uc3m

Final Project – Mortality Rates Prediction

Time Series Analysis

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Index

Acronyms

ARIMA – Autoregressive integrated moving average

ACF – Autocorrelation Factors

PACF – Partial Autocorrelation Factors

VaR – Value at risk

TVaR – Tail value at risk

BP test – Breusch Pagan test

LB test – Ljung-Box test

ADF test – Augmented Dickey Fuller test

KPSS test –Kwiatkowski–Phillips–Schmidt–Shin test

AICc – Akaike Information Criterion

BIC – Bayesian Information Criterion

B test – Bartlett test

1. Overview

In this Project, an insurance company risk for the next year is going to be calculated by the estimation of the mortality rates of the policyholders. The objective of the model is to properly evaluate the premium risk (the risk of having more claims than expected) so the economic capital can be estimated.

The insurance company to be modelled is composed by the following number of policies for each age:

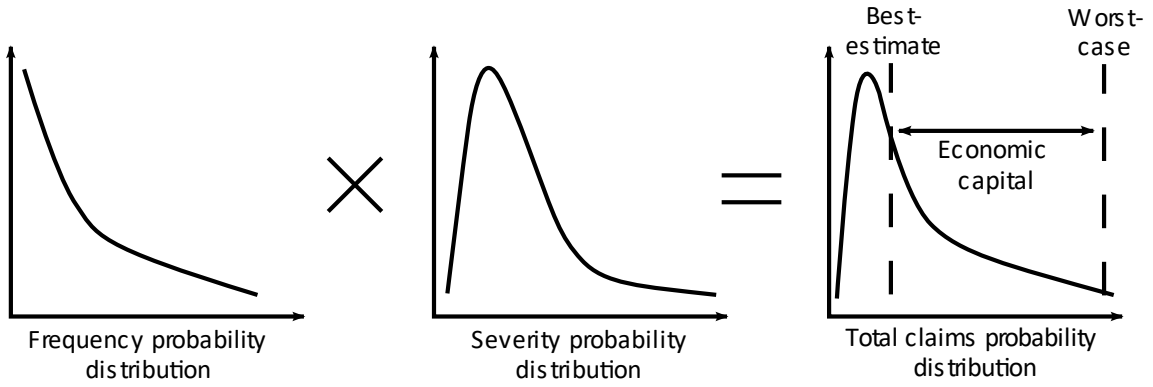
Age	Policies
67	902
68	659
69	1471
70	978
71	675
72	850
73	882
74	1035
75	995

The historical mortality rates of the policyholders age is available since 1935 up to 2021.

The project has four main parts:

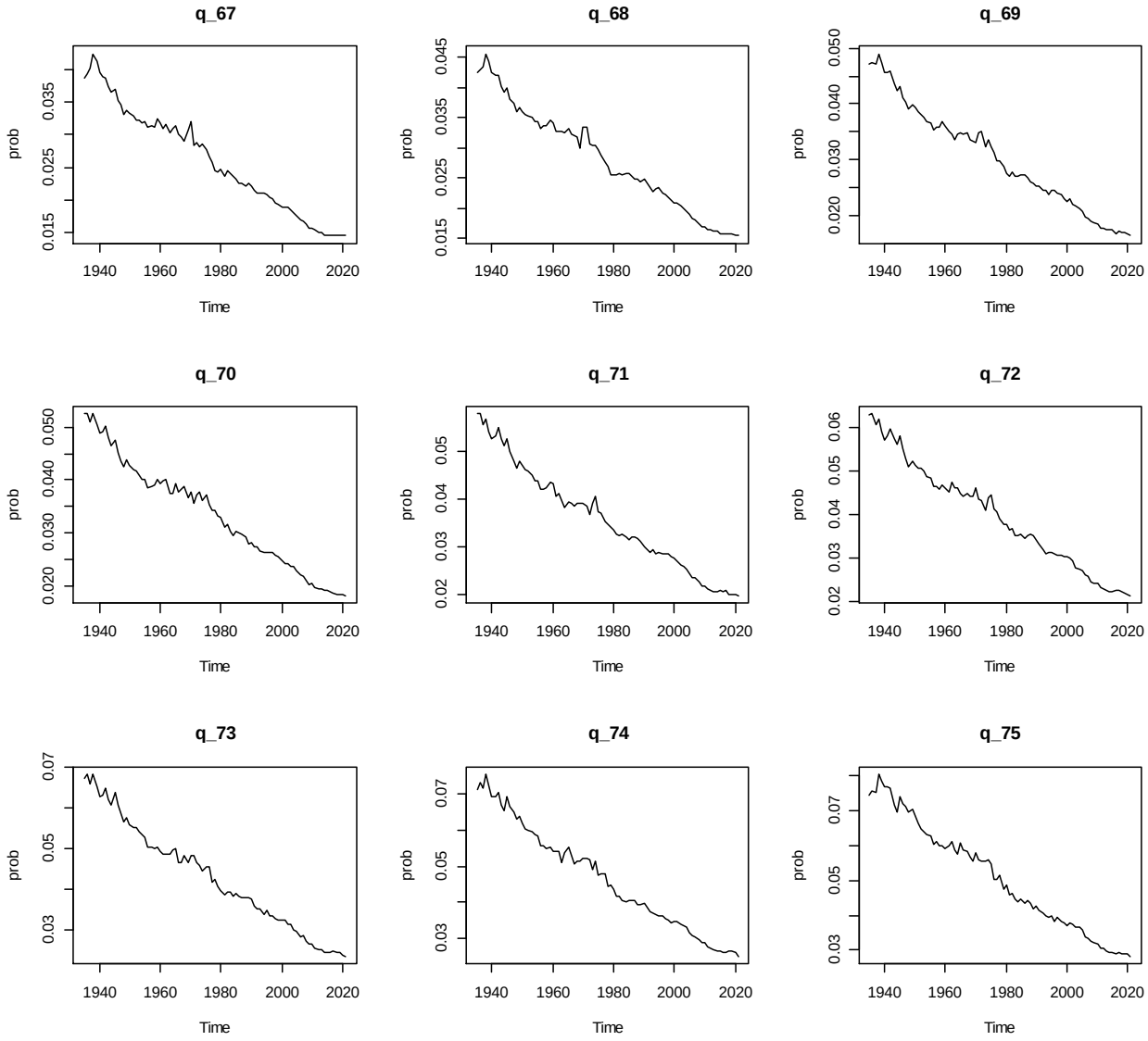
1. Mortality rates time series ARIMA functional form estimation by Box-Jenkins procedure
2. Estimation and diagnosis of the ARIMA model
3. One year period mortality rate prediction
4. Cost estimation and economic capital calculation

With this model, the expected cost will be evaluated together with the VaR_{99} (maximum expected cost in the 99% of the cases, or which is the same, maximum expected cost in 1 out 100 years) and $TVaR_{99}$ (expected cost if the cost surpasses the VaR_{99}). This way the worst-case scenarios will be quantified, and the company solvency capital requirement will be determined.



2. Available data

The following mortality time series are available for which we need to predict the 2022 value.



3. Functional form identification by Box-Jenkins procedure

According to previous studies, Lee and Carter model (1992), the log-mortality rates are ARIMA processes. Therefore, for all time series, the log-qx have been calculated. Additionally, it has been checked that the time series are not stationary (see the code lines 93 to 117) so directly the analysis will be performed on the differentiated time series. The diagnosis and all the test are coded in lines 155 to 274.

3.1. Mortality rate q_{67} ARIMA estimation

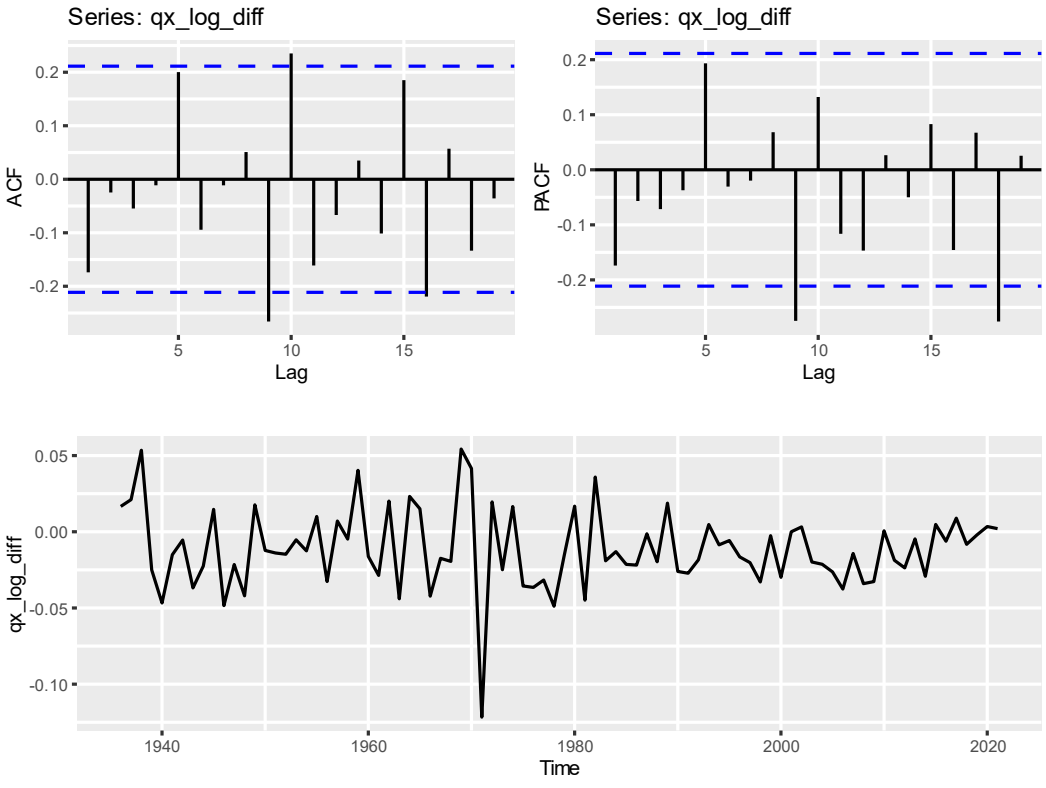
The ADF (Augmented Dickey-Fuller), PP (Phillips Perron) and KPSS (Kwiatkowski–Phillips–Schmidt–Shin) tests have been performed showing the following results for the I(0) and I(1) time series.

I(0)	p.value	stationary 0.05
ADF	0.75961	FALSE
PP	0.31752	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.04017	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

It is checked that all the test show the differentiated time series to be stationary for the significance level of 0.05.

For the log-mortality rates of the age 67, the differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

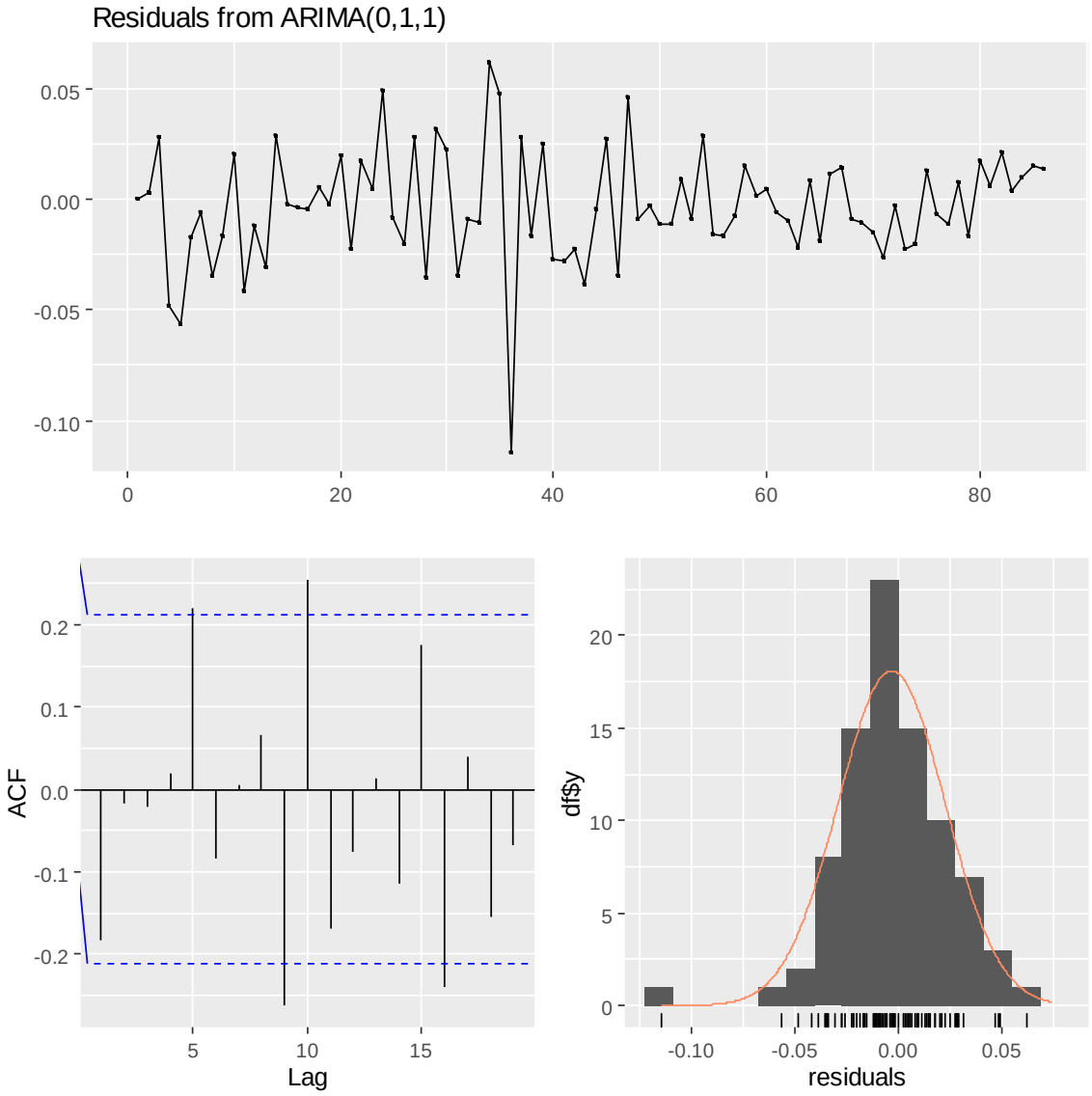
According to the ACF and PACF the most reliable form is (0,1,0). All the mentioned will be tested to confirm. All the results of the diagnosis up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_67 _log	q_67 _log	q_67 _log	q_67 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-380.58	-381.09	-381.45	-379.50
BIC	-375.81	-374.02	-374.38	-370.18
AR1	0.00000	-0.17491	0.00000	0.23707
AR1_H0_inf	#N/A	-0.38539	#N/A	-0.86543
AR1_H0_sup	#N/A	0.02675	#N/A	0.86011
AR1_H0	#N/A	NOT REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.20214	-0.43354
MA1_H0_inf	#N/A	#N/A	-0.47090	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.00162	0.83752
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01139	-0.01146	-0.01151	-0.01161
drift_H0_inf	-0.01709	-0.01425	-0.01593	-0.01916
drift_H0_sup	-0.00606	-0.00480	-0.00749	-0.01095
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.49746	0.56088	0.57427	0.62170
Normality_AD_pvalue	0.44161	0.45484	0.46396	0.50616
Normality_JB_pvalue	0.00000	0.00002	0.00015	0.00057
Uncorrelation_LB	0.02391	0.20978	0.26379	0.25722
Homocedasticity_BP_B	0.00008	0.98938	0.99784	0.99721
Zero_mean	0.98931	0.99584	0.98219	0.95597

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:



