Photon Quantum Computer. Example: Mach-Zehnder interferometer.

A photon passes through the first beam splitter (BS1) and propagates via 2 different paths to another beam splitter (BS2), which directs the particle to one of the two detectors. Along each path between the two BSs, there is a phase shifter (PSi : $i = 1, 2$).

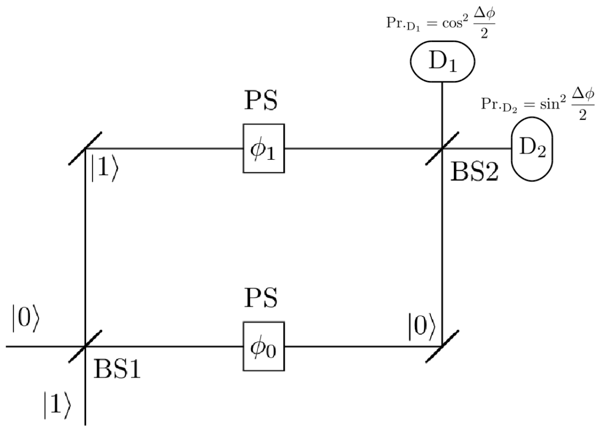

Figure 0.1: Mach-Zehnder interferometer. $\mathrm{Pr.}_{\mathrm{D}_i}$ is the probability of directing the particle to detector D_i.

If the initial particle is in path $|0\rangle$, it undergoes the following:

$$
\begin{aligned}
&|\psi(t_0)\rangle = |0\rangle \;\to\; \hat{U}_{\mathrm{BS1}}\,|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \;\to\; \hat{U}_{\mathrm{PS}}(\hat{U}_{\mathrm{BS1}}\,|0\rangle) = \frac{1}{\sqrt{2}}(e^{i\phi_0}\,|0\rangle + e^{i\phi_1}\,|1\rangle) \\
&= \frac{1}{\sqrt{2}} e^{i\frac{\phi_0+\phi_1}{2}} (e^{i\frac{\phi_0-\phi_1}{2}}\,|0\rangle + e^{-i\frac{\phi_0-\phi_1}{2}}\,|1\rangle) \;\to \\
&|\psi(t_f)\rangle = \hat{U}_{\mathrm{BS2}}\left(\hat{U}_{\mathrm{PS}}(\hat{U}_{\mathrm{BS1}}\,|0\rangle)\right) = \frac{1}{\sqrt{2}} e^{i\frac{\phi_0+\phi_1}{2}} \left(e^{i\frac{\phi_0-\phi_1}{2}} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + e^{-i\frac{\phi_0-\phi_1}{2}} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right) \\
&= \frac{1}{2} e^{i\frac{\phi_0+\phi_1}{2}} \left(2\cos\left(\frac{\phi_0-\phi_1}{2}\right)|0\rangle + 2i\sin\left(\frac{\phi_0-\phi_1}{2}\right)|1\rangle \right) = e^{i\frac{\phi_0+\phi_1}{2}} \left(\cos\frac{\Delta\phi}{2}\,|0\rangle + i\sin\frac{\Delta\phi}{2}\,|1\rangle \right),
\end{aligned}
\tag{0.1}
$$

where $\Delta\phi \equiv \phi_0 - \phi_1$. BS1 prepares the particle in a superposition of paths, PS1 and PS2 modify quantum phases and BS2 recombines the superpositions erasing the information about which path was taken from BS1.

The global phase $e^{i\frac{\phi_0+\phi_1}{2}}$, as mentioned in Section (2.3.3), is irrelevant, since it vanishes when we measure $\mathrm{Pr.}_{\mathrm{D}_i}$, $i = 1, 0$:

$$
\mathrm{Pr.}_{\mathrm{D}_1} = \mathrm{Pr.}_{|\psi(t_f)\rangle}(|0\rangle) = |\,\langle 0|\psi(t_f)\rangle\,|^2 = |e^{i\frac{\phi_0+\phi_1}{2}} \cos\frac{\Delta\phi}{2}|^2 = \cos^2\frac{\Delta\phi}{2}\,, \tag{0.2}
$$

$$
\text{and} \quad \mathrm{Pr.}_{\mathrm{D}_2} = \mathrm{Pr.}_{|\psi(t_f)\rangle}(|1\rangle) = |\,\langle 1|\psi(t_f)\rangle\,|^2 = |e^{i\frac{\phi_0+\phi_1}{2}} \sin\frac{\Delta\phi}{2}|^2 = \sin^2\frac{\Delta\phi}{2}\,. \tag{0.3}
$$

Notice that **this is a quantum circuit!** $\hat{U}_{\mathrm{BS}} \equiv \hat{U}_{\mathrm{Had}}$ and $\hat{U}_{\mathrm{PS}} \equiv \hat{U}_{\phi} \doteq \oplus_{k=0}^{n-1} e^{i\phi_k} = \begin{pmatrix} e^{i\phi_0} & \\ & e^{i\phi_1} \end{pmatrix}$. Hence, $|\psi(t_f)\rangle = \hat{U}_{\mathrm{Had}}\,\hat{U}_{\phi}\,\hat{U}_{\mathrm{Had}}\,|\psi(t_0)\rangle$.