Time Discretization for advection-diffusion Solver: IMEX Splitting Accuracy: BDF

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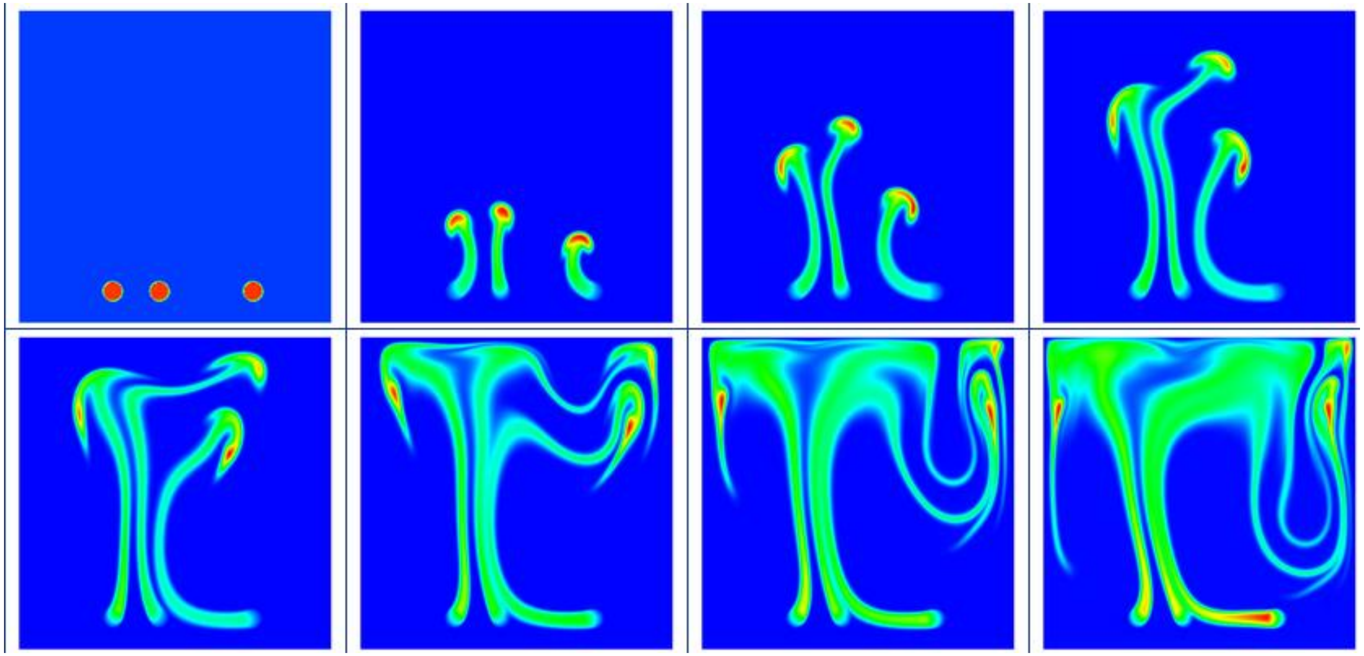
Contents

Abstract

Target:

- ❖ Advection- Diffusion
- ❖ Mathematical view
- ❖ Run the source code for time discretization and change the code



slides.30.25.pdf (colostate.edu)

Introduction

Advection Convection:

- Discuss the term of advection and diffusion
- Discuss about solving
- Run a code related time stepping
- Modify the Source Code

https://www.math.colostate.edu/~bangerth/videos.676.30.25.html

Advection and Diffusion

Advection **Diffusion**

Equation:

$$\frac{\partial u}{\partial t} - \Delta u = f$$

Parabolic-Implicit time stepping

$$\frac{\partial u}{\partial t} + \vec{\beta} \cdot \nabla u = f \qquad \frac{\partial^2 u}{\partial t^2} - \Delta u = f$$

