



# Final Project – Mortality Rates Prediction

Time Series Analysis

Asier Gil Sedano

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## **Acronyms**

ARIMA – Autoregressive integrated moving average

ACF – Autocorrelation Factors

PACF – Partial Autocorrelation Factors

VaR – Value at risk

TVaR – Tail value at risk

BP test – Breusch Pagan test

LB test – Ljung-Box test

ADF test – Augmented Dickey Fuller test

KPSS test –Kwiatkowski–Phillips–Schmidt–Shin test

AICc – Akaike Information Criterion

BIC – Bayesian Information Criterion

# 1. Overview

In this Project, an insurance company risk for the next year is going to be calculated by the estimation of the mortality rates of the policyholders. The objective of the model is to properly evaluate the premium risk (the risk of having more claims than expected) so the economic capital can be estimated.

The insurance company to be modelled is composed by the following number of policies:

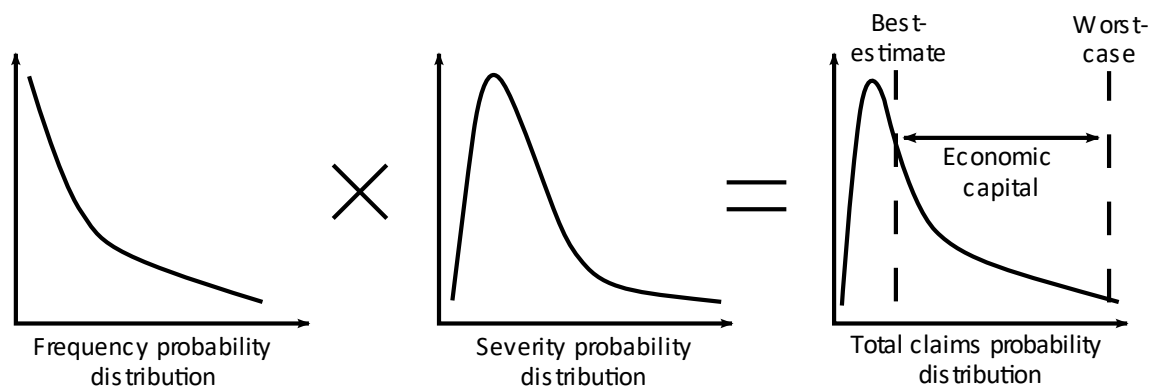
Age	Policies
67	902
68	659
69	1471
70	978
71	675
72	850
73	882
74	1035
75	995

The historical mortality rates of the policyholders age is available since 1935 up to 2021.

The project has four main parts:

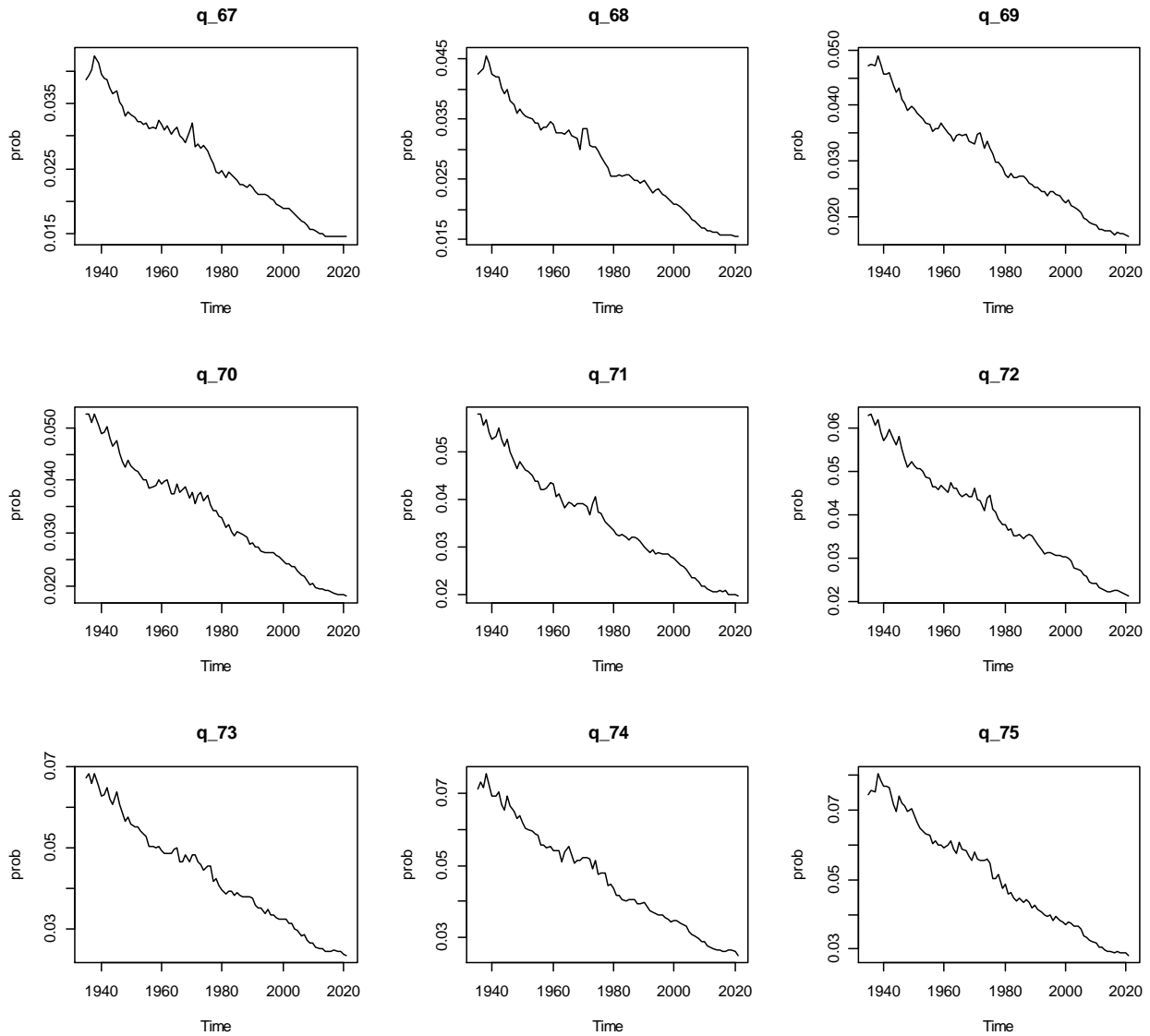
1. Mortality rates time series ARIMA functional form estimation by Box-Jenkins procedure
2. Estimation and diagnosis of the ARIMA model
3. One year period mortality rate prediction
4. Cost estimation and economic capital calculation

With this model, the expected cost will be evaluated together with the  $VaR_{99}$  (maximum expected cost in the 99% of the cases, or which is the same, maximum expected cost in 1 out 100 years) and  $TVaR_{99}$  (expected cost if the cost surpasses the  $VaR_{99}$ ). This way the worst-case scenarios will be quantified, and the company solvency capital requirement will be determined.



## 2. Available data

The following [mortality time series](#) are available for which we need to predict the 2022 value.



### 3. Functional form identification by Box-Jenkins procedure

According to previous studies, Lee and Carter model (1992), the log-mortality rates are ARIMA processes. Therefore, for all time series, the log-qx have been calculated. Additionally, it has been checked that the time series are not stationary (see the code lines 93 to 117) so directly the analysis will be performed on the differentiated time series. The diagnosis and all the test are coded in lines 155 to 274.

#### 3.1. Mortality rate $q_{67}$ ARIMA estimation

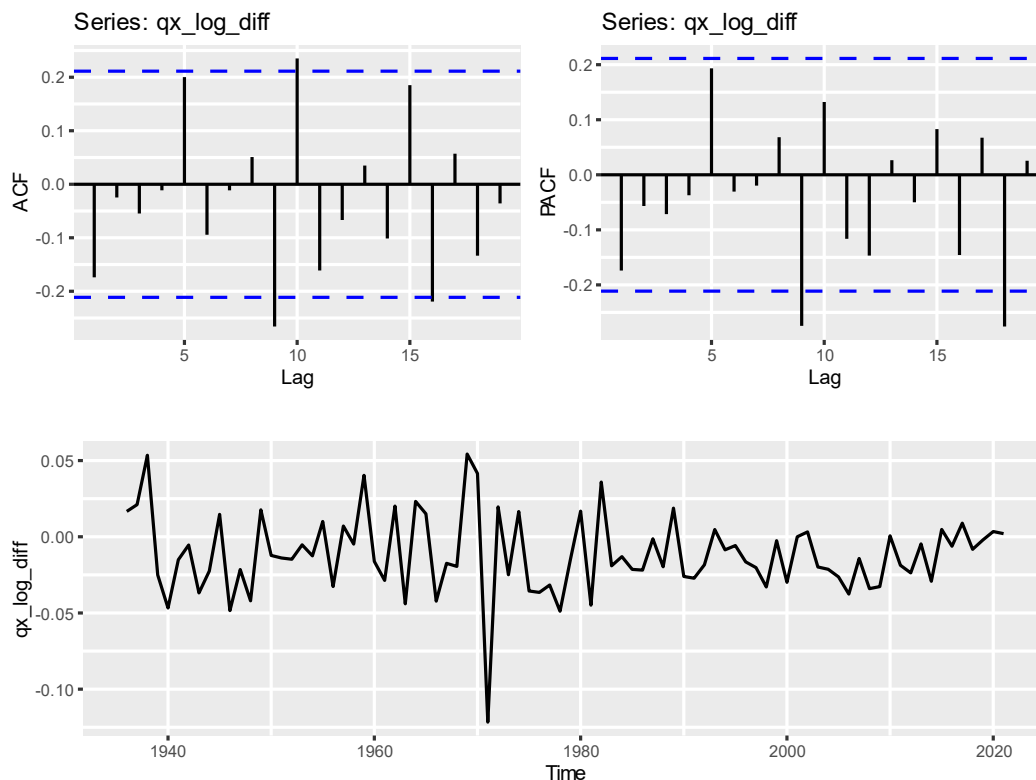
The ADF (Augmented Dickey-Fuller), PP (Phillips Perron) and KPSS (Kwiatkowski–Phillips–Schmidt–Shin) tests have been performed showing the following results for the  $I(0)$  and  $I(1)$  time series.

$I(0)$	p.value	stationary 0.05
ADF	0.75961	FALSE
PP	0.31752	FALSE
KPSS	0.01000	FALSE

$I(1)$	p.value	stationary 0.05
ADF	0.04017	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

It is checked that all the test show the differentiated time series to be stationary for the significance level of 0.05.

For the log-mortality rates of the age 67, the differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

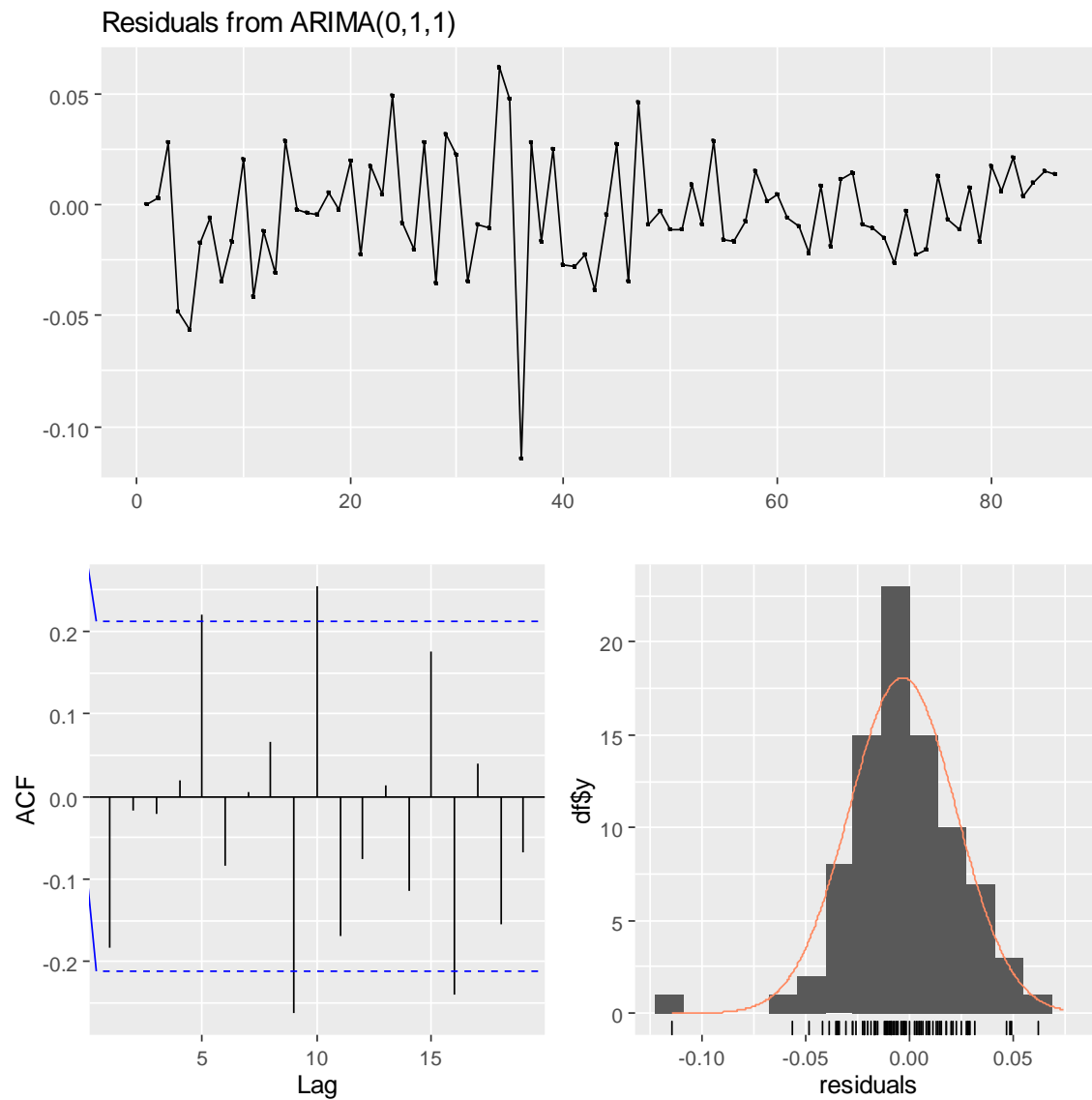
According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) proves to be the best fitting model with the following results:

time_series	q_67_log
ARIMA	(0,1,1)
AICc	-381.44546
BIC	-374.37510
AR1	0.00000
AR1_H0_inf	#N/A
AR1_H0_sup	#N/A
AR1_H0	#N/A
AR2	0.00000
AR2_H0_inf	#N/A
AR2_H0_sup	#N/A
AR2_H0	#N/A
MA1	-0.20214
MA1_H0_inf	-0.47090
MA1_H0_sup	-0.00162
MA1_H0	REJECTED
MA2	0.00000
MA2_H0_inf	#N/A
MA2_H0_sup	#N/A
MA2_H0	#N/A
drift	-0.01151
drift_H0_inf	-0.01593
drift_H0_sup	-0.00749
drift_H0	REJECTED
Normality_CVM_pvalue	0.57427
Normality_AD_pvalue	0.46396
Normality_JB_pvalue	0.00015
Incorrelation_LB	0.26379
Homocedasticity_BP_B	0.99784
Zero_mean	0.98219

