurement literature which focuses on identifying groups rather than disappearance of the middle categories. Gigliarano and Mosler (2009, 437) propose polarization indices that are functions of the inequality between groups, the inequality within groups, and the relative groups size. Thereby, the authors present measures of these three elements that are subsequently combined in an index. Gigliarano and Mosler (2009) apply those indices to data

[23] Permanyer and D'Ambrosio (2015, 312)

[24] When the "sub-distributions are not separately identified (by which is meant that the constituents of each group cannot be distinguished or separated from each other) but are embedded in a mixture, the Overlap measure is no longer useful, while fortunately the Trapezoid measure is, provided the mixture is bimodal" (Anderson, Linton, and Leo 2012, 54).

from German socio-economic panel that concerns the bi-variate distribution between income and education.

Another, increasingly used measure departs from the conceptual idea of alignment across scales (e.g. Converse 1964; Coser 1956; Galtung 1966). "[I]f people align along multiple, potentially divisive issues [scales], even if they do not take extreme positions on single issues [scales], the end result is a polarized society" (Baldassarri and Gelman 2008, 409). One of the measures proposed here is the average pair-wise correlation across scales (e.g. used in Baldassarri and Gelman 2008; Munzert and Bauer 2013).[25] Using this measure Baldassarri and Gelman (2008, 419) suggest that 'as the correlation between ideological dimensions [scales] increases, the distance between individuals that belong to the same cluster decreases, while the distance between people that belong to alternative clusters increases'. In principle the *average-pair wise correlation* should then account for both the aspect of dispersion and clustering in a multidimensional distribution.

To conclude, there are various polarization measures that have been developed to measure uni- or multidimensional polarization on different scale types. Often authors provide an example and apply their measures to phenomena such as self-assessed health, income or opinions. Thus, in principle interested researchers can choose from a wide variety of measures and apply them to their data. At the same time few of those measures are easily available for researchers to use and compare them.

5 Conclusion

In this study we pursue the question *How can we conceptualize/measure polarization?* Thereby, I surveyed contributions from the fields of Economics, Political Science and Sociology.

The concept of polarization - due to its general nature - has appeared as early as in the 1920s in the work of Ross. Thereafter, it turns up more or less explicitly in the works of other influential scholars such as Deutsch, Blau, Galtung or Marx. Nonetheless, one rarely finds explicit attempts to define the concept (e.g. Galtung 1996; Esteban and Ray 2012). In principle, however, any formula that aggregates positions on one or more scales and is used as a polarization measure can be regarded as an implicit definition of the concept. Such a formula fixes the number of scales that are used as input and the rules of how values on those scales are aggregated.

[25] DiMaggio, Evans, and Bryson (1996, 696) use Cronbach's alpha as measure of constraint and as an indicator of ideological cohesion. Alpha is defined as: $\alpha_j = \frac{k_j}{k_j - 1}[1 - \frac{\sum \sigma^2_{ij}}{\sigma^2_{yij}}]$ whereas α_j is Cronbach's alpha in country j. k is the number of items on the scale in country j and σ^2_{ij} is the diagonal co-variance for item i in country j. σ^2_{yij} is the sum of the diagonal co-variances and the non-diagonal co-variances for all items in country j.