Hyperbolic(1st and 2nd order)-Explicit time stepping

Advection-Diffusion(IMEX schemes)

$$\frac{\partial u}{\partial t} + \vec{\beta} \cdot \nabla u - \Delta u = f$$

Explicit time stepping

$$\frac{u^n_{\mathrm{adv}} - u^{n-1}}{k^n} + \vec{\beta} \cdot \nabla u^{n-1} = 0$$

Implicit time stepping

$$\frac{u^n_{\mathrm{diff}} - u^{n-1}}{k^n} - \Delta u^n_{\mathrm{diff}} = 0$$

$$\frac{u^n_{\mathrm{source}} - u^{n-1}}{k^n} = f$$

$$\frac{u^n - u^{n-1}}{k^n} = \frac{u^n_{\mathrm{adv}} - u^{n-1}}{k^n} + \frac{u^n_{\mathrm{diff}} - u^{n-1}}{k^n} + \frac{u^n_{\mathrm{source}} - u^{n-1}}{k^n}$$

$$u^n = u^{n-1} + \delta u^n_{\mathrm{adv}} + \delta u^n_{\mathrm{diff}} + \delta u^n_{\mathrm{source}}$$

– concurrently (in parallel) – by separate codes

Advection and Diffusion

Solve for one physical effect after the other

Separate for one physical effect after the other

Advection-Diffusion(Operator Splitting)

$$\frac{\partial u}{\partial t} + \vec{\beta}\cdot\nabla u - \Delta u = f$$

- Transport (explicit)
- Diffusion (implicit)

Explicit time stepping

$$\frac{u^n_{\text{adv}} - u^{n-1}}{k^n} + \vec{\beta}\cdot\nabla u^{n-1} = 0$$

Implicit time stepping

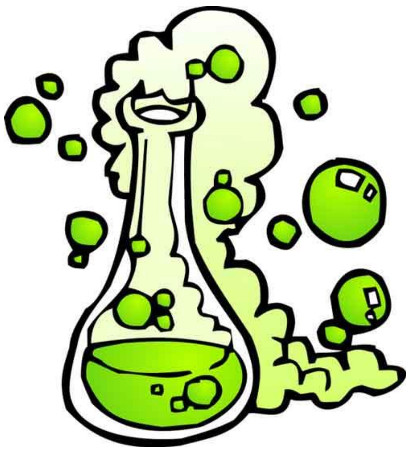
$$\frac{u^n_{\text{diff}} - u^n_{\text{adv}}}{k^n} - \Delta u^n_{\text{diff}} = 0$$

$$\frac{u^n_{\text{source}} - u^{n-1}_{\text{diff}}}{k^n} = f$$

$$u^n = u^n_{\text{source}}$$

first order in k^n - Lie Splitting

Higher order- Strang Splitting



Reaction(A+B>C)

- Solution variable: $u(x,t)=\{u_A(x,t),u_B(x,t),u_C(x,t)\}$
- Equation: $$\frac{\partial \vec{u}}{\partial t} - \Delta\vec{u} = \vec{f}(\vec{u})$$
- Reaction terms: $$\vec{f}(\vec{u}) = \begin{pmatrix} -k u_A u_B \\ -k u_A u_B \\ +k u_A u_B \end{pmatrix}$$

Advection and Diffusion

$$\frac{\partial \vec{u}}{\partial t} - \Delta \vec{u} = \vec{f}(\vec{u})$$

First order operator splitting ("Lie splitting"):

- First account for the effect of one time step's worth of diffusion (implicit):

$$\frac{\vec{u}^* - \vec{u}^{n-1}}{k^n} - \Delta \vec{u}^* = 0$$

- Then account for one time step's worth of reactions (local ODE):

$$\frac{\partial \vec{u}^{**}}{\partial t} = \vec{f}(\vec{u}^{**}), \quad \vec{u}^{**}(t_{n-1}) = \vec{u}^* \quad \rightarrow \quad \vec{u}^n = \vec{u}^{**}(t_n)$$

