Hypothesis Testing

Class Objectives

- Developing Null and Alternative Hypotheses
- Type I and Type II Errors- Explanation
- Population Mean: Sigma Known
- Population Mean: Sigma Unknown
- Population Proportion

Hypothesis Testing

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The null hypothesis, denoted by H_0 , is a tentative assumption about a population parameter
- The alternative hypothesis, denoted by Ha, is the opposite of what is stated in the null hypothesis
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H0 and Ha.

Developing Null and Alternative Hypotheses

- It is not always obvious how the null and alternative hypotheses should be formulated
- Care must be taken to structure the hypotheses appropriately so that the test conclusion provides the information the researcher wants
- The context of the situation is very important in determining how the hypotheses should be stated
- In some cases it is easier to identify the alternative hypothesis first. In other cases the null is easier
- Correct hypothesis formulation will take practice

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Example: A new manufacturing method is believed to be better than the current method.
- Alternative Hypothesis:
  - The new manufacturing method is better.
- Null Hypothesis:
  - The new method is no better than the old method.

Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
- Example: A new bonus plan, that is developed in an attempt to increase sales
- Alternative Hypothesis:
  - The new bonus plan increase sales
- Null Hypothesis:
  - The new bonus plan does not increase sales

Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
- Example:
  - A new drug is developed with the goal of lowering Cholesterol-level more than the existing drug
- Alternative Hypothesis:
  - The new drug lowers Cholesterol-level more than the existing drug
- Null Hypothesis:
  - The new drug does not lower Cholesterol-level more than the existing drug

Developing Null and Alternative Hypotheses

- Null Hypothesis as an assumption to be challenged
- We might begin with a belief or assumption that a statement about the value of a population parameter is true
- We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect
- In these situations, it is helpful to develop the null hypothesis first

Developing Null and Alternative Hypotheses

- Null Hypothesis as an Assumption to be Challenged
- Example:
  - The label on a milk bottle states that it contains 1000 ml
- Null Hypothesis:
  - The label is correct. $\mu \geq 1000$ ml
- Alternative Hypothesis:
  - The label is incorrect. $\mu < 1000$ ml

Null and Alternative Hypotheses about a Population Mean μ

- The equality part of the hypotheses always appears in the null hypothesis
- In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean)

One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed
$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu = \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$	$H_a: \mu \neq \mu_0$

Null and Alternative Hypotheses

- A major hospital in Chennai provides one of the most comprehensive emergency medical services in the world
- Operating in a multiple hospital system with approximately 10 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 8 minutes or less
- The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 8 minutes or less is being achieved.

Null and Alternative Hypotheses

H_0: $\mu \leq 8$

The emergency service is meeting the response goal; no follow-up action is necessary.

H_a: $\mu > 8$

The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

where: μ = mean response time for the population of medical emergency requests

Type I Error

- Because hypothesis tests are based on sample data, we must allow for the possibility of errors
- A Type I error is rejecting H_0 when it is true
- The probability of making a Type I error when the null hypothesis is called the level of significance
- Applications of hypothesis testing that only control the Type I error are often called significance tests

Type II Error

- A Type II error is accepting H_0 when it is false.
- It is difficult to control for the probability of making a Type II error.
- Statisticians avoid the risk of making a Type II error by using “do not reject H_0” and not “accept H_0”.

Type I and Type II Errors

	Population Condition	
Conclusion	**H0 True** $(\mu \leq 8)$	**H0 False** $(\mu > 8)$
Accept H0 (Conclude $\mu \leq 8$)	**Correct Decision**	**Type II Error**
Reject H0 (Conclude $\mu > 8$)	**Type I Error**	**Correct Decision**

