

Output Power and Linewidth

ECE 455 Optical Electronics

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ECE Illinois

Introduction

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

In this section, we will learn how to do the following things:

- Determine the gain of a laser amplifier
- Find the threshold gain of a cavity
- Predict the output power of a laser
- Determine the output mode of the laser

Unless otherwise stated, steady state ($\frac{d}{dt} = 0$) behavior may be assumed.

Conventions for this Section

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

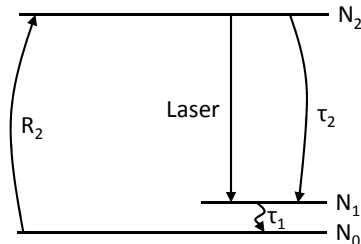
Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Assume the laser can be described by the diagram to the right.
- Some processes have been omitted to simplify the equations



Gain Saturation Derivation I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Begin by writing the laser rate equations:

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_2} - \frac{I\sigma_{se}}{h\nu} \left[N_2 - \frac{g_2}{g_1} N_1 \right] + R_2 = 0 \quad (1)$$

$$\frac{dN_1}{dt} = -\frac{N_1}{\tau_1} + \frac{I\sigma_{se}}{h\nu} \left[N_2 - \frac{g_2}{g_1} N_1 \right] + \frac{N_2}{\tau_{21}} = 0 \quad (2)$$

$$\frac{dN_p}{dt} = \frac{N_2}{\tau_{21}} + \frac{I\sigma_{se}}{h\nu} \left[N_2 - \frac{g_2}{g_1} N_1 \right] = 0 \quad (3)$$

In the absence of an optical field, the stimulated emission term may be ignored. In steady state, this yields:

$$N_2 = R_2\tau_2; \quad N_1 = \frac{N_2}{\tau_{21}}\tau_1 = \frac{R\tau_1\tau_2}{\tau_{21}} \quad (4)$$

Gain Saturation Derivation II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

For simplicity, let $\tau_1 \ll \tau_2$. This is a reasonable model for systems with a favorable lifetime ratio. With this assumption and Equation 4, we find that $N_1 \ll N_2$. Thus:

$$\Delta N_0 = N_2 - N_1 \approx N_2 = R_2 \tau_2 \quad (5)$$

Now consider the situation when the medium is in the presence of an optical field I . Stimulated emission can no longer be ignored.

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_2} - \frac{I\sigma_{se}}{h\nu} \left[N_2 - \frac{g_2}{g_1} N_1 \right] + R_2 = 0 \quad (6)$$

Gain Saturation Derivation III

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Combining this equation with Equation 5, we can write:

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_2} - \frac{I\sigma_{se}}{h\nu}N_2 + R_2 = 0 \quad (7)$$

Finally, solving for N_2 , we obtain:

$$\Delta N = \frac{R_2\tau_2}{1 + \frac{I\sigma_{se}\tau_2}{h\nu}} = \frac{\Delta N_0}{1 + I/I_{sat}} \quad (8)$$

where

$$I_{sat} = \frac{h\nu}{\sigma_{se}\tau_2} \quad (9)$$

Gain Saturation Derivation IV

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Finally, multiplying both side of Equation 8 by the stimulated emission cross section, $\sigma_{se}(\nu)$, we have

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + I/I_{sat}} \quad (10)$$

Returning to the original definition of the gain coefficient, we find:

$$\gamma \equiv \frac{1}{l} \frac{dl}{dz} = \frac{\gamma_0(\nu)}{1 + I/I_{sat}} \quad (11)$$

What happens in the limit that $I \rightarrow \infty$?

Homogeneous Saturation

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

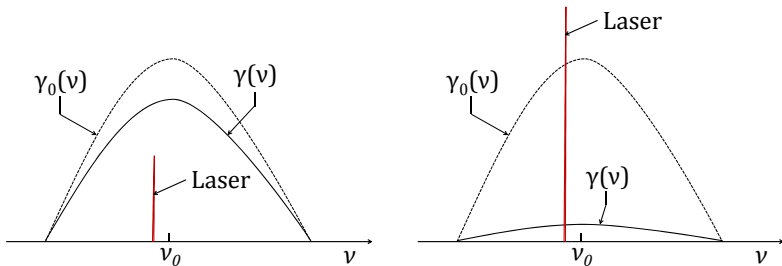
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Inhomogeneous Saturation

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

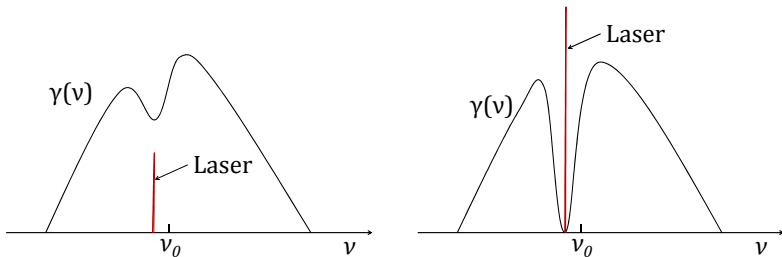
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Gain Saturation Summary

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Intense light reduces the population inversion
- Medium with reduced population inversion has reduced gain
- Homogeneous saturation reduces the *entire* gain spectrum uniformly
- Inhomogeneous saturation only reduces part of the gain spectrum

Application of Gain Saturation: STED Microscopy

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Stimulated Emission Depletion (STED) Microscopy is a method of overcoming the diffraction limit

$$d_{min} \approx \frac{\lambda}{2NA} \quad (12)$$

of confocal microscopy. Here $NA = n \cdot \sin(\theta)$ is the numerical aperture.

The basic idea of STED is as follows:

- ① Stain a sample with a gain medium (commonly a dye)
- ② Excite a volume of gain material with a pump laser
- ③ De-excite all but a small fraction of the pumped volume using stimulated emission
- ④ Scan beams around while collecting spontaneous emission signal to generate image

STED Microscopy: Excitation and De-excitation

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Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

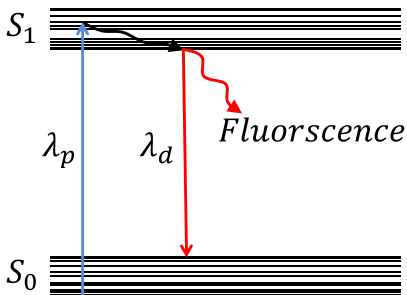
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



- Dye is excited by a laser at wavelength λ_p
- Spontaneous emission is suppressed in regions of the sample illuminated with a de-excitation laser (λ_d)
- Fluorescence only comes from regions not illuminated by the de-excitation laser.

STED Microscopy: Excitation and De-excitation

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

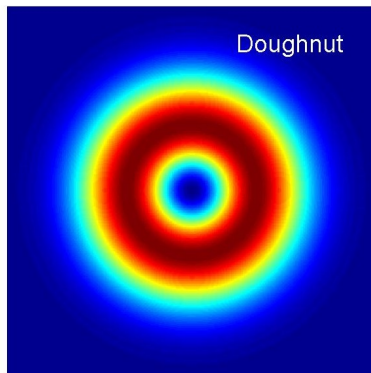
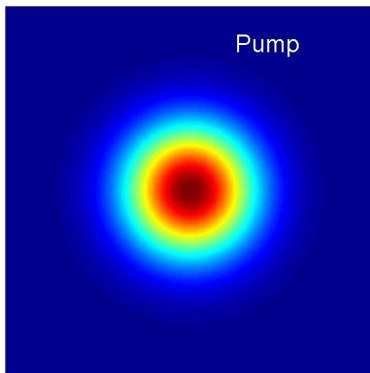
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Left: Pump beam at a shorter wavelength (λ_p) used to excite fluorescence in sample. **Right:** De-excitation beam at fluorescence wavelength (λ_d), used to de-populate upper state

STED Microscopy: Spot Size

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Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

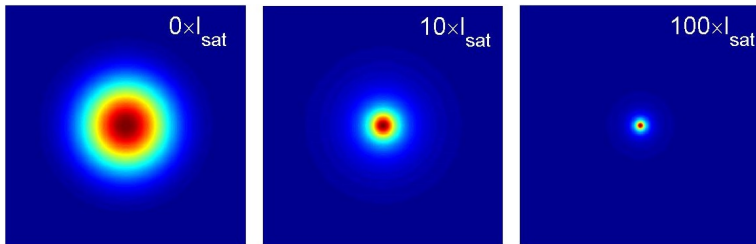
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Left: Fluorescence pattern with de-excitation beam off. This spot size is the normal diffraction limit for fluorescence microscopy. **Center:** Fluorescence pattern when the peak intensity of the de-excitation beam is $10 \cdot I_{sat}$. **Right:** Fluorescence pattern when the peak intensity of the de-excitation beam is $100 \cdot I_{sat}$. Note the decrease in size from regular diffraction limit.

