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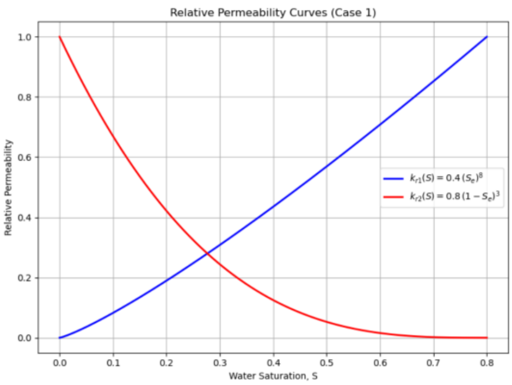


BUCKLEY-LEVERETT & SPONTANEOUS IMBIBITION

[FLUID AND FLOW IN POROUS MEDIA]

PROJECT SUMMARY

Programming for Analytical and Numerical Solutions

Ashwin Vel
CID: 06035193

The digital form of my work can be access here!

Buckley-Leverett

1. Analytical Solution

The Buckley–Leverett equation expresses conservation of water saturation under waterflooding conditions:

$$\frac{\partial S_1}{\partial t_D} + \frac{df_1}{dS_1}\frac{\partial S_1}{\partial x_D} = 0$$

where $f_1(S_1)$ is the fractional flow function and is defined as the ratio of water mobility to the total mobility and is typically an S-shaped function of saturation. By transforming the equation into dimensionless form, the governing equation simplifies considerably.

The analytical solution comprises three distinct sections:

- A constant state near the injection point where the saturation is at its maximum ($S_1=S^*$).
- A shock region where a discontinuity develops, determined by the tangent construction—yielding a shock speed (v_s)
- A rarefaction wave where the saturation changes smoothly, corresponding to the derivative of the fractional flow function.