Three Approaches for Hypothesis Testing

- P- Value

- Critical Value

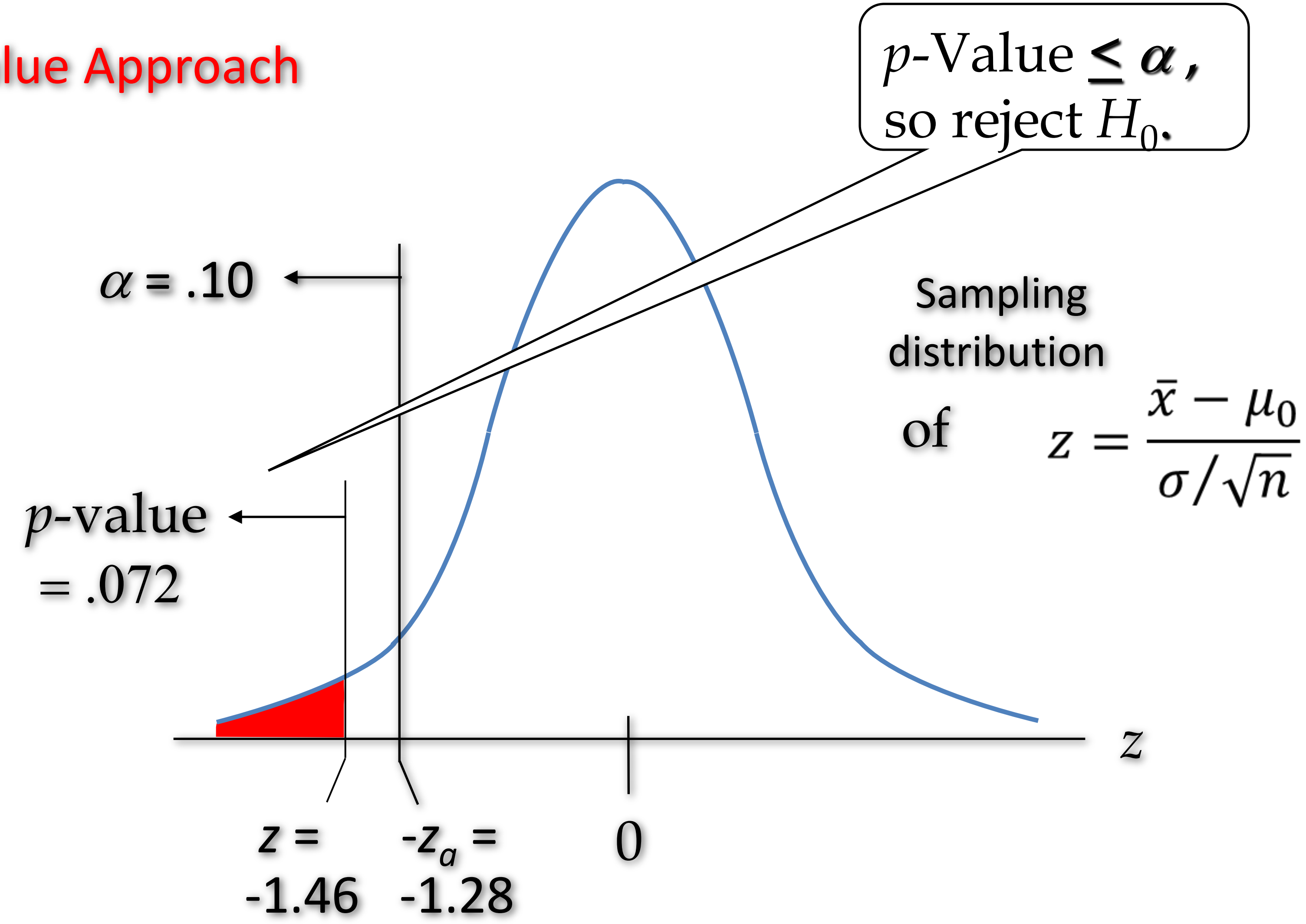
- Confidence Interval Value

p-Value Approach to One-Tailed Hypothesis Testing

- The p-value is the probability, computed using the test statistic, that measures the support (or lack of support) provided by the sample for the null hypothesis
- If the p-value is less than or equal to the level of significance α, the value of the test statistic is in the rejection region
- Reject H_0 if the p-value $\leq \alpha$

Lower-Tailed Test About a Population Mean: σ Known

p-Value Approach



p-Value Approach

Finding P Value

```
In [3]: stats.norm.cdf(-1.46)

Out[3]: 0.07214503696589378
```

