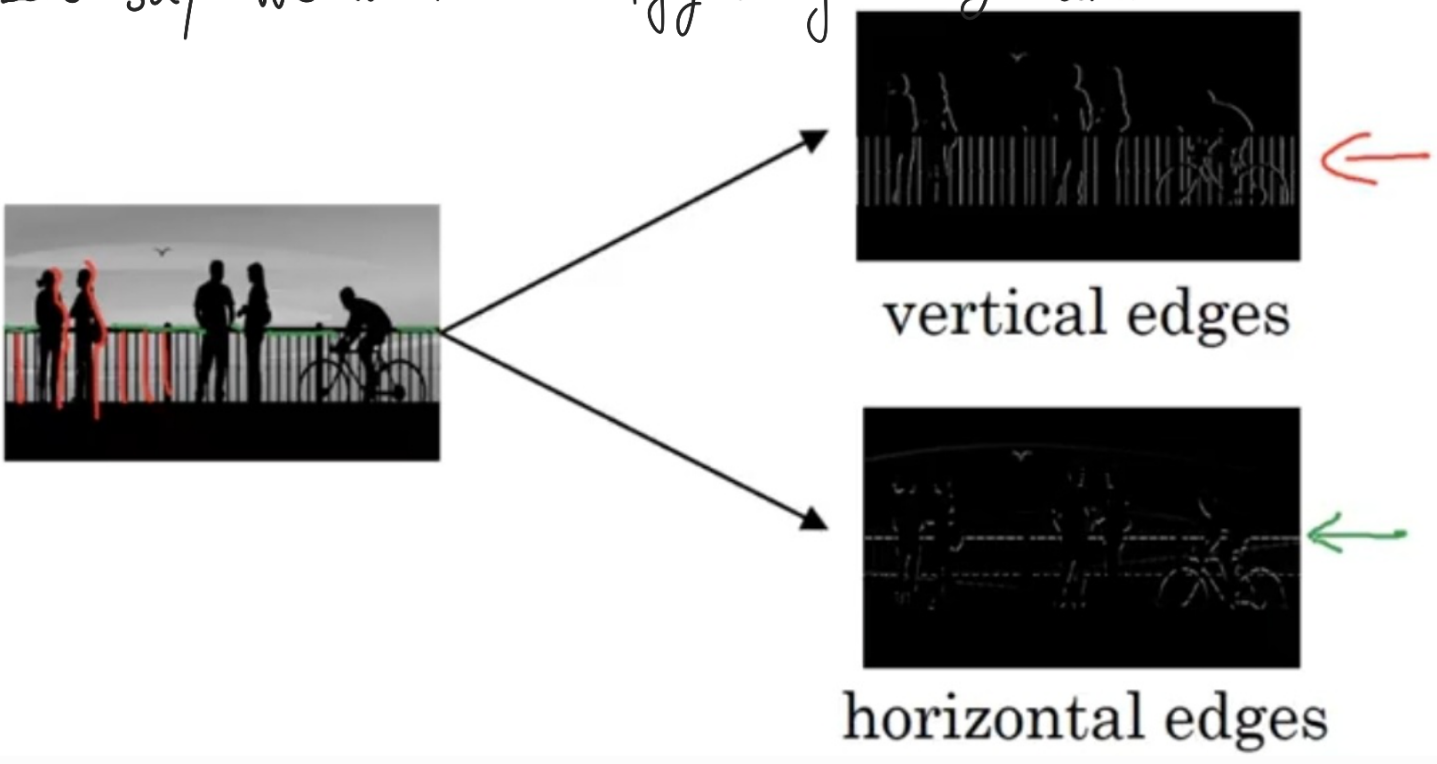
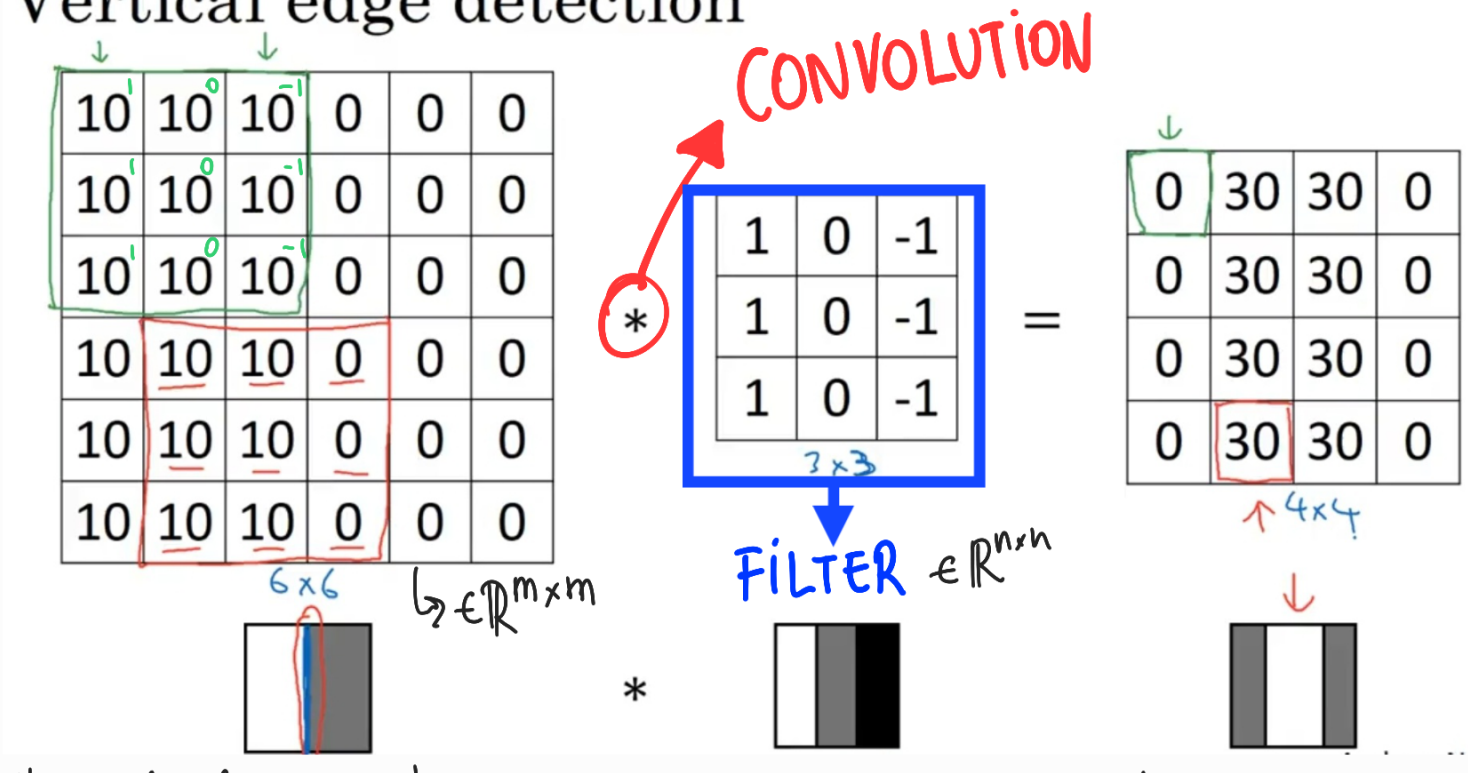
Let's say we want to identify the following 'lines':



