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CONTINUOUS OPINION DYNAMICS ON AN ADAPTIVE COUPLED RANDOM NETWORK

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Abstract

An agent-based model for opinion dynamics on an adaptive coupled random network is proposed. Based on Festinger's idea of "cognitive dissonance", in the proposed model an agent can either make opinion exchange with a neighbor according to the bounded confidence mechanism, or migrate towards another network position in case that the majority of the adjacent agents are beyond the confidence bound. Through numerical simulations, we test how the key factors, such as the interconnectivity of the two communities, the confidence bound or the communal tolerance to diversity, the initial distributions of the opinions, and the level of sense of community, affect the final opinion state of the system. The overall analyses show a general picture of the dynamics of opinions on an adaptive network with community structure. In particular, the results reveal that the clustering of similar agents has a bifurcating function for the opinion dynamics. Given that the inter-communal influence is high, the clustering fosters the global consensus; if the inter-communal influence is weak, the clustering would instead intensify polarization and thus hinder the formation of global consensus. The factors

of the communal tolerances and interconnectivity leverage the bifurcating effect.

Keywords: Continuous opinion dynamics, coupled random network, agent-based model, tolerance to diversity

1. Introduction

In the last decades, the studies on “opinion dynamics” have attracted enormous attention as a key branch of “social dynamics” (6), since the spreading of attitudes and opinions in society is critical for many social processes, e.g. the outbreak of political protests, the diffusion of innovations, the formation of fashions and fads, and the occurrence of stock-market bubbles and crashes. Generally speaking, the key theme of opinion dynamics is to study how the overall state of opinions evolves in a population through local interactions between individuals and among small groups. Therefore, as argued in (31), an opinion-dynamic model can inherently be characterized by three elements, namely the representation of opinions, the local rules of interactions, and the topological structure of the population. These three elements also provide a basic framework to review the past progresses of this research field and to foresee its future developments.

In the aforementioned three elements, the first two specify the microscopic models of opinion evolution in a population, irrespective of how the population is structured. In terms of their representation of opinions, the opinion-dynamic models can largely be divided into three categories, i.e. discrete models (in particular binary models) such as the voter model (2) and the Sznajd model (29), the continuous models such as the Deffuant model (7) and the Hegselmann and Krause (HK) model (16), and the vector models such as Axelrod’s culture dissemination model (1) and the vector extension of the HK model as specified in (10). The second key element of the opinion-dynamic models is the local interaction rules. Such interaction rules mimic the real-world mechanisms of social influence and opinion change. For example, the mechanism of homophilous interaction adopted in Deffuant’s “bounded confidence” model and the Axelrod model is conceptually rooted in the social comparison theory of social psychology (8); the interaction rule of the Sznajd model coincides with the well-discussed “conformity” phenomenon (23); and the majority rule models (18; 11) directly copy the notion of “majority rule” from political and decision sciences (21). To sum up, in

terms of the microscopic models, plenty of models have been investigated and fruitful results have been obtained. However, the existing models are often criticized for their oversimplification in mapping the actual human processes of opinion change and their insufficiency to cater for real-world situations. Consequently, a challenging direction for future research is to establish better linkage between the model and the reality (28). This furthermore demands more scholarly attention on models which are grounded on the well-examined social-psychological theories on persuasion and social influence (30) and can explain real-world phenomena better.

The third element is about the social structure in which the opinion dynamics take place. In this respect, we can see a trend that the research attention gradually moves from the well-mixed populations and regular lattices to “complex networks”. More recently, the opinion-dynamics on adaptive networks, or in other words, the co-evolution of opinion and network, has also come to the fore (14; 24; 17). These researches on the co-evolution of opinion and network may activate another key direction for the future developments of this research field, as the bi-directional influences between the dynamics on and of the social-network are common in real-world situations (15). Especially, in online social networks, the virtual social links are more-easily to establish and to break; and the proximity of ideas and opinions is a key factor in maintaining the links. Specific attention is deserved on the co-evolutionary dynamics of network and opinion in cyberspace.

This paper focuses on the latter thread of the aforementioned research directions, namely, the opinion-dynamics on adaptive networks. More specifically, the opinion-dynamics on adaptive modularized networks is concerned, as real-world social networks often exhibit modular structure of sparsely-interlinked communities(25). In literature, the opinion dynamics on such modular networks have been studied in a few endeavors, e.g. the Majority Rule model on coupled fully-connected graphs (20) and coupled random graphs (19), the cultural dissemination model on coupled random networks (5), and the Sznajd model on networks with community structures (27). However, the studies on opinion dynamics on the adaptive modular networks are largely inadequate. An exception is the work presented in (13; 12), in which Gargiulo and Huet developed a bounded-confidence model that embraces Festinger’s idea of “cognitive dissonance”(9) so that an agent either makes opinion exchange with a neighbor or moves towards another community. They found a lower threshold for consensus in their model than that in the original Deffuant model. Their contribution shows a good example for

endeavors in this direction; but further investigations are still needed.

Conceptually coincident with Gargiulo and Huet’s work, we in this paper propose another agent-based model upon the idea of “cognitive dissonance”. In the proposed model, agents make opinion exchanges and move on a modular network that is comprised of two communities. We are to examine how the factors such as the tolerance to opinion diversity, the interconnectivity of the two communities, the initial opinion distribution, and the “sense of community” influence the evolution of opinions as well as the change of sizes of the two communities. By analyzing our model, we identify an evolutionary pattern similar with that in Gargiulo and Huet’s model. Meanwhile, the results of the two models differ in a number of ways. For example, we find that the critical “confidence-bound” threshold to consensus in our model is between the critical thresholds in the original Deffuant model and in Gargiulo and Huet’s model. As a complement to the prior researches, this work partially reveals the richness of the opinion dynamics in adaptive social networks. Setting the real-world background as opinion spreading in today’s fast-growing online communities, the major aim of this work is to shed some light on the mechanisms of opinion evolution on modular online-social-networks (OSN) (4).

2. Model Description

We set the online communities as the real-world background of our model. Today the online communities are prominent due to the rapid growth of social-computing technologies such as instant messaging, bulletin board systems (BBS), social networking sites(SNS), and microblogging services. One social application, e.g. a BBS forum or an SNS site, can be regarded as an online community and each user can join multiple communities. However, as each individual’s attention is inherently limited she can actively participate in a limited number of communities. In this work the simplest case is taken into account. It is therefore assumed that there are two communities and each individual dwells in a single community although she may also contain sparser external links towards the other community. What’s more, owing to the fact of high mobility of the participants in online communities, in the proposed model we assume the agents communicate with other agents and move between the two communities in accordance with the idea of “cognitive dissonance”.

