Monin-Obukhov Similarity Theory as a Boundary Condition

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Overview

Recap

- Essential quantities: $\tau,\ u_*$
- Log-law of the wall
- Effect of roughness

Overview

Recap

- Roughness parameter z_0
- Wall model

As a boundary condition

Recap

Wall shear stress τ and friction velocity u_*

- Turbulent shear stress can be expressed as:

$$\tau_{ij} = -\rho\overline{u_i u_j}$$

  Reynolds stress tensor

- Friction velocity

$$u_* = \sqrt{\frac{|\tau|}{\rho}}$$

- Viscous length scale

$$\delta_\nu = \frac{\nu}{\text{friction velocity}}$$

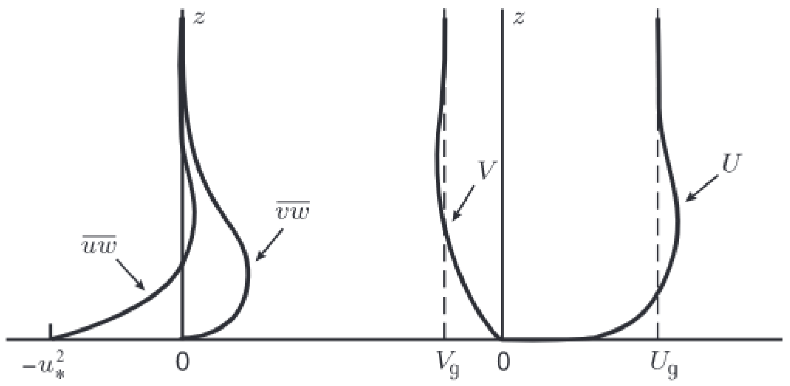


Figure 10.1 A sketch of profiles of kinematic shear stress (left) and mean wind (right) in the near-neutral ABL for z-independent U_g and V_g.

Observations from DNS of channel flows

$\delta_v =$ viscous length scale, $\quad \delta =$ displacement thickness

$$u^+ = \bar{U}/u_*, \quad y^+ = y/\delta_v$$

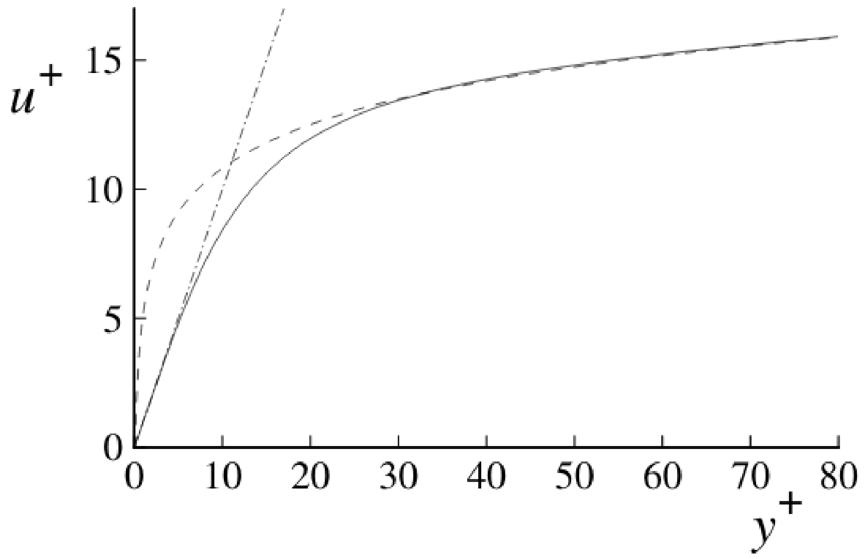


Figure 7.6: Near-wall profiles of mean velocity: solid line, DNS data of Kim *et al.*: Re = 13,750; dot-dashed line, $u^+ = y^+$; dashed line, the log law, Eqs. (7.43)–(7.44).

