CHAPTER 3

RANDOM WAVES

For the purpose of simplification in the analysis we assume that the waves are regular in nature. However, in actual they are random or irregular in their occurrence and behaviour. Successive waves observed at any given location have varying height, period and length. Analysis of such irregular waves measured at a particular site is necessary both from the point of knowing their details within a single sea state as well as for the purpose of deriving the largest wave height expected in the lifetime of a coastal or harbour facility. The former analysis is called Short Term Analysis while the latter is termed as Long Term Analysis.

3.1 Wave Spectrum Analysis

Introduction

The short term wave analysis is restricted to a single wave record observed for a short interval of time (say half an hour) for which the sea conditions are assumed to be stationary and are such that the waves properties can be studied around mean values. (Note: The reader not familiar with basic statistical definitions can refer to Bendat and Piersol, 1986 and Newland, 1975).

The simplified method of spectral analysis consists of characterizing the sea state by superposing a large number of linear progressive waves each with different height, period, length and random phase difference (See Figure 3.1).

Mathematically, this can be expressed as follows:

$$\eta(x,t) = \sum_{n=1}^{M} a_j \cos(k_j x - \omega_j t + \theta_j) \qquad 3.1$$

where

$\eta(x,t)$ = Sea surface elevation being considered at a point which is at a horizontal distance 'x' from any chosen origin and at time instant 't'.

M = Number of linear waves being added together.

a_j = Amplitude of the j^{th} wave.

k_j = Wave number of the j[th] wave = (2π)/Length of the j[th] wave

ω_j = Angular wave frequency = (2π)/Period of the j[th] wave

= (2π)x frequency in cycles per second of the j[th] wave

θ_j = Phase of the j[th] wave assumed to be uniformly distributed over the interval $(0,2\pi)$

The amplitude of the component wave is related to an important statistical function called the Spectral Density Function by the relationship.

$$a_j = \sqrt{2S_\eta(\omega_j)\Delta\omega} \quad 3.2$$

where

$S_\eta(\omega_j)$= Spectral density function corresponding to the frequency ω_j for sea surface (η).

$\Delta\omega$ = frequency step or interval used in calculating above mentioned function.

The graph of spectral density function versus wave frequency is called the wave spectrum and this has a general shape as shown in Figure 3.2.

The spectrum of waves established on the basis of Equations (3.1) and (3.2) is actually a simplified model of the generalized three dimensional representation of the sea surface. The wave spectrum has a number of practical applications. Once a wave spectrum is known, a variety of information can be deduced from it. The significant information obtained is that of the wave frequency composition in a given wave sample. Further the area under the wave spectrum gives the total energy of the irregular wave system per plan area and also the variance value of the water surface fluctuations. The wave spectra when multiplied by suitable transfer function yield the response spectra that are useful in structural design. The wave spectrum is also used in generating the random sea in a laboratory. The integration of the wave spectrum involving different powers of wave frequencies yields important design statistics like significant wave height and average zero cross period.

From Equations (3.1) and (3.2), it may be clear that a wave spectrum can be derived from a given time history of past observations of sea surface elevations.

Wave Spectra and Statistics

For the sake of economy as well as convenience in data collection and handling the wave records of instantaneous sea surface elevations are collected for about 10 to 30 minutes only within each 3 hours duration.

The spectral density function for such a short-term record can be calculated by two different methods.

Covariance Method

The surface (η) spectral density function $S_\eta(f)$ for wave frequency of f is obtained by taking the Fourier Transform of auto-correlation function $R_\mu(\tau)$ for all time lag values τ, i.e.

$$R_\eta(\tau) = \int_0^\infty \eta(t)\eta(t+\tau)dt \qquad 3.3$$

$$S_\eta(f) = \int_0^\infty R_\eta(\tau)\cos(2\pi f\tau)d\tau \qquad 3.4$$

where,

$\eta(t)$ = Sea surface elevation at time t

$\eta(t+\tau)$ = Sea surface elevation at time $t+\tau$

A part of an actual wave record is typically shown in Figure 3.3.An example of variation of $R_\eta(\tau)$ against various lag or τ values is given in Figure 3.4 from which it is evident that the 'auto-correlogram' shows an oscillatory decay for random ocean waves. As can be seen from this figure there is less correlation among the surface elevation values separated by larger time lags. The examples of wave spectral plots showing $S_\eta(f)$ versus f are given in earlier referred Figure 3.6.

Fast Fourier Transform

This is a faster method (FFT) to arrive at $S_\eta(f)$ values and is very useful when large data are required to be handled. This technique however is relatively complex and reference could be made to Bendat and Piersol (1986) and Newland (1975). However the principle involved in it is given below:

In the covariance method of obtaining $S_\eta(f)$ values, the same exponentials appear several times in the calculations. This can be avoided by taking total number of observations (N), say N = 2^m, where m is usually 10 or 11.

The actual formulae involved in the use of FFT are different than those used in the covariance method. In the FFT, $S_k(\omega_k)$ is calculated directly from the observed $\eta(t)$ value, as follows:

$$\eta_k = \frac{1}{N}\sum_{j=0}^{N-1}\eta_j e^{-i\left(\frac{2\pi jk}{N}\right)} \quad 3.5$$

where,

η_k = Discrete Fourier Transform of N values

N = Total number of observed η_j values

η_j = j^{th} value of the sea surface elevation

j = 0,.........,N-1

k = 0,.........,N-1

I = $(-1)^{1/2}$

Then, $$S_\eta(\omega_k) = \frac{2\pi}{T}\eta_k^{\ *}\eta_k \quad 3.6$$

If $$\omega_k = \frac{2\pi k}{T} = \Delta f_k \quad 3.7$$

where

ω_k = Circular wave frequency

T = Total duration of observations

Δ f = Frequency width = 2π/T

$\eta_k^{\ *}$ = Complex conjugate of η_k

f-w Conversions

The wave spectrum can be plotted either as a graph of $S_\eta(f)$ versus f, (where f is natural frequency in Hz), or that of $S_\eta(\omega)$ versus ω (where ω is radial frequency in radians/sec)

In any case, Energy in interval $\Delta\ \omega$ = Energy in interval df. Hence,

$$S_\eta(\omega)d\omega = S_\eta(f)df \qquad 3.8$$

Integration of Wave Spectrum

The spectrum obtained for each short-term sea state can be integrated to obtain the n^{th} spectral moment (m_n) as shown below:

$$m_n = \int_0^\infty f^n S_\eta(f)df \qquad 3.9$$

Where n = 0,1,2...

