



# **Simulation and implementation of a Conformal Finite Difference Time Domain method**

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# English Abstract

Some differential equations in the literature present arduous work to find the associated solution, even in some cases, the solution to said systems turn out to be impossible to find through analytical methods. In this situation, numerical methods plays an important role since they allow us to solve the system of interest through discrete operations with a low numerical error involved.

Among the numerous existing techniques to solve electromagnetism problems, the Finite Difference in Time Domain method (FDTD) stands out, however, when we consider complicated geometries, it is necessary to refine the method in search of better efficiency, that is where we can introduce the Conformal Finite Difference in Time Domain method (CFDTD), which can be studied as the modification of the FDTD by introducing a Perfect Electric Conductor (PEC) volume into the geometry to consider.

In this work, a simulation and implementation of the CFDTD method is made in both one and two dimensions, in the last one, considering a line or an area of PEC that interrupts the spatial mesh worked. The codes worked out were prepared with test-oriented development in the python language, these can be found in the associated GitHub repository presented in annexes.

# Resumen en Español

Algunas ecuaciones diferenciales en la literatura presentan un trabajo arduo para encontrar la solución asociada, incluso en algunos casos, la solución a dicho sistema resulta ser imposible de encontrar a través de métodos analíticos. Ante esta situación los métodos numéricos juegan un papel importante ya que nos permiten resolver el sistema de interés a través de operaciones discretas con un bajo error numérico de por medio.

Entre las diversas técnicas existentes para poder resolver problemas de electromagnetismo destaca el método de diferencias finitas en el dominio del tiempo (FDTD por sus siglas en inglés), sin embargo, al momento de considerar geometrías complicadas, es necesario refinar el método en búsqueda de una mayor eficiencia, allí es donde se puede introducir la técnica conforme de diferencias finitas (CFDTD), la cual puede ser estudiada como la modificación de FDTD al introducir un volumen de conductor eléctrico perfecto (PEC) en la geometría a considerar.

En el presente trabajo se realiza una simulación e implementación del método CFDTD tanto en una como en dos dimensiones, en este último caso, considerando una línea o un área de PEC que interrumpen en el mallado. Los códigos trabajados fueron realizados con desarrollo orientado por tests en el lenguaje python, estos pueden encontrar en el repositorio de GitHub asociado presentado en anexos.

# 1 | Introduction

IDK what to write down here

## 2 | Maxwell's equations and the FDTD method

Let's first start by introducing the Maxwell's equation of electromagnetism and the basic notions of the FDTD algorithm in one and two dimensions in the free space case.

### 2.1 Introduction to the finite differences

We want to find a function that is the solution to a specific differential equation, however, this is a hard problem in general and only rarely can an analytic formula be found for the solution. A finite difference method proceeds by replacing the derivatives in the differential equation with finite differences approximations [1, 2]. For example, let's consider the Taylor approximation for  $f(x + h)$  and  $f(x - h)$

$$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \mathcal{O}(h^3) = f(x) + hf'(x) + \mathcal{O}(h^2), \quad (2.1)$$

$$f(x - h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) + \mathcal{O}(h^3) = f(x) - hf'(x) + \mathcal{O}(h^2), \quad (2.2)$$

in both equations it is possible to isolate the derivative, then we obtain:

$$f'(x) = \frac{f(x + h) - f(x - h)}{2h} + \mathcal{O}(h), \quad (2.3)$$

$$f'(x) = \frac{f(x) - f(x - h)}{h} + \mathcal{O}(h). \quad (2.4)$$

If we ignore the order  $h$  terms, we obtain the first order approximation for the derivative of the function with an error proportional to  $h$ . However, if we want to improve



and reduce the error to order  $h^2$ , it's necessary to introduce the central finite difference approximation. If we consider  $h = \Delta x/2$  and subtract the equations 2.1 and 2.2 we can obtain the central finite difference as it follows

$$f'(x) = \frac{f\left(x + \frac{\Delta x}{2}\right) - f\left(x - \frac{\Delta x}{2}\right)}{\Delta x} + \mathcal{O}(\Delta x^2), \quad (2.5)$$

we obtain then the approximation searched by ignoring the quadratic order terms. Since the error decreases faster in this case for smaller  $\Delta x$ , the equation will be more efficient to work with, for this reason, this approximation will be used for the discretization of the Maxwell's equations.

## 3 | References

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