Optical Amplification

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Equation 11 is a separable differential equation and can be integrated as follows:

$$\int_{I_{in}}^{I_{out}} \frac{1}{I} + \frac{1}{I_{sat}} dI = \int_0^L \gamma_0 dz \quad (13)$$

When integrated, this gives:

$$\ln \left(\frac{I_{out}}{I_{in}} \right) + \frac{I_{out} - I_{in}}{I_{sat}} = \gamma_0 L \quad (14)$$

This is an exact, but transcendental equation, which can be difficult to use. In the next slides, two limits will be considered in which this expression becomes simpler.

Small Signal Limit

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

If $I_{out} \ll I_{sat}$, then, to an approximation, the upper and lower state populations are unaffected by the presence of the optical field.

$$\frac{dI}{dz} = \frac{\gamma_0 I}{1 + I/I_{sat}} \approx \gamma_0 I \quad (15)$$

Thus:

$$I_{out} = I_{in} e^{\gamma_0 L} \quad (16)$$

A small signal grows exponentially.

Saturated Gain Limit

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

If $I_{in} \gg I_{sat}$, then the intense optical field will drive the system to transparency. A fixed amount of power will be extracted per unit length. This process is commonly known as 'bleaching.'

$$\frac{dI}{dz} = \frac{\gamma_0 I}{1 + I/I_{sat}} \approx \gamma_0 I_{sat} \quad (17)$$

$$I_{out} = I_{in} + \gamma_0 I_{sat} L \quad (18)$$

A large signal grows linearly. *Why?*

The power available from a laser amplifier is:

$$P_{av} = A \gamma_0 I_{sat} L \quad (19)$$

Gain Saturation Picture

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Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

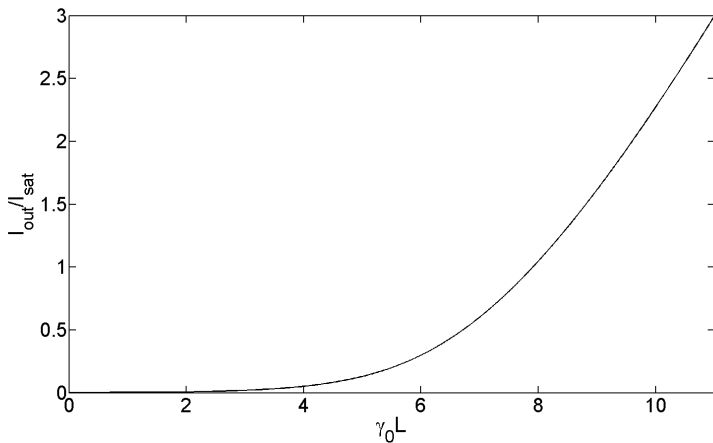
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



The Saturation Intensity

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- $I_{sat} = \frac{h\nu}{\sigma_{se} \tau_2}$ only valid with simplifying assumptions of this section
- Rigorous Definition:

$$I_{sat} \equiv \left\{ \begin{array}{l} \text{The intensity which causes the small} \\ \text{signal gain to fall by half} \end{array} \right\} \quad (20)$$

- Characteristic of the gain medium
- Scale of what intensities can be achieved with a given medium

Example: Saturation Intensities

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: Find the saturation intensity of the Helium-Neon (HeNe) laser. The relevant parameters are: $\lambda = 632.8 \text{ nm}$, $\tau_2 = 3 \times 10^{-8} \text{ s}$, and $\sigma_{se} = 3 \times 10^{-17} \text{ m}^2$

Solution: Simply use Equation 9

$$\begin{aligned} I_{sat} &= \frac{hc}{\lambda \sigma_{se} \tau_2} \\ &= \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s}) \cdot (299792458 \text{ m/s})}{(632.8 \text{ nm}) \cdot (3 \times 10^{-17} \text{ m}^2) \cdot (3 \times 10^{-8} \text{ s})} \\ &= 34.88 \text{ W/cm}^2 \end{aligned} \quad (21)$$

A typical bore radius for a HeNe laser is $w_0 = 0.5 \text{ mm}$. This implies an output of $P = \pi I_{sat} r^2 = 273 \text{ mW}$. Commercial HeNe lasers can be purchased with outputs ranging from 10 - 200 mW.

Example: Saturation Intensities

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: Find the saturation intensity of the titanium doped sapphire (Ti:Sapph) laser. The relevant parameters are: $\lambda = 800 \text{ nm}$, $\tau_2 = 3.8 \times 10^{-6} \text{ s}$, and $\sigma_{se} = 3.4 \times 10^{-23} \text{ m}^2$

Solution: Simply use Equation 9

$$\begin{aligned} I_{sat} &= \frac{hc}{\lambda \sigma_{se} \tau_2} \\ &= \frac{(6.626 \times 10^{-34} \text{ J-s}) \cdot (299792458 \text{ m/s})}{(800 \text{ nm}) \cdot (3.4 \times 10^{-23} \text{ m}^2) \cdot (3.8 \times 10^{-6} \text{ s})} \\ &= 192 \text{ kW/cm}^2 \end{aligned} \quad (22)$$

A typical beam waist for a HeNe laser is $w_0 = 0.3 \text{ mm}$. This implies an output of $P = \pi I_{sat} r^2 = 543 \text{ W}$. Commercial Ti:Sapph systems can be purchased with average output power from 100 mW - 10 W.

Threshold Gain Picture

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

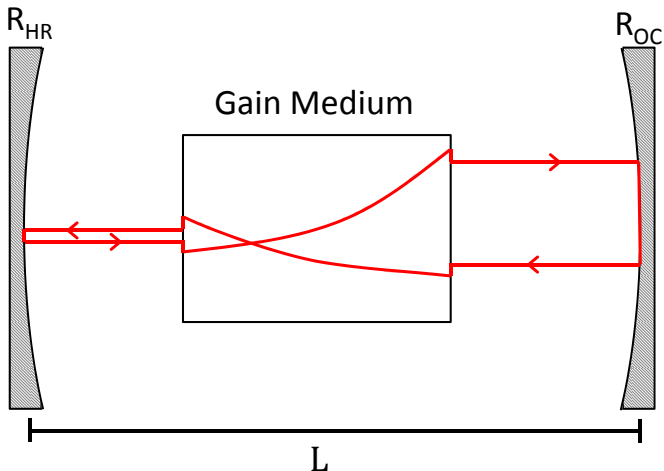
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Threshold Gain

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

If a laser is pumped above threshold, the round trip gain must be 1!

- If the round trip gain were less than one, the field would decay continuously, but then output power would decay to zero.
- If the round trip gain were greater than one, the field would grow continuously. Because the gain medium is being pumped with a finite amount of energy, this is impossible.
- The light intensity will saturate the gain.
- Inside a laser $\gamma = \gamma_{th} < \gamma_0$

Threshold of a Two-Mirror Cavity

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

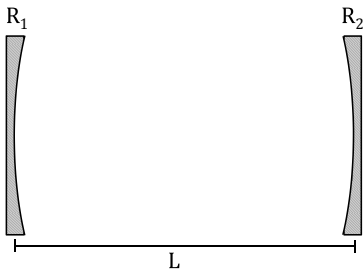
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



$$R_1 e^{\gamma_{th} L} R_2 e^{\gamma_{th} L} = 1 \quad (23)$$

$$\gamma_{th} = -\frac{1}{2L} \ln(R_1 R_2) \quad (24)$$

Threshold of a More Complicated Cavity

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

If the gain medium on the previous slide experiences a Beer's Law absorption loss, the threshold gain expressions become:

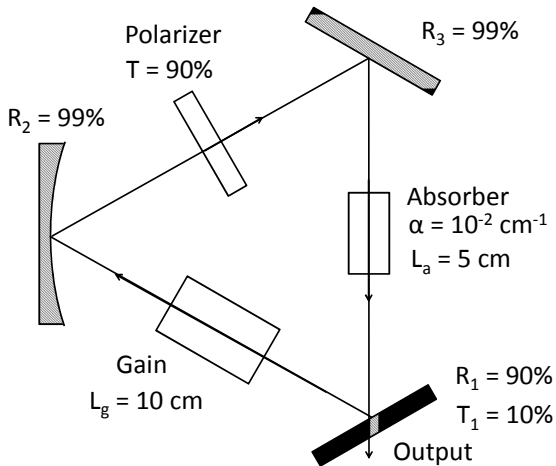
$$R_1 e^{(\gamma_{th} - \alpha)L_g} R_2 e^{(\gamma_{th} - \alpha)L_g} = 1 \quad (25)$$

$$\gamma_{th} = \alpha + \frac{1}{2L_g} \ln(R_1 R_2) \quad (26)$$

Example: Finding the Threshold

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Lecture 4

Problem: Find the threshold gain of the following cavity.