This is further used to derive the significant wave height (H_s) and average zero cross periods (T_z) as below: (Note: Average zero cross period refers to an average of all periods defined by upcrosses of the zeroth level or Mean Sea Level as explained in Figure 3.5.

$$H_s = \sqrt{4m_0} \qquad 3.10$$

$$T_z = \sqrt{\frac{m_0}{m_2}} \qquad 3.11$$

Sometimes a parameter called Spectral Width Parameter (ε) that shows whether a spectrum is narrow or broad banded is necessary to obtain, which is given by:

$$\varepsilon = \sqrt{1 - \frac{m_0^2}{m_0 m_4}} \qquad 3.12$$

The value of $\varepsilon < 0.75$ usually means a narrow banded spectrum.

Theoretical Spectra

When actual measurements of waves and their analysis as above are not intended, theoretical wave spectra would provide an approximate alternative.

There are several forms of such wave spectra proposed by different authors. Some of the important ones are given below.

Pierson-Muskowitz Spectrum

Kitaigorodskii had proposed a similarity hypothesis that the plots of the observed spectra have similar shapes if plotted in non-dimensional forms. Pierson-Muskowitz (1964) developed their spectrum based on this concept. They assumed that

$$S_\eta(\omega) = \phi(U, g, f) \tag{3.13}$$

where

U = Wind speed

f = Wave frequency,

$\omega = 2\pi f$

and further carried out dimensional analysis to arrive at a functional relationship for S(f).This involved constants that were determined by analysis of the data of North Atlantic Sea using curve fitting techniques. The resulting spectrum is

$$S_\eta(\overline{\omega}) = \frac{ag^2}{\overline{\omega}^5} e^{\left[-\beta\left(\frac{\overline{\omega}_0}{\overline{\omega}}\right)^4\right]} \tag{3.14}$$

where

α = Philip constant = 0.0081 (This is independent of U and wind fetch F)

β = 0.74

$\overline{\omega}_0$ = Frequency corresponding to the peak value of the energy spectrum

= $2\pi f_0$ = g/U_w

Characteristics wind speed $U_w = \left\{ \frac{H_s g \left(\beta / \alpha \right)^{1/2}}{2} \right\}^{1/2}$ 3.15

Equation (3.14) depends only on U_w and not on wind fetch or duration θ, Hence, it is valid for fully developed sea that is produced when wind of unlimited fetch and duration blows, in which case the resulting wave height are not restricted by F or θ and all further input of energy from the wind is dissipated in breakers and not in wave growth.

Several alternative forms of equation (3.14) are available.

$$S_\eta(\omega) = (1/2\pi) S_\eta(f); \omega = 2\pi f; \omega_0 = g / U_w$$

Equation (3.14) becomes:

$$S_\eta(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{\left\{ -\frac{\beta'}{f^4} \right\}} \qquad 3.16$$

where,

$$\beta' = 0.74 \left(\frac{g}{2\pi U_w} \right)^4 \quad ; \alpha = 0.0081 \qquad 3.17$$

Bretschneider Spectrum

Bretschneider (1963) had earlier developed a similar form of spectrum but he had given $S_\eta(f)$ as a function of significant wave height H_s and T_s (which was empirically related to peak-energy frequency f_0) that are obtained from the SMB curves. The spectrum is described as:

$$S_\eta(f) = \frac{5 H_s^2}{16 f_0 (f / f_0)^5} e^{\left\{ -\frac{5}{4} \left(\frac{f}{f_0} \right)^{-4} \right\}} \qquad 3.18$$

or,

$$S_\eta(f) = \frac{\alpha'}{f^5} e^{\left[-\frac{\beta''}{f^4} \right]} \qquad 3.19$$

where

$$\alpha' = \frac{5H_s^2 f_0^4}{16} \quad \text{and} \qquad \beta'' = \frac{5f_0^4}{4} \qquad 3.20$$

Bretschneider spectrum is useful for undeveloped or developing sea, which are more generally met with.

JONSWAP Spectrum

A group led by Hasselmann et al. (1973) conducted wave observations under the Joint North Sea Wave Project (JONSWAP). They analysed data collected in the North Sea and found out that the PM spectra underestimate the spectral peaks, which could be due to the assumption of fully developed sea conditions. Hence Hasselmann et al. suggested a new form of spectrum shown below that incorporates a peak enhancement factor (γ).

$$S(\omega) = \frac{\overline{\alpha} g^2}{\omega^5} e^{\left(-\overline{\beta}\frac{\omega_0^4}{\omega^4}\right)} \gamma^{e^{\left[-\frac{(\omega-\omega_0)^2}{2\omega_0\sigma^2}\right]}} \qquad 3.21$$

or

$$S(f) = \frac{\overline{\alpha} g^2}{2\pi^4 f^5} e^{\left(-\overline{\beta}\frac{f_0^4}{f^4}\right)} \gamma^{e^{\left[-\frac{(f-f_0)^2}{2f_0\sigma^2}\right]}} \qquad 3.22$$

where in

$$\overline{\beta} = 1.25 \qquad 3.22(a)$$

$$\overline{\alpha} = 0.066\left(\frac{gF}{U^2}\right)^{-0.22} \qquad 3.22(b)$$

$$\sigma = 0.07 \text{ if } \omega \le \omega_0 \qquad 3.22(c)$$

$$= 0.09 \text{ if } \omega > \omega_0 \qquad 3.22(d)$$

$$\omega_0 = \text{peak-energy frequency} = 2.84\left(\frac{gF}{U^2}\right)^{-0.33}$$

$$\gamma = 3.3 \text{ average} \qquad 3.23$$

Scott Spectrum

This is modified form of Darbyshire spectrum and is based on different data analysis (Scotts 1965). This could be expressed in terms of H_s.

$$S(\omega) = 0.214 H_s^2 e^{\left[\frac{(\omega-\omega_0)^2}{0.065(\omega-\omega_0+0.26)}\right]^{1/2}} \qquad 3.24$$

for $0.26 < \omega - \omega_0 < 1.65$

$= 0$ otherwise

ω_0 is obtained by $\frac{d}{d\omega} S(\omega) = 0$

This is found to be good for Indian Conditions (Dattatri (1978) and Narasimhan and Deo (1979)) along with the Scotts Wiegel Spectrum explained subsequently. See Figure 3.6 as an example.