Finding Z Value

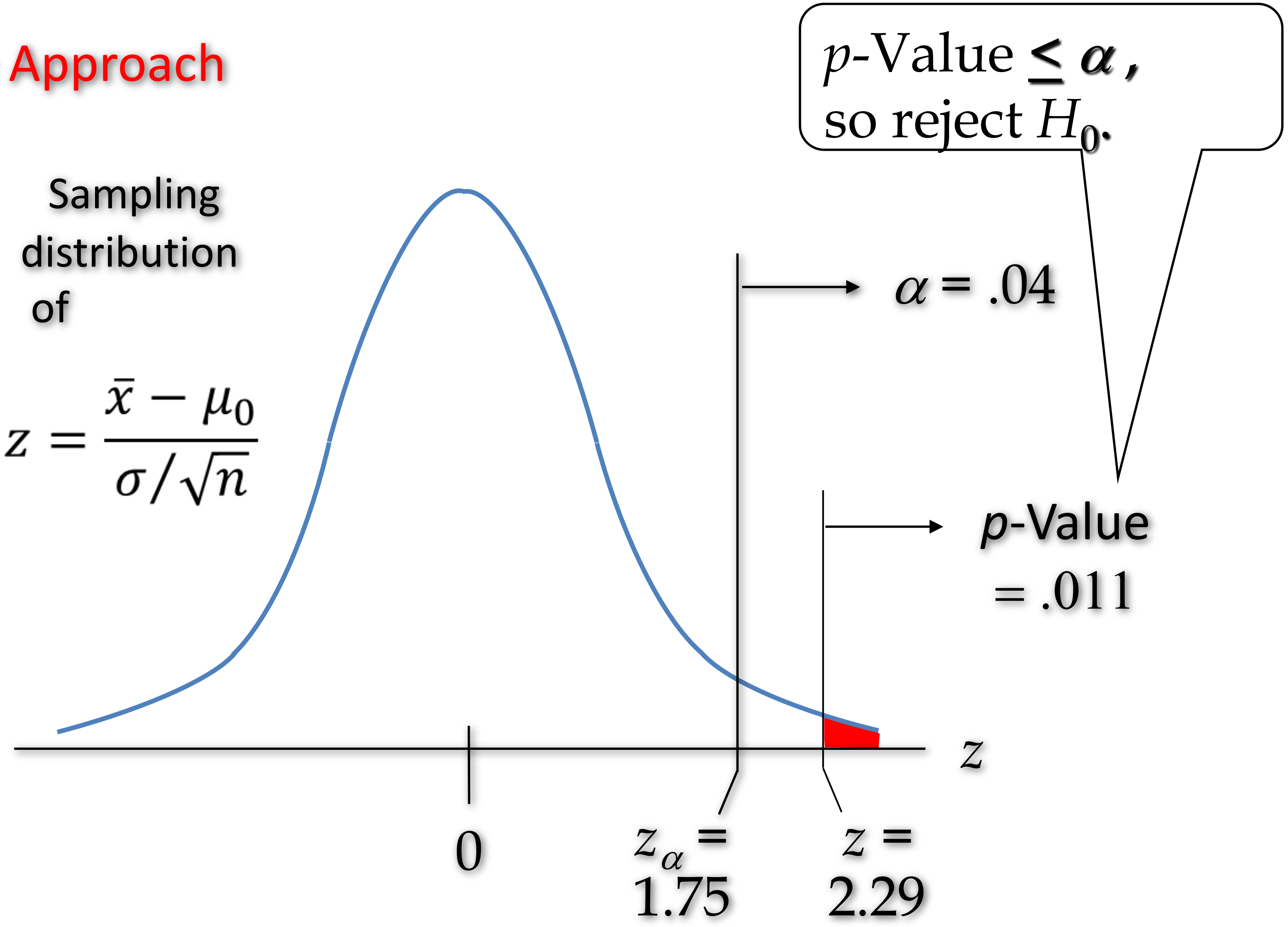
```
In [5]: stats.norm.ppf(0.1)

Out[5]: -1.2815515655446004
```

Upper-Tailed Test About a Population Mean : σ Known

p-Value Approach

Sampling distribution of

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$



p-Value Approach

```
In [4]: 1-stats.norm.cdf(1.75)

Out[4]: 0.040059156863817114
```

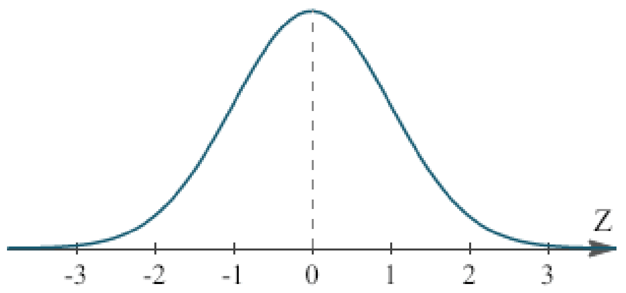
```
In [5]: 1-stats.norm.cdf(2.29)

Out[5]: 0.011010658324411393
```

