"Shape Grammars and the Generative Specification of Painting and Sculpture"
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Presented at IFIP Congress 71 in Ljubljana, Yugoslavia.  Selected as the Best Submitted Paper.
(IFIP stands for International Federation for Information Processing)

Published in the Proceedings: C V Freiman (ed.) *Information Processing 71* (Amsterdam: North-Holland, 1972) 1460-1465.

Republished in O R Petrocelli (ed.) *The Best Computer Papers of 1971* (Auerbach, Philadelphia, 1972) 125-135.  This is the version reproduced below.

# 6
# Shape Grammars and the Generative Specification of Painting and Sculpture

## by *George Stiny and James Gips*

A method of shape generation using shape grammars which take shape as primi-
tive and have shape specific rules is presented. A formalism for the complete,
generative specification of a class of non-representational, geometric paintings
or sculptures is defined, which has shape grammars as its primary structural com-
ponent. Paintings are material representations of two-dimensional shapes gener-
ated by shape grammars, sculptures of three-dimensional shapes. Implications
for aesthetics and design theory in the visual arts are discussed. Aesthetics
is considered in terms of specificational simplicity and visual complexity. In
design based on generative specifications, the artist chooses structural and
material relationships and then determines algorithmically the resulting art
objects.

We present a formalism for the complete specification of families of non-
representational, geometric paintings and sculptures. Formally defining the speci-
fication of an art object independently of the object itself provides a framework
in which theories of design and aesthetics can be developed. The specifications
introduced are algorithmic and made in terms of recursive schemata having shape
grammars as their basic formal component. This represents a departure from
previous mathematical approaches to the visual arts [1], [2] which have been in-
formal rather than effective and, except for Focillon [3], paradigmatic rather
than generative. The painting and sculpture discussed are material representa-
tions of shapes generated by shape grammars. Our underlying aim is to use
formal, generative techniques to produce good art objects and to develop under-
standing of what makes good art objects.

The class of paintings shown in Figure 6-1 is used as an explanatory example.
Over fifty classes of paintings and sculptures have been defined using generative
specifications and produced using traditional artistic techniques.

## PAINTING

Informally, the specification of painting consists of the definition of a lan-
guage of two-dimensional shapes, the selection of a shape in that language for

painting, the specification of a schema for painting the areas contained in the shape, and the determination of the location and scale of the shape on a canvas of given size and shape.
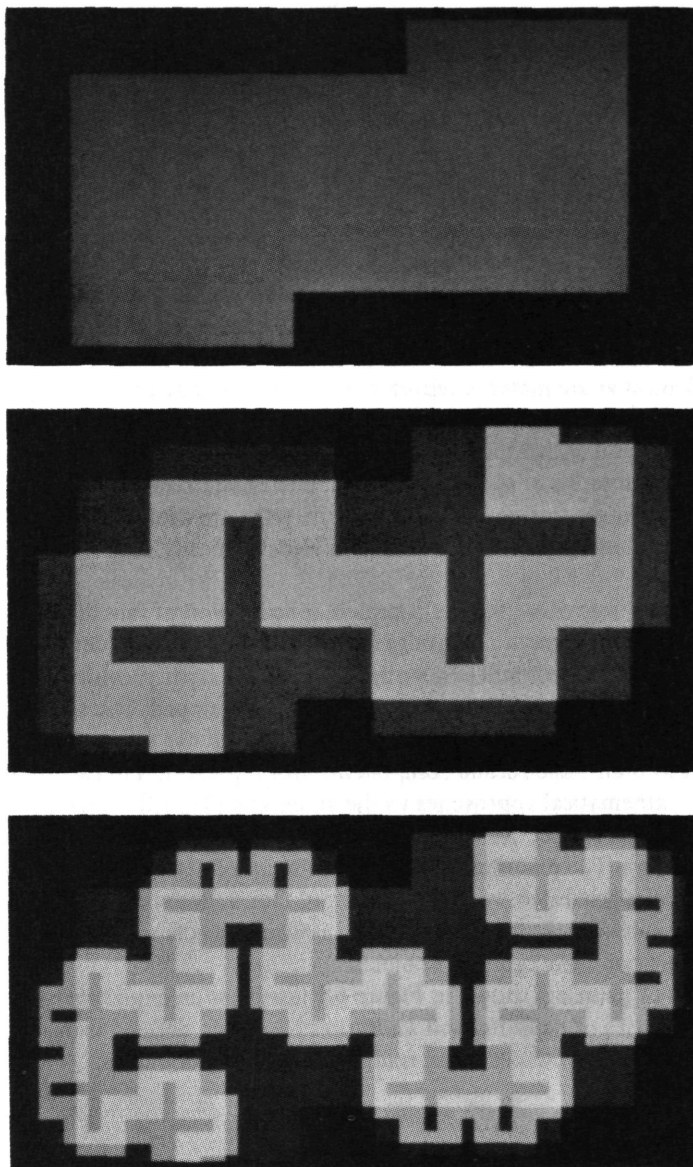