<u>CASE 1: WATER WET: NON- WETTING PHASE → WETTING</u>

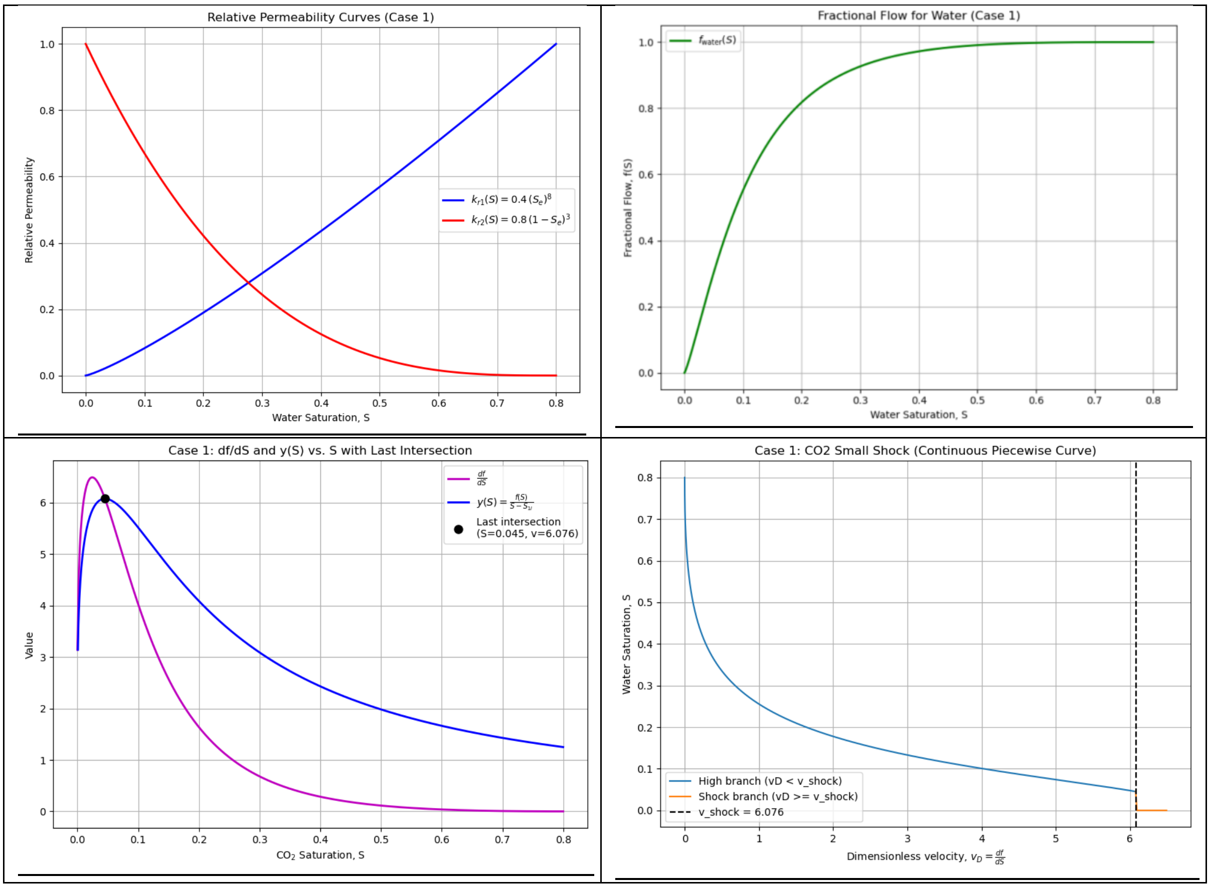
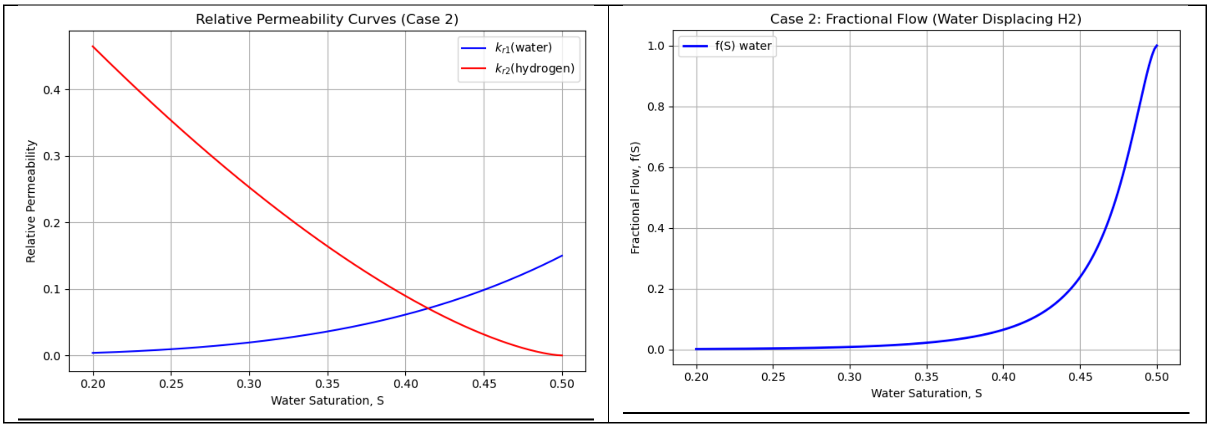


Figure 1 Case 1 Buckley- Leverett Analytical

The fractional flow function rises sharply with increasing CO_2 saturation because CO_2 mobility is higher than water. This is why there is a discontinuous jump (small shock) where the saturation changes rapidly over a very narrow range. This sharp front means that water remains almost immobile until a vD is reached, after which the fast-moving CO_2 rapidly displaces it.