Based on the prior assumptions, we can describe the proposed model as follows. In the model, N agents or nodes are split into a couple of communities, which are represented as C_A and C_B . Each community is initialized as a random ER network which contains $N/2$ agents and L_{in} internal edges. L_{out} edges are then randomly assigned to connect the two communities. Thus, the initial network is a coupled-random-network or CRN (19) which contains $2L_{in} + L_{out}$ edges; and the interconnectivity or the density of cross-community connections (denoted by v) can be defined as $v = L_{out}/(2L_{in} + L_{out})$. Furthermore, the opinions of the agents in each community are initialized as real values in the range of $[0.0, 1.0]$, following a specific distribution. Based on this initialization, the overall process of the proposed model can be specified below.

2.1. Overall process of the proposed model

In the previously-described network, the overall dynamics of opinion and network are based on two local procedures, i.e. the procedures for inter-agent opinion-exchange and agent migration in the network. In our model, both procedures rely on a concept of “tolerance to diversity”, which is denoted as a real value in the range of $(0.0, 1.0)$. Two values of such “tolerance”, namely ε_A and ε_B , are respectively assigned to the two communities C_A and C_B . Each value respectively measures the corresponding community’s overall attitude towards the markedly-different opinions. Two agents would be “incommunicable” if the opinion difference between them is beyond the tolerance threshold of the belonging community. Thus, opinion exchange and assimilation would become difficult in the case of low tolerance. For inter-agent opinion exchange, the original Deffuant model is adopted and the “tolerances to diversity” serve as the “confidence bounds” to leverage the process of opinion averaging. For agent migration, the “tolerance” values are used to gauge an agent’s satisfaction level in its current neighborhood. The agent would be “unsatisfied” if the majority of the neighbors become “incommunicable”. In such case, the agent tends to move towards another position in the network to alleviate the “cognitive dissonance” (9).

In this spirit, an iterative process is implemented in the suggested agent-based model, after the network and the opinions of the agents are initialized. At each time step of this process, an agent, which is denoted as agent i , is arbitrarily selected in the network as the focal agent. According to its “satisfaction” to the atmosphere in the neighborhood, the focal agent can either make opinion exchange with a neighbor or move towards another position

in the network. The atmosphere, which is dependent on the tolerance to diversity of the belonging community, is defined as the fraction of the focal agent's adjacent agents that is communicable with the focal agents. This atmosphere can be formulated by Eq. (1):

$$w = \frac{\#\{j \in \Gamma_i \mid |o_i - o_j| \leq \varepsilon\}}{\#\Gamma_i} \quad (1)$$

In Eq.(1), Γ_i represents the set of adjacent agents of the focal agent i . $\#\Gamma_i$ represents the total number of the adjacent agents. ε is the tolerance of the community in which the focal agent resides. o_i is agent i 's opinion. Thus, $\#\{j \in \Gamma_i \mid |o_i - o_j| \leq \varepsilon\}$ denotes the number of the adjacent agents that are communicable with the focal agent. We use a simple measure for the focal agent's satisfaction. In case of $w > 0.5$, the focal agent would be "satisfied" in that the majority of its neighbors are communicable and its decision is to stay at the same position and to exchange opinion with one randomly selected neighbor. Otherwise, the focal agent decides to move to another position.

The proposed model would repeat the preceding actions, until both the network structure and the opinion distribution are absorbed into a stationary state, or the rounds of iteration reach the pre-defined upper-limit T_{max} .

The agents' activities of opinion-exchange and migration are at the center of the preceding process. Next we describe these two mechanisms in more detail.

2.2. Opinion exchange mechanism

When $w > 0.5$ in Eq. (1), the focal agent i would conduct opinion exchange with an agent j that is arbitrarily-selected in the focal agent's neighborhood, obeying the "Opinion Exchange Mechanism" as described below:

- (ii) The opinions of i and j update with respect to Eq. (2), if one of the following two conditions is satisfied:
 - (b) In case of $j \in C_A$, the opinion difference between the two agents is within the tolerance threshold of C_A , i.e., $|o_i - o_j| \leq \varepsilon_A$;
 - (b) In case of $j \in C_B$, the opinion difference between the two agents is within the tolerance threshold of C_B , i.e., $|o_i - o_j| \leq \varepsilon_B$.
- (iii) Otherwise, the opinions of i and j keep unchanged.

The opinion averaging process (Eq. (2)) is formulated as follows:

$$\begin{aligned} o_i(t+1) &= o_i(t) + \phi(o_j(t) - o_i(t)) \\ o_j(t+1) &= o_j(t) + \phi(o_i(t) - o_j(t)) \end{aligned} \tag{2}$$

Where $\phi \in (0, 0.5)$ is the speed coefficient.

2.3. Neighborhood adjustment mechanism

The focal agent is unsatisfied with its current position, if $w \leq 0.5$. In this case, the agent moves towards a new position in the network, in accordance with the subsequent “Neighborhood Adjustment Mechanism”:

- (ii) With probability p the focal agent conducts edge-rewiring within the same community, i.e. removing an existing intra-community edge that is connected to an agent with the most different opinion in the focal agent’s neighborhood and establishing a new-edge to a random non-neighbor in the same community.
- (iii) With probability $1 - p$, the focal agent migrates to the opposite community by completely re-configuring the focal agent’s existing links. Suppose the focal agent is located in C_A , and it contains m intra-community links and n inter-community links. Then it marks itself as a member of C_B ; and it removes all the existing links and arbitrarily establishing m links to agents in C_B and n links to agents in C_A .

In this mechanism, the focal agent’s movement is leveraged by the parameter p , which partly reflects the “sense of community” (22). When p is high, the agents have high level of sense of community and they have high loyalty to the belonging community. Otherwise, the agents are more likely to move towards the other community as the “sense of community” is low.