**Figure 6-1.** *Urform I, II, and III* (Stiny, 1970. Acrylics on canvas, each canvas 30 ins. x 57 ins.) Colors are: darkest—blue, second darkest—red, second lightest—orange, lightest—yellow.

A *class of paintings* is defined by the double (S,M). S is a specification of a class of shapes and consists of a shape grammar, defining a language of two-dimensional shapes, and a selection rule. M is a specification of material representations for the shapes defined by S and consists of a finite list of painting rules and a canvas shape (limiting shape). Figure 6-2 shows the complete, generative specification of the class of paintings shown in Figure 6-1.
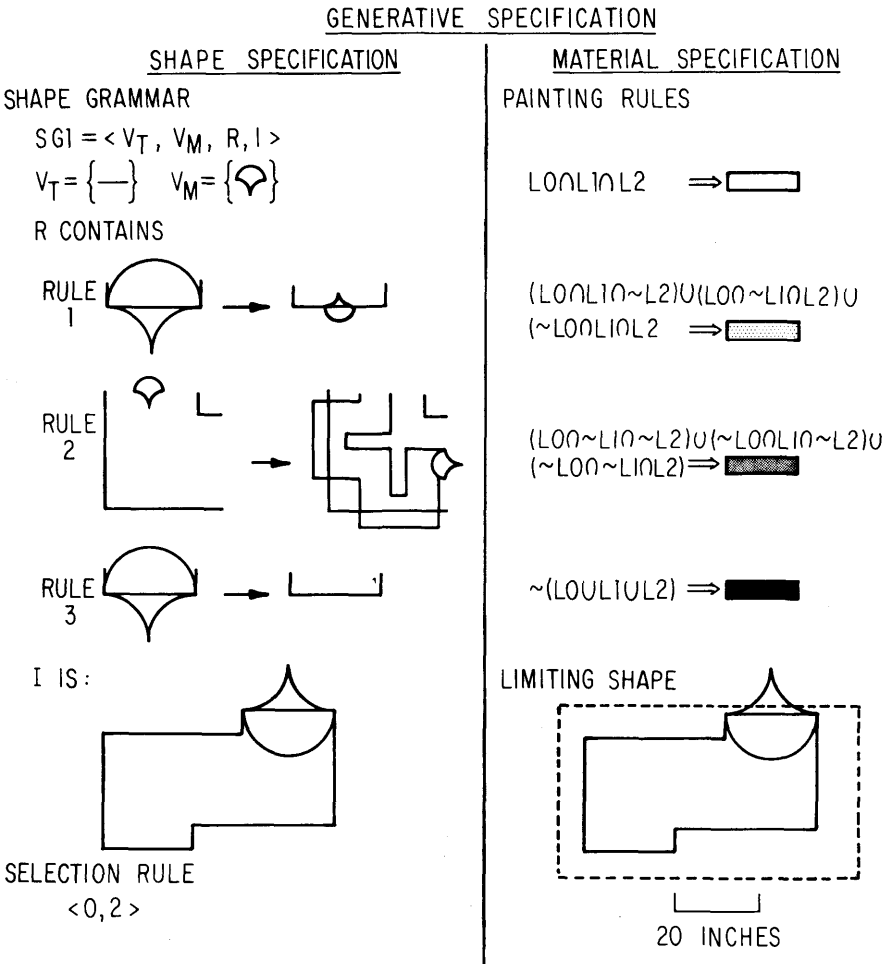
GENERATIVE SPECIFICATION