- The order could of course be reversed.

$$\frac{\partial \vec{u}}{\partial t} - \Delta \vec{u} = \vec{f}(\vec{u})$$

Second order operator splitting ("Strang splitting"):

- Half diffusion step:

$$\frac{\vec{u}^* - \vec{u}^{n-1}}{k^n/2} - \Delta \vec{u}^* = 0$$

- Full reaction step:

$$\frac{\partial \vec{u}^{**}}{\partial t} = \vec{f}(\vec{u}^{**}), \quad \vec{u}^{**}(t_{n-1}) = \vec{u}^* \quad \rightarrow \text{solve for } \vec{u}^{**}(t_n)$$

- Half diffusion step:

$$\frac{\vec{u}^n - \vec{u}^{**}(t_n)}{k^n/2} - \Delta \vec{u}^n = 0$$

- The order of sub-steps can be reversed.

More accuracy-BDF

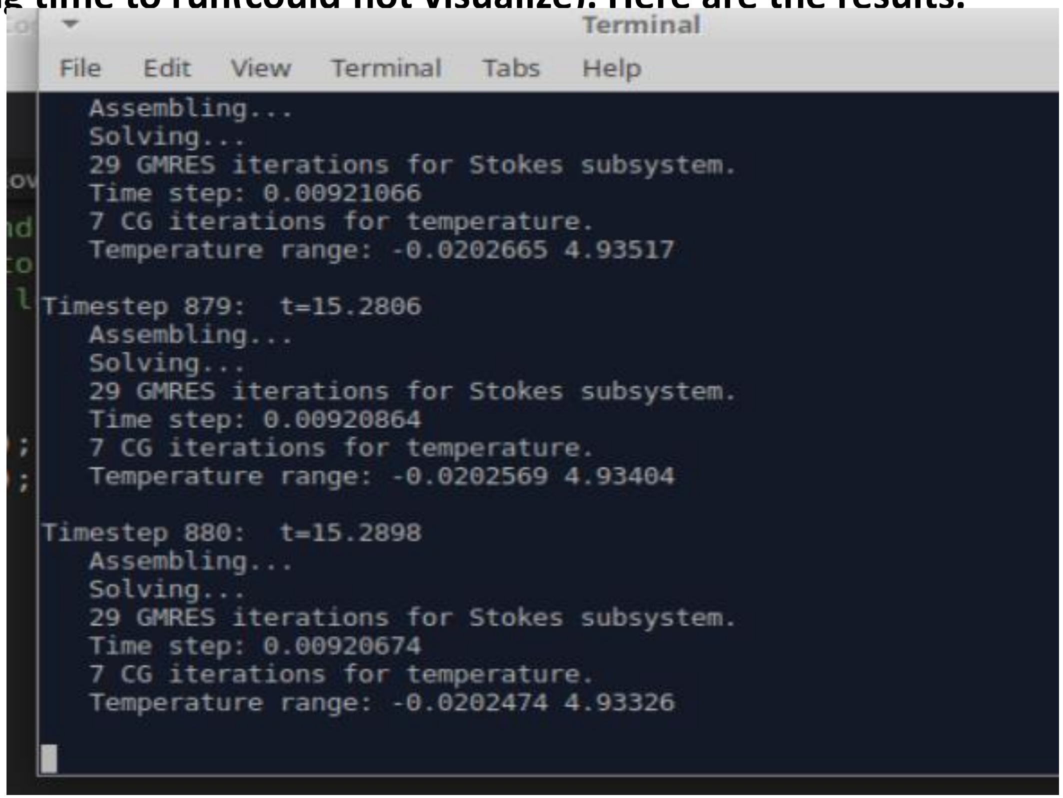
Conclusion

Advection and Diffusion:

- ❖ Solver Introduction
- ❖ Condition
- ❖ Accuracy

Result:Step-22

Took a long time to run(could not visualize). Here are the results.



Thank You