Chaos

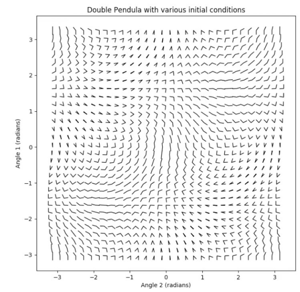
Complex behavior, arising in a deterministic nonlinear dynamic system, that exhibits two special properties:

- sensitive dependence on initial conditions
- characteristic structure…

Systems that exhibit chaos are ubiquitous; many of them are also simple, well-known, and "well-understood"

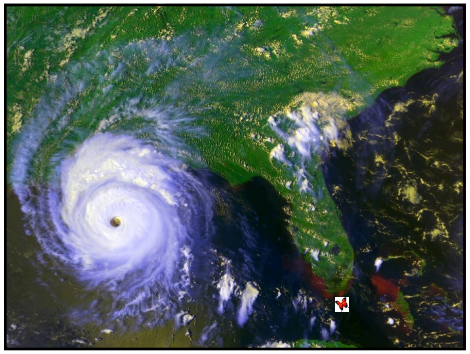
NB: I will post these slides on the course webpage

1



https://jnafzig.github.io/2018/02/05/double-pendulum.html

2



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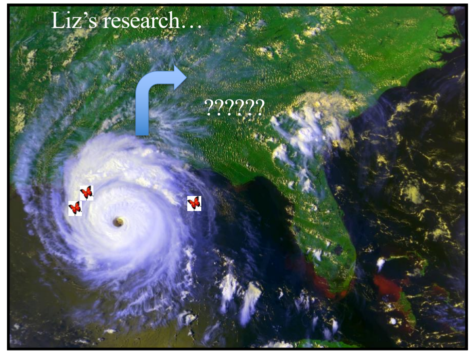

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Rhymes with Orange
31 January 2022

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What this has to do with computer science

- Dynamic(al) systems* are best described by ODE models
- *Chaotic* ODEs, **by definition**, cannot be solved in "closed form"
- Rather, they are *nonintegrable*
- So you have to solve them numerically—with an ODE solver like Euler's method or Runge-Kutta
- Before the advent of widely available programmable digital computers, this was a real pain, so the field only really took off in the 1970s.
- The computer is the laboratory for this field!! "Experimental mathematics"

*finite dimensional ones, that is…

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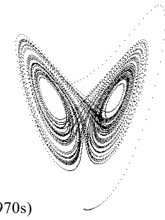
Some history:

- Huygens (late 1600s)
- Poincare (late 1800s)
- Lorenz (early 1960s)
- Yorke (early 1970s)
- Chaos Cabal at UCSC (mid 1970s)
- Wisdom (early 1980s)
- Strogatz (late 1980s)



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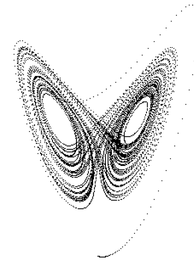

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Where nonlinear dynamics turns up

- Flows (of fluids, heat, …)
  - Eddy in creek
  - Weather
  - Vortices around marine invertebrates
  - Air/fuel flow in combustion chambers

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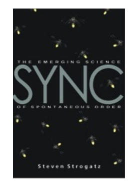
Where nonlinear dynamics turns up

- Driven nonlinear oscillators
  - Pendula
  - Hearts
  - Fireflies



- and lots of other electronic, chemical, & biological systems

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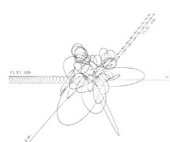
Where nonlinear dynamics turns up

- Classical mechanics
  - three-body problem
  - paired black holes
  - pulsar emission
  - ….
- Protein folding
- Population biology
- And many, many other fields (including computers!!)



Hut & Bahcall *Ap.J.* ***268****:319*

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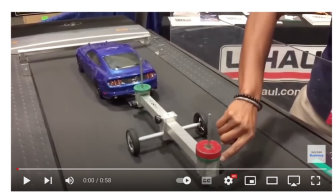
Bifurcations



Towing a trailer can be dangerous with the wrong weight distribution

https://www.youtube.com/watch?v=6mW_gzdh6to

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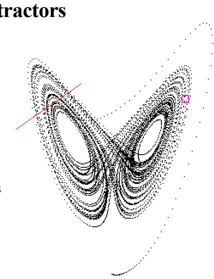
"Strange" or chaotic attractors



- *often* fractal
- covered densely by trajectories
- exponential divergence of neighboring trajectories…

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Courtesy of Mike Neuder

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A useful metaphor:

(image from wikipedia)

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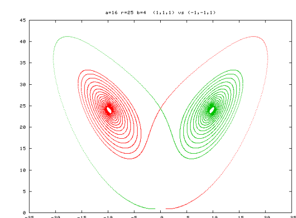

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- discrete time systems:
  - time proceeds in clicks
  - "maps"
  - modeling tool: differ*ence* equation
- continuous time systems:
  - time proceeds smoothly
  - "flows"
  - modeling tool: differ*ential* equations

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Apply now to Study Abroad!

Feb 1 – March 15 application deadlines for:

- Summer '23
- Fall '23
- AY '23-24 programs

Study, intern, or research abroad

Engineers Abroad Website:

- Find recommended programs by major & term
- Access pre-approved course list
- Explore & register for Global Engineering Minor
- Link to CU's Education Abroad website, including scholarship info

Request advising appointment: andrew.wingfield@colorado.edu

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Newton's method on $x^4 - 1 = 0$

From Strogatz

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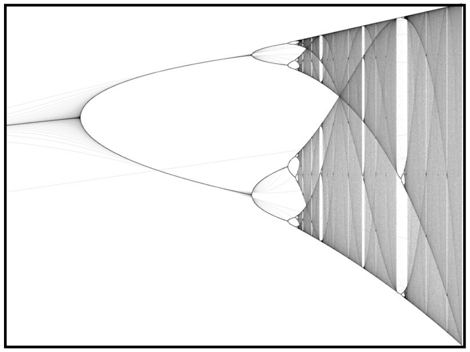

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- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- period-doubling cascade @ low R
- windows of order within the chaos, complete with their own period-doubling cascades (e.g., 3 to 6 to 12)
- *small copies of object embedded in it (fractal)*

(lots of other interesting stuff, too — e.g., Misiurewicz points)

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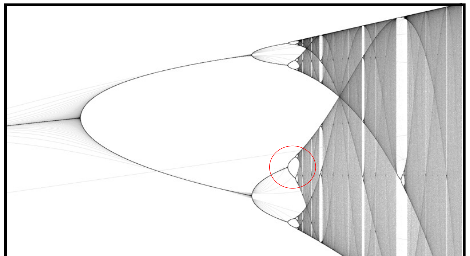

We'll come back to fractals later today

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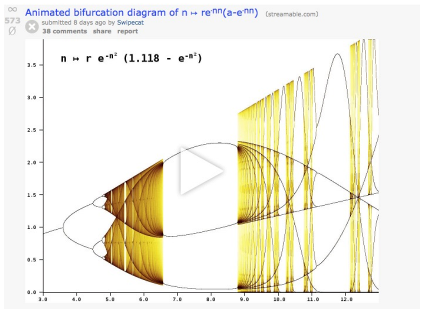

https://www.reddit.com/r/math/comments/77dhcv/animated_bifurcation_diagram_nn/?st=J8YF6UE4&sh=e25b4143

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Courtesy of Allison Brown
Best Poster prize, Experimental Chaos Conference, 2012

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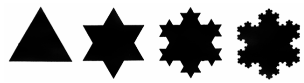
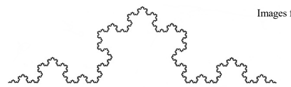
Fractals

- non-integer Hausdorff dimension
- self-similar





Images from Gleick

Examples: Cantor set, coastlines, trees, lungs, clouds, drainage basins, …

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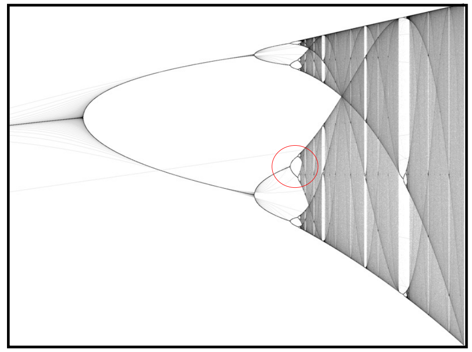

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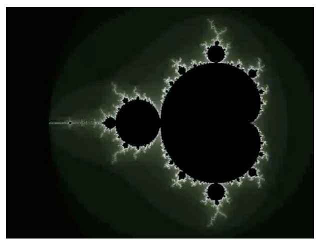
The Mandelbrot set



www.youtube.com/watch?v=G_GBwuYuOOs

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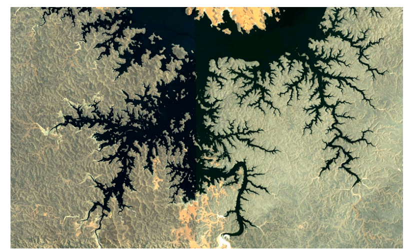
Fractals in the wild



http://paulbourke.net/fractals/googleearth/

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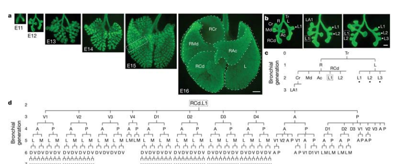
~23 generations of branches in the mammalian lung



2^{23} ~ 8 million branching events!

how does the epithelium fold 8 million times?

Metzger et al, Nature, 2008

Slide courtesy of Celeste Nelson

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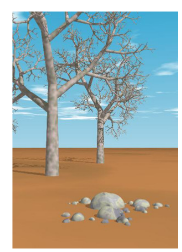
Fractals in computer graphics



Matthew Ward, WPI
davis.wpi.edu/~matt/courses/fractals/trees.html

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Fractals in maps

Newton's method on $x^4 - 1 = 0$

From Strogatz

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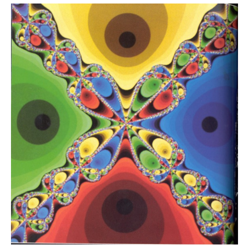
Another connection to chaos: fractal *basin* structure

- The basins
- Their boundaries



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- Fractal basin boundaries can occupy a lot of space
- and cause the trajectory destination to be very sensitive to the initial condition

***Not* same thing as SDOIC!**

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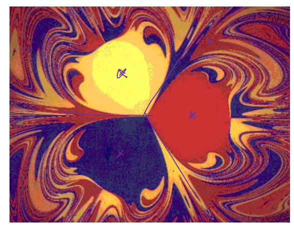
That doesn't just occur in mathematical models:



pendulum swinging over three magnets; Grebogi et al. *Science*

(scribbled Xs are where the magnets are)

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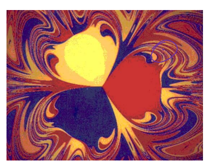
The dynamics of a physical pendulum are a *flow*…



…and basins in flows are connected…?!!

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Poincare section:

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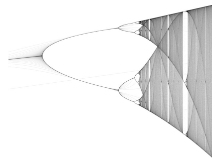
Fractal basin boundaries:

- Affected by changes in parameters, just like attractors are
- Often are ghosts of dead chaotic attractors, much as UPOs are ghosts of dead POs (periodic orbits)



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Fractal basin boundaries, cont.:

- Affected by changes in parameters, just like attractors are
- Often are ghosts of dead chaotic attractors, much as UPOs are ghosts of dead POs (periodic orbits)
- Can occupy lots of space

- Can arise in maps of 2D or more
- Can arise in flows of 3D or more (but *basin* has to be connected in flows; think Koch curve…)

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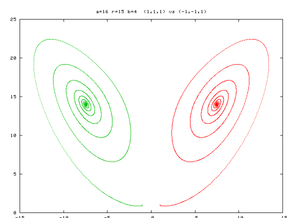
***r* = 15**



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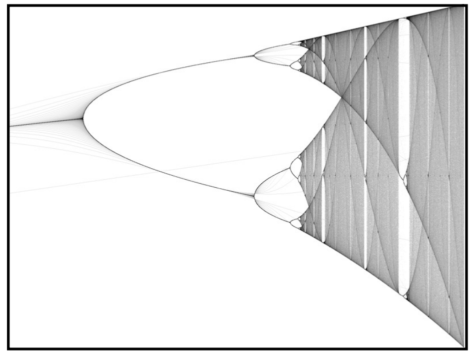

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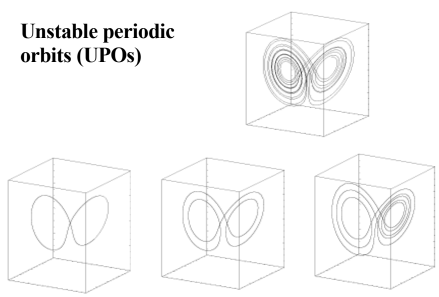
Unstable periodic orbits (UPOs)



Bradley/Mantilla, *Chaos* **12**:596

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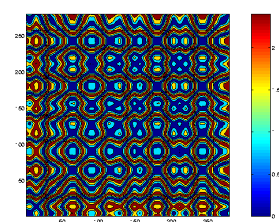

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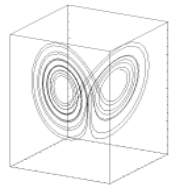
A *section*:



(cf., CAT scans)

Why: reduce complex, high-dimensional information while preserving its essence.

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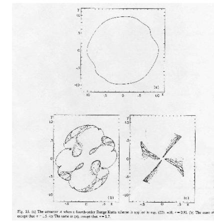

Different timestep

Lorenz, *Physica D* **35**:229

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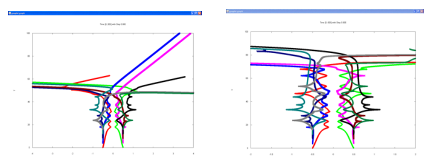

Different arithmetic

N. Ross Ph.D. thesis, Ucolorado, 2008

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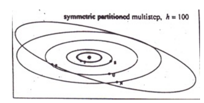

Different solver algorithm...

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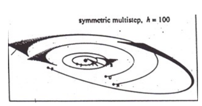

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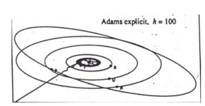

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Moral: numerical methods can run amok in "interesting" ways…

- can alter the dynamics qualitatively and quantitatively
- in a manner that resembles *real, physical* effects
- whence: algorithms, arithmetic system, timestep, etc.
- diagnosis: change the timestep, arithmetic, algorithm; if your results don't change, that should increase your confidence, but sadly it does not *prove* anything.

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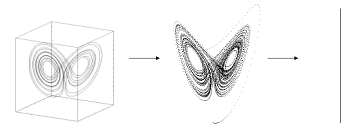
What if you can only measure one state variable?



(this is a *projection,* by the way — not a *section)*

Is the information about the shape of the attractor gone forever?

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Delay-coordinate embedding:

"Reinflate" that squashed data* to get a *topologically identical* copy of the original thing.

(Not *geometrically* identical; that would be a solution to control theory's *observer problem*…)

- technically, doesn't even have to be a measurement of a state variable…

The foundation of nonlinear time-series analysis!

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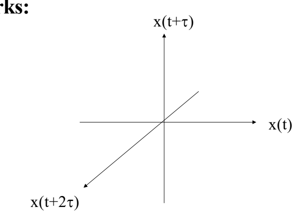
How delay-coordinate embedding works:



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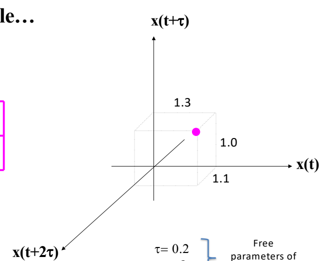
For example…

x	t
1.3	0.1
1.2	0.2
1.0	0.3
0.8	0.4
1.1	0.5
1.4	0.6
1.6	0.7



τ= 0.2
m = 3

Free parameters of the method

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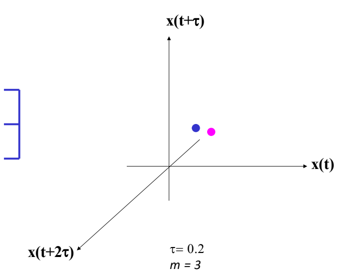
x	t
1.3	0.1
1.2	0.2
1.0	0.3
0.8	0.4
1.1	0.5
1.4	0.6
1.6	0.7



τ= 0.2
m = 3

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Takens* theorem:

For the right τ and enough dimensions (m), the dynamics in this *reconstruction space* are diffeomorphic to the original state-space dynamics.

* Whitney, Mane, …

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Diffeomorphisms and topology:

Diffeomorphic: transformation from the one to the other is 1:1, onto, differentiable, and has a differentiable inverse.

What that means:

- *qualitatively* the same shape

56

Diffeomorphisms and topology:

Diffeomorphic: transformation from the one to the other is 1:1, onto, differentiable, and has a differentiable inverse.

What that means:

- *qualitatively* the same shape
- have same dynamical invariants (e.g., λ)

Challenge: finding good values for τ and m (PS8-9)

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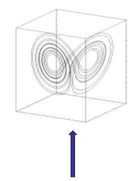
What does this look like, in practice?



Measure just the x coordinate…

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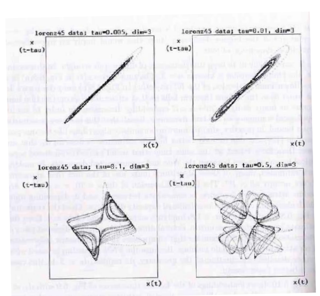
…and then embed:



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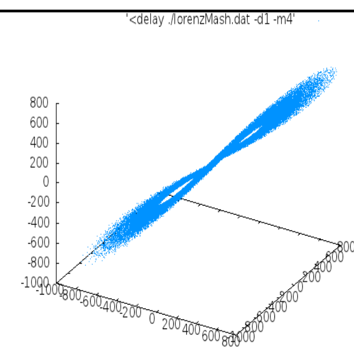

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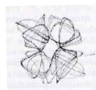
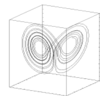
The point:

For the right τ and enough dimensions, the embedded dynamics are diffeomorphic to (have same topology as) the original state-space dynamics.





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Taking this to extremes:

Could theoretically reconstruct the dynamics of the weather of the western hemisphere from a single thermometer.

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Taking this to extremes:

Could theoretically reconstruct the dynamics of the weather of the western hemisphere from a single thermometer.

At least twice as many as the true dynamics

For the right τ and enough dimensions, the embedded dynamics are diffeomorphic to (have same topology as) the original state-space dynamics.

And there are nasty data requirements too; we'll come back to that.

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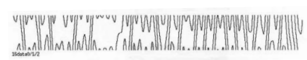
The driven pendulum:

Scalar time-series data:



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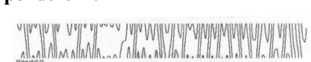
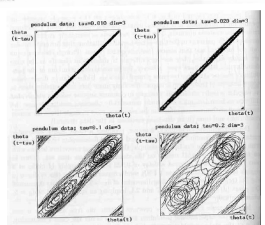
The driven pendulum:

Scalar time-series data:



Reconstructed dynamics:



Bradley, in *Intelligent Data Analysis*, Springer 2000.

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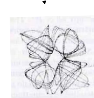
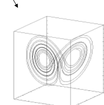
The important implication:

For the right τ and enough dimensions, the embedded dynamics are diffeomorphic to (have same topology as) the original state-space dynamics.





Dynamical invariants are the same!

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Back to Poincare sections:

Surface of section — Σ (“hyperplane”)

Why do this: to reduce info overload while preserving as much of the dynamically meaningful parts of that information as possible

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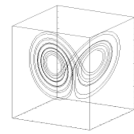
Sections:



• Σ in space

• Σ in time

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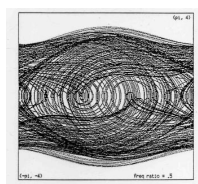

trajectory

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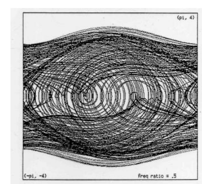
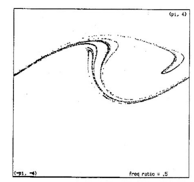


Trajectory



Section

70

Why slice:

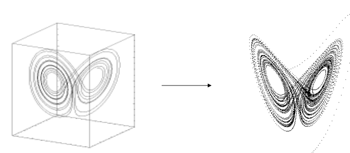
Preserve—and bring out!—the fundamental structure:

- periodicity
- “chaoticity”
- fractal features
- λ
- other dynamical invariants

Pathological slice directions may violate this, of course.

71

Not the same thing as a projection!

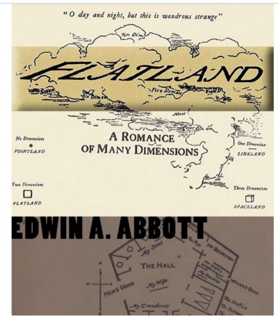


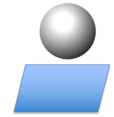
(like the ones that embedding undoes, for example)

72

Section vs projection:

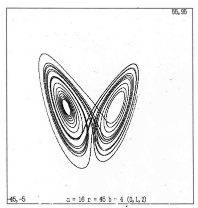
73

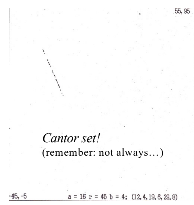




74

(Spatial) section of the Lorenz attractor

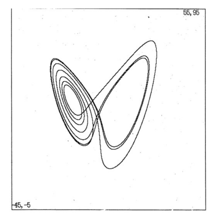




Cantor set!
(remember: not always…)

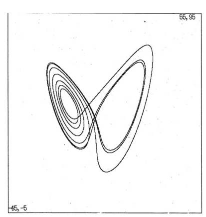
75

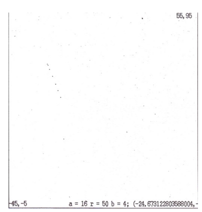
(Spatial) section of a UPO



?

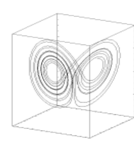
76





77

Sections:



- Σ in space
- Σ in time

78

Temporal sections of periodic orbits: some thought experiments

- pendulum rotating @ 1 Hz and strobe @ 1 Hz?
- pendulum rotating @ 1 Hz and strobe @ 2 Hz?
- pendulum rotating @ 1 Hz and strobe @ 3 Hz?
- pendulum rotating @ 1 Hz and strobe @ 1/2 Hz?
- pendulum rotating @ 1 Hz and strobe @ π Hz? (or some other irrational)

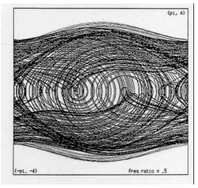
79

Best-ever temporal Poincare section demo…

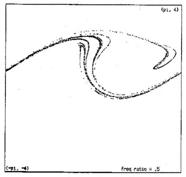
Also: https://twitter.com/buitengebieden_/status/1286788103060037632?s=21

80

Temporal section of a chaotic trajectory



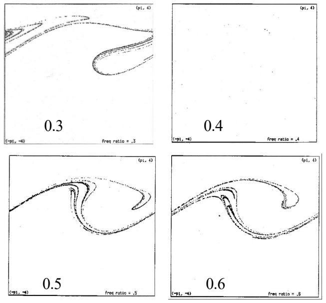
Trajectory (driven damped pendulum)



sampled at the drive frequency

81

What bifurcations look like on a temporal section



82

What temporal sections look like in reality…

83

Next chunk of the course: applications

1. Lorenz
2. Rossler
3. Pendulum
4. Chemical oscillators
5. Music & dance (!)
6. Secure communications
7. Radio tracking circuits (maybe)

84

First publication of "chaos in…"

1. Lorenz (1963) NB: "chaos" not so named yet
2. Rossler (1974)
3. Pendulum (1982)
4. Chemical oscillators (1980s)
5. Music & dance (1998)
6. Secure communications (1990)
7. Radio tracking circuitry (1993)

85

Physical reality?

- Lorenz: semiphysical
- Rossler: pure math exercise
- Others: physical, practical, useful…

86

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

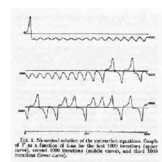
ABSTRACT

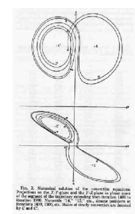
Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

87





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CHAOS THEORY

The Hidden Heroines of Chaos

10

Two women programmers played a pivotal role in the birth of chaos theory. Their previously untold story illustrates the changing status of computation in science.

https://www.quantamagazine.org/hidden-heroines-of-chaos-ellen-fetter-and-margaret-hamilton-20190520/

89

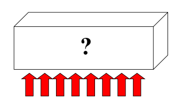
The application: hydrodynamics (…weather)

- Equations:

$x' = a(y-x)$

$y' = rx - y - xz$

$z' = xy - bz$

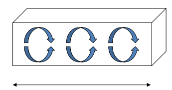


90

- State variables:
  - x convective intensity
  - y temperature
  - z deviation from linearity in the vertical convection profile

91

- Parameters:
  - a Prandtl number - fluids property
  - r Rayleigh number - related to ΔT
  - b aspect ratio of the fluid sheet



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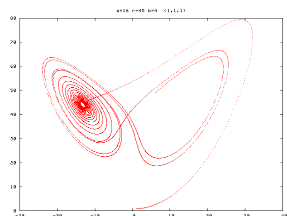
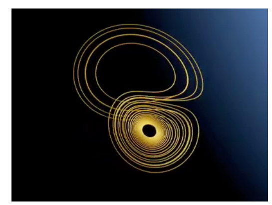
***r* = 45**



93



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Garden-variety chaotic attractor:

- Fractal structure
- Sensitive dependence on initial conditions
- Dense coverage

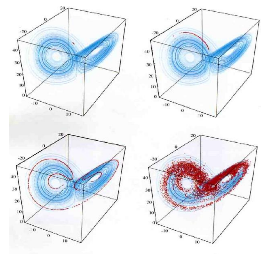
95

Garden-variety chaotic attractor:

- Fractal structure
- Sensitive dependence on initial conditions
- Dense coverage
- $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 < 0$; $|\lambda_3| > |\lambda_1|$

96

Growth *along* the attractor is subexponential



Also: https://www.youtube.com/watch?v=5xu-9D4ahVU

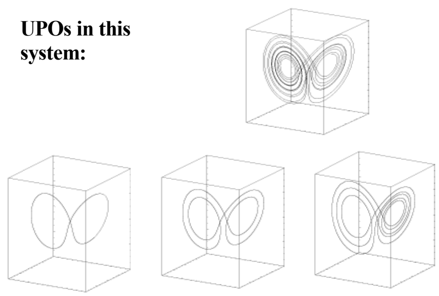
97

Garden-variety chaotic attractor:

- Fractal structure
- Sensitive dependence on initial conditions
- Dense coverage
- $\lambda 1 > 0, \lambda 2 = 0, \lambda 3 < 0; |\lambda 3| > |\lambda 1|$
- *Contains an infinite number of unstable periodic orbits, of all periods…*

98

UPOs in this system:



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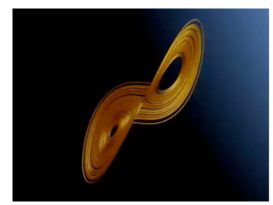
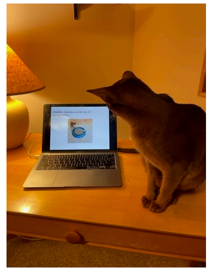


100

101

Stability: robustness in the face of perturbation...

102



103

Physical reality?

$x' = a(y-x)$
$y' = rx - y - xz$
$z' = xy - bz$

- Typical parameter values: $a = 16$, $r = 45$, $b = 4$
- But these equations are only a good approximation to reality if r is around 1.0

104

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- But these equations are only a good approximation to reality if r is around 1.0
- And they're an ODE truncation of a model that's a PDE, which *itself* is only a simplified approximation of the real dynamics….

105

Physical reality?

$x' = a(y-x)$
$y' = rx - y - xz$
$z' = xy - bz$

- Typical parameter values: $a=16$, $r=45$, $b= 4$
- But these equations are only a good approximation to reality if r is around 1.0
- And they're an ODE truncation of a model that's a PDE, which *itself* is only a simplified approximation of the real dynamics….
- But it's still fun to play with r ☺

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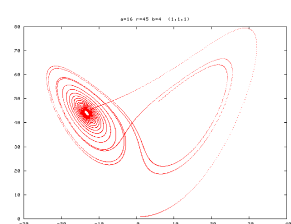
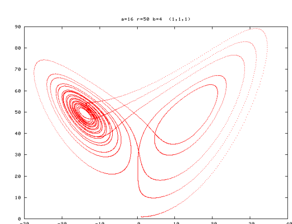
$r = 45$



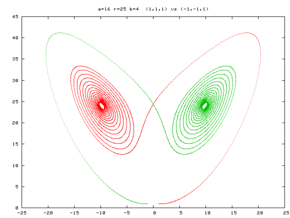
107

$r = 50$



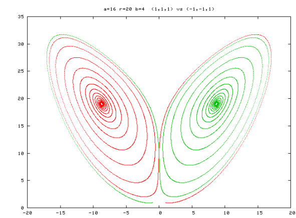
108

$r = 25$



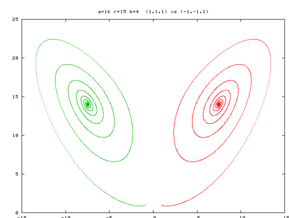
109

$r = 20$



110

$r = 15$



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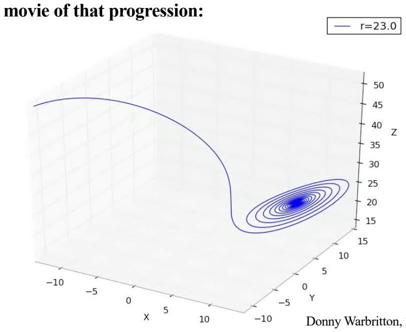
A movie of that progression:



Donny Warbritton, 2011

112

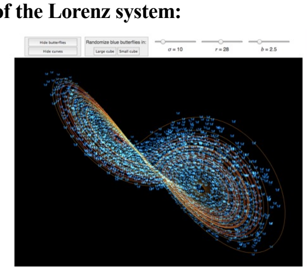
Another one:

Note to self: video starts 8 sec in; be patient

Ayoub Ghriss, 2019

113

Exploring the state and parameter space of the Lorenz system:



www.malinc.se/m/Lorenz.php

114

A *mechanical* computer…

(i.e., a physical analogue)

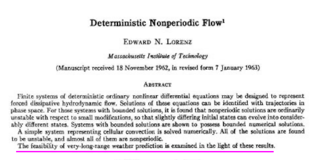
https://www.youtube.com/watch?v=7iNCfNBEJHo

115

Courtesy of Joshua Rines (2019):

116

Before we leave Lorenz…



Deterministic Nonperiodic Flow[1]

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(Manuscript received 18 November 1962, in revised form 7 January 1963)

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A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

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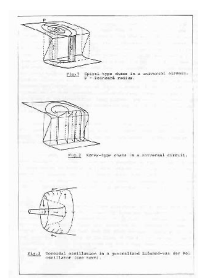
Applications:

1. Lorenz
2. *Rossler*
3. Pendulum
4. Chemical systems
5. Secure communications
6. Music & dance
7. Radios

118

Rossler's transformations:



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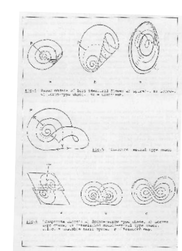
Rossler's transformations, cont.



