**CSCI 5454 Spring 2023**: Make up assignment for exam # 2
**Due Date:** Tuesday May 2 (no extensions possible)

---

**P1.** Suppose we insert $n = 2^k - 1$ keys from $1, \ldots, n$ uniformly at random into a binary search tree. Let $P_k$ probability that the tree is *perfectly* balanced: i.e, all paths from root to leaf are of the same length. Derive a recurrence for $P_k$. Calculate $P_5$.

**P2.** Suppose there are $n$ delicious sweets in front of you that you wish to consume. Each morning you toss a coin for each sweet that is left and consume it if the coin turns up heads or save it for another day if the coin turns up tails.
**(A)** What is the probability that after $k$ days of this, there are still candies left.
Express your answer as a function of $n, k$ and simplify as much as possible.
**(B)** What is the expected time taken to consume all the candies?
**Hint:** For a positive integer valued random variable $X$ we have

$$E(X) = P(X \geq 1) + P(X \geq 2) + P(X \geq 3) + \cdots$$

Express your answer as a function of $n, k$ and simplify as much as possible.
**(C)** Use Bernoulli's inequality $(1-x)^r \geq 1 + rx$ for any real valued $x \geq -1$ and integer $r \geq 1$ to prove that the expected time to consume all candies is $\leq \log_2(n) + c$ for some constant $c$ for $n \geq 4$.