

Time Discretization for advection-diffusion Solver: IMEX Splitting Accuracy: BDF

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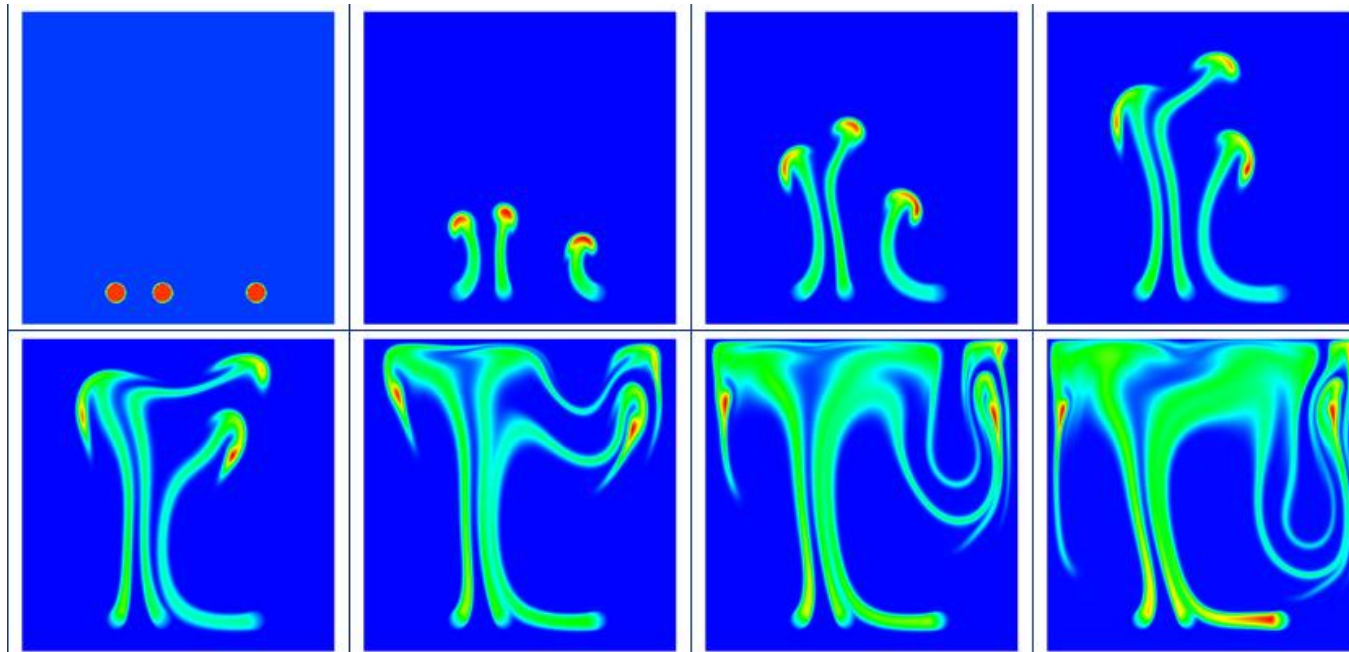
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Abstract

Target:

- ❖ Advection- Diffusion
- ❖ Mathematical view
- ❖ Run the source code for time discretization and change the code



Introduction

Advection Convection:

- ❖ Discuss the term of advection and diffusion
- ❖ Discuss about solving
- ❖ Run a code related time stepping
- ❖ Modify the Source Code



<https://www.math.colostate.edu/~bangerth/videos.676.30.25.html>

Advection and Diffusion

Advection

Diffusion

Equation:

$$\frac{\partial u}{\partial t} - \Delta u = f$$

Parabolic-Implicit time stepping

$$\frac{\partial u}{\partial t} + \vec{\beta} \cdot \nabla u = f$$

Hyperbolic(1st and 2nd order)-Explicit time stepping

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = f$$

Advection-Diffusion(IMEX schemes)

$$\frac{\partial u}{\partial t} + \vec{\beta} \cdot \nabla u - \Delta u = f$$

Explicit time stepping

$$\frac{u_{\text{adv}}^n - u^{n-1}}{k^n} + \vec{\beta} \cdot \nabla u^{n-1} = 0$$