In summary, the proposed model is conceptually based on Finstinger’s cognitive dissonance theory (9), as an agent tries to mitigate the “cognitive dissonance” by either changing the opinions to improve the coherence in the neighborhood or adjusting the social relations. This idea furthermore coincide with Gargiulo and Huet’s model (13). The major difference between the two models is twofold. First, in our model, the agent’s satisfaction is based on the opinion difference against its direct neighbors, while the opinion distance to the average opinion of the whole belonging community is compared in Gargiulo and Huet’s model. Thus, we emphasize more on a “niche” for

communication. Correspondingly, the judgment of agent’s satisfaction or comfort is based on the simple measure whether the majority of its neighbors are communicable, following the same idea of Schelling’s segregation model (26). Second, the agent’s movement is always across the community boundary in Gargiulo and Huet’s model. In comparison, the intra-community migration is an option in our model, as we attempt to examine in the impact of the “sense of community” or “community attachment”. With similarities in the conceptual basis and differences in model setting, our work complements Gargiulo and Huet’s efforts on opinion dynamics on adaptive modular networks.

3. Numerical Experiments and Result Analyses

With the above-described model, computational simulations are conducted to explore the possible patterns for the co-evolution of network and opinions. The parameters for running the simulations are specified below.

For the network size, we basically run the numerical simulations under $N = 200$ and $G = 2$. Thus, there are initially two communities and each contains 100 agents or nodes. For the density of edges, we set a fixed value of $L_{in} = 500$, and let L_{out} range from 10 to 250. Correspondingly, the interconnectivity ranges from about 0.01 to 0.2. Thirdly, we set the initial distribution of the agents’ opinions. In this work, two cases of the initial opinion distribution are taken into account, namely the unbiased and biased initial distribution. For the case of unbiased distribution, we suppose the opinion distribution obeys the uniform distribution between 0.0 and 1.0 for the agents in both communities at time 0, i.e., $o_{i \in A}(0) \sim U(0, 1)$ and $o_{i \in B}(0) \sim U(0, 1)$. For the case of biased distribution, we suppose the opinions in each community approximate a normal distribution but the distributions of the two communities have different mean and variance values, i.e. $o_{i \in A}(0) \sim N(\mu_A, \sigma_A)$ and $o_{i \in B}(0) \sim N(\mu_B, \sigma_B)$. In order to manifest the differences in the opinions of the two communities, we let $\mu_A < 0.5$ and $\mu_B > 0.5$. To note, we practically use approximate-normal-distributions to make sure all opinions are in the range of $[0.0, 1.0]$, by truncating the generated opinion-values that are out of the range.

Based on the preceding parameter setting, agent-based simulations are executed to explore the patterns of opinion evolution on such adaptive modular network. As in common with many opinion-dynamic models, in our model the evolution of opinions can generally end up with three possible

stationary phases, namely, symmetry or “consensus”, asymmetry or “polarization”, and disorder or “fragmentation”. The actual absorbing state is prominently affected by a few key parameters of the proposed model, in particular, the interconnectivity (v), the communal tolerances (ε_A and ε_B), the initial distribution of opinions, and the strength of community attachment (p). Subsequently, we are to examine how these key parameters affect the evolution of opinions. We carry out our inquiry in two steps. First, it is assumed that all the agent movements are across the community boundary, by letting $p = 0.0$. Under such setting, we test how the other key parameters except for p affect the opinion evolution. The influence of the strength of community attachment is examined at the second step by letting the p value varies.

3.1. Overall result and the effect of interconnectivity

Under the condition of $p = 0.0$, we first examine the effects of the interconnectivity and the communal tolerances on the absorbing state of opinions. As described above, we let the interconnectivity range from around 0.01 to 0.2 and conduct numerical experiments under different initial opinion-distributions and different tolerance levels. For the simplicity of discussion, we in this subsection consider the symmetric communal tolerances by letting $\varepsilon_A = \varepsilon_B = \varepsilon$. As for the initial opinion-distributions, both biased and unbiased distributions are taken into account. We in this subsection attempt to portray an general picture of the overall opinion dynamics and to analyze the effect of interconnectivity in particular.

In case of unbiased initial opinion-distributions, where the initial opinions obey the uniform distribution in both communities, the numerical experiments show a somehow unanticipated result that the interconnectivity does not exhibit significant influence on the absorbing state of the opinions. The stationary state of the opinions is mainly influenced by the tolerance levels of the two communities. Instead, when the initial opinion-distribution is biased, the influence of the interconnectivity is more apparent. Therefore, we place the focus on the case of biased initial opinion-distribution. We let the initial opinions in the two communities approximate the normal distribution ($o_{i \in A}(0) \sim N(\mu_A, \sigma_A)$ and $o_{i \in B}(0) \sim N(\mu_B, \sigma_B)$), and set the expectations as $\mu_A = 0.2$ and $\mu_B = 0.8$, and the standard deviations as $\sigma_A = \sigma_B = 0.3$. The phase diagram of the entire population’s final stationary opinions under these settings are illustrated in Fig. 1.

The upper part of Fig.1 is the contour plot of the opinion phases in the v - ε parameter space. The contour plot is drawn with respect to the average data from 100 realizations at each (ε, v) point. Using the number of the remaining opinions as the order parameter, we can basically differentiate three stationary phases, namely “fragmentation”, “polarization” and “consensus”. In the upper part of Fig.1, Regime *I* corresponds to the “fragmentation” phase, in which more than three incommunicable opinions finally coexist in the population. Regime *II* corresponds to the “polarization” phase, in which two opposite opinions survive at the final stage. The simulations show that, in this polarization phase, the opinions of all members of C_A are converged into 0.25, as the average of numerous realizations; and the agreed opinion of C_B is 0.75 on average. This reveals that the entire population are split into two opinion groups although an agreement is locally reached in each community. Regime *III* corresponds to the “consensus” phase, in which the opinions of the entire population are eventually agreed at 0.5 as the average of numerous realizations. The above results of the consensus at 0.5 and the polarization at 0.25 and 0.75 are consistent with the results observed in Deffuant et al.’s original bounded confidence model (7). Besides the previous three basic phases, it is interesting to note that there exists another transition-phase or mixed-phased between “polarization” and “consensus”, which is denoted as Regime “*II-III*” in the upper part of Fig.1. In this phase, the final absorbing opinion-state can be attracted to either “polarization” or “consensus” in two independent realizations under the same initial condition. This transition-phase will be further analyzed in the next subsection.