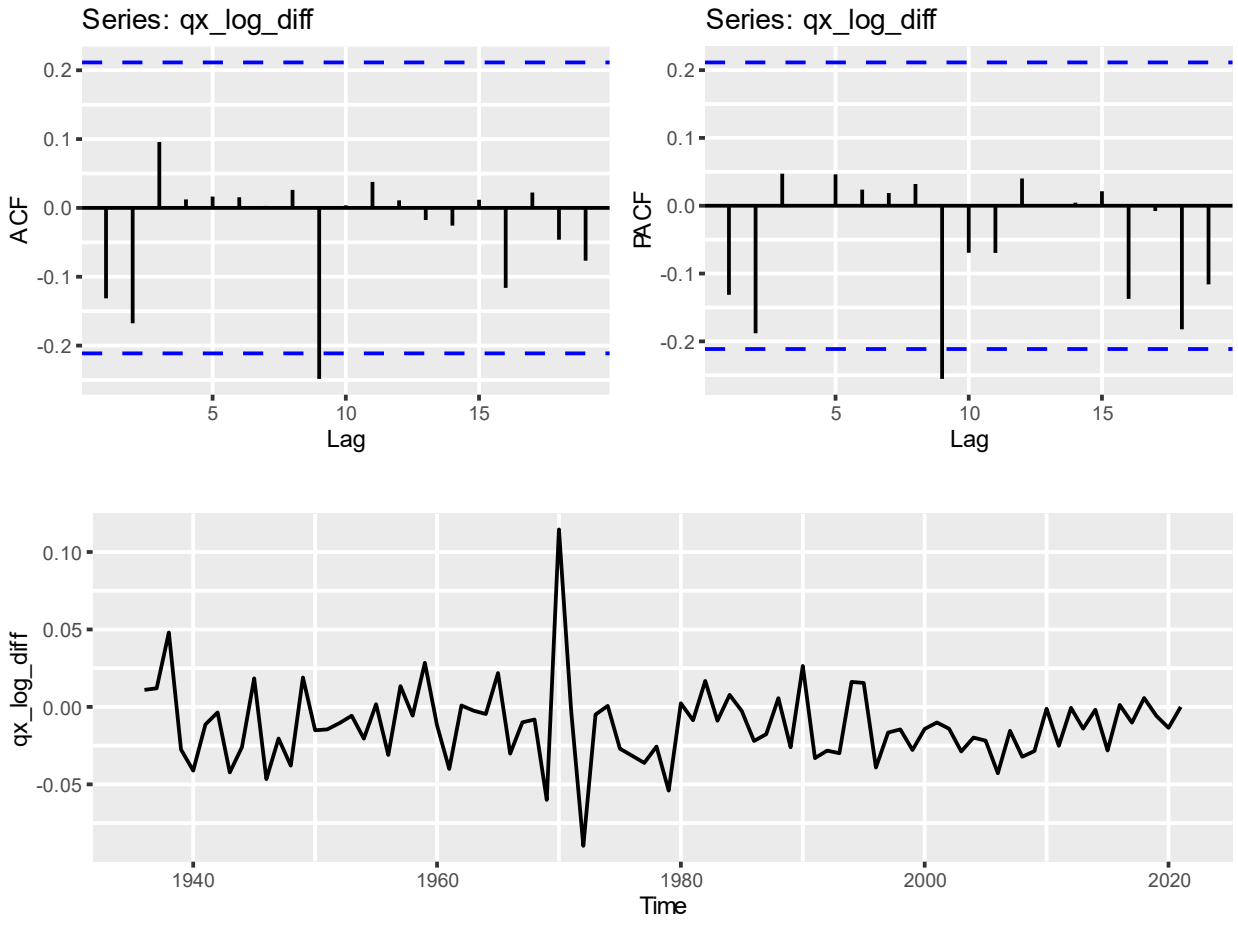
3.2. Mortality rate q_{68} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.69155	FALSE
PP	0.41466	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01523	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

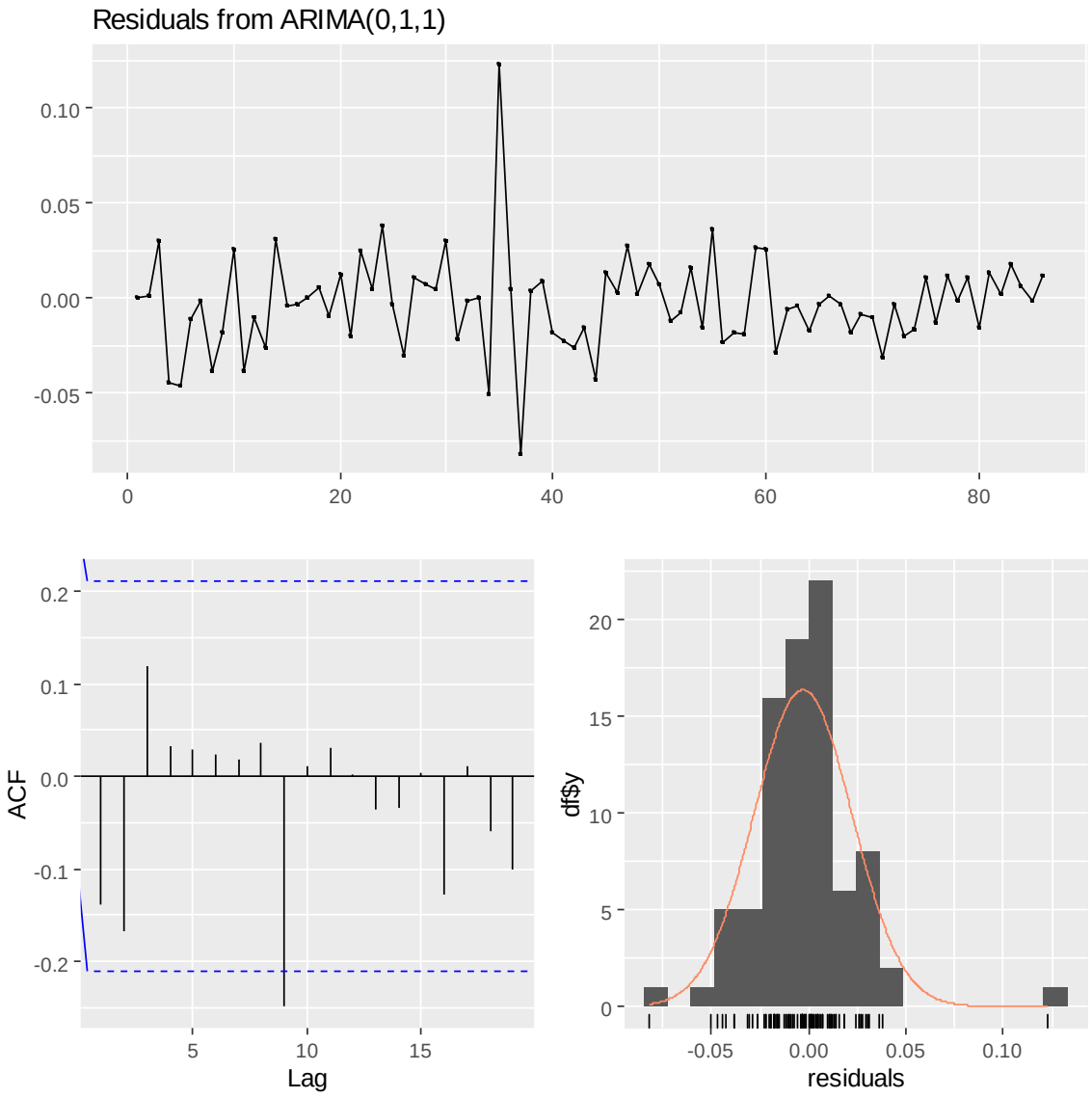
According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_68 _log	q_68 _log	q_68 _log	q_68 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-384.29	-383.64	-384.38	-382.94
BIC	-379.52	-376.57	-377.31	-373.62
AR1	0.00000	-0.13142	0.00000	0.32092
AR1_H0_inf	#N/A	-0.32630	#N/A	-0.86053
AR1_H0_sup	#N/A	0.07750	#N/A	0.86942
AR1_H0	#N/A	NOT REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.19205	-0.50283
MA1_H0_inf	#N/A	#N/A	-0.43452	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.00261	0.87481
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01179	-0.01183	-0.01188	-0.01198
drift_H0_inf	-0.01719	-0.01500	-0.01601	-0.02129
drift_H0_sup	-0.00655	-0.00580	-0.00751	-0.01343
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.33446	0.40394	0.43926	0.44613
Normality_AD_pvalue	0.26487	0.32407	0.36454	0.36896
Normality_JB_pvalue	0.00000	0.00000	0.00000	0.00000
Uncorrelation_LB	0.34223	0.43434	0.49218	0.56345
Homocedasticity_BP_B	0.00011	0.78753	0.58348	0.70464
Zero_mean	0.98939	0.99883	0.98894	0.95895

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:


Residuals from ARIMA(0,1,1)
0.10
0.05
0.00
-0.05
0
20
40
60
80
ACF
0.2
0.1
0.0
-0.1
-0.2
5
10
15
Lag
df$y
20
15
10
5
0
-0.05
0.00
0.05
0.10
residuals

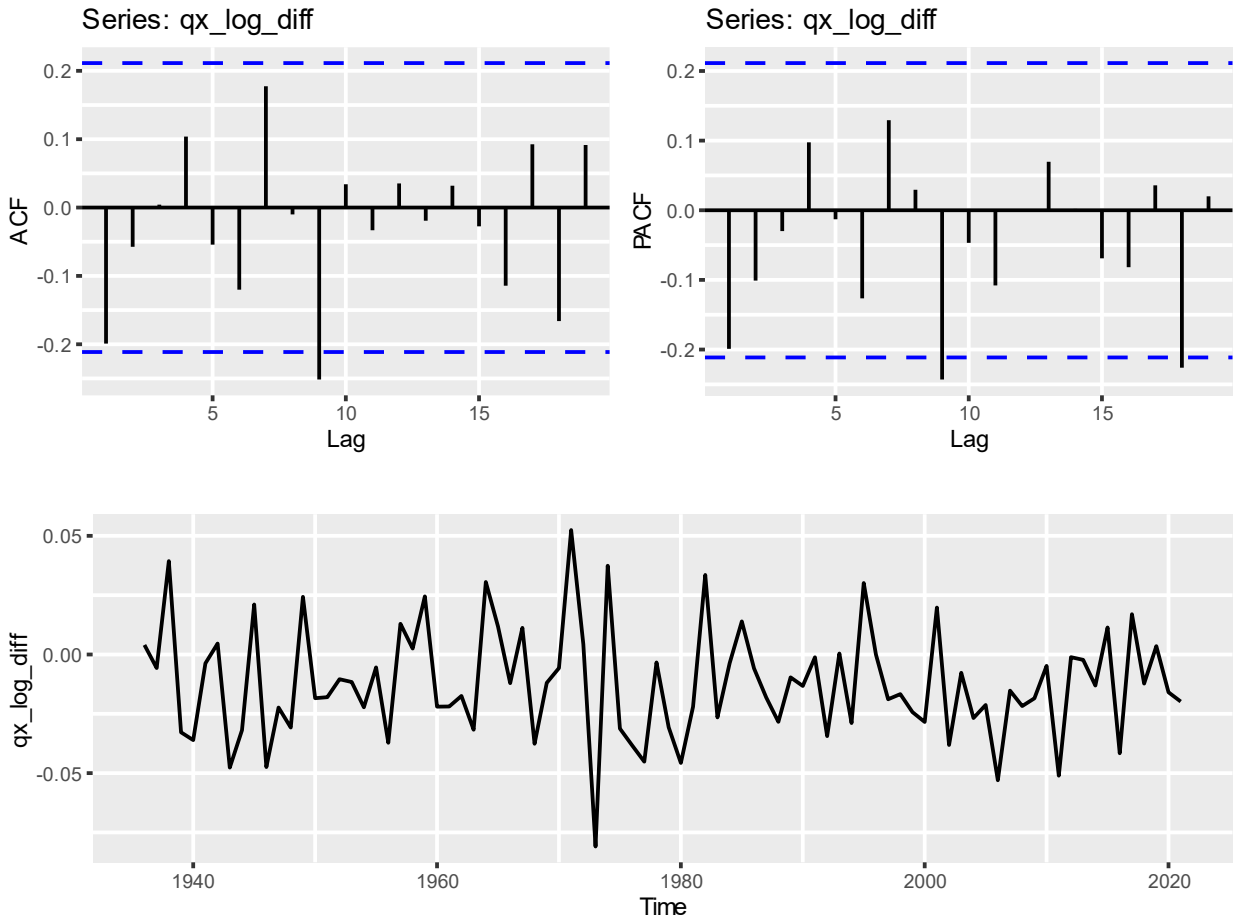
3.3. Mortality rate q_{69} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.56396	FALSE
PP	0.42881	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01857	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

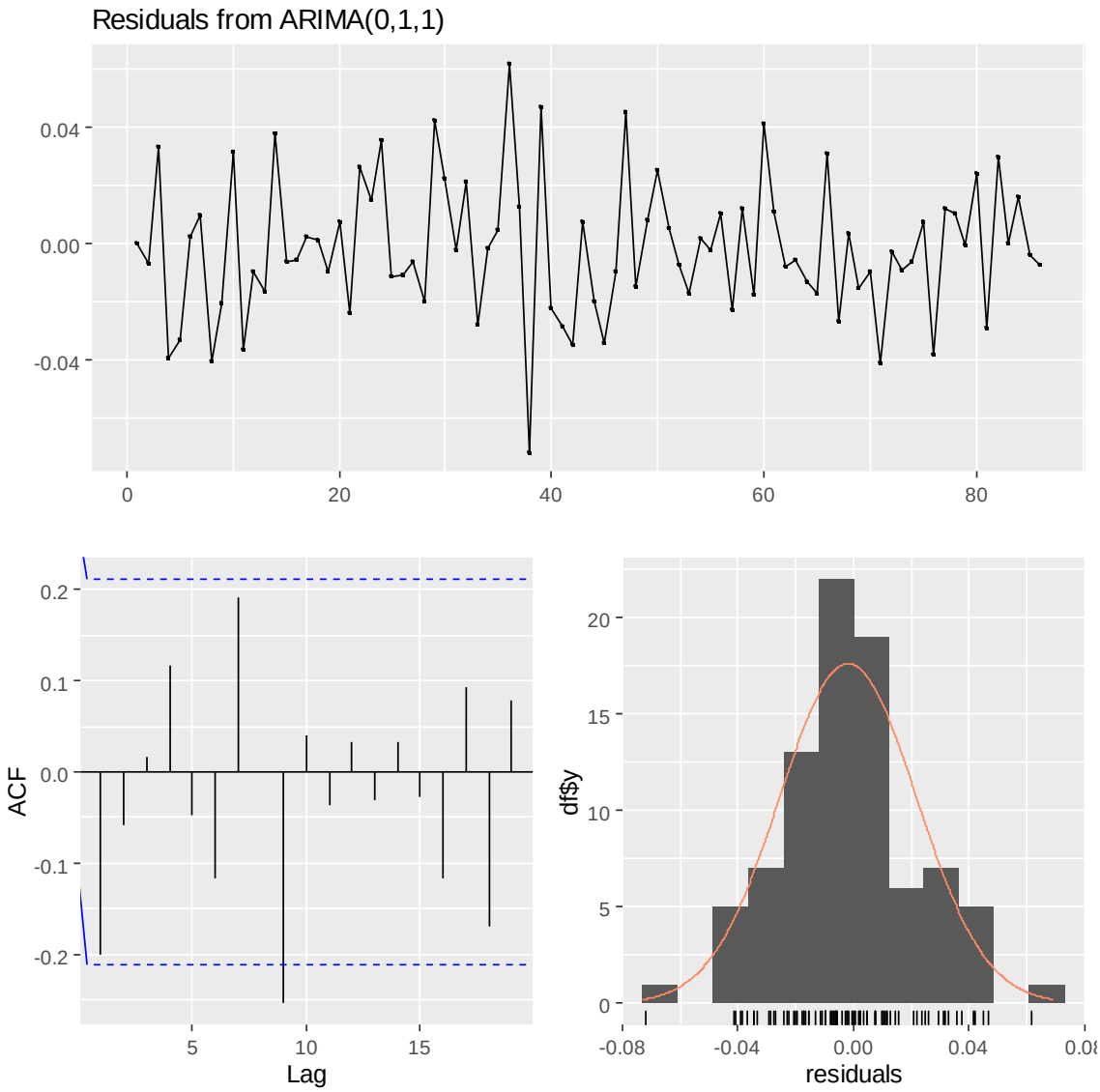
According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_69 _log	q_69 _log	q_69 _log	q_69 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-396.32	-397.63	-398.27	-396.20
BIC	-391.56	-390.56	-391.20	-386.88
AR1	0.00000	-0.19804	0.00000	0.13897
AR1_H0_inf	#N/A	-0.39990	#N/A	-0.86266
AR1_H0_sup	#N/A	0.01290	#N/A	0.86611
AR1_H0	#N/A	NOT REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.23209	-0.36184
MA1_H0_inf	#N/A	#N/A	-0.49352	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.01913	0.79766
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01223	-0.01224	-0.01226	-0.01228
drift_H0_inf	-0.01706	-0.01434	-0.01577	-0.01773
drift_H0_sup	-0.00654	-0.00617	-0.00824	-0.01059
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.75219	0.74609	0.76858	0.76416
Normality_AD_pvalue	0.81030	0.85382	0.87554	0.86531
Normality_JB_pvalue	0.51871	0.58847	0.57133	0.56436
Uncorrelation_LB	0.10170	0.21492	0.31720	0.35443
Homocedasticity_BP_B	0.03674	0.11356	0.12939	0.30118
Zero_mean	0.98900	0.99947	0.99118	0.98258

