Photon Quantum Computer. Example: Mach-Zehnder interferometer.

A photon passes through the first beam splitter (BS1) and propagates via 2 different paths to another beam splitter (BS2), which directs the particle to one of the two detectors. Along each path between the two BSs, there is a phase shifter (PSi : $i = 1, 2$).

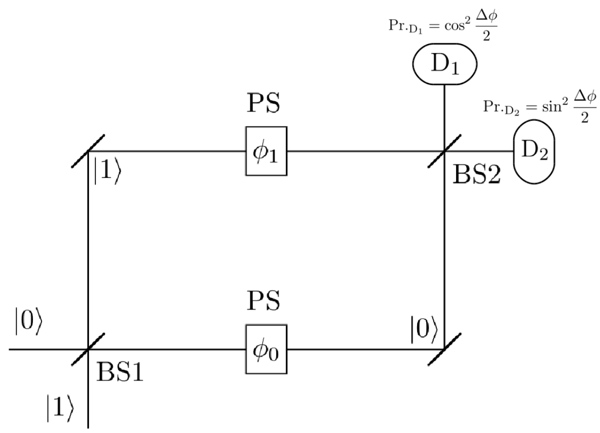

Figure 0.1: Mach-Zehnder interferometer. $\Pr_{\cdot \mathrm{D}_i}$ is the probability of directing the particle to detector D_i.

If the initial particle is in path $|0\rangle$, it undergoes the following:

$$
\begin{aligned}
&|\psi(t_0)\rangle = |0\rangle \;\rightarrow\; \hat{U}_{\mathrm{BS1}}\,|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \;\rightarrow\; \hat{U}_{\mathrm{PS}}(\hat{U}_{\mathrm{BS1}}\,|0\rangle) = \frac{1}{\sqrt{2}}(e^{i\phi_0}\,|0\rangle + e^{i\phi_1}\,|1\rangle) \\
&= \frac{1}{\sqrt{2}} e^{i\frac{\phi_0+\phi_1}{2}} (e^{i\frac{\phi_0-\phi_1}{2}}\,|0\rangle + e^{-i\frac{\phi_0-\phi_1}{2}}\,|1\rangle) \;\rightarrow \\
&|\psi(t_f)\rangle = \hat{U}_{\mathrm{BS2}}\left(\hat{U}_{\mathrm{PS}}(\hat{U}_{\mathrm{BS1}}\,|0\rangle)\right) = \frac{1}{\sqrt{2}} e^{i\frac{\phi_0+\phi_1}{2}} \left(e^{i\frac{\phi_0-\phi_1}{2}} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + e^{-i\frac{\phi_0-\phi_1}{2}} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) \\
&= \frac{1}{2} e^{i\frac{\phi_0+\phi_1}{2}} \left(2\cos\left(\frac{\phi_0-\phi_1}{2}\right)|0\rangle + 2i\sin\left(\frac{\phi_0-\phi_1}{2}\right)|1\rangle\right) = e^{i\frac{\phi_0+\phi_1}{2}} \left(\cos\frac{\Delta\phi}{2}\,|0\rangle + i\sin\frac{\Delta\phi}{2}\,|1\rangle\right),
\end{aligned}
\tag{0.1}
$$

where $\Delta\phi \equiv \phi_0 - \phi_1$. BS1 prepares the particle in a superposition of paths, PS1 and PS2 modify quantum phases and BS2 recombines the superpositions erasing the information about which path was taken from BS1.

The global phase $e^{i\frac{\phi_0+\phi_1}{2}}$, as mentioned in Section (2.3.3), is irrelevant, since it vanishes when we measure $\Pr_{\cdot \mathrm{D}_i}$, $i = 1, 0$:

$$
\Pr_{\cdot \mathrm{D}_1} = \Pr_{\cdot |\psi(t_f)\rangle}(|0\rangle) = |\,\langle 0|\psi(t_f)\rangle\,|^2 = |e^{i\frac{\phi_0+\phi_1}{2}} \cos\frac{\Delta\phi}{2}|^2 = \cos^2\frac{\Delta\phi}{2}\,, \tag{0.2}
$$

$$
\text{and} \quad \Pr_{\cdot \mathrm{D}_2} = \Pr_{\cdot |\psi(t_f)\rangle}(|1\rangle) = |\,\langle 1|\psi(t_f)\rangle\,|^2 = |e^{i\frac{\phi_0+\phi_1}{2}} \sin\frac{\Delta\phi}{2}|^2 = \sin^2\frac{\Delta\phi}{2}\,. \tag{0.3}
$$

Notice that **this is a quantum circuit!** $\hat{U}_{\mathrm{BS}} \equiv \hat{U}_{\mathrm{Had}}$ and $\hat{U}_{\mathrm{PS}} \equiv \hat{U}_{\phi} \doteq \oplus_{k=0}^{n-1} e^{i\phi_k} = \begin{pmatrix} e^{i\phi_0} & \\ & e^{i\phi_1} \end{pmatrix}$. Hence, $|\psi(t_f)\rangle = \hat{U}_{\mathrm{Had}}\,\hat{U}_{\phi}\,\hat{U}_{\mathrm{Had}}\,|\psi(t_0)\rangle$.

These quantum information processing physical schemes relying on optical techniques (manipulations of photons by beamsplitters and mirrors) are easiest to craft. This is because, despite the difficulty of producing single photons on demand and the fact that they do not directly interact with one another, but by the mediation of noise adding atoms, they are highly stable (hardly decohered by the environment) quantum information carriers [1, p. 49, l. 8-21].