120

Equations:

$x' = -y-z$

$y' = x+ay$

$z' = b+z(x-c)$

(PS5)

121

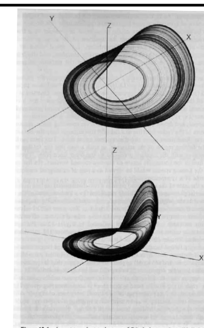


Figure 12.1 A post-transient trajectory of Rössler's equations (12.1) for the simply folded band attractor. Parameters are $a = 0.398$, $b = 2$, $c = 4$

Thompson & Stewart

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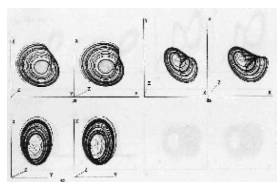
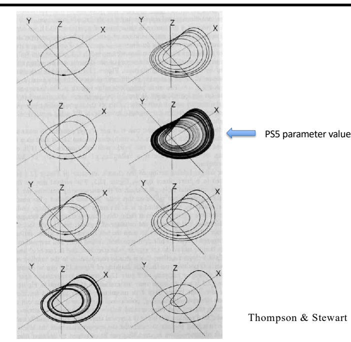


123



Thompson & Stewart

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Applications:

1. Lorenz
2. Rossler
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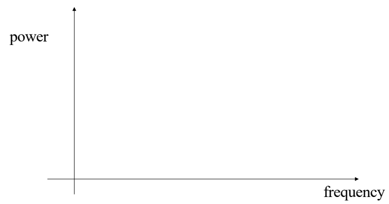
125

A physical demo, with Poincare section:

126

Aside: the power spectrum



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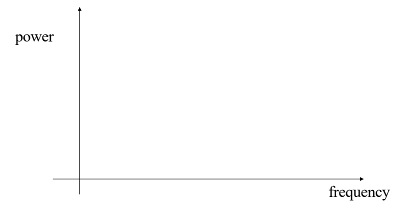
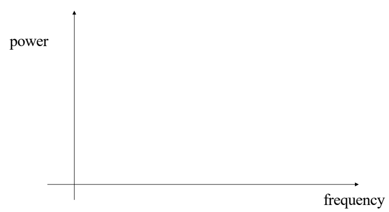
Pure tone?



128

Chord?



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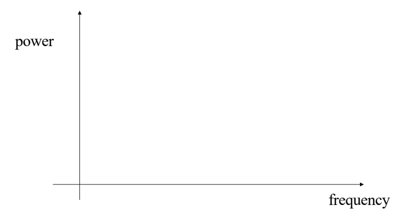
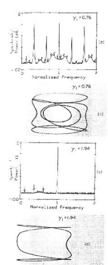
Periodic orbit (e.g., five-cycle)?



130



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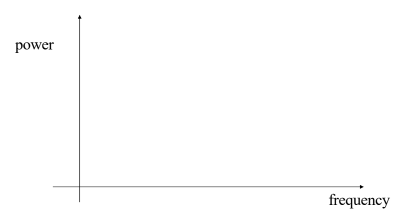
Noise?



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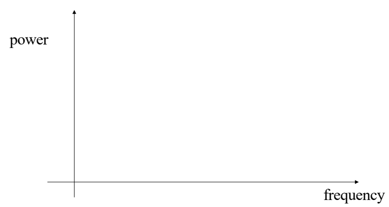
Chaos?



Caveat: frequency spectra are rarely useful (and often confusing) in analyzing chaotic systems!

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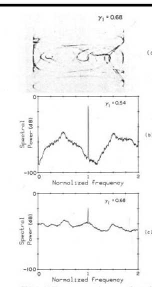

FIG. 6. (a) Phase-space portrait corresponding to the intermittent domain of Fig. 3 at $\omega=0.67$. (b) and (c) show the spectral power density corresponding to Figs. 4(d) and 5(a). Note different behavior as $\omega \to 0$. Note also that in this figure and all other power spectra the frequency axis is normalized to the drive frequency.

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PS4 driven pendulum equations:

(torque @ pivot)

$$ml\theta'' - \beta l\theta' + mg\sin\theta = A\sin\alpha t$$

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Parameter-space portrait:

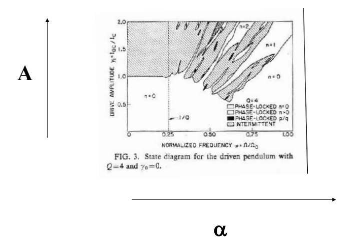

FIG. 3. State diagram for the driven pendulum with $Q=4$ and $\gamma_0=0$.

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My instructions for PS4:

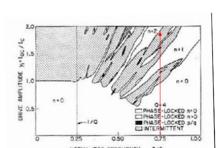
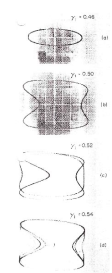


FIG. 3. State diagram for the driven pendulum with $Q=4$ and $\gamma_0=0$.

137



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How did they construct this plot?

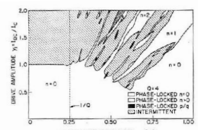

FIG. 3. State diagram for the driven pendulum with $Q=4$ and $\gamma_0=0$.

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TABLE I. Corresponding variables for pendula, Josephson junctions, and phase-locked loops.

	Pendulum	Josephson junction	Phase-locked loop
θ	Angular position	Quantum phase difference	Phase difference between the oscillators
$\dot{\theta}$	Angular velocity	$\frac{2\,eV}{\hbar}$	kV
a	Inertia momentum	$\hbar C/2e$	C/k
b	Viscous damping	$\hbar/2eR$	$1/kR$
$c\sin\theta$	Restoring torque	Josephson current I_J	Feedback current I_{FB}
c	·	Critical current I_c	V_1/R_S
$\Gamma(t)$	Applied torque	Applied current $I(t)$	Applied current $V_E(t)/R_E$
$\Omega_0=\sqrt{c/a}$	Natural frequency	$(2eI_c/\hbar C)^{1/2}$	$(kV_1/R_SC)^{1/2}$
$\tau_0=b/c$	Damping time	$\hbar/2eRI_c$	R_S/kRV_1
$Q=(ac/b^2)^{1/2}$	Quality factor	$(2eI_cR^2C/\hbar)^{1/2}$	$(kV_1CR^2/R_S)^{1/2}$

140

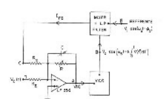


FIG. 14. Schematic diagram of the phase-locked loop used in our simulations. Typical circuit values are $R=11$ kΩ, $C=100$ nF, $k=4\pi\times10^3\,s^{-1}/V$ which lead to a natural frequency $\Omega_0/2\pi=600$ Hz and a $Q=4$. We also used $R_E=R_S=10$ kΩ and a reference frequency $\omega_r/2\pi=100$ kHz.

141

PS4 driven pendulum equations:

(torque @ pivot)

$$ml\theta'' - \beta l\theta' + mg\sin\theta = A\sin\alpha t$$

Easy to build with electronics, but not so easy in a machine shop…

142

Driven pendulum equations with *vertical oscillation* @ pivot:

$$ml\theta'' + \beta l\theta' + m\sin\theta\,[g - A\,\alpha\,\alpha\sin\,\alpha t]=0$$

*(that's the equation for **my** driven pendulum)*

143

Applications, part II:

1. Lorenz
2. Rossler
3. Pendulum
4. *Chemical systems*
5. Secure communications
6. Music & dance
7. Radio tracking circuitry

144

Spatiotemporal chaos in a chemical system:

Belousov-Zhabotinskii

145

Temporal (*) chaos in a chemical system:

Briggs-Rauscher

https://www.youtube.com/watch?v=8R33KWPmqlo

(*) the spatial angle is absent here because of the mixing

146

Applications, part II:

1. Lorenz
2. Rossler
3. Pendulum
4. Chemical systems
5. *Secure communications*
6. *Radio tracking circuitry*
7. *Music & dance*

147

Ways to *use* chaos:

- Communicating securely
- Improving a radio's frequency-tracking range
- Generating musical (or choreographic) variations

148

Aside: AM and FM transmission

149

Problems:

- Both AM and FM are easy to "intercept"
- Potential solutions:
  - "Hop" the carrier
  - Extra layers of circuitry
  - …
  - (or use digital signals and RSA ;-)

150

Use a *chaotic* carrier "wave"?

Fundamental property that makes this work:

151

- Two coupled Lorenz systems will synchronize
- Robust w.r.t. a small amount of noise
- Use this to transmit & receive information

$$\begin{array}{l} x' = a(y-x) \\ y' = rx - y - xz \\ z' = xy - bz \end{array} \longrightarrow \begin{array}{l} x' = a(y-x) \\ y' = r(x+\varepsilon x) - y - xz \\ z' = xy - bz \end{array}$$

- Chaotic carrier wave, so hard to intercept or jam

Pecora & Carroll *Phys. Rev. Lett* **64**:821; Strogatz § 9.6

152

Cuomo video:

https://www.youtube.com/watch?v=J-ca_bqWp4I

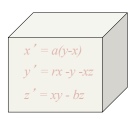
153

How to crack this?

$$\begin{array}{l} x' = a(y-x) \\ y' = rx - y - xz \\ z' = xy - bz \end{array} \longrightarrow \begin{array}{l} x' = a(y-x) \\ y' = r(x+\varepsilon x) - y - xz \\ z' = xy - bz \end{array}$$

154

"System identification"…



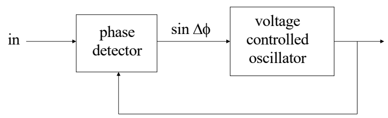
155

Applications:

1. Lorenz
2. Rossler
3. Pendulum
4. Chemical systems
5. Secure communications
6. *Radio tracking circuitry*
7. Music & dance

156

The phase-locked loop:



(need this to demodulate FM, among other things)

157

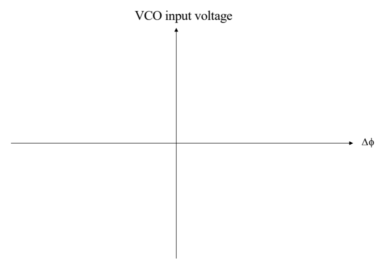
PLL is *isomorphic* to the pendulum:

TABLE I. Corresponding variables for pendula, Josephson junctions, and phase-locked loops.

	Pendulum	Josephson junction	Phase-locked loop
θ	Angular position	Quantum phase difference	Phase difference between the oscillators
$\dot{\theta}$	Angular velocity	$\frac{2\,eV}{\hbar}$	kV
a	Inertia momentum	$\hbar C/2e$	C/k
b	Viscous damping	$\hbar/2eR$	$1/kR$
$c\sin\theta$	Restoring torque	Josephson current I_J	Feedback current I_{FB}
c	·	Critical current I_c	V_1/R_S
$\Gamma(t)$	Applied torque	Applied current $I(t)$	Applied current $V_E(t)/R_E$
$\Omega_0=\sqrt{c/a}$	Natural frequency	$(2eI_c/\hbar C)^{1/2}$	$(kV_1/R_SC)^{1/2}$
$\tau_0=b/c$	Damping time	$\hbar/2eRI_c$	R_S/kRV_1
$Q=(ac/b^2)^{1/2}$	Quality factor	$(2eI_cR^2C/\hbar)^{1/2}$	$(kV_1CR^2/R_S)^{1/2}$

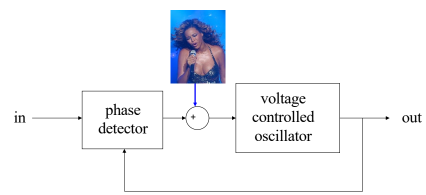
158

State space:



159

Phase modulation:



160

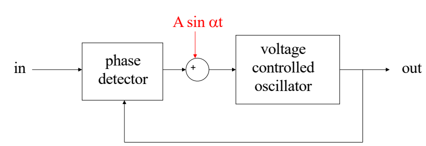
Phase-modulated PLL is isomorphic to the driven pendulum:

TABLE I. Corresponding variables for pendula, Josephson junctions, and phase-locked loops.

	Pendulum	Josephson junction	Phase-locked loop
θ	Angular position	Quantum phase difference	Phase difference between the oscillators
$\dot{\theta}$	Angular velocity	$\frac{2\,eV}{\hbar}$	kV
a	Inertia momentum	$\hbar C/2e$	C/k
b	Viscous damping	$\hbar/2eR$	$1/kR$
$c\sin\theta$	Restoring torque	Josephson current I_J	Feedback current I_{FB}
c	·	Critical current I_c	V_1/R_S
$\Gamma(t)$	Applied torque	Applied current $I(t)$	Applied current $V_E(t)/R_E$
$\Omega_0=\sqrt{c/a}$	Natural frequency	$(2eI_c/\hbar C)^{1/2}$	$(kV_1/R_SC)^{1/2}$
$\tau_0=b/c$	Damping time	$\hbar/2eRI_c$	R_S/kRV_1
$Q=(ac/b^2)^{1/2}$	Quality factor	$(2eI_cR^2C/\hbar)^{1/2}$	$(kV_1CR^2/R_S)^{1/2}$

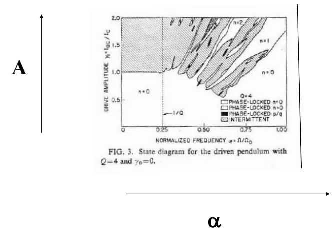
161

PS4:

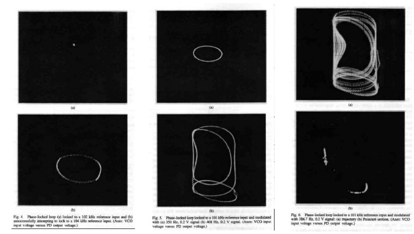


162

Parameter-space portrait:

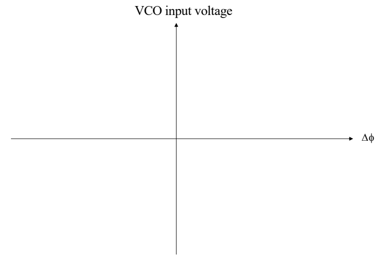


163



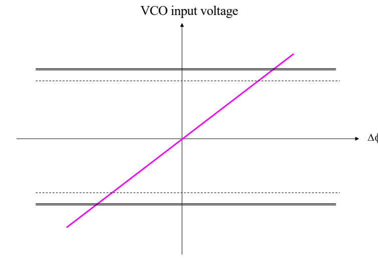
164

PLL state-space dynamics:



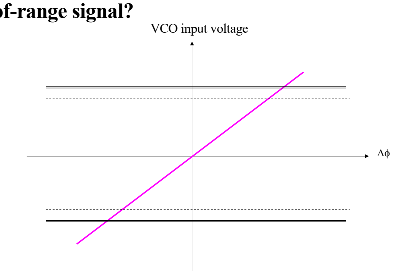
165

Normal capture dynamics:



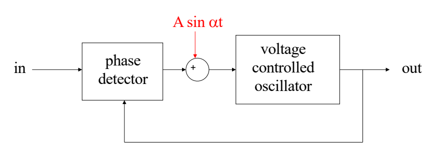
166

What if you wanted to capture an out-of-range signal?



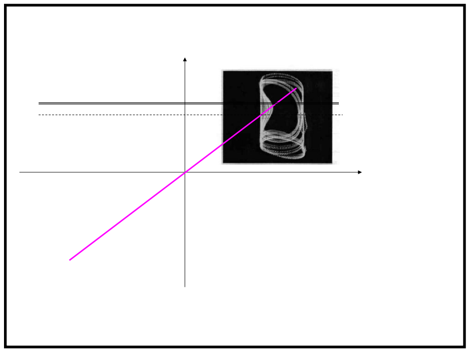
167

We can diddle the dynamics of the system by messing with A and α…



…can we use that to capture out-of-range signals?

168



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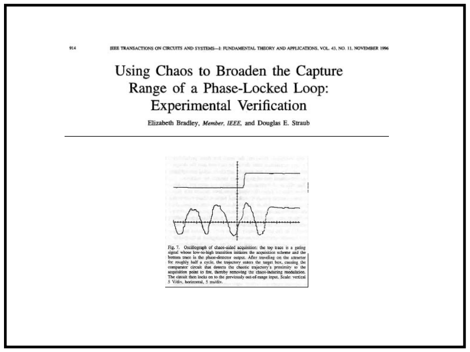
914 IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: FUNDAMENTAL THEORY AND APPLICATIONS, VOL. 43, NO. 11, NOVEMBER 1996

Using Chaos to Broaden the Capture Range of a Phase-Locked Loop: Experimental Verification

Elizabeth Bradley, *Member, IEEE,* and Douglas E. Straub

170

Applications:

1. Lorenz
2. Rossler
3. Pendulum
4. Chemical systems
5. Secure communications
6. Radio tracking circuitry
7. Music & dance

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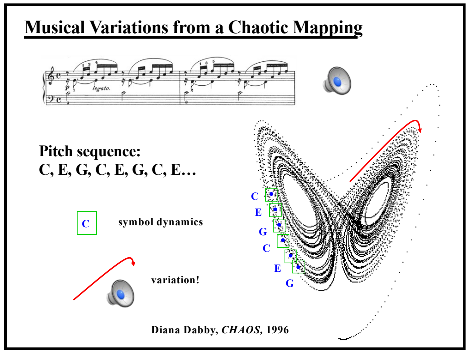

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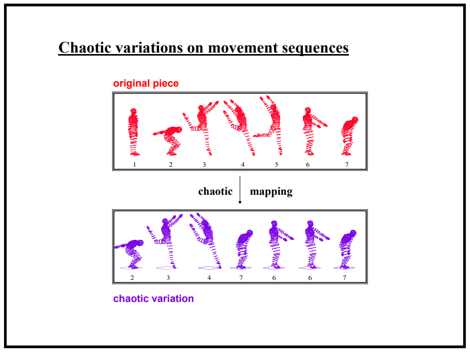

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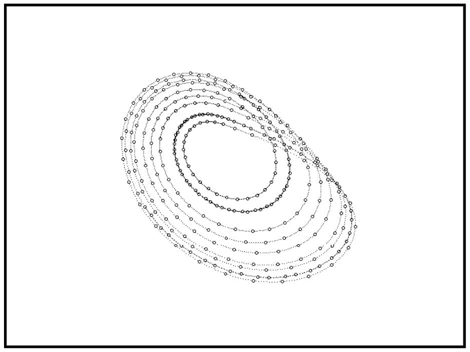

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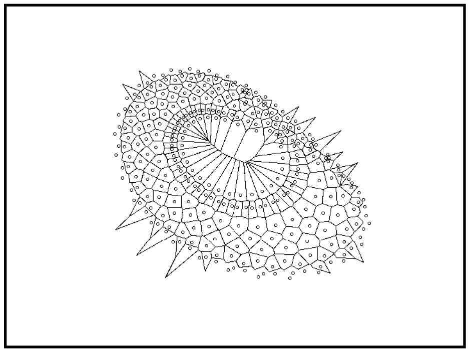

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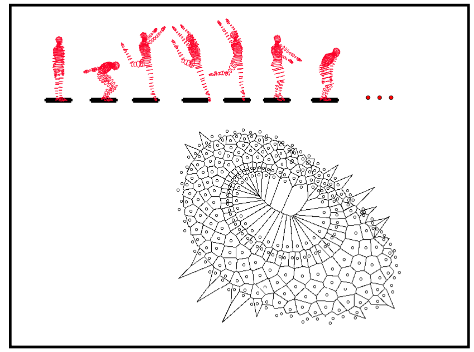

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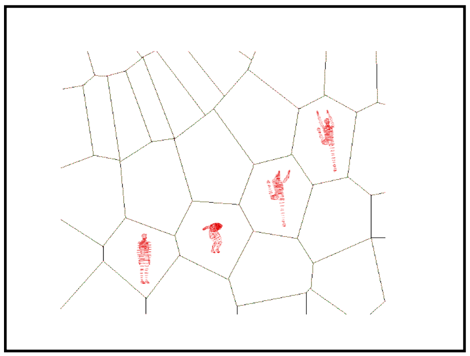

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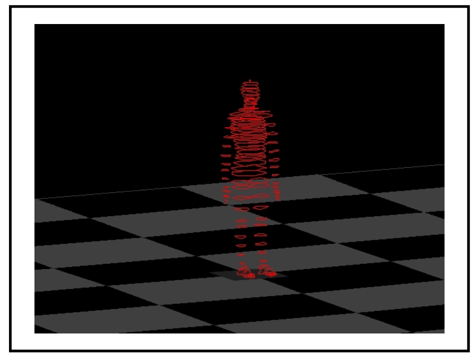

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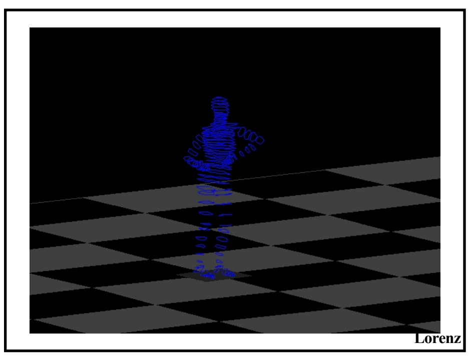

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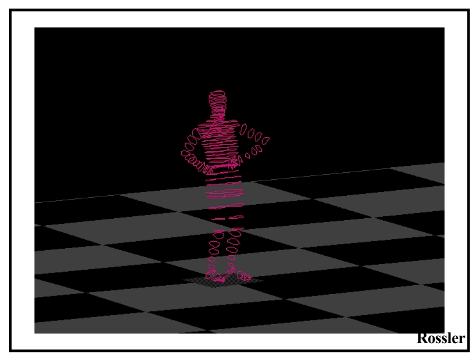

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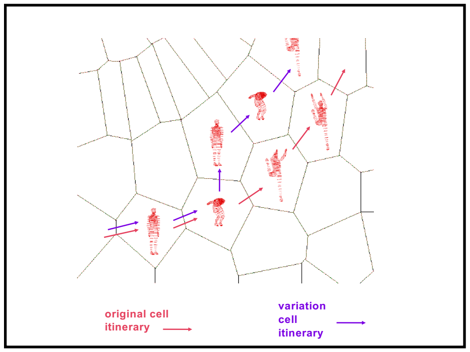

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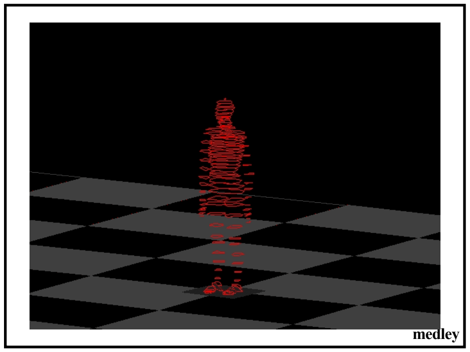

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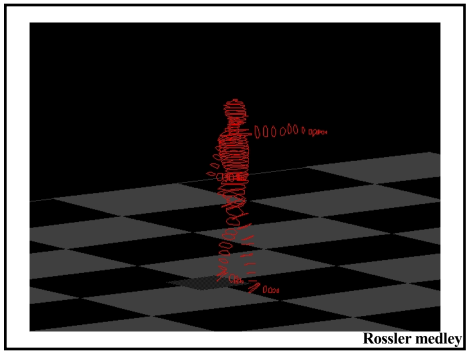

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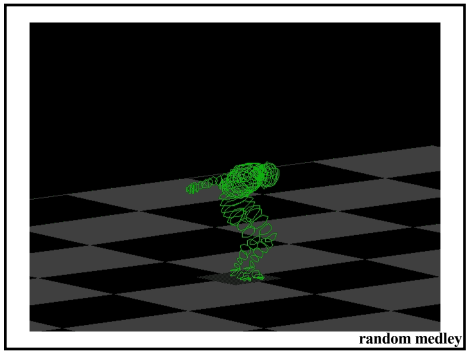

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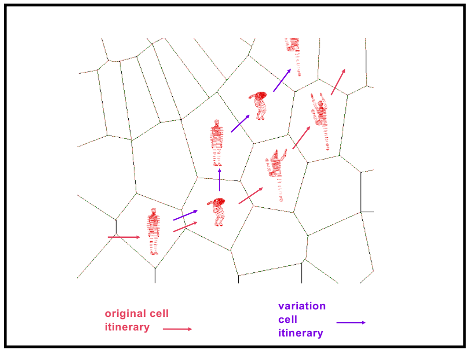

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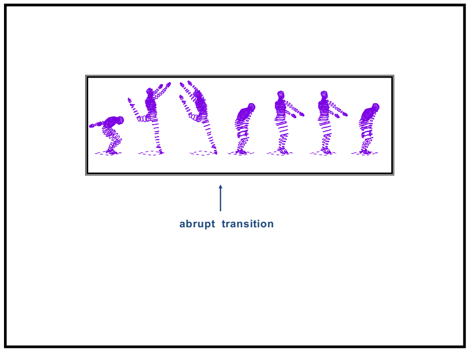

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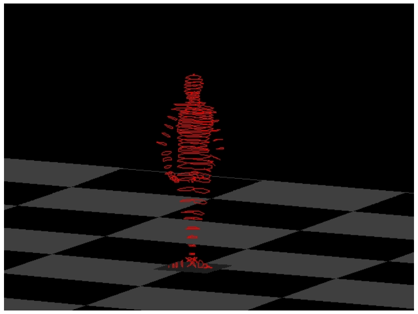

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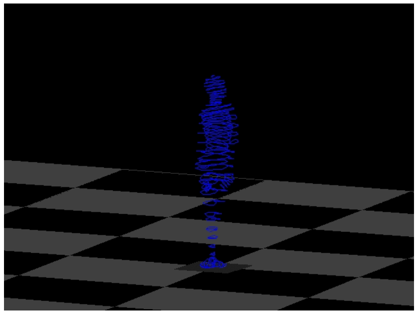

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What this does to text

Alice was beginning to get very tired of sitting by her sister
on the bank, and of having nothing to do: once or twice she had
peeped into the book her sister was reading, but it had no
pictures or conversations in it, `and what is the use of a book,'
thought Alice `without pictures or conversation?' .....

chaotic mapping

about stopping herself she found very tired of sitting by her sister on the bank,
and of having nothing to do: once or twice she had peeped into either |
question, | it didn't much matter which way she put it. She | felt that she was
dozing off, and had just begun to dream that she was walking hand in | hand
with Dinah, and saying to her very earnestly, `Now, Dinah, tell me me the truth:
did you ever ever eat a bat?'

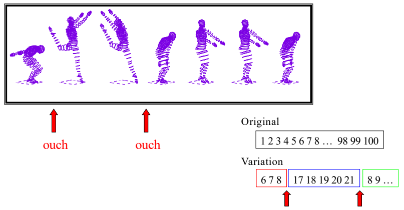
(| symbols inserted to show the shuffle breaks)

189

This "chunkwise shuffle" mechanism is used by human composers, too:

- Lukas Foss's *Baroque Variations*
- Sampling (Big Audio Dynamite, hip hop, …)

But abrupt transitions raise different issues in movement…



190

"Cutting and pasting is the essence of what hip-hop culture is all about for me. It's about drawing from what's around you, and subverting it and decontextualizing it." DJ Shadow[61]. "I look at all the different parts and see how I can organize them in a way. It's like maths. Very mathematic. It's like graphs!"
Blockhead[62].

[61] http://to-the-quick.binghamton.edu/issue%202/sampling.html.
[62] http://www.trip-hop.net/interview-10-Blockhead.html.

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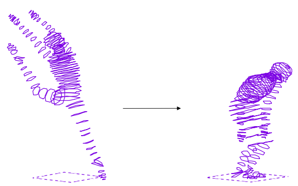
Interpolation



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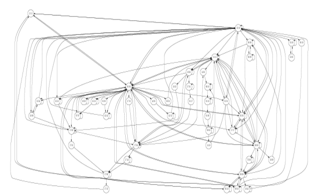
Corpus-based approach



- graph captures motions of one joint
- note: specific to the genre of the corpus!

