Algorithm $\mathcal{M}$

Input: A graph $G = (V, E)$, IN, BN, LN, FL $\subseteq V$

Reduce G according to the reduction rules.

if there is some unreachable $v \in \text{Free} \cup \text{FL}$ **then** return 0

if $V = \text{IN} \cup \text{LN}$ **then** return $|\text{LN}|$

Choose a vertex $v \in \text{BN}$ of maximum degree.

if $d(v) \geq 3$ **or** $(d(v) = 2$ and $N_{\text{FL}}(v) \neq \emptyset)$ **then**

  $\langle v \to \text{LN} \,||\, v \to \text{IN} \rangle$ (B1)

else if $d(v) = 2$ **then**

  Let $\{x_1, x_2\} = N_{\text{Free}}(v)$ such that $d(x_1) \leq d(x_2)$.

  if $d(x_1) = 2$ **then**

    Let $\{z\} = N(x_1) \setminus \{v\}$

    if $z \in \text{Free}$ **then**

      $\langle v \to \text{LN} \,||\, v \to \text{IN}, x_1 \to \text{IN} \,||\, v \to \text{IN}, x_1 \to \text{LN} \rangle$ (B2)

    else if $z \in \text{FL}$ **then** $\langle v \to \text{IN} \rangle$

  else if $(N(x_1) \cap N(x_2)) \setminus \text{FL} = \{v\}$ **and** $\forall z \in (N_{\text{FL}}(x_1) \cap N_{\text{FL}}(x_2))$,

    $d(z) \geq 3$ **then** (B3)

    $\langle v \to \text{LN} \,||\, v \to \text{IN}, x_1 \to \text{IN} \,||\, v \to \text{IN}, x_1 \to \text{LN}, x_2 \to \text{IN} \,||$

    $v \to \text{IN}, x_1 \to \text{LN}, x_2 \to \text{LN}, \quad N_{\text{Free}}(\{x_1, x_2\}) \to \text{FL}, N_{\text{BN}}(\{x_1, x_2\}) \setminus \{v\} \to \text{LN} \rangle$

  else $\langle v \to \text{LN} \,||\, v \to \text{IN}, x_1 \to \text{IN} \,||\, v \to \text{IN}, x_1 \to \text{LN}, x_2 \to \text{IN} \rangle$ (B4)

else if $d(v) = 1$ **then**

  Let $P = (v = v_0, v_1, \ldots, v_k)$ be a maximum path such that

    $d(v_i) = 2$, $1 \leq i \leq k$, $v_1, \ldots, v_k \in \text{Free}$.

  Let $z \in N(v_k) \setminus V(P)$.

  if $z \in \text{FL}$ **and** $d(z) = 1$ **then** $\langle v_0, \ldots, v_k \to \text{IN}, z \to \text{LN} \rangle$

  else if $z \in \text{FL}$ **and** $d(z) > 1$ **then** $\langle v_0, \ldots, v_{k-1} \to \text{IN}, v_k \to \text{LN} \rangle$

  else if $z \in \text{BN}$ **then** $\langle v \to \text{LN} \rangle$

  else if $z \in \text{Free}$ **then** $\langle v_0, \ldots, v_k \to \text{IN}, z \to \text{IN} \,||\, v \to \text{LN} \rangle$ (B5)