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:





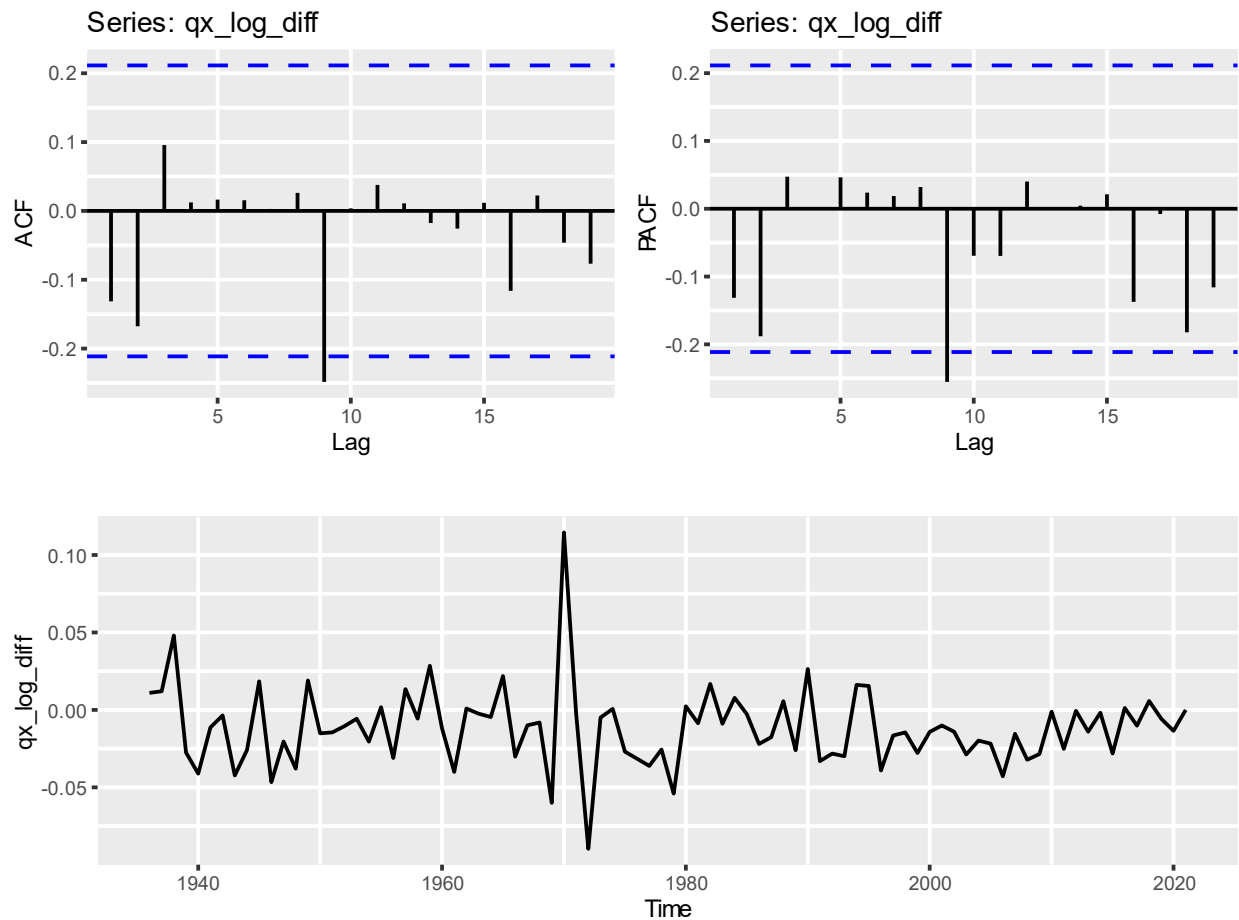
### 3.2. Mortality rate $q_{68}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.69155	FALSE
PP	0.41466	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01523	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

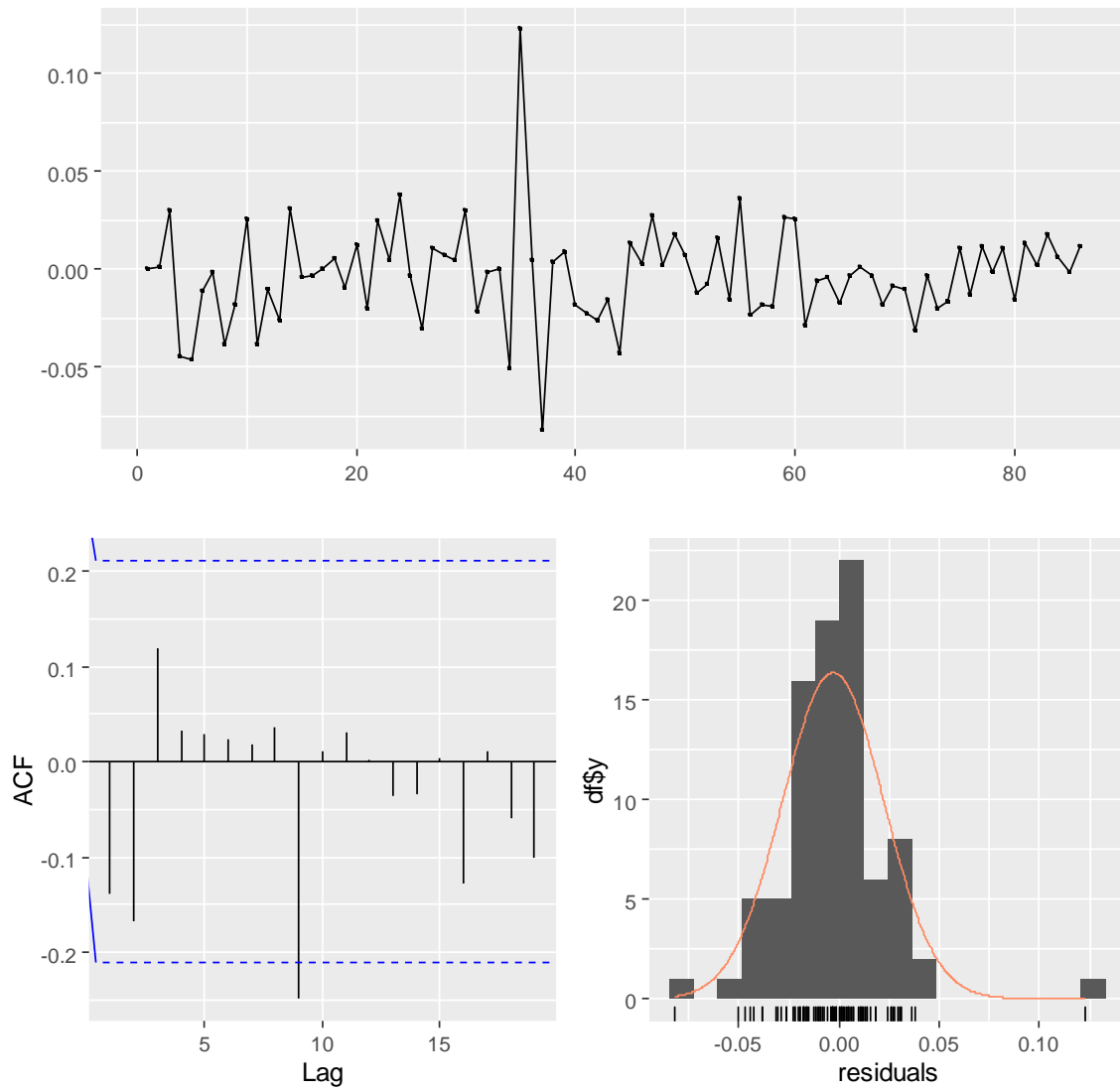
The ARIMA(0,1,1) proves to be the best fitting model with the following results:

<b>time_series</b>	q_68_log
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-384.38390
<b>BIC</b>	-377.31354
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.19205
<b>MA1_H0_inf</b>	-0.43452
<b>MA1_H0_sup</b>	-0.00261
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01188
<b>drift_H0_inf</b>	-0.01601
<b>drift_H0_sup</b>	-0.00751
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.43926
<b>Normality_AD_pvalue</b>	0.36454
<b>Normality_JB_pvalue</b>	0.00000
<b>Incorrelation_LB</b>	0.49218
<b>Homocedasticity_BP_B</b>	0.58348
<b>Zero_mean</b>	0.98894

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

Residuals from ARIMA(0,1,1)



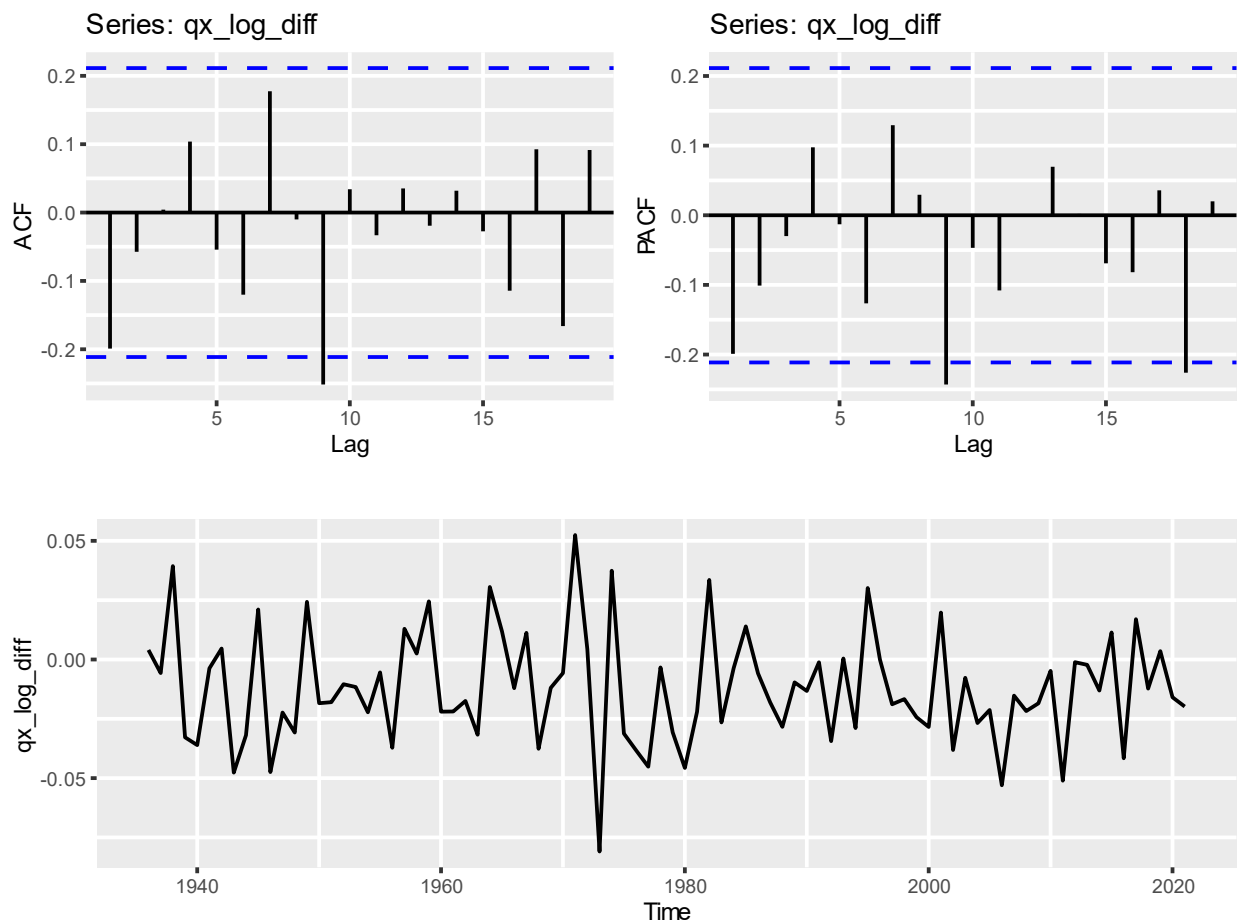
### 3.3. Mortality rate $q_{69}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.56396	FALSE
PP	0.42881	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01857	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