SHAPE SPECIFICATION | MATERIAL SPECIFICATION

SHAPE GRAMMAR

$SG1 = < V_T, V_M, R, I >$

$V_T = \{ — \}$   $V_M = \{ \heartsuit \}$

R CONTAINS

PAINTING RULES

RULE 1

$L \cap L \cap L2 \implies \boxed{\qquad}$

RULE 2

$(L \cap L \cap \sim L2) \cup (L \cap \sim L \cap L2) \cup$
$(\sim L \cap L \cap L2 \implies \boxed{\vdots\vdots}$

RULE 3

$(L \cap \sim L \cap \sim L2) \cup (\sim L \cap L \cap \sim L2) \cup$
$(\sim L \cap \sim L \cap L2) \implies \boxed{\blacksquare}$

$\sim (L \cup L \cup L2) \implies \blacksquare$

I IS:

SELECTION RULE
$<0, 2>$

LIMITING SHAPE

20 INCHES

**Figure 6-2.** Complete, generative specification of the class of paintings containing *Urform I, II, and III.*

## Shape Grammars

Shape grammars are similar to phrase structure grammars, which were introduced by Chomsky [4] in linguistics. Where phrase structure grammars are

defined over an alphabet of symbols and generate one-dimensional strings of symbols, shape grammars are defined over an alphabet of shapes and generate $n$-dimensional shapes. The definition of shape grammars follows the standard definition of phrase structure grammars [5].

*Definition.* A *shape grammar* (SG) is a 4-tuple: SG = $(V_T, V_M, R, I)$ where

1. $V_T$ is a finite set of shapes.
2. $V_M$ is a finite set of shapes such that $V_T^* \cap V_M = \phi$.
3. R is a finite set of ordered pairs (u,v) such that $u$ is a shape consisting of an element of $V_T^*$ combined with an element of $V_M$ and $v$ is a shape consisting of (A) the element of $V_T^*$ contained in $u$ or (B) the element of $V_T^*$ contained in $u$ combined with an element of $V_M$ or (C) the element of $V_T^*$ contained in $u$ combined with an additional element of $V_T^*$ and an element of $V_M$.
4. I is a shape consisting of elements of $V_T^*$ and $V_M$.

Elements of the set $V_T^*$ are formed by the finite arrangement of an element or elements of $V_T$ in which any element of $V_T$ may be used a multiple number of times with any scale or orientation. Elements of $V_T^*$ appearing in some $(u,v)$ of R or in I are called terminal shape elements (or *terminals*). Elements of $V_M$ are called non-terminal shape elements (or *markers*). Elements $(u,v)$ of R are called *shape rules* and are written u $\rightarrow$ v. I is called the *initial shape* and normally contains a $u$ such that there is a $(u,v)$ which is an element of R.

A shape is generated from a shape grammar by beginning with the initial shape and recursively applying the shape rules. The result of applying a shape rule to a given shape is another shape consisting of the given shape with the right side of the rule substituted in the shape for an occurrence of the left side of the rule. Rule application to a shape proceeds as follows: (1) find part of the shape that is geometrically similar to the left side of a rule in terms of both non-terminal and terminal elements; (2) find the geometric transformations (scale, translation, rotation, mirror image) which make the left side of the rule identical to the corresponding part in the shape; and (3) apply those transformations to the right side of the rule and substitute the right side of the rule for the corresponding part of the shape. Because the terminal element in the left side of a shape rule is present identically in the right side of the rule, once a terminal is added to a shape it cannot be erased. The generation process is terminated when no rule in the grammar can be applied.

