Atmospheric Boundary Layer turbulence theory

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Overview

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- Regions inside the boundary layer
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Monin-Obukhov similarity theory

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- "Buoyancy term"
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Possible compressible formulation

Appendix: project update

- Filtering
- Next steps?

References

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Turbulence fundamentals

Essential quantities

Wall shear stress τ and friction velocity u_*

Shear stress can be driven by:

- molecular diffusion

  $$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j}$$

  … Newton's law of viscosity

- turbulent diffusion

  $$\tau_{ij} = -\rho \overline{u_i u_j}$$

  … Reynolds stress tensor (statistical quantity)

Friction velocity

$$u_* = \sqrt{\frac{\tau}{\rho}}$$

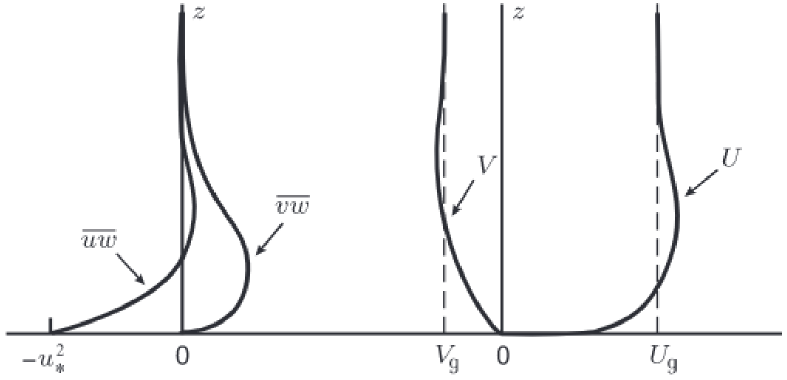

Figure 10.1 A sketch of profiles of kinematic shear stress (left) and mean wind (right) in the near-neutral ABL for z-independent U_g and V_g.

Observations from DNS of channel flows

N.B: y is the wall normal direction in engineering

$\delta_v =$ viscous length scale, $\quad \delta =$ displacement thickness

$$u^+ = \bar{U}/u_*, \quad y^+ = y/\delta_v$$

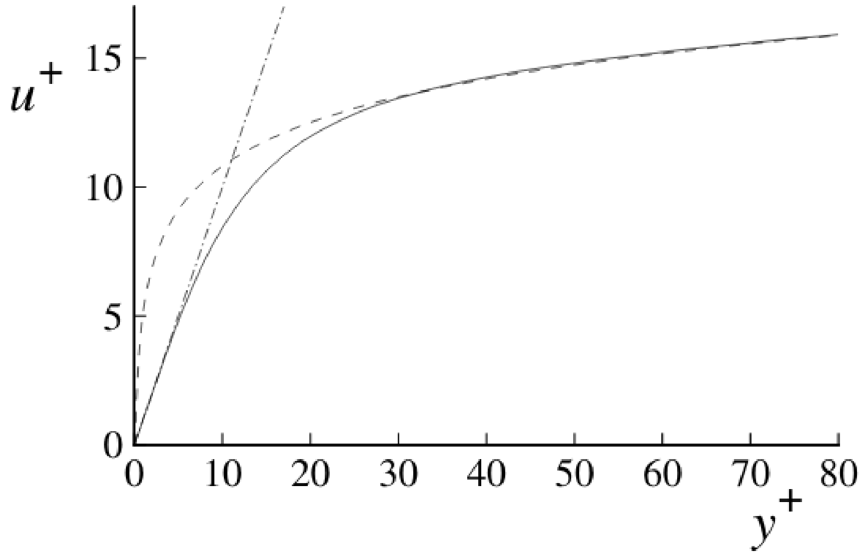

Figure 7.6: Near-wall profiles of mean velocity: solid line, DNS data of Kim *et al.*: Re = 13,750; dot-dashed line, $u^+ = y^+$; dashed line, the log law, Eqs. (7.43)–(7.44).