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:


Residuals from ARIMA(0,1,1)
0.04
0.00
-0.04
0
20
40
60
80
ACF
0.2
0.1
0.0
-0.1
-0.2
5
10
15
Lag
df$y
20
15
10
5
0
-0.08
-0.04
0.00
0.04
residuals

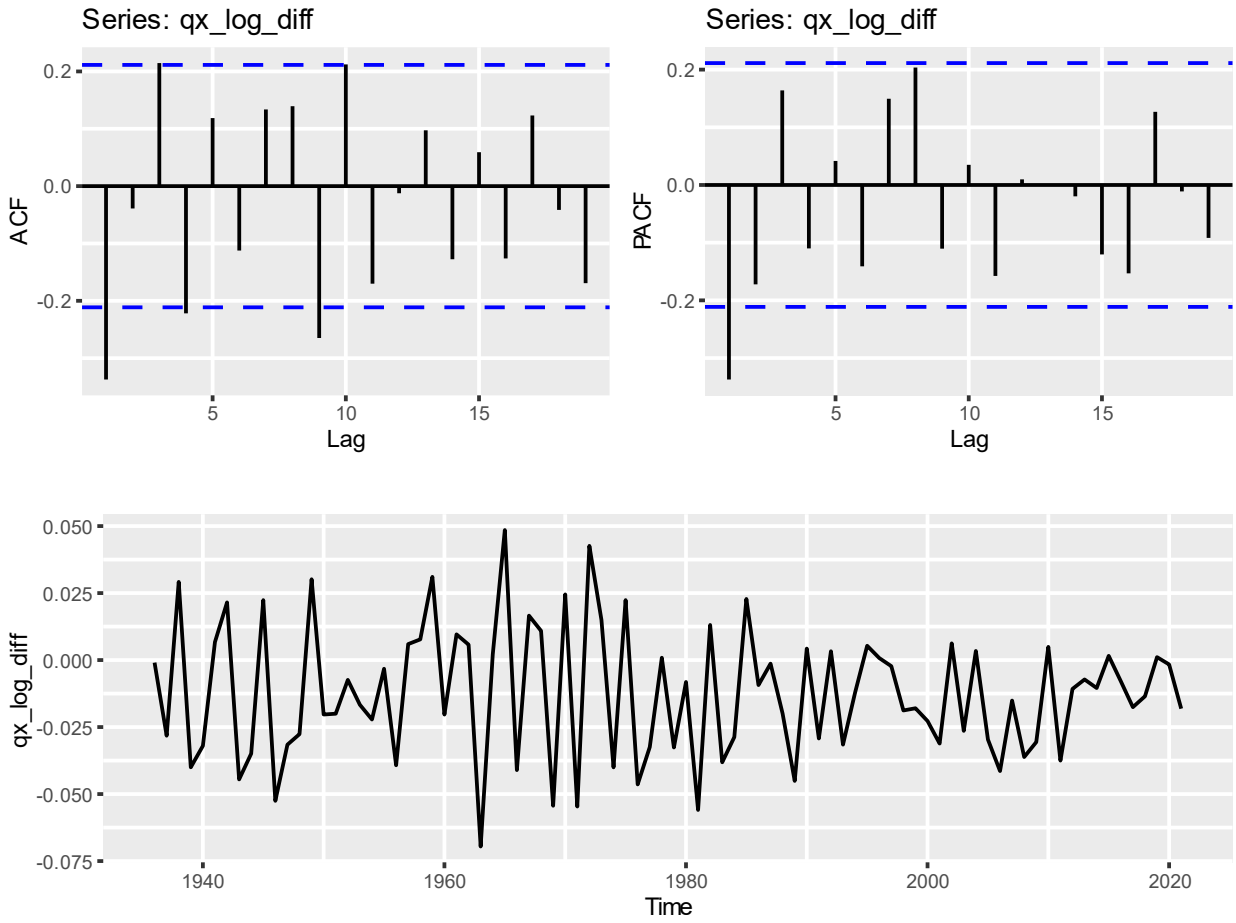
3.4. Mortality rate q_{70} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.81072	FALSE
PP	0.61122	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01000	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

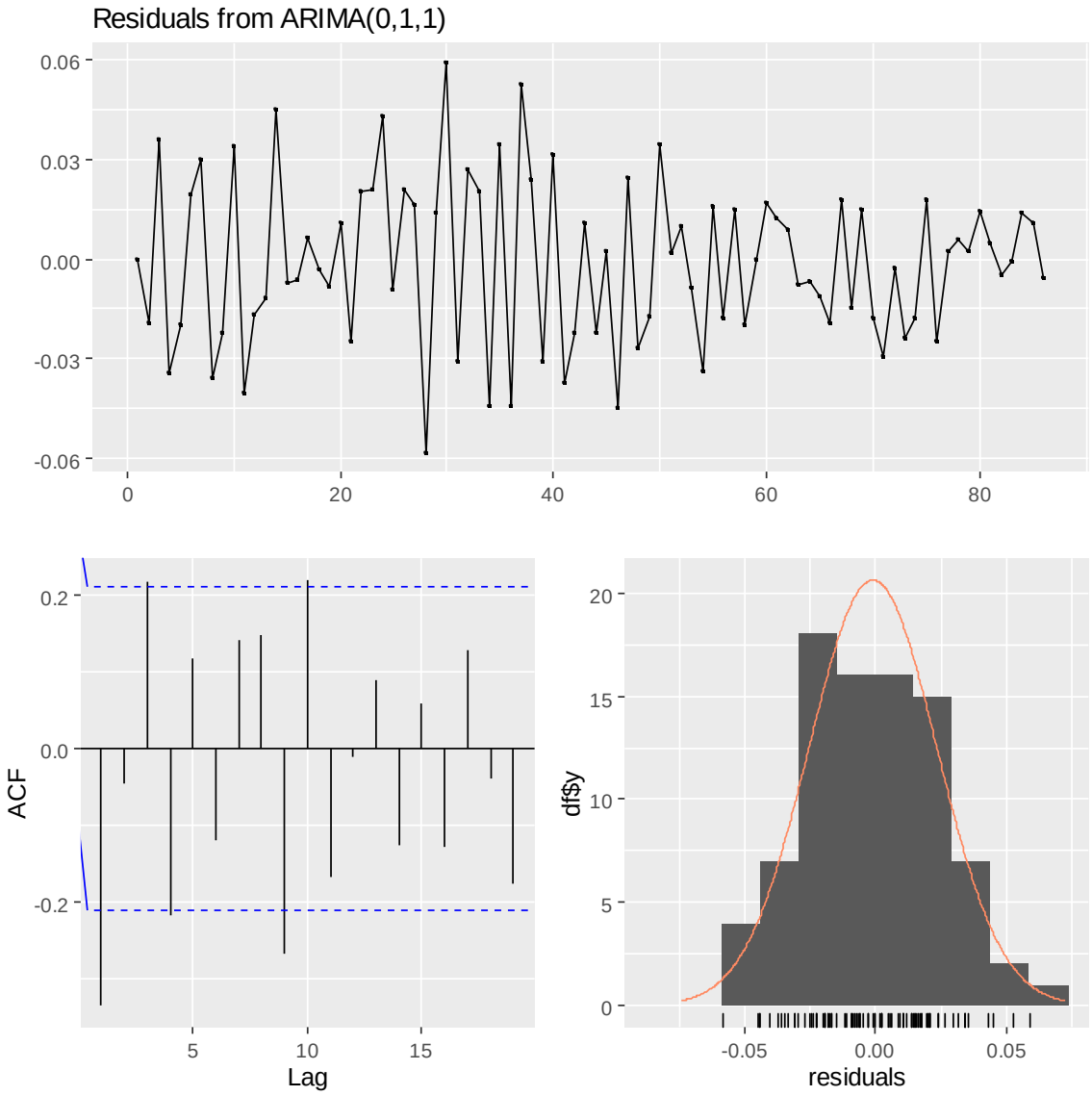
According to the ACF and PACF the most reliable form is (1,1,0), (0,1,1) or (1,1,1). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results, slightly better than ARIMA(1,1,0)

time_series	q_70 _log	q_70 _log	q_70 _log	q_70 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-390.88	-399.03	-400.19	-398.17
BIC	-386.11	-391.96	-393.12	-388.85
AR1	0.00000	-0.33445	0.00000	-0.10864
AR1_H0_inf	#N/A	-0.52623	#N/A	-0.67669
AR1_H0_sup	#N/A	-0.12959	#N/A	0.69004
AR1_H0	#N/A	REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.35757	-0.26838
MA1_H0_inf	#N/A	#N/A	-0.62077	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.16846	0.41490
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01243	-0.01245	-0.01247	-0.01247
drift_H0_inf	-0.01750	-0.01290	-0.01534	-0.01434
drift_H0_sup	-0.00745	-0.00566	-0.00937	-0.00799
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.86210	0.93385	0.92672	0.96391
Normality_AD_pvalue	0.93434	0.96271	0.91772	0.95743
Normality_JB_pvalue	0.61160	0.51841	0.46602	0.49018
Uncorrelation_LB	0.00007	0.10938	0.15678	0.19198
Homocedasticity_BP_B	0.00060	0.58374	0.41163	0.45469
Zero_mean	0.98972	0.99908	0.99543	0.99684

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:



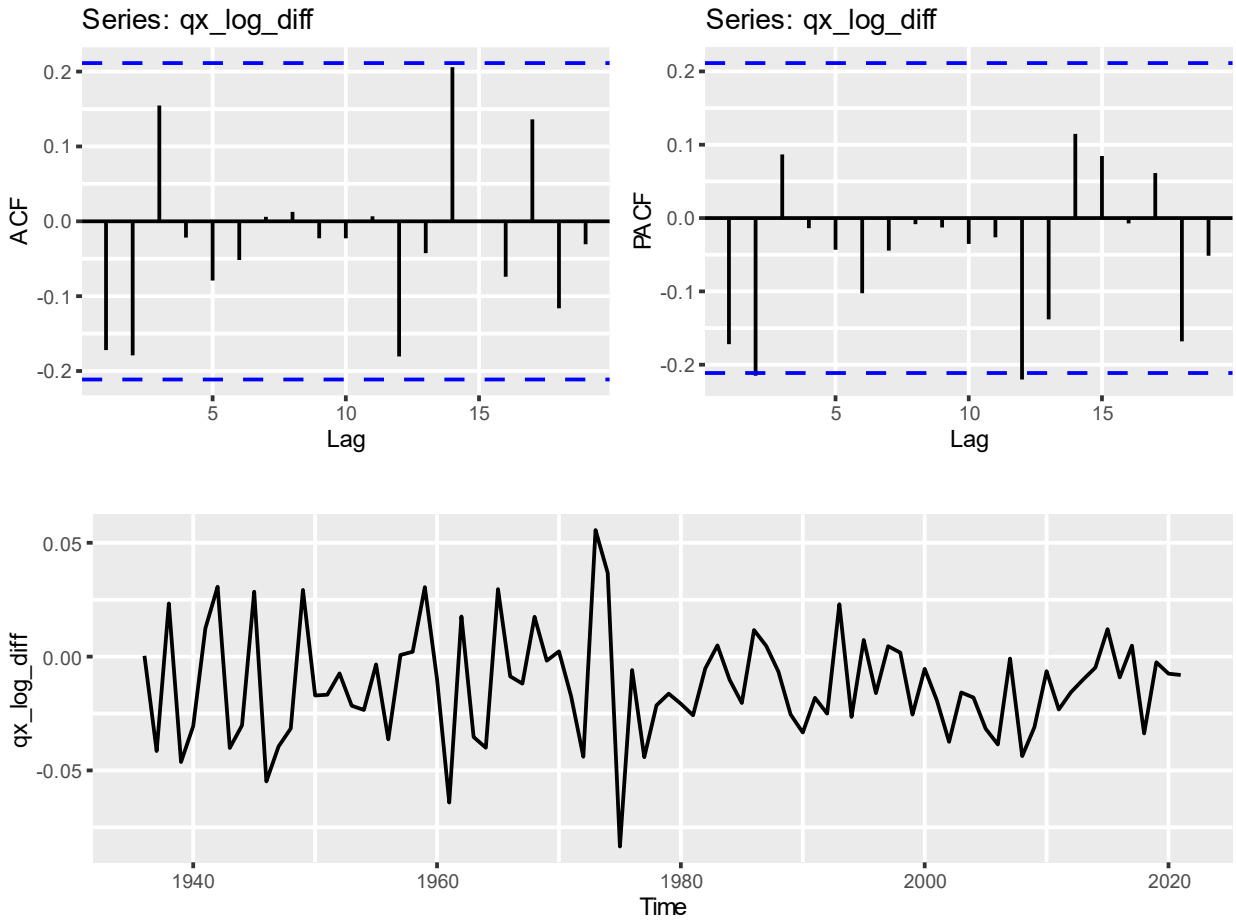
3.5. Mortality rate q_{71} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.46113	FALSE
PP	0.25084	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.0100	TRUE
PP	0.0100	TRUE
KPSS	0.1000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