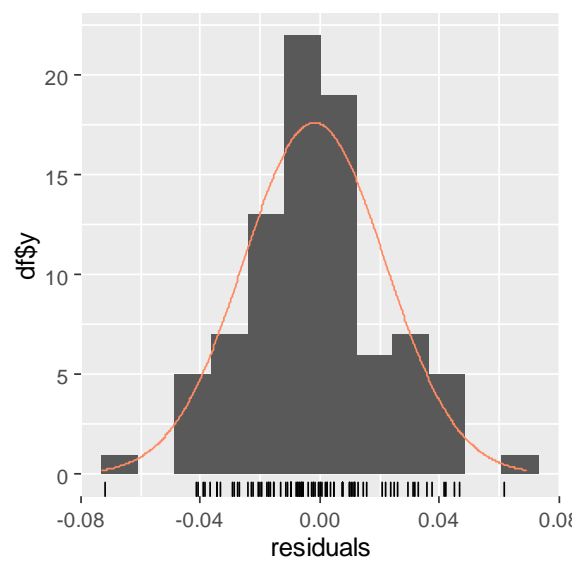
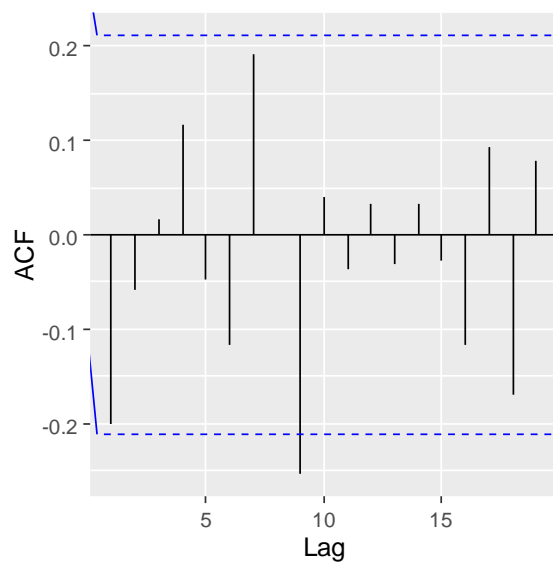
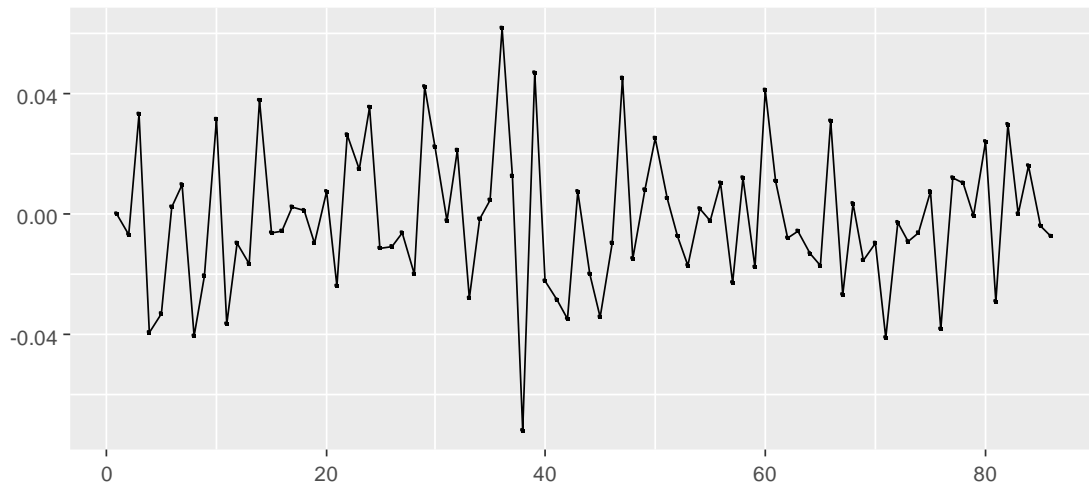
The ARIMA(0,1,1) proves to be the best fitting model with the following results:

<b>time_series</b>	<b>q_69_log</b>
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-398.27097
<b>BIC</b>	-391.20061
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.23209
<b>MA1_H0_inf</b>	-0.49352
<b>MA1_H0_sup</b>	-0.01913
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01226
<b>drift_H0_inf</b>	-0.01577
<b>drift_H0_sup</b>	-0.00824
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.76858
<b>Normality_AD_pvalue</b>	0.87554
<b>Normality_JB_pvalue</b>	0.57133
<b>Incorrelation_LB</b>	0.31720
<b>Homocedasticity_BP_B</b>	0.12939
<b>Zero_mean</b>	0.99118

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

Residuals from ARIMA(0,1,1)



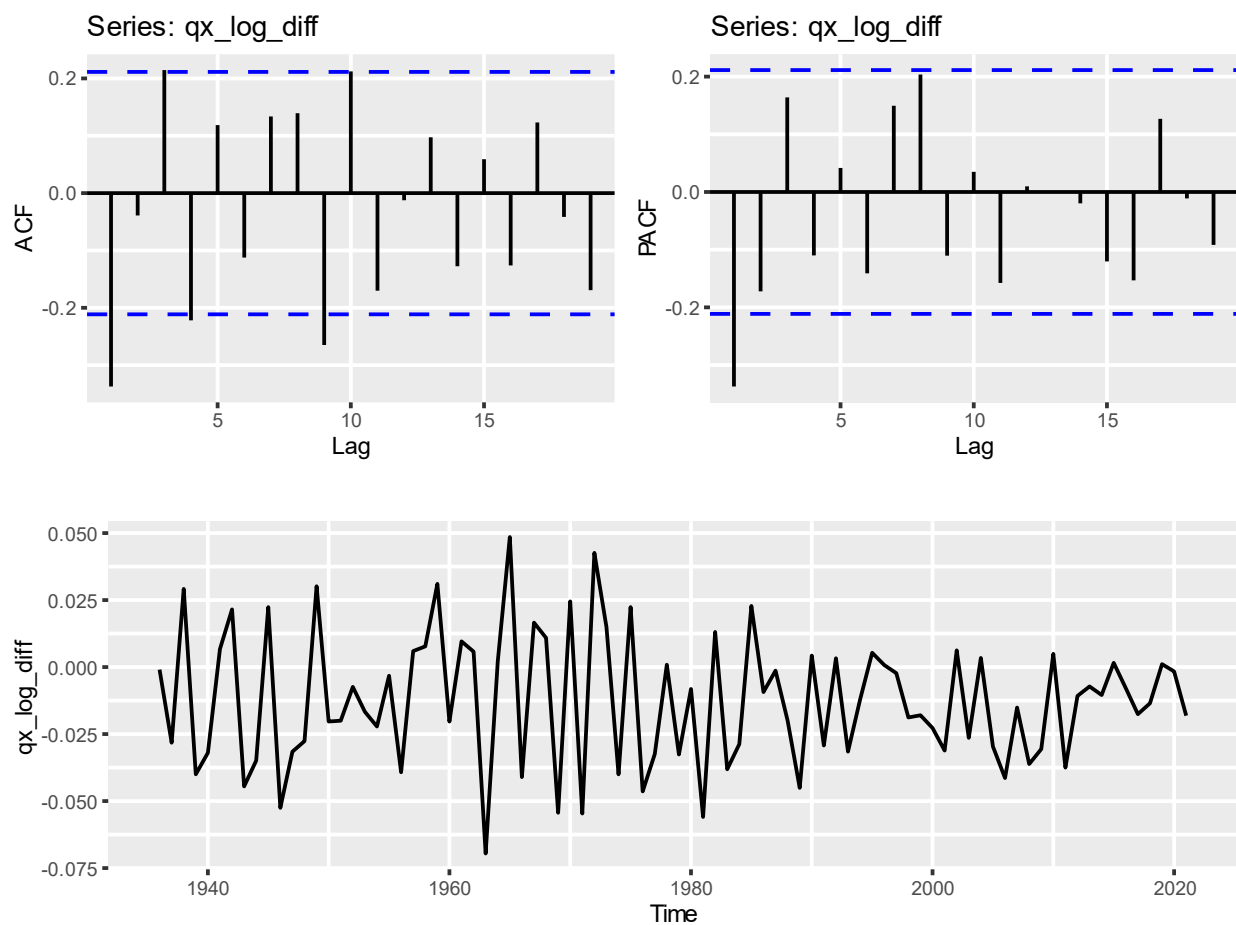
### 3.4. Mortality rate $q_{70}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.81072	FALSE
PP	0.61122	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01000	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) proves to be the best fitting model with the following results:

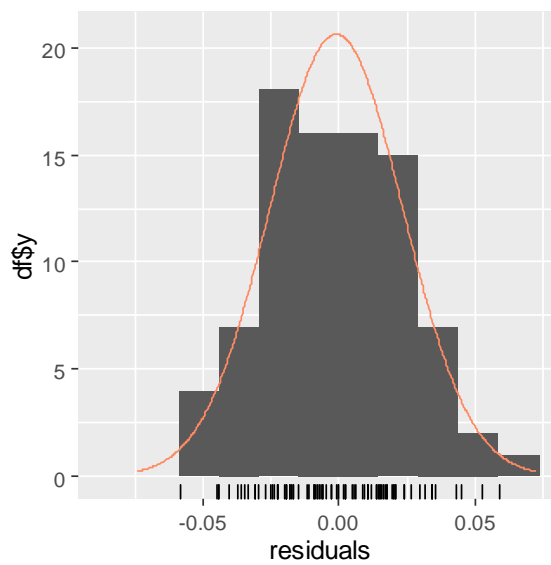
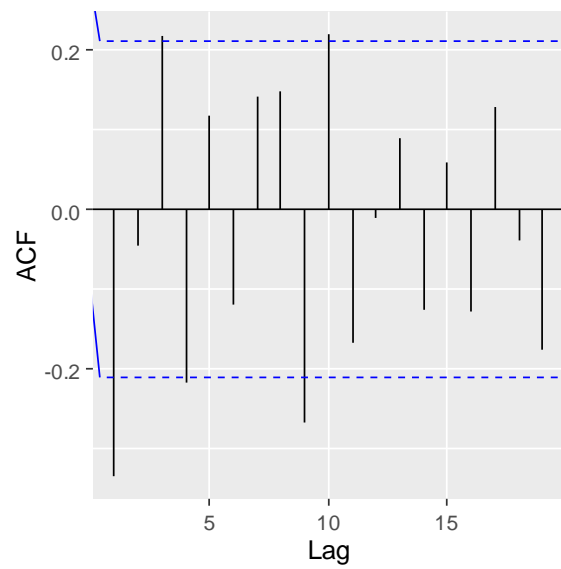
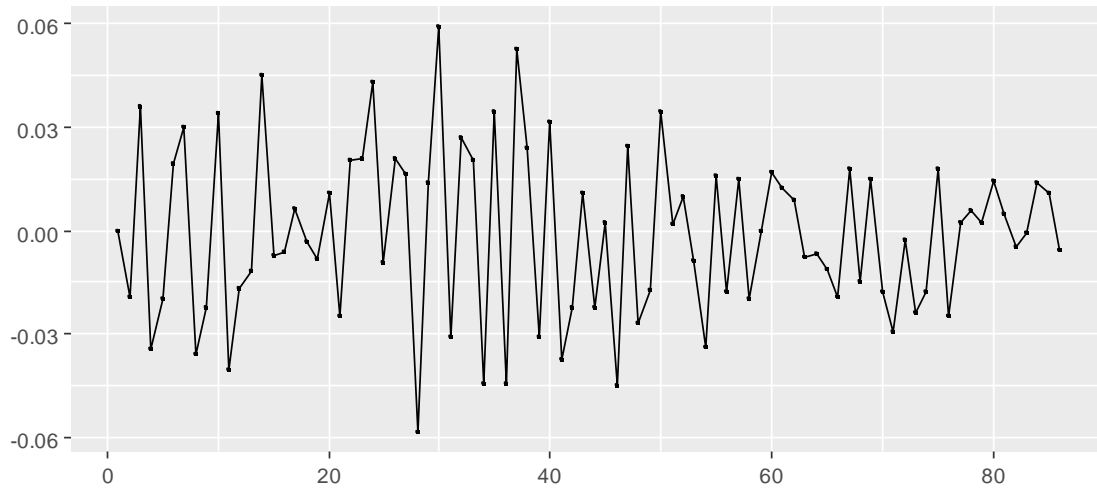
<b>time_series</b>	<b>q_70_log</b>
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-400.18847
<b>BIC</b>	-393.11811
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.35757
<b>MA1_H0_inf</b>	-0.62077
<b>MA1_H0_sup</b>	-0.16846
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01247
<b>drift_H0_inf</b>	-0.01534
<b>drift_H0_sup</b>	-0.00937
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.92672
<b>Normality_AD_pvalue</b>	0.91772
<b>Normality_JB_pvalue</b>	0.46602
<b>Incorrelation_LB</b>	0.15678
<b>Homocedasticity_BP_B</b>	0.41163
<b>Zero_mean</b>	0.99543

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:



Residuals from ARIMA(0,1,1)