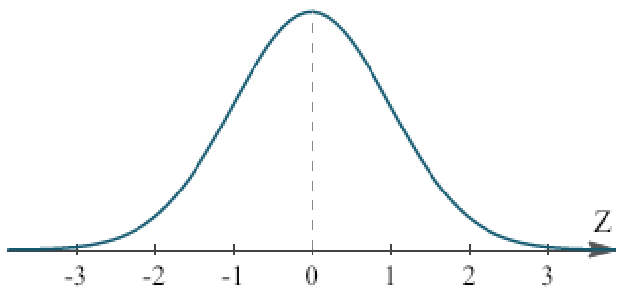
Critical Value Approach to One-Tailed Hypothesis Testing

- The test statistic z has a standard normal probability distribution.
- We can use the standard normal probability distribution table to find the z-value with an area of α in the lower (or upper) tail of the distribution.
- The value of the test statistic that established the boundary of the rejection region is called the critical value for the test.
- The rejection rule is:

  Lower tail: Reject H_0 if $z \leq -z_\alpha$

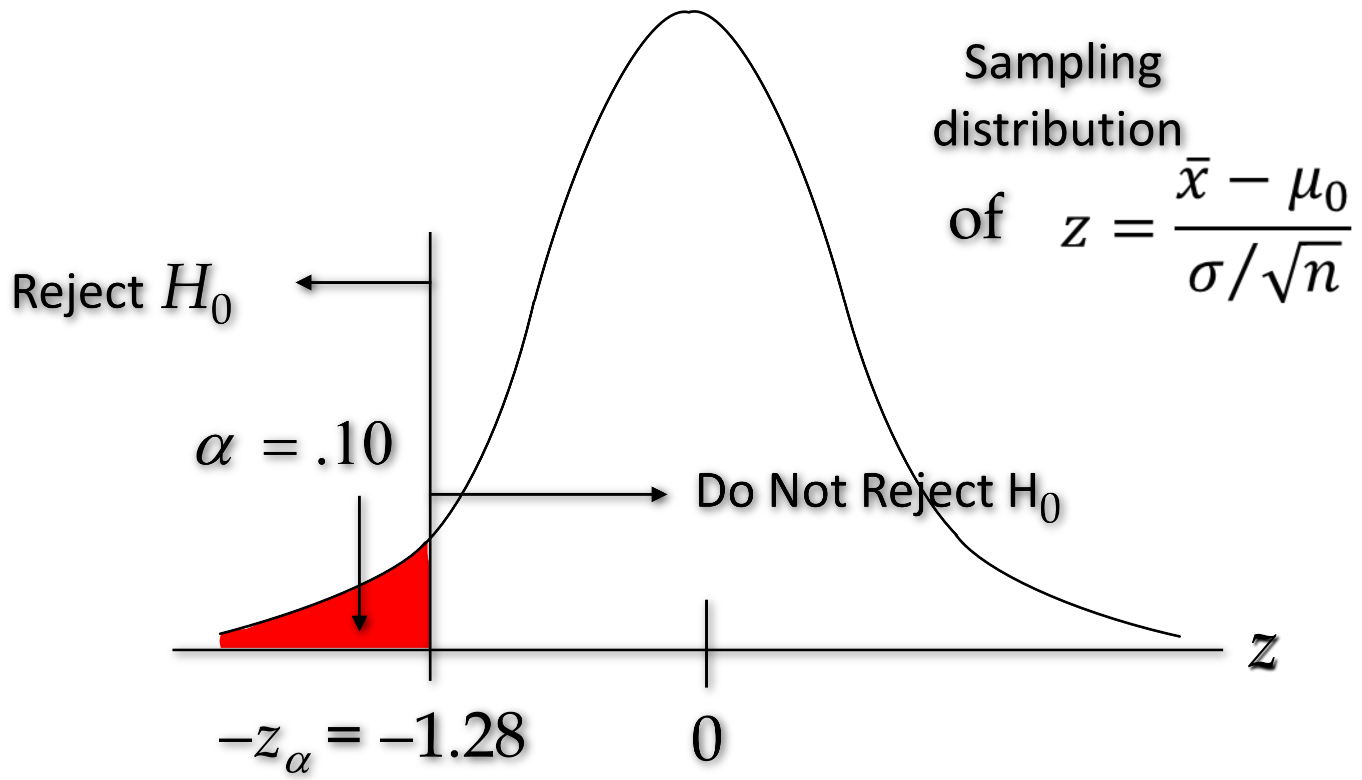
  Upper tail: Reject H_0 if $z \geq z_\alpha$