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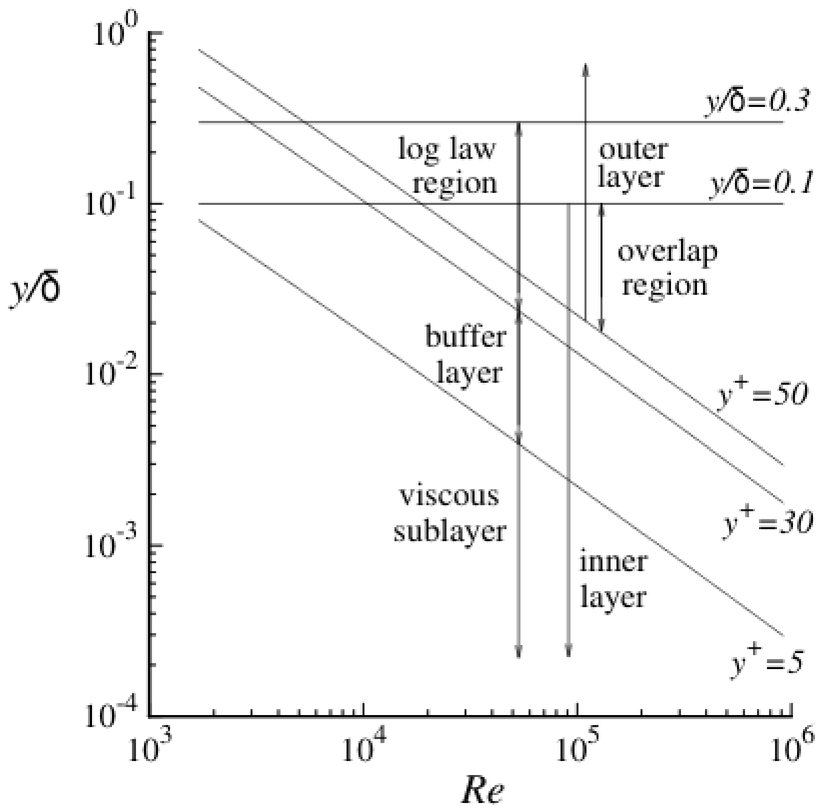


Figure 7.13: Regions and layers in turbulent channel flow as functions of Reynolds number.

Mean velocity profiles

In general (without any assumptions), on dimensional grounds,

$$\frac{\partial \bar{U}}{\partial y} = \frac{u_*}{y}\Phi(y/\delta_v, y/\delta)$$

For $y/\delta \ll 1$, *tends asymptotically to*

$$\frac{\partial \bar{U}}{\partial y} = \frac{u_*}{y}\Phi_1(y/\delta_v) \implies \frac{\partial u^+}{\partial y^+} = \frac{1}{y^+}\Phi_1(y^+)$$

Log-law of the wall

von Karman (1930) postulated that for high Re, $y/\delta \ll 1$, $y^+ \gg 1$,

negligible viscous effects, which implies velocity profile is free from dependence of ν or y/δ_v

$$\Phi_1 = \frac{1}{\kappa} \implies \frac{\partial u^+}{\partial y^+} = \frac{1}{\kappa y^+} \implies u^+ = \frac{1}{\kappa}\ln y^+ + B$$

Effect of roughness

In general, the velocity gradient depends on 3 parameters. Including s the roughness scale:

$$\frac{\partial \bar{U}}{\partial y} = \frac{u_*}{y}\Phi(y/\delta_v, y/\delta, s/\delta_v)$$

For the general case of roughness size $s \sim \delta_v$ we get,

$$u^+ = \frac{1}{\kappa}\ln(y/s) + B(s/\delta_v)$$

other relations exist for extremes cases of small and large roughness scale s.

Note

- The log-law is one among many established results in turbulence.
- A similar approach is used in developing Monin-Obukhov similarity theory.