CASE 2: WATER WET- WETTING PHASE →NON-WETTING



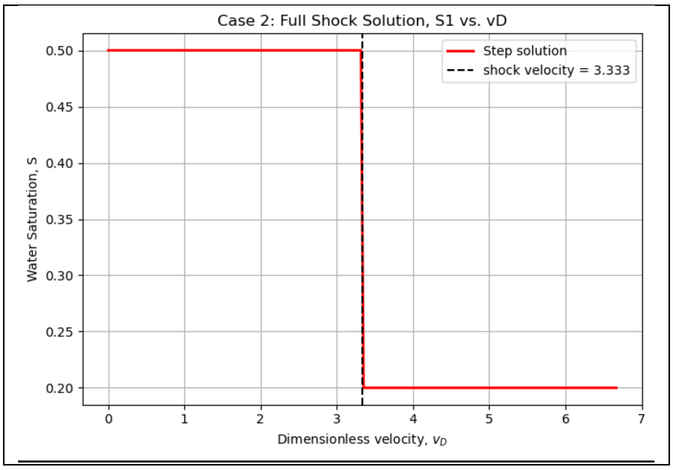


Figure 2 Case 2 Buckley- Leverett Analytical

At low water saturations, water remains nearly immobile because its relative permeability is extremely low. Because water's mobility is lower in a water-wet system as water goes and fill up the throat rather than the pore, the fractional flow increases very slowly. Thats the fractional flow function stays nearly flat until a critical threshold is reached, which is at S_{2r}. That why you can find any intersection between the derivate plot and the y plot. This all-shock behaviour reflects that water mobilizes only after a critical saturation is achieved, leading to a very abrupt displacement front.

