Department of Mathematics
MTL 106 (Introduction to Probability and Stochastic Processes)
Tutorial Sheet No. 3
Answer for selected Problems

1. (i) $P[\mid X \mid = k] = \begin{cases} \frac{1}{2n+1}, & k = 0 \\ \frac{2}{2n+1}, & k = 1, 2, \ldots, n \\ 0, & \text{otherwise} \end{cases}$

   (ii) $P[X^2 = k] = \begin{cases} \frac{1}{2n+1}, & k = 0 \\ \frac{2}{2n+1}, & k = 1^2, 2^2, \ldots, n^2 \\ 0, & \text{otherwise} \end{cases}$

   (iii) $P[\frac{1}{|X|+1} = k] = \begin{cases} \frac{1}{2n+1}, & k = 1 \\ \frac{2}{2n+1}, & k = \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n} \\ 0, & \text{otherwise} \end{cases}$

2. 0.51

3. Y is uniformly distributed random variable on the interval (a, b)

4. (i) $f_Y(y) = \frac{1}{|b|\sqrt{2\pi}\sigma} e^{\frac{-1}{2}\left(\frac{y-(a+\mu b)}{b\sigma}\right)^2}, \quad -\infty < y < \infty$

   (ii) $f_Z(z) = \begin{cases} \frac{1}{\sqrt{2\pi z}} e^{\frac{-z}{2}}, & z > 0 \\ 0, & \text{otherwise} \end{cases}$

5. $f_Y(y) = \frac{\alpha e^y}{(e^y+1)^{\alpha+1}}, \quad -\infty < y < \infty$

6.

7. $f_Y(y) = \begin{cases} \frac{\lambda e^{-1}}{2\sqrt{y}}\left(e^{\lambda\sqrt{y}} + e^{-\lambda\sqrt{y}}\right), & 0 < y < \frac{1}{\lambda^2} \\ \frac{\lambda e^{-1}}{2\sqrt{y}} e^{-\lambda\sqrt{y}}, & \frac{1}{\lambda^2} < y < \infty \\ 0, & \text{otherwise} \end{cases}$

8. (a) Y is a continuous type random variable. (b) $f_Y(y) = \begin{cases} e^{-y} + \frac{1}{y^2} e^{-1/y}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$.

9. $F(y) = \begin{cases} 0, & -\infty < y < 2 \\ \frac{y}{10}, & 2 \le y < 4 \\ 1, & 4 \le y < \infty \end{cases}$

10. $\alpha = e^{-\lambda}$

11. $f_Y(y) = \sqrt{\frac{2}{\pi}} \mid y \mid e^{-\frac{1}{2}y^4}, \quad -\infty < y < \infty$

12. $f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}}, & 0 < y < 1 \\ \frac{1}{8\sqrt{y}}, & 1 < y < 9 \\ 0, & \text{otherwise} \end{cases}$

13. $f_Y(y) = \begin{cases} \frac{1}{\pi\sqrt{1-y^2}}, & -1 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

14. Z has mixed type distribution where pmf is given by

$$P[Z=z] = \begin{cases} \frac{1}{4}, & z=-1,1 \\ 0, & \text{otherwise} \end{cases}$$

and density function given by

$$f_Z(z) = \begin{cases} \frac{1}{\pi(1+z^2)}, & -1<z<1 \\ 0, & \text{otherwise} \end{cases}$$

15. $P[X=x] = \begin{cases} \dfrac{\begin{pmatrix} n \\ x \end{pmatrix} p^x q^{n-x}}{\sum_{i=0}^{r-1} \begin{pmatrix} n \\ i \end{pmatrix} p^i q^{n-i}}, & x=0,1,\dots,r-1 \\ 0, & \text{otherwise} \end{cases}$

16. $f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma(\phi(\frac{\beta-\mu}{\sigma})-\phi(\frac{\alpha-\mu}{\sigma}))} exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2), & \alpha<x<\beta \\ 0, & \text{otherwise} \end{cases}$

17. $\exp(-3/8)$

18. $\frac{1}{2}$

19. (a) Bin(24,0.4) (b) Bin(26,0.6)

20. (a) U(0,1) (b) Exp(1)

21. (a) k=2

22. $1-\exp\left(-\frac{1}{3}\right)$, 30 minutes

23. (a) $e^{-\lambda}$

24.

25. $e^{\sigma^2 t^2/2}$

$$E(X^n) = \begin{cases} 0 & n-\text{odd} \\ \frac{n!}{(n/2)!2^{n/2}}\sigma^n & n-\text{even} \end{cases}$$

26. $P(-1.062 < X < 0.73) = \frac{2}{3}$

27.

28. $Y = \log(X)$

$f_Y(y) = \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{1}{4}(y-\exp(y))^2 + \exp(y)\right)$

30. $X =$ No. of games played, $P(X=k) = p_k(>0), k=4,5,6,7$ $E(X) = \frac{93}{16}$

32. (a) $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3}<y<\sqrt{3} \\ 0, & \text{otherwise} \end{cases}$; $P(-2<Y<2)=1.$

(b) 50

34. (a) $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} < y < \sqrt{3} \\ 0, & \text{otherwise} \end{cases}$; $P(-2 < Y < 2) = 1.$

(b) 50 (c) $\frac{\exp(t)-1}{t}$

35. $E(Y) = 1, \quad Var(Y) = 1 \;\; where \;\; Y = |X|$

36. (a) $X \sim P(\mu)$ (b) $\sum_{k=1}^{7} \frac{e^{-\mu}\mu^k}{k!}$; $\mu = 4$

37. 11.125

38. $E[X^2] = \lambda + \lambda^2, Var(X) = \lambda, E[X^3] = \lambda^3 + 3\lambda^2 + \lambda$