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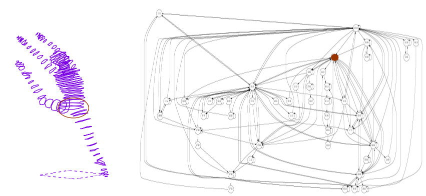
Initial state



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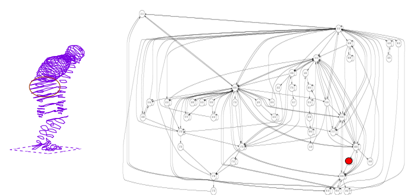
Target state



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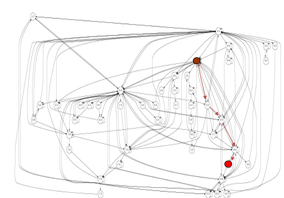
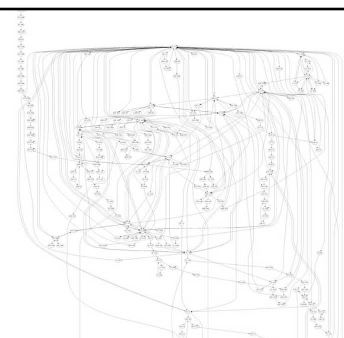
Graph search

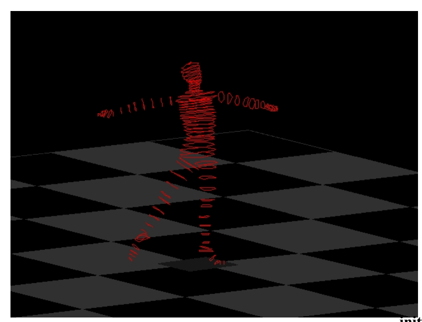


…for 44 joints in parallel!

196

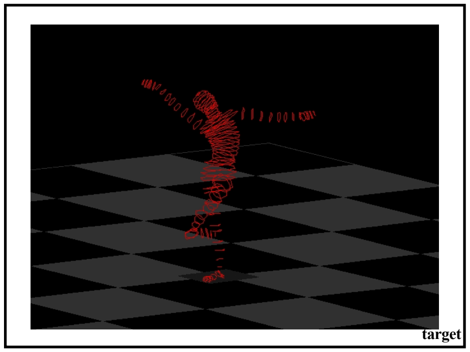


197



initial

198



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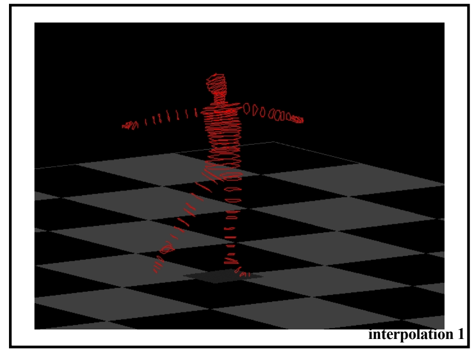

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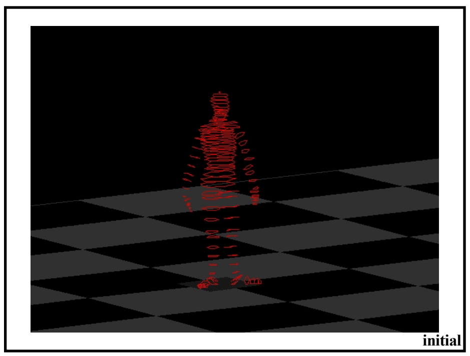

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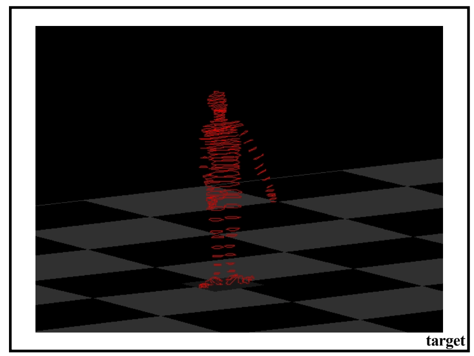

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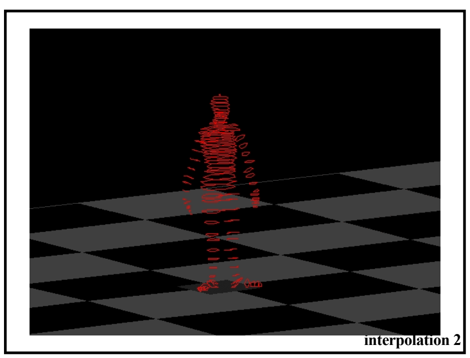

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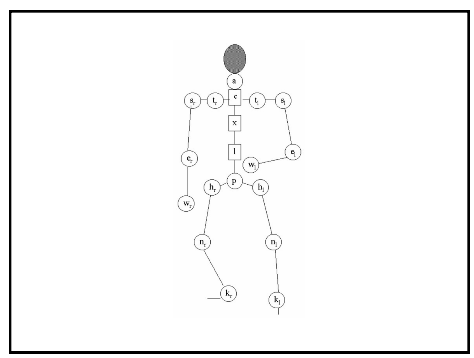
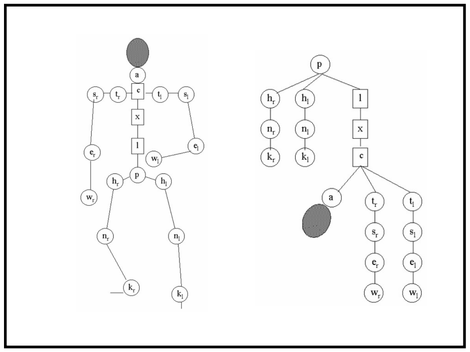
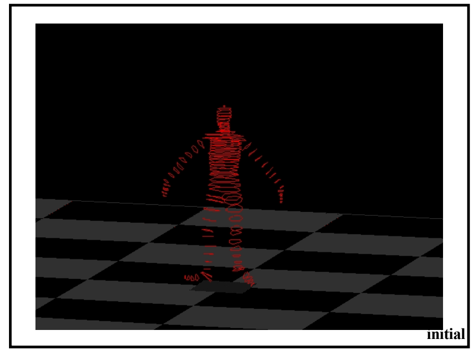


204



205



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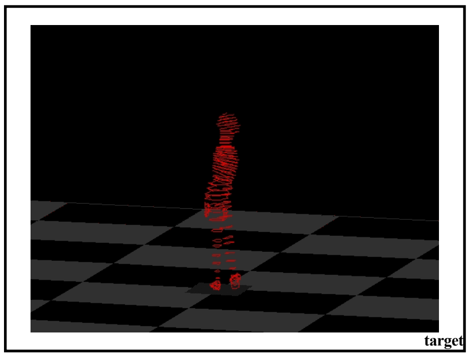
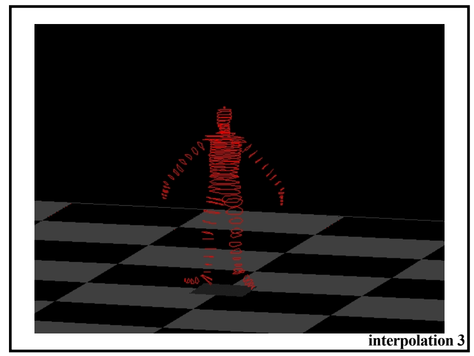


207



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"Chaographer" and "MotionMind"

- "stylistically consonant" movement sequences
  - variations
  - interpolations
- but meager corpus can create discursive paths
- removing constraints induced by topology and gravity: ballet → modern??
- *great* way to engage laymen/students in math, physics, computer science…

Other applications:

- flight simulators
- training (e.g., wargames)
- etc.

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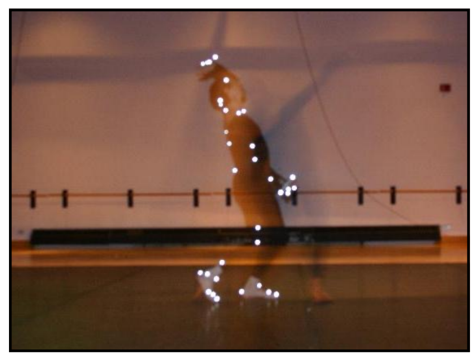

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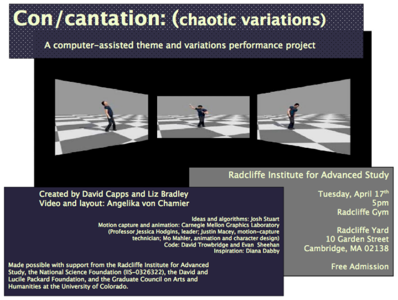

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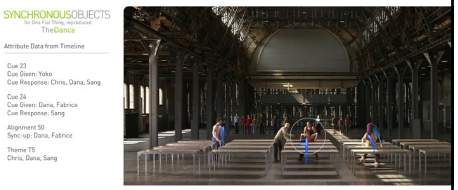
“chaos” vs. “complexity”



synchronousobjects.osu.edu

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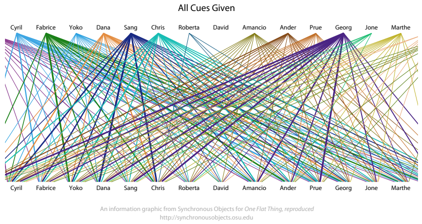

synchronousobjects.osu.edu/content.html

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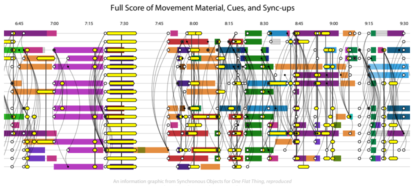

synchronousobjects.osu.edu/content.html

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Next chunk of this course: attractor characterization

1. attractor and basin definitions
2. state-space deformation math (PS7)
3. how to find attractors, basins
4. delay-coordinate embedding in more depth (PS8)
5. how to calculate λ (PS9)
6. how to calculate fractal dimensions (PS10)
7. how to find UPOs

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A useful metaphor:

(image from wikipedia)

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More broadly…

(image from wikipedia)

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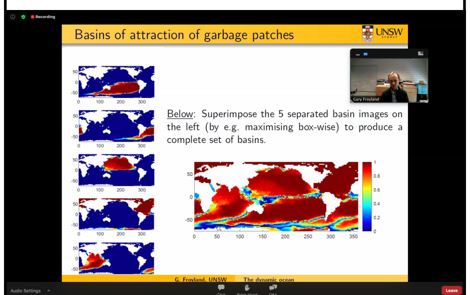

(Gary Froyland speaking at SIAM MPE, August 2020)

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Where we are in the "attractor characterization" chunk of this course:

1. √ attractor and basin definitions
2. √ state-space deformation math (PS7)
3. √ how to find attractors, basins
4. delay-coordinate embedding in more depth (PS8)
5. how to calculate λ (PS9)
6. how to calculate fractal dimensions (PS10)
7. how to find UPOs

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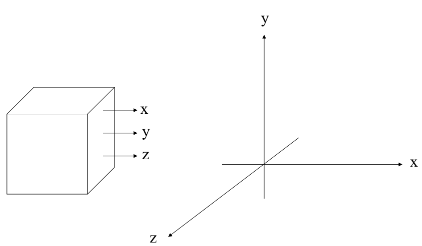
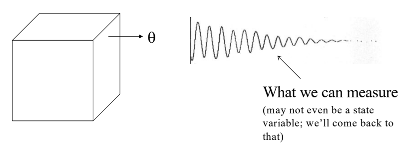
We've been assuming that we know and can measure all the state variables…



221

But often you can't.



The *observer* problem…

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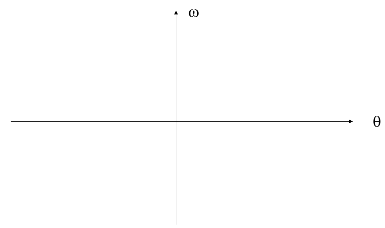
What we want:

The state-space picture!



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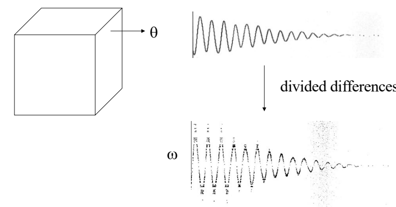
How to reconstruct the other state vars?



derivatives magnify noise!

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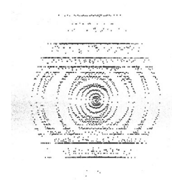
What that looks like in the state space:



Yucko.

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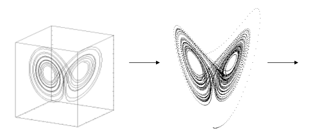
Recall: what we're asking here is to undo a projection



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Delay-coordinate embedding

"reinflate" that squashed data to get a *topologically identical* copy of the original thing.

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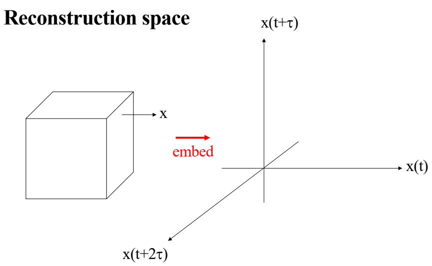
Reconstruction space



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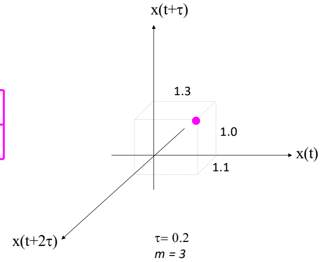
Mechanics

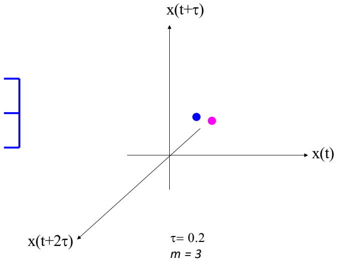
x	t
1.3	0.1
1.2	0.2
1.0	0.3
0.8	0.4
1.1	0.5
1.4	0.6
1.6	0.7



τ= 0.2
m = 3

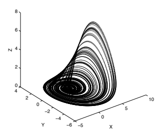
229

x	t
1.3	0.1
1.2	0.2
1.0	0.3
0.8	0.4
1.1	0.5
1.4	0.6
1.6	0.7

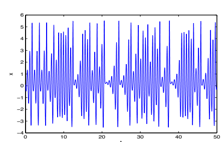


τ= 0.2
m = 3

230



Measure just the *x* coordinate…



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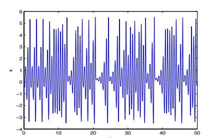
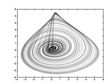
…and then embed:





τ = 150

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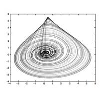
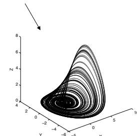
Takens* theorem

For the right τ and enough dimensions, the embedded dynamics are diffeomorphic to (have same topology as) the original state-space dynamics.



τ = 150



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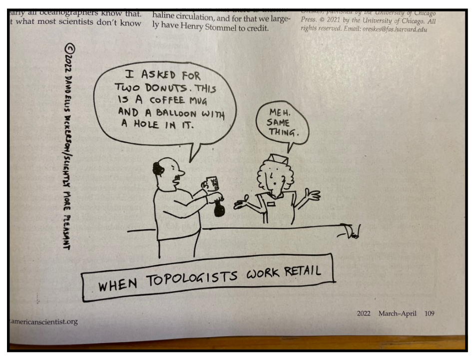

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www.shapeways.com/shops/henryseg

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Thought exercise: what if the trajectory were headed for a fixed point?

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Thought exercise: what if the trajectory were headed for a fixed point?

Will changing τ change this?

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Thought exercise: what if the trajectory were headed for a periodic orbit?

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Thought exercise: what if the trajectory were headed for a periodic orbit?

Additional condition in the Takens thm:
τ can't be a multiple of any period (if the dynamics are periodic).

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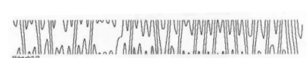
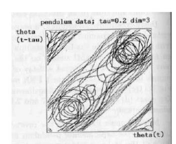
What if the data are from a chaotic attractor?

Scalar time-series data:



Reconstructed dynamics:



Bradley, in *Intelligent Data Analysis*, Springer 2000.

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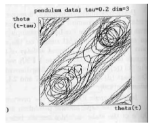
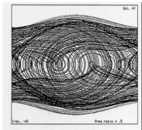
The point:



Same
topology



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Why embedding is useful, redux

Diffeomorphic: mapping from the one to the other is differentiable and has a differentiable inverse.

What that means:

- *qualitatively* the same shape
- have same dynamical invariants (e.g., λ)

Challenges:

- Finding good values for τ and m (PS8-9) Next up
- Enough data?
- Measuring the system without perturbing it

Later

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The two free parameters of the method:

For the right τ and enough dimensions, the embedded dynamics are diffeomorphic to (have same topology as) the original state-space dynamics.

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Picking τ

Takens only requires $\tau > 0$*;
in practice, though, that's inadequate.

* And not a multiple of any period

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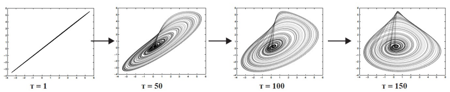
What you see when you change τ:



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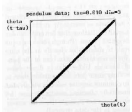
Picking τ

Takens only requires $\tau > 0$*;
in practice, though, that's inadequate.



- Noise
- Sensor error
- Floating-point arithmetic

* And not a multiple of any period

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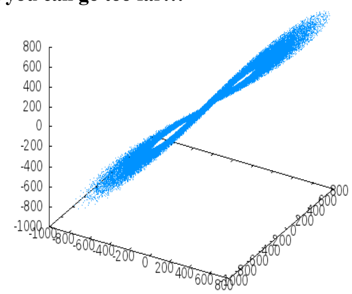
And you can go too far…



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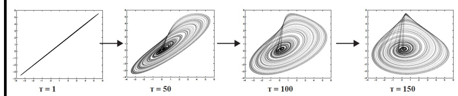

Back to those thought exercises:

- What would we see if the dynamics were at a fixed point and we changed τ?
- What if they were *on a limit cycle* and we changed τ?

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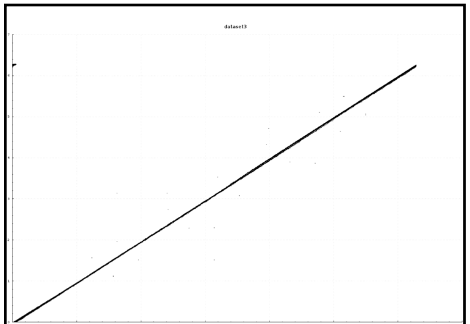

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Mathematical tools for choosing τ:

Idea:

- We want the different coordinates in the embedded space to be independent
- But not *too* independent or the attractor will get all folded up—and eventually the points will become uncorrelated.

There are lots of implementations of this, using tools that measure correlation in different ways.

The most common: average mutual information.

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Other methods for choosing τ:

Different ideas:

- Want to fill up as much space as possible with the embedded attractor
- Or maybe what really matters is the τ*m product (Pecora/Moniz)
- Could we use geometry, like curvature?
- Could we use *topology*?
- Many other approaches; see Bradley/Kantz 2015, Garland/Bradley 2015 for reviews

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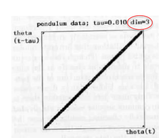
Choosing *m*

Takens+ theorem requires $m > 2d$: sufficient condition to ensure no crossings in reconstruction space.



What's wrong with this picture?

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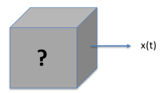
Choosing *m*

Takens+ theorem requires $m > 2d$: sufficient condition to ensure no crossings in reconstruction space.

Problem: we don't know d…



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Choosing *m*

$m > 2d$: sufficient to ensure no crossings in reconstruction space…

…but that may be overkill (i.e., a lower m might work fine)

"Embedology" paper: $m > 2\ d_{box}$
(box-counting dimension)
but you need to embed in order to calculate d_{box}

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The "asymptotic invariant" method

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The "false near neighbor" method

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The FNN algorithm:

- Compute every point's near(est) neighbor(s)
- Increment m
- Recompute neighbor(s)
- (M)any changes?
  - If not, done.
  - If so, loop.

Issues:
- Computational complexity
- Noise

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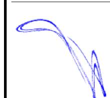
Software tools for choosing embedding parameters:



TISEAN
Nonlinear Time Series Analysis

Rainer Hegger
Holger Kantz
Thomas Schreiber

Go to Version 3.0.1 (released March 2007)

Go to Version 2.1 (released December 2000)

Implementations of most of the tools described in their book!
(url is on the course homepage; code is in the CSCI4446/5446 tile on the CSEL setup)

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TISEAN 3.0.1: Table of Contents

TISEAN 3.0.1

TISEAN home

Table of Contents

General Manual

Surrogates Manual

Tutorial

Usage Notes

Installation

Problems

All programs in alphabetical order

Sections

Generating time series

A few routines are provided to generate test data from simple equations. Since there are powerfull packages (for e
Helena Nusse and Jim Yorke) that can generate chaotic data, we have only included a minimal selection here.

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Embedding and Poincaré sections

Since the concept of phase space is at the heart of all the nonlinear methods in this package, phase space reconstruction plays an important role. Although delay and other embeddings are used inside most of the other programs, it is important to have these techniques also for data viewing, selection of parameters, etc. For delay embeddings, use delay.

Phase space reconstruction is discussed also in the the introduction paper.

Embed using delay coordinates	delay
Mutual information of the data	mutual
Poincaré section	poincare
Determine the extrema of a time series	extrema
Unstable periodic orbits	upo, upoembed
False nearest neighbours	false_nearest

Top

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Choosing τ with TISEAN

Description of the program: mutual

Estimates the time delayed mutual information of the data. It is the simplest possible realization. It uses a fixed mesh of boxes. No finite sample corrections are implemented so far.

Usage:

mutual [Options]

Everything not being a valid option will be interpreted as a potential datafile name. Given no datafile at all, means read stdin. Also
means stdin

Possible options are:

Option	Description	Default
-l#	number of data to use	whole file
-x#	number of lines to be ignored	0
-c#	column to be read	1
-b#	number of boxes for the partition	16
-D#	maximal time delay	20
-o[#]	output file name	without name given: 'datafile'.mut (or stdin.mut if data were read from stdin) without -o results are written to stdout.
-V#	verbosity level 0: only panic messages 1: add input/output messages	1
-h	show these options	none

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```
lizb> mutual -D 150 cache.dat -o cache.mutual

TISEAN 3.0.1 (C) R. Hegger, H. Kantz, T. Schreiber (1998-2007)

mutual: Estimates the time delayed mutual information
   of the data set

Using cache.dat as datafile, reading column 1
Use 775113 lines.
Opened cache.mutual for writing
…
```

Produces a two-column data file, which you then plot…

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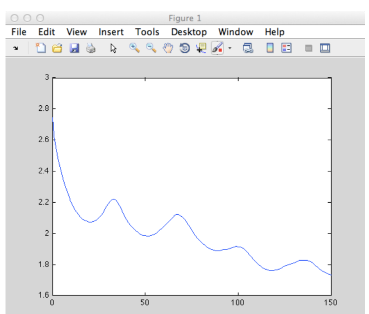

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Choosing *m* with TISEAN

Description of the program: false_nearest

This program looks for the nearest neighbors of all data points in m dimensions and iterates these neighbors one step (more precisely delay steps) into the future. If the ratio of the distance of the iteration and that of the nearest neighbor exceeds a given threshold the point is marked as a wrong neighbor. The output is the fraction of false neighbors for the specified embedding dimensions (see Kennel et al.).

Note: We implemented a new second criterion. If the distance to the nearest neighbor becomes smaller than the standard deviation of the data devided by the threshold, the point is omitted. This turns out to be a stricter criterion, but can show the effect that for increasing embedding dimensions the number of points which enter the statistics is so small, that the whole statistics is meanlingless. **Be aware of this!**

Usage:

false_nearest [Options]

Everything not being a valid option will be interpreted as a potential datafile name. Given no datafile at all, means read stdin. Also -
means stdin

Possible options are:

Option	Description	Default
-l#	number of data to use	whole file
-x#	ignore the first # rows	0
-c#	columns to be read	1
-m#	minimal embedding dimensions of the vectors	1
-M#,#	# of components, max. embedding dimension of the vectors	1,5
-d#	delay of the vectors	1
-f#	ratio factor	2.0
-t#	theiler window	0

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```
lizb> false_nearest data3 -M1,25 -o data3.fnn

TISEAN 3.0.1 (C) R. Hegger, H. Kantz, T.
Schreiber (1998-2007)

false_nearest: Determines the fraction of
false nearest neighbors.

Using data3 as datafile!
Using columns: 1
get_multi_series: first data item(s) used:
2.666059
Use 10000 lines.
Opened data3.fnn for writing
Start for dimension=1
Found 0 up to epsilon=6.281651e-05
Found 0 up to epsilon=8.883596e-05
Found 0 up to epsilon=1.256330e-04
…
```

Produces a multi-column data file, of which you then plot the first two columns…

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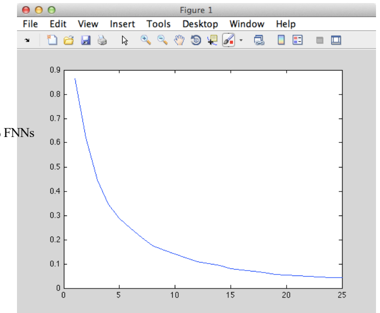

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If Δt is not uniform

~~Theorem (Takens): for τ>0 and m > 2d, reconstructed trajectory is diffeomorphic to the true trajectory~~

~~Conditions: evenly Δt, τ>0 and not sub/multiple of period~~

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How to work around that?

- Use a τ that's many multiples of the Δt
- Interpolate
- Hack & kludge (e.g., "Fuzzy delay-coordinate embedding")
- *Interspike interval embedding*

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Interspike interval embedding

idea: lots of systems generate spikes — hearts, nerves, etc.

if you assume that the spikes are the result of an integrate-and-fire system, then the Δt_i (the spike intervals) have a one-to-one correspondence to some state variable's *integrated* value…

in which case the embedding theorems still hold.

(with the Δt_i as the scalar data)

Sauer *Chaos* **5**:127

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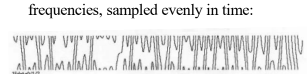
PS8: Several pendulum datasets, at different drive frequencies, sampled evenly in time:



- Construct ω using divided differences and plot ω versus θ.
- Use `mutual` and `false_nearest` to estimate m & τ
- Embed the θ data and plot the reconstructed trajectory. Write your own code to do this; do **not** use TISEAN's `delay`!
- Couple of thought experiments about m & τ

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Reading assignment

- Embedding, in general:
  1. *pp 1–3 (through section II A) of E. Bradley and H. Kantz, "Nonlinear time-series analysis revisited," Chaos* **25**:*097501 (2015).*
     You can download that paper from `arxiv.org/abs/1503.07493`
  2. *pp10–13 of Liz's time-series analysis notes* (through section 3.1)
  3. *section 3.2 of Kantz & Schreiber.*
  4. sections 12.4–5 of Strogatz.
  5. more detail: chapter 9 of Kantz & Schreiber
- Finding d_E and τ:
  1. *section II B of the Bradley/Kantz paper listed above*
  2. *sections 3.3–3.4 of Kantz & Schreiber*
  3. more detail: section 3.2 of Liz's time-series analysis notes

See the "assigned reading for PS8-10" handout on the course webpage

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Other useful papers

- H. Abarbanel, *Analysis of Observed Chaotic Data*, Springer, 1995. Another good book on time-series analysis, from embedding onwards. This is on library reserve.
- E. Bradley and H. Kantz, "Nonlinear Time-Series Analysis Revisited," *Chaos* **25**:097610 (2015). `DOI: 10.1063/1.4917289`.
- A. M. Fraser and H. L. Swinney, "Independent coordinates for strange attractors from mutual information," *Physical Review A* **33**:1134-1140 (1986). The original paper on the use of average mutual information in estimating τ.
- J. Garland and E. Bradley, "Prediction in Projection," *Chaos* **25**:123108 (2015) `DOI: 10.1063/1.4936242`.
- M. B. Kennel *et al.*, "Determining minimum embedding dimension using a geometrical construction," *Physical Review A*, **45**:3403-3411 (1992). The original paper on the use of average mutual information in estimating m. (A synopsis of this algorithm appears on page 17 of my notes on nonlinear time-series analysis, which you can find on the course webpage. If you want the full paper, it's in the "Coping..." collection.)

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Other useful papers, cont.

- N. Packard *et al.*, "Geometry from a Time Series," *Physical Review Letters* **45**:712 (1980). The first of the two original papers on embedding; also in the "Coping..." collection.
- T. Sauer *et al.*, "Embedology," *Journal of Statistical Physics*, **65**:579-616 (1991). The definitive paper about the broader field of embedding, but long & highly technical. Proves $m < 2d_{cap}$.
- T. Sauer, "Interspike interval embedding of chaotic signals," *Chaos*, **5**:127 (1995). An embedding method that uses the intervals between discrete events, rather than evenly-sampled measurements of the whole time line.
- F. Takens, "Detecting strange attractors in fluid turbulence," in *Dynamical Systems and Turbulence* (D. Rand and L.-S. Young, eds.), Springer, Berlin, 1981. The other original paper on delay-coordinate reconstruction.

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Where we are in the "attractor characterization" chunk of this course:

1. √ attractor and basin definitions
2. √ state-space deformation math (PS7)
3. √ how to find attractors, basins
4. √ delay-coordinate embedding in more depth (PS8)
5. how to calculate λ (PS9)
6. how to calculate fractal dimensions (PS10)
7. how to find UPOs

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Lyapunov exponents:

λ: Lya<u>PU</u>nov or Ly<u>A</u>Punov?

