Content

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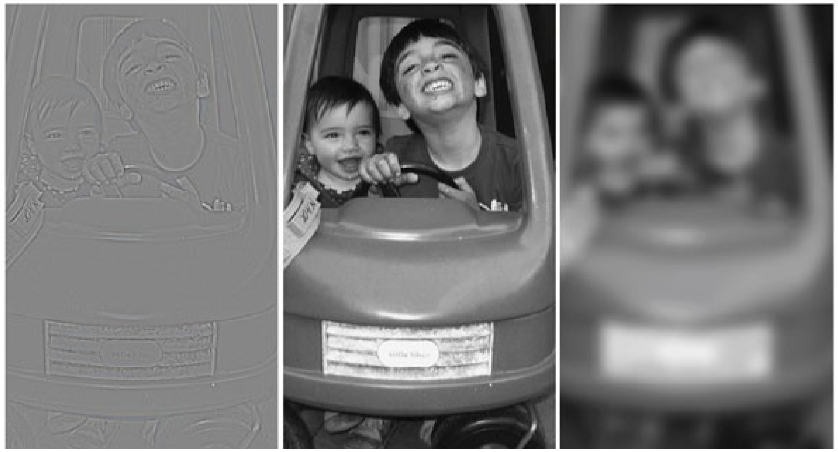
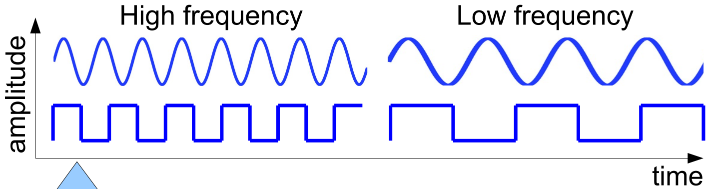
Spatial Frequency: example



High

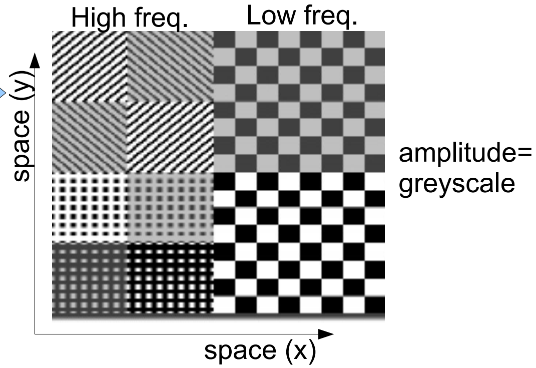
Low

Spatial Frequency



Frequency

- **in temporal domain**
- **in spatial domain**



amplitude=
greyscale

Small-scale texture is said to have a high spatial frequency.

Smoothing removes this, leaving low spatial frequencies.

Box mask

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Each pixel replaced by average of itself and its eight neighbours.

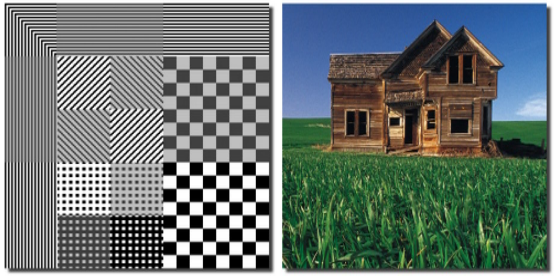
Generates a smoothed (blurred) image.

Useful.

Called “mean filter” or “box mask”.