The *language* defined by a shape grammar (L(SG)) is the set of shapes generated by the grammar that do not contain any elements of $V_M$. The language of a shape grammar is a potentially infinite set of finite shapes.

*Example.* In SG1, shown in Figure 6-2, $V_T$ contains a straight line; terminals consist of finite arrangements of straight lines. $V_M$ consists of a single element.

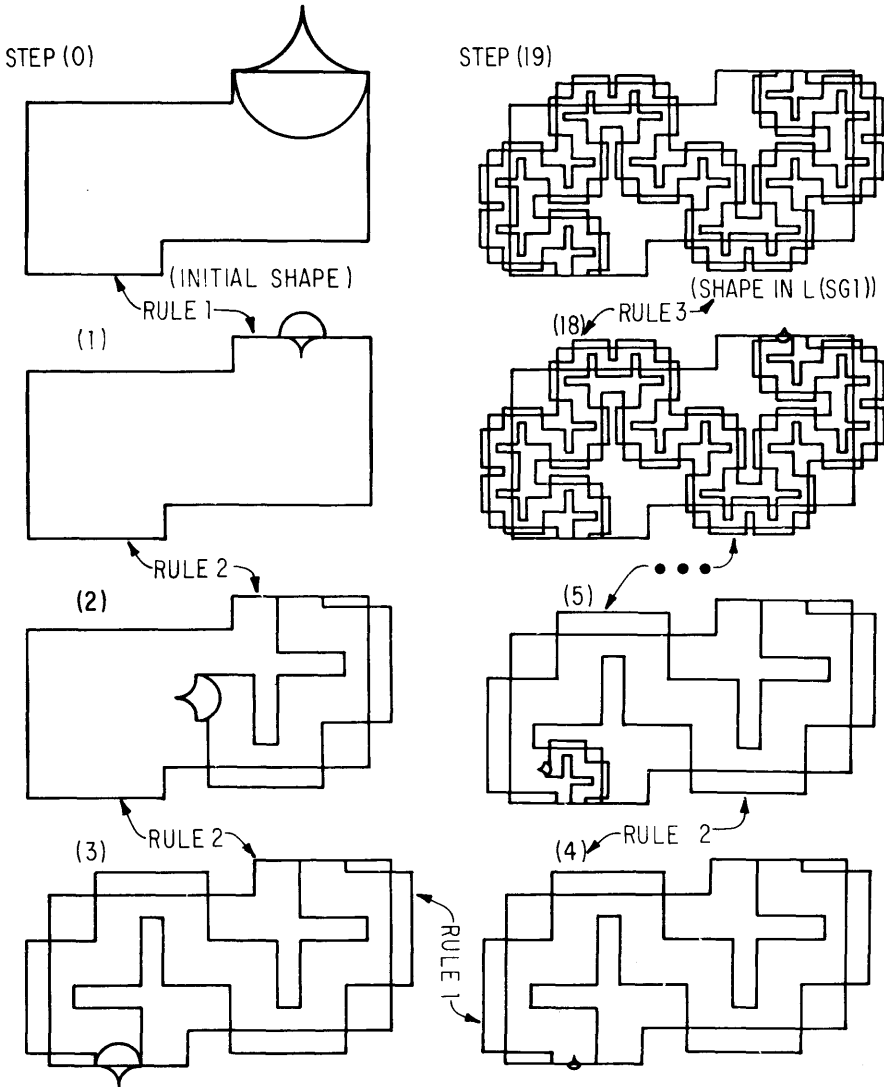R contains three rules—one of each type allowed by the definition. The initial shape contains one marker.