Furthermore, the boundaries of the opinion-phases are to be identified. With the increase of the parameters v and ε , the final state of the system gradually moves from “fragmentation” to “polarization” and then to “consensus”. As shown in Fig.1, the interconnectivity (v) doesn’t have significant influence on the boundary between the fragmentation and polarization phases. For all the levels of the interconnectivity ranging from 0.01 to 0.2, the phase transition from fragmentation to polarization would occur at around $\varepsilon = 0.182$. This phase-transition point of ε is quite stable in numerous realizations. In most realizations, the critical value is at 0.18; and in a few realizations, the value can reach 0.2. The critical value is averaged at $\varepsilon = 0.182$ under all different interconnectivity values. This result is consistent with Gargiulo and Huet’s work (13). The boundary between “polarization” and “consensus” is blurry due to the existence of the mixed-phase *II-III*. We therefore use the zone of Phase *II-III* to denote the “polarization-consensus” boundary. The

interconnectivity has more apparent influence on this boundary. In the case that the interconnectivity is at the level of 0.01, the phase transition occurs at around $\varepsilon = 0.43 \sim 0.46$. At this interconnectivity level, the dynamics of opinion evolution are close to Deffuant et al.’s original model. On the other side, in the case that the interconnectivity level is relatively high, the consensus is more easily to form. For example, if the interconnectivity is at 0.2, the corresponding phase transition would take place at around $\varepsilon = 0.3 \sim 0.37$. In general, this “polarization-consensus” boundary fluctuates irregularly from realization to realization. Correspondingly, the averaged results shown in Fig.1 exhibit a zigzagged area, revealing an indistinct boundary between polarization and consensus.

The three fundamental regimes are exemplified by the diagrams in the lower part of Fig.1, which illustrate typical final states of actual realizations under high ($v=0.167$) and low ($v=0.01$) interconnectivity levels. Diagrams (1), (3), and (5) are examples for the final states under $v=0.167$, indicating that the final state gradually moves from “fragmentation” ($\varepsilon = 0.1$) to “polarization” ($\varepsilon = 0.3$) and then to “consensus” ($\varepsilon = 0.4$). In comparison, diagrams (2) and (4) shows the final states under lower interconnectivity ($v=0.01$). From the illustrations we can see that the “consensus” is more difficult to obtain in case of low interconnectivity. As shown in diagram (4), the final state is still at the “polarization” regime when $\varepsilon = 0.4$.

The above-described results are based on simulations under the setting of $N=200$ and $L_{in} = 500$. To test the robustness of the above results, we execute additional simulations under different node-sizes and link-densities. First we examine the cases of different node sizes by keeping the density of links in each community constant. Thus, we respectively examine the cases of $N=100, 400, 500$, and 800 nodes, setting the corresponding amount of intra-community links $L_{in}=125, 2000, 3150$, and 8000 . In all these testing cases, the density of intra-community links is set to 10% of the fully-connected graph of the same size. Furthermore, by fixing the node-size at $N=200$, we examine the cases of increasing link densities at $L_{in}=1000, 1500, 2000$ and 4950 (where each community becomes fully-connected). In all the previous settings of N and L_{in} , we run the simulations under different v - ε pairs. The results of a good number of simulations show that the identified boundaries are quite robust. In other words, for different node-sizes and different link-densities within a community, the boundaries between the “fragmentation” and “polarization” phases and the regimes of the mixed “polarization-consensus” phase are statistically coherent to what are shown in Fig.1. These additional experiments

give a partial validation for the robustness of the above-described results.

To sum up, the numerical experiments reveal that the interconnectivity plays a vital role for boosting the convergence of opinions, given that the initial opinion-distributions are biased. Lower the interconnectivity is, the system is more likely to be attracted into the fragmentation phase. At an extremely-low level of the interconnectivity, the consensus can only be reached when the communal tolerance or “confidence bound” approaches 0.5, which is the critical point to consensus in the original bounded confidence model. From another aspect, the numerical simulations also show that the communal tolerance is positively related to the convergence of opinions. At each interconnectivity level, the stationary opinions are more likely to converge with the increase of the parameter ε . This phenomenon is basically in line with the previous bounded confidence models, as analyzed in (3) and (13). However, the effect of this parameter deserves further examinations. In next subsection, we pay more attention on the effect of communal tolerances, especially taking the possible asymmetry of the two communal tolerances into account.

3.2. Effect of communal tolerances

In the preceding subsection we have shown that the communal tolerances have a prominent effect on the final opinion state. In this subsection the effect of the communal tolerances is further examined under a fixed interconnectivity level at $v = 0.167$ or $L_{out} = 200$, letting the two tolerances independently range from 0.0 to 0.5 so that the two tolerances can be asymmetric. The numerical experiments are respectively executed under the biased and unbiased initial opinion-distributions.

The basic results of numerical simulations are plotted in Fig.2, which are based on the average data of 100 independent realizations at each pair of ε_A and ε_B ranging from 0.0 to 0.4. We omit the results under $\varepsilon_A > 0.4$ and $\varepsilon_B > 0.4$ in that the consensus of the stationary opinions can be confirmed under such parameter settings. In Fig.2, the left part or part (A) illustrates the contour plot of the stationary opinion-phases under the biased initial opinion-distributions ($o_{i \in A}(0) \sim N(0.2, 0.3)$, $o_{i \in B}(0) \sim N(0.8, 0.3)$), while the right part or part (B) refers to the contour plot of the stationary opinion-phases under the unbiased initial opinion-distributions ($o_{i \in A}(0) \sim U(0, 1)$, $o_{i \in B}(0) \sim U(0, 1)$).

An interesting phenomenon shown in Fig.2 is the existence of the mixed or transition phase between “polarization” and “consensus”, which has been mentioned in the preceding subsection. This phase is labeled as Regime *II-III* in both Fig.1 and Fig.2. In this mixed phase, for two realizations under the same initialization, the opinions can be absorbed into two different phases, i.e., either polarization or consensus, with different probabilities. Closer to the consensus regime in the parameter space, the probability to reach consensus becomes higher; on the contrary, the polarization is more likely to be reached when the parameters are closer to the polarization regime. Such mixed-phase indicates a bifurcating process attributed to the existence of two attractors of polarization and consensus. This bifurcating process can be inspected by tracking the dynamic processes of actual realizations.

Fig.3 shows the time-charts of two realizations starting from an exactly-identical initialization. The results exemplify the existence of the two-attractor phase of polarization and consensus. Both states of polarization and consensus shown in Fig.3 are the stationary states. In Chart (B), the consensus is surely a stationary opinion-state. For the case of Chart (A), the two predominant opinions at 0.25 and 0.75 are incommunicable as the difference between them is larger than the given tolerance 0.34. Therefore, this polarization state would also be a stationary state. In all the realizations in Regime *II-III*, the bifurcation to either polarization or consensus can be observed.