CASE 3: MIXED-WET

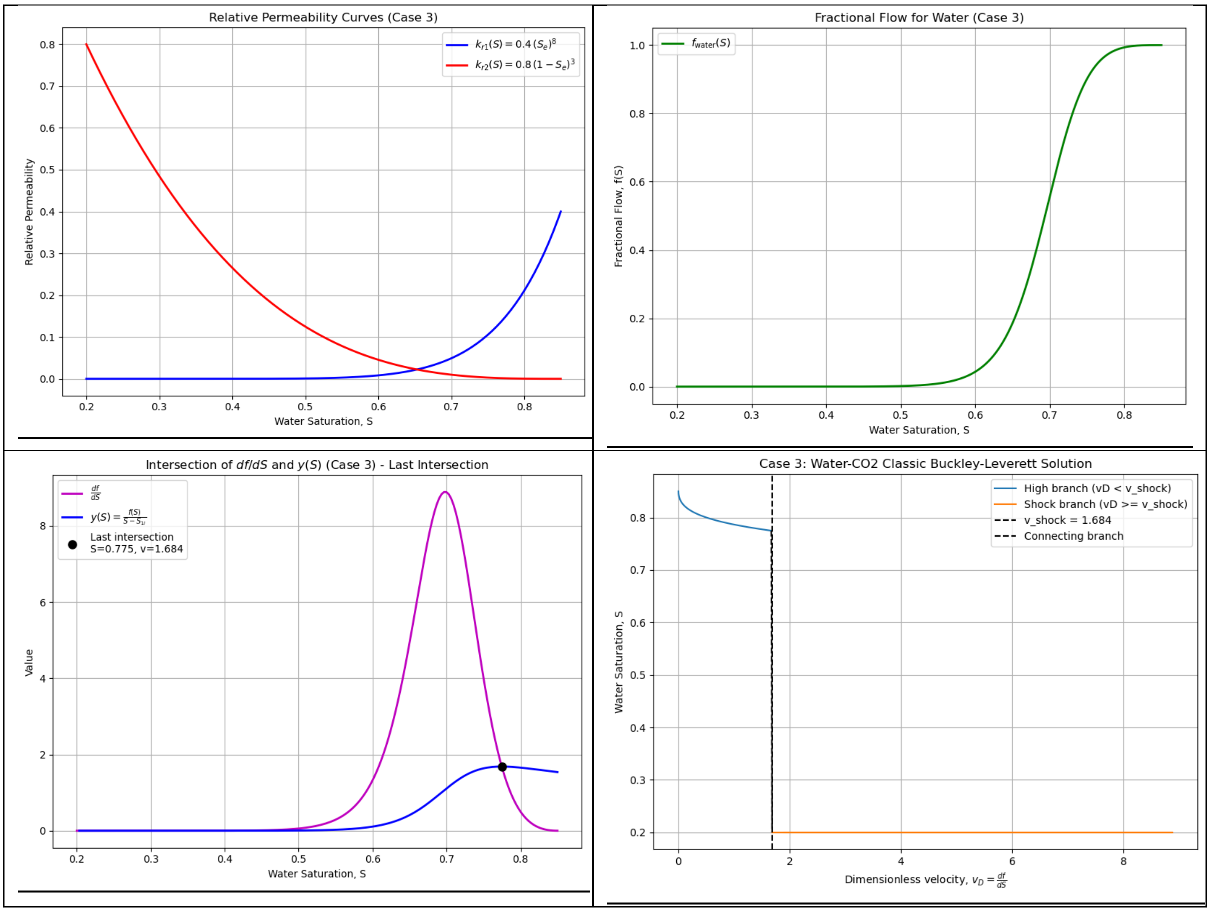


Figure 3 Case 3 Buckley- Leverett Analytical

In mixed-wet reservoirs, previous fluid contact with oil alters the wettability, leading to an intermediate balance between wetting and non-wetting behavior resulting in fractional flow function is less steep than in Case 1 but not as gradual as in Case 2. its slope is moderate, and it exhibits an inflection point where the initial part is no shock and after the inflection point, it starts having a shock. Hence why, at velocity lower than vD, we see a rarefaction section.

2. Numerical Solution

An explicit finite difference formulation of the Buckley–Leverett mass conservation equation is used. In dimensionless form, the saturation update for each grid block is given by:

$$S_j^{n+1} = S_j^n - \frac{\Delta t}{\Delta x}\left[f_{\text{water}}(S_j^n) - f_{\text{water}}(S_{j-1}^n)\right]$$

Parameter	Value
Domain Length, L	1.0 (dimensionless)
Spatial Step, Δx	0.01
Number of Grid Points, N	101

Time Step, Δt	0.001
Number of Time Steps, nmax	200
Final Time, T	0.2

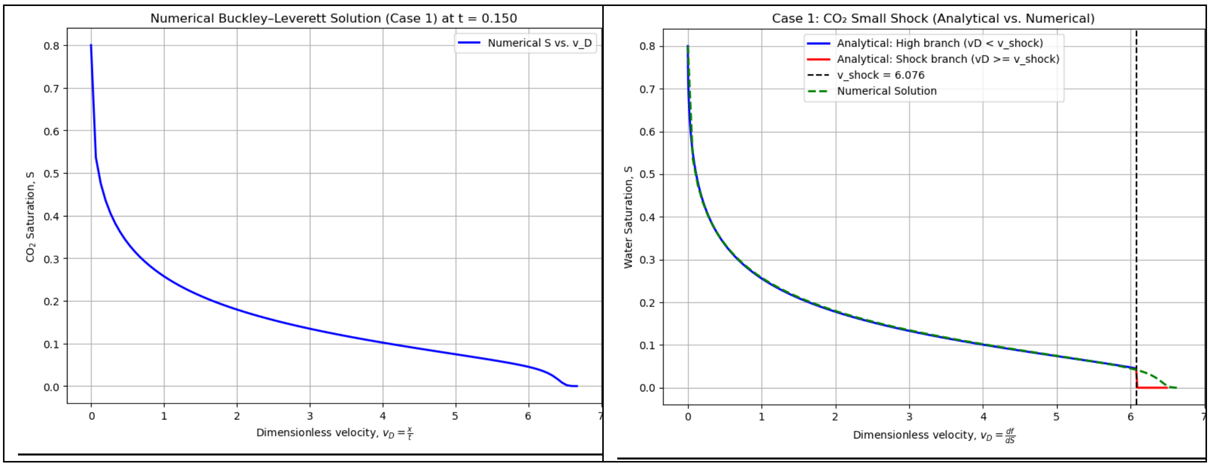


Figure 4 Case 1 Buckley- Leverett Numerical & Analytical Comparison

For numerical, a small numerical diffusion smooths the shock slightly, but overall the profile still captures the steep saturation gradient predicted by theory. The analytical solution indicates a “small shock,” meaning a narrow jump from high CO_2 saturation to lower CO_2 saturation over a small range of v_d. The numerical solution follows the same trend but appears slightly more diffused near the shock due to the explicit method’s inherent numerical dispersion. Despite this minor smoothing, both solutions agree on the overall shape and location of the saturation front, confirming that the numerical approach successfully replicates the key physics of CO_2 displacing water in a strongly water-wet system.