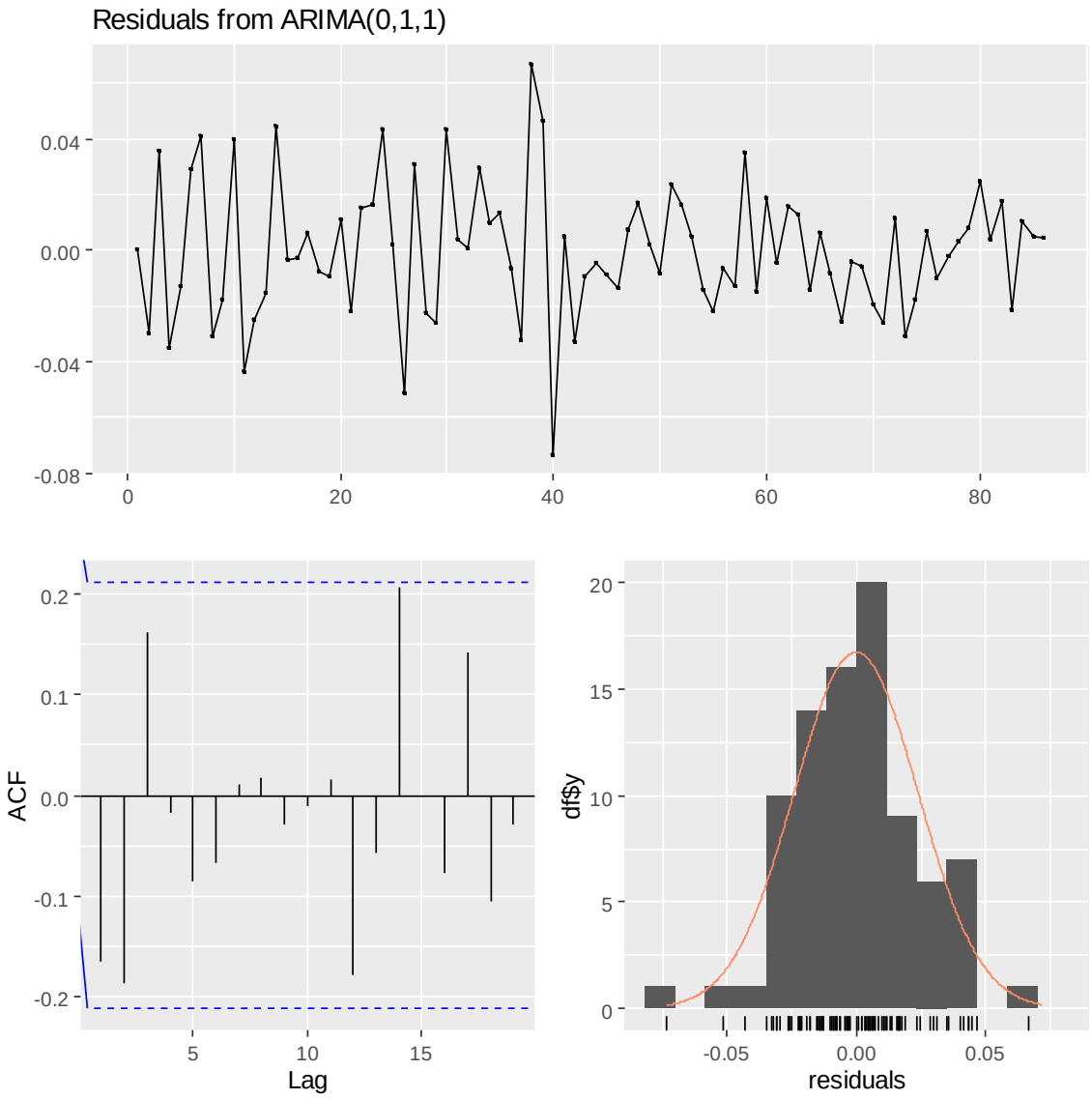
According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_71 _log	q_71 _log	q_71 _log	q_71 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-394.56	-394.97	-396.25	-396.33
BIC	-389.79	-387.90	-389.18	-387.01
AR1	0.00000	-0.17067	0.00000	0.82400
AR1_H0_inf	#N/A	-0.37347	#N/A	-0.93649
AR1_H0_sup	#N/A	0.04595	#N/A	0.85686
AR1_H0	#N/A	NOT REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.24286	-0.99995
MA1_H0_inf	#N/A	#N/A	-0.52452	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.03119	0.99261
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01249	-0.01252	-0.01253	-0.01232
drift_H0_inf	-0.01750	-0.01478	-0.01614	-0.07107
drift_H0_sup	-0.00734	-0.00653	-0.00871	-0.06883
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.91293	0.94389	0.96312	0.89052
Normality_AD_pvalue	0.91236	0.94104	0.95826	0.91262
Normality_JB_pvalue	0.60818	0.60701	0.52026	0.22172
Uncorrelation_LB	0.55977	0.77591	0.89443	0.85513
Homocedasticity_BP_B	0.00001	0.82362	0.86741	0.82451
Zero_mean	0.98984	0.99698	0.99405	0.86600

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:


Residuals from ARIMA(0,1,1)
0.04
0.00
-0.04
-0.08
0
20
40
60
80
ACF
0.2
0.1
0.0
-0.1
-0.2
5
10
15
Lag
df$y
20
15
10
5
0
-0.05
0.00
0.05
residuals

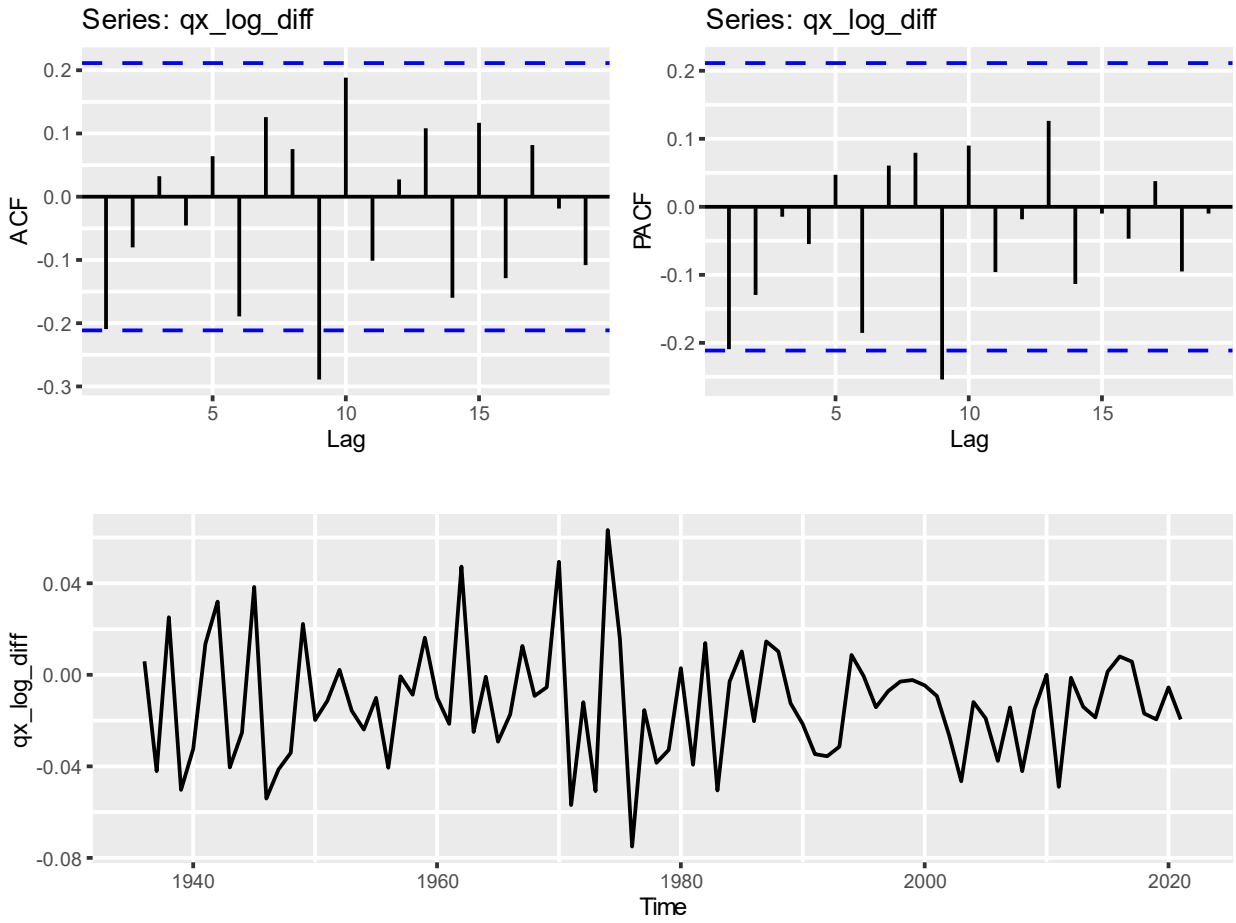
3.6. Mortality rate q_{72} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.65098	FALSE
PP	0.43575	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01000	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

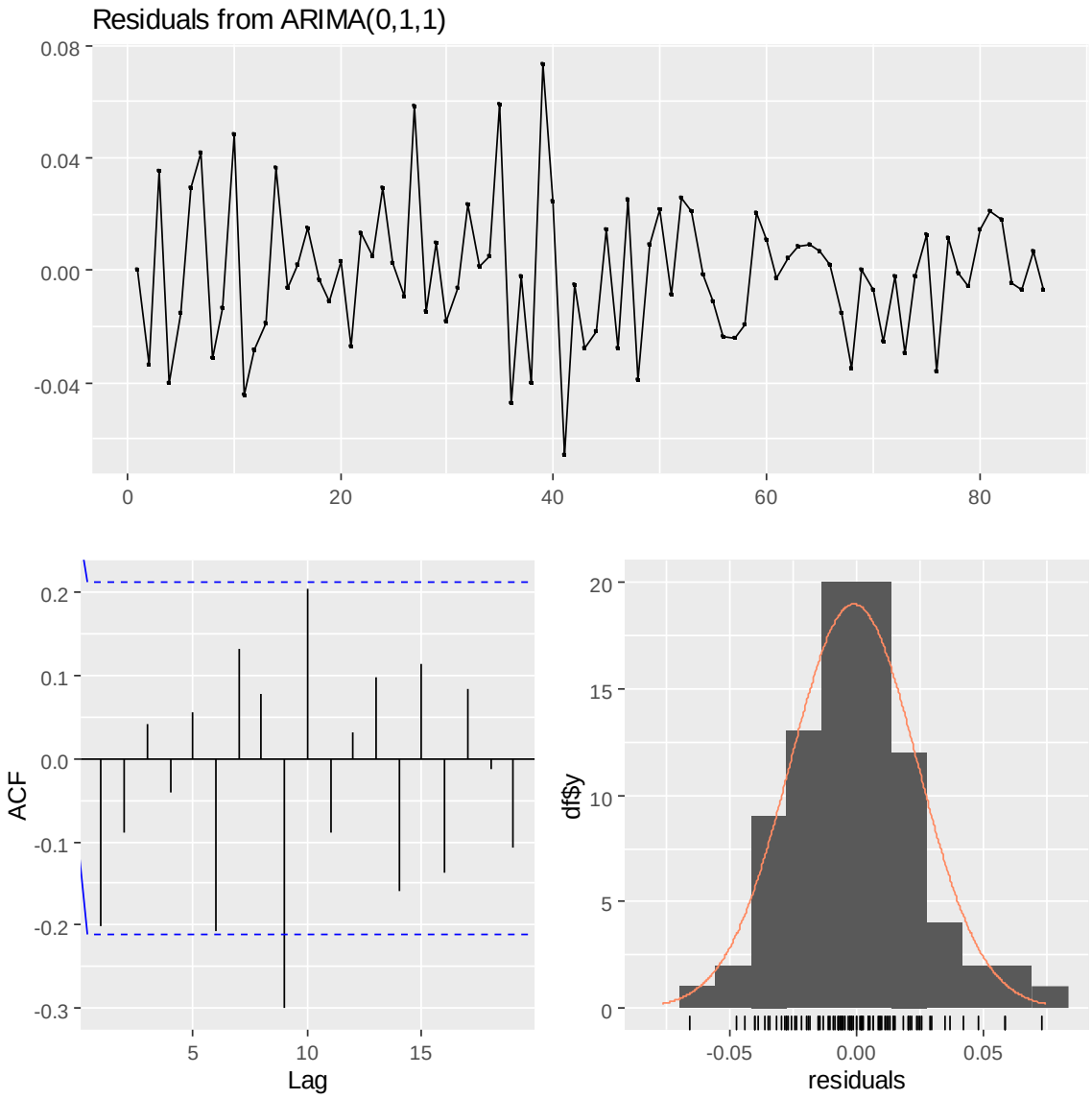
According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_72 _log	q_72 _log	q_72 _log	q_72 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-386.12	-387.81	-388.79	-387.12
BIC	-381.35	-380.73	-381.72	-377.80
AR1	0.00000	-0.20836	0.00000	0.33960
AR1_H0_inf	#N/A	-0.42394	#N/A	-0.76403
AR1_H0_sup	#N/A	0.00053	#N/A	0.84977
AR1_H0	#N/A	NOT REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.25911	-0.57710
MA1_H0_inf	#N/A	#N/A	-0.49469	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.04240	0.71376
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01253	-0.01256	-0.01255	-0.01255
drift_H0_inf	-0.01806	-0.01436	-0.01657	-0.02221
drift_H0_sup	-0.00727	-0.00613	-0.00857	-0.01571
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.88707	0.78892	0.78017	0.70116
Normality_AD_pvalue	0.88787	0.69273	0.65034	0.58521
Normality_JB_pvalue	0.22566	0.21293	0.22729	0.19459
Uncorrelation_LB	0.01297	0.19110	0.31635	0.27714
Homocedasticity_BP_B	0.00070	0.26159	0.07697	0.05239
Zero_mean	0.99060	0.99763	0.99740	0.99954

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:


Residuals from ARIMA(0,1,1)
0.08
0.04
0.00
-0.04
0
20
40
60
80
0.2
0.1
0.0
-0.1
-0.2
-0.3
ACF
5
10
15
Lag
20
15
10
5
0
df$y
-0.05
0.00
0.05
residuals

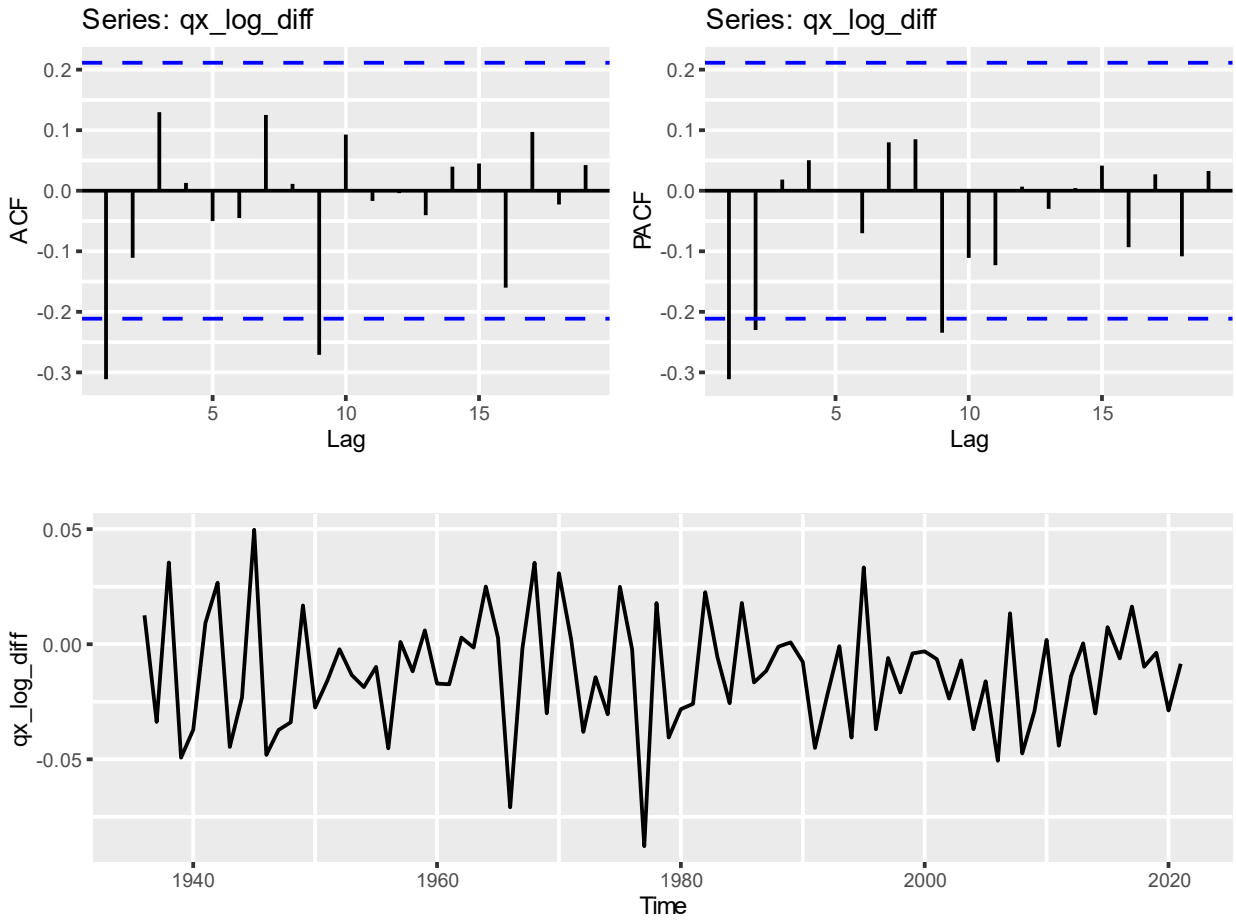
3.7. Mortality rate q_{73} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.58792	FALSE
PP	0.20170	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01000	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)
- (2,1,1)
- (2,1,0)
- (2,1,1)