In general (without any assumptions), on dimensional grounds,

$$\frac{\partial \bar{U}}{\partial y} = \frac{u_*}{y}\Phi(y^+, y/\delta)$$

Log-law of the wall

von Karman (1930) postulated that for

- high Reynolds number
- $y/\delta \ll 1$
- $y^+ \gg 1$

negligible viscous effects, which implies velocity profile is free from dependence of ν or y/δ_v

$$\Phi_1 = \frac{1}{\kappa} \implies \frac{\partial u^+}{\partial y^+} = \frac{1}{\kappa y^+} \implies u^+ = \frac{1}{\kappa}\ln y^+ + B$$

Effect of roughness

Roughness influences the "intercept" B

$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

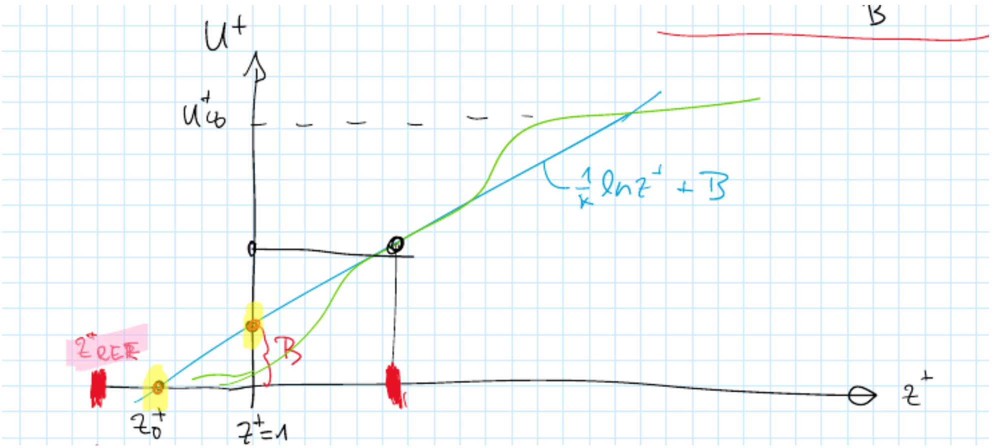
Monin-Obukhov similarity theory

- Monin-Obukhov similarity theory incorporates stratification effects with a correction ϕ_m

In **neutral** conditions, $\phi_m \approx 1$ and we recover the classical log-law.

$$\frac{\partial U}{\partial z} = \frac{u_*}{\kappa z}$$

$$U(z) = \frac{u_*}{\kappa} [\ln z - \ln z_0]$$



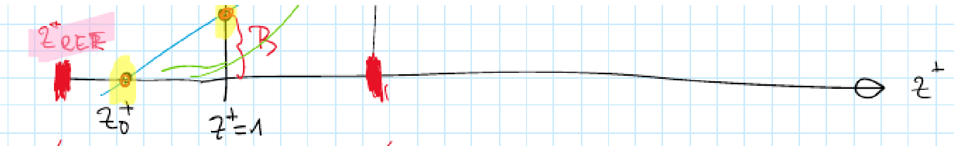
As a stress boundary condition (Moeng 1984)

$$\tau = \rho u_*^2 = \rho \left[\frac{\kappa U}{\ln(z/z_0)} \right]^2$$

evaluated at $z = \frac{\Delta_z}{2}$ half grid height

What if...

- Mesh starts at z_0 where on average $\bar{U} \to 0$
- Mesh coordinates are expressed with z_{ref}



$\bar{z}_0 = z_0 - z_{REF}$

$\bar{z} = z - z_{REF}$

$$\tau = \rho u_*^2 = \rho\left[\kappa U(z)\,\frac{1}{\ln\left(\frac{z}{z_0}\right)}\right]^2$$

$$= \rho\left[\kappa U(z)\,\frac{1}{\ln(z) - \ln(z_0)}\right]^2$$

$\bar{z}_0 = z_0 - z_{REF}$

$\bar{z} = z - z_{REF}$

$z_0 = \bar{z}_0 + z_{REF}$

$z = \bar{z} + z_{REF}$

$$\frac{1}{}$$

$$\ln(\bar{z}_0 + z_{REF}) - \ln(\bar{z} + z_{REF})$$

$$\frac{\kappa(}{\ln\left(\frac{\bar{z}_0 + z_{REF}}{\bar{z} + z_{REF}}\right)}$$