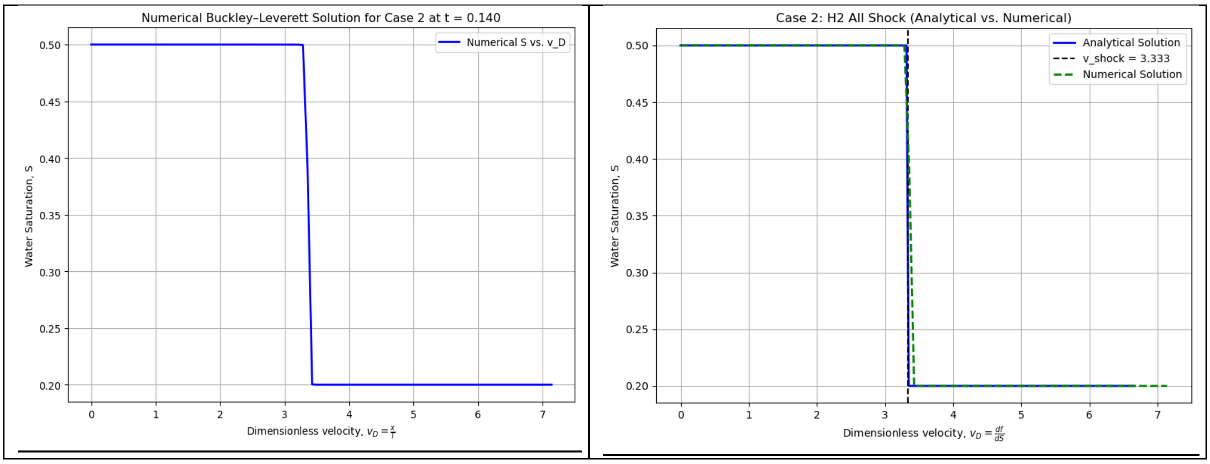


Figure 5 Case 2 Buckley- Leverett Numerical & Analytical Comparison

The explicit scheme produces an almost vertical step in the saturation vs. v_d, showing the “all-shock” nature of the displacement. Minor numerical diffusion may smooth the edge slightly, but the profile remains predominantly a single sharp jump. The numerical result matches well, capturing the near-vertical saturation front despite a tiny diffusive rounding at the shock. The smearing happens as a factor of sqrt(t).

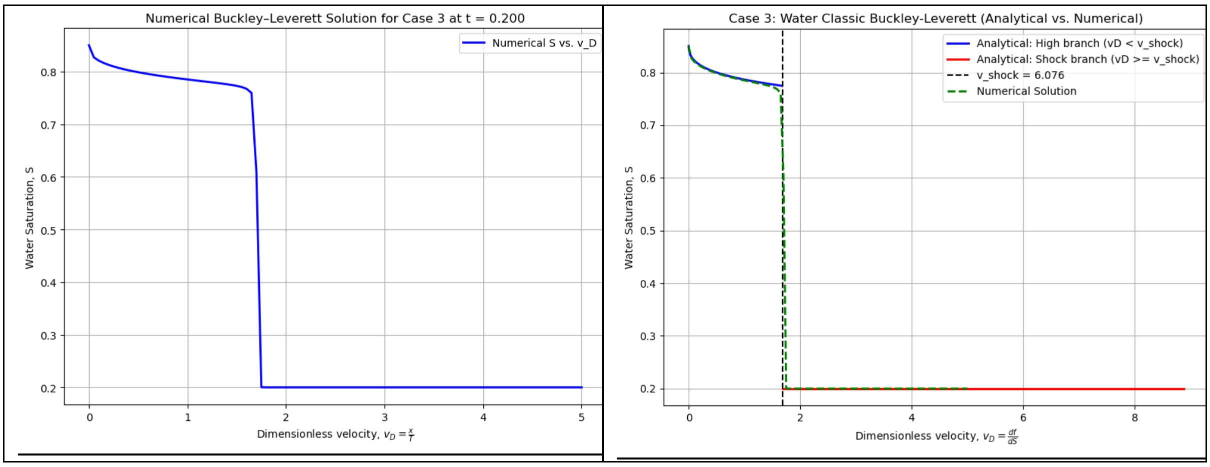


Figure 6 Case 3 Buckley- Leverett Numerical & Analytical Comparison

Numerically, an explicit scheme shows a partial shock front alongside a gently sloped region, reflecting moderate mobility contrasts. Very minor numerical diffusion softens the abrupt jump, yet both solutions confirm that certain saturations change rapidly (shock), while others vary smoothly (rarefaction), capturing the mixed-wet displacement behavior.