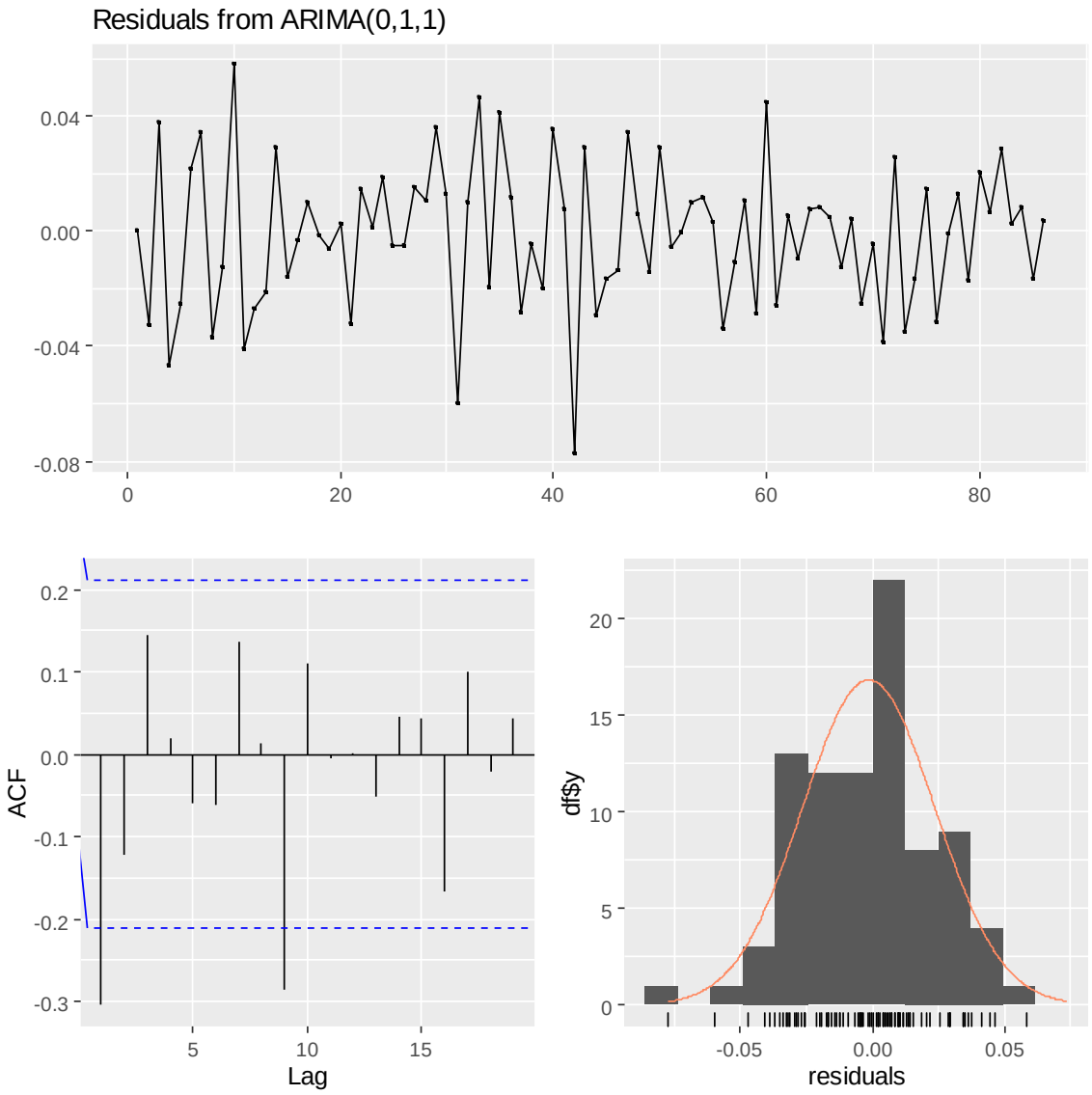
According to the ACF and PACF the most reliable form is (0,1,0), (1,1,1), (1,1,0), (1,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_73 _log	q_73 _log	q_73 _log	q_73 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-386.51	-393.14	-396.23	-394.05
BIC	-381.75	-386.07	-389.16	-384.73
AR1	0.00000	-0.31141	0.00000	0.03195
AR1_H0_inf	#N/A	-0.50965	#N/A	-0.57611
AR1_H0_sup	#N/A	-0.10436	#N/A	0.68737
AR1_H0	#N/A	REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.39198	-0.41704
MA1_H0_inf	#N/A	#N/A	-0.63862	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.19727	0.23129
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01226	-0.01234	-0.01236	-0.01236
drift_H0_inf	-0.01764	-0.01286	-0.01530	-0.01569
drift_H0_sup	-0.00683	-0.00559	-0.00935	-0.00979
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.95971	0.99466	0.99178	0.99164
Normality_AD_pvalue	0.97087	0.99674	0.98878	0.98748
Normality_JB_pvalue	0.87771	0.87999	0.99511	0.99550
Uncorrelation_LB	0.01854	0.08052	0.28037	0.28942
Homocedasticity_BP_B	0.01838	0.03291	0.33750	0.20594
Zero_mean	0.99082	0.98623	0.97290	0.97216

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:


Residuals from ARIMA(0,1,1)
0.04
0.00
-0.04
-0.08
0
20
40
60
80
0.2
0.1
0.0
-0.1
-0.2
-0.3
ACF
5
10
15
Lag
20
15
10
5
0
df$y
-0.05
0.00
0.05
residuals

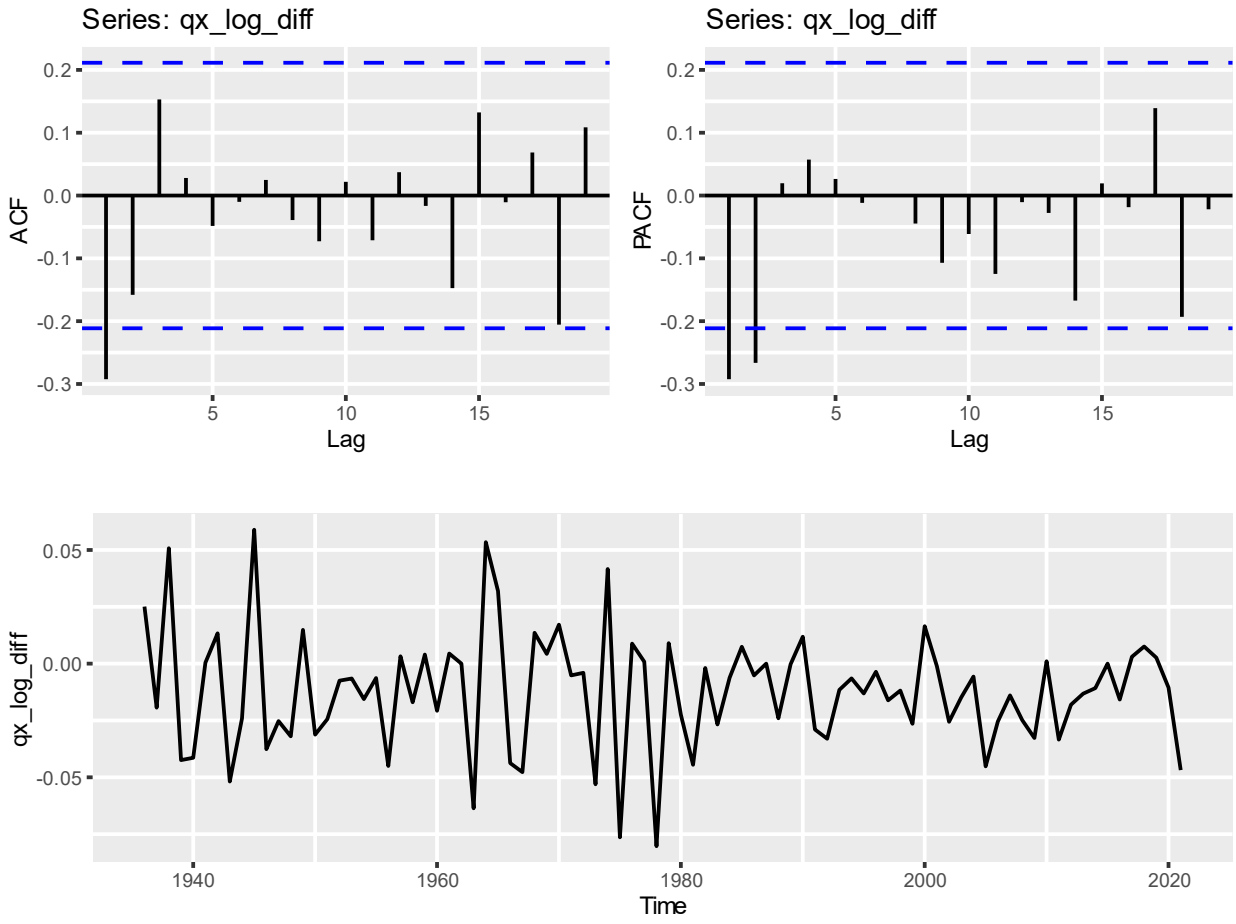
3.8. Mortality rate q_{74} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.50764	FALSE
PP	0.02272	TRUE
KPSS	0.01	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01731	TRUE
PP	0.01	TRUE
KPSS	0.1	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)
- (2,1,1)
- (2,1,0)
- (2,1,1)

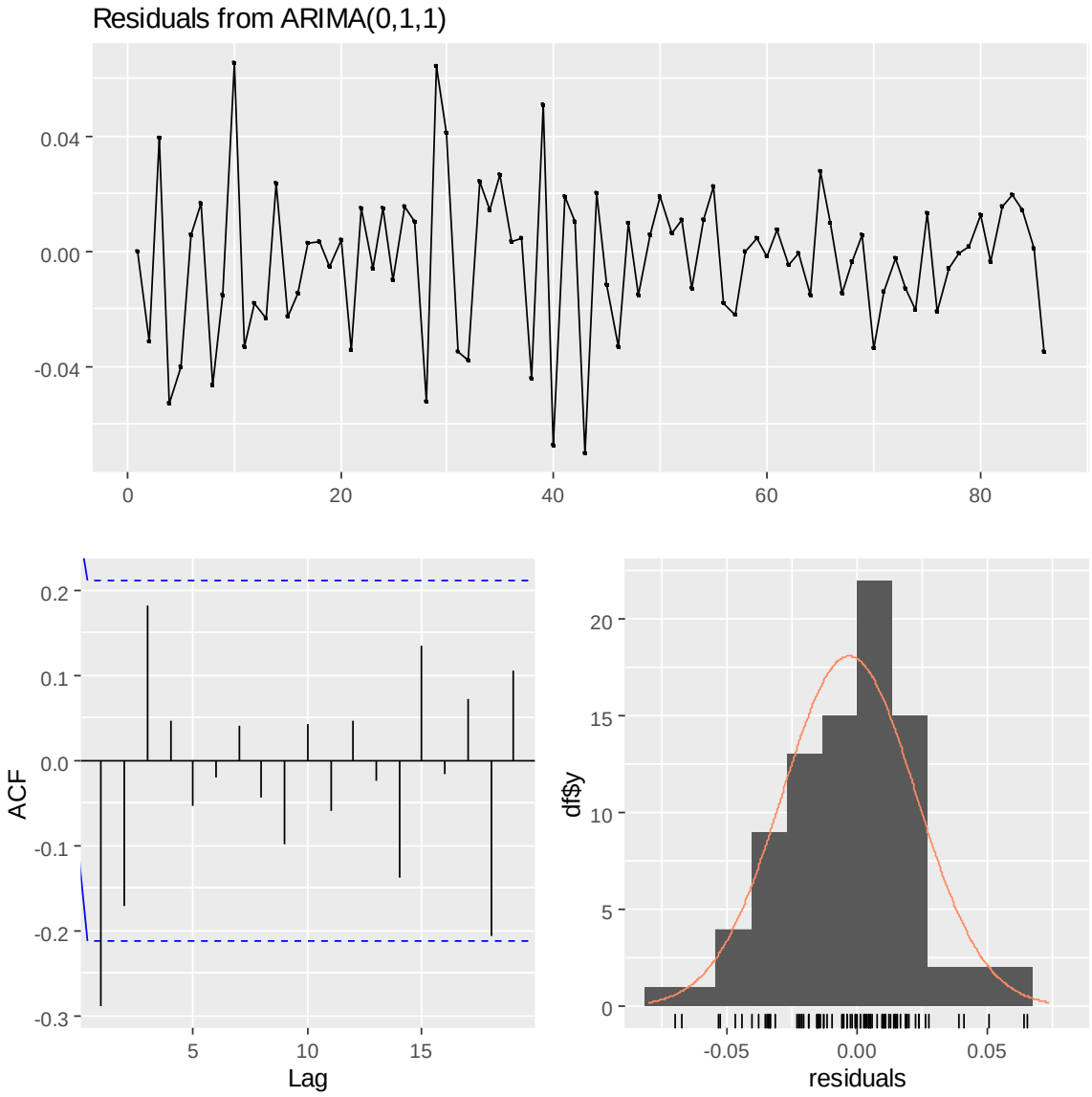
According to the ACF and PACF the most reliable form are (2,1,0), (0,1,1), (1,1,1), (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

In this case both the ARIMA(0,1,1) with drift and (2,1,0) with drift prove to be a good fit and provide good results in the diagnosis. In any case, since all the other time series are clearly an ARIMA(0,1,1) and the process will probably be the same, the ARIMA(0,1,1) with drift has been selected for modelling.

time_series	q_74 _log	q_74 _log	q_74 _log	q_74 _log
ARIMA	(1,1,0)	(0,1,1)	(0,1,2)	(2,1,0)
AICc	-387.47	-391.66	-389.65	-391.95
BIC	-380.40	-384.59	-380.33	-382.63
AR1	-0.30269	0.00000	0.00000	-0.38687
AR1_H0_inf	-0.48542	#N/A	#N/A	-0.58247
AR1_H0_sup	-0.08826	#N/A	#N/A	-0.18588
AR1_H0	REJECTED	#N/A	#N/A	REJECTED
AR2	0.00000	0.00000	0.00000	-0.27679
AR2_H0_inf	#N/A	#N/A	#N/A	-0.48076
AR2_H0_sup	#N/A	#N/A	#N/A	-0.08426
AR2_H0	#N/A	#N/A	#N/A	REJECTED
MA1	0.00000	-0.41636	-0.37988	0.00000
MA1_H0_inf	#N/A	-0.66639	-0.61425	#N/A
MA1_H0_sup	#N/A	-0.23485	-0.17937	#N/A
MA1_H0	#N/A	REJECTED	REJECTED	#N/A
MA2	0.00000	0.00000	-0.05835	0.00000
MA2_H0_inf	#N/A	#N/A	-0.33738	#N/A
MA2_H0_sup	#N/A	#N/A	0.14799	#N/A
MA2_H0	#N/A	#N/A	NOT REJECTED	#N/A
drift	-0.01207	-0.01212	-0.01215	-0.01206
drift_H0_inf	-0.01322	-0.01504	-0.01501	-0.01014
drift_H0_sup	-0.00555	-0.00925	-0.00930	-0.00405
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.66417	0.65720	0.67654	0.71482
Normality_AD_pvalue	0.72335	0.72732	0.72690	0.76951
Normality_JB_pvalue	0.39544	0.32767	0.30700	0.33890
Uncorrelation_LB	0.47134	0.79649	0.81999	0.96575
Homocedasticity_BP_B	0.41790	0.62931	0.29340	0.35359
Zero_mean	0.97608	0.92790	0.91629	0.95073