Monin-Obukhov similarity theory

Governing parameters and assumptions

Turbulence in the surface layer is determined by:

1. length scale $l \sim z$
2. velocity scale $u \sim u_*$
3. surface stress $\tau = \rho u_*^2$
4. buoyancy parameter $\sim g/\theta_0$
5. surface temperature flux Q_0
6. ~~surface flux of a conserved scalar~~ C_0

Buckingham Pi theorem

- $m = 5$ parameters
- $n = 3$ dimensions: length, time, temperature, ~~scalar~~

implies the model can be rewritten using $m - n = 2$ independent dimensionless quantities.

M-O functions

Similarity variable taken as z/L, where *Monin-Obukhov length*:

$$L = -u_*^3\theta_0/\kappa g Q_0$$

L is negative in unstable conditions ($Q_0 > 0$), positive in stable conditions ($Q_0 < 0$), and infinite at neutral ($Q_0 = 0$); thus, the range of the M-O independent variable is $-\infty < z/L < \infty$. Conditions are *near-neutral* when $|z/L|$ is sufficiently small, which occurs near the surface when $z \ll |L|$.

$$\frac{kz}{u_*}\frac{\partial U}{\partial z} = \phi_m\left(\frac{z}{L}\right), \quad -\frac{kzu_*}{Q_0}\frac{\partial \Theta}{\partial z} = \frac{kz}{T_*}\frac{\partial \Theta}{\partial z} = \phi_h\left(\frac{z}{L}\right)$$

L being a function of the turbulence statistic u_* restricts a closed form solution of the M-O functions.

These remained merely a theory until…

The 1968 Kansas experiment measured the LHS for different z and L and confirmed these functions are truly similar:

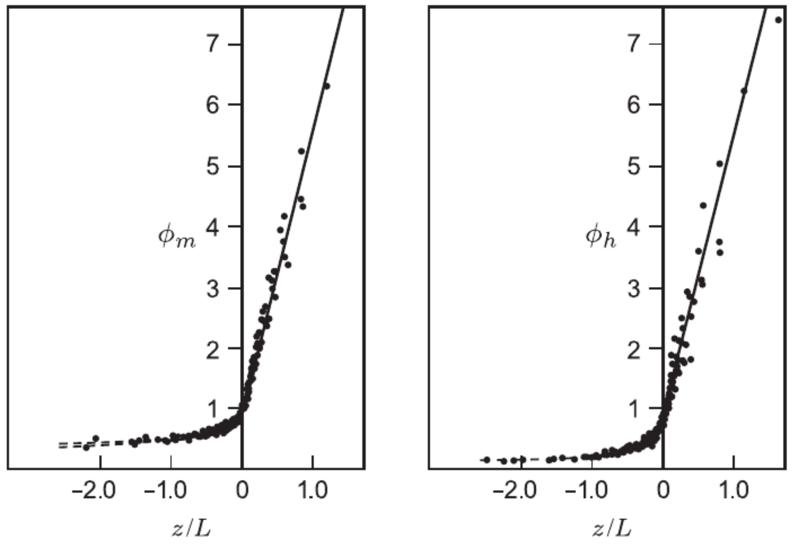


Figure 10.3 The M-O functions for mean wind shear (left) and mean potential temperature gradient (right), Eq. (10.12), from the 1968 Kansas experiment. From Businger *et al.* (1971).