**Figure 6-3.** Generation of a shape using SG 1.

The generation of a shape in the language, L(SG1), defined by SG1 is shown in Figure 6-3. Step 0 shows the initial shape. Recall that a rule can be applied to a shape only if its left side can be made identical to some part of the shape, with respect to both marker and terminal. Either rule 1 or rule 3 is applicable to the

shapes indicated in steps 0, 3, and 18. Application of rule 3 results in the removal of the marker, the termination of the generation process (as no rules are now applicable), and a shape in L(SG1). Application of rule 1 reverses the direction of the marker, reduces it in size by one-third, and forces the continuation of the generation process. Markers restrict rule application to a specific part of the shape and indicate the relationship in scale between the rule applied and the shape to which it is applied. Rule 2 is the only rule applicable to the shape indicated in steps 1, 2, and 4-17. Application of rule 2 adds a terminal to the shape, advances the marker, and forces the continuation of the generation process. Shape generation using SG1 may be regarded in this way: the initial shape contains two connected "⌐L" 's, and additional shapes are formed by the recursive placement of seven smaller "⌐L" 's on each "⌐L" such that all "⌐L" 's of the same size are connected. Notice that the shape produced in this way can be expanded outward indefinitely but is contained within a finite area. The language defined by SG1 is shown in Figure 6-4.
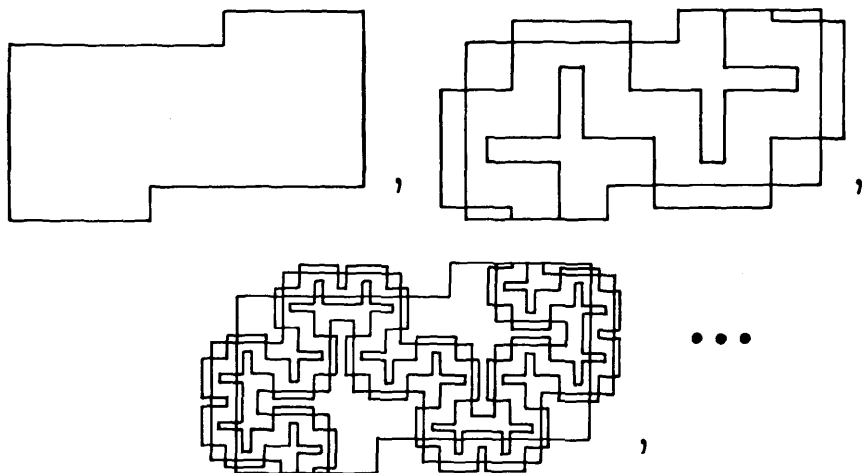


**Figure 6-4.** The language defined by SG1, L(SG1).

*Discussion.* SG1 defines a language containing rectilinear shapes of two dimensions. Grammars can be written to define languages containing shapes with demensions greater than two and can define curved as well as rectilinear shapes.

   In shape grammars, shape is assumed to be primitive, that is, definitions are made ultimately in terms of shape. These grammars use rules that are shape rather than property specific. The definition of shape grammars allows rules of three types. Where rule type B is logically redundant in the system, it was included because it was found useful in defining painting and sculpture formalisms. Different rule types consistent with the idea of shape grammars are possible and can define classes of grammars analogous to the different classes of phrase structure grammars [5].

Where we use shape grammars exclusively to generate shapes for painting and sculpture, they can also be used to simulate Turing machines and to generate musical scores, structural descriptions of chemical compounds, and the sentences—and their tree structures—in languages defined by phrase structure grammars. Grammar-grammars, where the sentences generated are themselves shape grammars, are possible. While no parsing algorithms have been developed, shape grammars seem applicable to the analysis, as well as the generation, of shapes.

**Selection Rules**

Painting requires a small class of shapes, which are not beyond its techniques for representation. Because a shape grammar can define a language containing a potentially infinite number of shapes ranging from the simple to the very (infinitely) complex, a mechanism (selection rule) is required to select shapes in the language for painting. The concept of level provides the basis for this mechanism and also for the painting rules discussed in the next section.