Spontaneous Imbibition

3. Analytical Solution

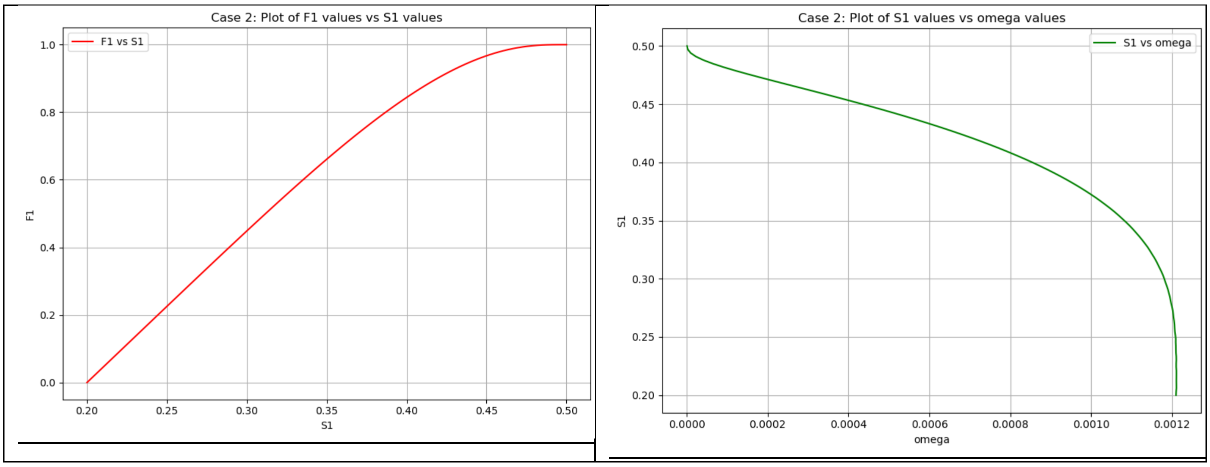


Figure 7 Case 2 Spontaneous Imbibition Analytical

In Case 2’s spontaneous imbibition under water-wet conditions, the governing PDE is dominated by capillary forces and behaves diffusively, so the water saturation profile scales with ω=x/sqrt(t).The inlet saturation S* (where capillary pressure is zero) transitions smoothly to the initial saturation Si, and the fractional flow F1(S) rises from near zero to one, reflecting increasing water mobility with saturation. Consequently, the ω-based solution is a continuous, monotonic curve. This profile captures how water imbibes into the medium by capillary diffusion, characteristic of a strongly water-wet system.