Example: Finding the Threshold

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Solution: Begin by setting the round-trip gain to one.

$$R_1 \exp(\gamma_{th} L_g) R_2 T R_3 \exp(-\alpha L_a) = 1 \quad (27)$$

$$\gamma_{th} = \alpha \frac{L_a}{L_g} - \frac{1}{L_g} \ln(R_1 R_2 T R_3) \quad (28)$$

$$= 0.0281 \text{ cm}^{-1} \quad (29)$$

Example: Forcing TEM₀₀ Operation I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

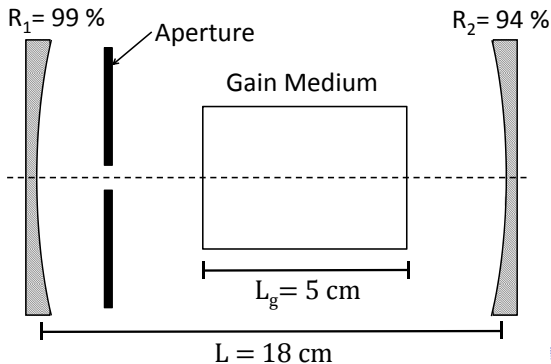
Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

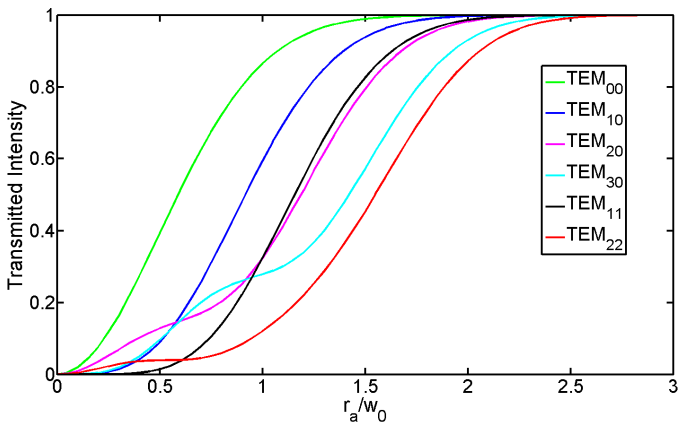
Problem: To stop higher-order modes from lasing, a circular aperture with radius $r_a = 1.5w(z)$ is placed in a laser cavity as shown below. Find the threshold gain for both the TEM₀₀ and TEM₁₁ modes with and without the aperture present.



Example: Forcing TEM₀₀ Operation II

ECE 455
Lecture 4

The plot below shows the transmitted intensity as a function of aperture size.



Example: Forcing TEM₀₀ Operation III

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Solution: Without the aperture, the threshold gains of each mode are identical:

$$\gamma_{th} = -\frac{1}{2L_g} \ln(R_1 R_2) = 0.0072 \text{ cm}^{-1} \quad (30)$$

Reading the plot on the previous page, we find $T_{00} = 0.9888$ for the TEM₀₀ mode. The threshold is then:

$$\gamma_{th,00} = -\frac{1}{2L_g} \ln(R_1 T_{00} R_2 T_{00}) = 0.0094 \text{ cm}^{-1} \quad (31)$$

Similarly for the TEM₁₁ mode, $T_{11} = 0.8260$. Therefore

$$\gamma_{th,11} = -\frac{1}{2L_g} \ln(R_1 T_{11} R_2 T_{11}) = 0.0454 \text{ cm}^{-1} \quad (32)$$

If you pump the laser such that $\gamma_{th,00} \leq \gamma_0 \leq \gamma_{th,11}$, only the TEM₀₀ mode will lase.

Saturation of Gain Media Inside a Cavity

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

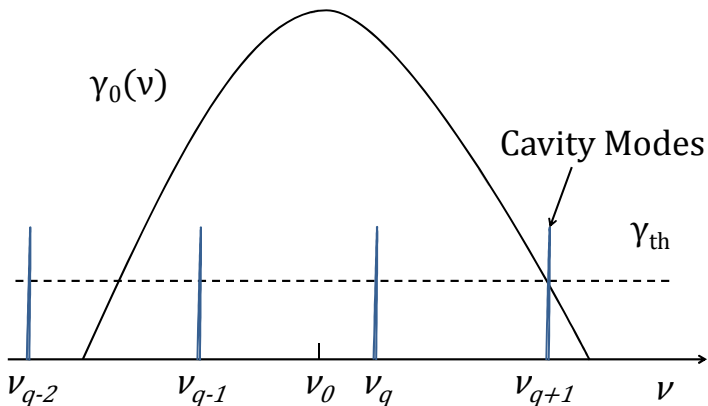
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Saturation of Homogeneous Medium in a Cavity

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

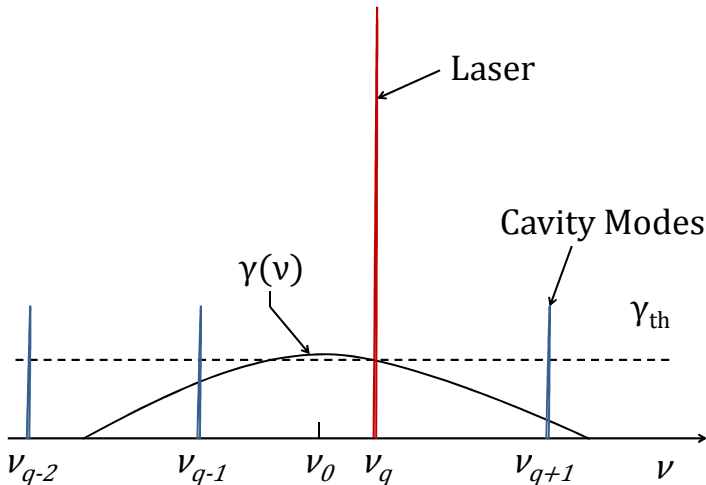
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Gain Clamping

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

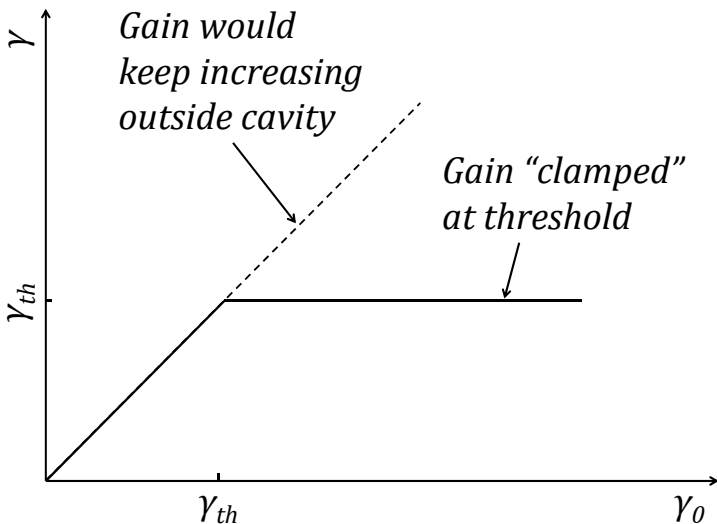
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Spontaneous Emission Power

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

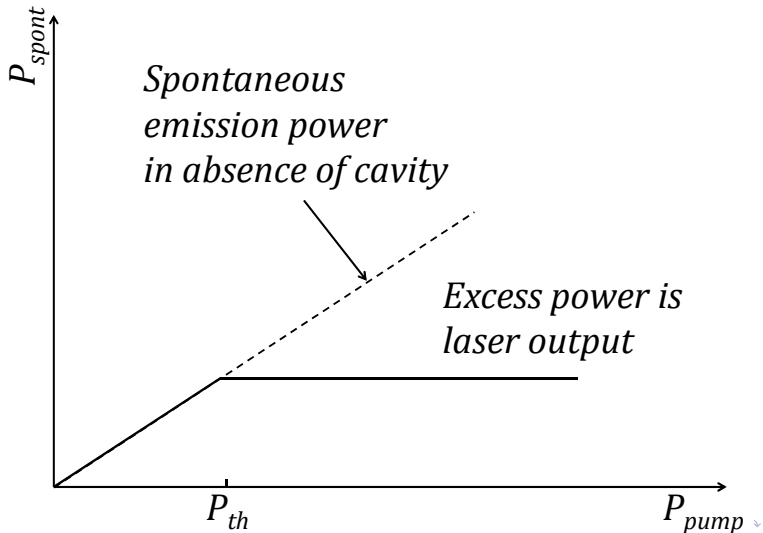
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Laser Emission Power

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

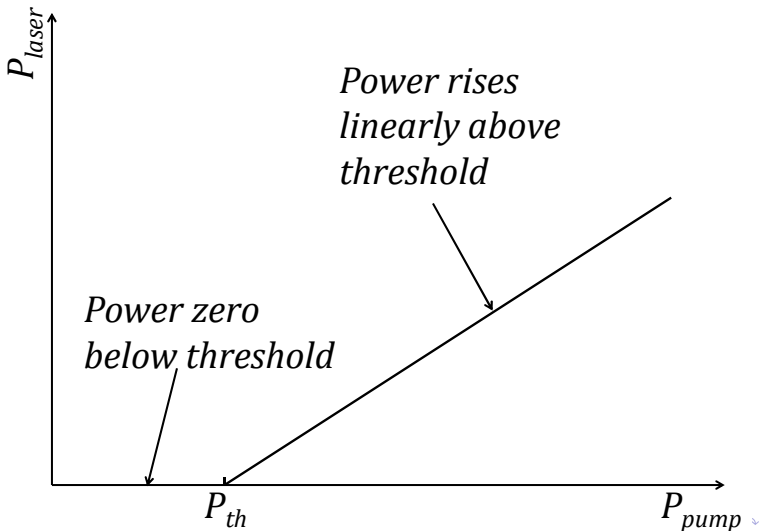
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