Lower-Tailed Test About a Population Mean: σ Known

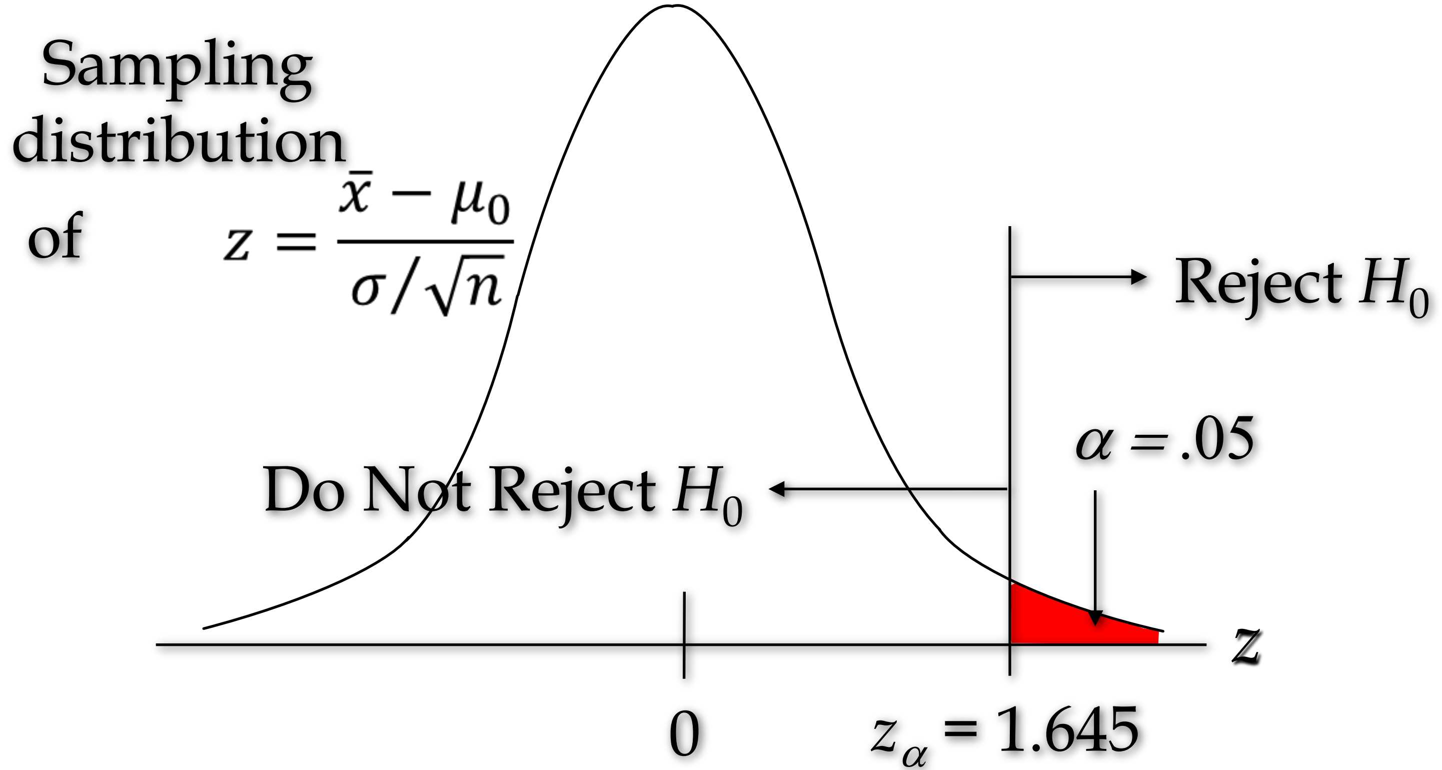
Critical Value Approach



```
In [6]: stats.norm.ppf(0.1)

Out[6]: -1.2815515655446004
```

Upper-Tailed Test About a Population Mean: σ Known

Critical Value Approach



```
In [7]: stats.norm.ppf(0.95)

Out[7]: 1.6448536269514722
```

Steps of Hypothesis Testing – P value approach

- Step 1. Develop the null and alternative hypotheses.
- Step 2. Specify the level of significance α.
- Step 3. Collect the sample data and compute the test statistic.
- *p*-Value Approach
- Step 4. Use the value of the test statistic to compute the *p*-value.
- Step 5. Reject H_0 if *p*-value $\leq \alpha$.

Steps of Hypothesis Testing

Critical Value Approach

- Step 4. Use the level of significance α to determine the critical value and the rejection rule.

- Step 5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0.