Content

- **Geometry of image formation**
  - perspective camera model
  - projective geometry
  - intrinsic and extrinsic parameters

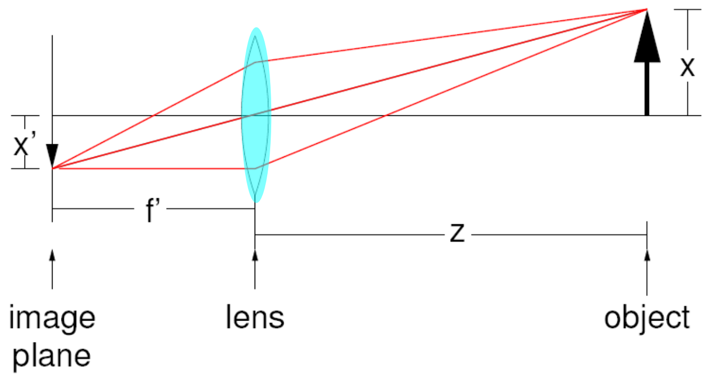
Geometric camera models

Given the coordinates of a point in the scene, what are the co-ordinates of this point in the image?

i.e. given P = (x,y,z) how do we calculate P'=(x',y',z')?

To answer, we need a mathematical model of the geometric projection implemented by the camera, often called simply a camera model.

Perspective (or pinhole) camera model



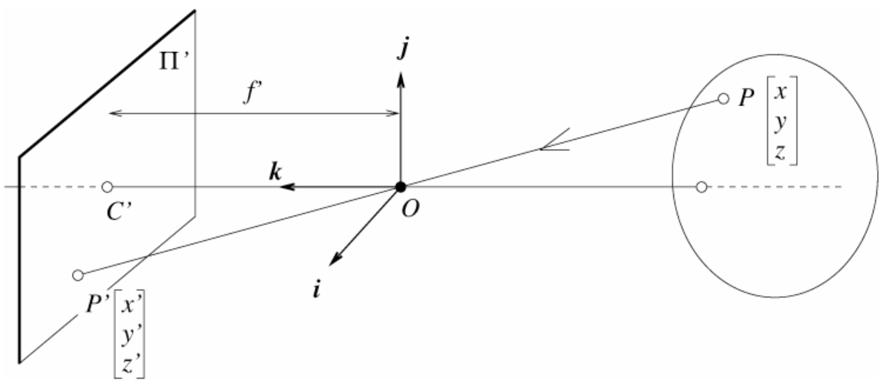
A lens follows the pinhole model for objects that are in focus.

The pinhole camera is therefore an acceptable mathematical approximation (i.e. a model) of image formation in a real camera.

For a pinhole camera everything is in focus regardless of image plane depth

Pinhole model arbitrarily assumes that image plane is at distance f '

Perspective camera model



P: a scene point with coordinates (x,y,z), P': its image with coordinates (x', y',z')

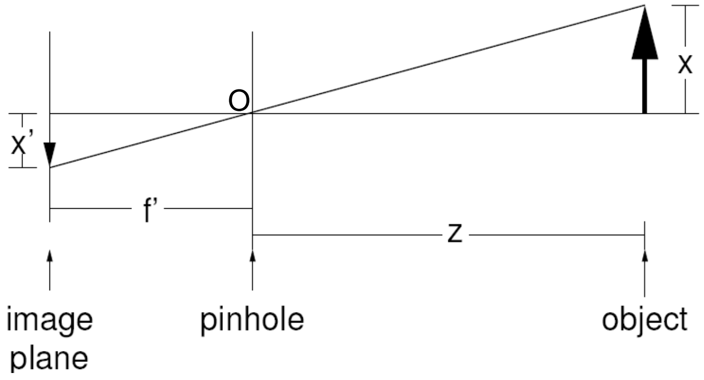
O: origin (pin hole / centre of lens)

Image plane Π' is located at a distance f' from the pinhole along the vector k

optical axis: line perpendicular to image plane and passing through O

C': image centre (intersection of optical axis and image plane)

Equation of projection (2D)