A Tale of Three Gains

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- γ_0 - Small Signal Gain
 - $\gamma_0 = \sigma_{se} \Delta N_0$
 - Set by material parameters and pumping rate
 - The gain in the absence of a strong optical field
- γ_{th} - Threshold Gain
 - $\gamma_{th} = \alpha - \frac{1}{2L} \ln(R_1 R_2) = \sigma_{se} \Delta N_{th}$
 - Determined only by the properties of the cavity
 - The gain which just balances cavity losses
- γ - Gain
 - $\gamma \equiv \frac{1}{I} \frac{dI}{dz} = \sigma_{se} \Delta N$
 - Set by both the cavity and the pump rate
- Inside a cavity
 - $\gamma = \gamma_0$ if $\gamma_0 < \gamma_{th}$
 - $\gamma = \gamma_{th}$ if $\gamma_0 > \gamma_{th}$

Conventions for this Section

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

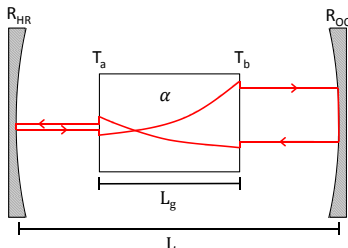
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



In this section we will consider the output power from a laser system as shown above:

- Window transmissions T_a and T_b
- Mirror reflectivities R_{hr} and R_{oc}
- Power taken from R_{hr} and $T_{oc} + R_{oc} = 1$
- Internal gain medium loss of α

The Low Loss Approximation I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

In the laser resonator, there is both a forward and backward propagating wave. Each wave works to saturate the medium. We may write:

$$\frac{dl_+}{dz} = \frac{\gamma_0 l_+}{1 + \frac{l_+ + l_-}{I_{sat}}} - \gamma_{th} l_+ \quad (33)$$

$$\frac{dl_-}{dz} = \frac{-\gamma_0 l_-}{1 + \frac{l_+ + l_-}{I_{sat}}} - \gamma_{th} l_- \quad (34)$$

In these equations, γ_{th} removes the portion of the gain which is used to overcome the cavity losses.

$$\gamma_{th} = \alpha_{int} - \frac{1}{2L_g} \ln(T_a^2 T_b^2 R_{hr} R_{oc}) \quad (35)$$

The Low Loss Approximation II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

If losses are low enough in the cavity, it may be assumed that $\frac{dI}{dz} = 0$. This implies $I_+ = I_-$. This is the Low Loss Approximation. If Equation 33 is set equal to zero and solved for I_+ , we find:

$$I_+ = \frac{I_{sat}}{2} \left[\frac{\gamma_0}{\gamma_{th}} - 1 \right] \quad (36)$$

I_+ is the intensity which must be present in the gain medium. To find how much is coupled out, one must multiply by T_b and T_2 .

$$I_{out} = \frac{T_{oc} T_b I_{sat}}{2} \left[\frac{\gamma_0}{\gamma_{th}} - 1 \right] \quad (37)$$

To find the total power, multiply by the area

$$P_{out} = A \frac{T_{oc} T_b I_{sat}}{2} \left[\frac{\gamma_0}{\gamma_{th}} - 1 \right] \quad (38)$$

Alternative Derivation of Output Power I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Once again assume the loss of cavity is small. Then the intracavity intensity is

$$I = I_{sat} \left(\frac{\gamma_0}{\gamma_{th}} - 1 \right) \quad (39)$$

The stimulated emission power from all atoms in the cavity is

$$\begin{aligned} P_e &= \sigma_{se} \Delta N_{th} \cdot I \cdot V \\ &= \gamma_{th} \cdot I \cdot V \end{aligned} \quad (40)$$

But only a fraction of this power will be useful. This fraction is:

$$T = \frac{-\frac{1}{2} \ln(R_{hr} R_{oc})}{\alpha L_g - \frac{1}{2} \ln(R_{hr} R_{oc})} = \frac{-\frac{1}{2} \ln(R_{hr} R_{oc})}{\gamma_{th} L_g} \quad (41)$$

Alternative Derivation of Output Power II

ECE 455
Lecture 4

Combining the results of Equations 39-41

$$P_{out} = P_e T \quad (42)$$

$$= \gamma_{th} \left[I_{sat} \left(\frac{\gamma_0}{\gamma_{th}} - 1 \right) \right] \cdot V \cdot \left[\frac{-\frac{1}{2} \ln(R_{hr} R_{oc})}{\gamma_{th} L_g} \right] \quad (43)$$

$$= I_{sat} \left(\frac{\gamma_0}{\gamma_{th}} - 1 \right) \cdot \frac{V}{L_g} \left[-\frac{1}{2} \ln(R_{hr} R_{oc}) \right] \quad (44)$$

$$= A I_{sat} \left(\frac{\gamma_0}{\gamma_{th}} - 1 \right) \left[-\frac{1}{2} \ln(R_{hr} R_{oc}) \right] \quad (45)$$

$$= A I_{sat} \left(\frac{\gamma_0}{\gamma_{th}} - 1 \right) \left[-\frac{1}{2} \ln(R_{hr} R_{oc}) \right] \quad (46)$$

Therefore the output intensity is

$$I_{out} = -\frac{1}{2} \ln(R_{hr} R_{oc}) I_{sat} \left(\frac{\gamma_0}{\gamma_{th}} - 1 \right) \quad (47)$$

Yet Another Derivation of Output Power I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

For the next derivation, we'll look at the cavity as a whole.
This derivation will require a few new variables

- ϕ - total number of photons in cavity
- V - mode volume (inside and outside the gain medium)
- τ_c - photon lifetime in cold (unpumped) cavity
- τ_{oc} - photon lifetime if output coupler were the only source of photon loss

There are also three equations to keep in mind:

$$\frac{1}{\tau_c} = \frac{1}{\tau_{oc}} + \frac{1}{\tau_{nc}} \quad (48)$$

$$I = \frac{c \phi h \nu}{n V} \quad (49)$$

$$\Delta N = \frac{\Delta N_0}{1 + I/I_{sat}} \quad (8)$$

Yet Another Derivation of Output Power II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

The rate of change of the number of photons can be written as:

$$\frac{d\phi}{dt} = \frac{L_g}{L} V \frac{I \sigma_{se}}{h\nu} \Delta N - \frac{\phi}{\tau_c} + \eta_{seed} A_{21} N_2 \quad (50)$$

$$= \frac{L_g}{L} V \frac{c}{n} \frac{\phi h\nu}{V} \frac{1}{h\nu} \sigma_{se} \Delta N - \frac{\phi}{\tau_c} + \eta_{seed} A_{21} N_2 \quad (51)$$

$$= \frac{L_g}{L} \frac{c}{n} \phi \sigma_{se} \Delta N - \frac{\phi}{\tau_c} + \eta_{seed} A_{21} N_2 \quad (52)$$

$$= \phi \left[\frac{L_g}{L} \frac{c}{n} \sigma_{se} \Delta N - \frac{1}{\tau_c} \right] + \eta_{seed} A_{21} N_2 \quad (53)$$

In steady state, the spontaneous emission into the cavity mode is negligible. Therefore:

$$\phi \left[\frac{L_g}{L} \frac{c}{n} \sigma_{se} \Delta N - \frac{1}{\tau_c} \right] = 0 \quad (54)$$

Yet Another Derivation of Output Power III

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Obviously $\phi = 0$ is a mathematical, but not physical solution, but according to Equations 8 and 49, ΔN is a function of ϕ . Substituting them into Equation 54 and solving for ϕ yields:

$$\phi = \frac{I_{sat} V}{h\nu} \left[\frac{L_g}{L} \tau_c \sigma_{se} \Delta N_0 - \frac{n}{c} \right] \quad (55)$$

The total output power is $P_{out} = \frac{\phi h\nu}{\tau_{oc}}$

$$P_{out} = I_{sat} V \left[\frac{L_g}{L} \frac{\tau_c}{\tau_{oc}} \sigma_{se} \Delta N_0 - \frac{1}{\tau_{oc}} \frac{n}{c} \right] \quad (56)$$

Rigrod Analysis

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

With $\alpha = 0$, the exact solution for output intensity was derived by Rigrod to be:

$$I = I_{sat} T_b T_{oc} \frac{\gamma_0 L_g + \frac{1}{2} \ln (R_{hr} R_{oc} T_a^2 T_b^2)}{\left(1 - \sqrt{R_{hr} R_{oc} T_a^2 T_b^2}\right) \left(1 + \sqrt{\frac{R_{oc} T_b^2}{R_{hr} T_a^2}}\right)} \quad (57)$$

Note that this equation is valid for ANY level of coupling.

Comparing Models

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

We've seen one and derived three expressions for the output power of the laser, but how are they related?