While scholars discussed the multidimensional nature of the polarization concept early on, the measurement literature started with single scales aka unidimensional polarization.
I started by outlining that conceptualizing and measuring polarization requires choices regarding how many and which kind of scales to incorporate as well as what distributional characteristics to focus on. Naturally, such choices depend on the phenomenon we are studying and one would probably make different choices when studying, for instance, income polarization as opposed to opinion polarization. Accordingly, there is no one-fits-all measure but scholars need choose or develop a measure that is aligned with their conceptual choices. A review of polarization subconcepts emphasizes that the concept is widely applied and that - in applied research - scholars tend to conceptualize/measure polarization differently depending on which kind of phenomenon they study.
The very short (non-exhaustive) overview of measures of polarization reveals a wide variety of measures that can be classified along the choices we outlined before. One of the more important distinctions is whether they are formally derived (based on a set of axioms) or not. From the three disciplines that we inspected more closely here, it seems as if most developments happen within economics, where scholars moved from developing unidimensional measures to multidimensional measures for particular scale combinations. Comparably less development is happening within Sociology and Political Science. In general, it is my impression that scholars are not yet aware of the diversity that exists within the measurement literature. For instance, political scientists seem to have hardly taken note of the developments in economics or at least do not use the corresponding measures developed by economists.
It appears that there are several venues for future research. First, the polarization literature seems to lack a comprehensive methodological assessment and comparison of different polarization measures. Simulation studies that compare different measures are hard to come by. This can probably be explained by the fact that empirical research on polarization is still a relatively young field (as compared to others) and by the fact that data simulation is technically challenging (especially if we want to flexibly produce multidimensional clustered data).
Moreover, the relevance of the polarization concept is often deduced from its relation to concflict and/or non-cooperation. My impression is that this link is a bit undertheorized and may change as a consequence of the phenomenon we study. Think of a scale of ethnic categories. As we mentioned one can make a sound argument that either a distribution with two equally sized ethnic groups leads to most conflicts or a distribution with one group dominating one or several smaller ones. Hence, insofar we see resulting conflict as something that constitutes polarization we may have to rethink our polarization concept and the corresponding measure.
Another aspect that we found is underemphasized in the polarization literature is group size.

One can argue that polarization is a matter of groups so that the contribution of isolated individuals should be negligible (Esteban and Ray 2012, 3). However, in everyday life we often live and interact within small groups, where the role of one or two "isolated" individuals is much more important. Any student that has lived in a shared flat can sing a song about such situations. In my view, group size should be an important aspect in any discussion of polarization. This currently does not seem too be the case.

Finally, it was previously suggested that "[i]n contrast to judging levels of polarization, identifying trends in polarization is an easier task" (Fiorina and Abrams 2008, 567). To me it seems as if both judgments are equally difficult. In themselves, polarization measures are often hard to interpret. For instance, we may use the variance as a polarization measure and find a difference of X between two groups or between two time points. However, insofar our polarization scale does not have a clear interpretation[26] in itself we need to anchor our scale using empirical cases. For instance, a study of polarization trends in the U.S. becomes more meaningful when we can compare U.S. trends to trends from other countries (cf. Munzert and Bauer 2013). This anchoring process and generally the interpretation of differences on polarization scales requires more scholarly attention.

[26] One way would be if we can express changes on our aggregate scale in terms of individual-level changes, i.e. if we would argue that a change of 1 in the variance can be more or less equalled with 100 individuals moving 1 unit further away from the distribution's mean.

6 Appendix

6.1 Software

6.1.1 Measures of polarization

Various R software packages can be used to calculate measures of polarization. The package `agrmt` (Ruedin 2016) contains functions to calculate agreement or consensus in ordered rating scales. The package `acid` (Sohn 2016) contains the functions `polarisation.ER` (Esteban & Ray 1994) and `polarisation.EGR` (Esteban, Gradin and Ray 2007) to calculate the respective polarization indices. The package `ineq` (Zeileis and Kleiber 2014) contains a set of inequality measures. The Herfindahl index can be computed relying on various packages, e.g. the `hhi` function in the `hhi` package (Waggoner 2018).

6.1.2 Data simulation

Simulating undimensional clustered data is fairly easy. The challenge lies in simulating both unidimensional and multidimensional data that has certain characteristics not only in terms of variance (the idea of extremeness as polarization) but also in terms of clustering. Various R package contain functions to simulate (clustered) data for varying dimensions. The package `clusterSim` contains the function `cluster.Gen` to generate random clusters “with known structure of clusters (optionally with noisy variables and outliers) (Walesiak and Dudek 2017). The package GenOrd contains the function `ordsample` to”Simulation of Discrete Random Variables with Given Correlation Matrix and Marginal Distributions" (Barbiero and Ferrari 2015). The package `MixSim` was programmed for simulation of “Data to Study Performance of Clustering Algorithms” (Melnykov, Chen, and Maitra 2017). The package `clusterGeneration` contains the `genRandomClust` function to “[g]enerate cluster data sets with specified degree of separation” (Qiu and Joe 2015). Besides R one can also simulate clustered data relying on Python (Source) (Mingle 2018)

6.2 Overview of some polarisation measures

Several articles provide helpful summarizes of polarization measures (e.g. Zhang and Kanbur 2001). Below we provide a non-exhaustive overview.