Scott-Wiegel Spectrum

Wiegel (1980) replaced the two constants (0.214 and 0.065) involved in the Scott spectrum by variables A' and B'.

$$S(\omega) = A' H_s^2 e^{-\left[\frac{(\omega-\omega_0)^2}{B'(\omega-\omega_0+0.26)}\right]^{1/2}} \qquad 3.25$$

A' and B' are given in tabular form as a function of H_s.

3.2 Wave Statistics

Short term Wave Statistics

Assuming that (i) probability distribution of instantaneous water surface fluctuations is Gaussian normal and (ii) the wave energy is confined to a narrow range of frequencies, the probability density function of individual wave heights (H), which is two times the wave amplitude, is given by a typical 'Rayleigh distribution'

$$p(H) = \frac{2H}{H_{rms}} e^{-(H/H_{rms})^2} \qquad 3.26$$

where H_{rms} = root-mean-square wave height of the record.

The probability distribution function for any numerical wave height being equal to or less than a particular wave height (H_1) can be derived from this.

$$P(H_1) = \int_0^{H_1} \frac{2H}{H_{rms}} e^{-\left(H/H_{rms}\right)^2} dH \qquad 3.27$$

$$= 1 - e^{-\left(H/Hrms\right)^2} \qquad 3.28$$

From this, probability that the wave height exceeds any particular value H_1 is

$$P(H_1) = e^{-\left(H_1/H_{rms}\right)^2} \qquad 3.29$$

Thus, knowing H_{rms} from the wave record, we can find the probability that a particular wave height will exceed a given value. Based on equation (3.26) it is possible to show that:

$$\left.\begin{array}{ll} \text{Mean wave height,} & H_m = 0.885\ H_{rms} \\ \text{Average of highest 33.3\% waves,} & H_{1/3} = 1.416\ H_{rms} \\ \text{Average of highest 10\% waves,} & H_{1/10} = 1.8\ H_{rms} \end{array}\right\}$$

$$\text{Maximum wave height,} \quad H_{max} = 2.172\ H_{rms} \qquad 3.30$$

Knowing any one particular wave height, we can compute other wave heights by the preceding relations.

An example of a typical short-term wave data analysis at an Indian location can be seen from Narasimhan and Deo (1979,1980,1981). Figure 3.7 shows how the observed data at Bombay High satisfactorily matches with the theoretical distribution of Rayleigh. The statistical distribution of wave periods can also be described by theoretical distributions; though its use is very much restricted in practice. Typically if T_z is average zero cross period, the probability density function of individual wave period (T_z) is given by (Bretschneider, 1977).

$$p(T) = 2.7\left(T^3/T_z^4\right)\exp\left(-0.675\left(T^4/T_z^4\right)\right) \qquad 3.31$$

There is generally a lack of correlation in the joint occurrence of wave height and wave period values. This can be seen from Figure 3.8 which shows that a given wave height can occur along with a range of values of wave periods and further that the largest waves are rarely associated with the longest periods.

Tucker Method

Tucker method is a quick and simple method for analyzing wave data and is based on observing only a few larger surface fluctuations of the sea state along with the total number of waves in that sea state. Tucker (1963) assumed the nature of wave spectra as narrow banded and gave the following expression to determine the root-mean-square wave height of the record.

$$H_{rms} = (2)^{1/2} H_1 (2\theta)^{-1/2} \left(1 + 0.289\theta^{-1} - 0.247\theta^{-2}\right)^{-1} \qquad 3.31a$$

$$= (2)^{1/2} H_2 (2\theta)^{-1/2} \left(1 - 0.289\theta^{-1} - 0.103\theta^{-2}\right)^{-1} \qquad 3.31b$$

H_1 = A+C ; H_2 = B+D ; $\theta = \log N_z$

where ,

A = Height of the highest crest in the given record above SWL

B = Height of the second highest crest above SWL

C = Depth of the lowest trough below SWL

D = Depth of the second lowest trough below SWL

N_z = Total number of zero up-crosses in the record.

For typical wave records collected at Bombay High, Figure 3.9 shows the extent of agreement between the H_{rms} values calculated by using Tucker method and those obtained directly from the entire record. It may be seen that despite the fact that the Tucker's method relies only on few observations of the highest and the second highest waves instead of the entire record, it gives satisfactory estimation of H_{rms} value.

Long-term Wave Height Statistics

Most of the structures are designed to withstand the design significant wave height having a return period of 100 years or so. Such a design wave can be derived from long-term statistical distribution of H_s values.

The pre-requisite for the long term description of the wave heights is that of collection of short term (or 3 hourly) wave records over duration of at least one year and preferably more. From each short-term wave record, a pair of significant wave height

(H_s) and average zero cross period (T_z) is derived. These data are often summarized in the form of a scatter diagram shown in Figure 3.10.

The mean, variance and other higher distribution moments of H_s are then calculated. These are used to establish one of the few theoretical long-term distribution of H_s as give below:

Gumbel Distribution (also called Type I distribution)

$$P(H_s) = e^{\left\{-e^{[-\alpha(H_s - u)]}\right\}} \qquad 3.32a$$

where,

$P(H_s)$ = cumulative probability distribution function for H_s.

$$\alpha = \pi / \left(6\sigma^2 H_s\right)^{1/2} \qquad 3.32b$$

$$u = \overline{H}_s - (0.5772)/\alpha \qquad 3.33$$

$\sigma^2_{H_s}$ = variance of all H_s values

$\overline{H}_s$ = mean of all H_s values.

Weibull Distribution (also called Type III$_L$ distribution):

$$P(H_s) = 1 - e^{-\left[\frac{H_s - A}{B}\right]} \qquad 3.34$$

where A,B,C are the parameters to be determined from data.

Log Normal Distribution:

It assumes that H_s values are normally distributed.

$$P(H_s) = \frac{1}{\sqrt{2\pi}} \int_{h=0}^{H_s} \frac{1}{Ch} e^{-\frac{1}{2}\left[\frac{\ln(h) - B}{C}\right]^2} dh \qquad 3.35$$

B, C are parameters that are determined from given H_s values.

Other relatively less important theoretical distributions are:

Fretchet Distribution (also called Type II distributions)

$$P(H_s) = e^{\left[(-H_s/B)^{-C}\right]} \qquad 3.36$$

Where B, C are dependent on data.

Upper bound Type III u distribution:

This is useful when there is an upper limit to the growth of H_s values at the site.

$$P(H_s) = e^{\left[-\left(\frac{A-H_s}{B}\right)^{C}\right]} \qquad 3.37$$

Where A, B, C are to be determined from the data.