- First three models assume gain medium is uniformly saturated, which implies low loss is necessary
- Rigrod analysis assumes $\alpha = 0$, but otherwise any amount of loss is accounted for
- Models should generally agree with each other, especially in the limit of low loss
- These are only order of magnitude estimates anyway, they do not include effects such as:
 - Gaussian beam shape
 - Non-uniform pumping
 - Nonlinear loss mechanisms
 - Many others
- The fit to data can be surprisingly good even with these simple models

Example: Comparing Output Power Models I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: A Ti:Sapphire ($I_{sat} = 192 \text{ kW-cm}^{-2}$ and $\alpha = 0$) with cavity parameters $L_g = 5 \text{ mm}$, $L = 1 \text{ m}$, $A = 0.07 \text{ mm}^2$, $R_{hr} = 0.99$, $R_{oc} = 0.94$, and $\gamma_0 = 1.1\gamma_{th}$. Find the output power of this laser using all four models.

Solution: According to the low-loss approximation:

$$\begin{aligned} P_{out} &= (0.07 \text{ mm}^2) \frac{(0.06) \cdot (1.0) \cdot (192 \text{ kW-cm}^{-2})}{2} [1.1 - 1] \\ &= 0.403 \text{ W} \end{aligned} \quad (58)$$

$$\begin{aligned} P_{out} &= (0.07 \text{ mm}^2) \left(-\frac{1}{2} \ln(0.99 \cdot 0.94) \right) (192 \text{ kW-cm}^{-2}) [1.1 - 1] \\ &= 0.4158 \text{ W} \end{aligned}$$

Example: Comparing Output Power Models II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Next we'll estimate the power using Rigrod's analysis. First, we'll need to find γ_0

$$\gamma_0 = 1.1\gamma_{th} = 1.1 \frac{1}{2(5 \text{ mm})} \ln(0.99 \cdot 0.94) = 0.0791 \text{ cm}^{-1} \quad (60)$$

Next, we insert this into the Rigrod formula to obtain:

$$\begin{aligned} P_{out} &= (0.07 \text{ mm}^2)(192 \text{ kW-cm}^{-2})(0.06)(1.0) \\ &\times \frac{(0.0791 \text{ cm}^{-1})(5 \text{ mm}) + \frac{1}{2} \ln(0.99 \cdot 0.94)}{(1 - \sqrt{0.99 \cdot 0.94}) \left(1 + \sqrt{\frac{0.94}{0.99}}\right)} \\ &= 0.4158 \text{ W} \end{aligned} \quad (61)$$

Example: Comparing Output Power Models III

ECE 455
Lecture 4

Finally, we'll estimate the output power using the rate equation approach.

$$\tau_c = \frac{2 \cdot (0.995 \text{ m} + 1.76 \cdot 0.005 \text{ m})}{(3 \times 10^8 \text{ m/s})(1 - 0.99 \cdot 0.94)} = 99.8 \text{ ns} \quad (62)$$

$$\tau_{oc} = \frac{2 \cdot (0.995 \text{ m} + 1.76 \cdot 0.005 \text{ m})}{(3 \times 10^8 \text{ m/s})(1 - 0.94)} = 115.4 \text{ ns} \quad (63)$$

These can now be placed into Equation 56

$$\begin{aligned} P_{out} &= (192 \text{ kW-cm}^{-2})(0.07 \text{ mm}^2)(5 \text{ mm}) \\ &\times \left[\frac{5 \text{ mm}}{1 \text{ m}} \frac{99.8 \text{ ns}}{115.4 \text{ ns}} (0.0755 \text{ cm}^{-1}) - \frac{1}{111.2 \text{ ns}} \frac{1.76}{3 \times 10^8 \text{ m/s}} \right] \\ &= 0.580 \text{ W} \end{aligned} \quad (64)$$

We see that all of the models predict similar output powers.

Example: Dust on the Optic

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: A piece of dust drifts near one face of the crystal and is burnt by the high-intensity light. The smoke coats one side of the crystal, causing a 0.1% scattering loss. Assume the laser is pumped with the same power as before. Find the new output power of the laser.

Solution: We'll use Rigrod's analysis

$$\begin{aligned} P_{out} &= (0.07 \text{ mm}^2)(192 \text{ kW-cm}^{-2})(0.06)(1.0)(0.95) \\ &\times \frac{(0.0791 \text{ cm}^{-1})(5 \text{ mm}) + \frac{1}{2} \ln(0.99 \cdot 0.94 \cdot 0.95^2)}{\left(1 - \sqrt{0.99 \cdot 0.94 \cdot 0.95^2}\right) \left(1 + \sqrt{\frac{0.94 \cdot 0.95^2}{0.99}}\right)} \\ &= 0.292 \text{ W} \end{aligned} \quad (65)$$

The Lesson: Outside the cavity, a 0.1% mirror loss would only have resulted in a 0.1% power loss. Inside the cavity, the effect of the loss is magnified. What is the output power with a 1% scattering loss?

Example: Dust on the Optic Redesign

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: Suppose the laser is redesigned so that $A = 0.0167 \text{ mm}^2$ and $\gamma_0 = 1.38\gamma_{th}$, with all other parameters remaining the same. Find the power before and after the dust burns on the optic

Solution: We'll use Rigrod's analysis, we find the output powers to be are:

$$P_{out} = 0.4150 \text{ W} \quad (66)$$

After the dust burns on, the output power is reduced to:

$$P_{out} = 0.3770 \text{ W} \quad (67)$$

The Lesson: Lasers operating well above threshold are less sensitive to perturbations. Small mode areas with high intensities are favored.

Output Coupler Optimization I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Before we start with equations, a few quick observations:

- All power scattered or coupled out at R_{hr} is wasted because the output is taken at R_{oc} . Hence R_{hr} should be as high as possible
- In the limit that R_{oc} is the only loss in the cavity, R_{oc} should be chosen as large as possible.
- If other loss mechanisms are present, than an optimal value exists for the output coupler. The power needs to be taken out of the cavity before it gets lost to scattering or reabsorption.

Output Coupler Optimization II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Recall that in the Low-Loss approximation, the output power of a laser is

$$I_{out} = \frac{T_{oc} I_{sat}}{2} \left[\frac{\gamma_0}{\gamma_{th}} - 1 \right] \quad (37)$$

The threshold gain may be written

$$2L_g \gamma_{th} = 2\alpha L_g - \ln(R_{hr}) - \ln(R_{oc}) \quad (68)$$

$$= 2\alpha L_g - \ln(R_{hr}) - \ln(1 - T_{oc}) \quad (69)$$

$$= \alpha' + T_{oc} \quad (70)$$

where the approximation $\ln(1 - T_{oc}) \approx -T_{oc}$ has been used and $\alpha' \equiv 2\alpha L_g - \ln(R_{hr})$ for convenience.

The equation for the output power is now:

$$I_{out} = \frac{T_{oc} I_{sat}}{2} \left[\frac{2L_g \gamma_0}{\alpha' + T_{oc}} - 1 \right] \quad (71)$$

Output Coupler Optimization III

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

We can take the derivative of Equation 71!

$$\frac{dI_{out}}{dT_{oc}} = \frac{I_{sat}}{2} \left[\frac{2L_g\gamma_0}{\alpha' + T_{oc}} - 1 - \frac{2L_g\gamma_0 T_{oc}}{(\alpha' + T_{oc})^2} \right] = 0 \quad (72)$$

The optimum value for the transmission is

$$T_{oc,opt} = (2L_g\gamma_0\alpha')^{1/2} - \alpha' \quad (73)$$

$$\approx 2L_g(\gamma_0\alpha)^{1/2} - 2\alpha L_g \quad (74)$$

Equation 74 is valid in the limit that $R_{hr} \rightarrow 1$. The corresponding output intensity for this coupling is:

$$I_{out,opt} = I_{sat}L_g \left(\gamma_0^{1/2} + \alpha^{1/2} \right)^2 \quad (75)$$

Output Coupler Optimization III

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

There is no one $T_{op,opt}$ for a give laser type. It changes depending on

- Desired output power
- Unavoidable cavity losses (R_{hr} and α)

A Tale of Many Efficiencies

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- There are many efficiencies in common use in laser physics
- Energy is lost to heat, not collected in the output beam

Wall Plug Efficiency

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Output power of laser divided by the amount of power the power company charges you for.
- Includes all losses, including electrical circuits, diffraction, spontaneous emission, mode overlap, auxiliary equipment and more.
- The CO₂ laser can have a wall plug efficiency around 30% and diode lasers can have wall plug efficiencies around 60%.
- Most lasers types have low wall plug efficiencies. Example: The Ar⁺ laser has a wall plug efficiency of about 0.5%.

