IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

χ^2 Test of Independence - II

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DEPARTMENT OF MANAGEMENT STUDIES

Agenda

- Using python to test the independence of variables
- Understanding goodness of fit test for Poisson

Example

- Record of 50 students studying in ABN School is taken at random, the first 10 entries are like this:

res_num	aa	pe	sm	ae	r	g	c
1	99	19	1	2	0	0	1
2	46	12	0	0	0	0	0
3	57	15	1	1	0	0	0
4	94	18	2	2	1	1	1
5	82	13	2	1	1	1	1
6	59	12	0	0	2	0	0
7	61	12	1	2	0	0	0
8	29	9	0	0	1	1	0
9	36	13	1	1	0	0	0
10	91	16	2	2	1	1	0

Example

Here :

- res_num = registration no.
- aa= academic ability
- pe = parent education
- sm = student motivation
- r = religion
- g = gender

Python code

In [1]:
```
import pandas as pd
import numpy as np
```

In [2]:
```
acad = pd.read_csv('AcademicAbilityData.csv')
```

In [3]:
```
acad
```

Out[3]:

	res_num	aa	pe	sm	ae	r	g	c
0	1	99	19	1	2	0	0	1
1	2	46	12	0	0	0	0	0
2	3	57	15	1	1	0	0	0
3	4	94	18	2	2	1	1	1
4	5	82	13	2	1	1	1	1
5	6	59	12	0	0	2	0	0
6	7	61	12	1	2	0	0	0
7	8	29	9	0	0	1	1	0
8	9	36	13	1	1	0	0	0
9	10	91	16	2	2	1	1	0
10	11	55	10	0	0	1	0	0
11	12	58	11	0	1	0	0	0

Hypothesis

- Test the hypothesis that “gender and student motivation” are independent

Python code

```
In [19]: #Cross table between gender and student's motivation
         obs =pd.pivot_table(acad[['g','sm']],index = 'g',columns='sm',aggfunc=len)
         obs
```

Out[19]:

sm	0	1	2
g			
0	10	13	6
1	4	9	8

Observed values

Gender	Student motivation			
	0 (Disagree)	1 (Not decided)	2 (Agree)	Row Sum
0 (Male)	10	13	6	29
1(Female)	4	9	8	21
Column Sum	14	22	14	50

Expected frequency (contingency table)

Gender	Student motivation		
	0	1	2
0	29*14/50= 8.12	12.76	8.12
1	5.88	9.24	5.88

Frequency Table

Gender	Student motivation		
	0	1	2
0	$f_o = 10$ $f_e = 8.12$	$f_o = 13$ $f_e = 12.76$	$f_o = 6$ $f_e = 8.12$
1	$f_o = 4$ $f_e = 5.88$	$f_o = 9$ $f_e = 9.24$	$f_o = 8$ $f_e = 5.88$

Chi sq. calculation

$$\chi^2 = \sum\sum \frac{(f_o - f_e)^2}{f_e}$$

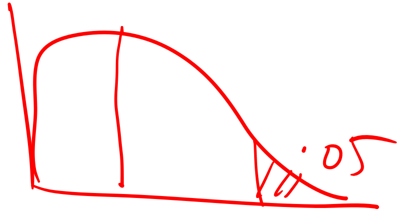
$= 0.435 + 0.005 + 0.554 + 0.601 + 0.006 + 0.764$

$= 2.365$

Python code

```
In [11]: ## Perform chi2 test to check independence
         from scipy.stats import chi2_contingency

In [14]: chi2, p,dof,tbl= chi2_contingency(obs)

In [15]: chi2
Out[15]: 2.3649585225939904

In [16]: p
Out[16]: 0.3065178579178871

In [17]: dof
Out[17]: 2
```

$\alpha = 5\% = 0.05$



$(2-1)(3-1) = 2$

Python code

```
In [11]: ## Perform chi2 test to check independence
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In [14]: chi2, p,dof,tbl= chi2_contingency(obs)

In [15]: chi2

Out[15]: 2.3649585225939904

In [16]: p

Out[16]: 0.3065178579178871

In [17]: dof

Out[17]: 2
```

Degrees of freedom = (2-1)*(3-1)

Python code

```
In [12]:  tbl

Out[12]: array([[ 8.12, 12.76,  8.12],
                [ 5.88,  9.24,  5.88]])
```