Having established a relationship between H_s and $P(H_s)$ as above, the probability of non-exceedance, $[P(H_s)]_{Tr}$ corresponding to a return period Tr of say 100 years, is calculated using the equation:

$$[P[H_s]]_{Tr} = 1 - \frac{1}{(number of H_s value \sin a year)Tr} \qquad 3.38$$

The value of H_s obtained by substituting above, $[P(H_s)]_{Tr}$ in any one of the Equations (3.32.a), (3.34), (3.34), (3.36) and (3.37) gives the design or long term significant wave height.

Examples of derivation of long term distribution of wave height, (H_s) and design wave height can be seen in Deo and Burrows (1986), Deo and Venugopal (1991), Goswami et al. (1991), Kirankumar et al. (1989), Pagrut and Deo (1992), Soni et al. (1989), Baba and Shahul Hameed (1989) and Baba and Kurian (1988).

Derivation of the distribution parameters like α and u using Equations (3.32b) and (3.33) involves use of what is called the method of moments to fit the data where the underlying equations are worked out by equating the sample moments to the population moments. Alternatively, a least squares approach as well as the method of maximum likelihood functions can also be employed to obtain the distribution parameters like α ,u, A, B, C (Sarpkaya and Issacson, 1981). However, these techniques are more laborious and need not necessarily mean a better accuracy in the resulting estimates.

For better accuracy in the estimation of design H_s values, some times, the derived distribution of observed H_s values (i.e. $P(H_s)$ versus H_s) is fitted to all the four theoretical distributions as mentioned above. Then the theoretical goodness of fit criteria, like the Chi-square test, Kolmogorov-Smirnov test or Confidence Bands are applied to choose one particular distribution that most closely fits the observed H_s distribution. The prediction of 100 years H_s value is thereafter made based on this 'best-fit' distribution. (See Soni et al. 1989 for more details). Following example will illustrate the simple method to get the design H_s having a return period of say 100 years.

Example: Annual data of significant wave heights collected for a site along the East coast of India is given below:

H_s (in m)	0	1	2	3	4	5
No. of observations	1198	999	322	112	15	2

Obtain the design H_s value corresponding to 100 years return using the Gumbel distribution.

Solution:

Gumbel distribution is

$$P(H_s) = e^{\left\{-e^{[-\alpha(H_s-u)]}\right\}}$$

where, $\alpha = \pi / \left(6\sigma^2 H_s\right)^{1/2}$ and $u = \overline{H}_s - (0.5772)/\alpha$

$$\overline{H}_s = \left(\sum H_s\right)/N$$

= [0.5(1198)+1.5(999)+2.5(322)+3.5(112)+4.5(15)+5.5(2)]/2648

= 1.27 m

$$\sigma_{H_s}^2 = [\sum\left(H_s - \overline{H}_s\right)^2]/N$$

$= [1198(0.5-1.27)^2+999(1.5-1.27)^2+322(2.5-1.27)^2+112(3.5-1.27)^2+15(4.5-1.27)^2+2(5.5-1.27)^2]/2648$

= 0.76

$$\alpha = \pi / \left(6\sigma^2 H_s\right)^{1/2}$$

$= \pi/[6(0.76)]^{1/2}$

$= 1.47$

$$u = \overline{H}_s - (0.5772)/\alpha$$

$= 1.27-(0.5772/1.47)$

$= 0.877$

For 100 years, $P(H_s) = 1-1(8*365*100) = 0.999\,99\,66$

Hence from $P(H_s) = e^{\left\{-e^{[-\alpha(H_s-u)]}\right\}}$, $H_s = u + [-\ln-\ln(0.999\,99\,66)]/\alpha$

$= 0.877 + [-\ln-\ln(0.999\,99\,66)]/1.47$

$= 9.44$ m

Long Term Distribution of Individual Wave Heights:

The long term distribution of individual wave heights was initially derived by Battjes and subsequently modified by Burrows as below:

$$[P(H)]_{LT} = 1.0 - \sum_{i=1}^{M^*} \exp\left(-2H^2/H_{si}^2\right)\overline{T}_{zi}^{-1}\, W_i \Big/ \overline{T}_z^{-1} \qquad 3.39$$

where,

$P(H)_{LT}$ = Long term distribution of individual wave heights

H = Individual wave heights

M^* = Corresponds to limiting value of H_s at the site say due to water depth

H_{si} = Midpoint Hs value corresponding to i^{th} row of (H_s, T_z) scatter diagram

(See Figure 3.11)

$$\overline{T}_{zi} = \sum_{allj} T_{zj}\left(W_{ij} \Big/ \sum_{allj} W_{ij}\right)$$

where,

T_{zj} = T_z value corresponding to the j_{th} column of (H_s, T_z) scatter diagram

W_{ij} = Total number of occurrence of H_s values in the (i, j) interval

W_i = $P(H_{is} + 0.5\Delta H_s) - P(H_{is} - 0.5\Delta H_s)$ obtained from the underlying observed and fitted distribution of H_s.

$$W = \sum_{all\,i} \sum_{all\,j} W_{ij}$$

P() = Cumulative distribution function of ()

ΔH_s = Class width of H_s in the (H_s, T_z) scatter diagram

$$\overline{T}_z = \sum_{i=1}^{M^*} T_{zi} W_i$$

A design individual wave height having a return period of 100 years is derived by reading the value of H from such a distribution curve that corresponds to a cumulative probability, $P(H) = 1 - 1/\left(\check{T}_z\, 3600 * 24 * 365 * 100)\right)$ since $\check{T}_z$ represents average number of waves per second in the long term. Figure 3.11 shows an example of long-term distribution of H values using the above equations.