$$\eta = \frac{P_{out}}{P_{PowerBill}} \quad (76)$$

Optical to Optical Efficiency

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Frequently used in optically pumped lasers because it can be conveniently measured
- Affected by such things as:
 - Surface reflections
 - Pump-mode overlap
 - Pumping of states other than laser transition
 - Scattering and absorption
 - Spontaneous emission
 - Quantum efficiency

$$\eta_{OO} = \frac{P_{out}}{P_{in}} \quad (77)$$

Quantum Efficiency

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

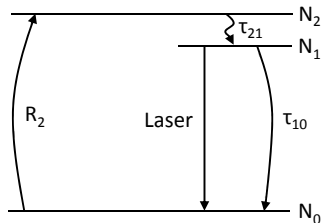
Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Sets the upper bound on the efficiency of the laser.
- Difference between energy levels is typically released as heat.
- Quantum efficiencies in excess of 100% are possible. Thermal energy may be removed by the laser; making the system a refrigerator!
- S. R. Bowman IEEE J. Quantum Electron. **35** 115 (1999).



$$\eta_Q = \frac{\nu_L}{\nu_{pump}} = \frac{\lambda_{pump}}{\lambda_L} \quad (78)$$

Slope Efficiency

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Describes how much power is added to the output for an infinitesimal increase in the pump power
- Units depend on pumping scheme
 - W/W for optical pumping
 - W/mA for electrical pumping
 - Other units possible

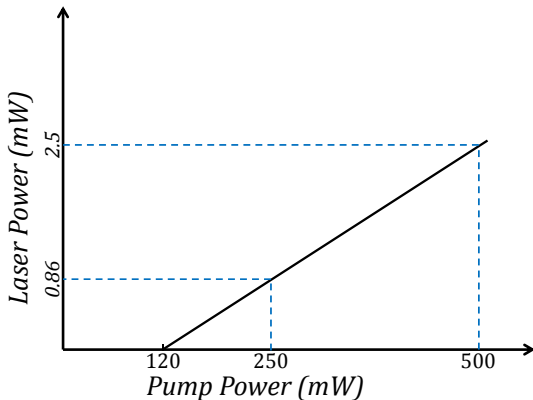
$$\eta_{slope} = \frac{dP_{out}}{dP_{pump}} \quad (79)$$

Example: Calculating Efficiency

ECE 455
Lecture 4

Gain
Saturation
Optical
Amplifiers
Threshold
Output Power
Optimization
Efficiency
Mode
Selection
Linewidth,
Stabilization,
and Tuning
Summary

Problem: An Erbium-doped fiber laser is pumped at 980 nm and lases at 1540 nm. Calculate the quantum, and the optical to optical, and the slope efficiencies at 250 mW and 500 mW pump power on the diagram (based on data from [1]).



Example: Calculating Efficiency

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Solution: The quantum efficiency is a property of the gain medium, independent of input or output power.

$$\eta_{QE} = \frac{980 \text{ nm}}{1540 \text{ nm}} = 0.6364 \quad (80)$$

The optical to optical efficiencies are:

$$\eta_{OO}(250 \text{ mW}) = \frac{0.86 \text{ mW}}{250 \text{ mW}} = 0.00344 \quad (81)$$

$$\eta_{OO}(500 \text{ mW}) = \frac{2.5 \text{ mW}}{500 \text{ mW}} = 0.005 \quad (82)$$

$$(83)$$

For this idealized data, the slope does not change between the points. The slope efficiency is therefore:

$$\eta_{slope} = \frac{2.5 - 0.86 \text{ mW}}{500 - 250 \text{ mW}} = 0.00656 \quad (84)$$

Selecting a Mode

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Our picture of laser oscillation looks like this:

- 1 The laser cavity defines a discrete number of optical modes, each with a its own characteristic frequency and physical shape.
- 2 Associated with each of the modes above is a gain. In addition to frequency dependent material gain, the gain is affected by frequency dependent mirror losses and mode dependent diffraction losses.
- 3 The mode with the highest net gain will depopulate the upper state of the lasing transition. Most other modes will no longer be able to lase.
- 4 If the mode selected is unable to fully depopulate the upper state (inhomogeneous broadening, spatial hole burning), then several modes may oscillate simultaneously.

Multiple Mode Lasers

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

In an ideal, homogeneously-broadened laser, only one mode will oscillate. However, if various modes are not competing for the same atoms, multiple modes may lase simultaneously.

Consider the following three scenarios:

- ① Not all emitters are capable of emitting the frequency of the mode with highest gain
- ② Some emitters fall in the node of a mode's standing wave pattern
- ③ The pumped area in the transverse direction is larger than the mode size

Secondary modes may or may not take power from the highest-gain mode

1 Inhomogeneous Broadening

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

The gain of an inhomogeneous medium saturates differently than a homogeneous medium. It can be show that the gain goes as:

$$\gamma = \frac{\gamma_0}{\left(1 + \frac{I}{I_{sat}}\right)^{1/2}} \quad (85)$$

For a linear cavity, we use the low loss approximation to find the output power of a given mode:

$$P_{out} = \frac{T_{oc} A I_{sat}}{2} \left[\left(\frac{\gamma_0}{\gamma_{th}} \right)^2 - 1 \right] \quad (86)$$

Inhomogeneous Saturation

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Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

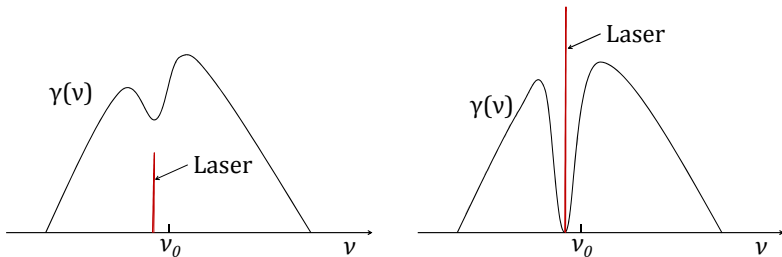
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Saturation of Inhomogeneous Medium in a Cavity

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

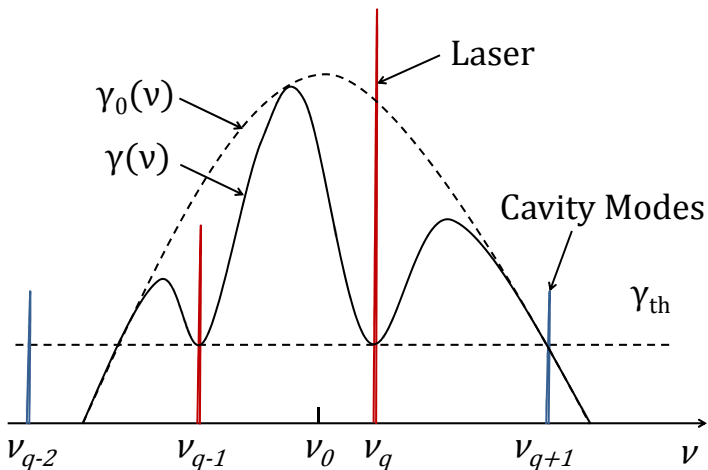
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



2 Multiple Longitudinal Modes

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

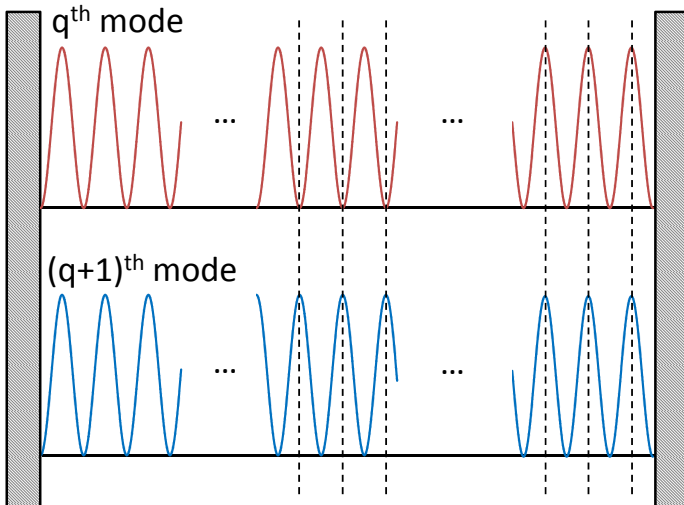
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



2 Multiple Longitudinal Modes

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Let λ_q be the wavelength of the mode with the highest gain

- In linear cavities, the forward and reverse propagating waves form a standing wave pattern

$$E(z) = \frac{E_0}{2} \left[e^{i\frac{2\pi}{\lambda_q}z} + e^{-i\frac{2\pi}{\lambda_q}z} \right] = E_0 \cos \left(\frac{2\pi}{\lambda_q}z \right) \quad (87)$$

- Population inversion is only saturated in regions of high intensity

$$I(z) = \frac{E^*(z)E(z)}{2\eta} = \frac{|E_0|^2}{2\eta} \cos^2 \left(\frac{2\pi}{\lambda_q}z \right) \quad (88)$$

- Standing wave pattern of adjacent longitudinal modes (λ_{q-1} and λ_{q+1}) must at some point be 180° out of phase with each other
- Adjacent longitudinal modes therefore encounter unsaturated gain medium

2 Multiple Longitudinal Modes

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Prevents entire gain volume from being used
- Output power lowered if laser is forced to oscillate with only one transverse mode
- Effect known as **spatial hole burning**
- Ring lasers are immune to spatial hole burning because there is no standing wave pattern