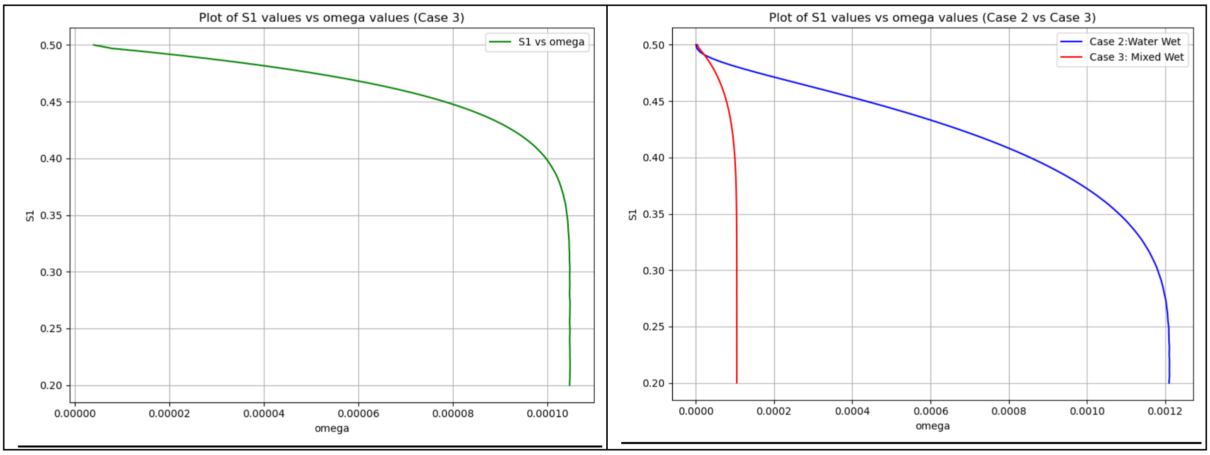


Figure 8 Case 3 Spontaneous Imbibition Analytical & Comparison

In Case 3 (mixed-wet), the spontaneous-imbibition solution is still driven by capillary diffusion, but water's mobility is restricted by the partial wetting conditions, so the ω-based saturation profile is more gradual than the strongly water-wet scenario of Case 2. In the second plot, comparing Case 2 (blue) and Case 3 (red) shows that the water-wet system (Case 2) imbibes more aggressively, yielding a steeper saturation decline with ω. By contrast, the mixed-wet system (Case 3) exhibits slower imbibition and a gentler slope, reflecting how water is partially inhibited from occupying the pore space due to less favorable wetting conditions.

4. Numerical Solution

This explicit forward-Euler method solves for the water saturation SSS at each time step, advancing the capillary-driven front. The final profile is then plotted against xxx and the similarity variable ω=x/sqrt(tfinal).

$$S_i^{n+1} = S_i^n + \frac{\Delta t}{\phi\,(\Delta x)^2}\left[D\left(\frac{S_i+S_{i-1}}{2}\right)\left(S_i^n - S_{i-1}^n\right) \;-\; D\left(\frac{S_{i+1}+S_i}{2}\right)\left(S_{i+1}^n - S_i^n\right)\right].$$

<u>CASE 2: WATER WET- WETTING PHASE →NON-WETTING</u>

Parameter	Value
Domain Length, L	1.0 m
Spatial Step, Δx	0.01
Number of Grid Points, N	101
Time Step, Δt	0.01
Number of Time Steps, nmax	20,000
Final Time, T	2000s