TABLE 1

A PART OF THE TYPICAL COMPUTER PRINTOUT FOR SPECTRAL ANALYSIS

(DATE 2.7.78)

LAG NO.	AUTO CORRELATION	FREQUENCY	SPECTRAL DENSITY
0	1.000000	0.0	0.091563
1	0.668255	0.010	0.099117
2	0.127182	0.020	0.061174
3	-0.345670	0.030	0.055347
4	-0.609706	0.040	0.065691
5	-0.605127	0.050	0.084348
6	-0.412819	0.060	0.223711
7	-0.125942	0.070	1.463952
8	0.116177	0.080	5.341101
9	0.282754	0.090	9.498674
10	0.377365	0.100	8.477833
11	0.373429	0.110	5.138504
12	0.253866	0.120	3.327433
13	0.076564	0.130	2.585230
14	-0.116333	0.140	2.156492
15	-0.258884	0.150	2.038771
16	-0.318416	0.160	1.873202
17	-0.271008	0.170	1.090199
18	-0.128157	0.180	0.598183
19	0.033781	0.190	0.640248
20	0.163707	0.200	0.566144
21	0.203942	0.210	0.299311
22	0.182920	0.220	0.221035
23	0.129335	0.230	0.245811
24	0.049667	0.240	0.219826
25	-0.014684	0.250	0.237477
26	-0.090852	0.260	0.289445
27	-0.155121	0.270	0.274819
28	-0.165966	0.280	0.238758
29	-0.097720	0.290	0.179577
30	0.005123	0.300	0.176351

TABLE II

SOME OF TH IMPORTANT WAVE STATISTICS OBTAINED FROM THE SPECTRAL ANALYSIS

No.	Date	Variance m^2	Standard Deviatio m^2	Third Central Moment m^2	Fourth Central Moment m^2	Zeroth spectral moment m^2	Second spectral moment m^2	Fourth spectral moment m^2	Significant Wave Height m	Average Zero cross period Sec	Spectral width parameter
1	2	3	4	5	6	7	8	9	10	11	12
1	17.6.78	0.998	0.999	-0.030	2.940	0.997	0.020	0.0013	3.99	7.02	0.83
2	18.6	0.645	0.803	-0.153	3.426	0.641	0.015	0.0012	3.20	6.55	0.84
3	19.6	0.530	0.728	-0.028	3.024	0.528	0.010	0.0006	2.91	7.23	0.84
4	20.6	0.652	0.80	0.057	3.345	0.651	0.016	0.0011	3.23	6.36	0.79
5	21.6	0.827	0.9[illegible]9	0.044	2.869	0.822	0.021	0.0015	3.63	6.25	0.79
6	22.6	0.977	0.[illegible]89	0.130	2.771	0.980	0.024	0.0015	3.96	6.36	0.78
7	23.6	1.069	1.034	0.034	2.969	1.066	0.024	0.0014	4.13	6.66	0.78
8	24.6	0.793	0.891	0.101	2.684	0.789	0.023	0.0016	3.55	5.84	0.76
9	25.6	0.732	0.856	-0.078	2.608	0.728	0.019	0.0013	3.41	6.17	0.78
10	26.5	0.514	0.717	-0.034	2.643	0.512	0.014	0.0009	2.86	5.96	0.75
11	27.6	0.391	0.626	-0.028	3.169	0.388	0.009	0.0006	2.49	6.63	0.81