- from ODEs
- from experimental data

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Lyapunov exponents: the big picture

- λ > 0 is a common definition of chaos
- λ is invariant under diffeomorphism
  Recall: for τ>0 and m > 2d, reconstructed trajectory is ***diffeomorphic*** to the true trajectory
- **So the λs of the reconstructed trajectory are the same as the λs of the original trajectory!**
  (with one caveat)

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Computing λ_i of a flow, if you have the equations:

run PS7 and compute the eigenvalues of the δ matrix.

issues:

- different initial conditions, run lengths (cf., local λs)
- limit as time goes to infinity (floating-point limits, ill-conditioned matrices, …)

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If you only have data:

Harder.

x	t
1.3	0.1
1.2	0.2
1.0	0.3
0.8	0.4
1.1	0.5
1.4	0.6
1.6	0.7

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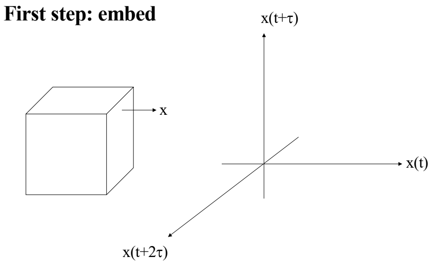
First step: embed



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Takens says:

The λ of the reconstructed trajectory is the same as the λ of the original trajectory

Theorem: for $\tau > 0$ and $m > 2d$, reconstructed trajectory is ***diffeomorphic*** to the true trajectory

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Problems to surmount in computing λ from data:

1. don't have equations
2. can't drop points at will
3. can't control timestep or traj length
4. data may be noisy
5. don't know m, τ

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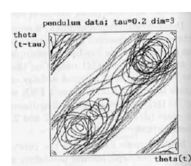
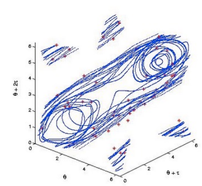
Noise:





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Problems to surmount in computing λ from data:

1. don't have equations
2. can't drop points at will
3. can't control timestep or traj length
4. data may be noisy
5. don't know m, τ

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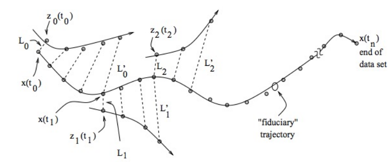
Wolf's algorithm for calculating λ_1:



284

Eckmann & Ruelle: issues

- these approximations to the Jacobian are local estimates, so you need to rebuild them in every patch
- and those patches can't be too big, lest they mush different dynamics together
- building an approximation to the Jacobian is numerically fraught
- especially in the face of a limited sampling of the dynamics (e.g., small patch)
- and it doesn't work at all if there's noise

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How about marrying the good features of those two approaches?

• Tracking the spreading of neighboring points is a good idea, but sensitive to noise & data issues
• Doing that over a bunch of points ➔ noise immunity!

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Kantz's algorithm:

1. Choose point K
2. Look at the points around it (ε)
3. Measure how far they are from K
4. Average those distances
5. Watch how that average grows with time (Δn)
6. Take the log, normalize over time ➔ S(Δn)
7. …?

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Kantz's algorithm:

1. Choose point K
2. Look at the points around it (ε)
3. Measure how far they are from K
4. Average those distances
5. Watch how that average grows with time (Δn)
6. Take the log, normalize over time ➔ S(Δn)
7. Repeat for lots of points K and average the S(Δn)

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Picking ε in Kantz's algorithm:

"ε should be as small as possible, but large enough so that each point has at least a few neighbors" (p70, Kantz & Schreiber)

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Kantz's algorithm:

1. Choose point K
2. Look at the points around it (ε)
3. Measure how far they are from K
4. Average those distances
5. Watch how that average grows with time (Δn)
6. Take the log, normalize over time ➔ S(Δn)
7. Repeat for lots of points K and average the S(Δn)

How to unwind the λ from that "stretching factor"?
Make a plot…

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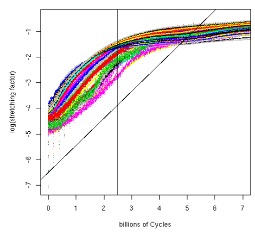
What S(Δn) looks like, in practice:



- The slope of the *scaling region* is the λ
- Why the asymptote?
- Different colors: different *m* values. Why did I do that?

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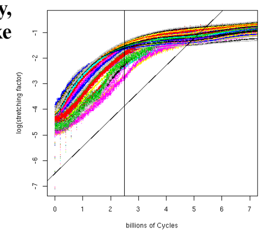
If you're lucky, things look like this.



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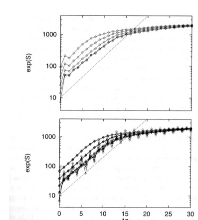
Or this:



This is fig 5.3 in Kantz & Schreiber

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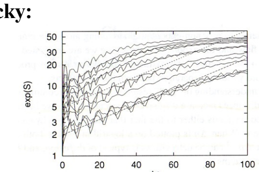
If you're not lucky:



- Scaling region…? (note only two fit lines)

This is fig 5.4 in Kantz & Schreiber

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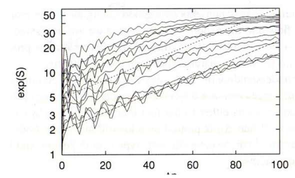
Aside: what do you think those oscillations might be?



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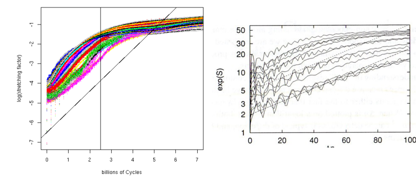

- The slope of the scaling region—~~iff one exists~~—is the λ.
- But what defines a scaling region, anyway?

➔ Be very very careful if you try to automate this…

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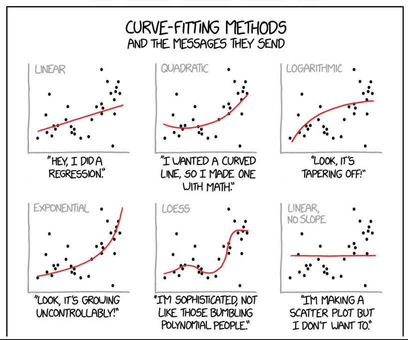

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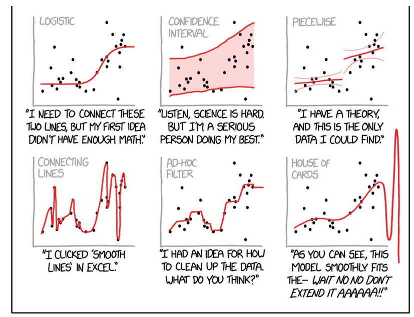

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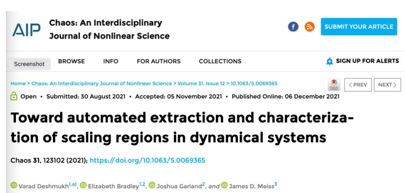

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Toward automated extraction and characterization of scaling regions in dynamical systems

Chaos **31**, 123102 (2021); https://doi.org/10.1063/5.0069365

Varad Deshmukh[1,a)], Elizabeth Bradley[1,2], Joshua Garland[2], *and* James D. Meiss[3]

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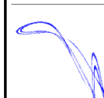
Software tools for calculating λ:



TISEAN
Nonlinear Time Series Analysis

Rainer Hegger
Holger Kantz
Thomas Schreiber

Go to Version 3.0.1 (released March 2007)

Go to Version 2.1 (released December 2000)

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TISEAN 3.0.1: Table of Contents

TISEAN 3.0.1

- TISEAN home
- Table of Contents
- General Manual
- Surrogates Manual
- Tutorial
- Usage Notes
- Installation
- Problems

All programs in alphabetical order

Sections

- Generating time series
- Linear tools
- Utilities
- Stationarity
- Embedding and Poincaré sections
- Prediction
- Noise reduction
- Dimension and entropy estimation
- Lyapunov exponents
- Surrogate data
- Spike trains
- XTisean
- Unsupported

Generating time series

A few routines are provided to generate test data from simple equations. Since there are powerfull packages (for e
Helena Nusse and Jim Yorke) that can generate chaotic data, we have only included a minimal selection here.

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Lyapunov exponents are an important means of quantification for unstable systems. They are however difficult to estimate from a time series. Unless low dimensional, high quality data is at hand, one should not attempt to calculate the full spectrum. Try to compute the maximal exponent first. The two implementations differ slightly. While lyap_k implements the formula by Kantz, lyap_r uses that by Rosenstein et al. which differs only in the definition of the neighbourhoods. We recommend to use the former version, lyap_k.

The estimation of Lyapunov exponents is also discussed in the introduction paper. A recent addition is a programm to compute finite time exponents which are not invariant but contain additional information.

Maximal exponent	lyap_k, lyap_r
Lyapunov spectrum	lyap_spec

Description of the program: lyap_k

The program estimates the largest Lyapunov exponent of a given scalar data set using the algorithm of Kantz.

Usage:

lyap_k [Options]

Everything not being a valid option will be interpreted as a potential datafile name. Given no datafile at all, means read stdin. Also - means stdin

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This syntax is different for some implementations

```
lizb> lyap_k -d 150 -m 3 -M 5 -o data3.lyapk data3

TISEAN 3.0.1 (C) R. Hegger, H. Kantz, T. Schreiber
(1998-2007)

lyap_k: Estimates the maximal Lyapunov exponent using
the Kantz algorithm

Using data3 as datafile, reading column 1
Use 10000 lines.
Opened data3.lyapk for writing
epsilon= 6.281651e-03
epsilon= 1.117053e-02
epsilon= 1.986433e-02
epsilon= 3.532432e-02
epsilon= 6.281651e-02
```

Produces a three-column data file with one section for the S(Δn) at each *m*, of which you then plot the first two columns:

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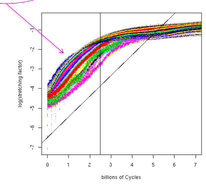
```
lyap_k -d 150 -m 3 -M 5 -o data3.lyapk data3
```



- Dimension matters: reconstructions with too-low *m* values aren't "embeddings" and may not have the correct topology, so their λ results won't be right.
- (That's about the structure of the dynamics, BTW, not the algorithm for computing the dynamical invariant.)
- Too-high m values also cause problems in practice.
- So there's a sweet spot.

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Option	Description	Default
-l#	number of data to be used	whole file
-x#	number of lines to be ignored	0
-c#	column to be read	1
-M#	maximal embedding dimension to use	2
-m#	minimal embedding dimension to use	2
-d#	delay to use	1
-r#	minimal length scale to search neighbors	(data interval)/1000
-R#	maximal length scale to search neighbors	(data interval)/100
-##	number of length scales to use	5
-n#	number of reference points to use	all
-s#	number of iterations in time	50
-t#	'theiler window'	0
-o#	output file name	without file name: 'datafile'.lyap (or stdin.lyap if the data were read from stdin)
-V#	verbosity level 0: only panic messages 1: add input/output messages 2: add statistics for each iteration	3
-h	show these options	none

Description of the Output:

For each embedding dimension and each length scale the file contains a block of data consisting of 3 columns

1. The number of the iteration
2. The logarithm of the stretching factor (the slope is the Lyapunov exponent if it is a straight line)
3. The number of points for which a neighborhood with enough points was found

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λ algorithms are sensitive to all of those parameters!

TISEAN supports exploration of those effects:

```
Opened data3.lyapk for writing
epsilon= 6.281651e-03
epsilon= 1.117053e-02
epsilon= 1.986433e-02
epsilon= 3.532432e-02
epsilon= 6.281651e-02
```

-r#	minimal length scale to search neighbors	(data interval)/1000
-R#	maximal length scale to search neighbors	(data interval)/100
-##	number of length scales to use	5

"interval" = "diam(A)"

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λ algorithms are also sensitive to traj length, data interval, …

Solution: vary Δn, downsample, use chunks, etc.

-s#	number of iterations in time	50

-l#	number of data to be used	whole file
-x#	number of lines to be ignored	0

307

λ algorithms are also sensitive to the position and length of the fiduciary trajectory

Solution: start from different ICs

(this is built into `lyap_k`)

-n#	number of reference points to use	all
-s#	number of iterations in time	50
-t#	'theiler window'	0

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Time and space: the Theiler window:

-n#	number of reference points to use	all
-s#	number of iterations in time	50
-t#	'theiler window'	0

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λ algorithms need good data…

Lots of it, sampled evenly in time, collected with a sensor whose output is a smooth measurement function on the state space.

Solution:

- Be aware of the theoretical limits on how many data points you need for this to work (Smith, Tsonis, …).
- Make sure your timebase is even and that the measurement function is smooth (e.g., no counter overflows!)
- Another good reason to downsample, use chunks, etc.— **to know if you've got enough data**

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PS9:

CSCI 4446 and CSCI 5446:
- Implement Wolf's algorithm
- Test it on a Lorenz trajectory
- Curtain call for PS7

CSCI 5446:
- Repeat the λ calculation with `lyap_k`

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Reading assignment

- Lyapunov exponents:
  1. *chapter 5 of Kantz & Schreiber*
  2. *Liz's notes on Wolf's algorithm.*
  3. section III 2 of Bradley/Kantz paper listed above
  4. section 10.5 of Strogatz covers Lyapunov exponents of 1D maps
  5. more detail: sections 3.3-4 of Parker & Chua (see the course webpage for a link to a pdf of this book).

See the "assigned reading for PS8-10" handout on the course webpage

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Other λ algorithms in TISEAN

Lyapunov exponents are an important means of quantification for unstable systems. They are however difficult to estimate from a time series. Unless low dimensional, high quality data is at hand, one should not attempt to calculate the full spectrum. Try to compute the maximal exponent first. The two implementations differ slightly. While lyap_k implements the formula by Kantz, lyap_r uses that by Rosenstein et al. which differs only in the definition of the neighbourhoods. We recommend to use the former version, lyap_k.

The estimation of Lyapunov exponents is also discussed in the introduction paper. A recent addition is a programm to compute finite time exponents which are not invariant but contain additional information.

Maximal exponent	lyap_k, lyap_r
Lyapunov spectrum	lyap_spec

Gives you a lot more information, which may or may not be trustworthy

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"All the λs…"

- recall: one λ for each dimension
- embedding: may add extra dimensions
- …which adds spurious λs!

(extra dimensions magnify noise, too…remember why?)

See § 11.2.4 and p209 of Kantz & Schreiber—and Parlitz paper on bibliography if you want more detail.

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Local vs. global λ:

λ is a *long-term average* property of an attractor
Locally, the behavior can be very different.

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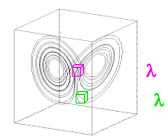
Local λs*:

- limited region of attractor
- short time span



* Called finite-time λs (FTLEs) by many others, incl Kantz & Schreiber

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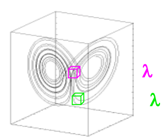
Local λs: issues



Caveat: averages filter out noise. Long runs filter out data and numerical effects. Local λ algorithms do neither…

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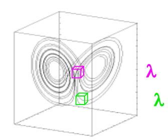
How to calculate these?



…unfortunately not easy with TISEAN

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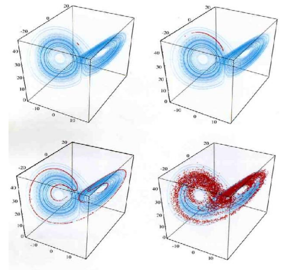
Coda: λs and information…



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Dimension:

1. Euclidean: minimum # of dimensions to specify point uniquely
2. Topological: "manifold dimension" — dimension of Euclidean space that manifold resembles locally
3. Dynamic system: # of state variables needed to uniquely describe the dynamics
4. Fractal…

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Fractal dimension:

- Capacity (pp167-169 and 177-181 of Parker & Chua; scan of this is on the course webpage)
  - Box counting
- Similarity
- Information
- Correlation (chapter 6 of Kantz & Schreiber; PS10 for CSCI 5446)
- Kth nearest neighbor
- Lyapunov
- …see §7.1 of Parker & Chua or §11.3 of Kantz & Schreiber for more examples

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Versions we'll play with for PS10:

CSCI 4446 and CSCI 5446: Box counting

CSCI 5446: Correlation (in TISEAN)

Reading assignment: on the "readings for PS8-10" sheet.

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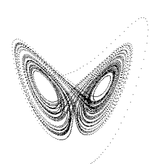
Challenge: computing dynamical invariants from a finite-length, finite-precision sampling of a trajectory from a dynamical system



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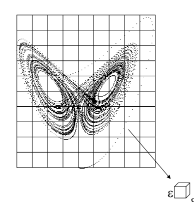
Capacity dimension: The naïve implementation…



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- initialize an n-dimensional array, m_i elements on a side, to all 0s
- set array elements to 1 as corresponding boxes are entered
- count 1s = $N(\varepsilon)$
- let ε go to 0…(ouch!)
- use the formula

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Naïve implementation: issues

- memory
- region size
- also sampling effects (length, point spacing, noise, etc.)

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Another way to calculate fractal dimension:

- compute auxiliary dimension D_A

(which is efficient because you can use Monte Carlo techniques to do it)

- and then exploit relationship between it, the ambient dimension of the space, and the capacity dimension
- Hunt & Sullivan paper (on PS10 bibliography); Parker & Chua pp180-181.

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Another way to calculate fractal dimension:

- compute auxiliary dimension D_A

(which is efficient because you can use Monte Carlo techniques to do it)

NB: in this case, we know this!

- and then exploit relationship between it, the ambient dimension of the space, and the capacity dimension
- Hunt & Sullivan paper (on PS10 bibliography); Parker & Chua pp180-181.

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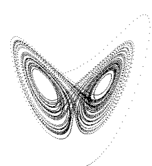
Hunt & Sullivan:



Need to change all A to A-bar

- have: a set A (e.g., an attractor)
- generate k random points
- count the # that are within ε of A & multiply by ε^d
- that gives you an estimate of $A(\varepsilon)$
- which you then use in a similar "scale-with-ε" formula to get D_A

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The formula…

$d_{cap} = n - D_A$

The derivation…

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Monte Carlo technique for estimating A(ε)

- have: a set A (e.g., an attractor)
- generate *k* random points
- count the # that are within ε of A
- that gives you an estimate of A(ε)
- which you then use in a similar formula
- and you can control the quality/effort of the A(ε) calculation…how?

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Monte Carlo technique for estimating A(ε)

- have: a set A (e.g., an attractor)
- generate k random points
- count the # that are within ε of A
- that gives you an estimate of A(ε)
- which you then use in a similar formula
- and you can control the quality/effort of the A(ε) calculation…how?
- what's the $$ step? how to improve?

332

Monte Carlo technique for estimating A(ε)

- have: a set A (e.g., an attractor)
- generate k random points
- count the # that are within ε of A
- that gives you an estimate of A(ε)
- which you then use in a similar formula
- and you can control the quality/effort of the A(ε) calculation…how?
- what's the $$ step? how to improve?
- how would this parallelize?

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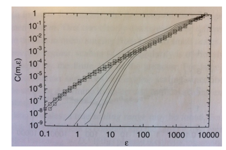
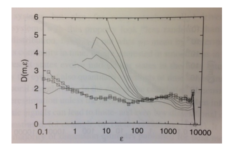
C(m,ε) and D(m,ε)…

Note: they're varying m, too…





And these are *good* ones…

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And d2 has lots of free parameters:

Option	Description	Default
-l#	number of data points to be used	whole file
-x#	number of lines to be ignored	0
-d#	delay for the delay vectors	1
-M#,#	# of components,maximal embedding dimension	1,10
-c#	columns to be read	1,...,# of components
-t#	theiler window	0
-R#	maximal length scale	(max data interval)
-r#	minimal length scale	(max data interval)/1000
-##	number of epsilon values	100
-N#	maximal number of pairs to be used (0 means all possible pairs)	1000
-E	use data that is normalized to [0,1] for all components	not set (use natural units of the data)
-o[#]	output file name (without extensions)	'datafile'[.c2][.d2][.h2][.stat] (or if data were read from stdin: stdin[.c2][.d2][.h2][.stat])
-V#	verbosity level 0: only panic messages 1: add input/output messages 2: add input/output messages each time output file is opened	1
-h	show these options	none

(which is good, though, because it makes it easy to do your due diligence!)

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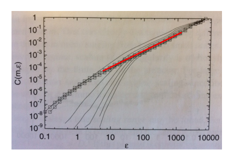
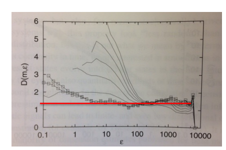
How to use those plots:



Caveat: D(ε) has all the the issues associated with numerical differentiation (cf., the first problem on PS6)



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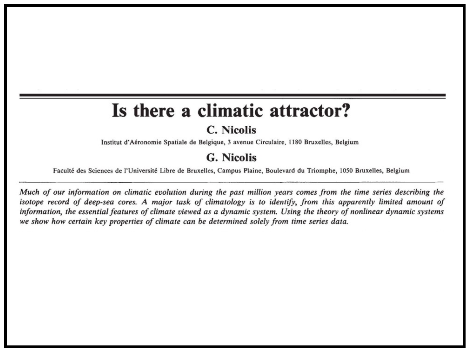

Is there a climatic attractor?

C. Nicolis

Institut d'Aéronomie Spatiale de Belgique, 3 avenue Circulaire, 1180 Bruxelles, Belgium

G. Nicolis

Faculté des Sciences de l'Université Libre de Bruxelles, Campus Plaine, Boulevard du Triomphe, 1050 Bruxelles, Belgium

Much of our information on climatic evolution during the past million years comes from the time series describing the isotope record of deep-sea cores. A major task of climatology is to identify, from this apparently limited amount of information, the essential features of climate viewed as a dynamic system. Using the theory of nonlinear dynamic systems we show how certain key properties of climate can be determined solely from time series data.

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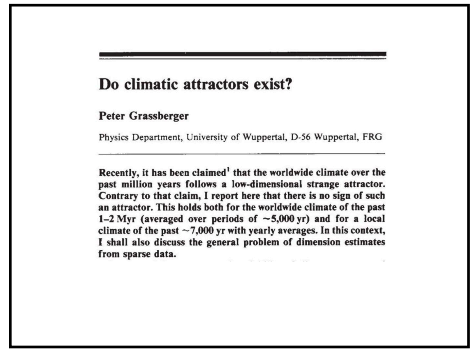

Do climatic attractors exist?

Peter Grassberger

Physics Department, University of Wuppertal, D-56 Wuppertal, FRG

Recently, it has been claimed[1] that the worldwide climate over the past million years follows a low-dimensional strange attractor. Contrary to that claim, I report here that there is no sign of such an attractor. This holds both for the worldwide climate of the past 1–2 Myr (averaged over periods of ~5,000 yr) and for a local climate of the past ~7,000 yr with yearly averages. In this context, I shall also discuss the general problem of dimension estimates from sparse data.

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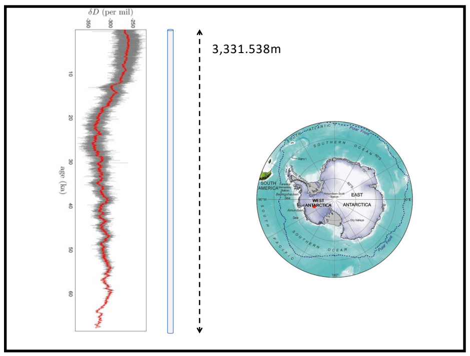

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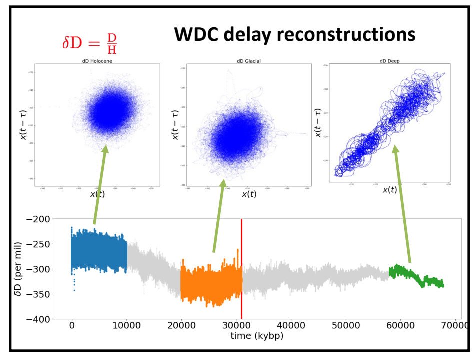

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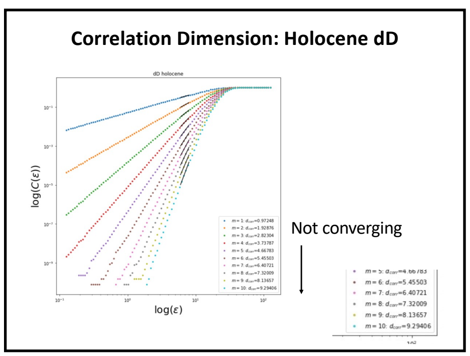

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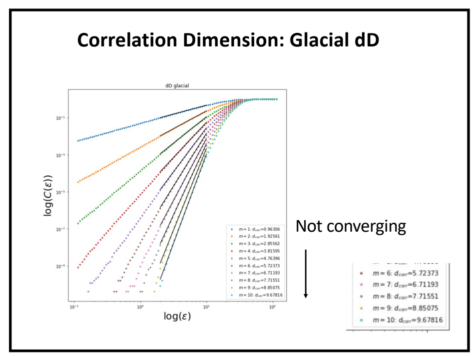

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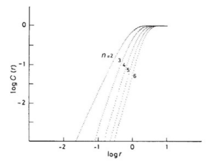

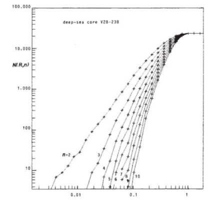

Nicolis & Nicolis   Grassberger

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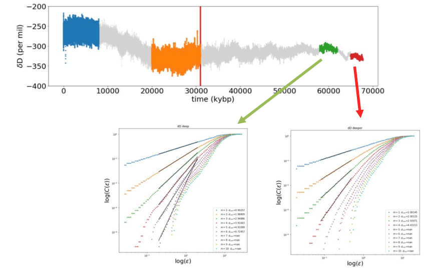
dD Correlation Dimension: Further Down



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So there's STILL not enough data…

- Smith, Tsonis limits
- Eckmann & Ruelle paper (PS10 biblio)

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Fractal dimension algorithms we covered:

- one brute-force algorithm
- an algorithm that uses a neat hack
- the algorithm that's under the hood of TISEAN's `d2` tool

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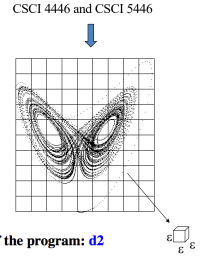
PS10:

CSCI 4446 and CSCI 5446



CSCI 5446
(and compare
results)

Description of the program: d2

This program estimates the correlation sum, the correlation dimension and the correlation entropy of a given, possibly n
assisted search algorithm and is quite fast as long as one is not interested in large length scales. All length scales are cor
is written every 2 min (real time, not cpu time). It is possible to set a maximum number of pairs. If this number is reach
scale will no longer be treated for the rest of the estimate.

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Reading assignment

- Fractal dimensions:
  1. *review sections 11.4 and 11.5 of Strogatz.*
  2. *chapter 6 of Kantz & Schreiber (through 6.7 only)*
  3. section 7.1 of Parker and Chua has some good discussion of other kinds of dimensions besides capacity and correlation. There's a scan of this section on the course webpage, as well as a link from which you can download the whole book.
  4. F. Hunt and F. Sullivan, "Efficient Algorithms for Computing Fractal Dimensions," in *Dimensions and Entropies in Chaotic Systems*, Springer-Verlag, Berlin, 1985, pp 74–81. A synopsis of this paper appears on pp 180–181 of Parker & Chua

See the "assigned reading for PS8-10" handout on the course webpage

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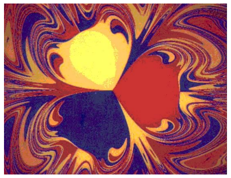
Recall: why fractal dimension…

Most (but not all) chaotic systems have some fractal state-space structure



(pendulum swinging over three magnets)

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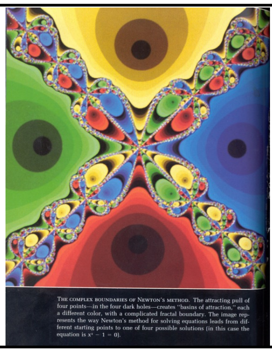

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Unstable periodic orbits:

- densely embedded in any chaotic attractor
- infinite number of them, of all periods

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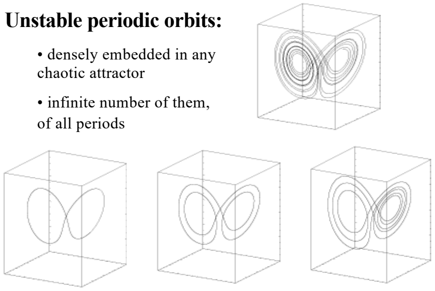
Unstable periodic orbits:

- densely embedded in any chaotic attractor
- infinite number of them, of all periods



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© 2006 Jos Leys and Etienne Ghys; www.josleys.com

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Algorithm:

- construct a (spatial) Poincare section of the attractor
- pick a point on Σ
- look for close returns (within ε)

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Algorithm:

- construct a Poincare section
- pick a point
- look for close returns (within ε)
- *repeat for all points on the section*

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Algorithm:

- construct a Poincare section
- pick a point
- look for close returns (within ε)
- repeat for all points on the section
- *bin them according to how many times they went around before returning*

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Algorithm:

- construct a Poincare section
- pick a point
- look for close returns (within ε)
- repeat for all points on the section
- bin them according to how many times they went around before returning
- *cluster and average*

Gunaratne 1989, So 1996 (on PS biblio)

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That concludes the "attractor characterization" chunk of the course:

1. attractor and basin definitions
2. state-space deformation math
3. how to find attractors, basins
4. embedding
5. how to calculate λ
6. how to calculate fractal dimensions
7. how to find UPOs

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Next up: Hamiltonian dynamics

- Lagrangians and Hamiltonians
- gravitation
- rigid-body dynamics
- *variational stuff*
- *oscillations*
- *perturbation theory*

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aka "Classical Mechanics"

- Problem Set 10: fractal dimension. Click here for a detailed list of the assigned reading for this topic and here for a scan of some of that reading (pp166-191 of Parker & Chua).
- Problem Set 11: playing with bike wheels, writing Lagrangians, and starting to explore the two-body problem for a binary star. This material is covered in the first few sections of the classical mechanics notes listed below. Click here for a picture defining true anomaly, here for an interactive simulator that you can use to explore orbits, and here for a wonderful lecture on dynamical toys like tops and rattlebacks.
- Some hints about presentations.
- Problem Set 12: integrating the two-body equations. See section 4 of the classical mechanics notes listed below. Here's an interesting link that Kristine Washburn found about a variant of this problem. You may also wish to check out the n-body section of Colonna's webpage (listed below). Here is the "Chaos Hits Wall Street" article that's on the reading assignment.
- Problem Set 13: integrating the three-body equations for a binary-field star collision. See section 4.2 of the classical mechanics notes listed below. The "visualization of dynamical systems" page in the "interesting links" list below has source code for a lovely visualization of this problem.