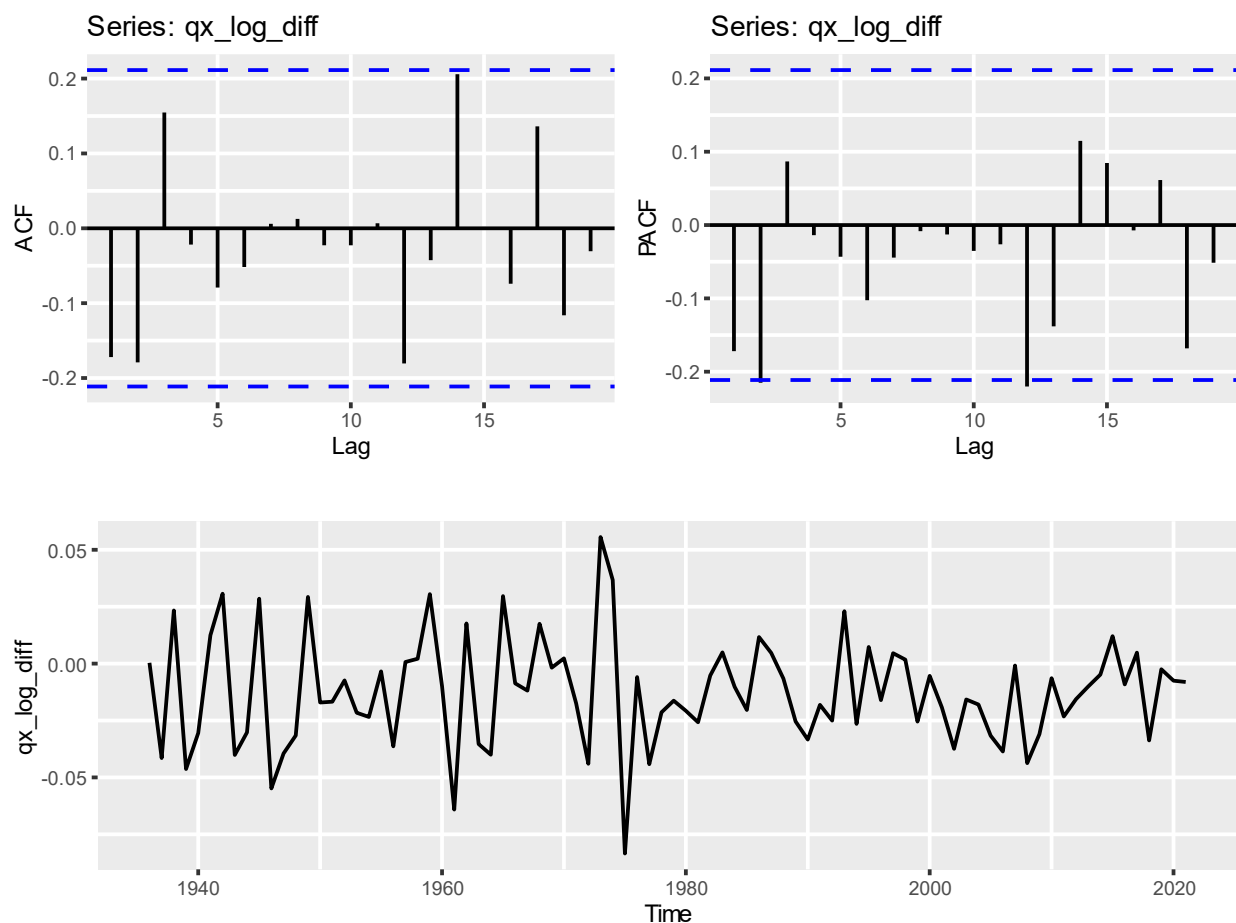
### 3.5. Mortality rate $q_{71}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.46113	FALSE
PP	0.25084	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.0100	TRUE
PP	0.0100	TRUE
KPSS	0.1000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

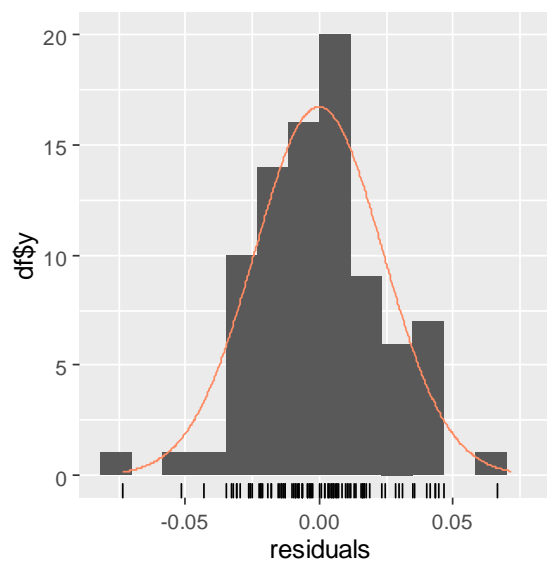
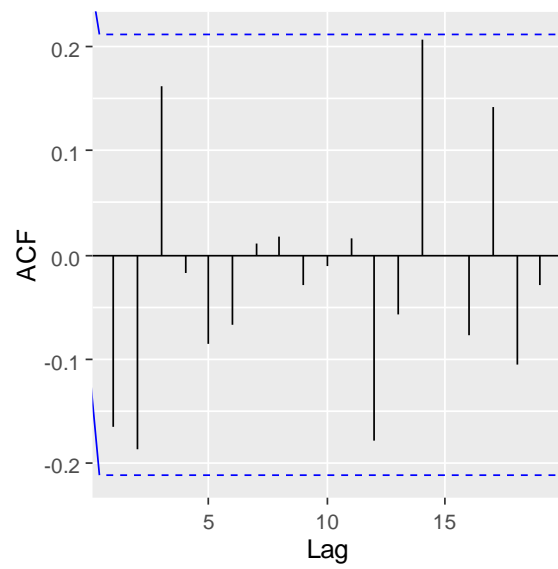
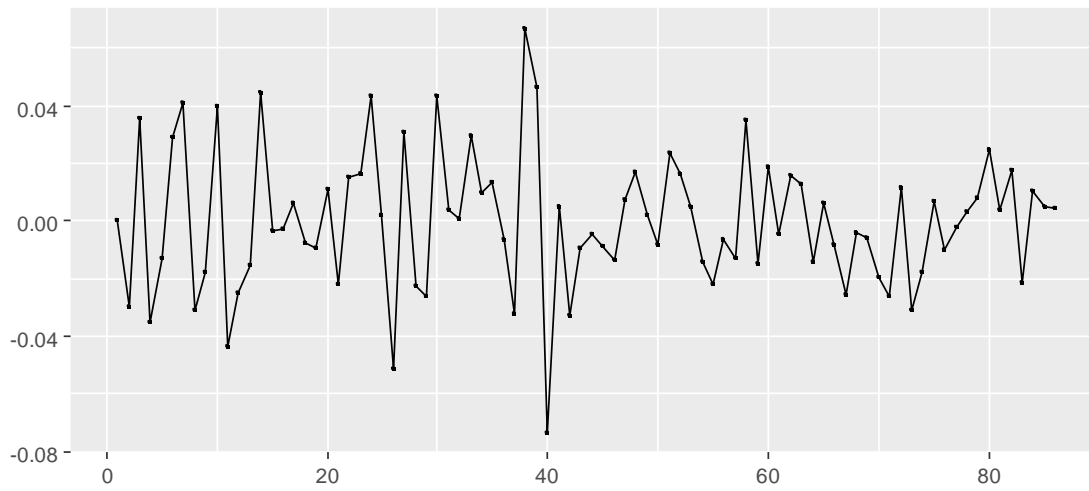
The ARIMA(0,1,1) proves to be the best fitting model with the following results:

<b>time_series</b>	<b>q_71_log</b>
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-396.24571
<b>BIC</b>	-389.17535
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.24286
<b>MA1_H0_inf</b>	-0.52452
<b>MA1_H0_sup</b>	-0.03119
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01253
<b>drift_H0_inf</b>	-0.01614
<b>drift_H0_sup</b>	-0.00871
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.96312
<b>Normality_AD_pvalue</b>	0.95826
<b>Normality_JB_pvalue</b>	0.52026
<b>Incorrelation_LB</b>	0.89443
<b>Homocedasticity_BP_B</b>	0.86741
<b>Zero_mean</b>	0.99405

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

Residuals from ARIMA(0,1,1)



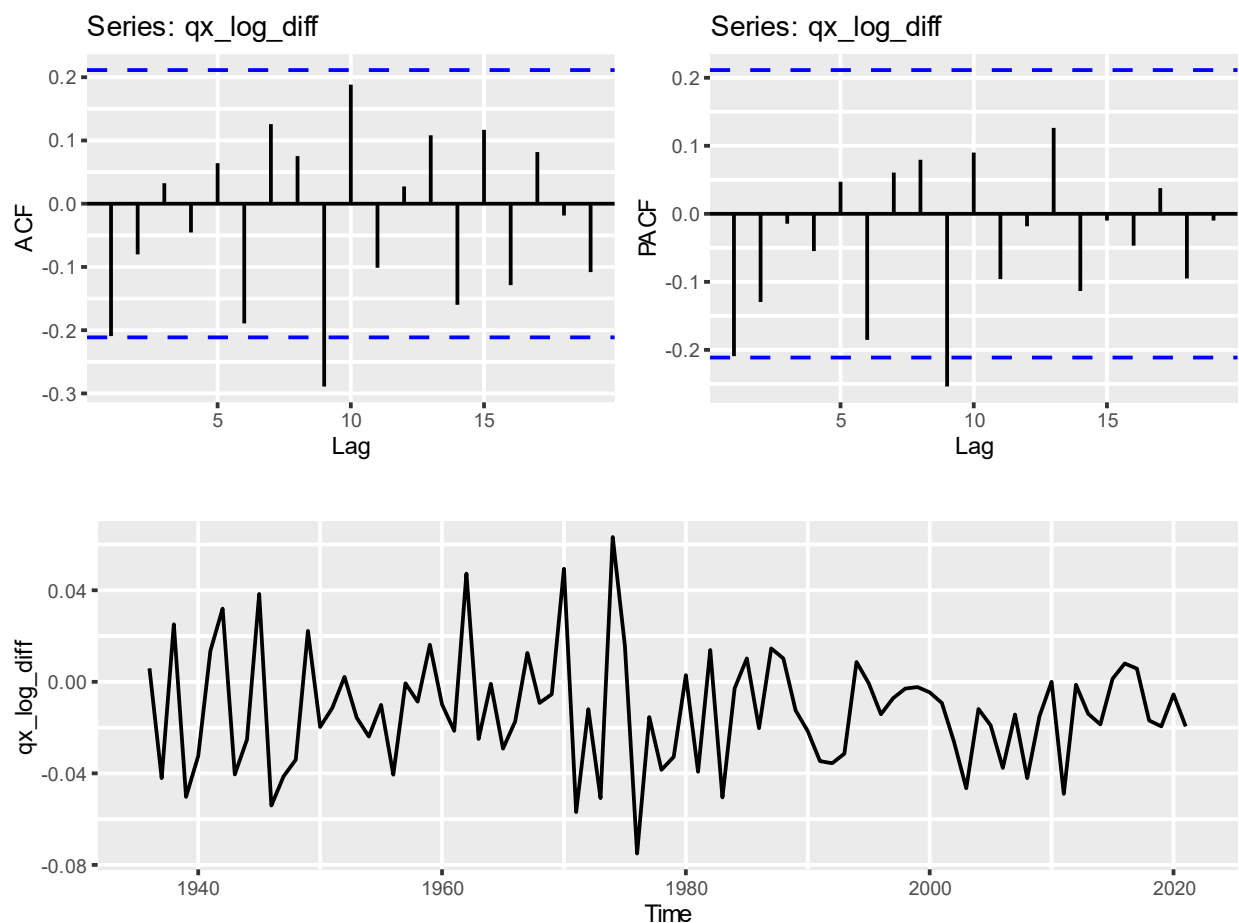
### 3.6. Mortality rate $q_{72}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.65098	FALSE
PP	0.43575	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01000	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)

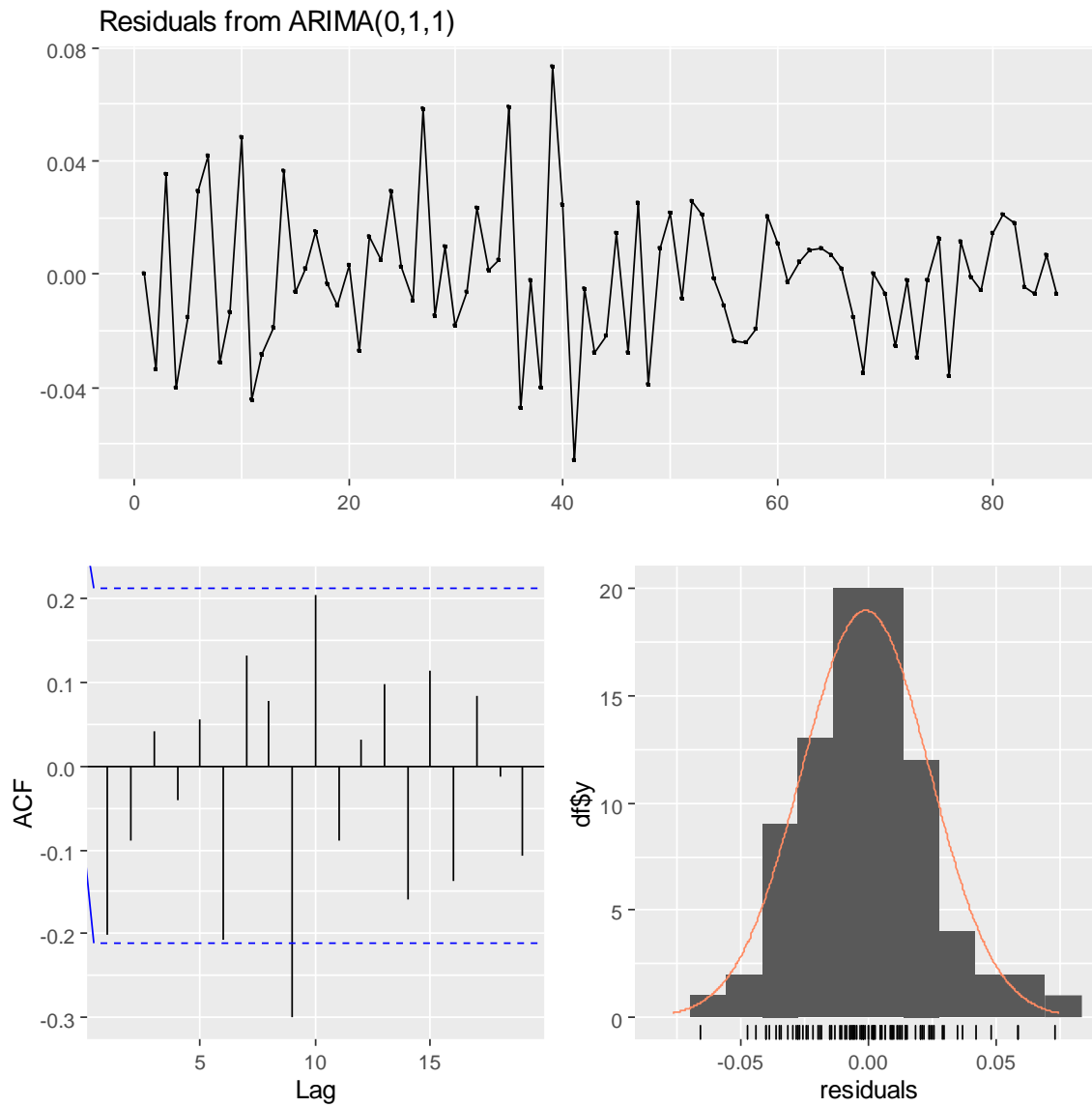
According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

