

Some other Positional Measure



Median is the most important positional measure. It divides the whole distribution into two equal parts. That is 50% observations are equal to or smaller than median and 50% observations are equal to or larger than the median . Other important positional measures are

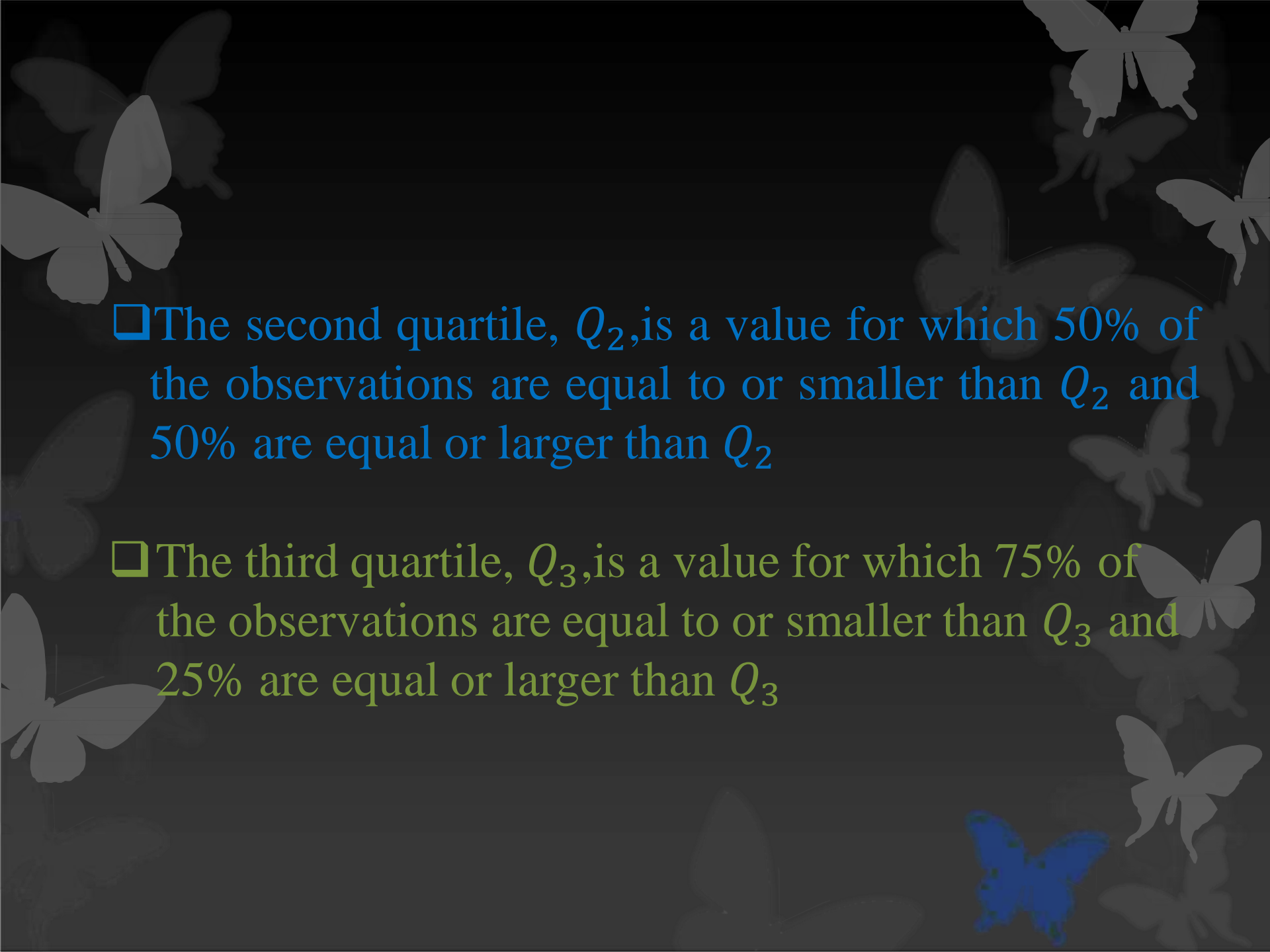
- Quartiles
- Deciles
- Percentiles

In general, Quartiles, Deciles and Percentiles are all known as quartiles.

Quartiles

Quartiles divide the ordered data into 4 equal parts. So there are three quartiles. They are Q_1 , Q_2 and Q_3 . Q_2 is the median, Q_1 and Q_3 are called the first and third quartiles.

□ The first quartile, Q_1 , is a value for which 25% of the observations are equal to or smaller than Q_1 and 75% are equal or larger than Q_1 .