Liz's videos and written materials:

- The Complexity Explorer version of this course. Click on "Lectures" to get to the course materials.
- The mapping of which videos and quizzes on that website go with which CSCI 4446/5446 lectures
- Notes on Taylor series and error in numerical methods (not required, but possibly useful)
- Notes on ordinary differential equations (ODEs) and solving them numerically
- Notes on the variational equation
- A book chapter and a review article about nonlinear time-series analysis
- A schematic of Wolf's algorithm for computing the largest positive Lyapunov exponent
- Notes on classical mechanics

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Lagrangians and Hamiltonians:

- different ways to write F=ma
- problem: write eqns of motion (EOM) for a Hamiltonian dynamical system

The approach:

- choose generalized coordinates
- write down kinetic energy
- write down potential energy
- put them together
- take some partial derivatives
- plug into a formula

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Generalized coordinates:

- *constraint forces* reduce the effective dimension of the problem
- figure out how many degrees of freedom the problem really has and pick one coordinate that "measures" the system state "along" each of them
- q_i: generalized coordinates
- if you can write an ODE in q_i and find its solution $q_i(t)$, you're done

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- *cyclic* coordinate: if missing from the Lagrangian
- serious implications: symmetry, conserved quantity

- *constant (*or *integral) of the motion*
- and this is obvious just from the symbols in the equation!

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The connection to chaos:

A system with *n* degrees of freedom is *integrable* <=> it has *2n* constants of the motion.

Noninetegrability is NASC for chaos.

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- *cyclic* coordinate: if missing from the Lagrangian
- serious implications: symmetry, conserved quantity

- *constant (*or *integral) of the motion*
- and this is obvious just from the symbols in the equation…
- …*iff* you picked the coordinates right.

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Hamiltonian mechanics

- Lagrangians and Hamiltonians
- gravitation
- rigid-body dynamics
- *variational stuff*
- *oscillations*
- *perturbation theory*

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Hamiltonian mechanics

- √ Lagrangians and Hamiltonians (section 2 in notes)
- gravitation (section 4 in notes)
- rigid-body dynamics (section 3 in notes)
- *variational stuff*
- *oscillations*
- *perturbation theory*

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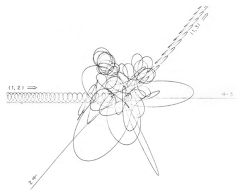

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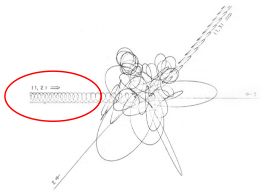
PS11 & 12:



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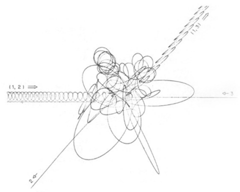
PS13:



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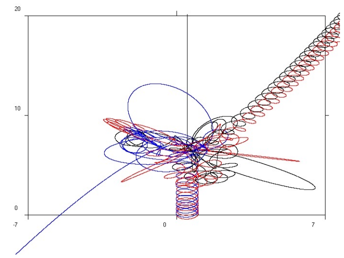

(Paul Kolesnikoff, 2005)

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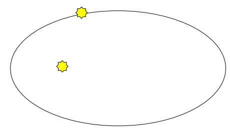
Binary stars:



These are remarkably common!

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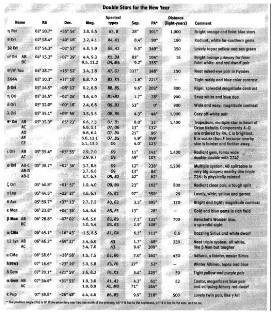

Double Stars for the New Year

Name		RA	Dec.	Mag.	Spectral types	Sep.	PA*	Distance (light-years)	Comment
η Per		$02^h\,50.7^m$	+55° 54′	3.8, 8.5	K3, B	28″	301°	1,000	Bright orange and faint blue stars
θ Eri		$02^h\,58.4^m$	−40° 18′	3.2, 4.1	A4, A1	8.4″	90°	160	Radiant, white far-southern gems
32 Eri		$03^h\,54.3^m$	−02° 57′	4.8, 5.9	G8, A2	6.9″	349°	350	Lovely topaz yellow and sea green
o^2 Eri	AB	$04^h\,15.3^m$	−07° 39′	4.4, 9.3	K1, DA	83″	104°	16	Bright orange primary far from
	BC			9.5, 11.2	DA, M4	9.2″	335°		faint white- and red-dwarf pair
θ^1/θ^2 Tau		$04^h\,28.7^m$	+15° 52′	3.4, 3.8	A7, G7	337″	348°	150	Neat naked-eye pair in Hyades
Σ644		$05^h\,10.3^m$	+37° 18′	6.8, 7.0	B2, K3	1.6″	221°	–	Tight ruddy and blue color contrast
β Ori		$05^h\,14.5^m$	−08° 12′	0.2, 6.8	B8, B5	9.6″	203°	800	Rigel, splendid magnitude contrast
η Ori		$05^h\,24.5^m$	−02° 24′	3.6, 4.9	B1+B2	1.7″	78°	900	Snug white and blue duo
δ Ori		$05^h\,32.0^m$	−00° 18′	2.4, 6.8	O9, B2	53″	0°	900	Wide and easy; magnitude contrast
λ Ori		$05^h\,35.1^m$	+09° 56′	3.5, 5.5	O8, B0	4.3″	44°	1,000	Cozy off-white pair
θ^1 Ori	AB	$05^h\,35.3^m$	−05° 23′	6.6, 7.5	O7, B1	8.8″	31°	1,400	Trapezium, multiple star in heart of
	AC			6.6, 5.1	O7, O6	13″	132°		Orion Nebula. Components A–D
	AD			6.6, 6.4	O7, B0	21″	96°		are ordered by RA, C is brightest.
	AE			6.6, 11.1	O7, B5	4.9″	349°		E is easier than F because nearby
	CF			5.1, 11.5	O6	4.0″	123°		star is fainter and farther away.
ι Ori	AB	$05^h\,35.4^m$	−05° 55′	2.9, 7.0	O9	11″	141°	1,400	Radiant gem, forms wide
	AC			2.9, 9.7	O9	49″	103°		double-double with Σ747
σ Ori	AB-C	$05^h\,38.7^m$	−02° 36′	3.7, 8.8	O9	12″	238°	1,200	Multiple system; AB splittable in
	AB-D			3.7, 6.6	O9	13″	84°		very big scopes; nearby dim triple
	AB-E			3.7, 6.3	O9, B2	42″	62°		Σ761 is physically related
ζ Ori		$05^h\,40.8^m$	−01° 57′	1.9, 4.0	O9, B0	23″	165°	800	Radiant close pair, a tough split
γ Lep		$05^h\,44.5^m$	−22° 27′	3.6, 6.3	F6, K2	97″	350°	29	Lovely, wide; yellow and garnet
θ Aur		$05^h\,59.7^m$	+37° 13′	2.7, 7.2	A0, G2	3.5″	305°	170	Bright and tight; magnitude contrast
ε Mon		$06^h\,23.8^m$	+04° 36′	4.4, 6.6	A5, F5	13″	28°	–	Gold and blue gems in rich field
β Mon	AB	$06^h\,28.8^m$	−07° 02′	4.6, 5.0	B3, B3	7.1″	133°	700	Herschel's Wonder Star,
	BC			5.0, 5.4	B3, B3	2.9″	108°		a splendid sight
α CMa		$06^h\,45.1^m$	−16° 43′	-1.5, 8.5	A1, DA	6.7″	111°	8.6	Dazzling Sirius and white dwarf
12 Lyn	AB	$06^h\,46.2^m$	+59° 27′	5.4, 6.0	A3	1.7″	68°	230	Neat triple system, all white,
	AC			5.4, 7.0	A3	9.4″	309°		like β Mon but tougher
ε CMa		$06^h\,58.6^m$	−28° 58′	1.5, 7.5	B2, B6	7.0″	161°	430	Adhara, a fainter, easier Sirius
h3945		$07^h\,16.6^m$	−23° 19′	5.0, 5.8	K3, F0	27″	52°	–	Winter Albireo, topaz and blue
δ Gem		$07^h\,20.1^m$	+21° 59′	3.6, 8.2	F0, K3	5.4″	225°	59	Tight yellow and purple pair
α Gem	AB	$07^h\,34.6^m$	+31° 53′	1.9, 3.0	A1, A2	4.3″	61°	52	Castor, magnificent blue pair
	AC			1.9, 8.9	A1, M0	71″	164°		and eclipsing-binary red dwarf
k Pup		$07^h\,38.8^m$	−26° 48′	4.4, 4.6	B6, B5	9.9″	318°	500	Lovely twin pair, like γ Ari

* The position angle (PA) is 0° if the secondary star lies due north of the primary, 45° if it lies to the northeast, 90° if it lies to the east, and so on.

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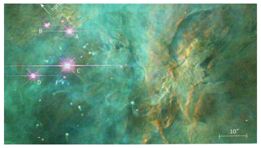
The "Trapezium":



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Hamiltonian mechanics

- Lagrangians and Hamiltonians
- gravitation
- rigid-body dynamics
- *variational stuff*
- *oscillations*
- *perturbation theory*

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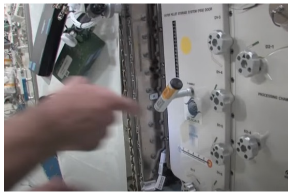

The context in which I'd really like to be doing that demo:
http://www.youtube.com/watch?v=1n-HMSCDYtM
(on the course webpage)

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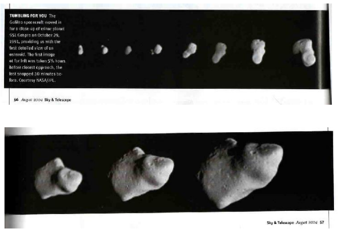
Rigid-body dynamics



New Horizons: all of Pluto's moons doing this too (except Charon, which is spin-locked)

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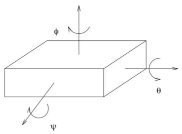
EOMs for a rigid body:



<u>State:</u> how much it's rotating around each axis: $[\omega_1\ \omega_2\ \omega_3]^T$

<u>Dynamics:</u> how those values evolve

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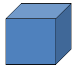
EOMs are "nice" if we use the *principal axes* as coordinates.

- triaxial

- symmetric



- axisymmetric

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The ELPH (**Euler Lagrange Poincare Hamill**) **Equations:**

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned}$$

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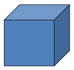
The ELPH equations for a *symmetric* body: ($I_1 = I_2 = I_3$)?



$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned}$$

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ELPH equations for a *triaxial* body?

(I_1 != I_2 != I_3)

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned}$$

p8 of class mech notes

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Thought experiment: what are the fixed points of the ELPH equations of a triaxial body?

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 \end{aligned}$$

And what are those, physically?

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ELPH equations for the triaxial case:

Stability analysis:
- find the fixed points
- linearize there…

$$\begin{aligned} 0 &= \omega_2\omega_3 \\ 0 &= \omega_3\omega_1 \\ 0 &= \omega_1\omega_2 \end{aligned}$$

$$\begin{aligned} \dot{\omega}_1 &= \frac{1}{6}\omega_2\omega_3 \\ \dot{\omega}_2 &= -\frac{3}{4}\omega_3\omega_1 \\ \dot{\omega}_3 &= \frac{2}{3}\omega_1\omega_2 \end{aligned}$$

$$J = \begin{bmatrix} 0 & \frac{1}{6}\omega_3 & \frac{1}{6}\omega_2 \\ -\frac{3}{4}\omega_3 & 0 & -\frac{3}{4}\omega_1 \\ \frac{2}{3}\omega_2 & \frac{2}{3}\omega_1 & 0 \end{bmatrix}$$

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At (1,0,0)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{3}{4} \\ 0 & \frac{2}{3} & 0 \end{bmatrix}$$

$$\begin{aligned} \dot{\omega}_1 &= 0 \\ \dot{\omega}_2 &= -3/4\,\omega_3 \\ \dot{\omega}_3 &= 2/3\,\omega_2 \end{aligned}$$

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At (0,1,0)

$$\begin{bmatrix} 0 & 0 & 1/6 \\ 0 & 0 & 0 \\ 2/3 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \dot{\omega}_1 &= 1/6\,\omega_3 \\ \dot{\omega}_2 &= 0 \\ \dot{\omega}_3 &= 2/3\,\omega_1 \end{aligned}$$

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At (0,0,1)

$$\begin{bmatrix} 0 & 1/6 & 0 \\ -3/4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \dot{\omega}_1 &= 1/6\,\omega_2 \\ \dot{\omega}_2 &= -3/4\,\omega_1 \\ \dot{\omega}_3 &= 0 \end{aligned}$$

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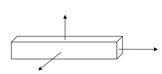
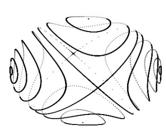
Dynamics of a triaxial body:





(fig from SICM)

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https://climate.nasa.gov/news/2948/milankovitch-orbital-cycles-and-their-role-in-earths-climate/

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The timing:

Liz's 8.06 problem set*: 33,236 years

The solutions to that problem set: "~23,000 years"

* 1987.

Recent consensus:

~25,770 yrs @ 23.469 degrees

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Implications of precession:

- stars
- Zodiac
- seasons…

Poll: when will the winter equinox happen in 12K years?

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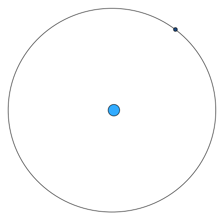
Another effect of nonspherical things pulling on each other: spin lock



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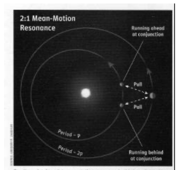
Lots of other tidal effects, such as *mean-motion resonances*:



Gravity maintains a 2:1 mean-motion resonance in this hypothetical two-planet system. When the inner planet is running slightly ahead of the outer planet at periastron, the gravity between the two planets pulls the inner planet back, so later conjunctions occur closer to periastron. If the inner planet falls behind at periastron, the outer planet's gravity will tug it forward. The inner planet thus librates (oscillates) around the perfect 2:1 ratio, but over long time scales, the inner planet orbits exactly twice as often as the outer planet.

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Figure 2. Physicists Wolfgang Pauli and Niels Bohr observe the motions of a Tippe Top toy at the inauguration of a physics institute in Lund, Sweden, in 1954. Due to the toy's geometry and construction, the top behaved differently from most, spinning first in one direction and then, after flipping over, in another. Long before this meeting, the two eminent scientists puzzled over the nature of the subatomic particle that we now call the neutrino. (Photograph by Erik Gustafson, courtesy the Emilio Segrè Visual Archives, Margrethe Bohr Collection, American Institute of Physics.)

395

Nonholonomic systems:

- the heavy top (or "tippie top")
- rattleback

396

Nonholonomic **systems:**

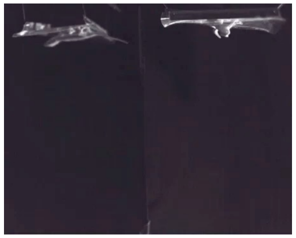
- the heavy top (or "tippie top")
- rattleback
- cats!

397

Nonholonomic **systems:**

- the heavy top (or "tippie top")
- rattleback
- cats, geckos, …

398



Bob Full, PolyPedal Lab, Berkeley
https://youtu.be/_oa46J4dBZ0

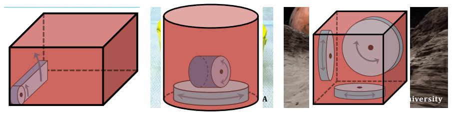
399

Nonholonomic **systems:**

- the heavy top (or "tippie top")
- rattleback
- cats, geckos, …
- attitude control
  - chair "ooching"
  - aerial gymnastics
  - holonomy motors
  - …

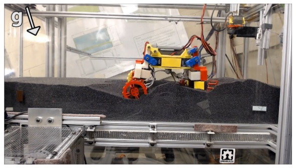
400

Holonomy motors (courtesy Stefan Waal)



401

401



A mini-rover escapes from terrain made of poppy seeds. Video by Shrivastava et al., Sci. Robot. 5, eaba3499 (2020)

https://www.nytimes.com/2020/05/13/science/nasa-rovers.html

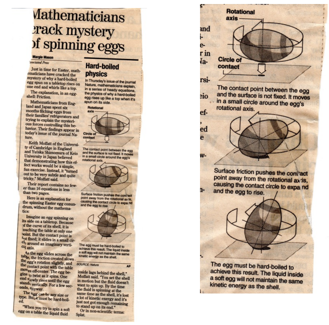
402

402

Nonholonomic systems:

- the heavy top (or "tippie top")
- rattleback
- cats, geckos, …
- attitude control
  - chair "ooching"
  - aerial gymnastics
  - holonomy motors
  - …

403



Mathematicians crack mystery of spinning eggs

Margie Mason
Associated Press

Just in time for Easter, mathematicians have cracked the mystery of why a hard-boiled egg spun on a tabletop rises on one end and whirls like a top.

The explanation, in an eggshell: Friction.

Mathematicians from England and Japan spent six months filching eggs from their families' refrigerators and trying to explain the mysterious forces controlling this behavior. Their findings appear in today's issue of the journal Nature.

Keith Moffatt of the University of Cambridge in England and Yutaka Shimomura of Keio University in Japan believed that demonstrating how this effect works would be a simple, fun exercise. Instead, it "turned out to be very subtle and quite tricky," Moffatt said.

Their report contains no fewer than 16 equations in less than two pages.

Here is an explanation for the spinning Easter egg conundrum, without the mathematics:

Imagine an egg spinning on its side on a tabletop. Because of the curve of its shell, it is touching the table at only one point. But the contact point is not fixed; it slides in a small circle around an imaginary vertical axis.

As the egg slides across the table, the friction created slows the egg's rotation slightly, and the contact point with the table moves off-center. The egg begins to twist as it spins. One end slowly rises until the egg stands vertically. For a few seconds, anyway.

The egg can be any size or type. But it must be hard-boiled.

"When you try to spin a soft egg on a table the liquid fluid inside lags behind the shell," Moffatt said. "You set the shell in motion but the fluid doesn't want to spin up. By the time the fluid is spinning at the same time as the shell, it's lost a lot of kinetic energy and it's just not got enough remaining to stand up on its end."

Or in non-scientific terms: Splat.

Hard-boiled physics

In Thursday's issue of the journal Nature, mathematicians explain, in a series of heady equations, the physics of why a hard-boiled egg rises up like a top when it's spun on its side.

The contact point between the egg and the surface is not fixed. It moves in a small circle around the egg's rotational axis.

Surface friction pushes the contact point away from the rotational axis, causing the contact circle to expand and the egg to rise.

The egg must be hard-boiled to achieve this result. The liquid inside a soft egg will not maintain the same kinetic energy as the shell.

SOURCE: Nature AP

404

Colorful language….

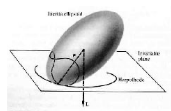


FIGURE 5–4
The motion of the inertia ellipsoid relative to the invariable plane

* Hence the jabberwockian sounding statement: the polhode rolls without slipping on the herpolhode lying in the invariable plane.

(Goldstein)

405

Symplectic integrators:

- preserve energy *or* momentum (but not both)
- "canonical transformation" - picking a new set of qs in the Lagrangian/Hamiltonian
- may make the math hard, but will not change the accuracy of the EOMs
- symplectic integrators are based on these transformations, rather than the Euler-ish "walk down the derivative" ideas
- the workhorse of the celestial mechanics world, like RK4 in dissipative dynamics

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Hamiltonian chaos: definition

If there are n constants of the motion in an n-dimensional Hamiltonian system, it is "integrable in the sense of Liouville," and it is **not** chaotic.

If you can *prove* that there are fewer than n of them, that amounts to a proof that the system is chaotic (that's what Poincare did for the three-body problem)

407

Hamiltonian chaos: properties

- no attractors
- many of our tools don't apply
- trajectories look very different
- *more complicated…*

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Hamiltonian chaos: examples

- driven pendulum with β=0
- the three-body problem (with the right ICs…)
- Fermi-Pasta-Ulam problem
- Henon-Heiles system
- (and many, many others!)

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Fermi, Pasta, and Ulam

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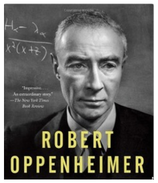


411

The arrow of time:

412

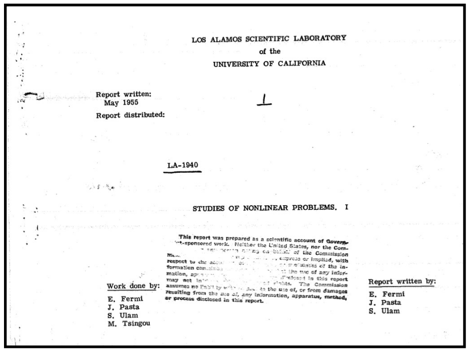
Fermi, Pasta, and Ulam

In 1953, FPU set out t… …uipartition and thermalization occur in simple classical nonlinear dynamical systems.

The "Hidden Figure," Mary Tsingou Menzel, who coded the problem. See T. Dauxois, Physics Today 61(1) 55-57 (2008)

413

413



LOS ALAMOS SCIENTIFIC LABORATORY
of the
UNIVERSITY OF CALIFORNIA

Report written:
May 1955
Report distributed:

1

LA-1940

STUDIES OF NONLINEAR PROBLEMS. I

This report was prepared as a scientific account of Government-sponsored work. Neither the United States, nor the Commission … of the Commission … express or implied, with respect to the … of the information … the use of any information … disclosed in this report … The Commission assumes no liability … to the use of, or from damages resulting from the use of, any information, apparatus, method, or process disclosed in this report.

Work done by:
E. Fermi
J. Pasta
S. Ulam
M. Tsingou

Report written by:
E. Fermi
J. Pasta
S. Ulam

414

What Fermi & Ulam thought would happen:

- If the springs were linear, the initial mode wouldn't change
- If they were nonlinear, energy would equipartition across all modes

What actually happened:

- With nonlinear springs, the energy spread out *and then reassembled in the original modes*

415

Modes in a 2D version of that system:

www.youtube.com/watch?v=9zecsL4rlUY

416

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Solitons:

"Solitary wave phenomena"

Examples:
- Travelling waves in shallow canals
- Traffic

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https://m.youtube.com/watch?v=7wm-pZp_mi0

421

Solitons:

“Solitary wave phenomena”

Examples:
- Travelling waves in shallow canals
- Traffic
- Clouds

422

423

Solitons:

“Solitary wave phenomena”

Examples:
- Travelling waves in shallow canals
- Traffic
- Clouds
- …and many others, including semiconductors…

424

Breathers:

Solitary wave phenomena that are stationary

425

What this looks like in the FPU system:

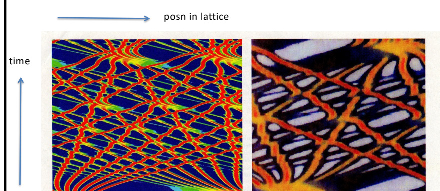
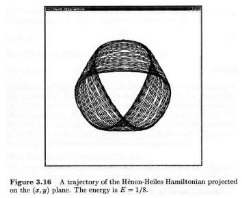


Figure 6. A system like the one Fermi, Pasta and Ulam modeled but with 256 masses *(left)* readily gives rise to solitons, which can propagate in either direction, exchange positions and eventually return the system to something that resembles its initial configuration. The motion of the solitons can be seen here by following the lines of hot colors, which denote large displacements of the masses. The horizontal axis corresponds to position along the series of springs and masses. The vertical axis corresponds to time, which begins at the bottom and progresses upward. The emergence of propagating solitons does not require that the masses be discrete: They also occur in a continuous analog of the Fermi-Pasta-Ulam system *(right)*. (Image at left from Zabusky, Sun and Peng 2006, courtesy of *Chaos*. Image at right courtesy of Zabusky.)

The color codes the displacement of each mass

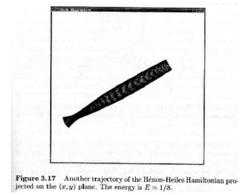
426

A trajectory with initial E = 1/8:



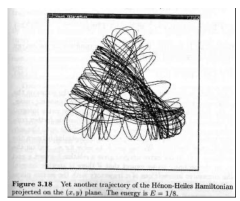
427

Another trajectory with initial E = 1/8:



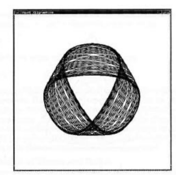
428

Yet another trajectory with initial E = 1/8:



429

These are *projections*.



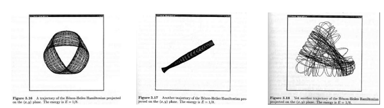
430

The dynamics actually live on a *torus* in 4D…

(NB: this is actually a closed orbit…)

431

How about an E = 1/8 phase portrait?

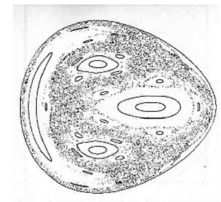


432

What would we see if we did a section of that 4D trajectory, rather than a projection?

433

Poincare section showing *lots* of E=1/8 trajectories from different ICs:



Here: Σ is the x=0 plane, with $p_x' > 0$

434

What those closed curves mean…

- something is conserved ("COM")
- some λ is zero

thought experiment: will there be more/fewer of them if the energy of the IC is lower? (hint: remember how Henon & Heiles designed the potential well…"

435

Lower energy: (E = 1/12)

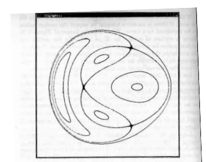


Figure 3.20 Surface of section for the Hénon-Heiles problem with energy $E = 1/12$.

436

Mid-range energy: (E = 1/8)

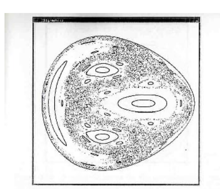


Figure 3.21 Surface of section for the Hénon-Heiles problem with energy $E = 1/8$.

437

Higher energy: (E=1/6)

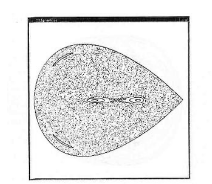


Figure 3.22 Surface of section for the Hénon-Heiles problem with energy $E = 1/6$.

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Patterns in Hamiltonian chaos:

Two kinds of trajectories:
- some on those invariant curves
- some that "fill" parts of space

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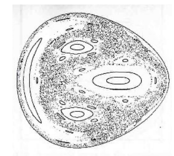
Patterns in Hamiltonian chaos:

- elliptic points
- n-periodic orbits
- invariant curves (aka *invariant tori*)
- island chains (around n-periodic orbits)
- hyperbolic points
- chaotic zones
- separatrices



Simultaneous existence of fixed points, periodic orbits, quasiperiodic orbits, and chaos—for different ICs.
But note that these aren't "attractors."

440

These patterns are generic.

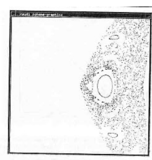


Figure 3.23 A surface of section for the non-axisymmetric top. The parameters are $A = 0.0000445\text{kg m}^2$, $B = 0.000035\text{kg m}^2$, $C = 0.000025\text{kg m}^2$, $g = 9.8\text{m s}^{-2}$, $M = 0.02$ kg, $R = 0.04$m. The energy and p_ψ are those of the top initially standing vertically with rotation frequency 35 rad/s. The angle θ, on the abscissa, runs from $-\pi$ to π. The momentum p_θ, on the ordinate, runs from $-0.001\text{kg m}^2\text{s}^{-1}$ to $0.001\text{kg m}^2\text{s}^{-1}$.

Non-axisymmetric top

441

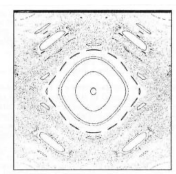


Figure 4.10 A section for the driven pendulum with the same parameters as were used to construct the tangle. The tangle expands to fill space, occupying at least part of the chaotic region.

Driven pendulum

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Not just in flows, either:

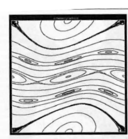

Figure 3.28 Surface of section for the standard map for $K = 0.6$. The section shows mostly regular trajectories, with a few dominant islands, but also shows a number of small chaotic zones.

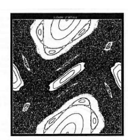

Figure 3.29 Surface of section for the standard map for $K = 1.4$. The dominant feature is a large chaotic zone. There are also some large islands of regular behavior. In this case there are also some interesting secondary islands - islands around islands.

Standard map

…and many other systems, including the three-body problem

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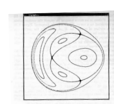
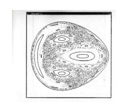
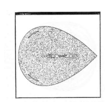
How these invariant curves are affected by parameter changes: the KAM theorem.

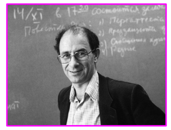


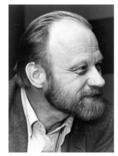




What happens when an *integrable* Hamiltonian (H_0) is perturbed

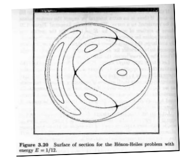
444





445

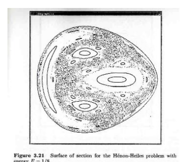
When the Hamiltonian is integrable:



All trajectories move on invariant curves (aka invariant tori)

446

If a small perturbation is made to the Hamiltonian, making it nonintegrable:



$H_0 + \varepsilon H_1$

447

If a small perturbation is made to the Hamiltonian, making it nonintegrable:

A bunch of the invariant tori have "broken down."