M-O functions and log-law

In **neutral** conditions, $z/L \to 0 \implies \phi_m \approx 1$ and we recover the classical log-law.

$$\frac{\partial U}{\partial z} = \frac{u_*}{\kappa z}$$

$$U(z) = \frac{u_*}{\kappa}[\ln z - \ln z_0]$$

Compare the term $\ln z_0$ to roughness parameter B. For **stable** and **unstable** regimes, curve fitting gives,

$$\begin{aligned} &\text{stable:} \quad \phi_m = 1.0 + 4.8\tfrac{z}{L}, \qquad \phi_h = 1.0 + 7.8\tfrac{z}{L}; \\ &\text{unstable:} \quad \phi_m = \left(1 - 19.3\frac{z}{L}\right)^{-1/4}, \quad \phi_h = \left(1 - 12\frac{z}{L}\right)^{-1/2}. \end{aligned} \qquad (10.14)$$

In stable conditions the functions proposed for ϕ_m and ϕ_h tend to be linear and thus integrate easily. For example, Hogstrom's ϕ_m and ϕ_h forms, Eqs. (10.14), give

$$U(z) = \frac{u_*}{k}\left[\ln\frac{z}{z_0} + 4.8\frac{z}{L}\right], \qquad \Theta(z) = \Theta(z_r) + \frac{T_*}{k}\left[\ln\frac{z}{z_r} + 7.8\frac{z}{L}\right], \quad (10.16)$$

Moeng's variant

for the wall boundary. If SGS effects are included, the vertical gradient of the velocity at $z_1 = 25$ m is required to compute the shear production term in (12). From the similarity formula, e.g., for the u-component,

$$\left(\frac{\partial \bar{u}}{\partial z}\right) = \frac{u^* \cos\phi}{\kappa(\frac{1}{2}z_1)} \Phi\left(\frac{\frac{1}{2}z_1}{L}\right) \quad \text{at} \quad z = \tfrac{1}{2}z_1, \tag{27}$$

the vertical gradient of $\bar{u}$ at $z = 12.5$ m is computed. Here $\Phi(z_1/L) = [1 - (15z_1/L)]^{-1/4}$ if the horizontal-mean surface heat flux is positive, and $\Phi(z_1/L) = 1 + (4.7z_1/L)$ if it is negative; u^* is the local friction velocity, defined as $(\tau_{xz}^2 + \tau_{yz}^2)^{1/4}$, and ϕ is the angle of surface stress away from the x-direction. The gradient at $z_1 = 25$ m is then obtained by interpolating those at 50 and 12.5 m.

The SGS vertical fluxes at the surface, derived from $\tau = C_D S_1 \bar{\mathbf{V}}_1$ and using $S_1 = \langle S_1 \rangle + S_1''$ (see Appendix; J. C. Wyngaard, personal communication, 1983), etc., are

$$(\tau_{xz})_0 = \langle \tau_{xz} \rangle_0 \frac{S_1\langle \bar{u}_1 \rangle + \langle S_1 \rangle(\bar{u}_1 - \langle \bar{u}_1 \rangle)}{\langle S_1 \rangle \langle \bar{u}_1 \rangle}, \tag{28}$$

$$S_1\langle \bar{v}_1 \rangle + \langle S_1 \rangle(\bar{v}_1 - \langle \bar{v}_1 \rangle)$$