Another noteworthy issue is whether the asymmetry of the tolerances of the two communities influences the final opinions. As shown in Fig.2, if the asymmetry is nonsignificant, the overall pattern of opinion dynamics is similar with the case of symmetric tolerances. However, with the increase of the asymmetry level, Regime *II* gradually diminishes and the area of fragmentation or Regime *I* expands. In particular, Regime *II* would vanish and the stationary opinion phase leaps from fragmentation to consensus with the increase of the tolerances, if $|\varepsilon_A - \varepsilon_B| > \Delta\varepsilon^*$. $\Delta\varepsilon^*$ is around 0.25 as imprecisely estimated from Fig.2.

This phenomenon is essentially correlated with the dynamics of community sizes over the asymmetry level of the two tolerances. The basic observation is that the community with higher tolerance to diverse opinions would attract more members. Thus, when the asymmetry level is high, the sizes of the two communities would also be highly asymmetric. The amount of the remaining members of the “intolerant” community would be small. Correspondingly, the opinions of this community would not have significant influence on the final opinions of the entire population.

In order to examine how the difference of the two tolerances affects the community sizes, we respectively set $\varepsilon_B = 0.1, 0.2, 0.3, 0.4$ and obtain four sequences of the final sizes of C_A under different ε_A values, which are sampled at the interval of 0.02 (i.e. $\varepsilon_A = 0.02, 0.04, \dots, 0.4$, respectively). The results are shown in Fig.4. To plot the four curves shown in Fig.4, we use the average data of 40 realizations at each ε_A value. Possibly due to the relatively-small set of samples, the curves in Fig.4 are not so smooth. But they unambiguously uncover the correlation between the final size of C_A and the difference between ε_A and ε_B . Starting from the size of 100 or one half of the entire population, the final size of C_A is solely determined by the value of $\varepsilon_A - \varepsilon_B$. If $\varepsilon_A \approx \varepsilon_B$, the final size of C_A is around 100. If ε_A is apparently less than ε_B , the final size of C_A would also be much smaller than 100. On the contrary, the final size of C_A would be apparently greater than 100 in case that ε_A is much larger than ε_B . What's more, we also test the final sizes of the two communities under other parameter settings. All the simulations show that the difference of the two tolerances is the predominant factor for the final sizes of the two communities. This result, together with the corresponding opinion dynamics, demonstrates a basic pattern for the co-evolution of network and opinion in the proposed model.

3.3. Effect of the initial distributions

Subsequent to the analyses on the effects of interconnectivity and communal tolerances, we examine the effect of the initial opinion-distributions. In this respect, the absorbing states of opinions under the biased and unbiased opinion-distributions are firstly compared in accordance with Fig.2. Two major differences can be observed for the final opinion-states under the biased and unbiased initial opinion-distributions. First, the polarization phase is actually different in the two cases. In the case of biased initial distributions, the opinions in C_A are absorbed into 0.25, while the opinions in C_B are absorbed into 0.75. In the case of unbiased initial distributions, however, it is arbitrary which community converges to 0.25 and which converges to 0.75. Correspondingly, in the mixed-phase or Regime II-III in Fig.2, there are actually three possible stationary phases in the case of unbiased initial distributions, namely 0.25 for C_A and 0.75 for C_B , 0.5 for both communities, and 0.75 for C_A and 0.25 for C_B . In contrast, if the initial distributions are biased, there are only two possible stationary phases, i.e. consensus at 0.5 and polarization at 0.25 for C_A and 0.75 for C_B .

The second difference can also be identified by comparing the two parts of Fig.2. The basic observation is that, in the case of the biased initial distributions, Regime *I* or “fragmentation” is smaller and simultaneously Regime *II-III* and Regime *III* (i.e. consensus) are larger. The smaller fragmentation regime and larger consensus regime under the unbiased-distributions reveal that the unevenness of the initial opinions is a factor to boost the convergence of final opinions. This result is counterintuitive at the first glance since one may consider the initial biases would hinder the formation of global consensus. This puzzle would be explained by further speculation that the initial biases foster the local convergence of opinions in the same community. Then, the opinion-influences through the inter-communal links may furthermore impel the locally-converged opinions to merge into the global consensus. The larger regime of the mixed-phase *II-III* reveals that the uneven initial distributions make the boundary between consensus and polarization blurry. This phenomenon can also be explained by the local-convergence process. The local-convergence triggers the bifurcating global process. The local-convergence may on one hand lead to the premature polarized opinions so as to block further inter-community influences, if the inter-community influence is relatively weak comparing with the local-convergence. This generates “polarization”. On the other hand, when the inter-community influences are prevailing, the local-convergence can instead foster the global consensus. This bi-directional function of local-convergence eventually enlarges the transition-phase.

Next we consider the influence of the degree of initial bias or unevenness. To do so, we generate multiple approximately-normal-distributions with different expectations (μ_A and μ_B) and standard deviations (σ_A and σ_B) as the initial opinion-distributions in the two communities. The expectation of an initial opinion-distribution reflects the prejudice of the community’s average opinion. If the expectation value is close to 0.5, the community’s average opinion is temperate. In contrast, the expectation value that is close to 1.0 or 0.0 implies the extremism of the community. The standard deviation stands for the steepness of the “prejudice”. When the deviation is low, the agents’ opinions are densely aggregated around the expectation value; and the opinions would be more dispersed in case of the higher deviation. Numerical simulations reveal that the expectations don’t have significant influence on the stationary states of the opinions. In comparison the deviation values have more significant influence on the final opinion states. Our observation is that the deviation or “steepness of prejudice” mainly affects the likeliness

to reach either consensus or polarization in the mixed phase (i.e. Regime *II-III* in Fig.1).

As an example, Fig.5 shows the fractions of realizations that are absorbed into polarization and consensus under different standard deviations, setting the other parameters as $\mu_A = 0.2$, $\mu_B = 0.8$, $v = 0.167$ and $\varepsilon_A = \varepsilon_B = 0.34$. The result reveals that with the increase of the σ value, the fraction of realizations to reach the consensus ascends from 0% to 80%, while the fraction to reach the polarization descends from 100% to 20%. In this sense, when the standard deviations of the two communities are low, the opinions of the entire population are most probably absorbed into the polarization state; instead, when the deviations are high, the consensus is more likely to occur. This indicates that the high steepness (i.e. the low deviation) is a factor that hampers the formation of consensus.