The background of the slide is dark gray with numerous light gray butterfly silhouettes scattered across it. One butterfly in the bottom right corner is highlighted in a vibrant blue color.

❑ The second quartile, Q_2 , is a value for which 50% of the observations are equal to or smaller than Q_2 and 50% are equal or larger than Q_2

❑ The third quartile, Q_3 , is a value for which 75% of the observations are equal to or smaller than Q_3 and 25% are equal or larger than Q_3

Deciles

Deciles divide the total ordered data into 10 equal parts. So there are nine deciles. They are denoted by D_1, D_2, \dots, D_9 .

□ D_5 is the median.

Percentile

Percentiles divide the total ordered data into 100 equal parts. So there are 99 Percentiles.

They are denoted by P_1, P_2, \dots, P_{99} .

- ☐ P_{50} is the median or 5th decile or 2nd quartile of the distribution.
- ☐ P_{25} is the first quartile
- ☐ P_{75} is the third quartile
- ☐ P_{20} is the second decile

Computation of Quartiles, Deciles and Percentiles from grouped data of continuous type

The following formulae are used for finding Quartiles, Deciles and Percentiles

The i th quartile, $Q_i = L_i + \frac{\frac{in}{4} - F_i}{f_i} \times c_i$, for $i=1,2,3$.

The j th decile, $D_j = L_j + \frac{\frac{jn}{10} - F_j}{f_j} \times c_j$, for $j=1,2,\dots,9$

The k th percentile, $P_k = L_k + \frac{\frac{kn}{100} - F_k}{f_k} \times c_k$, for $k=1,2,\dots,99$.

Example

The following frequency distribution refers to the number of hours worked per month of 50 workers of a factory. Compute Q_1 , Q_3 , D_3 , P_{65} and interpret the values you obtained.

No. of hours worked per month	Frequency
30-55	3
55-80	4
80-105	6
105-130	9
130-155	12
155-180	11
180-205	5

Solution

Class	Frequency f	Cumulative frequency F
30-55	3	3
55-80	4	7
80-105	6	13
105-130	9	22
130-155	12	34
155-180	11	45
180-205	5	50
Total	n =50	

First Quartile Q_1

Here $n=50$. First quartile Q_1 is the $\frac{n}{4}$ th ordered observation = 12.5th ordered observation which lies in the class interval 80-105.

We know that $Q_1 = L_1 + \frac{\frac{n}{4} - F_1}{f_1} \times c_1$

Here $L_1=80$, $\frac{n}{4}=12.5$, $F_1=7$, $f_1 = 6$ and $c_1=25$

Then $Q_1=102.92$ hours per month

Comment: 25% of the workers worked for 102.72 hours or less per month whereas 75% worked for more than 102.72 hours.

Third Quartile Q_3

Here $n=50$. Third quartile Q_3 is the $\frac{3n}{4}$ -th ordered observation = 37.5^{th} ordered observation which lies in the class interval 155-180.

We know that $Q_3 = L_3 + \frac{\frac{3n}{4} - F_3}{f_3} \times c_3$

Here $L_3=80$, $\frac{n}{4}=12.5$, $F_3=7$, $f_3 = 6$ and $c_3=25$

Then $Q_3=162.95$ hours per month

Comment: 75% of the workers worked for 162.95 hours or less per month whereas 25% worked for more than 162.95 hours.

Third Decile

Here $n=50$. Third decile D_3 is the $\frac{3n}{10}$ th ordered observation = 15th ordered observation which lies in the class interval 105-130.

We know that $D_3 = L_3 + \frac{\frac{3n}{10} - F_3}{f_3} \times c_3$

Here $L_3=105$, $\frac{3n}{10}=15$, $F_3=13$, $f_3 = 9$ and $c_3=25$

Then $Q_3=110.56$ hours per month

Comment: 30% of the workers worked for 110.56 hours or less per month whereas 70% worked for more than 110.56 hours per month.

Sixty fifth percentile P_{65}

Here $n=50$. Sixty fifth percentile P_{65} is the $\frac{65n}{100}$ th ordered observation = 32.5^{th} ordered observation which lies in the class interval 130-155.

We know that
$$P_{65} = L_{65} + \frac{\frac{65n}{100} - F_{65}}{f_{65}} \times c_{65}$$

Here $L_{65}=130$, $\frac{65n}{100}=32.5$, $F_{65}=22$, $f_{65} = 12$ and $c_{65}=25$

Then $Q_3=151.875$ hours per month

Comment: 65% of the workers worked for 151.875 hours or less per month whereas 35% worked for more than 151.875 hours per month.

