1 General density operator properties of $\hat{\rho}_{g_0}$

We will verify that Eq. (4.7) obeys the properties of the $\hat{\rho}$ density operators [13, p. 134, Eq. (4.2.23.1-3)].

1. The density operator $\hat{\rho}$ must be Hermitian:

$$\begin{aligned} \hat{\rho} = \hat{\rho}^{\dagger} \quad \leftrightarrow \quad \hat{\rho}^{\dagger}_{g_0} &\doteq \begin{pmatrix} (a)^* & (b-ic)^* \\ (b+ic)^* & (1-a)^* \end{pmatrix}^T = \begin{pmatrix} a^* & b^*+ic^* \\ b^*-ic^* & 1-a^* \end{pmatrix}^T \\ &= \begin{pmatrix} a^* & b^*-ic^* \\ b^*+ic^* & 1-a^* \end{pmatrix} \doteq \hat{\rho}_{g_0} \quad \leftrightarrow \quad a,b,c \in \mathbb{R}, \end{aligned} \tag{1.1}$$

   so that $(a^*, b^*, c^*) = (a, b, c)$.

2. From the normalization condition: $\mathrm{Tr}\hat{\rho} = 1 \leftrightarrow \mathrm{Tr}\hat{\rho}_{g_0} \doteq \mathrm{Tr}\begin{pmatrix} a & b-ic \\ b+ic & 1-a \end{pmatrix} = a + 1 - a = 1$.

3. The density operator $\hat{\rho}$ must have positive or null eigenvalues - or, equivalently, $\hat{\rho}$ must be a positive semidefinite matrix, $\hat{\rho} \geq 0$:

   $|\hat{\rho}_g - \lambda\hat{1}| = 0 = \begin{vmatrix} a-\lambda & b-ic \\ b+ic & 1-a-\lambda \end{vmatrix} = (a-\lambda)(1-a-\lambda) - (b+ic)(b-ic) = a - a^2 - a\lambda - \lambda + a\lambda + \lambda^2 - b^2 - c^2 = \lambda^2 - \lambda + a(1-a) - (b^2+c^2) \rightarrow \lambda = \frac{1}{2}(1 \pm \sqrt{1 + 4(a(a-1) + b^2 + c^2)}) \geq 0$.

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