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:



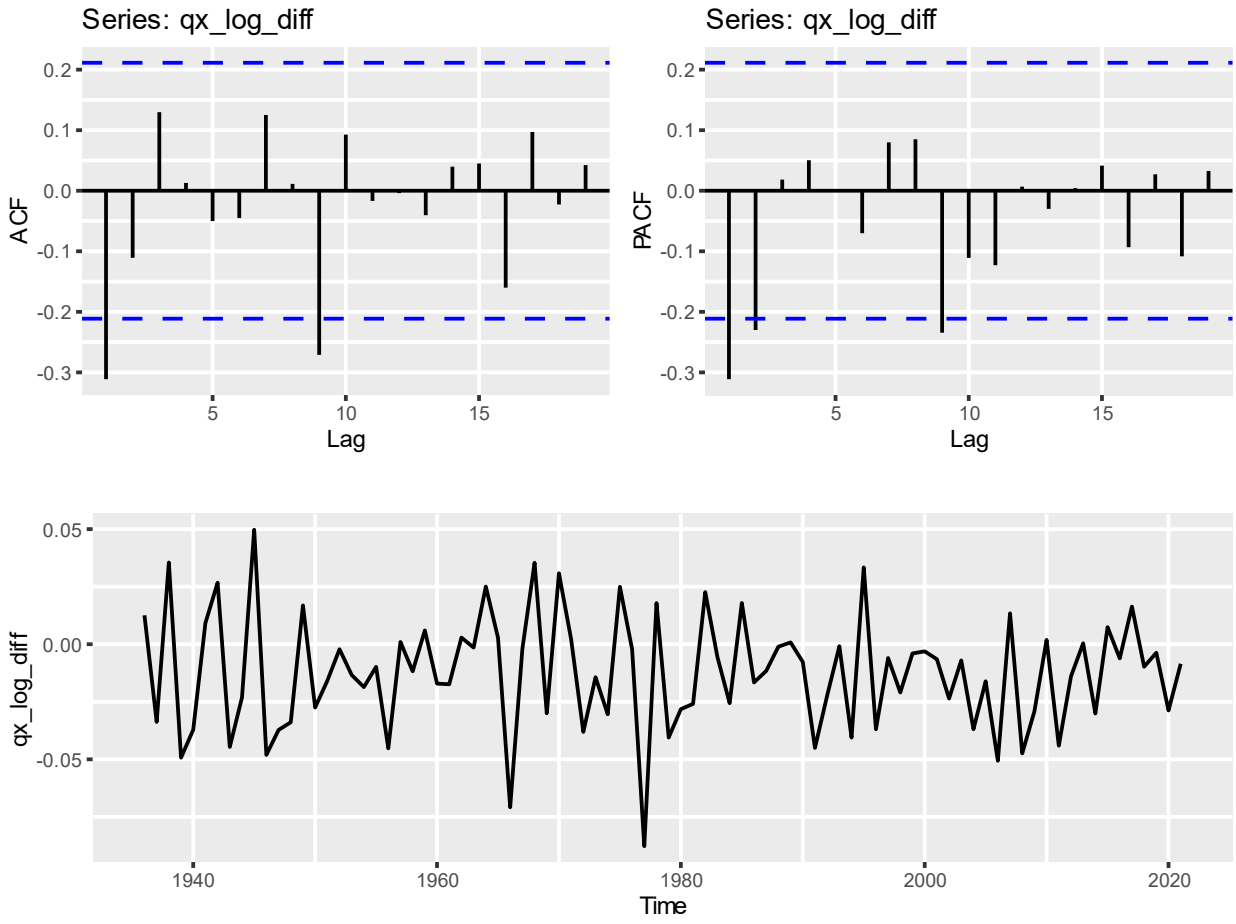
3.9. Mortality rate q_{75} ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.49425	FALSE
PP	0.02324	TRUE
KPSS	0.01	FALSE

I(1)	p.value	stationary 0.05
ADF	0.02318	TRUE
PP	0.01	TRUE
KPSS	0.1	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)
- (2,1,1)
- (2,1,0)
- (2,1,1)

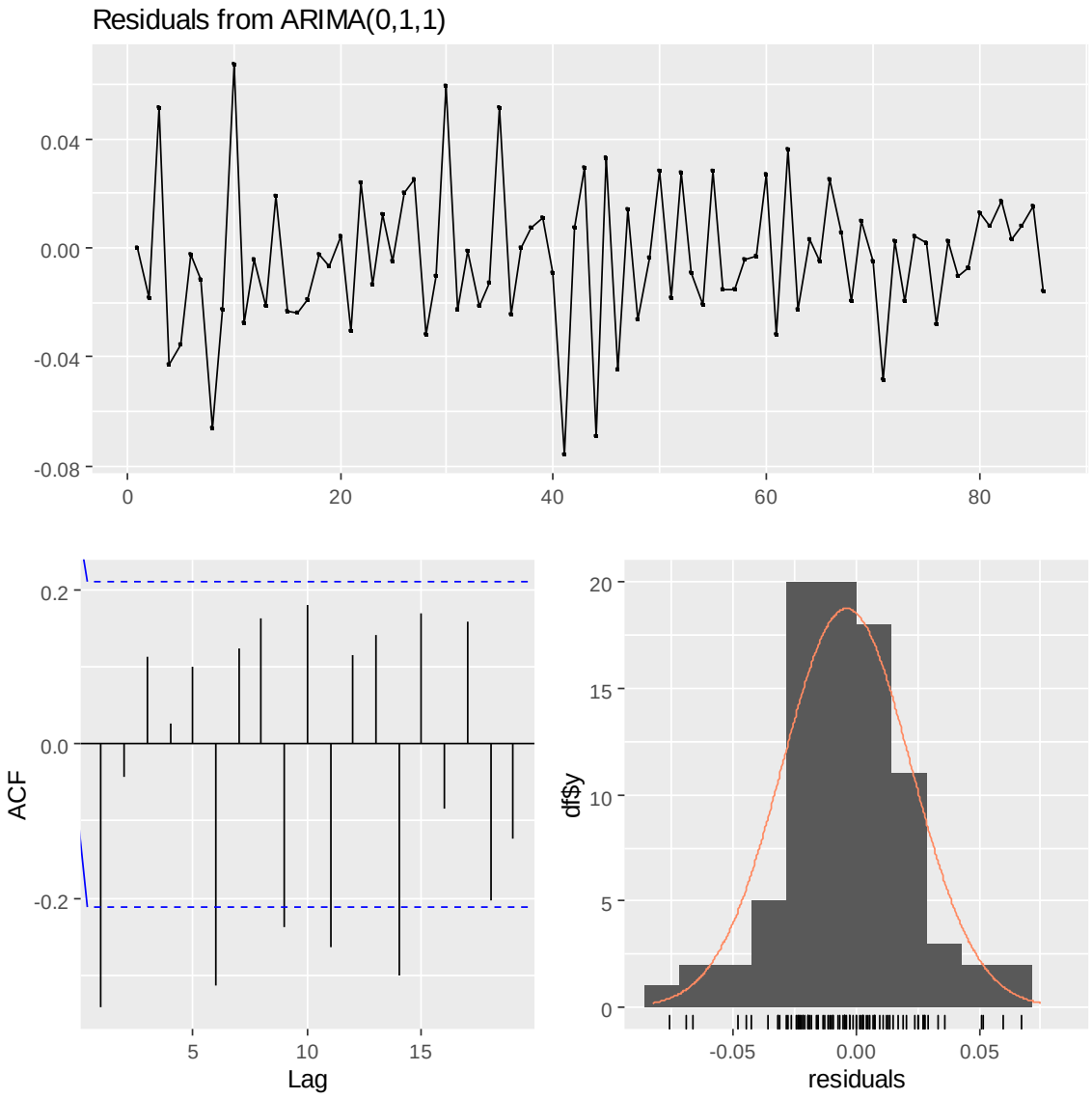
According to the ACF and PACF the most reliable form is (0,1,0), (0,1,1), (1,1,0), (1,1,1). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) with drift proves to be the best fitting model with the following results:

time_series	q_75 _log	q_75 _log	q_75 _log	q_75 _log
ARIMA	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)
AICc	-377.28	-384.92	-386.21	-384.10
BIC	-372.52	-377.85	-379.14	-374.78
AR1	0.00000	-0.32906	0.00000	-0.07907
AR1_H0_inf	#N/A	-0.51291	#N/A	-0.65369
AR1_H0_sup	#N/A	-0.12805	#N/A	0.72233
AR1_H0	#N/A	REJECTED	#N/A	NOT REJECTED
AR2	0.00000	0.00000	0.00000	0.00000
AR2_H0_inf	#N/A	#N/A	#N/A	#N/A
AR2_H0_sup	#N/A	#N/A	#N/A	#N/A
AR2_H0	#N/A	#N/A	#N/A	#N/A
MA1	0.00000	0.00000	-0.36161	-0.29575
MA1_H0_inf	#N/A	#N/A	-0.60699	-1.00000
MA1_H0_sup	#N/A	#N/A	-0.15844	0.33913
MA1_H0	#N/A	#N/A	REJECTED	NOT REJECTED
MA2	0.00000	0.00000	0.00000	0.00000
MA2_H0_inf	#N/A	#N/A	#N/A	#N/A
MA2_H0_sup	#N/A	#N/A	#N/A	#N/A
MA2_H0	#N/A	#N/A	#N/A	#N/A
drift	-0.01123	-0.01127	-0.01136	-0.01134
drift_H0_inf	-0.01659	-0.01254	-0.01448	-0.01389
drift_H0_sup	-0.00555	-0.00456	-0.00774	-0.00704
drift_H0	REJECTED	REJECTED	REJECTED	REJECTED
Normality_CVM_pvalue	0.45032	0.45231	0.56783	0.58878
Normality_AD_pvalue	0.39994	0.39269	0.49760	0.50966
Normality_JB_pvalue	0.02111	0.00204	0.00366	0.00357
Uncorrelation_LB	0.00048	0.05496	0.17215	0.16843
Homocedasticity_BP_B	0.09746	0.27820	0.29746	0.16568
Zero_mean	0.99162	0.98095	0.94285	0.95201

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

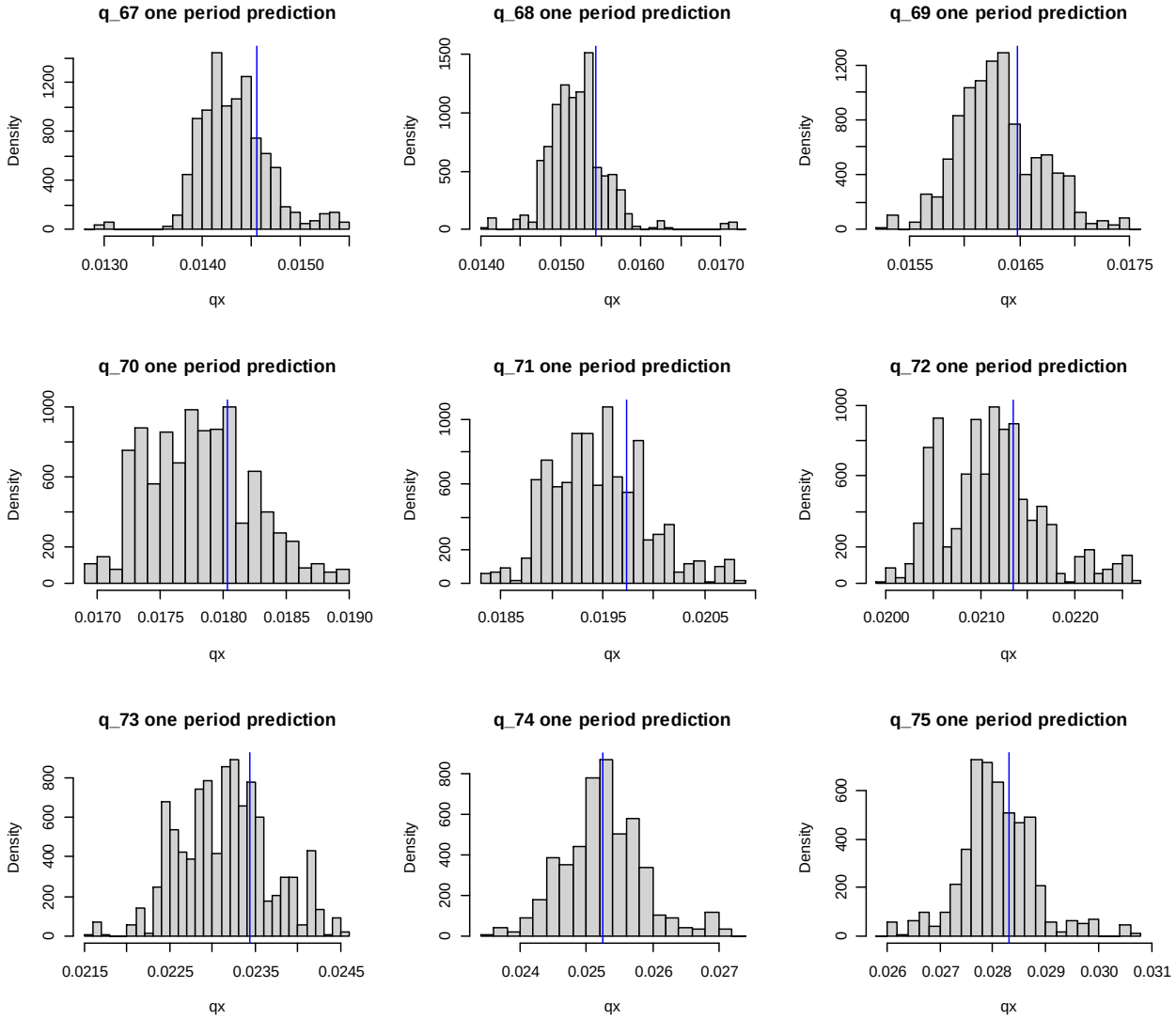

Residuals from ARIMA(0,1,1)
0.04
0.00
-0.04
-0.08
0
20
40
60
80
0.2
0.0
-0.2
ACF
5
10
15
Lag
20
15
10
5
0
df$y
-0.05
0.00
0.05
residuals

4. Next period mortality rate q_x prediction by bootstrap

Even if all the diagnosis checks are correct and the null hypothesis of the residuals to be white noise cannot be rejected, bootstrap will be used for the estimation instead of the gaussian method since it is more versatile and considers the uncertainties of the ARIMA parameters estimation as well as model specification uncertainty.

The bootstrap prediction algorithm is coded in lines 409 to 449.

The results of the estimation distribution and the 2021 value (marked with the vertical line in blue) are shown in the following graph:



5. Mortality prediction, cost and economic capital summary

The predictions shown in the previous part are used for the computation of the cost and the economic capital required to cover that cost. The calculation is coded in lines 480 to 538.