Contingency table

χ^2 Goodness of Fit Test

χ^2 Goodness-of-Fit Test

- The χ^2 goodness-of-fit test compares *expected* (theoretical) *frequencies* of categories from a population distribution to the *observed* (actual) *frequencies* from the distribution to determine whether there is a difference between what was expected and what was observed

χ^2 Goodness-of-Fit Test

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\text{df} = k - 1 - \text{p}$$

where: f_o = frequency of observed values

f_e = frequency of expected values

k = number of categories

p = number of parameters estimated from the sample data

Goodness of Fit Test: Poisson Distribution

1. Set up the null and alternative hypotheses.

   H_0: Population has a Poisson probability distribution

   H_a: Population does not have a Poisson distribution

2. Select a random sample and
   - Record the observed frequency f_i for each value of the Poisson random variable.
   - Compute the mean number of occurrences μ.

3. Compute the expected frequency of occurrences e_i for each value of the Poisson random variable.

Goodness of Fit Test: Poisson Distribution

4. Compute the value of the test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i}$$

where:

f_i = observed frequency for category i

e_i = expected frequency for category i

k = number of categories

Goodness of Fit Test: Poisson Distribution

5. Rejection rule:

p-value approach: Reject H_0 if p-value $\leq \alpha$

Critical value approach: Reject H_0 if $\boxed{\chi^2 \geq \chi^2_\alpha}$

where α is the significance level and there are $\underline{k - 2}$ degrees of freedom

$k - 1 - p$

Goodness of Fit Test: Poisson Distribution

- Example: Parking Garage

In studying the need for an additional entrance to a city parking garage, a consultant has recommended an analysis, that approach is applicable only in situations where the number of cars entering during a specified time period follows a Poisson distribution.

Goodness of Fit Test: Poisson Distribution

A random sample of 100 one- minute time intervals resulted in the customer arrivals listed below. A statistical test must be conducted to see if the assumption of a Poisson distribution is reasonable.

# Arrivals	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	1	4	10	14	20	12	12	9	8	6	3	1

Goodness of Fit Test: Poisson Distribution

- Hypotheses

H_0: Number of cars entering the garage during a one-minute interval is Poisson distributed

H_a: Number of cars entering the garage during a one-minute interval is <u>not</u> Poisson distributed

Python Code

In [1]:
```
import scipy
from scipy.stats import chi2
from scipy.stats import poisson
```

In [2]:
```
import pandas as pd
import numpy as np
```

In [3]:
```
data = pd.read_excel('P_distribution.xlsx')
data
```

Out[3]:

	Arrivals	Frequency
0	0	0
1	1	1
2	2	4
3	3	10
4	4	14
5	5	20
6	6	12
7	7	12
8	8	9
9	9	8
10	10	6
11	11	3
12	12	1

Goodness of Fit Test: Poisson Distribution

- Estimate of Poisson Probability Function

Total Arrivals = 0(0) + 1(1) + 2(4) + . . . + 12(1) = 600

Estimate of $\mu = 600/100 = 6$

Total Time Periods = 100

Hence,

$$= \frac{e^{-\mu} \mu^x}{x!}$$

$$f(x) = \frac{6^x e^{-6}}{x!}$$

$$\mu = \frac{\Sigma f n}{\Sigma f} = \frac{600}{100} = 6$$

Goodness of Fit Test: Poisson Distribution

- Expected Frequencies

x	$f(x)$	$nf(x)$	x	$f(x)$	$nf(x)$
0	.0025	.25	7	.1377	13.77
1	.0149	1.49	8	.1033	10.33
2	.0446	4.46	9	.0688	6.88
3	.0892	8.92	10	.0413	4.13
4	.1339	13.39	11	.0225	2.25
5	.1606	16.06	12+	.0201	2.01
6	.1606	16.06	Total	1.0000	100.00

Python code

```
In [4]: Observed_Freq = data['Frequency']

In [5]: total_arrival = 600
        total_time_period = 100
        mu = total_arrival/total_time_period

In [6]: Expected_Freq = []
        for i in range(len(Observed_Freq)):
            E_Freq = 100*poisson.pmf(i, mu)
            Expected_Freq.append(E_Freq)

In [7]: Expected_Freq

Out[7]: [0.24787521766663584,
         1.4872513059998145,
         4.461753917999444,
         8.923507835998894,
         13.385261753998332,
         16.062314104797995,
         16.06231410479801,
         13.767697804112569,
         10.32577335308442,
         6.883848902056284,
         4.130309341233764,
         2.2528960043093247,
         1.1264480021546681]
```