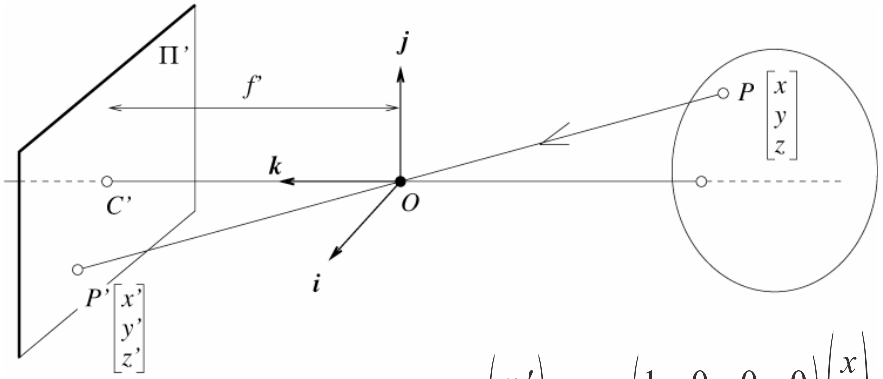
From similar triangles:

$$x'/x = f'/z$$

Hence:

$$x' = (f'/z)\, x$$

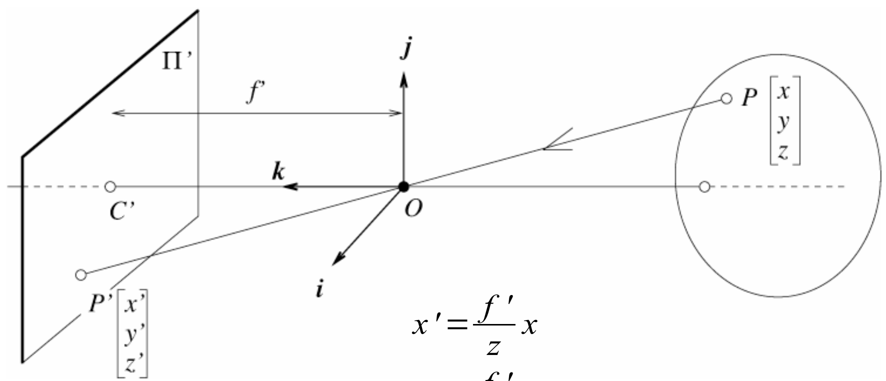
Equation of projection (3D)



We can re-write these equations in matrix form, it is a **convention** to use homogeneous coordinates:

$$\begin{pmatrix}x'\\y'\\z'\end{pmatrix} = \frac{f'}{z}\underbrace{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix}}_{\text{projection operator}}\begin{pmatrix}x\\y\\z\\1\end{pmatrix}$$

Equation of projection (3D)



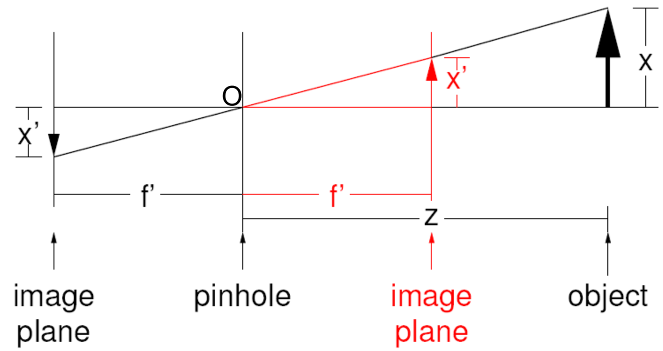
$$x' = \frac{f'}{z}x$$

Similarly, for y coordinates: $$y' = \frac{f'}{z}y$$

Since P' lies on image plane: $z' = f'$

All coordinates are relative to camera reference frame [mm] – i.e. axes rigidly attached to camera with origin at pinhole

Virtual image



A pinhole camera creates inverted images.

It is traditional to draw the image plane in front of the pinhole at the same distance from it as the actual image plane.

Resulting "virtual" image is identical to the real image except it is the right way up.

This does **not** change the mathematics of the perspective camera model.

Projective geometry

Euclidean geometry describes objects “as they are”.

It describes transformations within a 3D world (i.e. translations and rotations)

These mappings do not change the shape of an object (i.e. lengths, angles, and parallelism are preserved).

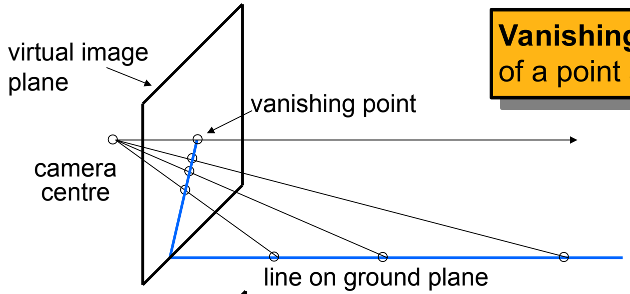
Projective geometry describes objects “as they appear”.