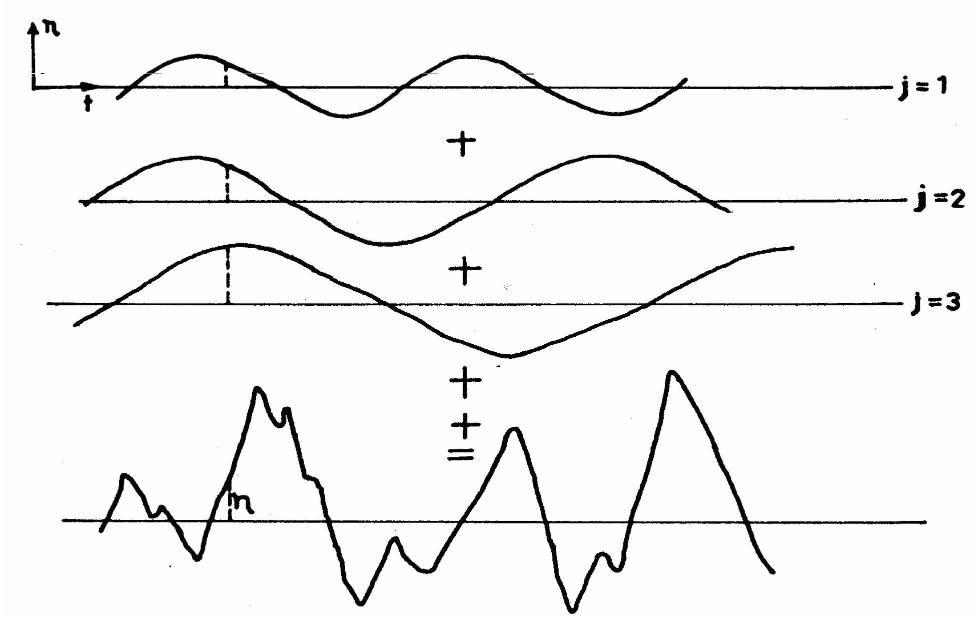

Fig 3.1 Superposition of Linear Waves

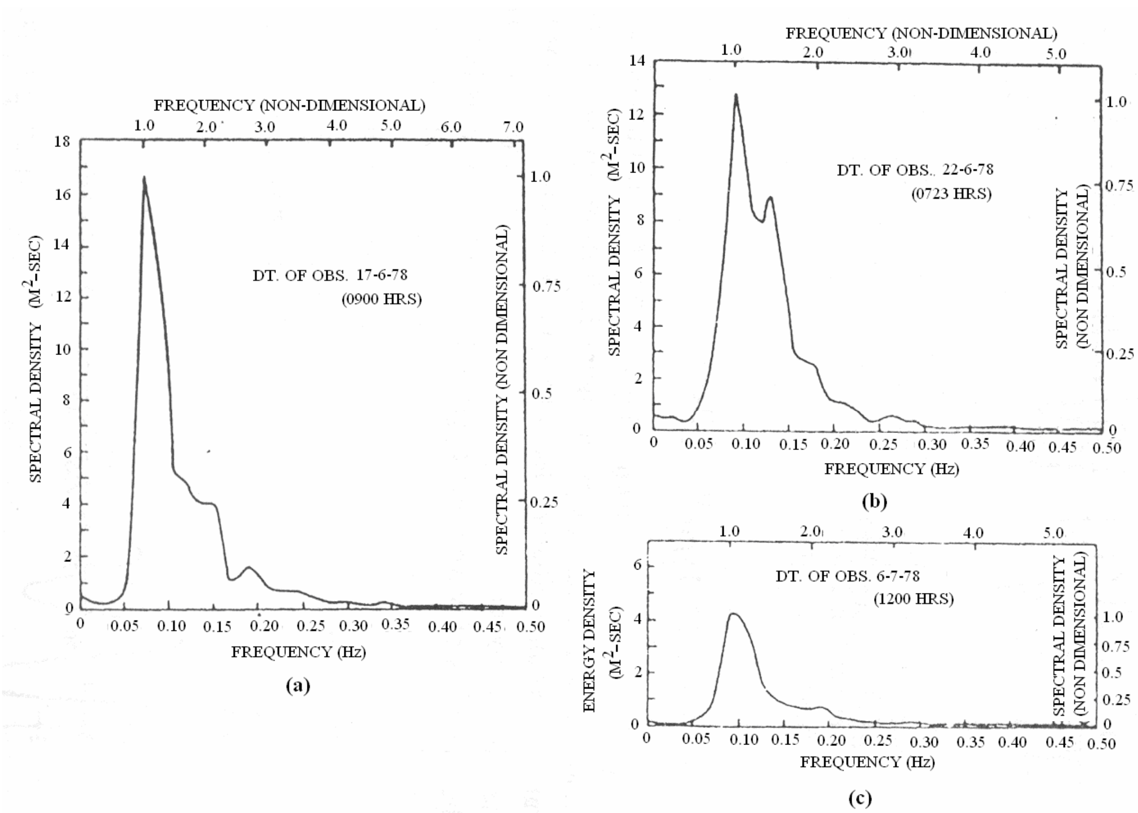

Fig.: 3.2 Examples of Wave Spectra

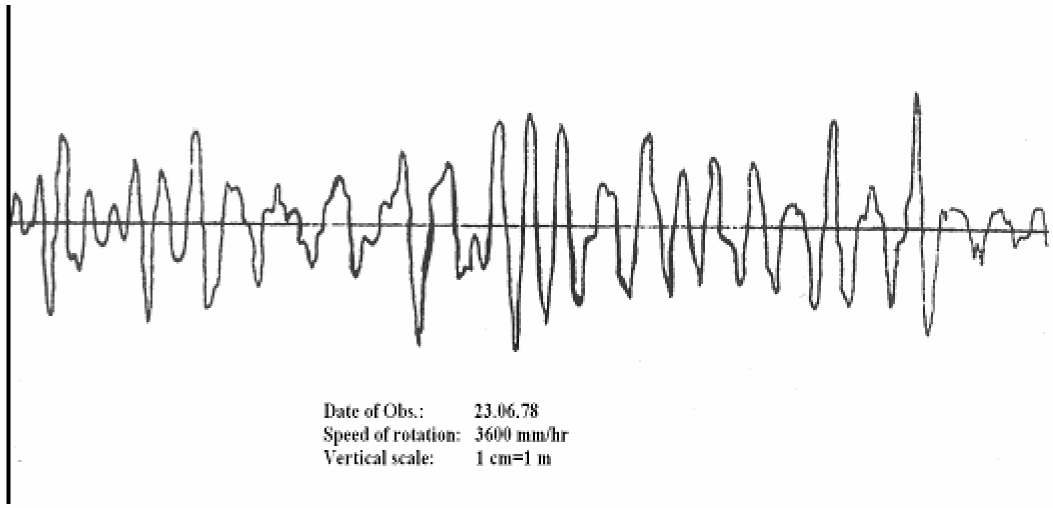

Fig 3.3 Typical Wave Record

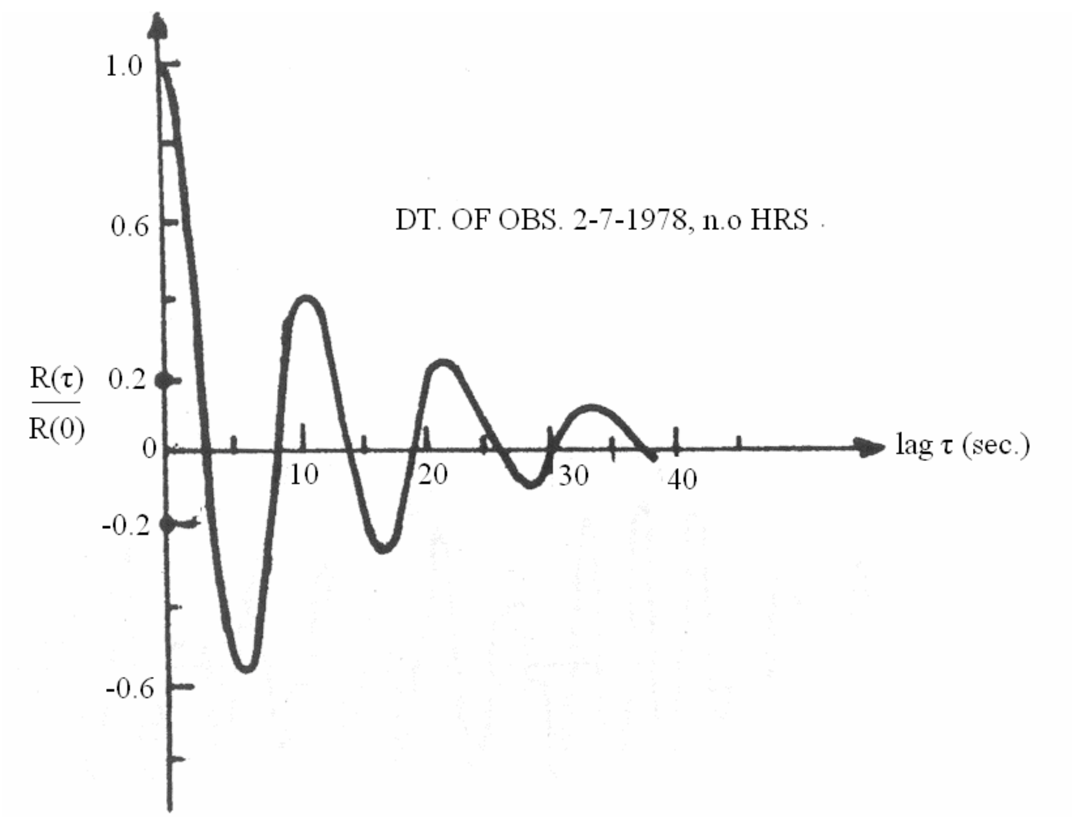

Fig 3.4 Correlogram

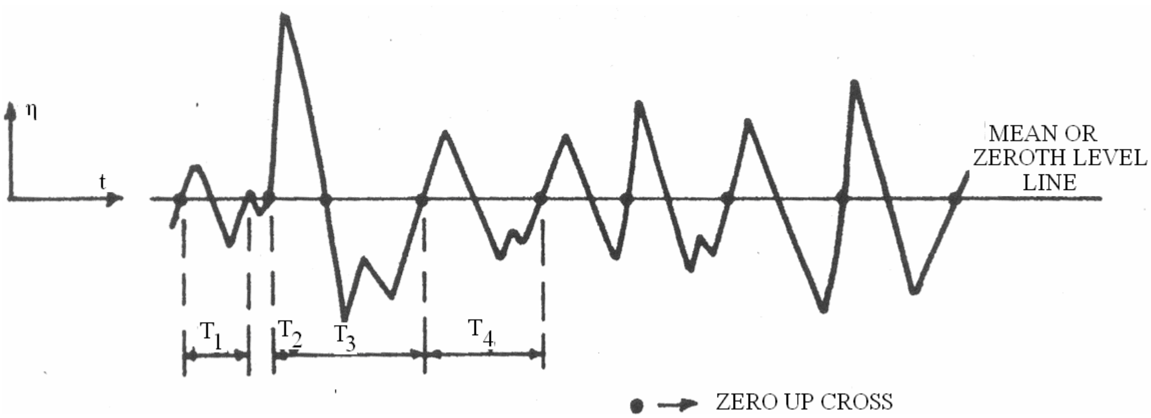

Fig 3.5 Individual Wave Periods