Python code

In [8]:
```
Expected_Freq_round_off = [round(elem, 2) for elem in Expected_Freq]
Expected_Freq_round_off
```

Out[8]:
```
[0.25,
 1.49,
 4.46,
 8.92,
 13.39,
 16.06,
 16.06,
 13.77,
 10.33,
 6.88,
 4.13,
 2.25,
 1.13]
```

In [9]:
```
df = pd.DataFrame(list(zip(Observed_Freq, Expected_Freq_round_off )), columns = ['Observed Frequency', 'Expected Frequency'])
df
```

Out[9]:

	Observed Frequency	Expected Frequency
0	0	0.25
1	1	1.49
2	4	4.46
3	10	8.92
4	14	13.39
5	20	16.06
6	12	16.06
7	12	13.77
8	9	10.33
9	8	6.88
10	6	4.13
11	3	2.25
12	1	1.13

Goodness of Fit Test: Poisson Distribution

- Observed and Expected Frequencies

i	f_i	e_i	f_i - e_i
0 or 1 or 2	5	6.20	-1.20
3	10	8.92	1.08
4	14	13.39	0.61
5	20	16.06	3.94
6	12	16.06	-4.06
7	12	13.77	-1.77
8	9	10.33	-1.33
9	8	6.88	1.12
10 or more	10	8.39	1.61

Python code

```
In [10]: obs_freq = [5, 10, 14, 20, 12, 12, 9, 8, 10]
         expected_freq = [6.20, 8.92, 13.39, 16.06, 16.06, 13.77, 10.33, 6.88, 8.39]

In [11]: scipy.stats.chisquare(obs_freq, expected_freq)

Out[11]: Power_divergenceResult(statistic=3.2738182931105193, pvalue=0.916017731732134)
```

Goodness of Fit Test: Poisson Distribution

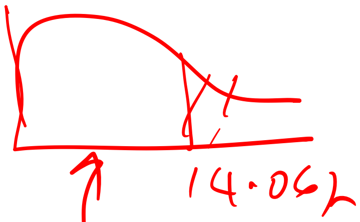
- Rejection Rule

With α = .05 and k - p - 1 = 9 - 1 - 1 = 7 d.f.

(where k = number of categories and p = number of population parameters estimated),

$$\chi^2_{.05} = 14.067$$

Reject H_0 if p-value $\leq$ .05 or $\chi^2 \geq 14.067$.



- Test Statistic

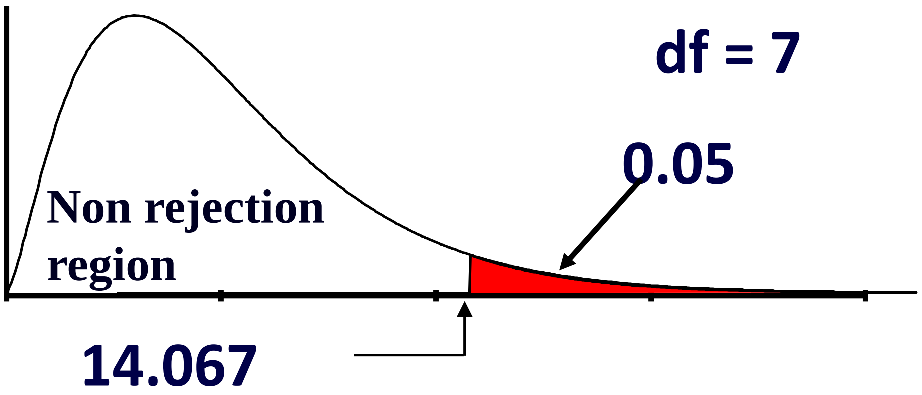
$$\chi^2 = \frac{(-1.20)^2}{6.20} + \frac{(1.08)^2}{8.92} + \ . \ . \ . \ + \frac{(1.61)^2}{8.39} = 3.268$$

Python code

In [4]:
```
from scipy.stats import chi2
chi2.ppf(0.95,7)
```

Out[4]: 14.067140449340167

Goodness of Fit Test: Poisson Distribution



$$\chi^2_{Cal} = \underline{3.268} < 14.067, \text{ do not reject } H_o.$$