3 Multiple Transverse Modes

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

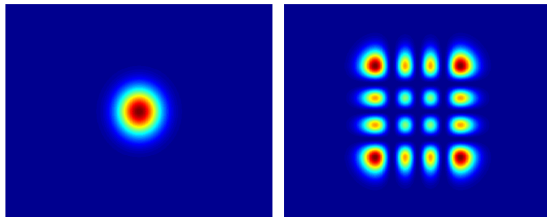
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



- TEM_{00} mode unable to saturate gain medium much beyond $w(z)$
- If pumped region is larger than $w(z)$, other transverse modes may oscillate
- Multiple transverse modes are generally undesirable because higher order modes diverge faster and can't be focused

3 Transverse Mode Intensity Overlap

ECE 455
Lecture 4

The following equation represents one possible metric for the overlap of the TEM_{mp} and $TEM_{m'p'}$ intensity profiles

$$\alpha_{mp,m'p'} = \frac{\iint \sqrt{I_{mp}(x,y)I_{m'p'}(x,y)} dx dy}{\iint I_{mp}(x,y) dx dy \iint I_{m'p'}(x,y) dx dy} \quad (89)$$

α	00	10	20	01	11	21	02	12	22
00	1.00	0.80	0.68	0.80	0.64	0.55	0.68	0.55	0.47
10	0.80	1.00	0.80	0.64	0.80	0.64	0.55	0.68	0.55
20	0.68	0.80	1.00	0.55	0.64	0.80	0.47	0.55	0.68
01	0.80	0.64	0.55	1.00	0.80	0.68	0.80	0.64	0.55
11	0.64	0.80	0.64	0.80	1.00	0.80	0.64	0.80	0.65
21	0.55	0.64	0.80	0.68	0.80	1.00	0.55	0.65	0.80
02	0.68	0.55	0.47	0.80	0.64	0.55	1.00	0.80	0.68
12	0.55	0.68	0.55	0.64	0.80	0.65	0.80	1.00	0.80
22	0.47	0.55	0.68	0.55	0.65	0.80	0.68	0.80	1.00

Gain
Saturation
Optical
Amplifiers
Threshold
Output Power
Optimization
Efficiency
Mode
Selection
Linewidth,
Stabilization,
and Tuning
Summary

3 Beam Profile of Laser with Multiple Transverse Modes

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

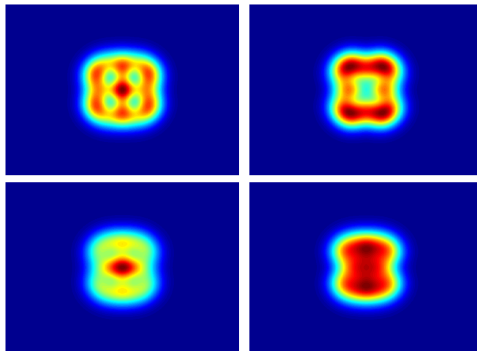
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



- Not easy to determine beam quality by eye
- Intensities of each mode chosen randomly for figure

Amplified Spontaneous Emission (ASE)

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Photons spontaneously emitted along the length of the cavity are amplified
- Although in the same general direction as the output, not in the same cavity mode
- Important when $\gamma L_g \gg 1$
- Generally undesirable
 - Adds noise
 - Difficult to focus
- Suppressed by laser or signal

Phase Noise

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

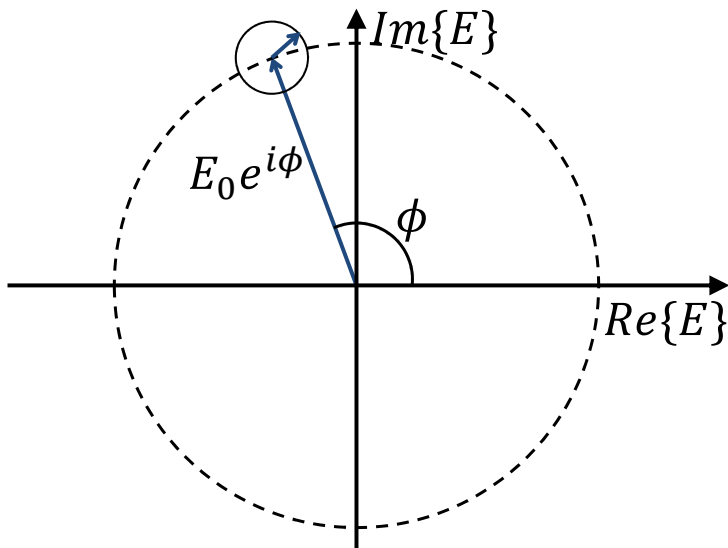
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Phase Noise

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- While the laser is operating, the spontaneous emission is clamped at its threshold value
- During operation, a photon may spontaneously emit into the lasing mode
- This photon's phase is unrelated to the phase of the lasing mode
- But the gain this photon will experience is the same as the already lasing mode
- The phase of the output undergoes a random walk
- This is the ultimate limit on the minimum linewidth of a laser

Schawlow-Townes Linewidth

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

The minimum linewidth of a laser was derived by Schawlow and Townes to be:

$$\Delta\nu_{laser} = \frac{\pi h\nu (\Delta\nu_c)^2}{P_{out}} \quad (90)$$

Where $\Delta\nu_c$ is the resonator bandwidth (FWHM). In practice, lasers have a much larger than this limit due to vibrations, power supply instabilities, and other considerations.

Example: Linewidth

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: A helium neon laser operates at 632.8 nm, is $L = 30$ cm long, has end mirror reflectivities of $R_{oc} = .98$ and $R_{hr} = .999$, and a power of 10 mW. Find the linewidth of the laser as predicted by Schalow and Townes.

Solution: The first step is to find the cavity FWHM:

$$\Delta\nu_c = \frac{c}{2nL} \left(\frac{1 - (R_{hr}R_{oc})^{1/2}}{\pi(R_{hr}R_{oc})^{1/4}} \right) \quad (91)$$

$$= 844 \text{ kHz} \quad (92)$$

The Schalow-Townes linewidth is then:

$$\Delta\nu_{laser} = \frac{\pi\hbar c(\Delta\nu_c)^2}{\lambda P_{out}} \quad (93)$$

$$= 7.02 \times 10^{-5} \text{ Hz} \quad (94)$$

A real HeNe laser has a bandwidth around 5 GHz. Other sources of noise dominate.

Mechanical Vibrations

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Mechanical vibrations may slightly change the distance between the end mirrors
- A change of $\lambda/(2F)$ is all it takes to bring a frequency from resonance to anti-resonance (where F is finesse)
- Small changes in mirror separation will result in output frequency changing slightly. Output power may change as the dominant mode tunes through the gain profile. See slide 80.
- If the mirrors are pulled far enough apart that the q^{th} mode no longer has the highest gain of all modes, laser will hop to another mode. See slide 81.
- Change in cavity resonance will sweep laser across gain bandwidth
- Vibrations are more difficult to control in large lasers

Mechanical Vibrations: Tuning

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

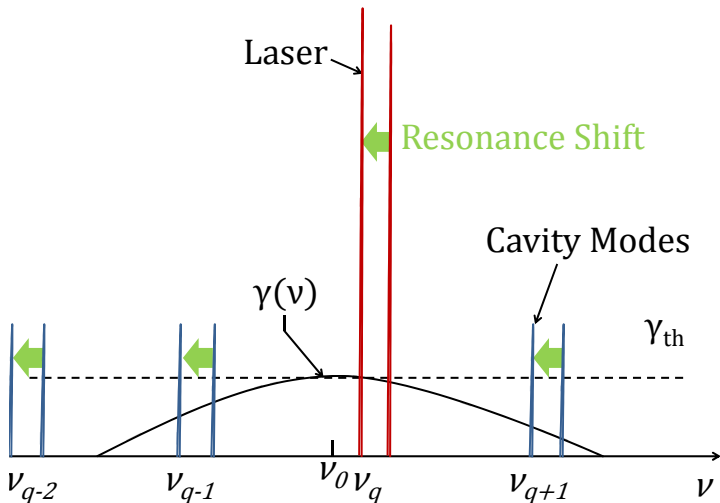
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

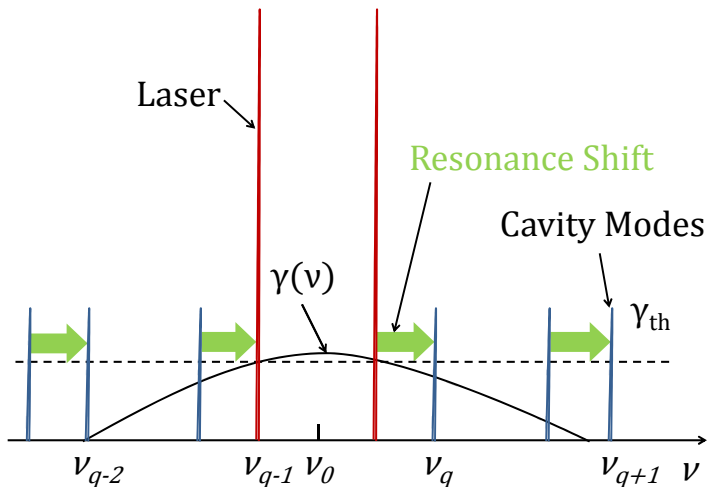
Summary



Mechanical Vibrations: Hopping

ECE 455
Lecture 4

Gain
Saturation
Optical
Amplifiers
Threshold
Output Power
Optimization
Efficiency
Mode
Selection
Linewidth,
Stabilization,
and Tuning
Summary



