2.4 Novel Enhanced Representation for Quantum Images

In the FRQI and MCQI representations, the color information is encoded by the superposition of one and three qubits, separately. Therefore, these quantum images will probably be retrieved based on multiple measurements. A novel enhanced quantum representation for digital images (NEQR) [30] that improves on the earlier models is now introduced. The new model uses the basis state of a qubit sequence to store the grayscale value of every pixel. Therefore, two qubit sequences, representing the grayscale and positional information of all of the pixels, are used in NEQR representation to store the whole image.

2.4.1 NEQR Representation and Initialization

NEQR representation uses the basis state of a qubit sequence to store the grayscale value of every pixel, instead of an angle encoded in a qubit, as in FRQI representation [30]. The representation of a $2^n \times 2^n$ NEQR image is defined as

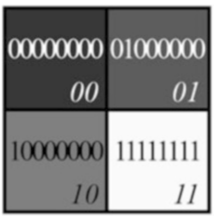
$$|I\rangle = \frac{1}{2^n} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^n-1} |f(y,x)\rangle |yx\rangle = \frac{1}{2^n} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^n-1} \bigotimes_{i=0}^{q-1} |C_{yx}^i\rangle |yx\rangle. \tag{2.55}$$

where $f(y, x)$ is the grayscale value, defined as

$$f(y,x) = C_{yx}^{q-1} C_{yx}^{q-2} \ldots C_{yx}^1 C_{yx}^0, \tag{2.56}$$

where $C_{yx}^i \in \{0, 1\}$ and $f(y, x) \in [0, 2^q - 1]$. An example of a 2×2 NEQR image and its quantum state is shown in Fig. 2.15.

The computational complexity of preparing an NEQR image exhibits an approximately quadratic decrease, i.e., $O(qn \cdot 2^{2n})$, compared to FRQI and MCQI images [30]. However, it should be stressed that NEQR representation uses more qubits to encode a quantum image. From its representation, q+$2n$ qubits are needed to construct the quantum image model for a $2^n \times 2^n$ image with gray range 2^q. The $2n$ qubits for position information is the same as for FRQI and MCQI representation. NEQR uses q qubits for color information, while FRQI and MCQI use one qubit



$$|I\rangle = \frac{1}{2}(|0\rangle \otimes |00\rangle + |64\rangle \otimes |01\rangle + |128\rangle \otimes |10\rangle + |255\rangle \otimes |11\rangle)$$
$$= \frac{1}{2}(|00000000\rangle \otimes |00\rangle + |01000000\rangle \otimes |01\rangle + |10000000\rangle \otimes |10\rangle + |11111111\rangle \otimes |11\rangle)$$

Fig. 2.15 A 2×2 NEQR image and its quantum state and three qubits, respectively. Researchers have devised improved NEQR (INEQR) and generalized QIR (GQIR) by resizing the quantum image to an arbitrary size for wider applications [8].

The first step in preparing an NEQR image is similar to that for an FRQI image, as presented in Sect. 2.2.1, hence, it is not repeated here. In the second step, the grayscale value for every pixel is set. This step is divided into 2^{2n} sub-operations to store the grayscale information for every pixel. During image retrieval from the quantum image, every pixel should be recovered individually by quantum measurements over the computational basis. After all pixels are recovered, the accurate classical image will be retrieved from the NEQR image model [30].

2.4.2 Color Transformations on NEQR Images

This section will discuss how to use the NEQR representation for QIMP, including complement color transformation as well as color transformation on the quantum image [30].

2.4.2.1 Complement Color Transformation

Complement color transformation changes all the grayscales of the pixels in an NEQR image to the complement values on 2^q [30]. The complement color transformation U_C is defined as

$$U_C = X^{\otimes q} \otimes I^{\otimes 2n}, \tag{2.57}$$

where X denotes the NOT gate and I represents the identity gate, as presented in Fig. 1.5.

For a quantum image $|I\rangle$, U_C takes q NOT gates for each color qubit and $2n$ identity gates for others. Therefore, this operation inverts all the color qubits in the NEQR model and changes the grayscale value of every pixel in the image to its opposite value. Equation (2.58) produces the result of the U_C operation on the quantum image $|I\rangle$ as

$$\begin{aligned} U_C(|I\rangle) &= U_C\left(\frac{1}{2^n}\sum_{y=0}^{2^n-1}\sum_{x=0}^{2^n-1}|f(y,x)\rangle|y\rangle|x\rangle\right) \\ &= \frac{1}{2^n}\sum_{y=0}^{2^n-1}\sum_{x=0}^{2^n-1}\left(\bigotimes_{i=0}^{q-1}\left(X|C_{yx}^i\rangle\right)|y\rangle|x\rangle\right) \\ &= \frac{1}{2^n}\sum_{y=0}^{2^n-1}\sum_{x=0}^{2^n-1}|2^q-1-f(y,x)\rangle|y\rangle|x\rangle. \end{aligned} \tag{2.58}$$