3.4. Effect of the sense of community

In the previous investigations, we assume all the agent movements are across the boundary of the communities by setting $p = 0$. In this subsection the effect of “sense of community” is then tested by letting p range from 0.0 to 1.0. As previously-described, high p implies high community attachment. We can therefore examine how such “sense of community” or “community attachment” influences the final network and opinions. For the network structure, it is obvious that the sizes of the two communities are more likely to be even for higher p or higher “sense of community”. For the final opinion state, the simulations reveal that the high “sense of community” is a factor that boosts the formation of global consensus.

Similar with the effect of the degree of the initial opinion-bias, the “sense of community” mainly affects the probability of consensus formation in the region of the mixed phase. One may imagine that the high mobility of agents can impel the opinion exchange across the communal boundary and hence boost the convergence of opinions. Nevertheless, the actual numerical simulations show the opposite trend. As shown in Fig.6, when the “sense of community” is high, the final opinions are more likely to be absorbed into consensus. A reasonable explanation is that the high mobility of inter-communal movements causes the aggregation of agents with similar opinions and meanwhile enlarges the opinion gap between the two communities, as the inter-communal movements of the agents inherently constitute a homophilous linking process and this process propels polarization.

3.5. Short comparison with Gargiulo and Huet's model

It is interesting to further compare the phase-transition boundaries in our model with those in Gargiulo and Huet's model (13), owing to the intrinsic similarity between the two models.

First, consider the phase boundary between fragmentation and polarization. In Gargiulo and Huet's model, the critical threshold for this phase-transition is around 0.182. This result is very similar with the critical tolerance levels in our model. As shown in Fig.1, in the case of symmetric tolerances (i.e. $\varepsilon_A = \varepsilon_B$), the phase boundary is also at around 0.182 for all different interconnectivities ranging from 0.01 to 0.2. Under asymmetric tolerances, as shown in Fig.2, the boundary between fragmentation and consensus is not so stable with the variance of the two tolerances. But the two phases are roughly divided by a line that links around (0.1, 0.3) and (0.3, 0.1) in the $\varepsilon_A - \varepsilon_B$ parameter space, when the interconnectivity is around 0.167.

For phase transition between polarization and consensus, the critical threshold is fixed at $\varepsilon = 0.267$ as specified by Gargiulo and Huet in their paper. However, in our model, the boundary between polarization and consensus is a zonal area that is significantly influenced by the interconnectivity v . If we use the same criterion as in Gargiulo and Huet's work to regard the 50% fraction of realizations with more than one opinion clusters as the phase-transition point, the critical threshold is around $\varepsilon_A = \varepsilon_B = 0.32$ if $v = 0.2$. If $v = 0.01$, the critical tolerance threshold would be around $\varepsilon_A = \varepsilon_B = 0.42$. This result is quite different from that in Gargiulo and Huet's model. As a matter of fact, we find that the interconnectivity have apparent effect on the phase-transition between polarization and consensus. In case that the inter-communal links are sparse, the phase-transition from polarization to consensus is similar with that in Deffuant et al.'s original model (7), as the critical value is close to 0.5. On the contrary, with denser inter-communal links, the critical tolerance threshold becomes close to the critical confidence-bound in Gargiulo and Huet's model. Generally speaking, in our model, the critical threshold is between that in the original Deffuant model and Gargiulo and Huet's model. A possible explanation for this phenomenon is that the existence of the homophilous communities prevents the occurrence of isolated agents and small clusters, as same as in Gargiulo and Huet's model; on the other side, the smaller amount of the communities in our model makes the threshold larger than that in Gargiulo and Huet's model. In all, this result

demonstrates the richness of interesting phenomena in opinion dynamics on adaptive modular networks.

4. Conclusion

In this work, an agent-based model, which is based on the idea of cognitive dissonance, is developed and analyzed. A few interesting results have been obtained from numerical simulations, as described in previous sections. In all, the above-mentioned results can be combined into an overall portrait for opinion dynamics in an adaptive network with community structure.

In general, the community structure of the network affects the final state of opinions by fostering local convergence of opinions. Furthermore, such local convergence of opinions has a bifurcating effect on the absorbing opinion-state. In case that the locally-converged opinions are communicable with one another, such local convergence boosts the global consensus of opinions, in that the existence of the communities hinders the formation of minor clusters and isolated agents. On the contrary, in case that the inter-communal influence is weak so that the local convergence in each community is independently reached, the local convergence may instead lead to the global polarization. The agent movements intensify the local convergence, as the basic process of agent movements is to foster the clustering of agents with similar opinions and to disconnect the dissimilar agents. The factors of the communal tolerances and interconnectivity leverage the bifurcating effect of the community-structure on the stationary opinion-state. The high interconnectivity ensures the high probability for opinion exchange between the two communities. Thus the majority of opinions in the two communities would influence each other. Such mutual influence boosts the global convergence of the opinions. The tolerances to diverse opinions have a straightforward effect on the final opinion state, since high tolerance boosts opinion convergence. Furthermore, the effects of the initial opinion-distributions and the degree of community attachment are also examined in this work.

The above experiments and analyses also imply a few issues for future work. First, for the simplicity of analyses, we in this work just consider the case that the network is comprised of two communities. It is worthwhile to extend our model to study opinion dynamics on an adaptive network with multiple communities. Second, in this work, we mainly adopt the simulative approach to examine the opinion dynamics. It would be a promising research issue to pursue an analytical result for the observed phenomena. Finally, as

pointed out in the model-description section, the real-world background of our model is the online communities. The results obtained in this paper can partly explain the highly-biased attitudes and opinions in the online communities. A further subject of inquiry is to examine the influence of mass media on the opinion dynamics on such modular networks. Our ongoing investigation is to study whether and how the mass media in one community can influence the opinions in another community that is loosely-connected to the first one. This would potentially have practical value in, for example, marketing, to examine how the advertisements in one community can be penetrated into another one in the context of social media.