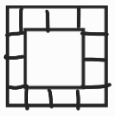
Vertical edge detection



We multiply each possible $n \times n$ square of the image by the filter, obtaining then, a matrix $\in \mathbb{R}^{(m-n+1)\times(m-n+1)}$

PADDING

Add extra cells to all the borders, with value = 0.

Ex: Padding = 1 ⇒ 

3.1 - Zero-Padding

Zero-padding adds zeros around the border of an image:

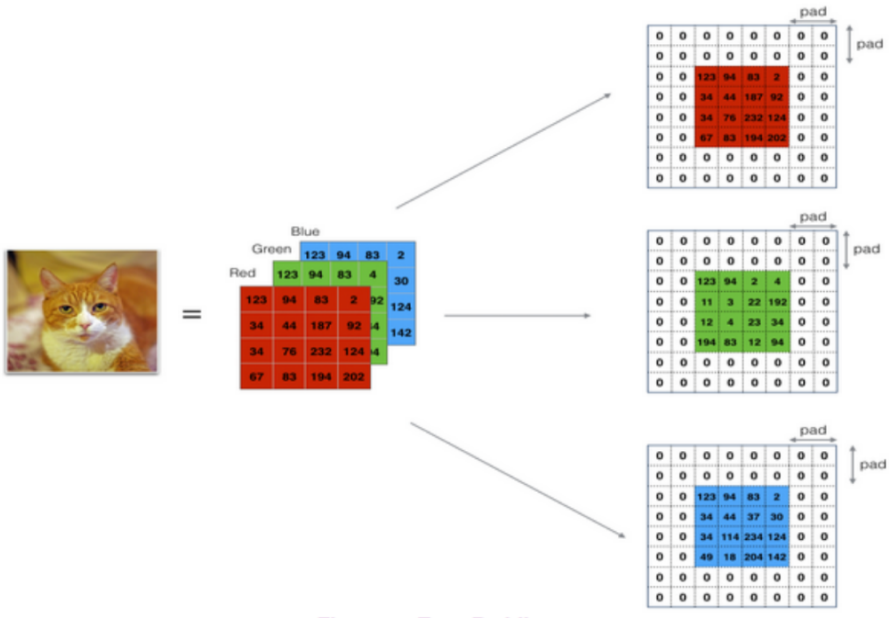


Figure 1 : **Zero-Padding**
Image (3 channels, RGB) with a padding of 2.

The main benefits of padding are:

- It allows you to use a CONV layer without necessarily shrinking the height and width of the volumes. This is important for building deeper networks, since otherwise the height/width would shrink as you go to deeper layers. An important special case is the "same" convolution, in which the height/width is exactly preserved after one layer.
- It helps us keep more of the information at the border of an image. Without padding, very few values at the next layer would be affected by pixels at the edges of an image.

Why convolutions?

Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

Sparsity of connections: In each layer, each output value depends only on a small number of inputs.

Valid and Same convolutions

↳ no padding

"Valid": $n \times n$ $*$ $f \times f$ $\rightarrow$ $n-f+1 \times n-f+1$

6×6 $*$ 3×3 $\rightarrow$ 4×4

"Same": Pad so that output size is the same as the input size.

$n+2p-f+1 \times n+2p-f+1$

$n+2p-f+1 = n \Rightarrow p = \frac{f-1}{2}$

STRIDE CONVOLUTIONS | Moving filter s cells

Strided convolution

2	3	7 ^{3}	4 ^{4}	6 ^{4}	2	9
6	6	9 ^{1}	8 ^{0}	7 ^{2}	4	3
3	4	8 $^{-1}$	3 ^{0}	8 ^{3}	9	7
7	8	3	6	6	3	4
4	2	1	8	3	4	6
3	2	4	1	9	8	3
0	1	3	9	2	1	4

7×7

$*$

3	4	4
1	0	2
-1	0	3

3×3

$=$

91		

stride = 2

Summary of convolutions

$n \times n$ image $\quad f \times f$ filter

padding p $\quad$ stride s

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Multiple filters