KAM theorem: small ε ==> most of the invariant tori are preserved

(the meat of the result and the bulk of the proof)

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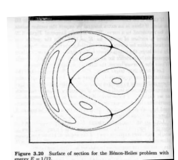
Rotation numbers and invariant tori…



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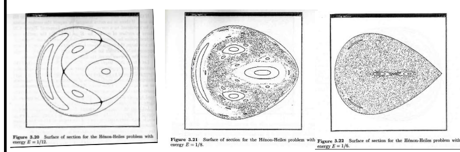
Invariant tori:

- one invariant torus for each rational rotation number
- infinite number…
- dense
- space between any two tori
- and torus between any two
- this is true in *rotation-number* space **and** *phase space*
- called "cantori" because of this Cantor-esque distribution

450

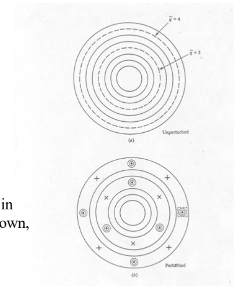
Torus breakdown with increasing ε



ε (amount of perturbation)

451

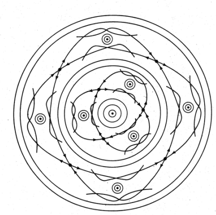
A schematized version of that:



KAM: the order in which they break down, and into what

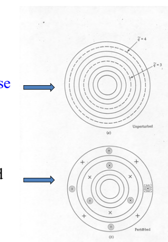
452

What they break down *into*:



453

KAM stuff summary:

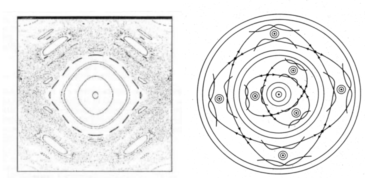


- trajectories of H_0 run around on these
- trajectories of $H_0 + \varepsilon H_1$ run around on those
- KAM: how these turn into those.

(Which torus do you think goes last?)

454

Where does that "tori within tori" stuff happen?



455

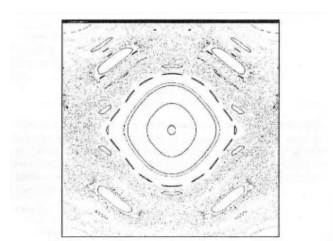
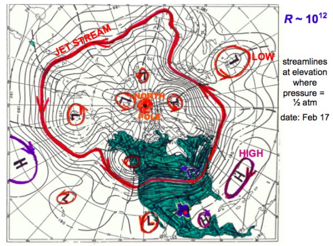


Figure 4.10 A section for the driven pendulum with the same parameters as were used to construct the tangle. The tangle expands to fill space, occupying at least part of the chaotic region.

456

The polar night jet: observations…



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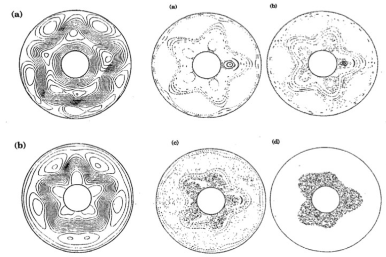
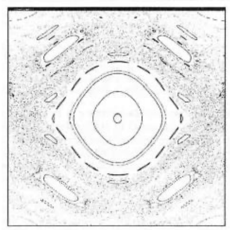
The polar night jet: lab experiments…



"Chaos and mixing in a geostrophic flow," Behringer/Meyers/Swinney, *Phys. Fluids A* **3**:1243 (1991)

458

Trajectories cannot cross KAM curves…



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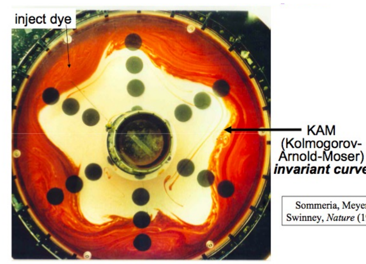
KAM curves = barriers to transport!

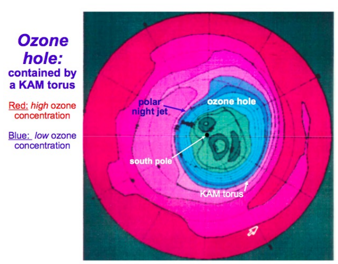


Sommeria, Meyers, Swinney, *Nature* (1989)

Slide from Harry Swinney

460

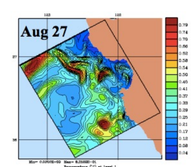
So what?



Slide from Harry Swinney

461

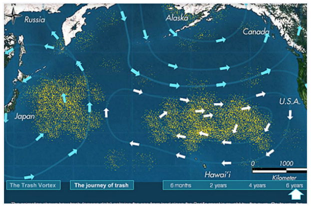
So what, part II:



http://robinson.seas.harvard.edu/PAPERS/pfjl_mit_me_dec5_05.pdf

462

So what, part III:



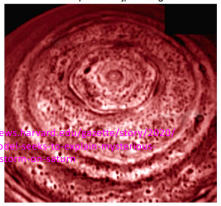
momfull.wordpress.com/2012/09/17/pacific-garbage-patch-from-space/

(the most depressing google image search ever…)

463

Not just on earth, either…

29 OCTOBER 2006: ***NASA's Casini spacecraft reveals***
"bizarre 6-sided feature encircling the north pole of Saturn"
http://saturn.jpl.nasa.gov/home/index.cfm



https://news.harvard.edu/gazette/story/2020/10/3d-model-seeks-to-explain-mysterious-hexagon-storm-on-saturn

Each side
13,800 km

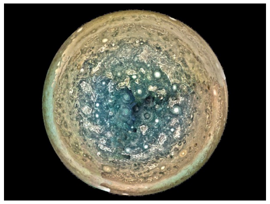
Period
10h 39min 24s

Latitude
78° North

Slide from Harry Swinney

464

…but not on Jupiter:



Slide from NASA's Juno mission

465

A good paper about this stuff:

CHAOS **20**, 017514 (2010)

Invariant-tori-like Lagrangian coherent structures in geophysical flows

Francisco J. Beron-Vera,[1,a)] María J. Olascoaga,[1] Michael G. Brown,[1] Huseyin Koçak,[2] and Irina I. Rypina[3]
[1]*Rosenstiel School of Marine and Atmospheric Science, University of Miami, 4600 Rickenbacker Cswy., Miami, Florida 33149, USA*
[2]*Department of Computer Science and Department of Mathematics, University of Miami, 1365 Memorial Dr., Coral Gables, Florida 33146, USA*
[3]*Woods Hole Oceanographic Institution, 266 Woods Hole Rd., Woods Hole, Massachusetts 02543, USA*

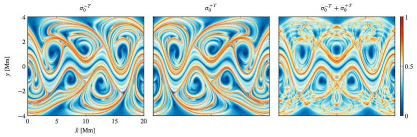
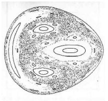


FIG. 2. (Color) FTLE fields for a Bickley jet perturbed by three Rossby-like waves as seen in a reference frame translating with the phase speed of one of the waves. Left, middle, and right panels show at $t=0$ the FTLE field computed backward in time, the FTLE field computed forward in time, and the field resulting from adding these two fields, respectively. Time integration $T \approx 11$ d, which is equal to the largest period in the superimposed wave field. The backward- and forward-time FTLEs are normalized by the maximum value in the domain.

466

Patterns in Hamiltonian chaos:



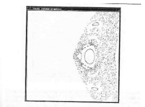


Figure 3.23 A surface of section for the non-axisymmetric top. The parameters are $A = 0.0000445\text{kg m}^2$, $B = 0.000035\text{kg m}^2$, $C = 0.000025\text{kg m}^2$, $g = 9.8\text{m s}^{-2}$, $M = 0.02$ kg, $R = 0.04$m. The energy and p_ϕ are those of the top initially standing vertically with rotation frequency 35 rad/s. The angle θ, on the abscissa, runs from $-\pi$ to π. The momentum p_θ, on the ordinate, runs from $-0.001\text{kg m}^2\text{s}^{-1}$ to $0.001\text{kg m}^2\text{s}^{-1}$.

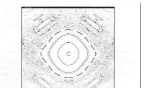


Figure 4.10 A section for the driven pendulum with the same parameters as were used to construct the tangle. The tangle expands to fill space, occupying at least part of the chaotic region.

467

- elliptic points
- n-periodic orbits
- invariant curves
- island chains
- hyperbolic points
- chaotic zones
- separatrices

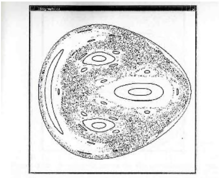


Figure 3.21 Surface of section for the Hénon-Heiles problem with energy $E = 1/8$.

468

How to automate that pattern-analysis process?

- elliptic points
- n-periodic orbits
- invariant curves
- island chains
- hyperbolic points
- chaotic zones
- separatrices

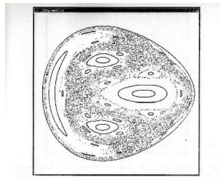


Figure 3.21 Surface of section for the Hénon-Heiles problem with energy $E = 1/8$.

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QUARTERLY OF APPLIED MATHEMATICS

Vol. XXVII OCTOBER 1969 No. 3

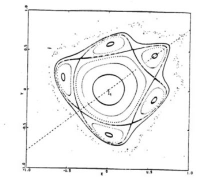
NUMERICAL STUDY OF QUADRATIC AREA-PRESERVING MAPPINGS*

BY

M. HÉNON

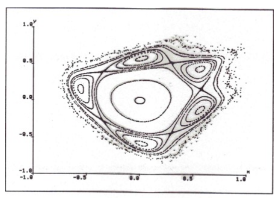
C.N.R.S., Institut d'Astrophysique, Paris

$$x_{n+1} = x_n \cos\alpha - (y_n - x_n^2) \sin\alpha$$
$$y_{n+1} = x_n \sin\alpha + (y_n - x_n^2) \cos\alpha$$



1

470



471

```
;;;KAM has no natrual language capability.
;;;Stylish output only.

(symbolic-describe fp)

The portrait has an elliptic fixed point at (0.0 0.0)
Surrounding the fixed point is a regular
region bounded by a KAM curve with rotation number
between 1/5 and 1/4.  Outside the regular region
lies a chain of 5 islands.  The island chain is
bounded by a KAM curve with rotation number between
4/21 and 5/26 The outermost region is occupied by
chaotic orbits which eventually escape.
```

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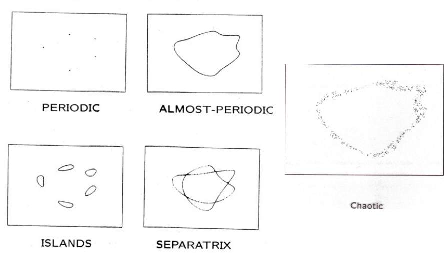
POSSIBLE ORBIT SHAPES

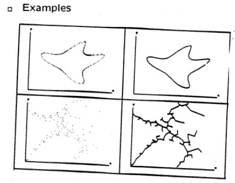


3

473

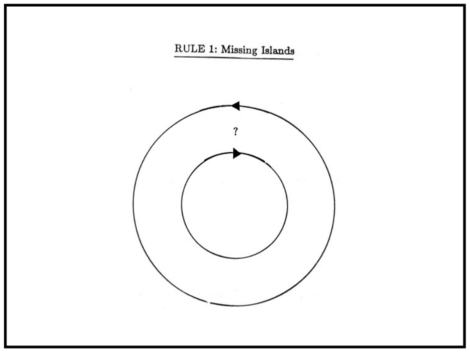
ORBIT RECOGNITION

- Key Idea: Euclidean MINIMAL SPANNING TREE (EMST)
- Branching Structure
- Examples

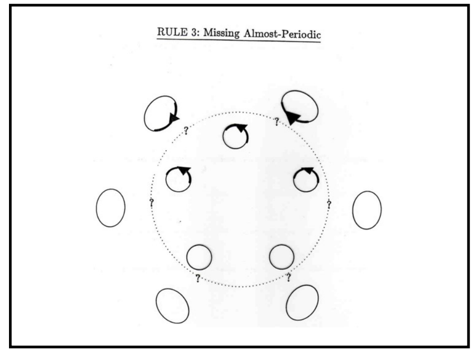


12

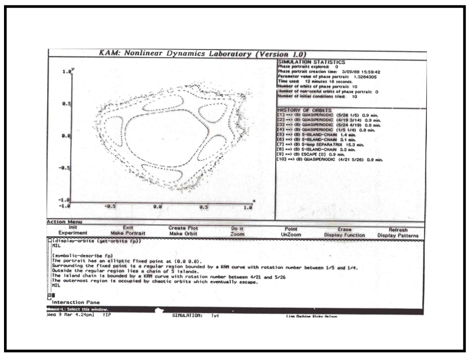
474



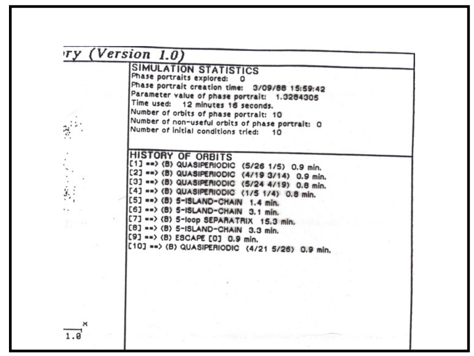
475



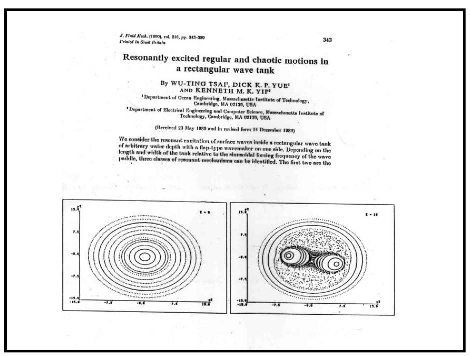
476



477



478



J. Fluid Mech. (1990), vol. 216, pp. 343–380
Printed in Great Britain
343

Resonantly excited regular and chaotic motions in a rectangular wave tank

By WU-TING TSAI[1], DICK K. P. YUE[1] AND KENNETH M. K. YIP[2]

[1] Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

[2] Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

(Received 23 May 1989 and in revised form 14 December 1989)

We consider the resonant excitation of surface waves inside a rectangular wave tank of arbitrary water depth with a flap-type wavemaker on one side. Depending on the length and width of the tank relative to the sinusoidal forcing frequency of the wave paddle, three classes of resonant mechanisms can be identified. The first two are the

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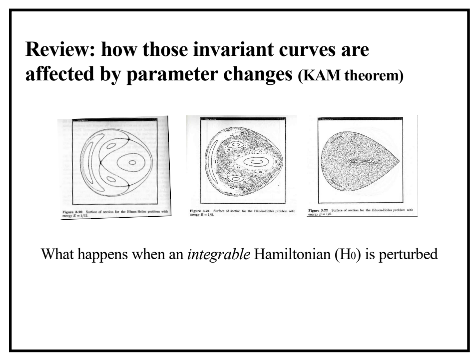
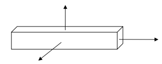


480

Review: dynamics of a triaxial body



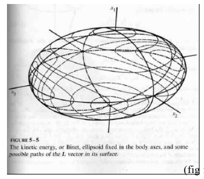


FIGURE 5–5
The kinetic energy, or Binet, ellipsoid fixed in the body axes, and some *possible paths of the L vector in its surface.*

(fig from Goldstein)

481

Recall: tidal equilibria of satellites

- spin lock
- spin axis perpendicular to plane of orbit
- and lots of others

(All of these have shorter time constants if the orbit eccentricities are larger, BTW)

482

Combining those three ideas, we can understand solar system chaos!

- rotation of Hyperion
- and many other moons and asteroids and planets and Kuiper belt objects
- the Kirkwood gaps
- orbit of Pluto
- obliquity of Mars
- ……..

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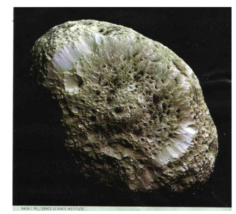
Hyperion:



484

Studying Hyperion’s rotation, step 1

- fix spin axis
    - write eqns
    - simulate (*symplectic* integrator)
    - sample @ periapse; plot θ vs θ’

(Wisdom/Klavetter)

485

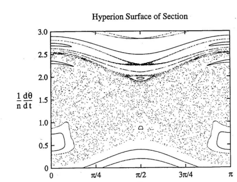


FIGURE 1. Surface of section for Hyperion (using $\alpha = \sqrt{3(B - A/C}$ = 0.89 and e = 0.1). The rate of change of the orientation is plotted versus the orientation at every periapse passage. The spin axis is fixed perpendicular to the orbit plane.

Note: spin axis is fixed in this approximation, so no tumbling; this plot is about what face Hyperion turns to Saturn at periapse.

486

Patterns in Hamiltonian chaos:

- elliptic points
- *n*-periodic orbits
- invariant curves
- hyperbolic points
- chaotic zones
- separatrices

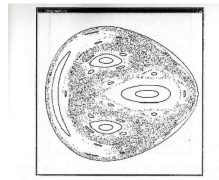


Figure 3.21 Surface of section for the Hénon-Heiles problem with energy $E = 1/8$.

487

The rest of Hyperion's dynamics:

- fix spin axis
  - write eqns
  - simulate (*symplectic* integrator)
  - sample @ periapse; plot θ vs θ'
- relax `spin axis fixed' assumption
- do stability analysis

488

Results: Hyperion is attitude unstable on the big torus that represents the spin-locked equilibrium ➔ tumbles chaotically!

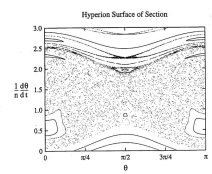


FIGURE 1. Surface of section for Hyperion (using $\alpha = \sqrt{3(B - A/C} = 0.89$ and $c = 0.1$). The rate of change of the orientation is plotted versus the orientation at every periapse passage. The spin axis is fixed perpendicular to the orbit plane.

489

Voyager *saw* this.

Ap. J. **97**:570
Ap. J. **98**:1855

So did Cassini!

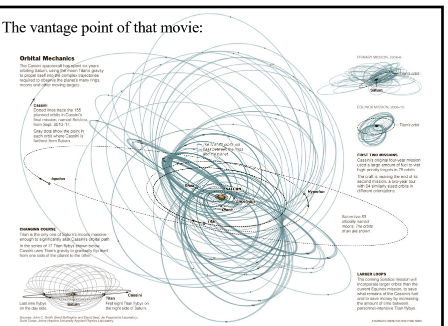
www.nasa.gov/mov/122121main_PIA06243-full-movie.mov

490

www.nasa.gov/mission_pages/cassini/multimedia/pia06243.html

491

The vantage point of that movie:



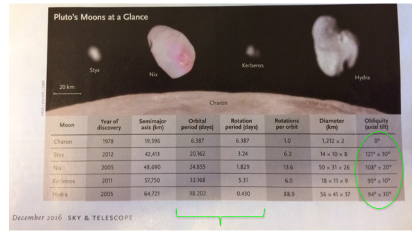
492

Galileo saw similar things:

From *Sky & Telescope*

493

So did New Horizons:



Moon	Year of discovery	Semimajor axis (km)	Orbital period (days)	Rotation period (days)	Rotations per orbit	Diameter (km)	Obliquity (axial tilt)
Charon	1978	19,596	6.387	6.387	1.0	1,212 ± 2	0°
Styx	2012	42,413	20.162	3.24	6.2	14 × 10 × 8	121° ± 30°
Nix	2005	48,690	24.855	1.829	13.6	50 × 31 × 26	108° ± 20°
Kerberos	2011	57,750	32.168	5.31	6.0	18 × 11 × 9	95° ± 10°
Hydra	2005	64,721	38.202	0.430	88.9	56 × 41 × 37	94° ± 30°

December 2016 SKY & TELESCOPE

494

That is *not* true for all moons in the solar system.

- fix spin axis
  - write eqns
  - simulate (symplectic integrator!)
  - sample @ periapse; plot θ vs θ'
- relax 'spin axis fixed' assumption
- do stability analysis

dictates whether the tidal equilibrium is stable or unstable (and it is stable for most moons, like ours)

495

Phobos:

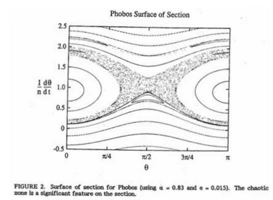


FIGURE 2. Surface of section for Phobos (using α = 0.83 and e = 0.015). The chaotic zone is a significant feature on the section.

496

Deimos:

is in an almost-circular orbit, so the chaotic zones are much thinner…but they're still there.

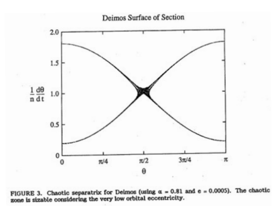
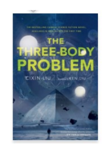


FIGURE 3. Chaotic separatrix for Deimos (using α = 0.81 and e = 0.0005). The chaotic zone is sizable considering the very low orbital eccentricity.

497

The difference:

- Hyperion is attitude unstable at the spin-locked equilibrium
- Phobos & Deimos are not, so we don't see them tumbling chaotically
- Neither are we, thankfully…



498

However!!

- KAM theorem: if the system is nonintegrable, all of those invariant tori are surrounded by chaotic zones
- so *almost all* satellites had to cross those zones in order to get to their current stable orbits (Ap. J. **94**:1350)
- some (e.g., Hyperion) are still tumbling
- "*almost all*"…except perfectly spherical objects in perfectly circular orbits (whose Hamiltonians are integrable and all of whose invariant tori are intact) or objects that started out near the stable orbits (inside any chaotic zones)
- And that includes the earth…

499

Neat! use of nonlinear dynamics techniques to analyze the equations of motion of solar-system bodies, and draw some interesting conclusions — about past, present, and future.

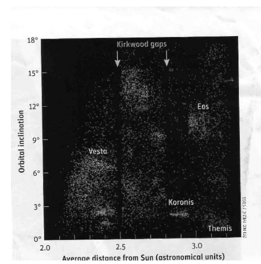
500

Chaos in the solar system:

- rotation of Hyperion
- and many other moons and asteroids and planets and Kuiper belt objects
- the Kirkwood gaps
- orbit of Pluto
- obliquity of Mars
- ……..

501

Kirkwood gaps:



From *Sky & Telescope*

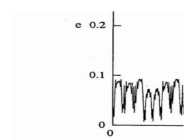
502

Kirkwood gaps:

- occur near resonances with Jupiter
(resonances…p/q…rotation numbers…KAM curves…)
- aka "mean-motion commensurabilities"
- and resonances are *almost always* associated with chaotic zones (thanks, KAM)
- research question: write EOMs for asteroids there and integrate them for a long time

503

A very expensive computation!



…so stop when it looks like things have settled out.

504

Was that the whole story? Nope.

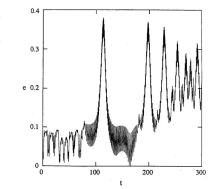


FIGURE 4. Eccentricity versus time for a chaotic trajectory near the 3/1 commensurability. Time is measured in millennia. A short 10,000 year integration could give a very poor idea of the nature of this trajectory.

505

Going even longer:

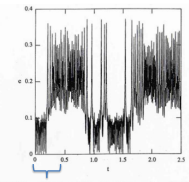
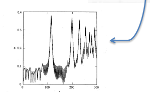


FIGURE 5. Eccentricity of a typical chaotic trajectory over a longer time interval. the time is now measured in millions of years. Bursts of high eccentricity behavior are interspersed with intervals of irregular low eccentricity behavior, broken by occasional spikes.



506

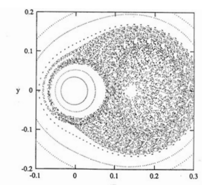
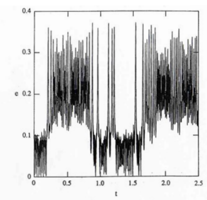


FIGURE 7. Surface of section corresponding to the trajectory of Fig. 5. The coordinates are $x = e\cos(\bar{\omega} - \bar{\omega}_J)$, and $y = e\sin(\bar{\omega} - \bar{\omega}_J)$. The orbital eccentricity is the radius from the origin. The trajectory is free to explore a rather large chaotic zone, but sometimes spends a period of time near the islands close to the origin.

507

What does chaos have to do with the Kirkwood gaps?



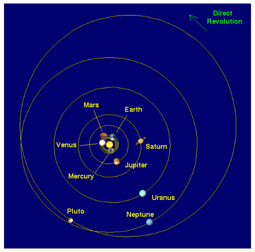
Higher eccentricity

FIGURE 5. Eccentricity of a typical chaotic trajectory over a longer time interval. the time is now measured in millions of years. Bursts of high eccentricity behavior are interspersed with intervals of irregular low eccentricity behavior, broken by occasional spikes.

Wisdom, *Nuclear Phys. B* **2:**391

508

What are the implications of high-e orbits?



csep10.phys.utk.edu/astr161/lect/solarsys/revolution.html

509

Evidence in favor of the conjecture:

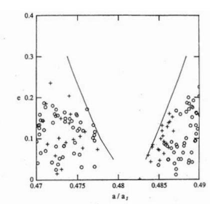


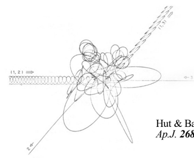
FIGURE 9. Comparison of the actual distribution of asteroids with the outer boundaries of the chaotic zone. There is both a chaotic region and quasiperiodic region in the gap, but trajectories of both types are planet crossing.

Wisdom, *Nuclear Phys. B* **2:**391

510

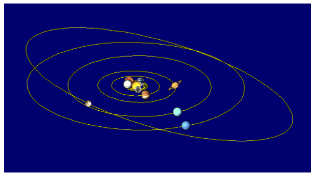
Solar system stability:

- recall: two-body problem not chaotic
- but three (or more) can be
- that prize from King of Sweden…



Hut & Bahcall
Ap.J. **268**:319

511



512

513

Is Pluto a planet?

"My view is that a useful working definition of a planet is that

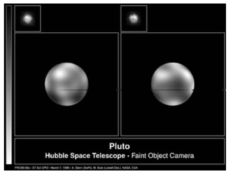
1. a planet orbits the Sun (rather than a moon that orbits a planet) and
2. it must be big enough that its own gravity pulls it together into a sphere - i.e. it must be round.

From looking at many objects in the solar system, we can generally say that objects greater than about 400 km across tend to be more or less spherical (rather than irregular or 'potato-shaped'). Pluto, although smaller than several moons, qualifies under this definition. About a handful of asteroids also qualify. I would call all of these minor planets."

(Fran Bagenal, http://dosxx.colorado.edu/plutohome.html)

514

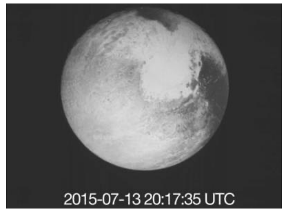
Pluto: what we knew before July 2015



515

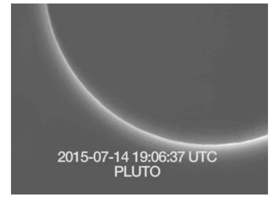
New Horizons:

516



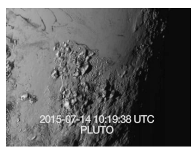
517

It has an atmosphere:



518

It has amazing geology:



Check out:
`https://www.nasa.gov/mission_pages/newhorizons/images/index.html`

519

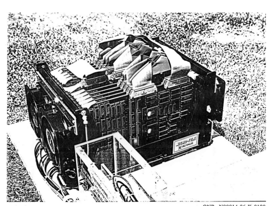
Figuring out Pluto's dynamics:
the n-body problem

- issue: expense
- solution:
  - smart integrators that can take long steps and not mess up the dynamics
  - a special-purpose computer tailored for this problem

520

Exploring that issue, circa 1980:

- write the *n*-body equations for the solar system
- solve them on a special-purpose computer

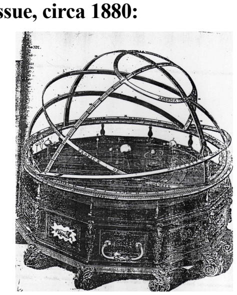


The *digital orrery*
(Wisdom & Sussman)

521

Exploring that issue, circa 1880:

- an *orrery*



522

Even earlier…

en.wikipedia.org/wiki/Antikythera_mechanism

523

Image: American Scientist

524

Numerical Evidence That the Motion of Pluto Is Chaotic

GERALD JAY SUSSMAN AND JACK WISDOM

The Digital Orrery has been used to perform an integration of the motion of the outer planets for 845 million years. This integration indicates that the long-term motion of the planet Pluto is chaotic. Nearby trajectories diverge exponentially with an e-folding time of only about 20 million years.

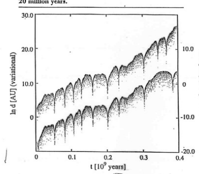


Figure 3: The exponential divergence of nearby trajectories is indicated by the average linear growth of the logarithms of the distance measures as a function of time. In the upper trace we see the growth of the variational distance around a reference trajectory. In the lower trace we see how two Plutos diverge with time. The distance saturates near 80AU when the Plutos are on opposite sides of the Sun. The variational method of studying neighboring trajectories does not have the problem of saturation. Note that the two methods are in excellent agreement until the two-trajectory method has nearly saturated.

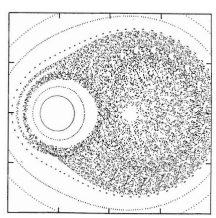
Science **241**:433

525

Should we worry?

- No.

526



527

More chaos in the solar system:

- obliquity of Mars (Touma & Wisdom, *Science* **259**:1294)

www.solarviews.com

- etc.

