Coding exercise for Buckley-Leverett and spontaneous imbibition solutions

Due 12 noon Friday 28th February

This is an individual project.

Consider the following situations and relative permeabilities. In all cases consider horizontal one-dimensional flow.

You need to solve the two partial differential equations for advective flow and spontaneous imbibition both analytically, and numerically using finite differences.

Compare the solutions and comment in the results, both in terms of comparison between numerical and analytical approaches, but also in terms of the physical processes represented.

For advective flow solve

$$\frac{\partial S_1}{\partial t_D} + \frac{df_1}{dS_1}\frac{\partial S_1}{\partial x_D} = 0$$

where

$$f_1 = \frac{1}{1 + \frac{\mu_1}{\mu_2}\frac{k_{r2}}{k_{r1}}}$$

with the boundary conditions: $S_1(x_D,t_D) = S_{1i}$ for $t_D=0$, $x_D>0$ and $S_1(x_D,t_D) = 1-S_{2r}$ for $t_D>0$, $x_D=0$.

The analytical solution should follow the procedure followed in class to find $S_1(v_D)$ where $v_D=x_D/t_D$. For the finite difference solution you can choose a suitable numerical scheme and discretization, but explain what you did. For one time t_D plot $S_1(v_D)$ and compare with the analytical solution.

For spontaneous imbibition solve

$$\phi\frac{\partial S_1}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial S_1}{\partial x}$$

where

$$D = -\frac{\frac{k_{r1}}{\mu_1}\frac{k_{r2}}{\mu_2}}{\frac{k_{r1}}{\mu_1} + \frac{k_{r2}}{\mu_2}} K \frac{dP_c}{dS_1}$$

with the boundary conditions: $S_1(x,t) = S_{1i}$ for $t=0$, $x>0$ and $S_1(x,t) = S_1^*$ for $t>0$, $x=0$ where $P_c(S_1^*)=0$.

The analytical solution should follow the procedure followed in class to find $S_1(\omega)$ where $\omega = x/\sqrt{t}$ and $\omega = dF_1/S_1$. Find an imbibition fractional flow $F_1(S_1)$ that obeys:

$$F_1(S_1) = 1 - \frac{\phi}{2C^2}\int_{S_1}^{S_1^*}\frac{(\beta - S_1)D(\beta)}{F_1(\beta)}d\beta$$

$$C^2 = \frac{\phi}{2}\int_{S_{1i}}^{S_1^*} \frac{(\beta - S_{1i})D(\beta)}{F_1(\beta)} d\beta$$

This has to be solved through numerical integration and iterate to find F_1. For an initial guess choose $F_1 = (S_1 - S_{1i})/(S_1^* - S_{1i})$. Usually around 4 iterations allow a converged solution.

For the finite difference solution you can choose a suitable numerical scheme and discretization, but explain what you did. For one time t plot $S_1(\omega)$ and compare with the analytical solution.

The relative permeabilities and capillary pressures are described by the following models:

$$S_e = \frac{(S_1 - S_{1i})}{(1 - S_{2r} - S_{1i})}$$

$$k_{r1} = k_{r1}^{max} S_e^a$$

$$k_{r2} = k_{r2}^{max}(1 - S_e)^b$$

$$P_c = P_2 - P_1 = P_c^{max} \frac{\left(\frac{S_1^*}{S_{1i}}\right)^{-c} - \left(\frac{S_1}{S_{1i}}\right)^{-c}}{\left(\frac{S_1^*}{S_{1i}}\right)^{-c} - 1};\ P_c \geq 0$$

1. **CO_2 injection into an aquifer. Primary drainage: a non-wetting phase displaces the wetting phase.**

The CO_2 is the non-wetting phase. Here we will represent a case when we have an invasion-percolation like advance at the pore scale and study how the CO_2 progresses. Only consider advective flow – there is no spontaneous imbibition.

2. **Water displaces hydrogen in a storage site. This is secondary imbibition where a wetting phase displaces a non-wetting phase.**

H_2 is the non-wetting phase. This represents the withdrawal of hydrogen from a storage site, where water moves in from the aquifer to trap hydrogen. Here we have percolation-like advance with trapping. Consider both advective flow and spontaneous imbibition.

3. **Water displacing CO_2 in a depleted oil-field. Here the CO_2-brine system is mixed-wet.**

Water displaces CO_2 but previous contact with oil renders the system mixed-wet. We have a combination of percolation-like imbibition and drainage processes. Consider both advective flow and spontaneous imbibition.

The table below shows the values of the parameters to use.

Case/Property	**Case 1**	**Case 2**	**Case 3**
Phase 1	CO_2	Water	Water
Phase 2	Water	Hydrogen	CO_2
μ_1	0.1 mPa.s	1 mPa.s	0.4 mPa.s
μ_2	1 mPa.s	0.1 mPa.s	0.1 mPa.s
k_{r1}^{max}	1	0.15	0.4
k_{r2}^{max}	1	1	0.8
a	1.2	4	8
b	3	1.5	3
S_{1i}	0	0.2	0.2
S_{2r}	0.2	0.5	0.15
K	Not needed	$10^{-13}\,m^2$	$10^{-13}\,m^2$
ϕ	Not needed	0.25	0.25
P_c^{max}	Not needed	100 kPa	100 kPa
c	Not needed	0.3	0.3
S_1^*	Not needed	0.5	0.5

You need to write a short report (around 5 pages) presenting your results and explaining them physically. While you need to upload your code to Github, you will be marked only on the report you submit.

70% is for getting correct solutions; 20% for the explanations and 10% for the presentation of the report.

This is a difficult exercise: you can still obtain good marks even if some things have not been done.