The ARIMA(0,1,1) proves to be the best fitting model with the following results:

<b>time_series</b>	<b>q_72_log</b>
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-388.78772
<b>BIC</b>	-381.71737
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.25911
<b>MA1_H0_inf</b>	-0.49469
<b>MA1_H0_sup</b>	-0.04240
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01255
<b>drift_H0_inf</b>	-0.01657
<b>drift_H0_sup</b>	-0.00857
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.78017
<b>Normality_AD_pvalue</b>	0.65034
<b>Normality_JB_pvalue</b>	0.22729
<b>Incorrelation_LB</b>	0.31635
<b>Homocedasticity_BP_B</b>	0.07697
<b>Zero_mean</b>	0.99740

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:



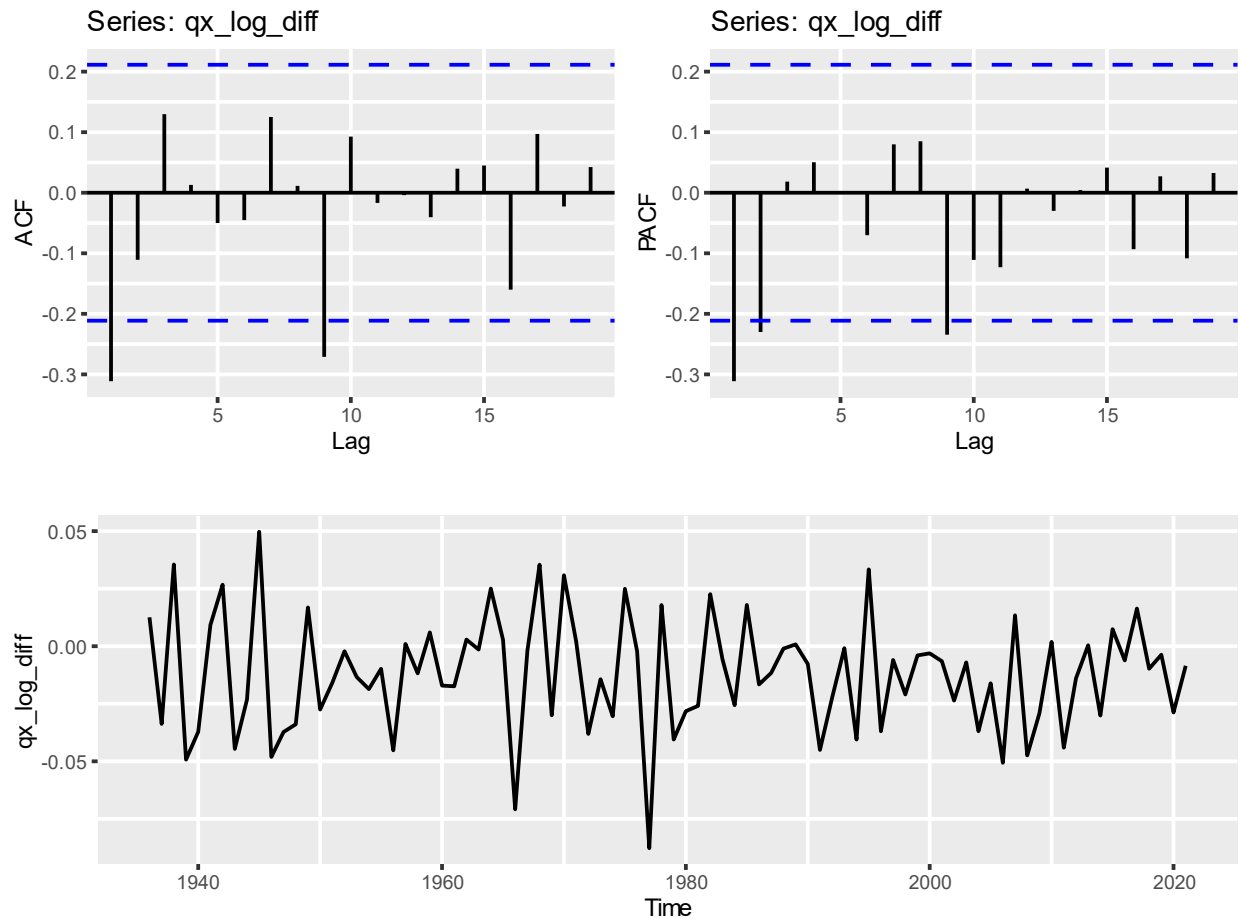
### 3.7. Mortality rate $q_{73}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.58792	FALSE
PP	0.20170	FALSE
KPSS	0.01000	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01000	TRUE
PP	0.01000	TRUE
KPSS	0.10000	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)
- (2,1,1)
- (2,1,0)
- (2,1,1)



According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

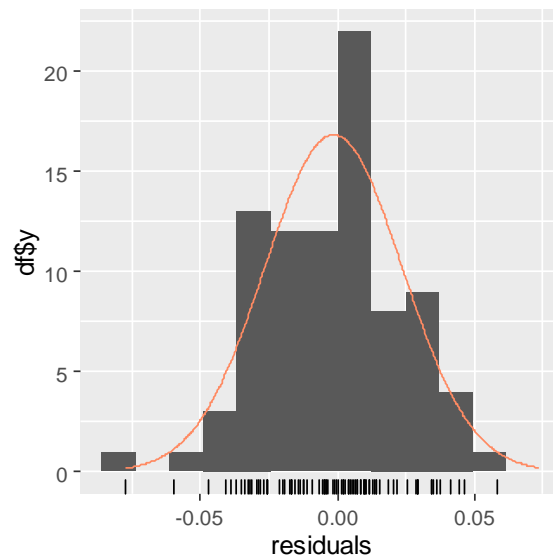
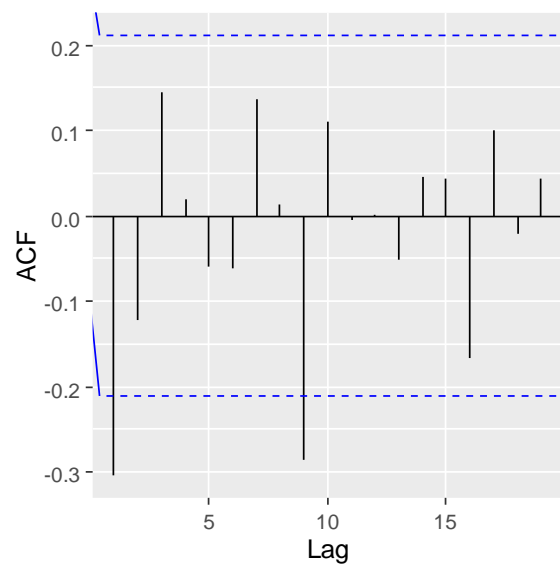
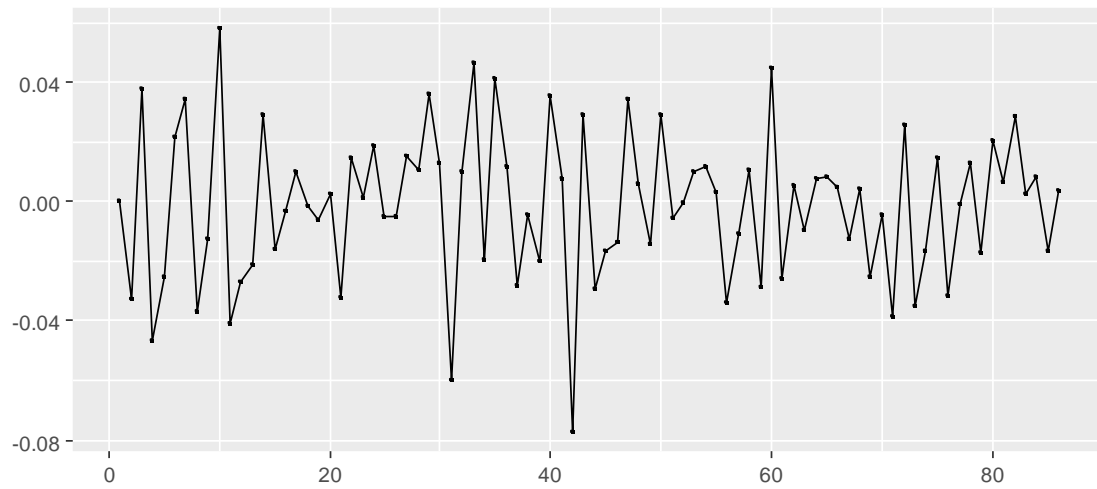
The ARIMA(0,1,1) proves to be the best fitting model with the following results:

<b>time_series</b>	<b>q_73_log</b>
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-396.23492
<b>BIC</b>	-389.16456
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.39198
<b>MA1_H0_inf</b>	-0.63862
<b>MA1_H0_sup</b>	-0.19727
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01236
<b>drift_H0_inf</b>	-0.01530
<b>drift_H0_sup</b>	-0.00935
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.99178
<b>Normality_AD_pvalue</b>	0.98878
<b>Normality_JB_pvalue</b>	0.99511
<b>Incorrelation_LB</b>	0.28037
<b>Homocedasticity_BP_B</b>	0.33750
<b>Zero_mean</b>	0.97290

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

Residuals from ARIMA(0,1,1)



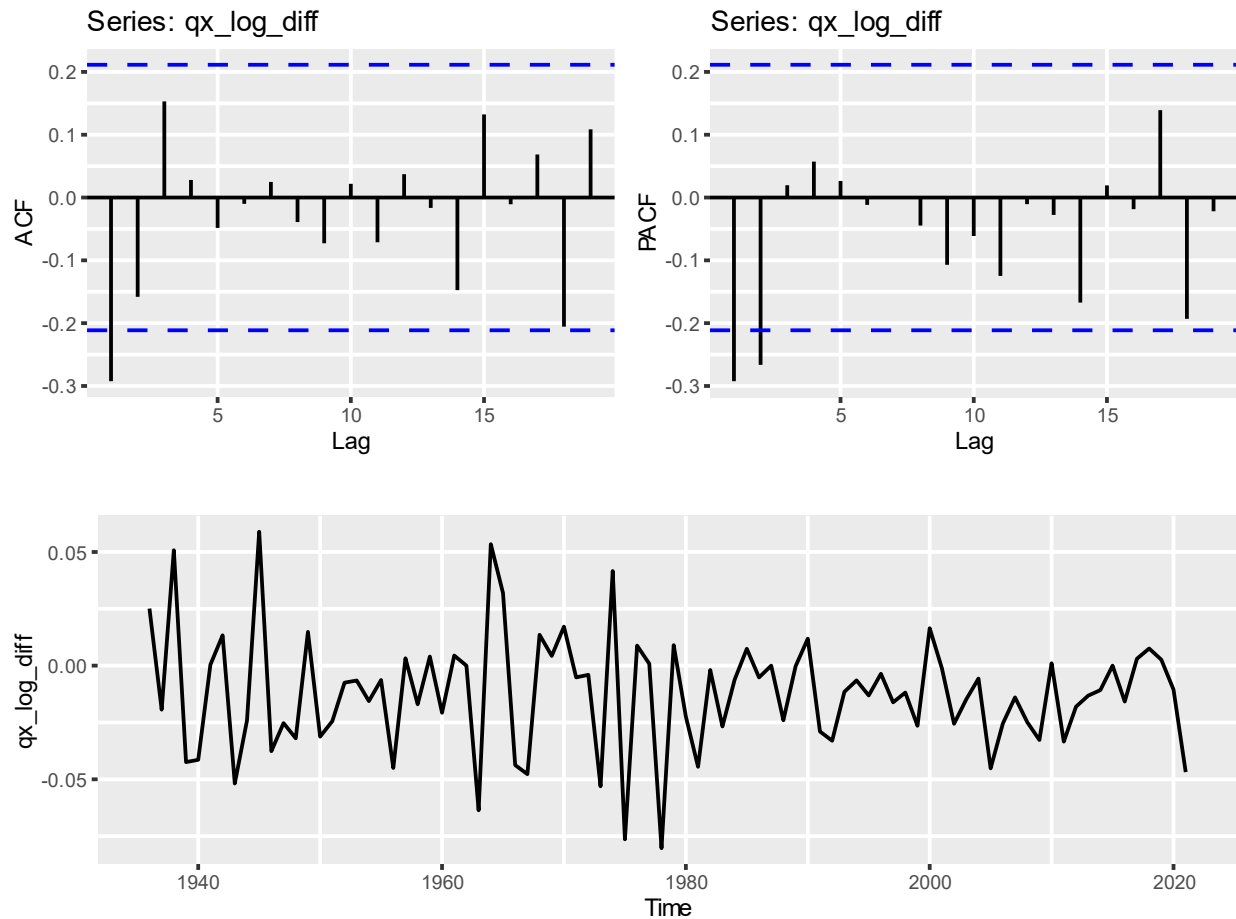
### 3.8. Mortality rate $q_{74}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.50764	FALSE
PP	0.02272	TRUE
KPSS	0.01	FALSE

