Numerical Analysis
Newton's method

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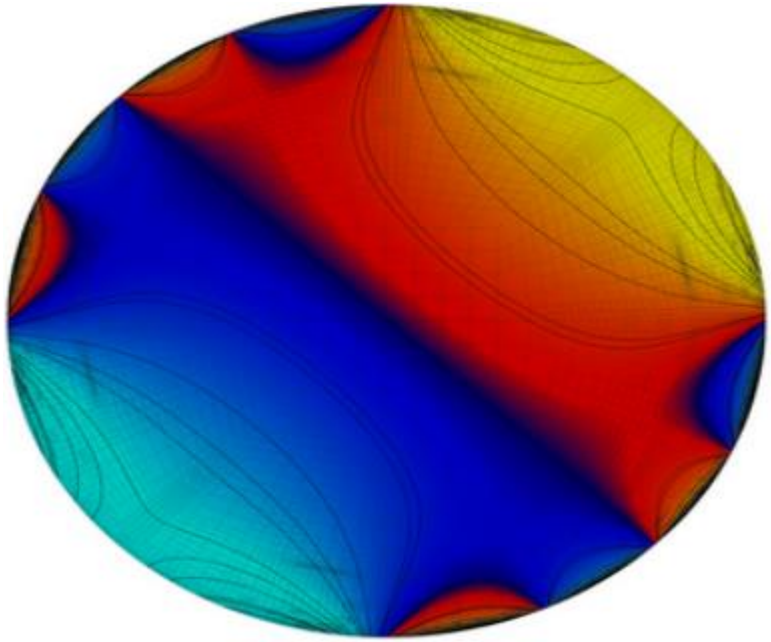
Contents

- Abstract
- Introduction
- Examples
- Concluding Remarks
- Reference

Abstract

Target:

- ❖ Define the Newton's Method
- ❖ Code explanation
- ❖ Result discussion

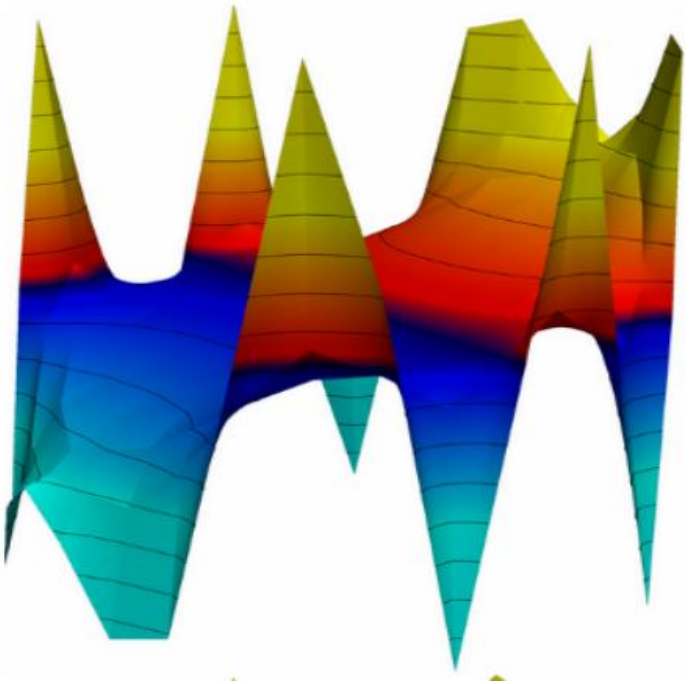


slides.31.6.pdf (colostate.edu)

Introduction

Newton's Method:

- Mathematical View
- Terms of codes
- Results



Wolfgang Bangerth's video lectures (colostate.edu)

Newton's Method

Solver Selection

$$A_k \, \delta U_k = F_k$$

$$A_{k,ij} = \left(\nabla \phi_i, \frac{A}{\sqrt{1+|\nabla u_{k,h}|^2}} \nabla \phi_j \right) - \left(\nabla \phi, \frac{A(\nabla u_{k,h} \cdot \nabla \phi_j)}{\left(1+|\nabla u_{k,h}|^2\right)^{3/2}} \nabla u_k \right)$$

Solver

Conjugate Gradient

A is symmetric- (derivative of a non-linear operator)

A is Positive- (convex energy function is minimized)

Newton's Method

Boundary Values of δu_k

$$u_0|_{\partial\Omega} = g \qquad (u_k+\alpha_k \delta u_k)|_{\partial\Omega} = g$$

δu_k have zero boundary

Way of Convergence

$$U_{k+1} = U_k+\delta U_k$$

Does not follow this rule always

$$U_{k+1} = U_k+\alpha_k \delta U_k$$

Relaxing the iteration, it takes

Line search- algorithm for α_k

quadratic convergence if $\alpha_k = 1$

slower convergence if $\alpha_k < 1$

may not converge if $\alpha_k = 1$

frequently converge if $\alpha_k < 1$

α_k depends on residual function

$$R(u) = L(u)-f = 0$$

If

$$\|R(u_k+\alpha_k \delta u_k)\| \le c\|R(u_k)\|$$

then use this α_k; otherwise set $\alpha_k := \alpha_k/2$ and try again

Compute $U_{k+1} = U_k+\alpha_k \delta U_k$

α_k=0.1 is taken for simplicity

Newton's Method

Implementation

1. Start with a coarse mesh
2. Do Newton iterations
3. If $\|R(u_k)\| \leq \text{tol}$ then stop
4. Save the solution on the current mesh
5. Refine the mesh
6. Go to step 2

Result:

Took a long time to run(could not visualize). Here are the results.

```
 Residual: 0.00508832
 Residual: 0.00457899
 Residual: 0.00412073
 Residual: 0.0037084
 Residual: 0.00333737
******** Refined mesh 11 ********
 Initial residual: 0.00342093
 Residual: 0.00307852
 Residual: 0.00277045
 Residual: 0.00249323
 Residual: 0.00224379
 Residual: 0.00201932
******** Refined mesh 12 ********
 Initial residual: 0.00202458
 Residual: 0.00182205
 Residual: 0.00163979
 Residual: 0.00147577
 Residual: 0.00132816
 Residual: 0.00119532
******** Refined mesh 13 ********
 Initial residual: 0.00115921
 Residual: 0.00104326
 Residual: 0.000938907
```

Conclusion

- Solver selection
- Methodology
- Conditionals

Thank You