The models used for the prediction of the log-mortality are as follows:

time_series	ARIMA	MA1	drift
q_67 _log	(0,1,1)	-0.20214	-0.01151
q_68 _log	(0,1,1)	-0.19205	-0.01188
q_69 _log	(0,1,1)	-0.23209	-0.01226
q_70 _log	(0,1,1)	-0.35757	-0.01247
q_71 _log	(0,1,1)	-0.24286	-0.01253
q_72 _log	(0,1,1)	-0.25911	-0.01255
q_73 _log	(0,1,1)	-0.39198	-0.01236
q_74 _log	(0,1,1)	-0.41636	-0.01212
q_75 _log	(0,1,1)	-0.36161	-0.01136

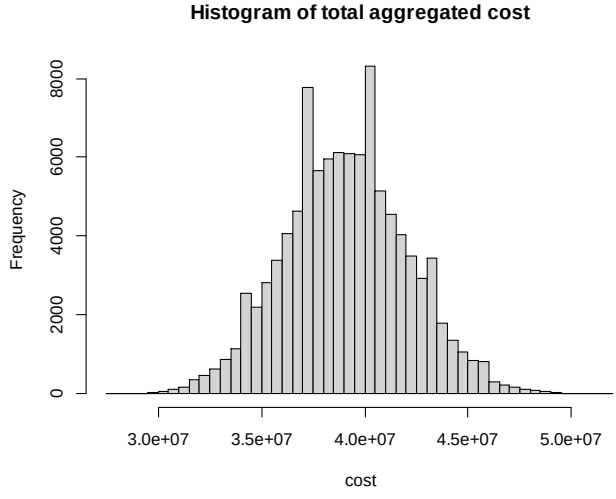
The following table summarizes all the data predicted. The expected mortality rate q_x and the conditional standard deviation as well as the VaR_{99} and $TVaR_{99}$ for the cost by age groups.

Age	qx_current	qx_estimated	qx_estimated_sd	qx_VaR99	qx_TVaR99	n_policies	cost_estimated	cost_sd	cost_VaR99	cost_TVaR99	economic_capital
67	0.01455	0.01433	0.00036	0.01536	0.01542	902	2,972,509	825,705	5,060,000	5,512,539	2,087,492
68	0.01543	0.01521	0.00038	0.01704	0.01711	659	2,306,376	723,342	4,140,000	4,557,033	1,833,624
69	0.01648	0.01631	0.00038	0.01739	0.01744	1471	5,518,827	1,122,437	8,280,000	8,842,602	2,761,173
70	0.01803	0.01781	0.00041	0.01888	0.01892	978	4,010,229	951,867	6,440,000	6,963,739	2,429,771
71	0.01974	0.01946	0.00046	0.02073	0.02077	675	3,025,271	827,874	5,060,000	5,539,776	2,034,730
72	0.02134	0.02111	0.00052	0.02253	0.02256	850	4,128,424	968,228	6,440,000	6,962,181	2,311,576
73	0.02343	0.02315	0.00053	0.02443	0.02448	882	4,697,543	1,036,329	7,360,000	7,874,576	2,662,457
74	0.02524	0.02526	0.00061	0.02697	0.02704	1035	6,016,671	1,165,096	8,970,000	9,535,517	2,953,329
75	0.02831	0.02809	0.00070	0.03048	0.03057	995	6,416,471	1,207,581	9,430,000	9,996,869	3,013,529

Additionally, the following summary table is provided with the aggregated model considering all the policies globally and running all together.

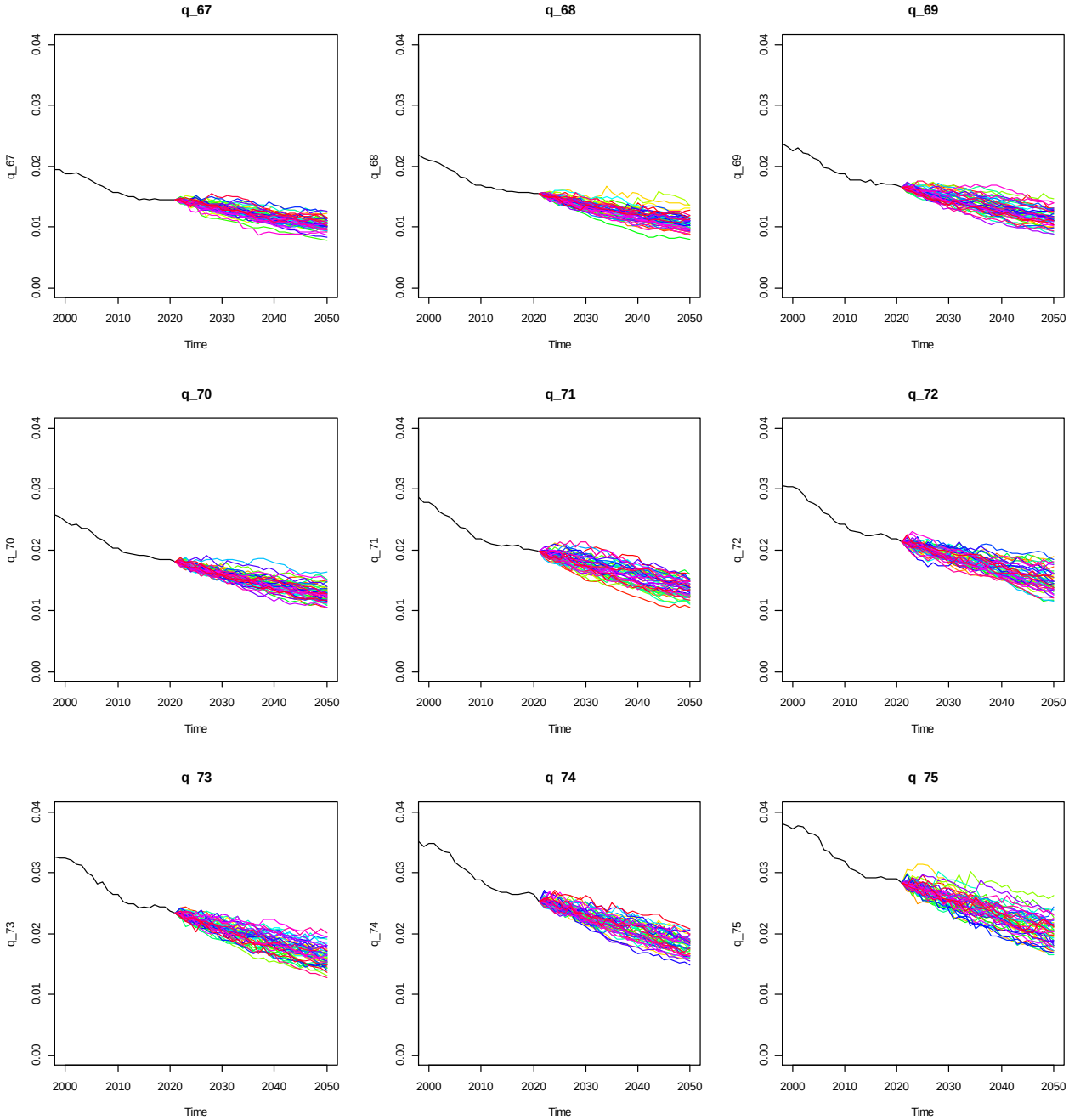
cost_total_expected	cost_total_sd	cost_total_VaR99	cost_total_TVaR99	total_economic_capital
39,091,145	2,980,131	46,230,000	47,374,907	7,138,855

The total cost distribution is as follows:



6. Further periods mortality rates prediction

Additionally, further mortality rates estimation predictions are provided for a time horizon greater than one period up to year 2050. The following graphs show these predictions for the ages previously studied with the estimated models. The prediction is performed by the same bootstrap technique used for the one period prediction.