I(1)	p.value	stationary 0.05
ADF	0.01731	TRUE
PP	0.01	TRUE
KPSS	0.1	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)
- (2,1,1)
- (2,1,0)
- (2,1,1)

According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

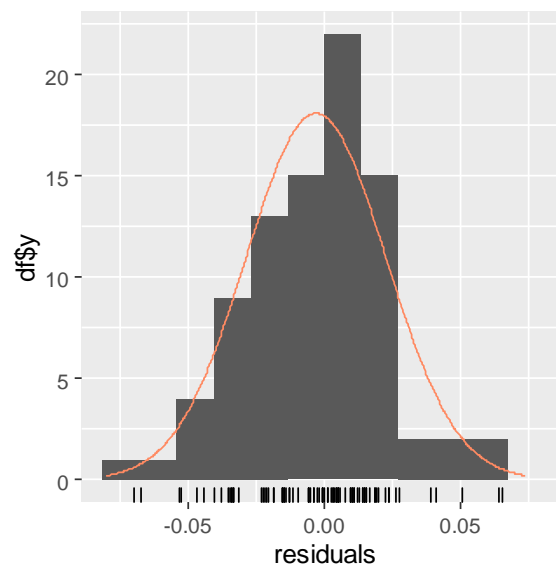
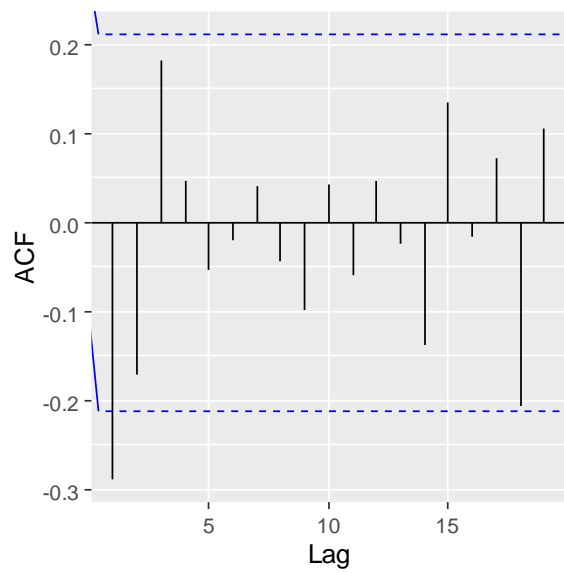
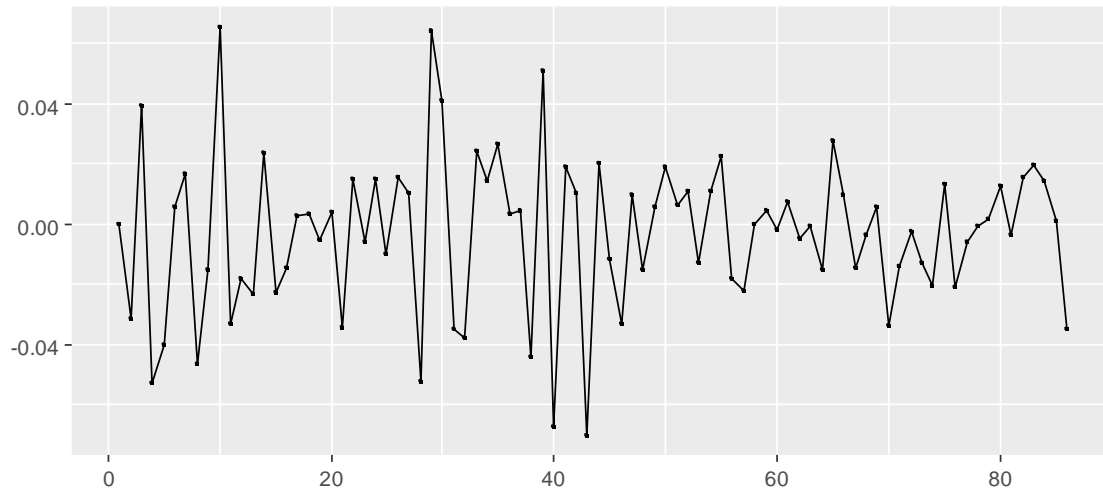
In this case both the ARIMA(0,1,1) and (2,1,0) prove to be a good fit and provide good results in the diagnosis. In any case, since all the other time series are clearly an ARIMA(0,1,1) and the process will probably be the same, the ARIMA(0,1,1) has been selected for modelling.

<b>time_series</b>	<b>q_74_log</b>	<b>q_74_log</b>
<b>ARIMA</b>	(0,1,1)	(2,1,0)
<b>AICc</b>	-391.65912	-391.95475
<b>BIC</b>	-384.58876	-382.63119
<b>AR1</b>	0.00000	-0.38687
<b>AR1_H0_inf</b>	#N/A	-0.58247
<b>AR1_H0_sup</b>	#N/A	-0.18588
<b>AR1_H0</b>	#N/A	REJECTED
<b>AR2</b>	0.00000	-0.27679
<b>AR2_H0_inf</b>	#N/A	-0.48076
<b>AR2_H0_sup</b>	#N/A	-0.08426
<b>AR2_H0</b>	#N/A	REJECTED
<b>MA1</b>	-0.41636	0.00000
<b>MA1_H0_inf</b>	-0.66639	#N/A
<b>MA1_H0_sup</b>	-0.23485	#N/A
<b>MA1_H0</b>	REJECTED	#N/A
<b>MA2</b>	0.00000	0.00000
<b>MA2_H0_inf</b>	#N/A	#N/A
<b>MA2_H0_sup</b>	#N/A	#N/A
<b>MA2_H0</b>	#N/A	#N/A
<b>drift</b>	-0.01212	-0.01206
<b>drift_H0_inf</b>	-0.01504	-0.01014
<b>drift_H0_sup</b>	-0.00925	-0.00405
<b>drift_H0</b>	REJECTED	REJECTED
<b>Normality_CVM_pvalue</b>	0.65720	0.71482
<b>Normality_AD_pvalue</b>	0.72732	0.76951
<b>Normality_JB_pvalue</b>	0.32767	0.33890
<b>Incorrelation_LB</b>	0.79649	0.96575
<b>Homocedasticity_BP_B</b>	0.62931	0.35359
<b>Zero_mean</b>	0.92790	0.95073

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

Residuals from ARIMA(0,1,1)



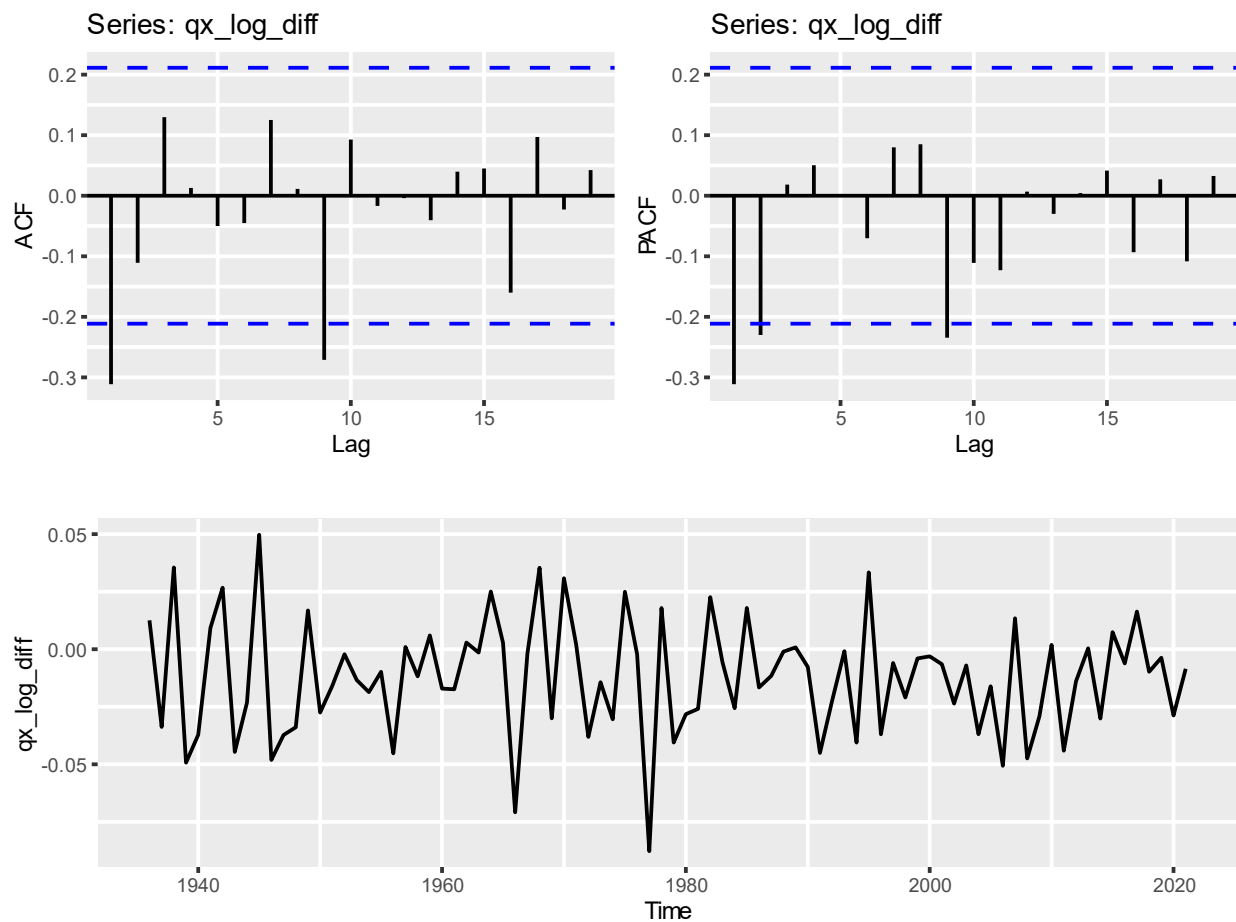
### 3.9. Mortality rate $q_{75}$ ARIMA estimation

The following stationary test have been performed.

I(0)	p.value	stationary 0.05
ADF	0.49425145	FALSE
PP	0.02324218	TRUE
KPSS	0.01	FALSE

I(1)	p.value	stationary 0.05
ADF	0.02318762	TRUE
PP	0.01	TRUE
KPSS	0.1	TRUE

The differentiated time series visualization and its ACF and PACF are shown:



According to Box-Jenkins procedure, the time series appears to be an ARIMA of orders:

- (0,1,0)
- (1,0,1)
- (0,1,1)
- (1,1,1)
- (2,1,1)
- (2,1,0)
- (2,1,1)

According to the ACF and PACF the most reliable form is (0,1,0). In any case all mentioned will be tested to confirm. All the results of the diagnosis for the time series up to an order (2,1,2) are summarized in the annex.