Implicit time stepping

$$\frac{u_{\text{diff}}^n - u^{n-1}}{k^n} - \Delta u_{\text{diff}}^n = 0$$

$$\frac{u_{\text{source}}^n - u^{n-1}}{k^n} = f$$

$$\frac{u^n - u^{n-1}}{k^n} = \frac{u_{\text{adv}}^n - u^{n-1}}{k^n} + \frac{u_{\text{diff}}^n - u^{n-1}}{k^n} + \frac{u_{\text{source}}^n - u^{n-1}}{k^n}$$

$$u^n = u^{n-1} + \delta u_{\text{adv}}^n + \delta u_{\text{diff}}^n + \delta u_{\text{source}}^n$$

– concurrently (in parallel) – by separate codes

Advection and Diffusion

Solve for one physical effect after the other

Separate for one physical effect after the other

Advection-Diffusion(Operator Splitting)

$$\frac{\partial u}{\partial t} + \vec{\beta} \cdot \nabla u - \Delta u = f$$

- Transport (explicit)
- Diffusion (implicit)

Explicit time stepping Implicit time stepping

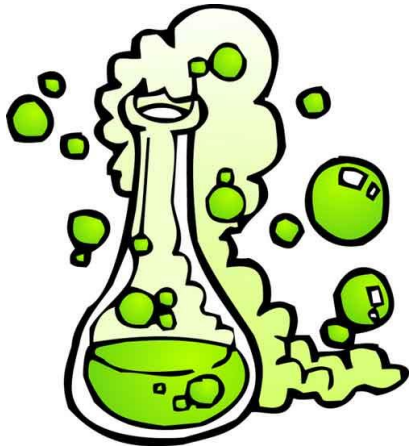
$$\frac{u_{\text{adv}}^n - u^{n-1}}{k^n} + \vec{\beta} \cdot \nabla u^{n-1} = 0 \rightarrow \frac{u_{\text{diff}}^n - u_{\text{adv}}^n}{k^n} - \Delta u_{\text{diff}}^n = 0$$

$$\frac{u_{\text{source}}^n - u_{\text{diff}}^{n-1}}{k^n} = f$$

$$u^n = u_{\text{source}}^n$$

first order in k^n - Lie Splitting

Higher order- Strang Splitting



Reaction($A+B \rightarrow C$)

- Solution variable: $u(x,t) = [u_A(x,t), u_B(x,t), u_C(x,t)]$

- Equation: $\frac{\partial \vec{u}}{\partial t} - \Delta \vec{u} = \vec{f}(\vec{u})$

- Reaction terms: $\vec{f}(\vec{u}) = \begin{pmatrix} -k u_A u_B \\ -k u_A u_B \\ +k u_A u_B \end{pmatrix}$

Advection and Diffusion

$$\frac{\partial \vec{u}}{\partial t} - \Delta \vec{u} = \vec{f}(\vec{u})$$

First order operator splitting (“Lie splitting”):

- First account for the effect of one time step's worth of diffusion (implicit):

$$\frac{\vec{u}^* - \vec{u}^{n-1}}{k^n} - \Delta \vec{u}^* = 0$$

- Then account for one time step's worth of reactions (local ODE):

$$\frac{\partial \vec{u}^{**}}{\partial t} = \vec{f}(\vec{u}^{**}), \quad \vec{u}^{**}(t_{n-1}) = \vec{u}^* \quad \rightarrow \quad \vec{u}^n = \vec{u}^{**}(t_n)$$

- The order could of course be reversed.

$$\frac{\partial \vec{u}}{\partial t} - \Delta \vec{u} = \vec{f}(\vec{u})$$

Second order operator splitting (“Strang splitting”):

- Half diffusion step:

$$\frac{\vec{u}^* - \vec{u}^{n-1}}{k^n/2} - \Delta \vec{u}^* = 0$$

- Full reaction step:

$$\frac{\partial \vec{u}^{**}}{\partial t} = \vec{f}(\vec{u}^{**}), \quad \vec{u}^{**}(t_{n-1}) = \vec{u}^* \quad \rightarrow \text{solve for } \vec{u}^{**}(t_n)$$

- Half diffusion step:

$$\frac{\vec{u}^n - \vec{u}^{**}(t_n)}{k^n/2} - \Delta \vec{u}^n = 0$$

- The order of sub-steps can be reversed.

