Navier Stokes Equation (Step 57)

Submitted by,

Asif Istiak; ID: 20205083

Mechanical Design Engineering

Andong National University

Submitted to,

Professor See Jo Kim

Mechanical Design Engineering

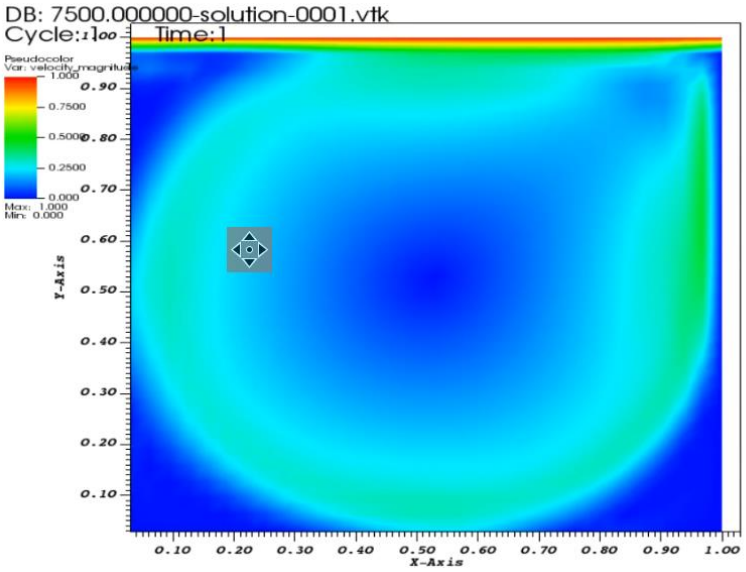
Andong National University

Contents

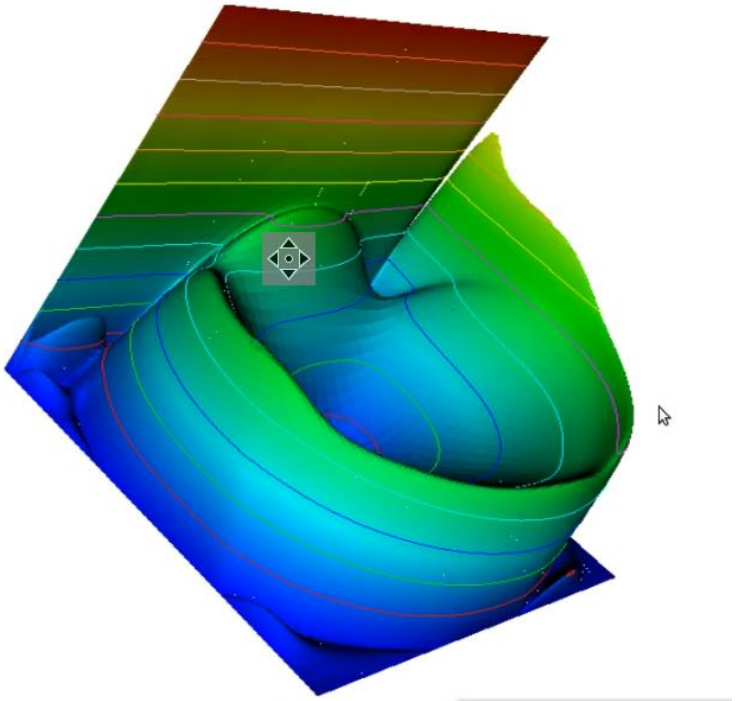
Abstract

NSE:

- Short discussion of step-57



2D



3D

Introduction

NSE:

- Discuss the theory
- Explain the code
- Change the source code
- Compare the result

Step-57

Navier Stokes equation:

$$\begin{aligned} -\nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p &= \mathbf{f} \\ -\nabla\cdot\mathbf{u} &= 0. \end{aligned}$$

Linearization:

$$F(\mathbf{u},p) = \begin{pmatrix} -\nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p - \mathbf{f} \\ -\nabla\cdot\mathbf{u} \end{pmatrix}.$$

Newton's iteration on a vector function can be defined as

$$\mathbf{x}^{k+1} = \mathbf{x}^k - (\nabla F(\mathbf{x}^k))^{-1} F(\mathbf{x}^k),$$

Final Linearization system

$$\begin{aligned} -\nu\Delta\delta\mathbf{u}^k + \mathbf{u}^k\cdot\nabla\delta\mathbf{u}^k + \delta\mathbf{u}^k\cdot\nabla\mathbf{u}^k + \nabla\delta p^k &= -F(\mathbf{x}^k), \\ -\nabla\cdot\delta\mathbf{u}^k &= \nabla\cdot\mathbf{u}^k, \end{aligned}$$