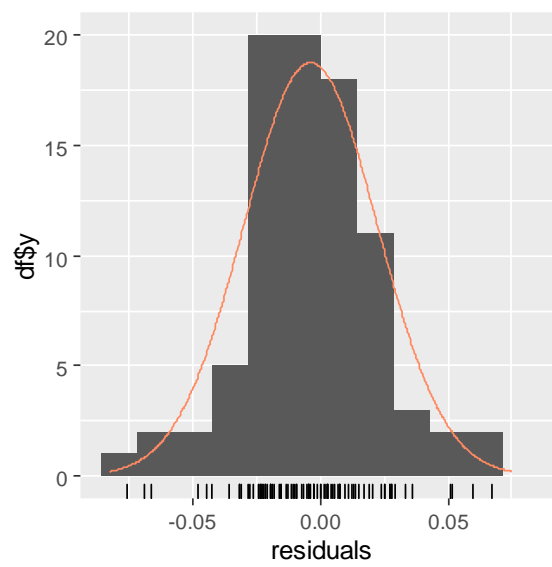
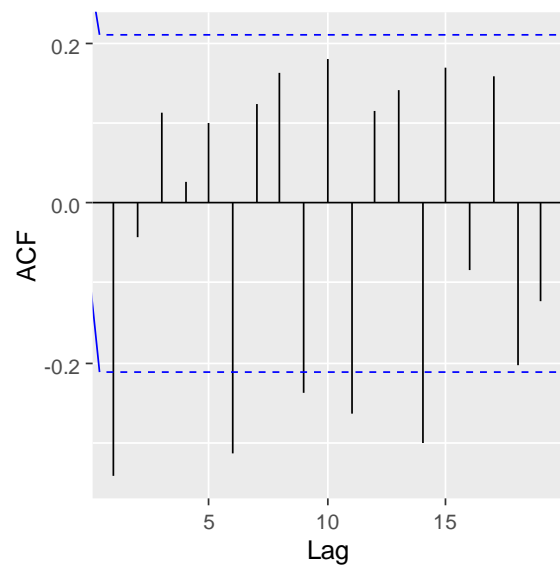
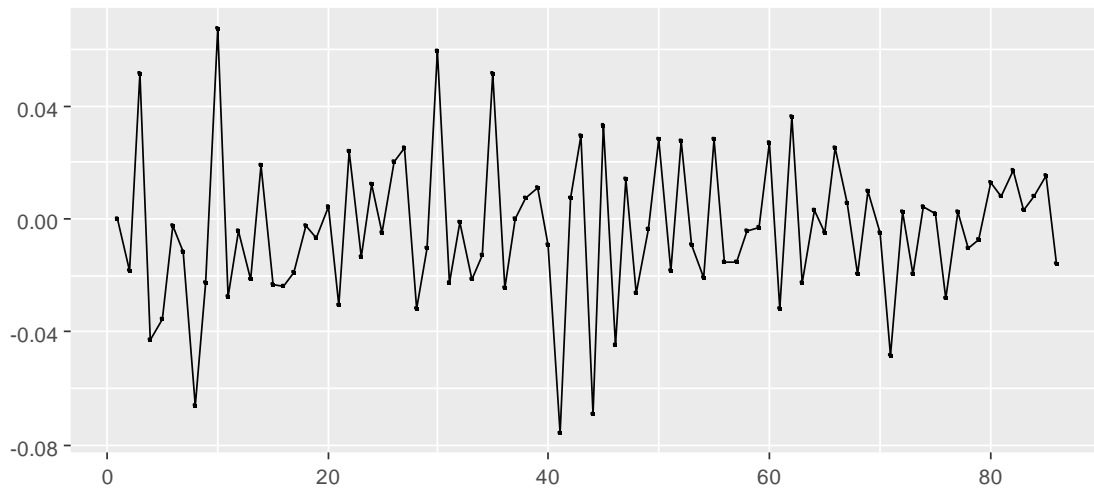
The ARIMA(0,1,1) proves to be the best fitting model with the following results:

<b>time_series</b>	<b>q_73_log</b>
<b>ARIMA</b>	(0,1,1)
<b>AICc</b>	-396.23492
<b>BIC</b>	-389.16456
<b>AR1</b>	0.00000
<b>AR1_H0_inf</b>	#N/A
<b>AR1_H0_sup</b>	#N/A
<b>AR1_H0</b>	#N/A
<b>AR2</b>	0.00000
<b>AR2_H0_inf</b>	#N/A
<b>AR2_H0_sup</b>	#N/A
<b>AR2_H0</b>	#N/A
<b>MA1</b>	-0.39198
<b>MA1_H0_inf</b>	-0.63862
<b>MA1_H0_sup</b>	-0.19727
<b>MA1_H0</b>	REJECTED
<b>MA2</b>	0.00000
<b>MA2_H0_inf</b>	#N/A
<b>MA2_H0_sup</b>	#N/A
<b>MA2_H0</b>	#N/A
<b>drift</b>	-0.01236
<b>drift_H0_inf</b>	-0.01530
<b>drift_H0_sup</b>	-0.00935
<b>drift_H0</b>	REJECTED
<b>Normality_CVM_pvalue</b>	0.99178
<b>Normality_AD_pvalue</b>	0.98878
<b>Normality_JB_pvalue</b>	0.99511
<b>Incorrelation_LB</b>	0.28037
<b>Homocedasticity_BP_B</b>	0.33750
<b>Zero_mean</b>	0.97290

All the diagnosis has been properly tested so the AR parameter and the constant are significant, the residuals are normal, uncorrelated, constant in variance and have zero mean.

The residuals can be summarized in the following graph:

Residuals from ARIMA(0,1,1)



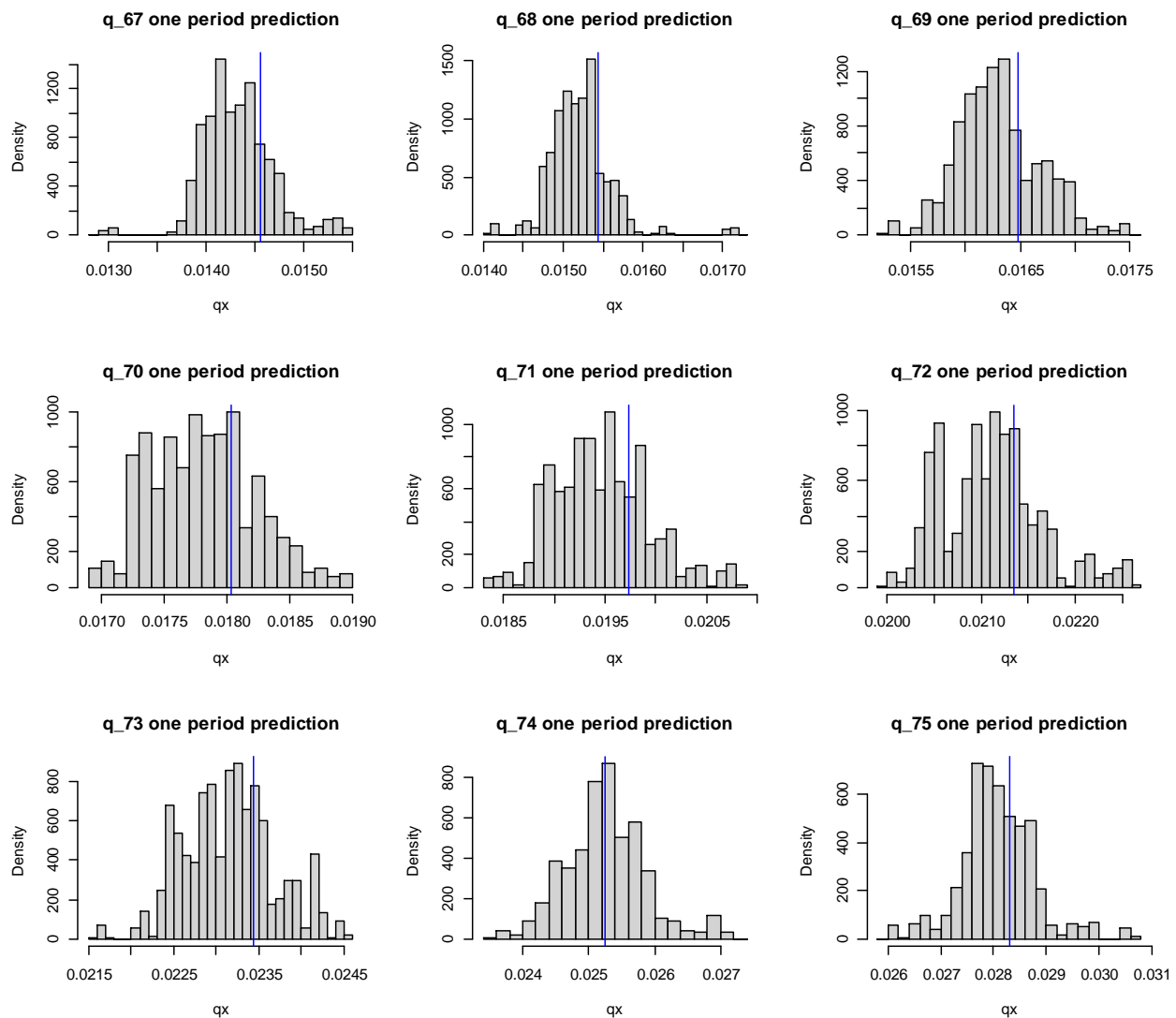


## 4. Next period qx prediction by bootstrap

Even if all the diagnosis checks are correct and the null hypothesis of the residuals to be white noise cannot be rejected, bootstrap will be used for the estimation instead of the gaussian method since it is more versatile and considers the uncertainties of the ARIMA parameters estimation as well as model specification uncertainty.

The bootstrap prediction algorithm is coded in lines 409 to 449.

The results of the estimation distribution and the 2021 value (marked with the vertical line in blue) are shown in the following graph:



## 5. Mortality prediction, cost and economic capital summary

The predictions shown in the previous part are used for the computation of the cost and the economic capital required to cover that cost. The calculation is coded in lines 480 to 538.

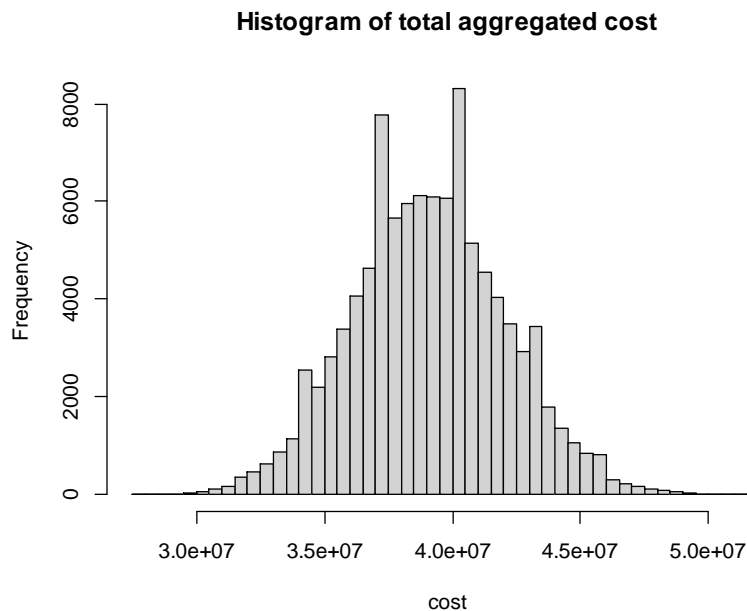
The following table summarizes all the data predicted. The expected  $q_x$  and the marginal standard deviation as well as the  $VaR_{99}$  and  $TVaR_{99}$  for the cost by age groups.

Age	$q_x$ _current	$q_x$ _expected	$q_x$ _expected_sd	$q_x$ _VaR99	$q_x$ _TVaR99	n_policies	cost_expected	cost_sd	cost_VaR99	cost_TVaR99	economic_capital
67	0.01455	0.01433	0.00036	0.01536	0.01542	902	2,972,509	825,705	5,060,000	5,512,539	2,087,492
68	0.01543	0.01521	0.00038	0.01704	0.01711	659	2,306,376	723,342	4,140,000	4,557,033	1,833,624
69	0.01648	0.01631	0.00038	0.01739	0.01744	1471	5,518,827	1,122,437	8,280,000	8,842,602	2,761,173
70	0.01803	0.01781	0.00041	0.01888	0.01892	978	4,010,229	951,867	6,440,000	6,963,739	2,429,771
71	0.01974	0.01946	0.00046	0.02073	0.02077	675	3,025,271	827,874	5,060,000	5,539,776	2,034,730
72	0.02134	0.02111	0.00052	0.02253	0.02256	850	4,128,424	968,228	6,440,000	6,962,181	2,311,576
73	0.02343	0.02315	0.00053	0.02443	0.02448	882	4,697,543	1,036,329	7,360,000	7,874,576	2,662,457
74	0.02524	0.02526	0.00061	0.02697	0.02704	1035	6,016,671	1,165,096	8,970,000	9,535,517	2,953,329
75	0.02831	0.02809	0.00070	0.03048	0.03057	995	6,416,471	1,207,581	9,430,000	9,996,869	3,013,529

Additionally, the following summary table is provided with the aggregated model considering all the policies globally and running all together.

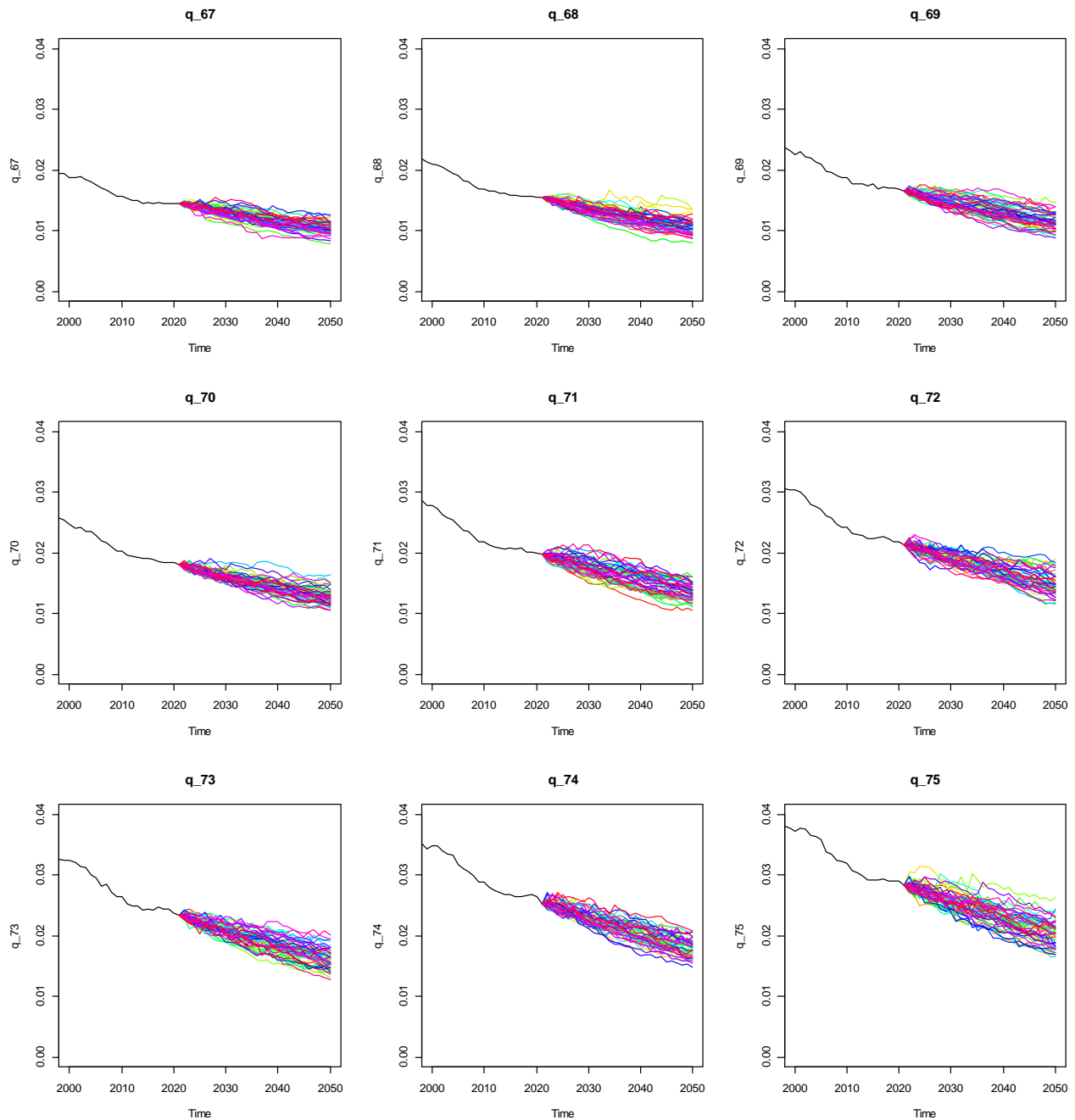
cost_total_expected	cost_total_sd	cost_total_VaR99	cost_total_TVaR99	total_economic_capital
39,091,145	2,980,131	46,230,000	47,374,907	7,138,855

The total cost distribution is as follows:



## 6. Further periods mortality rates prediction

Additionally, further mortality rates estimation predictions are provided for a time horizon greater than one period up to year 2050. The following graphs show these predictions for the ages previously studied with the estimated models. The prediction is performed by the same bootstrap technique used for the one period prediction.