Frequency Stabilization

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

**Linewidth,
Stabilization,
and Tuning**

Summary

Lamb Dip Stabilization

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Doppler-broadened absorption cell inside laser cavity
- Loss is lowest when laser
- Iodine-stabilized HeNe lasers are common frequency standards

Tuning a Laser

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Tunable lasers useful for spectroscopy
- Use etalons, gratings, or other filters to increase loss at all wavelengths which are not desired
- Carefully designed so frequency tunes continuously and doesn't 'hop'
- Commonly used with broadband gain media, including: dye lasers, $\text{Ti:Al}_2\text{O}_3$, and external cavity diode lasers (ECDLs)

Grating Feedback Configurations

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

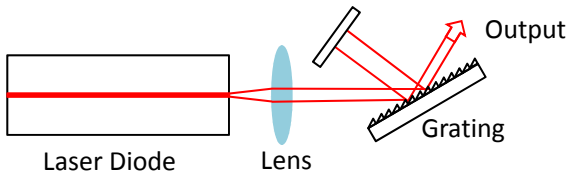
Output Power

Optimization
Efficiency

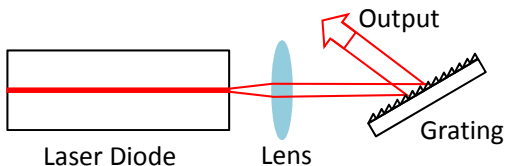
Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Littman-Metcalf



Littrow

Fabry-Perot Etalon

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Designed to have an FSR much larger than the gain bandwidth
May be used to both tune laser and narrow linewidth Can also
be used to stabilize absolute frequency, but are effected by
temperature changes

Lasing Off Line Center I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- Consider output power predicted by the low loss approximation

$$P_{out} = A \frac{T_{oc} T_b I_{sat}}{2} \left[\frac{\gamma_0}{\gamma_{th}} - 1 \right] \quad (38)$$

- Lasers can be forced to lase at a specific wavelength with an etalon, diffraction grating, or other filter. The output power would then be:

$$P_{out}(\lambda) = A \frac{T_{oc} T_b}{2} \frac{hc}{\lambda \sigma_{se}(\lambda) \tau} \left[\frac{\sigma_{se}(\lambda) \Delta N}{\gamma_{th}} - 1 \right] \quad (95)$$

$$= A \frac{T_{oc} T_b}{2} \frac{hc}{\lambda \tau} \left[\frac{\Delta N}{\gamma_{th}} - \frac{1}{\sigma_{se}(\lambda)} \right] \quad (96)$$

Lasing Off Line Center II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- In general the variation of σ_{se} is much faster than the variation of λ
- Even if the filter is perfectly lossless, output power will still drop significantly as the laser is tuned away from the gain center
- In macroscopic cavities, the FSR is generally much smaller than the linewidth, hence there will always be a mode close to line center.

In Equation 96, as $\sigma_{se} \rightarrow 0$, $P_{out} \rightarrow -\infty$. What does this mean?

Example: Lasing Off Line Center I

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

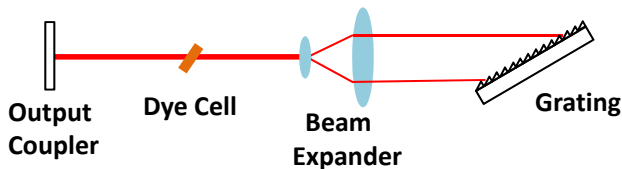
Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Problem: A prism-tuned dye laser is shown below, along an absorption and relative emission spectrum of the dye. Find the output power as a function of wavelength. Assume the tuning mechanism has a flat spectral response. Other necessary parameters are $L_g = 5$ mm, $R_{oc} = .9$, $R_{grating} = 0.68$, $n=1.3$, $\Delta N_0 = 10^{16}$ cm $^{-3}$, $r_{beam} = 100$ μ m, and $\tau_{21} = 4.08$ ns.



Example: Lasing Off Line Center II

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

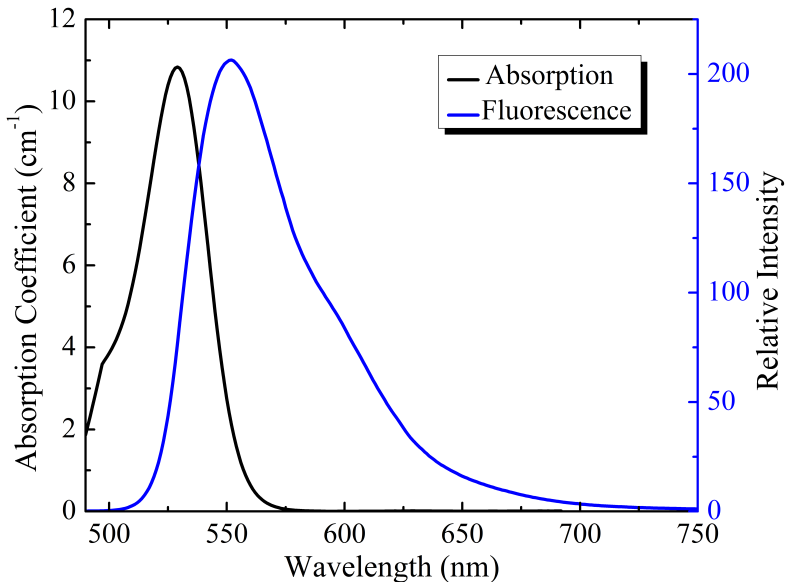
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



Example: Lasing Off Line Center III

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

Solution: The first step will be to calculate the gain spectrum. First, the lineshape function is derived from the fluorescence data:

$$g_{\lambda}(\lambda) = \frac{I_{em}(\lambda)}{\int I_{em}(\lambda) d\lambda} \quad (97)$$

which can be evaluated with a computer. The next step is convert the lineshape into a stimulated emission cross section

$$\sigma_{se}(\lambda) = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu) = \frac{1}{\tau_{21}} \frac{\lambda^4}{8\pi n^3 c} g_{\lambda}(\lambda) \quad (98)$$

where the formula on the right contains only known quantities. The saturation intensity is:

$$I_{sat}(\lambda) = \frac{hc}{\lambda \sigma_{se}(\lambda) \tau_{21}} \quad (99)$$

Example: Lasing Off Line Center IV

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

The small-signal gain of the system is:

$$\gamma_0(\lambda) = \sigma_{se}(\lambda)\Delta N \quad (100)$$

The threshold gain is:

$$\gamma_{th}(\lambda) = \alpha(\lambda) - \frac{1}{2L_g} \ln R_{oc} R_{grating} \quad (101)$$

Finally, the output power is:

$$P(\lambda) = AT_{oc} \frac{I_{sat}(\lambda)}{2} \left(\frac{\gamma_0(\lambda)}{\gamma_{th}(\lambda)} - 1 \right) \quad (102)$$

Note all of the quantities which are dependent on wavelength. A plot of the power vs wavelength is shown on the next slide. Compare with the fluorescence spectra on slide 90.

Example: Lasing Off Line Center

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

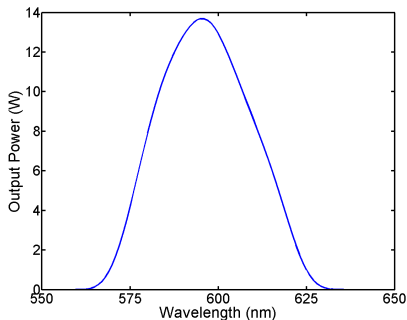
Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary



The Lessons:

- Compared to the fluorescence, the peak power has been shifted to the red (longer wavelengths) because of the absorption at shorter wavelengths
- Output power is a *nonlinear* function of gain.

Master Oscillator Power Amplifier

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- High-powered lasers require a larger gain medium and pump
- More difficult to stabilize large lasers rather than small ones
- Build a small laser which is highly stable, and then amplify it

Summary

ECE 455
Lecture 4

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary

- A population inversion increases the photon density of the optical field, but the photon density decreases the population inversion
- An ideal homogeneously broadened laser will oscillate at exactly one frequency
- Our output power equations are only approximate. We assumed uniform pumping as well as uniform saturation of the entire gain medium. Neither of these is a particularly good assumption.
- The linewidth in unstabilized lasers is usually limited by 'technical noise'

ECE 455
Lecture 4

[1]

Gain
Saturation

Optical
Amplifiers

Threshold

Output Power

Optimization
Efficiency

Mode
Selection

Linewidth,
Stabilization,
and Tuning

Summary