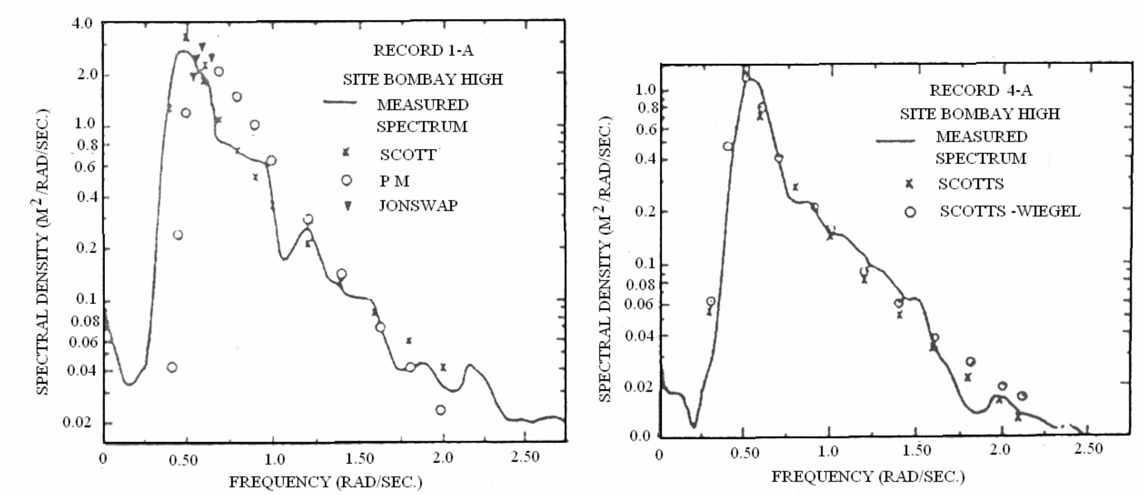

Fig 3.6 Comparison of Wave Spectra

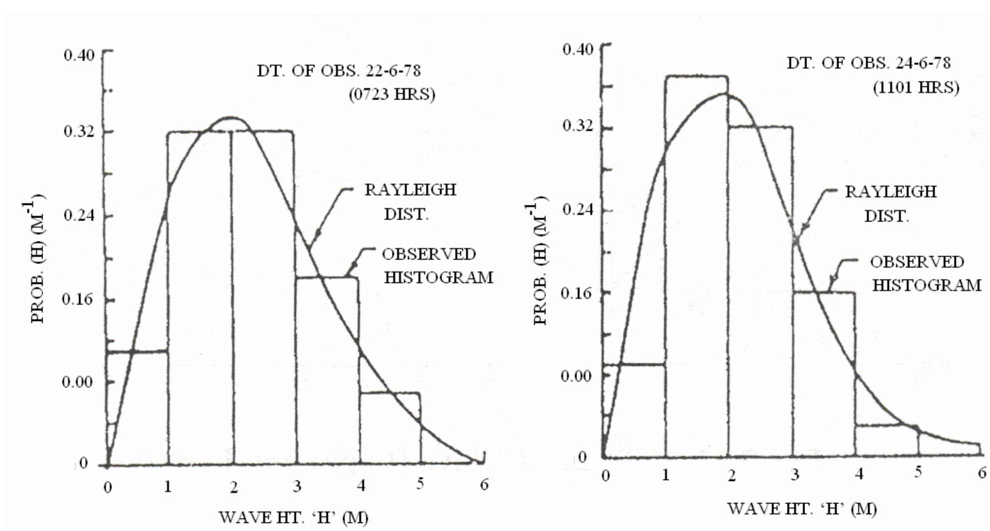

Fig 3.7 Wave height Distribution

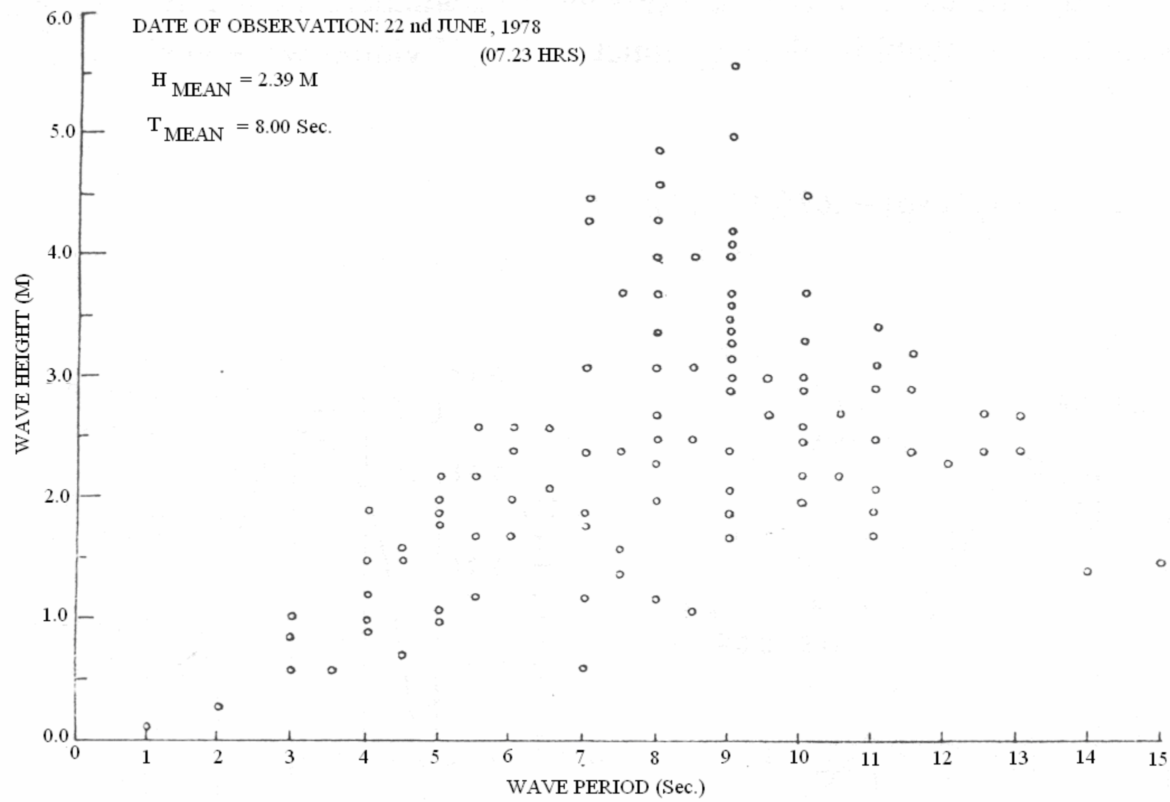

Fig 3.8 Joint Distribution of Wave Height and Period

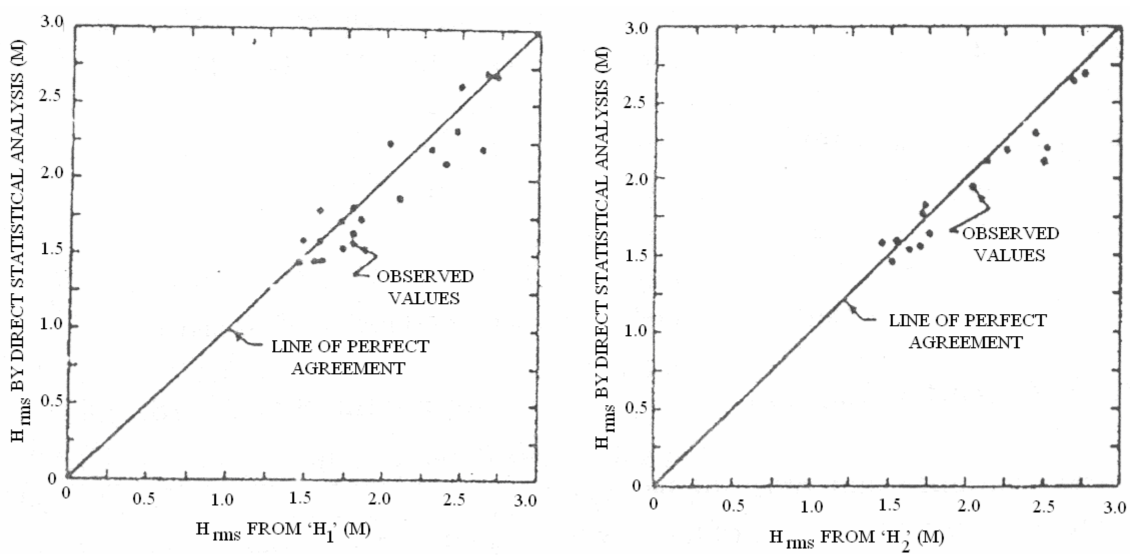

Fig 3.9 Tucker Method

H_s(m) \ T_z (sec)	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17