## 7. Annex – Diagnosis Results

[See online](#)

time_series	ARIMA	AICc	BIC	AR1	AR1_H0_inf	AR1_H0_sup	AR1_H0	AR2	AR2_H0_inf	AR2_H0_sup	AR2_H0	MA1	MA1_H0_inf	MA1_H0_sup	MA1_H0	MA2	MA2_H0_inf	MA2_H0_sup	MA2_H0	drift	drift_H0_inf	drift_H0_sup	drift_H0	Normality_CVM_pvalue	Normality_AD_pvalue	Normality_JB_pvalue	Uncorrelation_LB	Homocedasticity_BP_B	Zero_mean
q_67_log	(0,1,0)	-380.58	-375.81	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01139	-0.01709	-0.00606	REJECTED	0.49746	0.44161	0.00000	0.02391	0.00008	0.98931
q_67_log	(1,1,0)	-381.09	-374.02	-0.17491	-0.38539	0.02675	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01146	-0.01425	-0.00480	REJECTED	0.56088	0.45484	0.00002	0.20978	0.98938	0.99584
q_67_log	(0,1,1)	-381.45	-374.38	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.20214	-0.47090	-0.00162	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01151	-0.01593	-0.00749	REJECTED	0.57427	0.46396	0.00015	0.26379	0.99784	0.98219
q_67_log	(1,1,1)	-379.50	-370.18	0.23707	-0.86543	0.86011	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.43354	-1.00000	0.83752	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01161	-0.01916	-0.01095	REJECTED	0.62170	0.50616	0.00057	0.25722	0.99721	0.95597
q_67_log	(0,1,2)	-379.47	-370.14	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.19789	-0.45100	-0.00224	REJECTED	-0.04904	-0.29601	0.18458	NOT REJECTED	-0.01159	-0.01540	-0.00733	REJECTED	0.61445	0.49997	0.00066	0.26129	0.99832	0.96287
q_67_log	(1,1,2)	-378.82	-367.30	0.92207	-0.96271	0.89593	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-1.15596	-1.22835	0.82904	NOT REJECTED	0.15597	-0.36341	0.31213	NOT REJECTED	-0.01201	-0.15535	-0.15183	REJECTED	0.46164	0.32291	0.00039	0.23130	0.99949	0.58081
q_67_log	(2,1,0)	-379.21	-369.89	-0.18737	-0.40526	0.02203	NOT REJECTED	-0.06253	-0.29464	0.13444	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01151	-0.01357	-0.00477	REJECTED	0.57239	0.46474	0.00028	0.24738	0.99919	0.98336
q_67_log	(2,1,1)	-377.25	-365.73	0.20575	-1.16821	0.81369	NOT REJECTED	-0.01187	-0.34578	0.23821	NOT REJECTED	-0.40130	-1.00000	0.98959	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01161	-0.01831	-0.01048	REJECTED	0.62345	0.50790	0.00067	0.25476	0.99689	0.95650
q_67_log	(2,1,2)	-381.24	-367.58	1.18585	-1.12126	1.22744	NOT REJECTED	-0.78431	-0.98270	0.69630	NOT REJECTED	-1.45530	-1.56451	1.00390	NOT REJECTED	0.99998	-0.84957	1.00000	NOT REJECTED	-0.01121	-0.02341	-0.01443	REJECTED	0.65442	0.55780	0.00358	0.22353	0.98783	0.99614
q_68_log	(0,1,0)	-384.29	-379.52	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01179	-0.01719	-0.00655	REJECTED	0.33446	0.26487	0.00000	0.34223	0.98939	0.98939
q_68_log	(1,1,0)	-383.64	-376.57	-0.13142	-0.32630	0.07750	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01183	-0.01500	-0.00580	REJECTED	0.40394	0.32407	0.00000	0.43434	0.78753	0.99883
q_68_log	(0,1,1)	-384.38	-377.31	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.19205	-0.43452	-0.00261	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01188	-0.01601	-0.00751	REJECTED	0.43926	0.36454	0.00000	0.49218	0.58348	0.98894
q_68_log	(1,1,1)	-382.94	-373.62	0.32092	-0.86053	0.86942	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.50283	-1.00000	0.87481	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01198	-0.02129	-0.01343	REJECTED	0.44613	0.36896	0.00000	0.56345	0.70464	0.95895
q_68_log	(0,1,2)	-383.97	-374.64	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.13732	-0.38623	0.06931	NOT REJECTED	-0.14985	-0.40776	0.08958	NOT REJECTED	-0.01200	-0.01546	-0.00810	REJECTED	0.39323	0.32978	0.00000	0.64148	0.81132	0.95061
q_68_log	(1,1,2)	-382.22	-370.70	-0.34150	-0.90062	0.87554	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.19599	-1.12366	0.77966	NOT REJECTED	-0.20653	-0.46335	0.21037	NOT REJECTED	-0.01195	-0.01245	-0.00484	REJECTED	0.36841	0.32356	0.00000	0.67643	0.65711	0.96668
q_68_log	(2,1,0)	-384.60	-375.27	-0.15890	-0.37854	0.05762	NOT REJECTED	-0.19024	-0.39378	-0.01334	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01193	-0.01260	-0.00502	REJECTED	0.35090	0.30757	0.00000	0.66515	0.77401	0.97325
q_68_log	(2,1,1)	-382.46	-370.94	-0.36145	-1.07685	0.80275	NOT REJECTED	-0.21514	-0.42342	0.11400	NOT REJECTED	0.21207	-1.00000	0.97369	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01189	-0.01164	-0.00324	REJECTED	0.32504	0.29345	0.00000	0.67675	0.70605	0.98613
q_68_log	(2,1,2)	-387.96	-374.30	1.13145	-1.24063	1.20754	NOT REJECTED	-0.70200	-0.98253	0.61030	NOT REJECTED	-1.45150	-1.55342	1.10912	NOT REJECTED	1.00000	-0.78173	1.00000	NOT REJECTED	-0.01146	-0.02497	-0.01545	REJECTED	0.42159	0.38601	0.00000	0.86840	0.88659	0.93557
q_69_log	(0,1,0)	-396.32	-391.56	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01223	-0.01706	-0.00654	REJECTED	0.75219	0.81030	0.51871	0.03674	0.98900	0.98900
q_69_log	(1,1,0)	-397.63	-390.56	-0.19804	-0.39990	0.01290	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01224	-0.01434	-0.00617	REJECTED	0.74609	0.85382	0.58847	0.21492	0.11356	0.99947
q_69_log	(0,1,1)	-398.27	-391.20	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.23209	-0.49352	-0.01913	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01577	-0.00824	REJECTED	0.76858	0.87554	0.57133	0.31720	0.12939	0.99118
q_69_log	(1,1,1)	-396.20	-386.88	0.13897	-0.86266	0.86611	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.36184	-1.00000	0.79766	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01228	-0.01773	-0.01059	REJECTED	0.76416	0.86531	0.56436	0.35443	0.30118	0.98258
q_69_log	(0,1,2)	-396.25	-386.92	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.21893	-0.47639	-0.01804	REJECTED	-0.04390	-0.31492	0.18672	NOT REJECTED	-0.01228	-0.01595	-0.00858	REJECTED	0.75592	0.85690	0.55302	0.37578	0.36813	0.98290
q_69_log	(1,1,2)	-396.02	-384.50	0.91438	-0.95237	0.89053	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-1.18253	-1.22210	0.81244	NOT REJECTED	0.18253	-0.42279	0.28279	NOT REJECTED	-0.01228	-0.14454	-0.14161	REJECTED	0.69803	0.71426	0.23275	0.38096	0.69086	0.69326
q_69_log	(2,1,0)	-396.32	-386.99	-0.21869	-0.00836	-0.44054	REJECTED	-0.10051	-0.33284	0.08359	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01279	-0.00539	REJECTED	0.73978	0.84634	0.50735	0.42580	0.44133	0.99247
q_69_log	(2,1,1)	-394.10	-382.57	-0.10031	-1.16126	0.79930	NOT REJECTED	-0.07845	-0.37396	0.23155	NOT REJECTED	-0.12025	-1.00000	0.99998	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01226	-0.01390	-0.00662	REJECTED	0.74525	0.84935	0.51885	0.42191	0.34922	0.98894
q_69_log	(2,1,2)	-393.81	-380.15	1.26298	-1.57724	1.36652	NOT REJECTED	-0.85322	-0.99451	0.78782	NOT REJECTED	-1.42888	-1.54824	1.62243	NOT REJECTED	0.99999	-0.92915	1.00000	NOT REJECTED	-0.01224	-0.02489	-0.01619	REJECTED	0.76546	0.83393	0.50544	0.37986	0.45862	0.99374
q_70_log	(0,1,0)	-390.88	-386.11	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01243	-0.01750	-0.00745	REJECTED	0.86210	0.93434	0.61160	0.00007	0.00060	0.98972
q_70_log	(1,1,0)	-399.03	-391.96	-0.34445	-0.52623	-0.12959	REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01245	-0.01290	-0.00566	REJECTED	0.93385	0.96271	0.51841	0.10938	0.58374	0.99908
q_70_log	(0,1,1)	-400.19	-393.12	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.35757	-0.62077	-0.16846	REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01247	-0.01534	-0.00937	REJECTED	0.92672	0.91772	0.46602	0.15678	0.41163	0.99543
q_70_log	(1,1,1)	-398.17	-388.85	-0.10864	-0.67669	0.69004	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.26838	-1.00000	0.41490	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01247	-0.01434	-0.00799	REJECTED	0.96391	0.95743	0.49018	0.19198	0.45469	0.99684
q_70_log	(0,1,2)	-398.34	-389.02	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.38914	-0.65865	-0.18165	REJECTED	0.07527	-0.18498	0.30244	NOT REJECTED	-0.01245	-0.01566	-0.00903	REJECTED	0.97348	0.97444	0.51629	0.21388	0.46343	0.99956
q_70_log	(1,1,2)	-396.19	-384.66	0.21134	-0.97790	0.90993	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.59636	-1.35680	0.66377	NOT REJECTED	0.15084	-0.45869	0.47895	NOT REJECTED	-0.01244	-0.01926	-0.01233	REJECTED	0.96896	0.97222	0.53932	0.20636	0.55265	0.99865
q_70_log	(2,1,0)	-399.34	-390.02	-0.39128	-0.60866	-0.18186	REJECTED	-0.16842	-0.38109	0.01688	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01245	-0.01111	-0.00476	REJECTED	0.96117	0.95321	0.49859	0.23993	0.43609	0.99828
q_70_log	(2,1,1)	-400.88	-389.36	-1.01820	-1.31623	0.57721	NOT REJECTED	-0.41132	-0.59933	0.24723	NOT REJECTED	0.66403	-1.00000	0.99999	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	-0.01242	-0.00828	-0.00192	REJECTED	0.86613	0.82317	0.33700	0.95989	0.89304	0.98420
q_70_log	(2,1,2)	-399.21	-385.55	-1.18658	-1.53325	0.78300	NOT REJECTED	-0.59176	-0.96654	0.37536	NOT REJECTED	0.87701	-1.22278	1.45464	NOT REJECTED	0.21506	-0.69170	0.99999	NOT REJECTED	-0.01240	-0.00771	-0.00093	REJECTED	0.80190	0.69599	0.15062	0.55588	0.90078	0.98022
q_71_log	(0,1,0)	-394.56	-389.79	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01249	-0.01750	-0.00734	REJECTED	0.91293	0.91236	0.60818	0.59977	0.00001	0.98984
q_71_log	(1,1,0)	-394.97	-387.90	-0.17067	-0.37347	0.04595	NOT REJECTED	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	0.00000	#N/A	#N/A	#N/A	-0.01252	-0.01478	-0.00653	REJECTED	0.94389	0.94104	0.60701	0.77591	0.82362	0.99698
q_71_log	(0,1,1)	-396.25	-389.18	0.00000	#N/A	#N/A	#N/A																						