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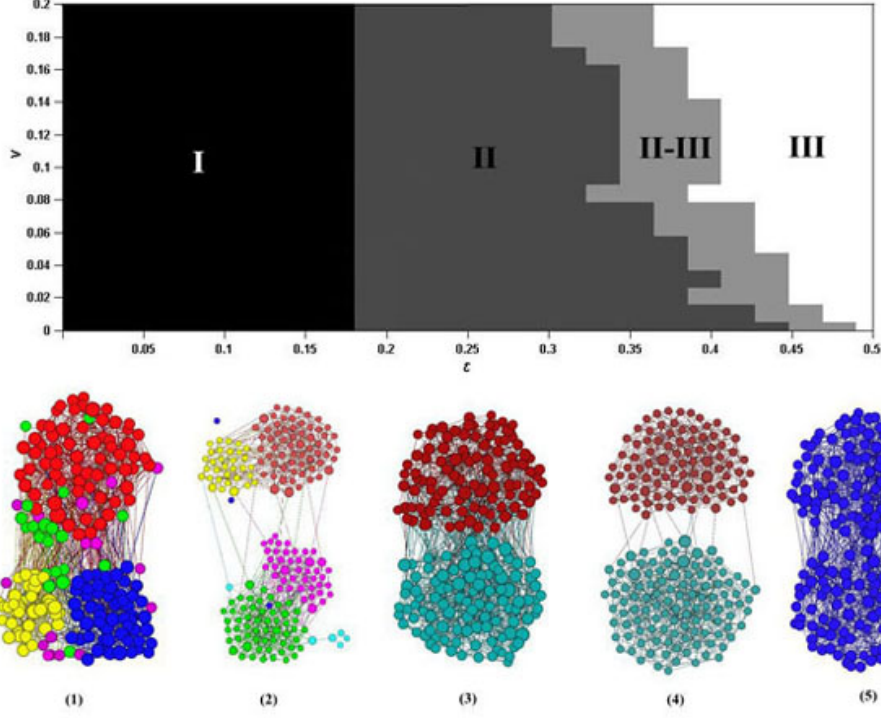


Figure 1: (Color online) The final opinion phases in the $v - \varepsilon$ parameter space. The upper part shows the contour plot of the final opinion distributions. With the number of the final opinions being the order parameter, four phases are identified through numerical experiments. Regime *I* corresponds to the fragmentation phase; Regime *II* corresponds to the polarization phase; Regime *III* corresponds to the consensus phase; and Regime *II-III* corresponds to the mixed phase between polarization and consensus. The lower part illustrates the results of typical realizations under different parameters, with the opinions of the nodes represented by different colors or grayscale: (1) fragmentation under $L_{out} = 200$ ($v = 0.167$), $\varepsilon = 0.1$; (2) fragmentation under $L_{out} = 10$ ($v = 0.01$), $\varepsilon = 0.1$; (3) polarization under $L_{out} = 200$ ($v = 0.167$), $\varepsilon = 0.3$; (4) polarization under $L_{out} = 10$ ($v = 0.01$), $\varepsilon = 0.4$; (5) consensus under $L_{out} = 200$ ($v = 0.167$), $\varepsilon = 0.4$.

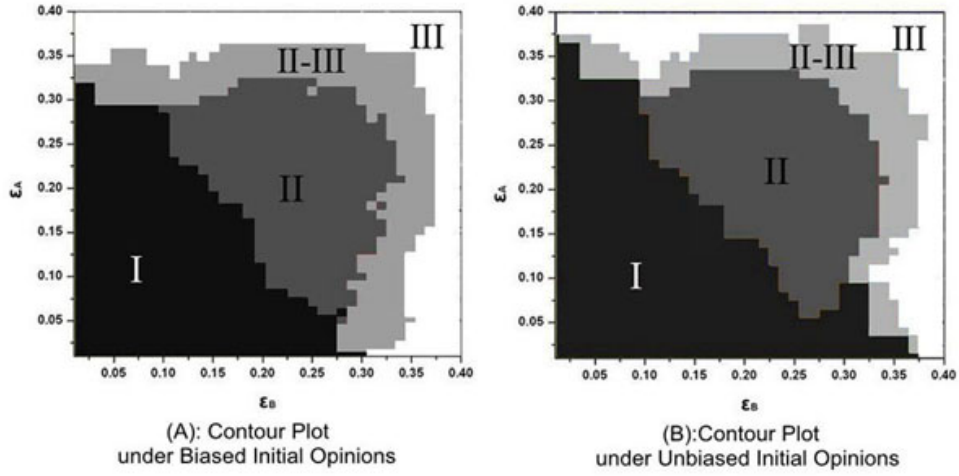


Figure 2: The final opinion phases in the ε_A - ε_B parameter space. (A) Contour plot of stationary phases under the biased initial opinion-distributions ($o_{i \in A}(0) \sim N(0.2, 0.3)$, $o_{i \in B}(0) \sim N(0.8, 0.3)$). (B) Contour plot of stationary phases under the unbiased initial opinion-distributions ($o_{i \in A}(0) \sim U(0, 1)$, $o_{i \in B}(0) \sim U(0, 1)$). In both cases, $N=200$, $L_{in} = 500$, $L_{out} = 200$. Four phases can be identified, with the number of remaining opinions being the order parameter. “I” corresponds to “fragmentation”; “II” corresponds to “polarization”; “III” corresponds to “consensus”; and “II-III” corresponds to the mixed phase of polarization and consensus.