0.0–0.5				3	4	1	1	2		2			1	
0.5–1.0	2	56	220	191	136	103	59	40	18	9	9	1	1	2
1.0–1.5		74	235	259	150	53	14	2	1	1				
1.5–2.0		4	33	34	36	8	2							
2.0–2.5			2		2									

Fig 3.10 Scatter Diagram (Wave data off Machilipatnam-From 9.5.83 – 18.12.83)

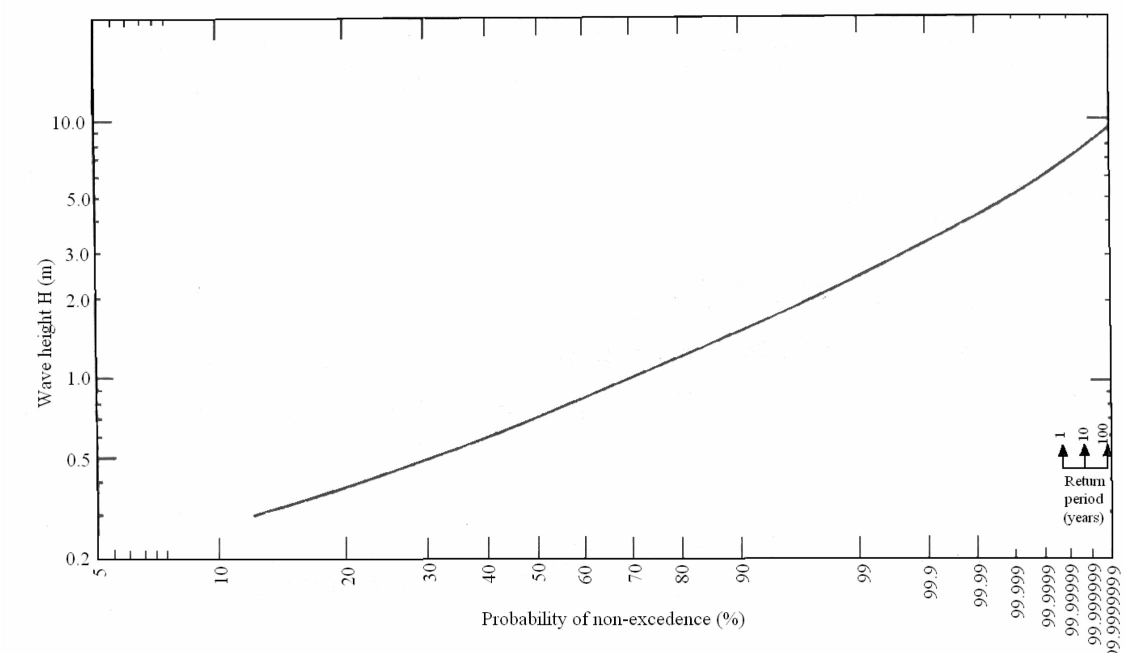

Fig. 3.11 Long Term Distribution of ‘H’
(Underlying H_s distribution – Weibull)