2019/19

$X(t_1)$ $t_1 = t_1$

Stochastic Process

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Defn: Let $(\Omega, \mathcal{F}, P)$ be a prob. space. The collection of rvs $\{X(t), t \in T\}$ defined on $(\Omega, \mathcal{F}, P)$ is called a stochastic process.

$\to$ Parameter space

State Space

		Disc	Cont
S	Disc	No. of stud registered on nth sem	No. of students entering a class till time t.
	Cont	Temp over diff hours	Weight of a stud over time.

T

POISON PROCESS

$\{X(t) ; t \geq 0\}$ $\longrightarrow$ No. of events occuring upto & including time t.

(Discrete state Continuous parameter)

$\boxed{X(t_1) \sim P(\lambda t)}$

Observed values of stochastic process is time series

Properties of Stochastic Process

① Independent increments :-

For arbitrary $0 \leq t_1 < t_2 < \dots < t_n < \dots$

For every n,

if the rvs

$$X(t_1) - X(t_0) \; ; \; X(t_2) - X(t_1), \dots, X(t_n) - X(t_{n-1})$$

are mutually independent.

Then, $\{X(t), t \geq 0\}$ has independent increment properties.

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(2) Stationary Property :-

(a) Wide sense :-

If (1) $E(X(t))$ is not a f^n of t.

(2) $E(X^2(t)) < \infty$

(3) $Cov(X(t), X(s))$ depends only $|t-s|$

Then $\{X(t), t \geq 0\}$ has wide-sense stationary property.

(b) Strict-sense :-

$t_1 < t_2 < \dots < t_n < \dots$

For ~~evals~~ every n, $\forall h > 0$

If $(X(t_1), X(t_2), \dots X(t_n)) \stackrel{d}{=} (X(t_1+h), X(t_2+h), \dots, X(t_n+h))$

(3) Memoryless ~~process~~ property / Markov Process.

→ Poison process is a special case of M

$$P(X(t_n) \leq x \mid X(t_0) = x_0, X(t_1) = x_1, \dots x_{t_{n-1}})$$

$$= P(X(t_n) \leq x \mid X(t_{n-1}) = x_{n-1})$$

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POISON PROCESS

Let $\{N(t), t \geq 0\}$ be a discrete state cont. time stochastic process defined on a prob space $(\Omega, \mathcal{F}, P)$

Assume:

(i) $N(0) = 0$ a.s.

(ii) for all $0 < t_1 < t_2 < \dots < t_n < \dots$
$N(t_i) - N(t_{i-1})$, $i = 1, 2, \dots$
~~can b~~ are independent and stationary.

(iii) For all $s, t \geq 0$, $0 \leq s < t$, $\lambda > 0$
$$N(t) - N(s) \sim P(\lambda(t-s))$$

Then $\{N(t), t \geq 0\}$ is a Poisson process.

$\boxed{\text{MARKOV PROCESS}}$

Discrete time & Discrete state	Constant time & Discrete state
Discrete time Markov chain	Continuous time Markov Chain

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DTMC

Defn :- Let $\{X_n, n=0,1,2,\ldots\}$ be a discrete state discrete time stochastic process with state space $S=\{0,1,2,\ldots\}$

If $x_0, x, x_0, \ldots, x_n$,

$$P\{X_{n+1}=x \mid X_0=x_0, X_1=x_1, \ldots, X_n=x_n\}$$

$$= P\{X_{n+1}=x \mid X_n=x_n\}$$

Latest information.

Future information $n=1,2,\ldots$

Then, $\{X_n, n=0,1,2,\ldots\}$ is said to be DTMC.

Ex :- Let $\{X_1, X_2, \ldots\}$ be a seq. of iid rvs with common pmf, $P\{X_i=0\}=1-p$, $P\{X_i=1\}=p$, $0<p<1$

Define $S_0=0$

$$S_n=\sum_{i=1}^{n} X_i, \quad n=1,2,\ldots$$

So, $\{S_n, n=0,1,2,\ldots\}$ is disc state disc time stochastic process. The state space $S=\{0,1,2,\ldots\}$, the parameter space $T=\{0,1,2,\ldots\}$

$$P\{S_{n+1}=x \mid S_0=x_0, S_1=x_1, \ldots, S_n=x_n\}$$

$$= P\left\{\sum_{i=1}^{n+1} X_i = x \;\middle|\; S_0=x_0, X_1=x_1, \ldots X_1+X_2=x_2, X_1+X_2+X_3=x_3 \ldots \sum_{i=1}^{n} X_i = x_n\right\}$$

$$= P\{S_0=x_0, X_1=x_1, X_2=x_2-x_1, X_3=x_3-x_2, \ldots X_{n+1}=x-x_n\}$$

$$= P\{X_1 = x_1\} \cdot P\{X_2 = x_2 - x_1\} \, P\{X_3 = x_3 - x_2\} \ldots P\{X_{n+1} = x - x_n\}$$

$$\cancel{P\{X_1 = x_1\}, P(X_2 = x_2 - x_1)} \ldots P(X_n = x_n - x_{n-1}).$$

$$= P\{X_{n+1} = x - x_n\}$$

$$= P\{S_{n+1} = x \mid S_n = x_n\} = P($$

① Independent Increments;

$S_{n+1} - S_n$, $n = 1, 2, \ldots$ independent r.v.s

② Increments are stationary:

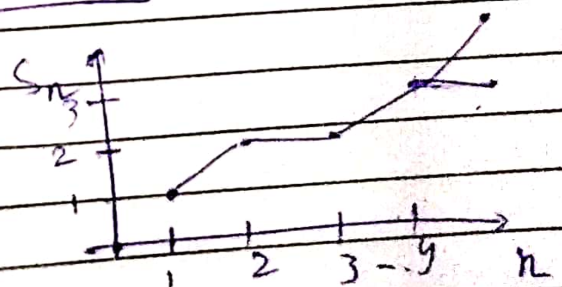
$X_{n+1}, \ldots$ $n = 1, 2, \ldots$ i.i.d. r.v.s

$$X_1, X_8, X_7 \xrightarrow[d]{n=4} (X_5, X_9, X_{11})$$

③ Markov property,

$\boxed{\text{Sample Path}}$ → Shows all the forecasted paths.

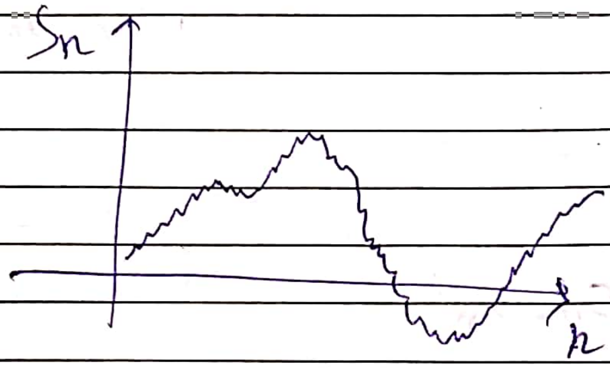
$X_1 = 1, X_2 = 1, X_3 = 0$



Sample path of brownian motion is nowhere differentiable

$S_0 = 0$, $S_n = \sum_{i=1}^{n} X_i$

$P(X_i = 1) = p$, $P(X_i = -1) = 1-p$.



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CYLINDER SPEED → 200-400

Idle gear → No effect on output speed of final gear