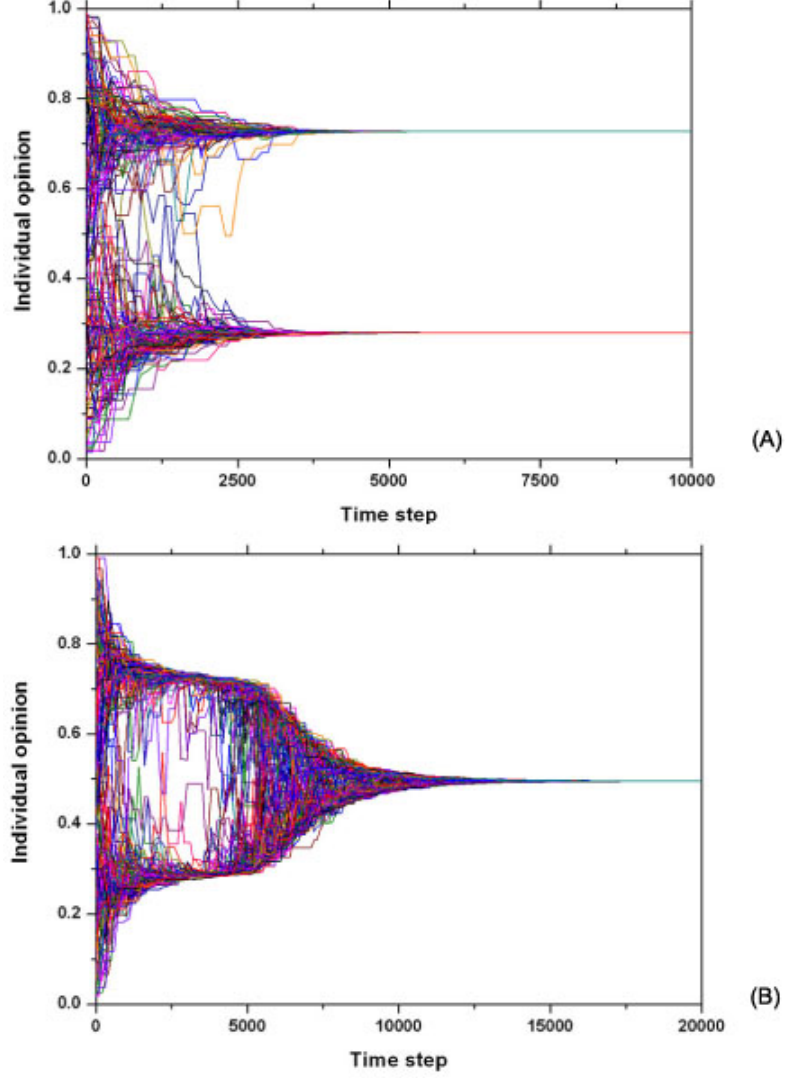


Figure 3: (Color online) The time-charts for two independent realizations under an identical initialization. The basic parameters are $N=200$, $L_{in} = 500$, $L_{out} = 200$, and $\varepsilon_A = \varepsilon_B = 0.34$. The initial opinions of the population are generated by approximating $o_{i \in A}(0) \sim N(0.2, 0.3)$ and $o_{i \in B}(0) \sim N(0.8, 0.3)$. The identical initial-opinions are used in the illustrated two realizations. The results show that in one realization the opinions of the population are split into two values at around 0.25 and 0.75 as shown in chart (A), and in another realization the consensus at round 0.5 is reached as shown in chart (B).

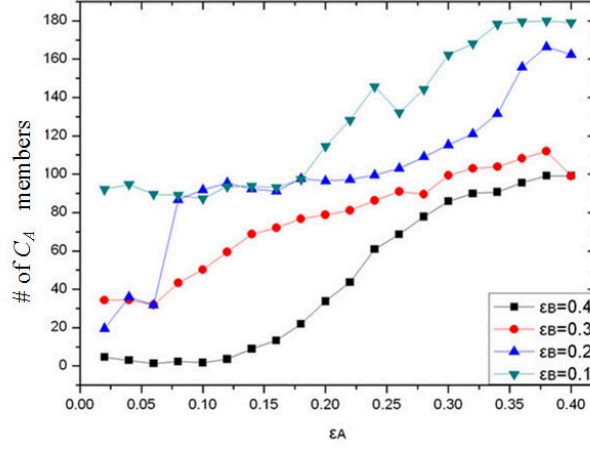


Figure 4: (Color Online) The final size of community C_A as a function of ε_A under different ε_B values. Curves are based on average data of 40 realizations at each ε_A value, which is sampled at the interval of 0.02. The other parameters are set as $N=200$, $L_{in} = 500$, $L_{out} = 200$, $o_{i \in A}(0) \sim N(0.2, 0.3)$ and $o_{i \in B}(0) \sim N(0.8, 0.3)$. The results show that the final size of C_A is determined by the difference of the tolerances of the two communities.

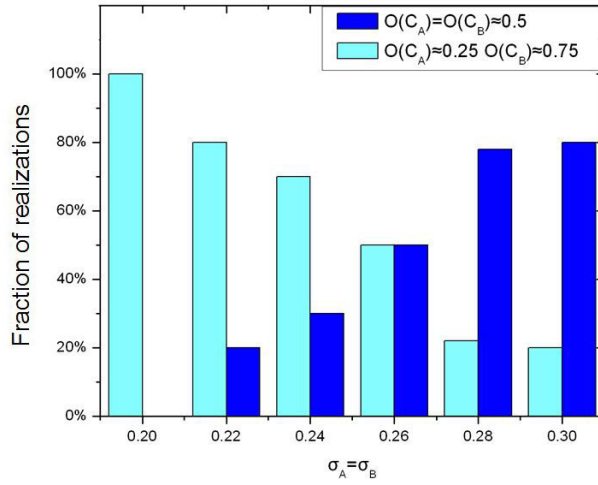


Figure 5: The effect of the degrees of initial opinion-biases on the final opinion-states. The histogram depicts the fractions of realizations to reach consensus (dark or blue columns) and polarization (light or cyan columns) under different deviation values of the initial approximately-normal-distributions. The parameters are set as: $N=200$, $L_{in} = 500$, $L_{out} = 200$, $\varepsilon_A = \varepsilon_B = 0.34$, $o_{i \in A}(0) \sim N(0.2, \sigma_A)$ and $o_{i \in B}(0) \sim N(0.8, \sigma_B)$. $\sigma = \sigma_A = \sigma_B$ ranges from 0.2 to 0.3.

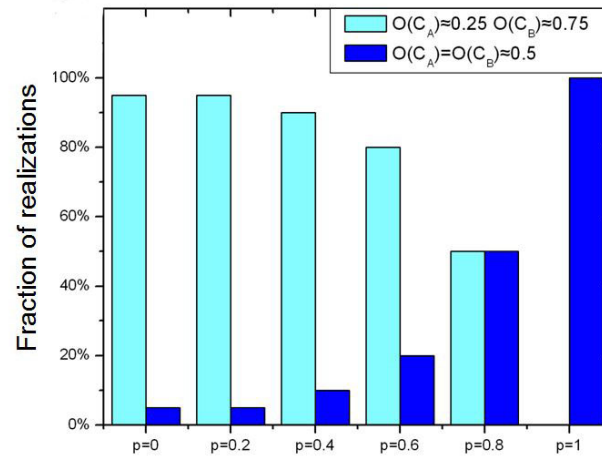


Figure 6: The fractions of realizations to reach consensus (dark or blue columns) and polarization (light or cyan columns) under different “sense of community” levels or p values. The basic parameters are: $\mu_A = 0.2$, $\mu_B = 0.8$, $\sigma_A = \sigma_B = 0.3$, $v = 0.167$, $\varepsilon_A = \varepsilon_B = 0.34$)