The *level* of a terminal in a shape is analogous to the depth of a constituent in a sentence defined by a context freephrase structure grammar. Level assignments are made to terminals during the generation of a shape using these rules:

1. The terminals in the initial shape are assigned level 0.
2. If a shape rule is applied, and the highest level assigned to any part of the terminal corresponding to the left side of the rule is N, then
   (a) If the rule is of type A, any part of the terminal enclosed by the marker in the left side of the rule is assigned N.
   (b) If the rule is of type B, any part of the terminal enclosed by the marker in the left side of the rule is assigned N and any part of the terminal enclosed by the marker in the right side of the rule is assigned N + 1.
   (c) If the rule is of type C, the terminal added is assigned N + 1.
3. No other level assignments are made.

Parts of terminals may be assigned multiple levels. The marker must be a closed shape in order for rules 2a and 2b to apply. Rules 1 and 2c are central to level assignment; rules 2a and 2b are necessary for boundary conditions. The terminals belonging to each of the three levels defined by level assignment in the example are shown individually in Figure 6-5.

A *selection rule* is a double $(m,n)$ where $m$ and $n$ are integers. $m$ is the minimum level required and $n$ is the maximum level allowed in a shape generated by a shape grammar for it to be a member of the class defined by S. Because the terminals added to a shape during the generation process cannot be erased and level assignments are permanent, the selection rule is used as a halting algorithm for shape generation. Where a single painting is to be considered uniquely, as is traditional, the class can be defined to contain only one element.

Where several paintings are to be considered serially or together to show the repeated use or expansion of a motif, as has become popular [6], the class can be defined to contain multiple elements.
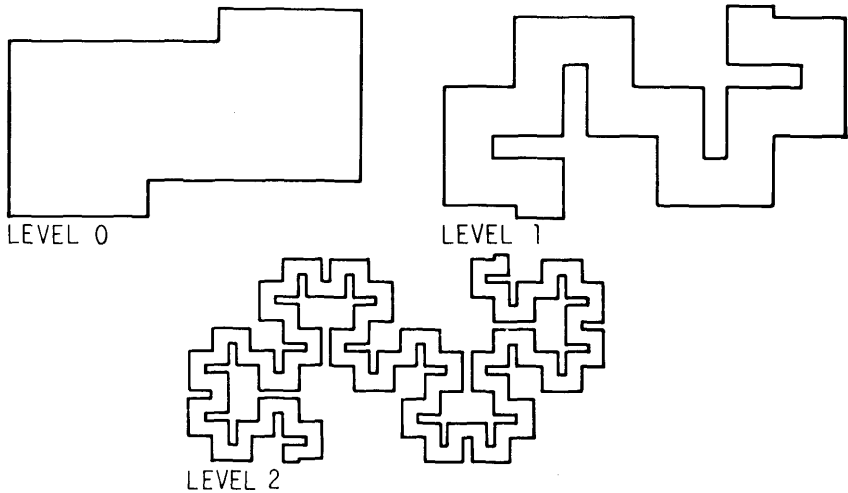


LEVEL 0                                   LEVEL 1

LEVEL 2

**Figure 6-5.** The terminals that form the boundaries of the first three levels of shapes generated by SG1.

The class of shapes containing just the three shapes in Figure 6-4 is specified by the double (SG1,(0,2)). The minimum level required is 0 (all shapes in L(SG1) satisfy this requirement) and the maximum level allowed is 2 (only three shapes in L(SG1) satisfy this requirement). (SG1,(2,2)) specifies the class containing only the most complex shape in Figure 6-4.

**Painting Rules**

Painting rules define a schema for painting the areas contained in a shape. Structurally equivalent areas can be painted identically by specifying these areas in terms of the level assignments to the terminals which form their boundaries.

Painting rules indicate how the areas contained in a shape are painted by considering the shape as a Venn diagram as in naive set theory. The terminals of each level in a shape are taken as the outline of a set in the Venn diagram. As parts of terminals may be assigned multiple levels, sets may have common boundaries. Levels 0, 1, 2, ... $n$ are said to define sets L0, L1, L2, ... Ln respectively, where $n$ is given in the selection rule.