528

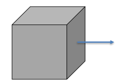
Hamiltonian dynamics, redux:

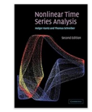
- Lagrangians and Hamiltonians
- gravitation
- rigid-body dynamics
- bit of theory (e.g., integrability, KAM theorem)
- phase portraits
- examples & applications:
  - FPU, solitons, Henon-Heiles, polar night jet
  - the KAM program
  - solar system dynamics

529

Back to the real world.

- friction (at human time scales)
- noise
- sensors that produce quantized measurements
  - in time
  - in space
- and state variables that you may not know and/or can't measure (though you **can** measure *something*…)





530

Back to the real world.

- friction
- noise
- sensors that produce quantized measurements
  - in time
  - in space
- and state variables that you may not know and/or can't measure (though you **can** measure *something*…)
- how to analyze time-series data, in view of all of that?
- and do we have any hope of modeling or predicting the dynamics?

Slippery word…

531

Announcements:

Hit the record button!

- PS11 due today; PS12 due on 4/25
- Planning/hoping for in-person class on Thursday, but check your email when you roll out of bed, just in case… 🤞
- No office hour on Thursday

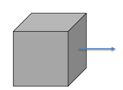
532

Projects and surveys:

- We have several projects that will use surveys to answer their research question [music, graphics, video games…]
- I will schedule those presentations on May 2
- Those people will need to have their work done and the surveys set up by April 27
- Please set aside some time that weekend to TAKE those surveys.
- Be aware that survey design is not at all trivial, so you'll need to do a bit of reading in the HCIish literature on how to do that.
- Keeping them short (5 min, max) will greatly increase the probability that people will fill them out.

533

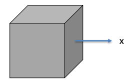
Naïve prediction:



x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
1.0	0.6
?	0.7

534

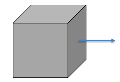
Naïve prediction: what's the "model"?



x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
1.0	0.6
?	0.7

535

Naïve prediction: what's the "model"?



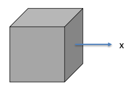
x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
1.0	0.6
?	0.7

Stationarity!

How to know that that assumption is valid…?

536

Naïve prediction: what's the "model"?



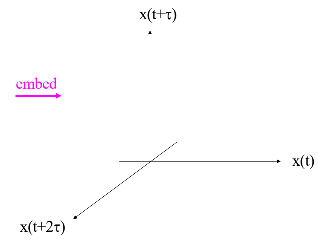
x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
1.0	0.6
?	0.7

General issue with prediction of scalar values: x is only *part* of the system state…

537

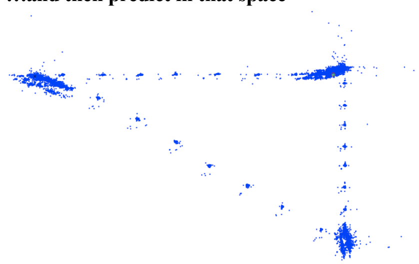
Address that by embedding…

x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
1.0	0.6



538

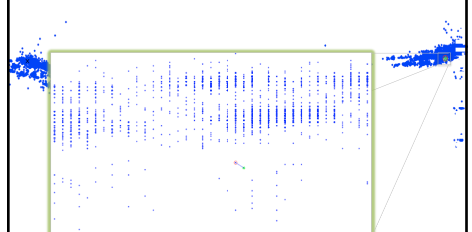
…and then predict in that space



aka "Lorenz's method of analogues" (*J. Atm. Sci.* **26**:636; extended to embeddings by Pikovsky somewhat later.)

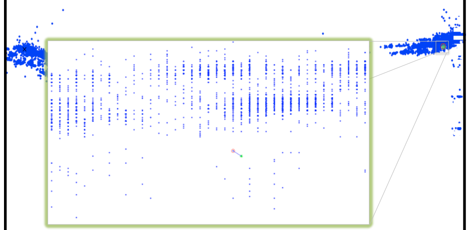
539

How LMA works:



540

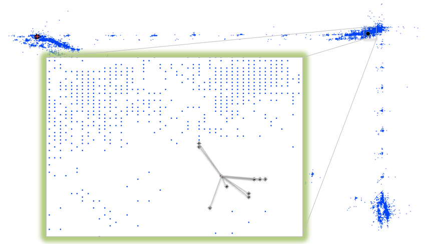
How LMA works:



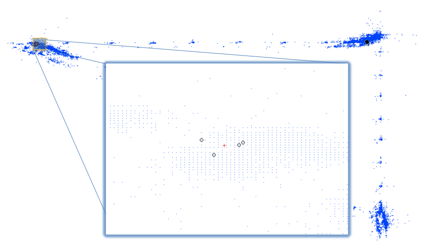
Issue: highly sensitive to noise. How to fix?

541

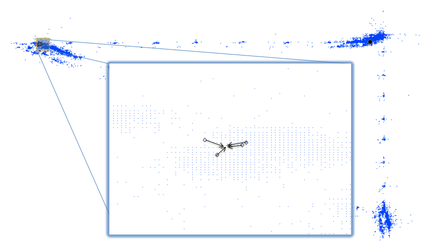
A *k*-nearest neighbor modification of LMA



542

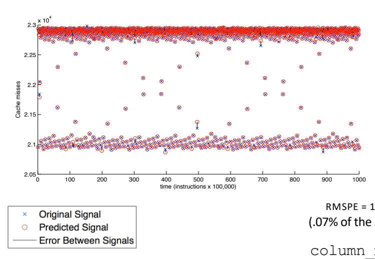


543



544

Using that idea to predict *computer* dynamics



RMSPE = 11.566
(.07% of the average)

`column_major`
cache misses

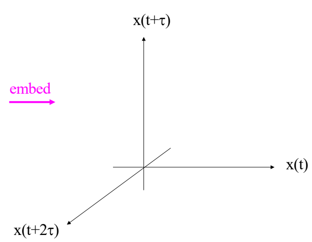
Garland/Bradley *Intelligent Data Analysis* 2011

545

Another embedding-based prediction strategy:

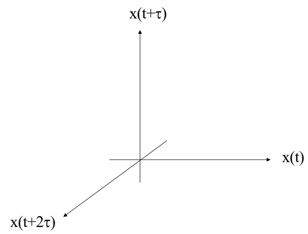
x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
0.3	0.6

embed →



546

Repeat for lots of patches, covering the attractor:



547

How do you use *that* to predict?

(This is not that far from what many of the early time-series prediction papers in the NLD literature did.)

548

Better patch models:

- linear
- quadratic
- radial basis functions
- …

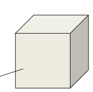
549

Some issues:

- patch size
- overlap?
- position and orientation
- determining which patch a point is in

550

What you'd really like to do, of course…



x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
0.3	0.6

The underlying ODE. (mechanistic model that captures the dynamics everywhere in state space)

551

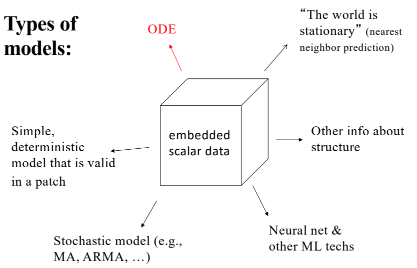
What you *can* get:

x	t
1.0	0.1
0.9	0.2
0.8	0.3
0.5	0.4
0.2	0.5
0.3	0.6

Local models of where the *solution* will go next.

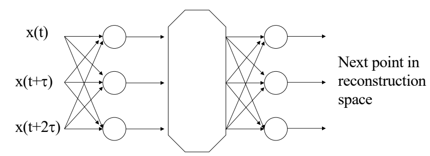
552

Types of models:



553

Neural nets: (tangent)



554

What the neurons do:

• perform weighted sum of their inputs and "fire" if it's over some threshold:

(like real neurons)
(NB: nonlinear!!)

555

What the neurons do:

• perform weighted sum of their inputs and "fire" if it's over some threshold

• weights are adjustable

• they start out randomized and then get *trained* on a *training set* of data

556

Training:

• weights start out randomized.

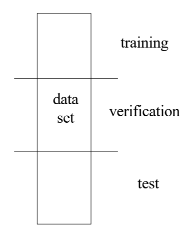
• then you take a vector for which you *know* the next point

• feed it to the net

• and use the difference between the net output and the known answer to adjust the weights

557

How the data is used:

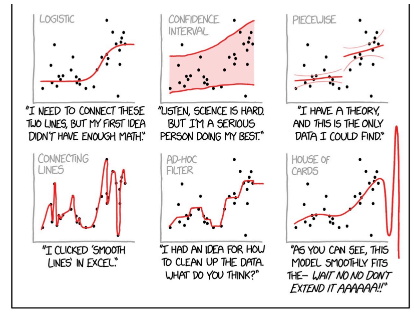


558

That is essentially a nonlinear regression fit.

- if you have enough spline pieces, polynomial terms, fft terms, etc., can fit almost anything
- NNs & Co. use different pieces, but same idea
- But…
  - how know when training has converged?
  - how know *whether* it will converge?
  - overfitting

559



560

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- But…
  - how know when training has converged?
  - how know *whether* it will converge?
  - overfitting
  - *ML methods don't give you explanations, intuition*
  - *And there are major data issues (esp. bias)*

561

Federal 'Extreme Vetting' Plan Castigated by Tech Experts

By THE ASSOCIATED PRESS NOV. 16, 2017, 8:24 P.M. E.S.T.

Leading researchers castigated a federal plan that would use artificial intelligence methods to scrutinize immigrants and visa applicants, saying it is unworkable as written and likely to be "inaccurate and biased" if deployed.

The experts, a group of more than 50 computer and data scientists, mathematicians and other specialists in automated decision-making, urged the Department of Homeland Security to abandon the project, dubbed the "Extreme Vetting Initiative."

562

CCC
Computing Community Consortium
Catalyst

Big Data, Data Science, and Civil Rights

Solon Barocas, Elizabeth Bradley, Vasant Honavar, and Foster Provost

Abstract
Advances in data analytics bring with them civil rights implications. Data-driven and algorithmic decision making increasingly determine how businesses target advertisements to consumers, how police departments monitor individuals or groups, how banks decide who gets a loan and who does not, how employers hire, how colleges and universities make admissions and financial aid decisions, and much more. As data-driven decisions increasingly affect every corner of our lives, there is an urgent need to ensure they do not become instruments of discrimination, barriers to equality, threats to social justice, and sources of unfairness. In this paper, we argue for a concrete research agenda aimed at addressing these concerns, comprising five areas of emphasis: (i) Determining if models and modeling procedures exhibit objectionable bias; (ii) Building awareness of fairness into machine learning methods; (iii) Improving the transparency and control of data- and model-driven decision making; (iv) Looking beyond the algorithm(s) for sources of bias and unfairness—in the myriad human decisions made during the problem formulation and modeling process; and (v) Supporting the cross-disciplinary scholarship necessary to do all of that well.

563

That is essentially a nonlinear regression fit.

- if you have enough spline pieces, polynomial terms, fft terms, etc., can fit almost anything
- NNs & Co. use different pieces, but same idea
- But…
  - how know when training has converged?
  - how know *whether* it will converge?
  - overfitting
  - *ML methods don't give you explanations, intuition*
  - *And there are major data issues (esp. bias)*
  - *How to incorporate closed-form knowledge?*

564

- *Symbolic AI*
  - logic systems
  - planners, theorem provers
  - rule-based systems
  - qualitative reasoning
  - …
- *Statistical AI*
  - machine learning
  - neural nets
  - support vector machines
  - Bayesian techniques
  - …

565

- Symbolic AI:
  - reasons generally and reports on its reasoning
  - but someone has to feed it the operative knowledge
  - and "knowledge engineering" is hard.
- Statistical AI:
  - works really well, but requires lots of information to learn from (training sets, priors, …)
  - captures (possibly hidden) biases in those data
  - output = statistics, not explanations

566

Time-series prediction examples:

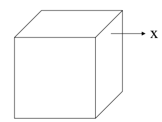
1. roulette revisited
2. the Santa Fe competition

Note: the Berreby paper that's in the PS12 assigned reading is online; see the PS12 bullet on the course webpage

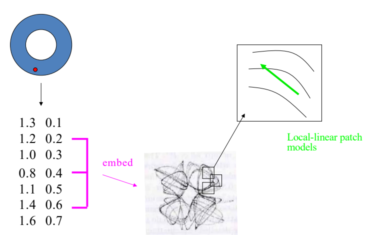
567

What if we measured time-series data from a roulette wheel?

Packard, Shaw, Farmer, Crutchfield, …
"Chaos Cabal" at UC Santa Cruz



568



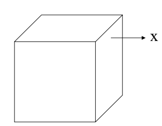
569

What if we measured time-series data from a roulette wheel?

Packard, Shaw, Farmer, Crutchfield, …
"Chaos Cabal" at UC Santa Cruz

thence

SANTA FE INSTITUTE



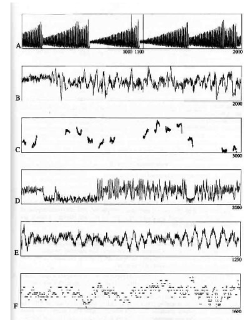
Berreby, "Chaos hits Wall Street" (assigned reading!)
Bass, *The Eudaemonic Pie* (aka *The Newtonian Casino*)

570

The Santa Fe competition

- Weigend & Gershenfeld, 1992
- put a bunch of data sets up on an ftp server
- and invited all comers to predict their future
- chronicled in *Time Series Prediction: Forecasting the Future and Understanding the Past,* Santa Fe Institute, 1993 (from which many of the images on the following half-dozen slides were reproduced)

571



572

The Santa Fe competition: data

- Laboratory laser
- Medical data (sleep apnea)
- Currency rate exchange
- RK4 on some chaotic ODE
- Intensity of some star
- A Bach fugue

573

Winner #1:

In his competition entry, shown in Figure 3, Sauer used a careful implementation of local-linear fitting that had five steps:

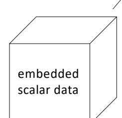
1. Low-pass embed the data to help remove measurement and quantization noise. This low-pass filtering produces a smoothed version of the original series. (We explained such filtered embedding at the end of Section 4.1.)
2. Generate more points in embedding space by (Fourier-) interpolating between the points obtained from Step 1. This is to increase the coverage in embedding space.
3. Find the k nearest neighbors to the point of prediction (the choice of k tries to balance the increasing bias and decreasing variance that come from using a larger neighborhood).
4. Use a local SVD to project (possibly very noisy) points onto the local surface. (Even if a point is very far away from the surface, this step forces the dynamics back on the reconstructed solution manifold.)
5. Regress a linear model for the neighborhood and use it to generate the forecast.

574

Where does that one fit in?

"The world is stationary" (nearest neighbor prediction)

Simple, deterministic model that is valid in a patch



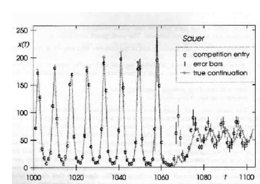
Other info about structure

Stochastic model (e.g., MA, ARMA, …)

ML

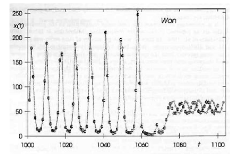
575

Sauer's results:



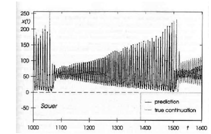
576

Winner #2:

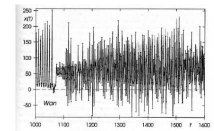


577

Further out:



Sauer



Wan

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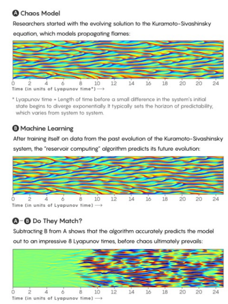
More recent work using *reservoir computing:*



Model-Free Prediction of Large Spatiotemporally Chaotic Systems from Data: A Reservoir Computing Approach

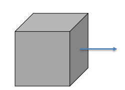
Jaideep Pathak, Brian Hunt, Michelle Girvan, Zhixin Lu, and Edward Ott
Phys. Rev. Lett. **120**, 024102 – Published 12 January 2018

Image: Quanta magazine

579

Back to the real world.

- friction (at human time scales)
- noise
- sensors that produce quantized measurements
  - in time
  - in space
- and state variables that you may not know and/or can't measure.

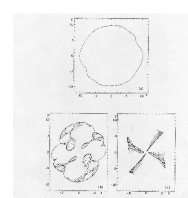


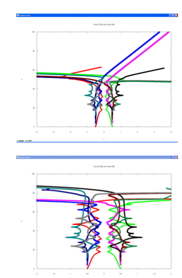
580

Issues: noise

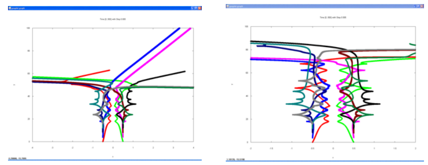
- SDOIC amplifies dynamical noise in a chaotic system, but…
- *Dynamical* noise = bad bad bad

581





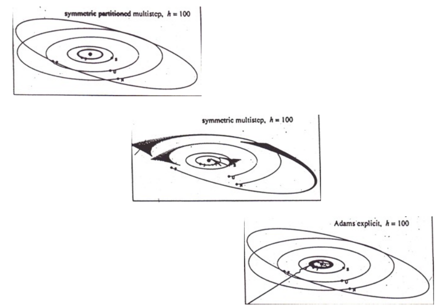
582



Different arithmetic

N. Ross Ph.D. thesis, Ucolorado, 2008

583



584

Issues: noise

- SDOIC amplifies dynamical noise in a chaotic system, but…
- *Dynamical* noise = bad bad bad (NB: mitigated by the shadowing lemma)
- *Additive* noise not so nasty; it simply moves where you *see* the points

585

Issues: noise

- SDOIC amplifies dynamical noise in a chaotic system, but…
- *Dynamical* noise = bad bad bad (NB: mitigated by the shadowing lemma)
- *Additive* noise not so nasty; it simply moves where you *see* the points
- But how to know which one's at work, what's the real dynamics, and what's noise?
- And how to get rid of it? Can't just fft and LPF!

586

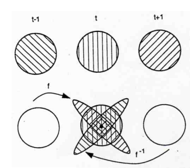
Getting rid of noise

Linear filtering: a bad idea if the system is chaotic

Nonlinear alternatives:

- use the stable and unstable manifold structure on a chaotic attractor…

587



Farmer & Sidorowich, in *Evolution, Learning and Cognition*, World Scientific, 1983

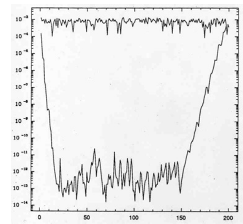
588

Idea:

- If you have a model of the system, you can simulate what happens to each point in forward *and backward* time
- If your system has transverse stable and unstable manifolds, that does useful things to the noise balls
- Since all three versions of that data should be identical at the middle time, can average them
- → noise reduction!
- Works best if manifolds are perpendicular, but requires only transversality
- Can extend further forward/backward in time

589

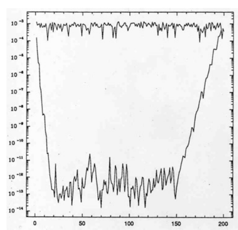
Results:



Farmer & Sidorowich, in *Evolution, Learning and Cognition*, World Scientific, 1983

590

Note those scaling regions at the beginning and end of the time series.
What's going on with those?



591

Idea:

- If you have a model of the system, you can simulate what happens to each point in forward *and backward* time
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- → noise reduction!
- Works best if manifolds are perpendicular, but requires only transversality
- Can extend further forward/backward in time

592

Comments:

- If you have a perfect model, can get the noise down close to where machine ε starts to bite you
- If you use a local-linear patch model, can get several orders of magnitude reduction
- Nice idea to use the transversality of the manifolds
- This is a *nonlinear* filtering strategy

593

Comments, cont.:

- Again, linear filtering is bad
- Farmer calls it "bleaching" the data
- Uses simple frequency-domain knowledge ("noise is high frequency; signal is low")
- Farmer's scheme uses the *known geometry of the dynamics* to reduce the noise
- Another idea: use the *topology*…

594

Another idea:

"The topology"…

The "Betti numbers"

- how many pieces
- how many holes
- …

595

www.shapeways.com/shops/henryseg

596

Another idea:

"The topology"…

The "Betti numbers"

- how many pieces
- how many holes
- …
- but how to do this when you only have *samples* of the set?

597

Computational Topology *aka* Topological Data Analysis

How:

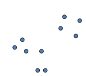
- introduce resolution parameter
- count components, holes, etc. at different resolutions
- deduce topology from patterns therein

V. Robins Ph.D. thesis, UColorado, 1999

598

B_0: the number of connected components

- how many "lumps" in a data set:
- ε-connectedness (after Cantor)
- ε-connected components



599

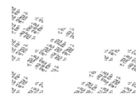
Aside: A nice way to compute that…

- The minimal spanning tree

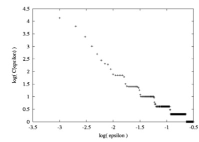
600

Connected components: examples

If the data points are samples of a disconnected fractal like this:



The number of connected components looks like this:



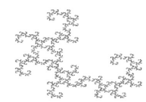
(note obvious tie-in to fractal dimension…)

Robins *et al.*, *Physica D* **139**:276, *Nonlinearity* **11**:913

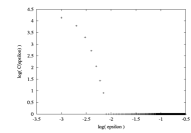
601

Connected components: examples

If the data points are samples of a connected set like this:



The number of connected components looks like this:



602

Doing that with holes (B_1):

- The alpha shape

603

Using TDA to analyze the polar ice pack:

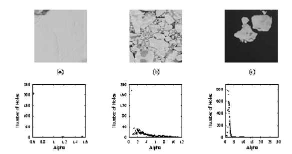
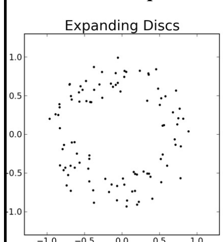


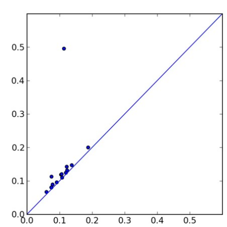
Figure 6: Using α-shapes to distinguish different ice/water distributions: (a) almost-solid ice (b) a lead filled with various-size floes (c) open water with a few big floes. The shape of the b_1 curve below each image is a direct measure of the morphology of the ice and water. Scales are dissimilar.

V. Robins, J. Abernethy, N. Rooney, and E. Bradley, "Topology and intelligent data analysis" *Intelligent Data Analysis* **8**:505-515 (2004)

604

Tracking the birth and death of topological features: the *persistence diagram*





Movie courtesy of Liz Munch

605

Using TDA to forecast solar flares:

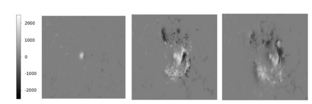
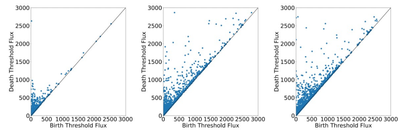


Figure 1: Three observations of line-of-sight magnetograms of sunspot #AR 12673, which produced multiple major (M-class and X-class) flares as it crossed the disk of the Sun in September 2017: (a) at 0000 UT on 9/1, (b) at 0900 UT on 9/5, about 24 hours before producing an X-class solar flare, and (c) at 1000 UT on 9/7, around the time of an M-class flare.

606

Using TDA to forecast solar flares:



0000 UT on 9/1/17 | 0900 UT on 9/5/17 | 1000 UT on 9/7/17

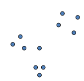
V. Deshmukh, T. Berger, J. D. Meiss, and E. Bradley, "Shape-based Feature Engineering for Solar Flare Prediction," *IAAI 2021*

607

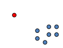
What does all of that have to do with filtering?

- attractors of dynamical systems are *perfect*
- i.e., they contain no *isolated points*
- so any isolated points that exist are probably noisy

608



- how many "lumps" in a data set:
- ε-connectedness (after Cantor)
- ε-connected components
- ε-isolated points:



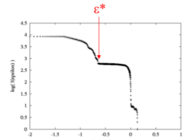
609

What does all of that have to do with filtering?

- attractors of dynamical systems are *perfect*
- i.e., they contain no *isolated points*
- so any isolated points that exist are probably noisy
- could we use that C(ε) curve to find them?

610

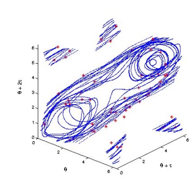
The effect of noise is to add isolated points to the set and a shoulder to the C(ε) curve:

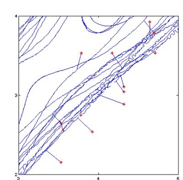


So if you know that the object is connected, you can reasonably assume that any isolated points are noisy, and remove them by pruning with ε = ε*

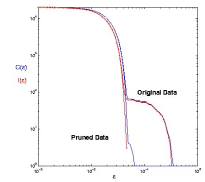
Robins *et al.*, *Intelligent Data Analysis* **8**:505, *Chaos* **14**:305

611

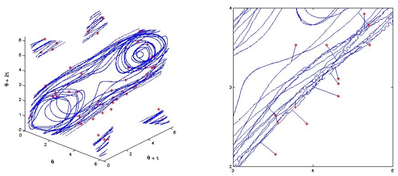




612



613



Note: this is what's known as "shot" noise — only a few of the points are affected, and by large amounts.
What about other kinds of noise?
And how can we evaluate the results in those situations?

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Evaluating these results using dynamical invariant calculations:

Noise Type	Parameters	λ of Noisy Data	λ of Filtered Data
Constant	0.5	5.7	3.2
Constant	1	35.5	3.2
Constant	2	82.2	3.4
Constant	3	81.3	2.9
Uniform	[0.5 4]	65.9	2.8
Uniform	[2 4]	81.9	4.7
Uniform	[2 6]	90.7	3.0
Gaussian	$\sigma = 1$, $x = 3$	73.5	2.9
Gaussian	$\sigma = 1.75$, $x = 2.25$	68.2	2.5
Gaussian	$\sigma = 2$, $x = 4$	78.2	3.5

Aside: what *should* noise do to λ? To d_{corr}?

615

Some issues here:

- Doesn't work equally well for all kinds of noise.
- Doesn't work in real time. You have to have the whole data set before you can find the ε-isolated points.
- How could we do better than just *removing* those points?

616

Filtering in nonlinear & chaotic systems: review

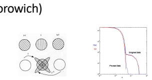
Linear filtering is a bad idea.
- Superposition breaks in nonlinear systems, making transforms problematic
- Chaotic systems have meaningful signal at **all** frequencies, so tossing everything in some band of the spectrum is problematic

Don't bleach chaotic data

Chaos 3, 771 (1993); https://doi.org/10.1063/1.165936

James Theiler

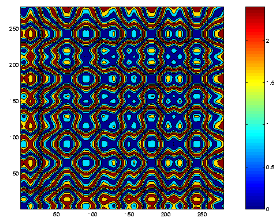
Use the known properties of the dynamics
- Geometry (Farmer/Sidorowich)
- Topology (Robins)
- Many others…



617

In the field of dynamical systems, we typically don't have "big data."

618



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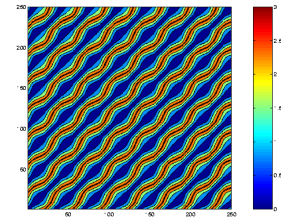
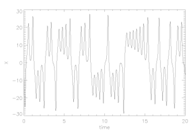
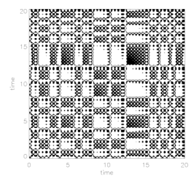


620

A signal and its recurrence plot:





621

Recurrence plots:

• introduced in 1987 by Eckmann, Kamphorst and Ruelle (*Europhysics Lett.* **4**:973-977)

• visualization of the recurrence matrix of a sequence: vec(x)1, vec(x)2, …vec(x)N

• (*i,j*)th pixel color codes distance between *i*th and *j*th sequence elements
- -"thresholded" – black if $\delta_l < \| x_i - x_j \| < \delta_h$
- -"unthresholded" – color bar for different distances

• symmetric

• clearly expose (quasi)periodicity

• preserve the dynamical invariants of the system (McGuire et al., *Physics Lett. A* **237**:43-47)

622

Applications

• physiological data (EMG, EEG, EKG, individual neuron data, breathing rate, blood pressure, …)
• driven systems
• amino acids in a protein
• noisy sequences
• nonstationary sequences
• free software: home.netcom.com/~eugenek/download.html

Contact me for references and/or see *Chaos* **12**:596-600.

623

Challenge: Understanding RP structure

e.g., to identify a dynamical system, compare two of them, analyze their behavior, …

RQA (Zbilut & Webber, *Physics Lett. A* **171**:199-203) defines several measures on the black points in an RP:
• % recurrence, the percentage of black points
• % determinism, the percentage of those black points that are contained in lines parallel to (but excluding) the main diagonal
• entropy, as calculated by binning and sorting those diagonal lines according to length
• trend, which measures how quickly the RP "pales" away from the main diagonal, and
• the reciprocal of the longest line length found in the computation of % determinism

624

The problem with RQA:

Its lumped statistical nature means that it cannot elucidate the spatiotemporal details of the dynamics.

625

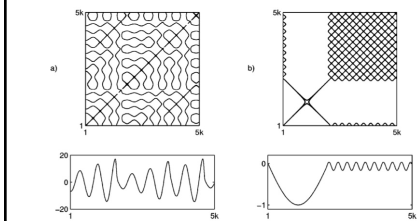
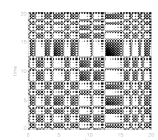


FIG. 3. RQA results on structurally dissimilar TRPS can be almost identical. These two very different TRPS, one (a) from the Rössler system and one (b) a sine-wave signal of varying period, have equal or near-equal values of REC (2.1%) and DET (42.9% for the Rössler data and 45.8% for the varying-period sine wave). Both TRPS were generated from 5000-measurement time series using an embedding dimension of 1 and a minimum line length of 5.

626

Understanding RP structure:

- combine RQA and PCA to pick out "important directions"
- geometry:
  - linear decomposition
  - UPOs!