Implementation in Nek5000 for neutral conditions

$$\tau = \rho u_*^2 = \rho \left[\frac{\kappa U}{\ln(z/z_0)}\right]^2$$

which evaluated at $z = \frac{1}{2}z_1$

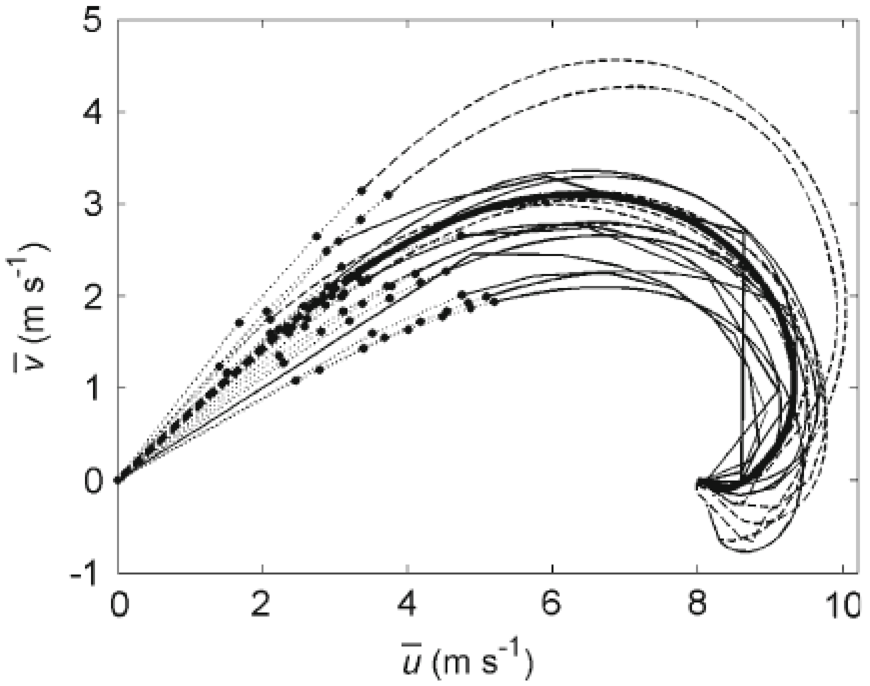
```
! --------Wall normal cordinate: `y`
      KAPPA = 0.41
      y0 = 0.1   ! << y_max
! --------Calculate Moeng's model parameters
      ie = gllel(eg)
      u1_2 = (vx(ix,2,iz,ie) + vx(ix,1,iz,ie))/2
      w1_2 = (vz(ix,2,iz,ie) + vz(ix,1,iz,ie))/2
      absu = sqrt(u1_2**2 + w1_2**2)
      y1_2 = (ym1(ix,2,iz,ie) + ym1(ix,1,iz,ie))/2

! --------Calculate Stresses
      trx = -KAPPA**2*(u1_2*absu)/(log(y1_2/y0)**2)
      try = 0.0
      trz = -KAPPA**2*(w1_2*absu)/(log(y1_2/y0)**2)
```