6.2.1 Variance and Kurtosis

Variance and kurtosis are among the simplest of polarization measures and often used to measure the concept on single scales.

The variance (sample):

$$v(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

The kurtosis (Wikipedia contributors 2017b):

$$\mathrm{K}URT(x) = \frac{\mu_4}{\sigma^4} = \frac{\mathrm{E}[(X-\mu)^4]}{(\mathrm{E}[(X-\mu)^2])^2},$$

where μ_4 is the fourth central moment and σ is the standard deviation. The kurtosis can be calculated relying, e.g. relying on the `kurtosis` function in the `moments` package (Lukasz Komsta 2015). The variance can be calculated using the `var` function in the `stats` package

The use of both measures to gauge polarization has been criticized (see e.g. Downey and Huffman 2001)

6.2.2 Ethnolinguistic Fractionalization Index (ELF) and HHI

The ethnolinguistic fractionalization index (Alesina et al. 2003) is normally calculated as 1 - Herfindahl-Hirschman Index (HHI) (cf. Hirschman 1964, 1980; Herfindahl 1950):

$$ELF_j = 1 - \sum_{i=1}^{N} s_{ij}^2$$

, where s_{ij} is the share of group i (i = 1...N) in country j.

Conceptually, the ELF gives the probability that two randomly selected individuals drawn from a given population will not belong to the same group. Analogue the HHI gives the probability that two individuals drawn from a given population will belong to the same group.

The Herfindahl index can be computed relying on various packages, e.g. the `hhi` function in the `hhi` package (Waggoner 2018).

6.2.3 Gini coefficient

The Gini coefficient (Gini 1921, @Gini1912–bg) is the most used measure of inequality and a measure of statistical dispersion. While not a measure of polarization itself it is strongly related measures such as by Esteban and Ray (1994). There are various formulas to calculate the Gini and it is usually defined mathematically based on the Lorenz curve. Another way is to "consider the Gini coefficient as half of the relative mean absolute difference, which is a mathematical equivalence.[...] The mean absolute difference is the average absolute difference of all pairs of items of the population, and the relative mean absolute difference is the mean absolute difference divided by the average, to normalize for scale." (Wikipedia contributors 2017a; Sen 1973 or 1977 second edition):

$$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|}{2\sum_{i=1}^{n}\sum_{j=1}^{n}x_j}$$

However, by neglecting (giving equal weight to) "the population frequency in each category, inequality measurement departs from the study of differentiation [polarization]" (Esteban and Ray 1994, 821). From a latter perspective "the formation of categories or groups, or 'subsocieties,' the population frequency in each category also [should also carry] carries weight" (Esteban and Ray 1994, 821). However, "the axioms of inequality measurement (or equivalently, second-order stochastic dominance for mean- normalized distributions) fail to adequately distinguish between"convergence" to the global mean and "clustering" around "local means." (Esteban and Ray 1994, 821)

6.2.4 Leik 1966

Leik (1966, 86) suggests a measure of ordinal dispersion that is defined as follows:

$$D = \frac{\sum_{i=1}^{i} d_i}{MAX\sum_{i=1}^{i} d_i} = \frac{2\sum_{i=1}^{i} d_i}{m-1}$$

where m is the number of categories of the ordinal scale. d_i is a difference that is defined as

$$d_i = \begin{cases} F_i \text{ if } F_i \leq 0.50 \\ 1 - F_i \text{ otherwise} \end{cases}$$

where F_i is the relative frequency distribution. D is "free of sample size, number of choice options, central tendency, and assumptions about intervals between choice options; yet it accurately reflects the degree to which choices are spread over the set of options available. Furthermore, because the measure is a sum divided by its maximum possible value, D is a percentage, hence a ratio scale variable" (Leik 1966, 86).
Leik (1966, 87) suggests that a simple measure of *ordinal consensus* would be $D - 1$. Relatedly, a measure of *interval consensus* is:

$$\text{Interval Consensus} = 1 - \left(\frac{2\sigma}{m-1} \right)^2$$

Leik (1966) contrasts the two measures and suggests that "Ordinal Consensus appears to be preferable as a measure of consensus to one based on variance" (Leik 1966, 88).[27]

See function`Leik` in package `agrmt` (Ruedin 2016).