More accuracy-BDF

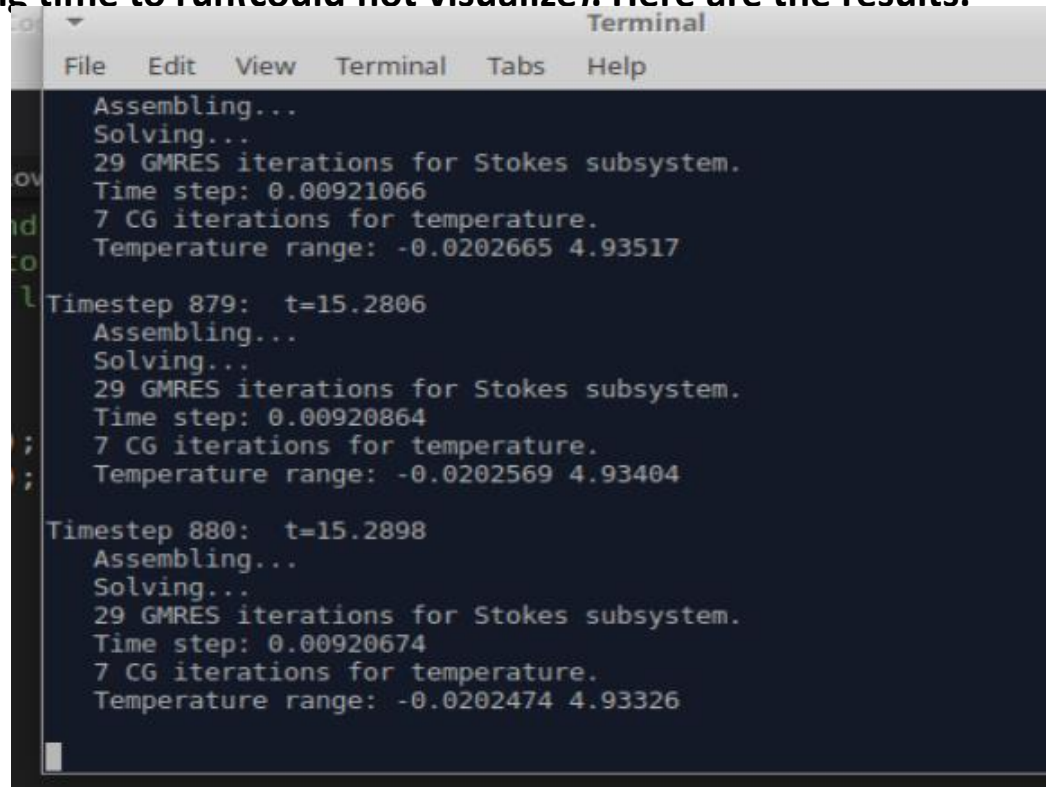
Conclusion

Advection and Diffusion:

- ❖ Solver Introduction
- ❖ Condition
- ❖ Accuracy

Result:Step-22

Took a long time to run(could not visualize). Here are the results.

A screenshot of a terminal window titled "Terminal" with a menu bar containing "File", "Edit", "View", "Terminal", "Tabs", and "Help". The terminal displays the output of a simulation. It shows the process of assembling and solving the system for two consecutive time steps. For each step, it reports the number of GMRES iterations for the Stokes subsystem (29), the time step value, the number of CG iterations for temperature (7), and the temperature range. The results for step 879 and step 880 are shown, with values being very similar.

```
Assembling...
Solving...
29 GMRES iterations for Stokes subsystem.
Time step: 0.00921066
7 CG iterations for temperature.
Temperature range: -0.0202665 4.93517

Timestep 879: t=15.2806
Assembling...
Solving...
29 GMRES iterations for Stokes subsystem.
Time step: 0.00920864
7 CG iterations for temperature.
Temperature range: -0.0202569 4.93404

Timestep 880: t=15.2898
Assembling...
Solving...
29 GMRES iterations for Stokes subsystem.
Time step: 0.00920674
7 CG iterations for temperature.
Temperature range: -0.0202474 4.93326
```


Thank You