7. Annex – Diagnosis Results

See online

time_series	ARIMA	AICc	BIC	AR1	AR1_H0_inf	AR1_H0_sup	AR1_H0	AR2	AR2_H0_inf	AR2_H0_sup	AR2_H0	MA1	MA1_H0_inf	MA1_H0_sup	MA1_H0	MA2	MA2_H0_inf	MA2_H0_sup	MA2_H0	drift	drift_H0_inf	drift_H0_sup	drift_H0	Normality_CVM_pvalue	Normality_AD_pvalue	Normality_JB_pvalue	Uncorrelation_LB	Homocedasticity_BP_B	Zero_mean
q_67 _log	(0,1,0)	-380.58	-375.81	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01139	-0.01709	-0.00606	REJECTED	0.49746	0.44161	0.00000	0.02391	0.00008	0.98931
q_67 _log	(1,1,0)	-381.09	-374.02	-0.17491	-0.38539	0.02675	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01146	-0.01425	-0.00480	REJECTED	0.56088	0.45484	0.00002	0.20978	0.98938	0.99584
q_67 _log	(0,1,1)	-381.45	-374.38	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.20214	-0.47090	-0.00162	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01151	-0.01593	-0.00749	REJECTED	0.57427	0.46396	0.00015	0.26379	0.99784	0.98219
q_67 _log	(1,1,1)	-379.50	-370.18	0.23707	-0.86543	0.86011	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.43354	-1.00000	0.83752	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01161	-0.01916	-0.01095	REJECTED	0.62170	0.50616	0.00057	0.25722	0.99721	0.95597
q_67 _log	(0,1,2)	-379.47	-370.14	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.19789	-0.45100	-0.00224	REJECTED	-0.04904	-0.29601	0.18458	NOT REJECTED	-0.01159	-0.01540	-0.00733	REJECTED	0.61445	0.49997	0.00066	0.26129	0.99832	0.96287
q_67 _log	(1,1,2)	-378.82	-367.30	0.92207	-0.96271	0.89593	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-1.15596	-1.22835	0.82904	NOT REJECTED	0.15597	-0.36341	0.31213	NOT REJECTED	-0.01201	-0.15535	-0.15183	REJECTED	0.46164	0.32291	0.00039	0.23130	0.99949	0.58081
q_67 _log	(2,1,0)	-379.21	-369.89	-0.18737	-0.40526	0.02203	NOT REJECTED	-0.06253	-0.29464	0.13444	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01151	-0.01357	-0.00477	REJECTED	0.57239	0.46474	0.00028	0.24738	0.99919	0.98336
q_67 _log	(2,1,1)	-377.25	-365.73	0.20575	-1.16821	0.81369	NOT REJECTED	-0.01187	-0.34578	0.23821	NOT REJECTED	-0.40130	-1.00000	0.98959	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01161	-0.01831	-0.01048	REJECTED	0.62345	0.50790	0.00067	0.25476	0.99689	0.95650
q_67 _log	(2,1,2)	-381.24	-367.58	1.18585	-1.12126	1.22744	NOT REJECTED	-0.78431	-0.98270	0.69630	NOT REJECTED	-1.45530	-1.56451	1.00390	NOT REJECTED	0.99998	-0.84957	1.00000	NOT REJECTED	-0.01121	-0.02341	-0.01443	REJECTED	0.65442	0.55780	0.00358	0.22353	0.98783	0.99614
q_68 _log	(0,1,0)	-384.29	-379.52	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01179	-0.01719	-0.00655	REJECTED	0.33446	0.26487	0.00000	0.34223	0.00011	0.98939
q_68 _log	(1,1,0)	-383.64	-376.57	-0.13142	-0.32630	0.07750	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01183	-0.01500	-0.00580	REJECTED	0.40394	0.32407	0.00000	0.43434	0.78753	0.99883
q_68 _log	(0,1,1)	-384.38	-377.31	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.19205	-0.43452	-0.00261	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01188	-0.01601	-0.00751	REJECTED	0.43926	0.36454	0.00000	0.49218	0.58348	0.98894
q_68 _log	(1,1,1)	-382.94	-373.62	0.32092	-0.86053	0.86942	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.50283	-1.00000	0.87481	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01198	-0.02129	-0.01343	REJECTED	0.44613	0.36896	0.00000	0.56345	0.70464	0.95895
q_68 _log	(0,1,2)	-383.97	-374.64	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.13732	-0.38623	0.06931	NOT REJECTED	-0.14985	-0.40776	0.08958	NOT REJECTED	-0.01200	-0.01546	-0.00810	REJECTED	0.39323	0.32978	0.00000	0.64148	0.81132	0.95061
q_68 _log	(1,1,2)	-382.22	-370.70	-0.34150	-0.90062	0.87554	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.19599	-1.12366	0.77966	NOT REJECTED	-0.20653	-0.46335	0.21037	NOT REJECTED	-0.01195	-0.01245	-0.00484	REJECTED	0.36841	0.32356	0.00000	0.67643	0.65711	0.96668
q_68 _log	(2,1,0)	-384.60	-375.27	-0.15890	-0.37854	0.05762	NOT REJECTED	-0.19024	-0.39378	-0.01334	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01193	-0.01260	-0.00502	REJECTED	0.35090	0.30757	0.00000	0.66515	0.77401	0.97325
q_68 _log	(2,1,1)	-382.46	-370.94	-0.36145	-1.07685	0.80275	NOT REJECTED	-0.21514	-0.42342	0.11400	NOT REJECTED	0.21207	-1.00000	0.97269	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01189	-0.01164	-0.00324	REJECTED	0.32504	0.29345	0.00000	0.67675	0.70605	0.98613
q_68 _log	(2,1,2)	-387.96	-374.30	1.13145	-1.24063	1.20754	NOT REJECTED	-0.70200	-0.98253	0.61030	NOT REJECTED	-1.45150	-1.55342	1.10912	NOT REJECTED	1.00000	-0.78173	1.00000	NOT REJECTED	-0.01146	-0.02497	-0.01545	REJECTED	0.42159	0.38601	0.00000	0.86840	0.88659	0.93557
q_69 _log	(0,1,0)	-396.32	-391.56	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01223	-0.01706	-0.00654	REJECTED	0.75219	0.81030	0.51871	0.10170	0.03674	0.98900
q_69 _log	(1,1,0)	-397.63	-390.56	-0.19804	-0.39990	0.01290	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01224	-0.01434	-0.00617	REJECTED	0.74609	0.85382	0.58847	0.21492	0.11356	0.99947
q_69 _log	(0,1,1)	-398.27	-391.20	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.23209	-0.49352	-0.01913	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01577	-0.00824	REJECTED	0.76858	0.87554	0.57133	0.31720	0.12939	0.99118
q_69 _log	(1,1,1)	-396.20	-386.88	0.13897	-0.86266	0.86611	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.36184	-1.00000	0.79766	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01228	-0.01773	-0.01059	REJECTED	0.76416	0.86531	0.56436	0.35443	0.30118	0.98258
q_69 _log	(0,1,2)	-396.25	-386.92	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.21893	-0.47639	-0.01804	REJECTED	-0.04390	-0.31492	0.18672	NOT REJECTED	-0.01228	-0.01595	-0.00858	REJECTED	0.75592	0.85690	0.55302	0.37578	0.36813	0.98290
q_69 _log	(1,1,2)	-396.02	-384.50	0.91438	-0.95237	0.89053	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-1.18253	-1.22210	0.81244	NOT REJECTED	0.18253	-0.42279	0.28279	NOT REJECTED	-0.01228	-0.14454	-0.14161	REJECTED	0.69803	0.71426	0.23275	0.38096	0.69086	0.69326
q_69 _log	(2,1,0)	-396.32	-386.99	-0.21869	-0.44054	-0.00836	REJECTED	-0.10051	-0.33284	0.08359	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01279	-0.00539	REJECTED	0.73978	0.84634	0.50735	0.42580	0.44133	0.99247
q_69 _log	(2,1,1)	-394.10	-382.57	-0.10031	-1.16126	0.79930	NOT REJECTED	-0.07845	-0.37396	0.23155	NOT REJECTED	-0.12025	-1.00000	0.99998	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01390	-0.00662	REJECTED	0.74525	0.84935	0.51885	0.42191	0.34922	0.98894
q_69 _log	(2,1,2)	-393.81	-380.15	1.26298	-1.57724	1.33652	NOT REJECTED	-0.85322	-0.99451	0.78782	NOT REJECTED	-1.42888	-1.54824	1.62243	NOT REJECTED	0.99999	-0.92915	1.00000	NOT REJECTED	-0.01224	-0.02489	-0.01619	REJECTED	0.76546	0.83393	0.50544	0.37986	0.45862	0.99374
q_70 _log	(0,1,0)	-390.88	-386.11	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01243	-0.01750	-0.00745	REJECTED	0.86210	0.93434	0.61160	0.00007	0.00060	0.98972
q_70 _log	(1,1,0)	-399.03	-391.96	-0.33445	-0.52623	-0.12959	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01245	-0.01290	-0.00566	REJECTED	0.93385	0.96271	0.51841	0.10938	0.58374	0.99908
q_70 _log	(0,1,1)	-400.19	-393.12	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.35757	-0.62077	-0.16846	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01247	-0.01534	-0.00937	REJECTED	0.92672	0.91772	0.46602	0.15678	0.41163	0.99543
q_70 _log	(1,1,1)	-398.17	-388.85	-0.10864	-0.67669	0.69004	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.26838	-1.00000	0.41490	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01247	-0.01434	-0.00799	REJECTED	0.96391	0.95743	0.49018	0.19198	0.45469	0.99684
q_70 _log	(0,1,2)	-398.34	-389.02	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.38914	-0.65865	-0.18165	REJECTED	0.07527	-0.18498	0.30244	NOT REJECTED	-0.01245	-0.01566	-0.00903	REJECTED	0.97348	0.97444	0.51629	0.21388	0.46343	0.99956
q_70 _log	(1,1,2)	-396.19	-384.66	0.21134	-0.97790	0.90993	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.59636	-1.35680	0.66377	NOT REJECTED	0.15084	-0.45869	0.47895	NOT REJECTED	-0.01244	-0.01926	-0.01233	REJECTED	0.96986	0.97222	0.53932	0.20636	0.55265	0.99865
q_70 _log	(2,1,0)	-399.34	-390.02	-0.39128	-0.60866	-0.18186	REJECTED	-0.16842	-0.38109	0.01688	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01245	-0.01111	-0.00476	REJECTED	0.96117	0.95321	0.49859	0.23993	0.43609	0.99828
q_70 _log	(2,1,1)	-400.88	-389.36	-1.01820	-1.31623	0.57721	NOT REJECTED	-0.41132	-0.59933	0.24723	NOT REJECTED	0.66403	-1.00000	0.99999	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01242	-0.00828	-0.00192	REJECTED	0.86613	0.82317	0.33700	0.59589	0.89304	0.98420
q_70 _log	(2,1,2)	-399.21	-385.55	-1.18658	-1.53325	0.78300	NOT REJECTED	-0.59176	-0.96654	0.37536	NOT REJECTED	0.87701	-1.22278	1.45464	NOT REJECTED	0.21506	-0.69170	0.99999	NOT REJECTED	-0.01240	-0.00771	-0.00093	REJECTED	0.80190	0.69599	0.15062	0.55588	0.90078	0.98022
q_71 _log	(0,1,0)	-394.56	-389.79	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01249	-0.01750	-0.00734	REJECTED	0.91293	0.91236	0.60818	0.55977	0.00001	0.98984
q_71 _log	(1,1,0)	-394.97	-387.90	-0.17067	-0.37347	0.04595	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01252	-0.01478	-0.00653	REJECTED	0.94389	0.94104	0.60701	0.77591	0.82362	0.99698
q_71 _log	(0,1,1)	-396.25	-389.18	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.24286	-0.52452	-0.03119	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01253	-0.01614	-0.00871	REJECTED	0.96312	0.95826	0.52026	0.89443	0.86741	0.99405
q_71 _log	(1,1,1)	-396.33	-387.01	0.82400	-0.93649	0.85686	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.99995	-1.00000	0.99261	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01232	-0.07107	-0.06883	REJECTED	0.89052	0.91262	0.22172	0.85513	0.82451	0.86600
q_71 _log	(0,1,2)	-395.77	-386.45	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.17253	-0.43037	0.02637	NOT REJECTED	-0.16825	-0.45435	0.02620	NOT REJECTED	-0.01252	-0.01552	-0.00941	REJECTED	0.99061	0.99467	0.51777	0.96269	0.55108	0.98595
q_71 _log	(1,1,2)	-394.86	-383.34	-0.57804	-0.87275	0.83759	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.40071	-1.11305	0.74434	NOT REJECTED	-0.28185	-0.54299	0.24129	NOT REJECTED	-0.01249	-0.01125	-0.00448	REJECTED	0.97972	0.97764	0.43475	0.99182	0.59747	0.98221
q_71 _log	(2,1,0)	-396.79	-387.46	-0.20636	-0.44317	-0.00084	REJECTED	-0.21314	-0.42549	-0.01763	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01251	-0.01222	-0.00536	REJECTED	0.99614	0.99604	0.56820	0.98856	0.72322	0.98531
q_71 _log	(2,1,1)	-395.25	-383.73	-0.57252	-1.08860	0.75240	NOT REJECTED	-0.27901	-0.48245	0.07288	NOT REJECTED	0.38805	-1.00000	0.94913	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01248	-0.01026	-0.00322	REJECTED	0.98285	0.98977	0.51679	0.99759	0.72277	0.97782
q_71 _log	(2,1,2)	-392.96	-379.29	-0.55580	-1.26339	1.08911	NOT REJECTED	-0.33226	-0.98775	0.59425	NOT REJECTED	0.37147	-1.39799	1.23085	NOT REJECTED	0.06206	-0.88414	1.00000	NOT REJECTED	-0.01248	-0.01026	-0.00271	REJECTED	0.98683	0.99302	0.54628	0.99797	0.87057	0.97747
q_72 _log	(0,1,0)	-386.12	-381.35	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01253	-0.01806	-0.00727	REJECTED	0.88707	0.88787	0.22566	0.01297	0.00070	0.99060
q_72 _log	(1,1,0)	-387.81	-380.73	-0.20836	-0.42394	0.00053	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01256	-0.01436	-0.00613	REJECTED	0.78892	0.69273	0.21293	0.19110	0.26159	0.99763
q_72 _log	(0,1,1)	-388.79	-381.72	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.25911	-0.49469	-0.04240	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01255	-0.01657	-0.00857	REJECTED	0.78017	0.65034	0.22729	0.31635	0.07697	0.99740
q_72 _log	(1,1,1)	-387.12	-377.80	0.33960	-0.76403	0.84977	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.57710	-1.00000	0.71376	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01255	-0.02221	-0.01571	REJECTED	0.70116	0.58521	0.19459	0.27714	0.05239	0.99954
q_72 _log	(0,1,2)	-387.12	-377.80	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.23792	-0.48631	-0.02112	REJECTED	-0.08448	-0.34652	0.12260	NOT REJECTED	-0.01255	-0.01600	-0.00893	REJECTED	0.71637	0.59744	0.24115	0.29728	0.24144	0.99943
q_72 _log	(1,1,2)	-385.27	-373.75	-0.63052	-0.92129	0.89173	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.39754	-1.24073	0.76886	NOT REJECTED	-0.24459	-0.49915	0.34902	NOT REJECTED	-0.01254	-0.01151	-0.00408	REJECTED	0.68815	0.60641	0.20487	0.46288	0.34092	0.99480
q_72 _log	(2,1,0)	-387.01	-377.69	-0.23409	-0.44256	-0.02605	REJECTED	-0.12722	-0.35946	0.06783	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01254	-0.01289	-0.00540	REJECTED	0.79234	0.69081	0.27835	0.33461	0.47447	0.99758
q_72 _log	(2,1,1)	-384.93	-373.41	-0.85089	-1.23702	0.77328	NOT REJECTED	-0.23670	-0.43569	0.24042	NOT REJECTED	0.63393	-1.00000	1.00000	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01253	-0.00981	-0.00204	REJECTED	0.70296	0.65130	0.14938	0.49464	0.67827	0.99260
q_72 _log	(2,1,2)	-382.97	-369.30	-0.59727	-1.32489	1.25415	NOT REJECTED	0.04356	-0.96942	0.65494	NOT REJECTED	0.36563	-1.57786	1.16672	NOT REJECTED	-0.28510	-0.97169	1.00000	NOT REJECTED	-0.01254	-0.01141	-0.00449	REJECTED	0.67690	0.59179	0.20500	0.44260	0.69680	0.99595
q_73 _log	(0,1,0)	-386.51	-381.75	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01764	-0.00683	REJECTED	0.95971	0.97087	0.87771	0.01854	0.01838	0.99082
q_73 _log	(1,1,0)	-393.14	-386.07	-0.31141	-0.50965	-0.10436	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01234	-0.01286	-0.00559	REJECTED	0.99466	0.99674	0.87999	0.08052	0.03291	0.98623
q_73 _log	(0,1,1)	-396.23	-389.16	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.39198	-0.63862	-0.19727	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01236	-0.01530	-0.00935	REJECTED	0.99178	0.98878	0.99511	0.28037	0.33750	0.97290
q_73 _log	(1,1,1)	-394.05	-384.73	0.03195	-0.57611	0.68737	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.41704	-1.00000	0.23129	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01236	-0.01569	-0.00979	REJECTED	0.99164	0.98748	0.99550	0.28942	0.20594	0.97216
q_73 _log	(0,1,2)	-394.07	-384.75	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.37911	-0.62467	-0.18533	REJECTED	-0.02294	-0.28292	0.21418	NOT REJECTED	-0.01236	-0.01528	-0.00954	REJECTED	0.99192	0.98676	0.99559	0.29785	0.19702	0.97176
q_73 _log	(1,1,2)	-392.32	-380.80	-0.58837	-0.94804	0.89640	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.23103	-1.34943	0.60167	NOT REJECTED	-0.28854	-0.56420	0.43104	NOT REJECTED	-0.01235	-0.01064	-0.00469	REJECTED	0.99051	0.99018	0.98194	0.37856	0.03757	0.97446
q_73 _log	(2,1,0)	-395.51	-386.19	-0.38124	-0.58112	-0.17113	REJECTED	-0.22711	-0.45009	-0.01886	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01232	-0.01059	-0.00467	REJECTED	0.97987	0.98854	0.96114	0.47096	0.09470	0.98382
q_73 _log	(2,1,1)	-393.27	-381.75	-0.42880	-1.19728	0.60163	NOT REJECTED	-0.24224	-0.49791	0.24226	NOT REJECTED	0.05041	-1.00000	0.89725	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01232	-0.01031	-0.00416	REJECTED	0.97944	0.98750	0.96109	0.47306	0.03080	0.98603
q_73 _log	(2,1,2)	-391.25	-377.58	-0.24472	-1.14035	0.99555	NOT REJECTED	-0.39013	-0.92268	0.40248	NOT REJECTED	-0.12475	-1.44997	0.84193	NOT REJECTED	0.23830	-0.71856	1.00000	NOT REJECTED	-0.01231	-0.01077	-0.00413	REJECTED	0.97618	0.98843	0.97507	0.51361	0.07415	0.99718
q_74 _log	(0,1,0)	-381.64	-376.88	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01206	-0.01767	-0.00681	REJECTED	0.70713	0.68190	0.31017	0.21421	0.00076	0.99127
q_74 _log	(1,1,0)	-387.47	-380.40	-0.30269	-0.48542	-0.08826	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01207	-0.01322	-0.00555	REJECTED	0.66417	0.72335	0.39544	0.47134	0.41790	0.97608
q_74 _log	(0,1,1)	-391.66	-384.59	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.41636	-0.66639	-0.23485	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01212	-0.01504	-0.00925	REJECTED	0.65720	0.72732	0.32767	0.79649	0.62931	0.92790
q_74 _log	(1,1,1)	-389.55	-380.22	0.06504	-0.51149	0.68594	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.46505	-1.00000	0.14770	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01214	-0.01581	-0.00995	REJECTED	0.66198	0.72546	0.31815	0.80487	0.32083	0.92179
q_74 _log	(0,1,2)	-389.65	-380.33	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.37988	-0.61425	-0.17937	REJECTED	-0.05835	-0.33738	0.14799	NOT REJECTED	-0.01215	-0.01501	-0.00930	REJECTED	0.67654	0.72690	0.30700	0.81999	0.29340	0.91629
q_74 _log	(1,1,2)	-388.31	-376.79	-0.58759	-0.90487	0.89101	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.22712	-1.35388	0.60124	NOT REJECTED	-0.32722	-0.58589	0.44467	NOT REJECTED	-0.01213	-0.01024	-0.00462	REJECTED	0.76698	0.80037	0.33293	0.89618	0.22136	0.92337
q_74 _log	(2,1,0)	-391.95	-382.63	-0.38687	-0.58247	-0.18588	REJECTED	-0.27679	-0.48076	-0.08426	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01206	-0.01014	-0.00405	REJECTED	0.71482	0.76951	0.33890	0.96575	0.35359	0.95073
q_74 _log	(2,1,1)	-389.70	-378.18	-0.38546	-1.07884	0.53837	NOT REJECTED	-0.27636	-0.51798	0.19907	NOT REJECTED	-0.00154	-1.00000	0.76676	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01206	-0.01005	-0.00404	REJECTED	0.71424	0.76904	0.33863	0.96560	0.16615	0.95059
q_74 _log	(2,1,2)	-387.77	-374.11	-0.03067	-1.04777	0.95162	NOT REJECTED	-0.38435	-0.87614	0.29758	NOT REJECTED	-0.36933	-1.46514	0.72321	NOT REJECTED	0.26969	-0.66309	0.99999	NOT REJECTED	-0.01202	-0.01145	-0.00518	REJECTED	0.62071	0.70476	0.31280	0.98277	0.40488	0.96583
q_75 _log	(0,1,0)	-377.28	-372.52	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01123	-0.01659	-0.00555	REJECTED	0.45032	0.39994	0.02111	0.00048	0.09746	0.99162
q_75 _log	(1,1,0)	-384.92	-377.85	-0.32906	-0.51291	-0.12805	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01127	-0.01254	-0.00456	REJECTED	0.45231	0.39269	0.00204	0.05496	0.27820	0.98095
q_75 _log	(0,1,1)	-386.21	-379.14	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.36161	-0.60699	-0.15844	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01136	-0.01448	-0.00774	REJECTED	0.56783	0.49760	0.00366	0.17215	0.29746	0.94285
q_75 _log	(1,1,1)	-384.10	-374.78	-0.07907	-0.65369	0.72233	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.29575	-1.00000	0.33913	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01134	-0.01389	-0.00704	REJECTED	0.58878	0.50966	0.00357	0.16843	0.16568	0.95201
q_75 _log	(0,1,2)	-384.17	-374.85	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.38420	-0.64532	-0.19570	REJECTED	0.05105	-0.20622	0.28316	NOT REJECTED	-0.01131	-0.01455	-0.00803	REJECTED	0.60843	0.52449	0.00360	0.16372	0.17499	0.96208
q_75 _log	(1,1,2)	-382.31	-370.79	0.39942	-0.95243	0.91780	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.79248	-1.38166	0.62469	NOT REJECTED	0.24477	-0.44608	0.53317	NOT REJECTED	-0.01116	-0.02244	-0.01463	REJECTED	0.63778	0.55156	0.00386	0.16775	0.16191	0.98367
q_75 _log	(2,1,0)	-384.65	-375.32	-0.37895	-0.59038	-0.16731	REJECTED	-0.14899	-0.35687	0.04942	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01131	-0.01067	-0.00395	REJECTED	0.65871	0.58805	0.00411	0.22837	0.09932	0.96471
q_75 _log	(2,1,1)	-382.79	-371.27	0.60294	-1.28682	0.71102	NOT REJECTED	0.26256	-0.41469	0.34063	NOT REJECTED	-0.99996	-1.00000	0.97776	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01188	-0.08953	-0.08696	REJECTED	0.43766	0.34606	0.00281	0.07754	0.79548	0.40327
q_75 _log	(2,1,2)	-381.08	-367.42	0.81069	-1.28802	1.09566	NOT REJECTED	0.10181	-0.84162	0.68226	NOT REJECTED	-1.22616	-1.46396	0.98124	NOT REJECTED	0.22617	-0.89493	0.99999	NOT REJECTED	-0.01181	-0.13581	-0.13314	REJECTED	0.48668	0.41060	0.00486	0.14644	0.60460	0.45251