6.2.5 Esteban and Ray (1994): ER Index

Esteban and Ray (1994) formally derive a polarization measure for discrete distributions.[28] The formula is below:

$$P(\pi, \mathbf{y}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i \pi_j \mathbf{T}(\mathbf{I}(\pi_i), a(\delta(y_i, y_j)))$$

Conceptually, P depends on *identification* $\mathbf{I}$ which is a function of individual i's group size π_i. Identification reflects the idea that an individual feels a sense of identification with others in the group, a sense that increases with the number of individuals in this group (reflected by π_i). P also depends on *alienation* $a(\delta(y_i, y_j))$ where $\delta(y_i, y_j)$ simply describes the absolute distance between two individuals i and j. The *effective antagonism* is given by a cardinal function $T(\mathbf{I}, a)$

[27] Leik (1966, 88) also points to the problem of statistical significance as without "assumptions about the intervals, allowing algebraic derivation of, e.g., a moment generating function, there seems to be no way to derive sampling variability of the index".

[28] "According to the authors polarization measurement starts with Esteban and Ray (1991), Esteban, Gradin and Ray (1998,2007) and Duclos, Esteban and Ray (2004)" (Esteban and Ray 2012)

that combines both identification and alienation. The choice of $\mathbf{I}(\dot{})$, $a(\dot{})$ and $\mathbf{T}(\dot{})$ will yield a particular measure of polarization:

$$P^*(\pi, \mathbf{y}) = K \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i^{1+\alpha} \pi_j |y_i - y_j|$$

where K is a multiplicative constant that is used for normalization, and α restricts identification function to the form p^α where $\alpha > 0$. As α goes to zero P approximates the Gini coefficient (defined on log incomes). π is a vector of shares and $\mathbf{y}$ is a vector of means. y_i and y_j are the means in the respective groups and π_i and π_j are the proportions of observations in those groups.

6.2.6 Blair and Lacy 2000: d^2

d^2 is a measure of dispersion for discrete data:

$$d^2 = \sum_{i=1}^{k-1} (F_i - 1/2)^2$$

where F_i is the cumulative frequency for the ith category; that is $F_i = \sum_{j=1}^{i} p_j$ with p_j representing the sample proportion for the jth of the k categories (Blair and Lacy 2000, 255–56).

6.2.7 Wolfson 1994: W Index

Wolfson (1994) departs from the observation that the quite common notion of the "disappearing middle class" is typically associated with the idea of increasing income inequality. At the same time conventional scalar measures of inequality do not reflect the idea that observations disappear from the middle of a distribution (cf. Levy and Murnane 1992). Consequently, Wolfson (1994, 356) suggests that "summary measures based on concepts like polarization should be given equal space along with Lorenz-consistent inequality measures when describing trends in income distribution" as it may be "closer to the general public's vernacular concept of inequality".

The index by Wolfson (1994) is derived from the Lorenz curve and may be written as follows (see Zhang and Kanbur 2001, 89):

$$W = 2(2T - Gini)/(m/\propto) = 2(\propto^* - \propto^L)/m$$

where $T = 0.5 - L(0.5)$ and $L(0.5)$ denotes the income share of the bottom half of the population; m is the median income; $\propto$ is the mean income; $\propto^*$ is the distribution-corrected mean income which is given by the actual mean times (1-Gini); and $\propto^L$ is the mean income of the bottom half of the population.The maximum polarization occurs when half the population has zero income and the other half has twice the mean (see also Wolfson 1997, and @Foster2010–zx).

Software wise the index can be computed relying on the `polar.aff` in the `affluenceIndex` package (Alicja Wolny-Dominiak 2017).