Moeng's variant

for the wall boundary. If SGS effects are included, the vertical gradient of the velocity at $z_1 = 25$ m is required to compute the shear production term in (12). From the similarity formula, e.g., for the u-component,

$$\left(\frac{\partial \bar{u}}{\partial z}\right) = \frac{u^* \cos\phi}{\kappa(\frac{1}{2}z_1)} \Phi\left(\frac{\frac{1}{2}z_1}{L}\right) \quad \text{at} \quad z = \tfrac{1}{2}z_1, \qquad (27)$$

the vertical gradient of $\bar{u}$ at $z = 12.5$ m is computed. Here $\Phi(z_1/L) = [1 - (15z_1/L)]^{-1/4}$ if the horizontal-mean surface heat flux is positive, and $\Phi(z_1/L) = 1 + (4.7z_1/L)$ if it is negative; u^* is the local friction velocity, defined as $(\tau_{xz}^2 + \tau_{yz}^2)^{1/4}$, and ϕ is the angle of surface stress away from the x-direction. The gradient at $z_1 = 25$ m is then obtained by interpolating those at 50 and 12.5 m.

The SGS vertical fluxes at the surface, derived from $\tau = C_D S_1 \bar{\mathbf{V}}_1$ and using $S_1 = \langle S_1 \rangle + S_1''$ (see Appendix; J. C. Wyngaard, personal communication, 1983), etc., are

$$(\tau_{xz})_0 = \langle \tau_{xz} \rangle_0 \frac{S_1\langle \bar{u}_1 \rangle + \langle S_1 \rangle(\bar{u}_1 - \langle \bar{u}_1 \rangle)}{\langle S_1 \rangle \langle \bar{u}_1 \rangle}, \qquad (28)$$

$$S_1\langle \bar{v}_1 \rangle + \langle S_1 \rangle(\bar{v}_1 - \langle \bar{v}_1 \rangle)$$

Implementation in Nek5000 for neutral conditions

$$\tau = \rho u_*^2 = \rho \left[\frac{\kappa U}{\ln(z/z_0)}\right]^2$$

which evaluated at $z = \frac{1}{2} z_1$

```
! --------Wall normal cordinate: `y`
      KAPPA = 0.41
      y0 = 0.1   ! << y_max
! --------Calculate Moeng's model parameters
      ie = gllel(eg)
      u1_2 = (vx(ix,2,iz,ie) + vx(ix,1,iz,ie))/2
      w1_2 = (vz(ix,2,iz,ie) + vz(ix,1,iz,ie))/2
      absu = sqrt(u1_2**2 + w1_2**2)
      y1_2 = (ym1(ix,2,iz,ie) + ym1(ix,1,iz,ie))/2

! --------Calculate Stresses
      trx = -KAPPA**2*(u1_2*absu)/(log(y1_2/y0)**2)
      try = 0.0
      trz = -KAPPA**2*(w1_2*absu)/(log(y1_2/y0)**2)
```

Thank you for your attention!

Any questions?

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References

1. Pope, S.B. Turbulent Flows. Cambridge University Press, 2000.
2. Wyngaard, John C. Turbulence in the Atmosphere. Cambridge: Cambridge University Press, 2010. https://doi.org/10.1017/CBO9780511840524.
3. Monin, A S, and A M Obukhov. “Basic Laws of Turbulent Mixing in the Surface Layer of the Atmosphere,” 1954, 30.
4. Moeng, Chin-Hoh. “A Large-Eddy-Simulation Model for the Study of Planetary Boundary-Layer Turbulence.” Journal of the Atmospheric Sciences 41, no. 13 (July 1, 1984): 2052–62. https://doi.org/10.1175/1520-0469(1984)041<2052:ALESMF>2.0.CO;2.