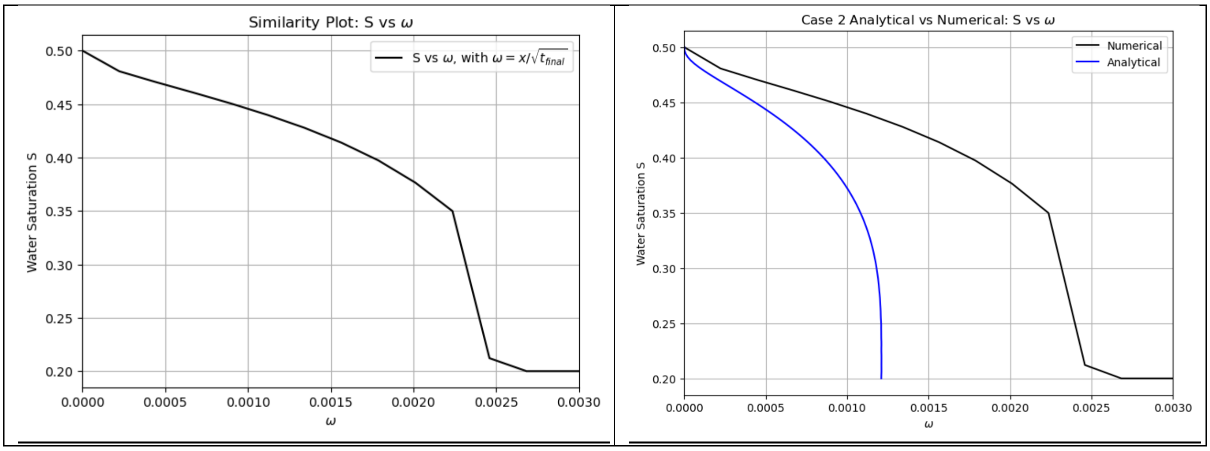


Figure 9 Case 2 Spontaneous Imbibition Numerical & Comparison with Analytical

In the left plot, we see the final saturation profile versus the similarity variable omega, which is defined as x divided by the square root of the final time. Water saturation starts at the boundary saturation S* near omega equals zero and transitions smoothly toward the initial saturation Sinit at higher omega, reflecting capillary-driven imbibition in a strongly water-wet medium. In the right plot, the numerical solution (blue curve) is compared with the analytical solution (black curve). Both display a similar shape, showing a gradual decline in saturation from S* to Sinit. Minor differences arise from numerical diffusion and grid resolution, but the good overall agreement confirms that the numerical scheme effectively reproduces the key diffusive behavior of spontaneous imbibition.

CASE 3: MIXED WET- WETTING PHASE →NON-WETTING

Parameter	Value
Domain Length, L	1.0 m
Spatial Step, Δx	0.001
Number of Grid Points, N	1000
Time Step, Δt	1.0 s
Number of Time Steps, nmax	4000
Final Time, T	3000s

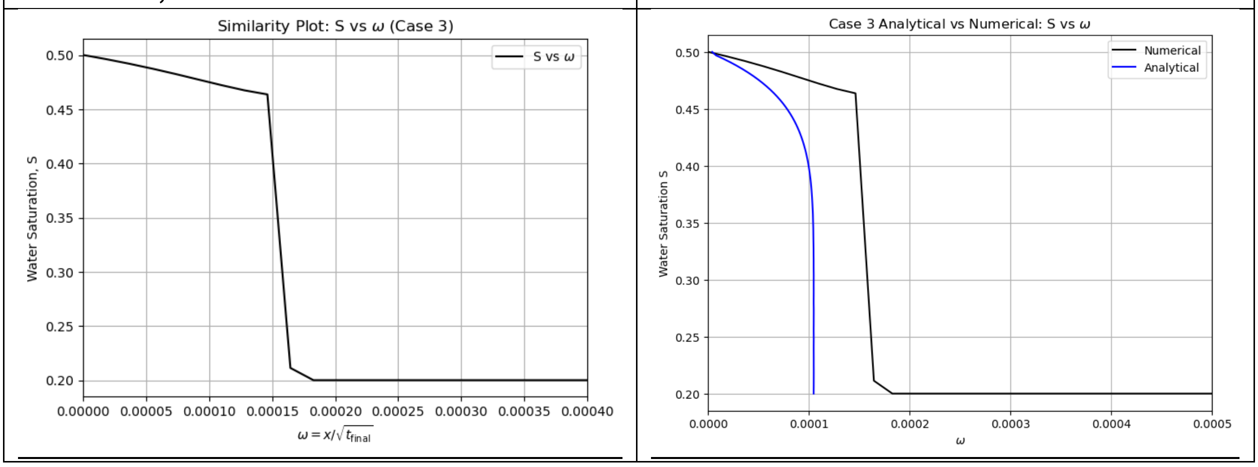


Figure 10 Case 3 Spontaneous Imbibition Numerical & Comparison with Analytical

In a mixed-wet system, water's mobility is partially restricted by less favorable wetting conditions. Consequently, the imbibition front exhibits a more abrupt transition from the boundary saturation (where capillary pressure is zero) toward the initial saturation, compared to a strongly water-wet medium. This leads to a steep drop in saturation versus the similarity variable, and while the numerical method slightly smooths the transition, the overall shape confirms that mixed-wet conditions cause a rapid but still partially hindered capillary-driven invasion of water.

Unlike Buckley Leverett, there is no flux boundary condition. So there is no priori how much is being imbibed. So the constant C imposed in analytical is the solution to the problem. This is why in numerical, the front is either moving too fast in my cases.

References

1. Blunt, M. J. (2017). *Multiphase flow in permeable media: A pore-scale perspective.* Cambridge University Press.
2. Blunt, M. J., & Krevor, S. (2012). *Prediction of reservoir performance & EOR.* Imperial College London, Centre for Petroleum Studies.
3. Sonny. (2017, March 30). *Buckley–Leverett in Python.* [Blog post]. https://sonny-qa.github.io/2017/03/30/Bucky-Leverett-Python/