6.2.8 Wang Tsui 2000: WT Index

Wang and Tsui (2000) generalize a new class of indices (TW index hereafter) based on the Wolfson index using the two partial ordering axioms of increased bipolarity and increased spread. The formula is (see Zhang and Kanbur 2001, 89):

$$TW = \frac{\theta}{N} \sum_{i=1}^{K} \pi_i \left| \frac{y_i - m}{m} \right|^r$$

where N is the number of total population, π_i is the number of population in-group i, K is the number of groups, y_i is the mean value in group i, and m is the median income. θ is a positive constant scalar and $r \in (0, 1)$. Zhang and Kanbur (2001, 89) set r to 0.5.

6.2.9 Apouey 2007

Apouey (2007) suggest two polarization indices for ordinal scales. On ordinal scales all the information is given "given either by the distribution of cumulative numbers $N = (N_1, ..., N_{C-1}, N_C)$ or by that of cumulative proportions $F = (F_1, ..., F_{C-1}, 1)$, two indices of polarization are conceivable: one is a function of $N;, P_1(N)$, and the other one is a function of $F, P_1(F)$." (Apouey 2007, 881). The formulas are as follows:

$$P_1(N) = K_1 N_C^{\alpha} \left[\left(\frac{1}{2} \right)^{\alpha} - \frac{1}{C-1} \sum_{c=1}^{C-1} \left| F_c - \frac{1}{2} \right|^{\alpha} \right]$$

$$P_2(F) = K_2 \left[\left(\frac{1}{2} \right)^{\alpha} - \frac{1}{C-1} \sum_{c=1}^{C-1} \left| F_c - \frac{1}{2} \right|^{\alpha} \right]$$

where K_1 and K_1 are strictly positive constants and $\alpha > 0$.

6.2.10 Reynal-Querol 2001/2002: RQ Index

The polarization index by Reynal-Querol (Reynal-Querol 2001, 2002; Jose G Montalvo and Reynal-Querol 2005) is a measure of polarization for nominal scales. The RQ can be written as (cf. Montalvo and Reynal-Querol 2010, 127):

$$RQ = 1 - \sum_{i=1}^{N} \left[\frac{0.5 - \pi_i}{0.5} \right]^2 \pi_i$$

, where π_i is the proportion of individuals in each (ethnic) group and N is the number of (ethnic) groups. The RQ can also be written as

$$IRC1 = \sum_{i=1}^{N} f(\pi_i)$$

and thus critically depends on the properties of f which reaches a minimum for $\pi = 1/6$ and a maximum for $\pi = 1/2$ and is convex for $\pi < 1/3$ and concave for $\pi < 1/3$ concave for $\pi > 1/3$ (see Figure 1). Montalvo and Reynal-Querol (2010, 128) discuss differences of the RQ index to measures such as the ELF and the ER.

6.2.11 Duclos, Esteban and Ray (2004): DER Index

Duclos, Esteban, and Ray (2004) formally derive a measure that can be used for cases in which a (income) distribution can be described by density functions.

The measure is the following:

$$P_a(f) = \int \int f(x)^{1+\alpha} f(y) |y - x| dy dx$$

6.2.12 Eiik 2001: Agreement on ordered rating scale

Van Der Eijk (2001) proposes a measure of agreement for ordered rating scales.

$$A = U \cdot \left(1 - \frac{(S-1)}{(K-1)} \right)$$

where S is the number of non-empty categories, and K is the total number of categories in the rating scale.

"As the original empirical distribution is decomposed into layers, the degree of its agreement can be described as the weighted average of the agreement in each of its constituent layers (from here on to be distinguished by an index i), where the proportions of cases contained in the layers are used as weights":

$$A = \sum_i w_i \cdot A_i$$

The function `agreement` in the package `agrmt` (Ruedin 2016) contains functions to calculate agreement or consensus in ordered rating scales.

6.2.13 Dalton 2008

Party system polarization by Dalton (2008, 906). Underlying data is the average perceived party position (average across all individuals in a country) on the left-right scale and the party vote share.

$$PI = \sqrt{\sum (\text{party vote share}_i) * ((\text{party L/R score}_i \text{ˇ party system average L/R score})/5)^2}$$

Golder (2010) uses a multiplier of 1.6 instead.