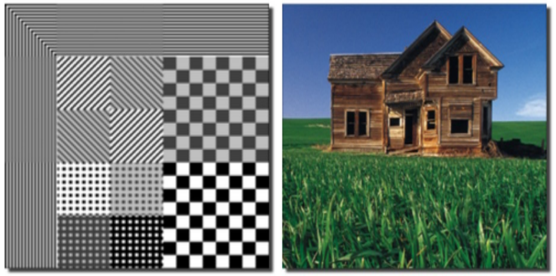
Smoothing mask example

Original Images



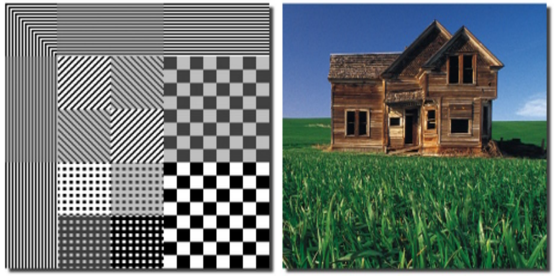
Images convolved with 3x3 box mask

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

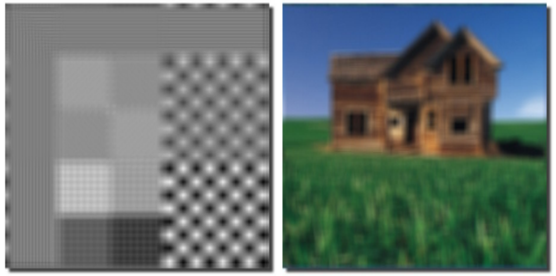


Smoothing mask example

Original Images

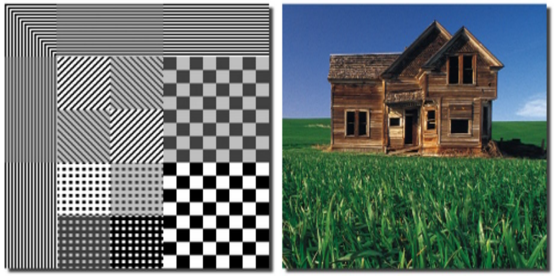


Images convolved with 17x17 box mask

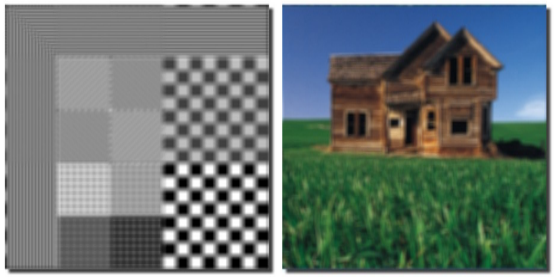


Smoothing mask example

Original Images



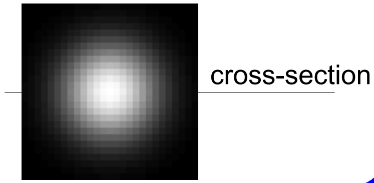
Images convolved with 9x9 box mask

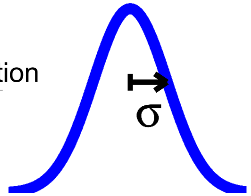


Gaussian mask

The box mask is not very good at smoothing as it has sharp edges.

The Gaussian mask is more effective as it falls off gently at the edges. It gives more weight to nearby pixels.





$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-(x^2+y^2)}{2\sigma^2}\right)$$

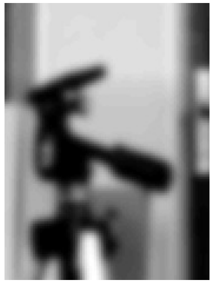
The Gaussian mask effectively removes any spatial frequencies with a period or wavelength smaller than the mask dimensions (defined by σ = standard deviation, or “**scale**”).

Smoothing with Gaussian mask

Original image

σ=1

σ=2



σ=4

“Scale” = the standard deviation of the Gaussian (i.e. σ)

as this parameter goes up:

- the size of the mask needs to increase (mask width should be ≥6σ)
- more pixels are involved in the average
- the image gets more blurred (more high frequencies suppressed)
- noise is more effectively suppressed

Separability of the Gaussian mask

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-(x^2+y^2)}{2\sigma^2}\right)$$

$$= \left\{\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right)\right\} \left\{\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-y^2}{2\sigma^2}\right)\right\}$$

i.e. 2D Gaussian is the product of two 1D Gaussians.
Each 1D Gaussian is a function of one variable only (either x or y)