Initial Guess

$$\begin{aligned} -\nu_1\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p &= \mathbf{f}, \\ -\nabla\cdot\mathbf{u} &= 0, \end{aligned}$$

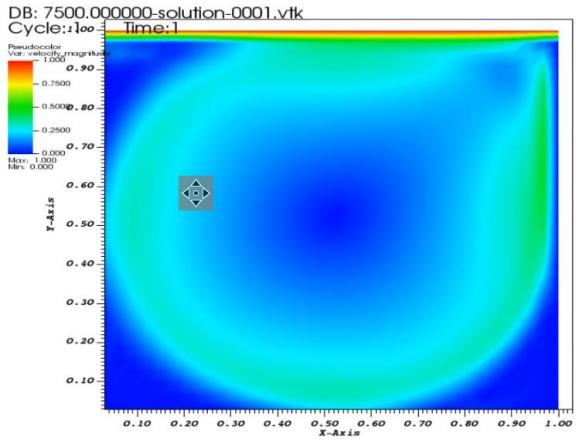
Preconditioner

$$\begin{pmatrix} A & B^T \\ B & 0 \end{pmatrix} \begin{pmatrix} U \\ P \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}.$$

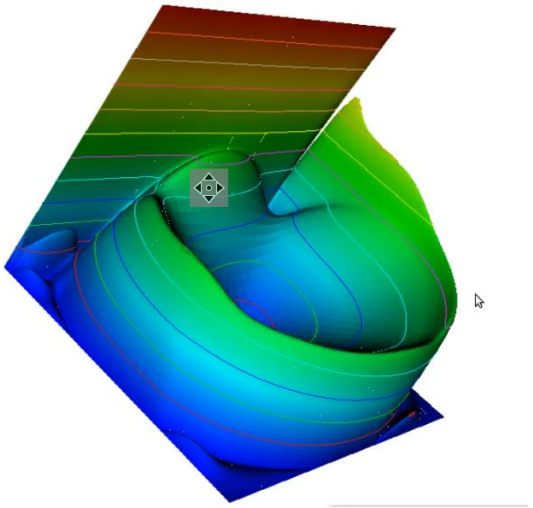
Test case

$$\begin{aligned} (u(x,y), v(x,y)) &= (1,0) && \text{if } y = 1 \\ (u(x,y), v(x,y)) &= (0,0) && \text{otherwise.} \end{aligned}$$

Abstract

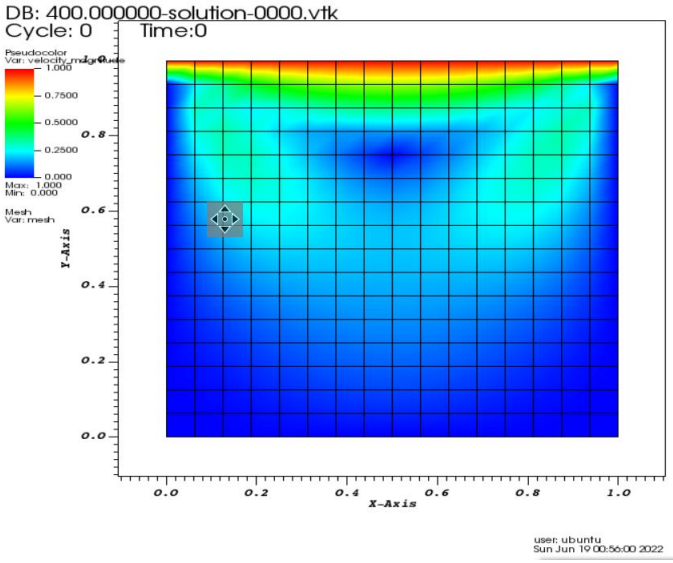


Re=7500

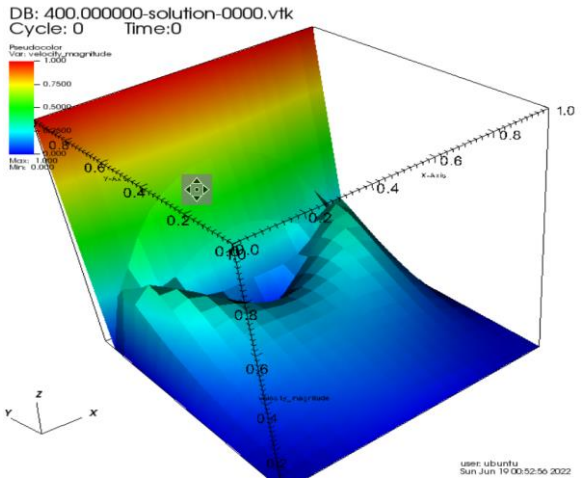


2D

3D



Re=400



Conclusion

NSE:

- Discuss the theory
- Explain the code
- Change the source code
- Compare the result

Thank You