A *painting rule* has two sides separated by a double arrow ($\Rightarrow$). The left side of a painting rule defines a set using the sets determined by level assignment and the usual set operators, for example, union ($\cup$), intersection ($\cap$), complementa-

tion ($\sim$), and exclusive or ($\otimes$). The sets defined by the left sides of the paint-
ing rules of M must partition the universal set. The right side of a painting rule
is a rectangle painted in the manner the set defined by the left side of the rule
is to be painted. The rectangle gives implicitly medium, color, texture, edge
definition, etc. Because the left sides of painting rules form a partition, every
area of the shape is painted in exactly one way. Any level in a shape may be
ignored by excluding the corresponding set from the left sides of the rules.

Using the set notation, all possible overlap configurations in a shape can be
specified independently of their shape. The effect of the painting rules in the
example is to count set overlaps. Areas with three overlaps are painted lightest,
two overlaps second lightest, one overlap second darkest, and zero overlaps
darkest.

### The Limiting Shape

The *limiting shape* defines the size and shape of the canvas on which a
shape is painted. Traditionally the limiting shape is a single rectangle, but this
need not be the case. For example, the limiting shape can be the same as the
outline of the shape painted or it can be divided into several parts. The limiting
shape is designated by broken lines, and its size is indicated by an explicit
notation of scale. The initial shape of the shape grammar in the same scale is
located with respect to the limiting shape. The initial shape need not be located
within the limiting shape. Informally, the limiting shape acts as a camera view-
finder. The limiting shape determines what part of the painted shape is repre-
sented on a canvas and in what scale.

## SCULPTURE

Sculpture is the material representation of three-dimensional shapes and is
defined analogously to painting. A *class of sculptures* is defined by the double
(S,M). S is a specification of a class of shapes and consists of a shape grammar,
defining a language of three-dimensional shapes and a selection rule. M is a
specification of material representations and consists of a finite list of sculpting
rules and a limiting shape. Sculpting rules take the same form as painting rules
with medium, surface, edge, etc., given implicitly in a rectangular solid. The
limiting shape is three-dimensional.

## AESTHETICS

Generative specifications of painting and sculpture have wide implications in
aesthetic theory, a theory that regards the art object as a coherent, structured
whole. In this context, aesthetics proceeds by the analysis of that whole into
its determinate parts toward a definition of the relationship of part to part and
part to whole in terms of "unified variety" [7], "order" and "complexity" [8],

[9], "a series of planned harmonies", "an internal organizing logic", "the play of hidden rules" [3], etc. The relationship between the wealth of visual information presented in an art object and the parsimony of structural and material information required to determine that object seems central to this aesthetics. Wealth of visual information may be associated with "variety" and "complexity" and is taken to mean *visual complexity*. Parsimony of structural and material information may be associated with "order" and "an internal organizing logic" and is taken to mean *specificational simplicity*. Visual complexity and specificational simplicity have been studied independently in other contexts [10], [11]. With a generative specification of art objects, investigations such as these can be used as the starting point for the development of a formal, mathematical aesthetics. We believe that painting and sculpture that have a high visual complexity which does not totally obscure an underlying specificational simplicity make for good art objects. The use of the words "beautiful" and "elegant" to describe computer programs, mathematical theorems, or physical laws is in the spirit of this aesthetics—parsimonious specification supporting complex phenomena.

## DESIGN

The formalism defined for the specification of painting and sculpture gives a complete description of a class of paintings or sculptures which is independent of the members of the class and is made in terms of a generative schema. For design theory in the visual arts, this means that the definition and solution of design problems can be based on the specification of an art object instead of the object itself. Generative specifications provide a well-defined means of expressing the artist's decisions about shapes and their organization and representation, in the design of non-representational, geometric art. Once the decisions are made as to the relationships that are to underly a class of paintings or sculptures, a generative specification is defined and the structural and material consequences of the relationships are determined algorithmically. This enables the artist to obtain art objects with specificational simplicity and visual complexity which are faithful to these relationships and which would be difficult to design by other means.

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