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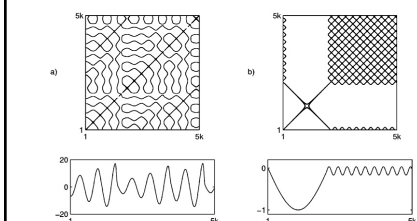


FIG. 3. RQA results on structurally dissimilar TRPS can be almost identical. These two very different TRPS, one (a) from the Rössler system and one (b) a sine-wave signal of varying period, have equal or near-equal values of REC (2.1%) and DET (42.9% for the Rössler data and 45.8% for the varying-period sine wave). Both TRPS were generated from 5000-measurement time series using an embedding dimension of 1 and a minimum line length of 5.

628

CHAOS VOLUME 8, NUMBER 4 DECEMBER 1998

Recurrence plots of experimental data: To embed or not to embed?

Joseph S. Iwanski
Department of Applied Mathematics, University of Colorado, Boulder, Colorado 80309-0526
Elizabeth Bradley[a)]
Department of Computer Science, University of Colorado, Boulder, Colorado 80309-0430

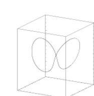
(Received 13 March 1998; accepted for publication 17 August 1998)

A recurrence plot is a visualization tool for analyzing experimental data. These plots often reveal correlations in the data that are not easily detected in the original time series. Existing recurrence plot analysis techniques, which are primarily application oriented and completely quantitative, require that the time-series data first be embedded in a high-dimensional space, where the embedding dimension d_E is dictated by the dimension d of the data set, with $d_E \geqslant 2d+1$. One such set of recurrence plot analysis tools, recurrence quantification analysis, is particularly useful in finding locations in the data where the underlying dynamics change. We have found that for certain low-dimensional systems the same results can be obtained with no embedding. © *1998 American Institute of Physics*. [S1054-1500(98)00604-1]

But…norm matters $*%$(*

629

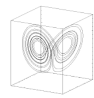
Unstable periodic orbits:



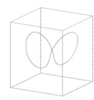
- infinite number of UPOs, of all periods, densely embedded in chaotic attractor
- dynamical invariant; can be used to calculate other invariants (Auerbach *et al.*, *PRL* **58**:2387)
- orbit on the attractor is the closure of the set of UPOs – trajectory hops from one to the other (various Cvitanovic Papers in *PRL, Nonlinearity*)

630

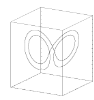
What UPOs look like:



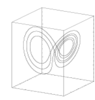
full trajectory



one-cycle



two-cycle



five-cycle

(periodicity depends on plane of section)

631

Aside: Finding UPOs:

- basic algorithm (Gunaratne et al., *Phys.Rev.Lett.* **63**:1):
  - generate Poincare section
  - find "close returns" (heuristic parameter)
  - bin according to period
  - average
- *O(n³)* at least
- can use estimates of local dynamics to improve this (So et al., *Phys.Rev.Lett.* **76**:4705, *Phys.Rev.E* **55**:5398)
- if have eqns, can obviously do better (flows: Guckenheimer & Meloon, *SIAM J.Sci.Comp.* **22**:951; maps: Biham & Wenzel, *Phys.Rev.Lett.* **63**:819)

632

Some associated issues…

A UPO's stability (transverse Lyapunov exponents) affects whether — and for how long — it will turn up in a given orbit (and therefore whether or not that algorithm can find it)

→ different trajectories visit different UPOs in different order
→ no finite-length trajectory visits all UPOs
→ hard to find high-period UPOs

NB: these are fundamental properties; all UPO-finding algorithms suffer from the same problems.

BUT…

There is evidence that a system's short, low-period UPOs provide "good" descriptions of its dynamics (Auerbach et al., *Phys.Rev.Lett.* **58**:2387)

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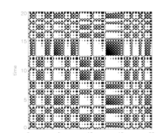
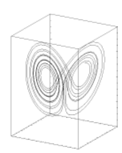
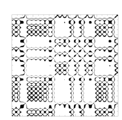


Perhaps the set of unstable n-periodic orbits that lie within a chaotic attractor are a useful geometric basis set for RPs of any orbit on that attractor?

634





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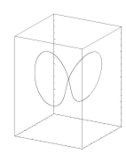

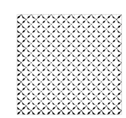

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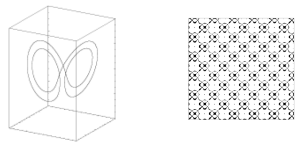

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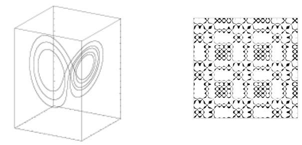

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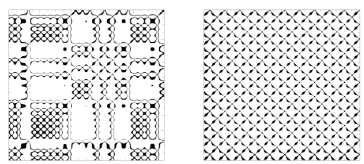

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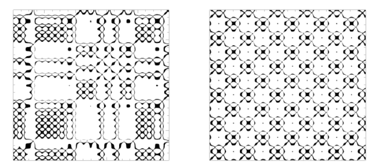

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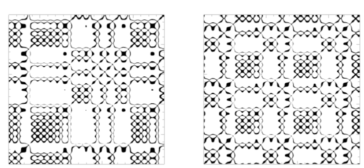


641

Why this connection is useful:

- find UPOs…in *O(n log n)* time
- see exactly when trajectory visits each
- makes compositional relationship clear
  - two-cycle as "part of" five-cycle
  - trajectory as made up of UPO segments
- independent of coordinates (*x, xy, xyz*, …)
- independent of norm and threshold corridor (mostly)
- identify system
- compare two systems

642

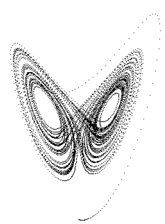
- understanding RP structure is a challenge
- finding UPOs is computationally intensive
- each is the other's solution:
  - UPO as geometric basis for RP structure
  - RP as way to find UPOs
- nice way to see the compositional relationships, identify and compare systems, etc.

643

Un/stable manifold structure on a 3D chaotic attractor:

644

How that evolves with time:



645

Aside: How to find *those*, numerically?

646

***Using* that structure:**

- Farmer's noise reduction scheme
- Spacecraft orbits

647

The un/stable manifolds of a point are curves.

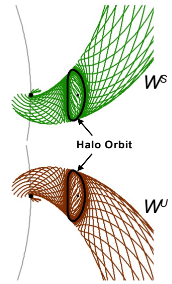
What do the un/stable manifolds of a *periodic orbit* look like?

648

Want to get a spacecraft onto a "halo orbit," which is a UPO of the dynamics.

Unstable Periodic Orbits (UPOs) have invariant manifolds:

- Stable Invariant Manifold (W^S)
  - *The set of all trajectories a particle could use to arrive onto the UPO.*
- Unstable Invariant Manifold (W^U)
  - *The set of all trajectories a particle could take after a small perturbation from the UPO.*

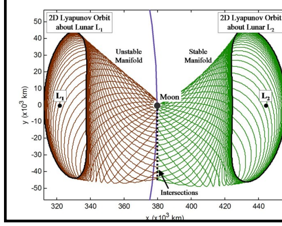


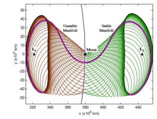
Jeff Parker, PhD thesis, UColorado 2008

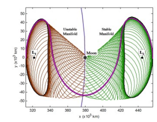
649

Low-energy (cheap) orbit transfers!

- Depart along $W^U{}_{L1}$ & arrive on $W^S{}_{L2}$







Jeff Parker, PhD thesis, UColorado 2008

650

Using the un/stable manifold structure:

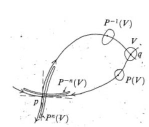
- Farmer's noise reduction scheme
- Spacecraft orbits
- Proofs of chaos

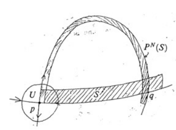
651

What happens to a ball V of ICs near a heteroclinic point q of a map Λ:

652

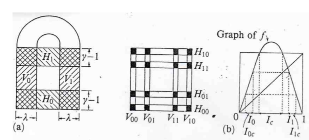
From Holmes:





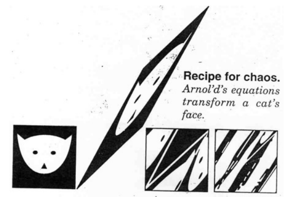
653

Smale's Horseshoe: a stylized version of that



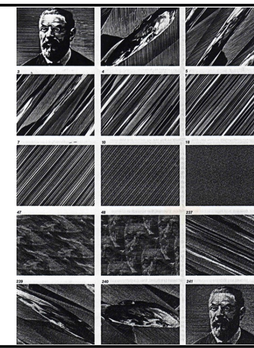
654

Similar stretch'n'fold maps:



655

Another interesting one…

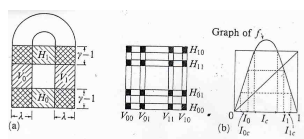


Crutchfield *et al.*
Chaos **255**:46

656

Back to horseshoes:

• *invariant set* of the map: points that lie in the intersection of all forward iterates



657

Proposition:

The invariant set of the horseshoe map contains:

1. countably infinite set of periodic orbits, of all periods
2. uncountably infinite set of *nonperiodic* orbits, including countably infinite many homo- and heteroclinic orbits
3. a *dense orbit* (aka chaos)

658

Chaos in maps:

Can extend that result to any map that has the same qualitative features — need only structural stability of the map (Λ) and some contraction conditions.

And if the map has a transverse homoclinic point, it automatically has those features.

659

Why is this useful?

If a system has horseshoes in its state-space dynamics (or Poincare sections thereof), can easily prove it's chaotic.

Need contraction conditions to prove that there's a chaotic *attractor.*

~~*So finding horseshoes* amounts to proving that the system is chaotic!*~~

(and horseshoes happen when stable & unstable manifolds intersect transversely)

* Finding horseshoes: Mel'nikov's method

660

All of that came from transverse intersections of un/stable manifolds.

- noise reduction
- homoclinic tangles
- spacecraft orbits (cheap or even free)
- horseshoes, Poincare recurrence and chaotic cats
- specific orbit properties, including proofs of chaos

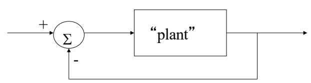
661

Transverse un/stable manifolds also let you control chaos!

662

Control Theory 101:

- negative feedback



663

Control Theory 101:

- negative feedback
- setpoint
- control variable
- strategies: P/I/D

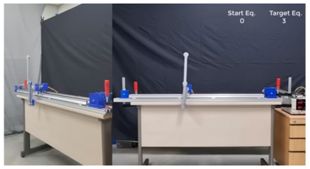
664

Controlling nonlinear systems:

- **much** harder; all that nice PID stuff works only with linear systems…..

665

Controlling nonlinear systems:



https://www.youtube.com/watch?v=I5GvwWKkBmg

Thanks to Zach for finding this ☺

666

Controlling nonlinear systems:

- much harder; all that nice PID stuff works only with linear systems
- but Hartman-Grobman ⟹ linear control *does* work…locally
- "locally" depends on eigenstuff and the sensor & actuator capabilities

667

Controlling chaotic systems:

1. have transverse un/stable manifolds
2. that situation is locally controllable
3. *what points might be useful targets?*
4. *what about the nonlocal part of the control?*

668

Controlling chaotic systems:

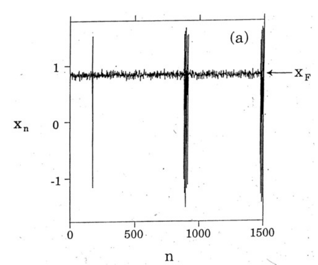
1. have transverse un/stable manifolds
2. that situation is locally controllable
3. *what points might be useful targets?*
   UPOs, for instance
4. *what about the nonlocal part of the control?*
   Dense attractor coverage

669

Balancing a chaotic system on a UPO:

1. find one
2. linearize there
3. design the controller and calculate the controllable region
4. let the system go
5. detect when it enters the controllable region
6. and turn on the local-linear controller

670



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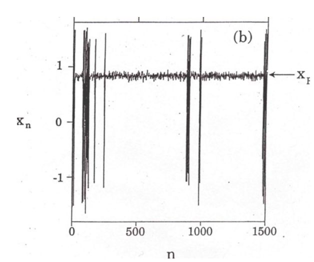

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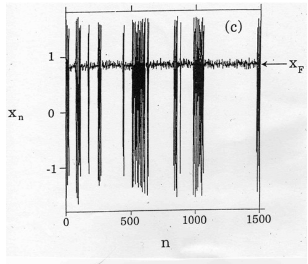


673

A real-world application of that:

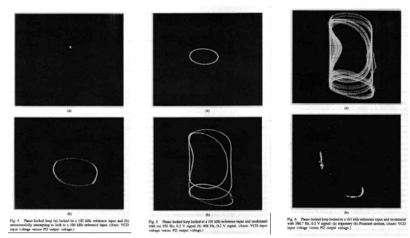
914 IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: FUNDAMENTAL THEORY AND APPLICATIONS, VOL. 43, NO. 11, NOVEMBER 1996

Using Chaos to Broaden the Capture Range of a Phase-Locked Loop: Experimental Verification

Elizabeth Bradley, *Member, IEEE*, and Douglas E. Straub

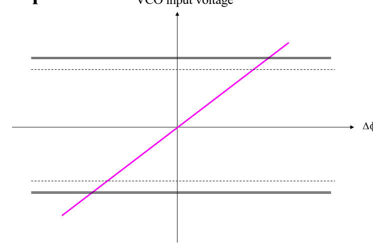
674

Bifurcations in the dynamics of the PLL



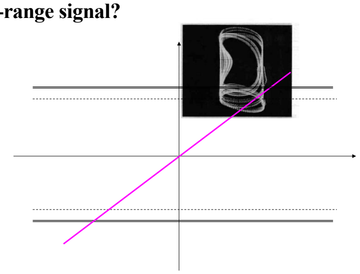
675

Normal capture dynamics, in the PLL state space:



676

What if you wanted to capture an out-of-range signal?



677

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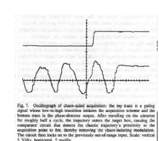

678

Problem:

1. find one
2. linearize there
3. design the controller and calculate the controllable region
4. let the system go
5. detect when it enters the controllable region
6. and turn on the local-linear controller

But…how long will that take?

679

Tuesday 25 April

Presentations

680

Presentations

- 4/25, 4/27, and 5/2, during the normal class period
- 8 min for presentation + 2 min for Q&A
- Schedule:
  - T 4/25: Cara, Nathan, Madi [+2min], Paul, Akshit
  - Th 4/27: David, Joey, Patrick [+2 min], Frank, John O., Tirthankar, Pawin
  - T 5/2: Carolyn, Zach [+2min], Alex, Brian, Gowri, John M., Emily
- See presentation hints handout on course webpage
- Practice, practice, practice!
- We'll project from my computer to avoid projector ick. Send me your slides by 6am on the day of your presentation (pdf or ppt/x format) and meet me between 9-9:20am to test them out
- 4446 people: be ready to take notes and email them to me by midnight on the day of the presentation

681

The endgame, part I

5446 students:

Submit your final paper on Canvas before 5pm on Thursday 4 May. Please look at the project handouts on the course webpage for guidelines & requirements on this paper.

Everybody:

Final exam here on Tuesday 9 May at 4:30pm. Mostly concepts and drawings—both understanding them and producing them

How to study: review the readings, the slides, your notes, and your problem sets. The MOOC may also be useful.

682

The endgame, part II

Everybody:

The final deadline for all problem sets is also 5pm on the last day of classes. **Note: problem sets received after that time will not be graded!**

I will harvest the unit test grades from the Complexity Explorer website the day after the final, when I'm working on grades.

I will have the grades done and uploaded by the posted deadline for this course.

683

- FCQs are really important—to CU and to me
- We're going to take the first 10 min of class today for filling them out
- You know the drill: colorado.campuslabs.com/courseeval
- p.s. they don't seem to have managed to fix the bug that makes this not work well with Safari, so use Firefox or Chrome

684

Thursday 27 April

Presentations

685

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• 8 min for presentation + 2 min for Q&A

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687

Tuesday 2 May

Presentations

688

Presentations

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• 8 min for presentation + 2 min for Q&A

• Schedule:

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• See presentation hints handout on course webpage

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• We'll project from my computer to avoid projector ick. Send me your slides by 6am on the day of your presentation (pdf or ppt/x format) and meet me between 9-9:20am to test them out

• 4446 people: be ready to take notes and email them to me by COB tomorrow (Wednesday) — these are worth a few points in your grade, BTW.

689

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691

Thursday 4 May

692

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694

OGY Control:

1. find a UPO
2. linearize there
3. design the (local linear) controller and calculate the controllable region
4. let the system go
5. detect when it enters the controllable region
6. and turn on the controller from step 3

695

OGY Control:

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But…how long will that take?

696

Targeting:

- rather than just "letting the system go."
- actively exploit SDOIC to get to control region faster
- leverage = opportunity, but also danger…

697

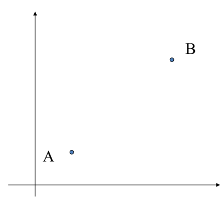
Targeting:

- rather than just "letting the system go."
- actively exploit SDOIC to get to control region faster
- leverage = opportunity, but also danger…

- Shinbrot: a nice pure-math approach
- Bradley: an ugly but useful AI approach
- many many others since

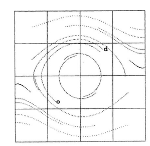
698

Control goal:



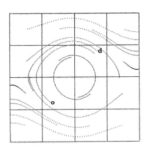
699

Map out phase portrait:



700

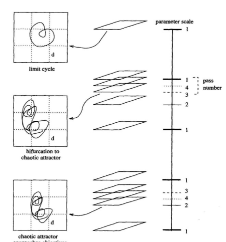
Map out phase portrait:



Automated phase portrait analysis
(cf., KAM…)

701

Map out parameter effects:

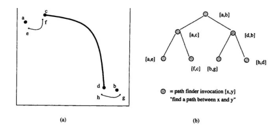


702

The *General Problem Solver* (GPS) approach to search:

(early 1960s)

703



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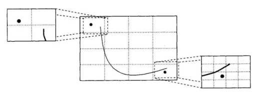
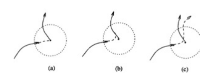


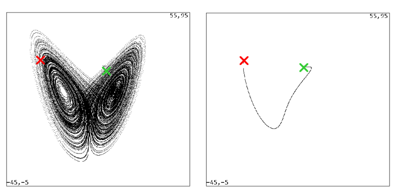
Figure 3. Refining regions between search passes: a gross path between origin and destination cells is first found, then each endpoint cell is recentered around the pair of points that will be an (origin destination) pair on the next search pass. A new grid division is then computed in each region, reflecting the local dynamics therein

705

Navigate the path by changing the control parameter at the segment junctions:



706



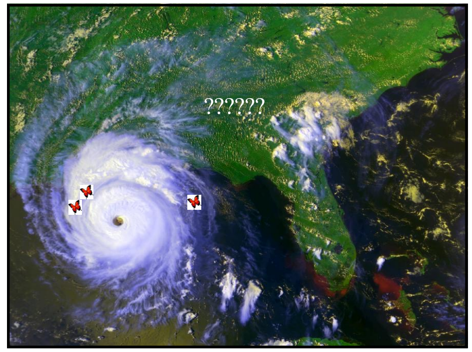
707

Erik Bollt's PhD thesis:

708

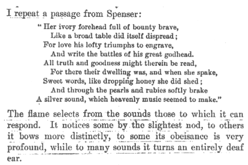
Can we do that kind of thing in spatially extended systems, too?

709



710

Exploiting SDOIC in *spatiotemporal* systems:


I repeat a passage from Spenser:

“Her ivory forehead full of bounty brave,
Like a broad table did itself dispread;
For love his lofty triumphs to engrave,
And write the battles of his great godhead.
All truth and goodness might therein be read,
For there their dwelling was, and when she spake,
Sweet words, like dropping honey she did shed;
And through the pearls and rubies softly brake
A silver sound, which heavenly music seemed to make.”

The flame selects from the sounds those to which it can respond. It notices some by the slightest nod, to others it bows more distinctly, to some its obeisance is very profound, while to many sounds it turns an entirely deaf ear.

711

Flow control

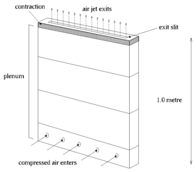
- Idea: co-opt system’s natural instability to “grow” small perturbation into desired downstream effect
- Motivation: automotive, aerospace, combustion, …
- Challenge: control the convective instability of a spatiotemporally extended nonlinear system (ouch)

712

The target system: planar jet

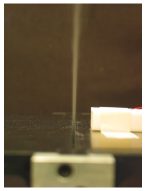
Plenum section : 1m x 0.4m x 1.5cm

- Flow treatment screens & contraction
- Aspect ratio of exit slit 160:1

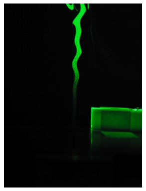


713

End view



room lighting



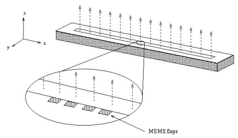
stop-action laser “slice”

aerosolized canola oil

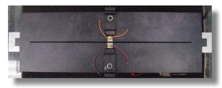
714

Actuation

Transverse velocity perturbation at the base of the boundary layer:

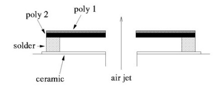


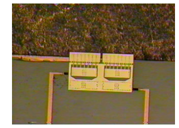
(MicroElectroMechanical Systems)



715

MEMS actuators





Video : overhead view at 2Hz, 10V

Area of individual flap is 1.0 x 0.25mm

(IEEE Trans. Adv. Packaging 26:268)

716

The Butterfly effect in action…

no forcing *6Hz forcing*

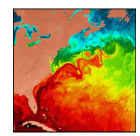
Re ~ 25

Forcing generates coherent structures that enhance entrainment and mixing

(*Exp.Fluids* 37:22)

717

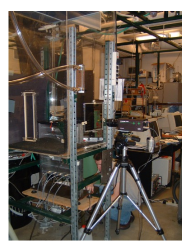
What does this have to do with reality?



718

However…

- Physically simplified
- Heavily instrumented
- Tightly isolated



So "scaling" this stuff to real-world applications is decades off, and applying it to things like hurricanes is, in practice, impossible.

719

Complexity: the flip side of chaos

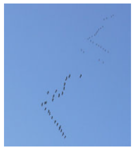
- chaos: complicated behavior from simple systems
- complexity: simple behavior from complicated systems

(way too pat, but it's a useful first-order approximation)

720

Complexity & emergence…

wikipedia



wikipedia

www.borneo.com.au

`Also: vimeo.com/31158841`

721

Complex systems: features

- lots of (identical) simple agents
- simple, *local* rules for their behavior and interaction
- no omniscience, in view or in action
- global “emergent” phenomena
- robustness!
- reductionist analysis doesn’t work

722

Automaton: machine that processes info logically, based on some encoded instructions

723

Cellular automaton:

- cells
- each has a state
- FSM determines next state, based on states of cell and of its neighbors
- time proceeds in clicks

(John von Neumann, Stan Ulam)

724

Simulation of life: what do we need to match?

- self reproduction
- *what else?*

725

Simulation of life:

- self reproduction
- Aristotle: “can nourish self and decay”
- Wikipedia:

Life (cf. biota) is a characteristic that distinguishes objects that have signaling and self-sustaining processes from those that do not,[1][2] either because such functions have ceased (death), or else because they lack such functions and are classified as inanimate.[3][4] Biology is the science concerned with the study of life.

726

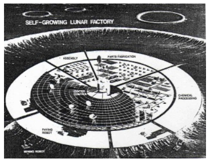
Creating automata that meet these requirements:

- the field called "Artificial Life"

Within fifty to a hundred years a new class of organisms is likely to emerge. These organisms will be artificial in the sense that they will originally be designed by humans. However, they will reproduce, and will evolve into something other than their original form; they will be "alive" under any reasonable definition of the word. . . . The advent of artificial life will be the most significant historical event since the emergence of human beings. . . .

727

A self-replicating lunar factory:



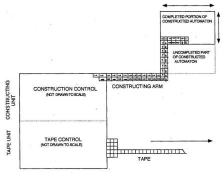
(1980 NASA study, cited in Levy)

728

http://www.youtube.com/watch?v=E3keLeMwfHY

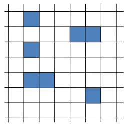
729

A simple version of that:



730

Another simplified ansatz: Conway's Game of Life



At each time step, every cell…

- Dies from overcrowding (if alive & > 3 live neighbors)
- Dies from loneliness (if alive & < neighbors)
- Is born (if dead & exactly 3 live neighbors)
- Or continue in current state

731

Dr. Conway in his Princeton office in 1993. Dith Pran/The New York Times

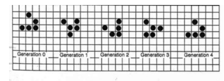
732

Conway's Game of Life

- simulation was nontrivial
- most ICs settled quickly into stable patterns
- some settled into periodic orbits
- some went into long chaotic orbits
- some moved across the board!

733

Gliders:

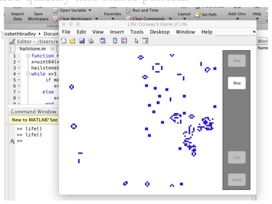


The motion of a glider in Life. In the course of the four generations required for the glider to recapture its original form, the pattern displaces itself one square diagonally.

734

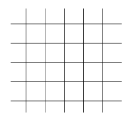
Conway's Game of Life

- simulation **<u>was</u>** nontrivial…
- most ICs settled quickly into stable patterns
- some settled
- some went in
- some moved



735

That was a 2D cellular automaton.



736

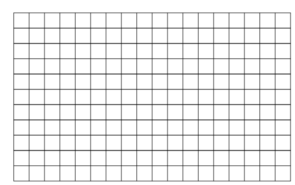
1D cellular automaton:

- cell state = 0 or 1
- FSM: next state = f(state of cell and of cells in some neighborhood)

737

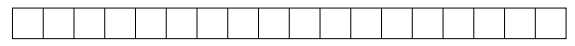
Space-time graph of behavior:

time



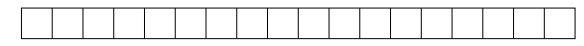
738

Next-state rules:



739

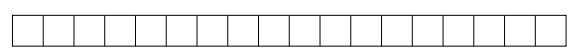
How to get a 1D cellular automaton to have some desired *global* behavior?



• computing the majority?

740

How to get a 1D cellular automaton to have some desired *global* behavior?



• computing the majority?
• or oscillating?
• this is a new kind of programming task/paradigm…….

741

The "local majority" rule:

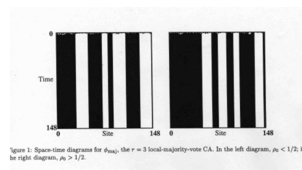


Figure 1: Space-time diagrams for ϕ_{maj}, the $r = 3$ local-majority-vote CA. In the left diagram, $\rho_0 < 1/2$; in the right diagram, $\rho_0 > 1/2$.

Mitchell & Crutchfield, *PNAS* 92:10742 (1995)

742

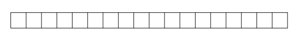
The obvious choices don't work.

• lots of (identical) simple agents
• simple, *local* rules for their behavior and interaction
• no omniscience!
• **global "emergent" phenomena**
• **reductionist analysis doesn't work**

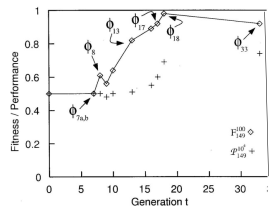
So how to find the right rule?

743

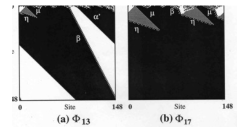
Use a GA to search the space of rules…



744



745



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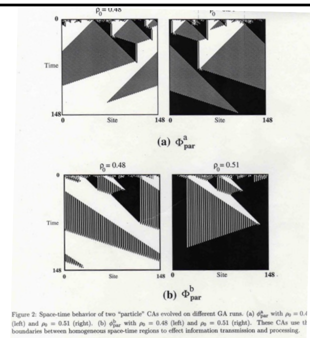
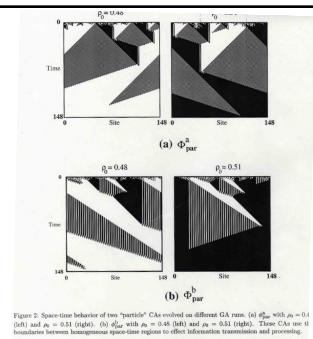


747

M&C also came up with a Feynmann-diagram like way to describe and categorize these space-time plots…

748



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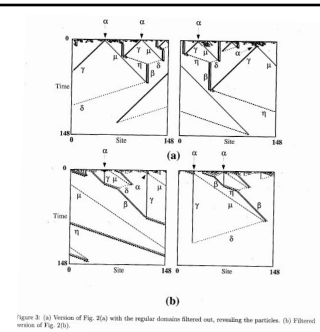

750

Chaos vs complexity in the context of dance…

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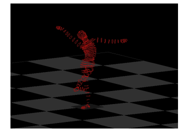
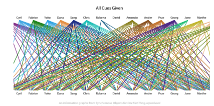
synchronousobjects.osu.edu
https://www.youtube.com/watch?v=_U9seLPSr3U
```

751

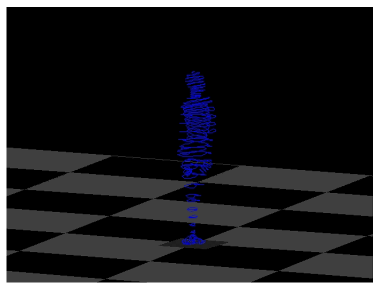
Chaos vs complexity in the context of dance…





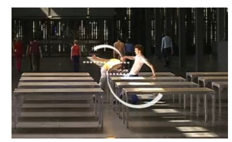
752

Chaos and dance



753

Complexity and dance



754