Physical aspects

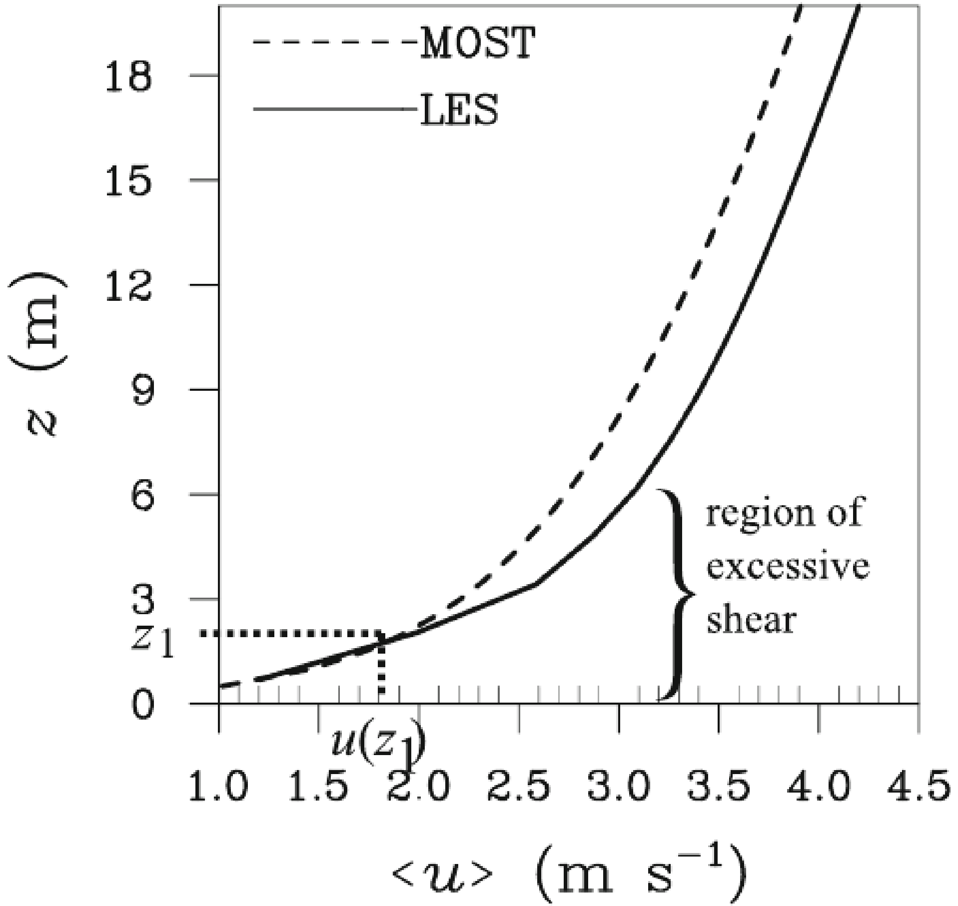
Meaning of z/L

$$z/L = \frac{\text{buoyant production rate of turb. kinetic energy}}{\text{shear production rate of turb. kinetic energy}}$$



Effect of boundary conditions

- Average turning angle of the Ekmann spiral (bottom left)
- Log-layer mismatch (bottom right)



Possible compressible formulation

Buoyancy frequency / stability

$$\frac{dp}{dz} = RT\frac{d\rho}{dz} + R\rho\frac{dT}{dz}.$$

With the pressure gradient given by the hydrostatic balance (11.7), it follows that the density and temperature gradients are related by

$$\frac{1}{\rho}\frac{d\rho}{dz} + \frac{1}{T}\frac{dT}{dz} + \frac{g}{RT} = 0,$$

and the force on the fluid parcel can be expressed in terms of the temperature gradient

$$F \simeq -\frac{\rho g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right)h.$$

$$N^2 = -\frac{g}{\rho}\left(\frac{d\rho}{dz} + \frac{\rho g}{\gamma RT}\right) \tag{11.9a}$$

$$= +\frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right) \tag{11.9b}$$

is a positive quantity, the force recalls the particle toward its initial level, and the stratification is stable. As we can clearly see, the relevant quantity is not the actual temperature gradient but its departure from the adiabatic gradient $-g/C_p$. As in the previous case of a stably stratified incompressible fluid, the

The *potential temperature*, denoted by θ, is defined as the temperature that the parcel would have if it were brought adiabatically to a given reference pressure.[3] From Eq. (11.6a), we have

$$\frac{p}{p_0} = \left(\frac{T}{\theta}\right)^{\gamma/(\gamma-1)}$$

and hence

$$\theta = T\left(\frac{p}{p_0}\right)^{-(\gamma-1)/\gamma}. \tag{11.10}$$

The corresponding density is called the *potential density*, denoted by σ:

$$\sigma = \rho\left(\frac{p}{p_0}\right)^{-1/\gamma} = p_0/\mathrm{R}\,\theta. \tag{11.11}$$

The definition of the stratification frequency (11.9b) takes the more compact form:

$$N^2 = -\frac{g}{\sigma}\frac{d\sigma}{dz} = +\frac{g}{\theta}\frac{d\theta}{dz}. \tag{11.12}$$

Appendix: project update

Filtering

- Excessive filtering in last meeting
- Filtering parameters were reduced
- If time permits … some visuals

Next steps?

Stay with neutral stratification

- Oscillations
  - Sponge layer
  - Rayleigh radiative BC
- Improved boundary condition: Robin conditions in literature
- Comparison of statistics
  - Generated but unsure of what is plotted.
  - Need contact with a Ph.D. student using Nek5000.

Thank you for your attention!

Any questions?

slides will be uploaded: ashwin.info.tm/talks