6.2.14 Permaneyer 2012

Permanyer (2012) introduces two measures that are somewhat related to the RQ index.

$$P^b_{N,a}(f) = \sum_{i=1}^{N} \sum_{j \neq i} \pi_i^{1+\alpha} \pi_j (\mu_i + \mu_j)$$

where $\alpha \in (0, 1]$ and μ_i is the mean value of the radicalism distribution $f_i(x)$. Here alienation is a between group phenomenon only. Moreover, when $\alpha = 0$ then $P^b_{N,a}(f) = \sum_{i=1}^{N} \sum_{j \neq i} \pi_i \pi_j (\mu_i + \mu_j)$ which can be regarded as some sort of weighted fractionalization index (Permanyer 2012, 53).

Permanyer (2012, 58) also introduces an index that accounts for within group distances.

$$P_{N,a}(f) = \sum_{i=1}^{N} \int \int f_i^{1+\alpha}(x) f_i(y) |x - y| dy dx + \sum_{i=1}^{N} \sum_{j \neq i} \int \int f_i^{1+\alpha}(x) f_i(y) |x + y| dy dx$$

where $\alpha \in [\frac{1}{3N-2}, 1]$.

6.2.15 Apouey and Silber 2013

Apouey and Silber (2013) develop measures of polarization on two ordinal scales (e.g. health and socio-economic status). There approach generalizes previous uni-variate approaches (Abul Naga and Yalcin 2008; Apouey 2007; Reardon 2009). The formula is as follows:

$$B_1 = 1 - \frac{\left(2\sum_{h=1}^{H}\sum_{y=1}^{Y}|F_{h,y} - 0.5|\right) - 1}{HY - 1}$$

where $F_{h,y}$ are the elements of the observed cumulative probabilities from which the elements of a maximal bi-polarized matrix (= 0.5) are subtracted. H are the indices of the categories of the first variable, Y of the second variable.
They also consider a second measure that is:

$$B_2 = \frac{1}{HY - 1}\sum_{h=1}^{H}\sum_{y=1}^{Y} 4F_{h,y}(1 - F_{h,y})$$

which is an extension of (Reardon 2009).

6.2.16 Permanyer and D'Ambrosio 2015

Permanyer and D'Ambrosio (2015) also develop a measure of polarization on a nominal and an ordinal scale (cf. Fusco and Silber 2014, 847). The formula is as follows:

$$AP = \sum_{i=1}^{I}\sum_{h=1}^{I} S_i^{1+\alpha} S_h (1 - \theta_{ih})$$

where $\alpha = 0.71$ and $\theta_{ih} = \sum_{j=1}^{K} Min(p_{ij}p_{hj})$ where the subscripts i and h refer to unordered population subgroups, I being the total number of such groups and j to the various health categories, J being the total number of such categories (Permanyer and D'Ambrosio 2015).

6.2.17 Fusco and Silber 2014

Fusco and Silber (2014) propose two measures of multidimensional social polarization. The concrete example they use is self-assessed health (SAH) - first dimension - among immigrants

and non-immigrants - second dimension. The first focuses on the overlap of the distributions of different subgroups (segmentation or stratification approach). The second approach lays the emphasis on bi-polarization and "considers that the social polarization of SAH will be maximal if the individuals belonging to a given population subgroup are in the same health category but this category corresponds either to the lowest or to the highest health status" (Fusco and Silber 2014, 842). The series of indices is shown below:

$$POLOR^1 = 1 - \frac{\sum_{i=1}^{I} s_i \tau_i^1}{\tau^1}$$

$$POLOR^2 = 1 - \frac{\sum_{i=1}^{I} s_i \tau_i^2}{\tau^2}$$

$$POLOR^3 = 1 - \frac{\sum_{i=1}^{I} s_i \tau_i^3}{\tau^3}$$

$$POLOR^4 = 1 - \frac{\sum_{i=1}^{I} s_i \tau_i^4}{\tau^4}$$

$$SP^{\alpha} = \frac{2^{\alpha}}{(J-1)} \sum_{i=1}^{I} s_i \sum_{j=1}^{J-1} |P_{ij} - P_j|^{\alpha} \forall \alpha > 0$$

$$SP^{Gini} = \sum_{i=1}^{I} \sum_{h>i}^{I} s_i s_h \sum_{j=1}^{J-1} |P_{ij} - P_{hj}|$$