It describes the transformation from the 3D world to a 2D image (i.e. scaling and shear in addition to translations and rotations)

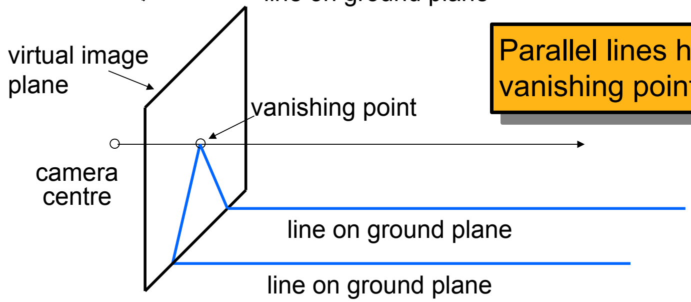
This mapping does distort the shape of an object (i.e. lengths, angles, and parallelism are not preserved).

Some properties of (i.e. distortions caused by) projective geometry are described on the following slides...

Vanishing points

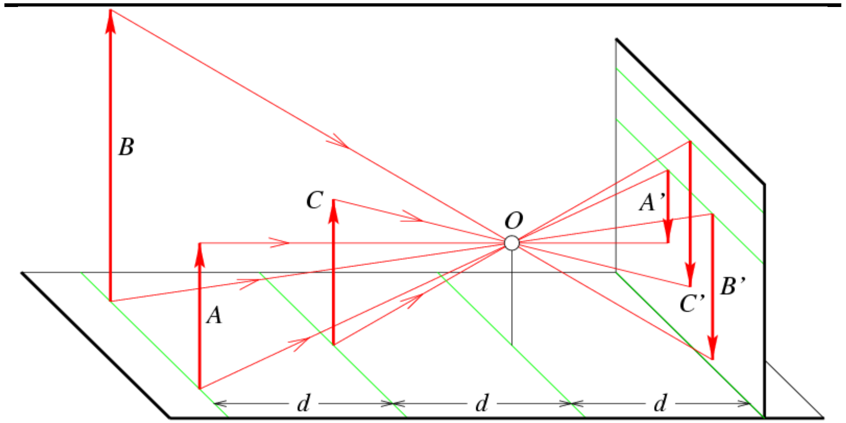


Vanishing point = projection of a point at infinity



Parallel lines have the same vanishing point

Distant objects appear smaller



The apparent size of an object depends on its distance

In world:

A and C have equal length, B twice this length

In image:

B’ and C’ have equal length, A' half this length

Example of projective distortion

Reality is distorted by projection:

In world:

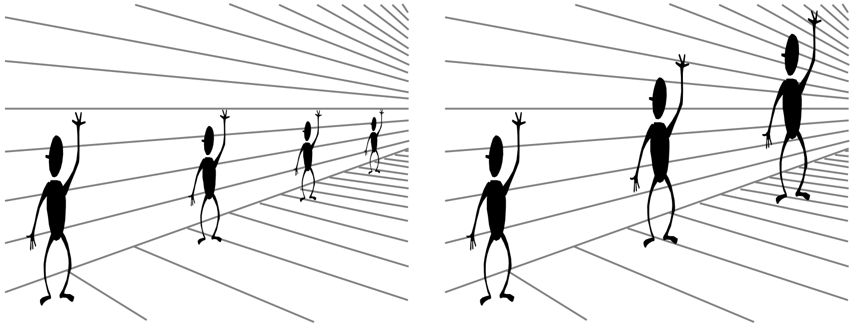
- rail tracks parallel
- ties equal in length
- ties perpendicular to tracks
- ties evenly spaced

In image:

- rail tracks converge at a vanishing point
- ties get shorter with distance
- ties not at right angles to track
- ties get closer together at longer distances.

Vanishing points provide cues to size

Our visual system is good at detecting vanishing points and using this to extract information about depth and object size. Can be used in computer vision too.



Vanishing points provide cues to size

Our visual system is good at detecting vanishing points and using this to extract information about depth and object size. Can be used in computer vision too.