Black's equation to calculate the heater reliability with current density and ambient temperature ($T_{ambient}$):

$$MTTF(J, T_{ambient}) = \frac{A_{MTTF}}{J^n} \cdot exp(\frac{E_A}{k_B T_{ambient}}) \tag{1}$$

To convert Black's equation from ambient temperature to local heater temperature (T_{joule}), we need:

$$MTTF(T_{joule}) = A_{MTTF} \cdot exp(\frac{E_A}{k_B T_{joule}}) \tag{2}$$

Next step is to determine the failure-rate and activation energy from the following 2 figures

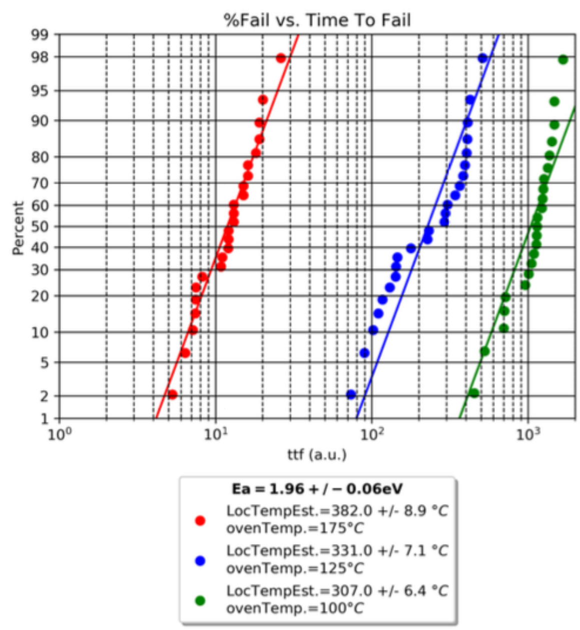

Figure 1: Cumulative distribution of failure vs. time to failure at three oven temperatures. Both the oven temperature and the estimated local heater temperature are shown.

1 Arhenius Weibull Distribution

Failure rate is be estimated using Weibull distribution. It is given by:

$$P_{failure}(t, T_{joule}) = 1 - exp(-[\frac{t}{MTTF(T_{joule})}]^{\beta}) \tag{3}$$

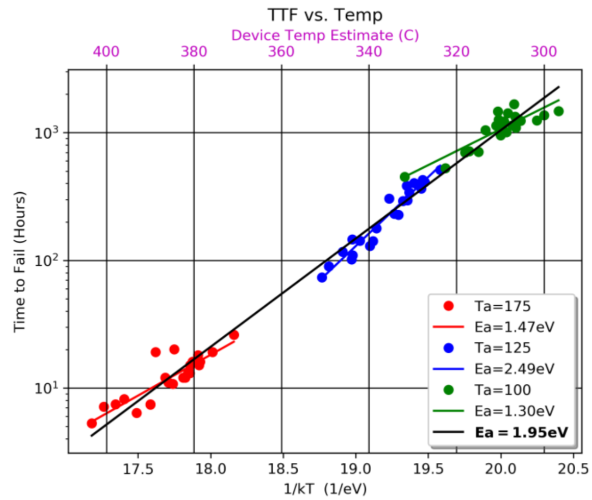

Figure 2: Time to failure vs local temperature for each individual device. Devices were stressed at three different oven temperatures.

Estimating β from GF's publication:

$$\begin{aligned} \frac{P_{failure1}}{P_{failure2}} &= \frac{1 - exp(-[\frac{MTTF(T_{joule})}{MTTF(T_{joule})}]^{\beta})}{1 - exp(-[\frac{MTTF(T_{joule})/2}{MTTF(T_{joule})}]^{\beta})} \\ \Rightarrow \frac{0.6321}{P_{failure2}} &= \frac{1 - exp(-1)}{1 - exp(-1 \cdot [0.5]^{\beta})} \end{aligned} \tag{4}$$

For t = MTTF(T_{joule}), we have a failure-rate of 63.21% is around 2500 a.u. for the blue curve. For t = MTTF(T_{joule})/2, we have $P_{failure,rate_2} = 7.5\%$.

$$\frac{0.6321}{0.075} = \frac{0.6321}{1 - exp(-1 \cdot [0.5]^{\beta})} \Rightarrow \beta \quad = 3.68 \tag{5}$$

The final lifetime $\tau_{lifetime}$ is:

$$\tau_{lifetime} = MTTF(T_{joule})[-1 \cdot log_e(1 - P_{failure,rate}(T_{joule}))]^{1/\beta} \tag{6}$$

2 Arhenius Lognormal Distribution

Reliability function ($R(t)$) and failure-rate ($P_{failure}(t)$) using Lognormal distribution. It is given by:

$$\begin{aligned} R(t) &= \int_{ln(t)}^{\infty} \frac{1}{\sigma' \sqrt{2\pi}} exp(-\frac{1}{2}[\frac{x-\mu'}{\sigma'}]^2)\, dx \\ P_{failure}(t) &= 1 - R(t) \\ MTTF(T_{joule}) &= exp(\mu') \\ MTTF(T_{joule}) &= A \cdot exp(\frac{E_A}{k_B T_{joule}}) \end{aligned} \tag{7}$$