6.3 The concepts of cleavage and polarization

The concept of "cleavage" can be regarded as a conceptual relative of "polarization" and research relevant to the idea of polarization still often carries the label "cleavage" today. The cleavage concept seems to go back to Lipset and Rokkan (1967) who investigated "when the one type of cleavage will prove more salient than the other, what kind of alliances they have produced and what consequences these constellations of forces have had for consensus-building within the nation-state" (Lipset and Rokkan 1967, 6). Lipset and Rokkan (1967) use the term "cleavage" to refer to "conflict groups based on perceptions of association in opposition to other such groupings among large segments of a population" (Zuckerman 1975, 234). In their conception

cleavages develop in the social realm and are politicized when they develop into large scale conflicts and become tied to political parties (Zuckerman 1975, 234). In their seminal work Lipset and Rokkan (1967, 35) differentiate between four cleavages: center-periphery, state-church, land-industry and owner-worker and explain the contrasts between Western party systems by national historical developments of these cleavages. According to the authors, especially the first three are responsible for national developments in divergent directions.[29]
Independently from this key study, the concept of "cleavage" has remained vague and ambiguous in the contemporary literature (Zuckerman 1982, 1975). Scholars reacted to this ambiguity by extending the concept through qualifications such as political, cultural, institutionalized or non-institutionalized cleavages - a development that withdrew considerable theoretical value from the concept itself (Bartolini and Mair 2007, 198). To reverse this trend, Bartolini and Mair (2007) make another attempt in defining the concept. According to the authors, the concept of "cleavage" incorporates three levels: "[A]n *empirical* element, which identifies the empirical referent of the concept, and which we can define in socio-structural terms; a *normative* element, that is the set of values and beliefs which provides a sense of identity and role to the empirical element, and which reflect the self-consciousness of the social group(s) involved; and an organisational/behavioral element, that is the set of individual interactions, institutions, and organisations, such as political parties, which develop as part of the cleavage" (Bartolini and Mair 2007, 199). Subsequently, the term "cleavage" should be restricted to the "indication of a dividing line in a polity which refers to and combines all three aspects" (Bartolini and Mair 2007, 200).
Some scholars relieve the "cleavage" concept from its strong tie to social structural variables (in contrast to Bartolini and Mair 2007, 198). For instance, Dahl (1966, 48ff) defines it in terms of differences in political attitudes and behavior (cf. Zuckerman 1975, 235). Similarly, Rae and Taylor (1970, 1, 23) conceive "cleavage" as "the criteria which divide the members of a community or sub-community into groups with important political differences at specific times and places" and differentiate between *trait cleavages* such as race or caste, *attitudinal cleavages* such as ideology or preference and *behavioral cleavages* such as those elicited through voting or organizational membership (Rae and Taylor 1970, 1).
When we restrict the concept of "cleavage" to attitudes as (partly) does Dahl (1966) and Rae and

[29]Moreover, the authors established the famous "freezing hypothesis" stating that the "the party systems of the 1960's reflect, with few but significant exceptions, the cleavage structures of the 1920's" (Lipset and Rokkan 1967, 50). They argue that the basic differences between party systems go back to the early phases of competitive politics and emerged before the final phase of mass mobilization. This is also the reason why the fourth cleavage (owner-worker) proved to be of less relevance (Lipset and Rokkan 1967, 35). Methodologically, the authors base these insights on a systematic overview of the comparative history of partisan oppositions in European polities up to the 1920s.

Taylor (1970) with the term “attitudinal cleavage” we are essentially speaking about divisions in public opinion. However, even in this latter understanding, *cleavage* is a dichotomous concept with individuals of a society located on either side of the division line, the “cleavage”. More generally, the cleavage concept with its deadlocked dimensions of owners vs. workers, church vs. state, urban vs. rural and center vs. periphery looks slightly outdated with regard to the increasingly difficulty to divide publics into these categories.

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