Tut - 8

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Q.1.) Let Y_n be the no. of arrivals when the hair of n^{th} person was cutted.

$\therefore X_n$ = no. of persons in the shop just after the cut of n^{th} person haircut.

$n = 1, 2, 3 \ldots$

$\therefore$ State space possible are $1, 2, 3, 4, 5, 6$.

$P(Y_n = 0) = 0.3 \quad P(Y_n = 1) = 0.4 \quad P(Y_n = 2) = 0.3$.

$\therefore X_1 = Y_0 + Y_1 - 1$

$X_2 = Y_0 + Y_1 + Y_2 - 2$

$X_3 = Y_0 + Y_1 + Y_2 + Y_3 - 3$.

Now this Y_0, Y_1 Y_i's are iid's.

$$\therefore P(X_{n+m} = k \mid X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n).$$

$$= \frac{P(X_{n+m} = k, X_n = i_n, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1)}{P(X_1 = i_1, \ldots, X_n = i_n)}$$

$$= \frac{P(X_{n+m} - X_n = k - i_n, X_n - X_{n-1} = i_n - i_{n-1}, \ldots, X_2 - X_1 = i_2 - i_1)}{P(X_n - X_{n-1} = i_n - i_{n-1}, X_{n-1} - X_{n-2} = i_{n-2} - i_{n-1} \ldots)}$$

$$= P(X_{n+m} - X_n = k - i_n)$$

$$= \boxed{P(X_{n+m} = k \mid X_n = i_n)}$$

0.7 become

if 1 or 2 arrive in both case $X_n = 6$.

hence it follows the memory less property hence it is a markov's process.

And the state space are also discrete, $\therefore$ it is a markov chain.

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Q.2-) $\max(x,y) = \left(\frac{x+y}{2}\right) + \left|\frac{x-y}{2}\right|$.

X_0 is independent of Z_n and $P(X_0 = 0) = 1$.

$P(Z_n = 1) = p$ $\quad P(Z_n = -1) = q$ $\quad P(Z_n = 0) = 1-(p+q)$.

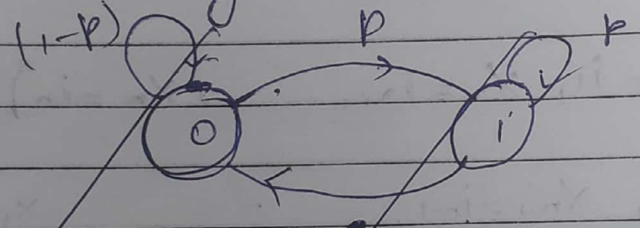
∴ $X_n = \max(0, X_{n-1} + Z_n)$. $\quad n = 1, 2, \ldots$

∴ $X_n = \left(\frac{X_{n-1}+Z_n}{2}\right) + \left|\frac{X_{n-1}+Z_n}{2}\right|$.

Now $X_n - X_{n-1} = (Z_n \text{ or } -X_{n-1})$.

and X_{n-1} = again a f^n of Z_n.

~~X_n can take only two values. (0,1)~~



⇒ ~~Matrix =~~

$$\begin{array}{c|ccccccc} & & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & p & & & & & & \\ 2 & & & & & & & \\ 3 & & & & & & & \\ 4 & & & & & & & \\ 5 & & & & & & & \\ 6 & & & & & & & \end{array}$$

$X_n = \langle 0, 1, 2, 3, \ldots \rangle$.

$$\text{Matrix} = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & \\ 0 & 1-p & p & 0 & 0 & 0 & \ldots \\ 1 & q & 1-(p+q) & \ldots & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \end{array}$$

Q.4.) Markov chain is $\{X_n, n = 0, 1, \ldots\}$.

$S = \{1, 2, 3\}$ one step matrix :-

$$P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.3 & 0.4 & 0.3 \\ 2 & 0.6 & 0.2 & 0.2 \\ 3 & 0.5 & 0.4 & 0.1 \end{array}$$

$$\pi_0 = \begin{array}{ccc} (0.7, & 0.2, & 0.1) \\ 1 & 2 & 3 \end{array}.$$

① $P(X_2 = 3)$, ~~[illegible]~~ X_2 means in two steps $(0-1-2)$. reach the state 3.

$$P^2 = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.1 \end{pmatrix} \times \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.1 \end{pmatrix}$$

$$= \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.48 & 0.32 & 0.20 \\ 2 & 0.40 & 0.36 & 0.24 \\ 3 & 0.44 & 0.32 & 0.24 \end{array}$$

$$\text{Ans} = 0.20 \times 0.7 + 0.24 \times 0.2 + 0.24 \times 0.1$$

$$= 0.14 + 0.048 + 0.024$$

$$= 0.14 + 0.072$$

$$= \boxed{0.212}$$

$0.048 + 0.024 = 0.072$

$0.140 + 0.072 = 0.212$

② $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

$$= \begin{array}{cccc} 2 & 3 & 3 & 2 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 0 & \longrightarrow 1 & \longrightarrow 2 & \longrightarrow 3 \end{array}$$

$$= 0.2 \times 0.2 \times 0.1 \times 0.4$$

$$= \frac{16}{10000} = \boxed{0.0016}$$

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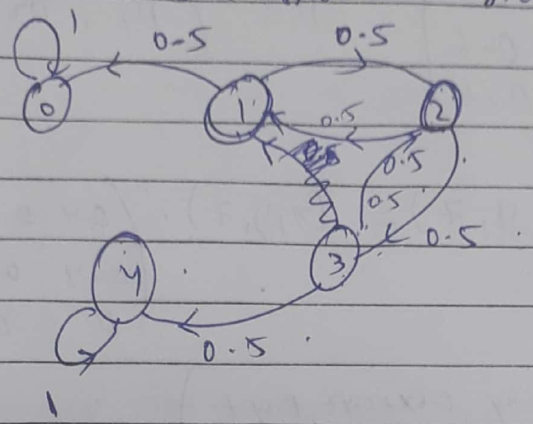
Q.5.) a.) If we draw the state transition diagram for this then.



State 0 → $F_{00} = 1$ hence Recurrent and null recurrent.

1 → $F_{11} < 1$ hence transient.

2 → $F_{22} < 1$ hence ~~tran~~ transient.

3 → $F_{33} < 1$ Transient.

4 → $F_{44} = 1$ null Recurrent.

b) 1.b.) $T = \{1, 2, 3\}$. $R = \{0, 4\}$.

~~B = (I~~

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{pmatrix} \end{array}$$

$$M = (I-B)^{-1} = \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{pmatrix}\right]^{-1}$$

$$M = \overbrace{\begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & -0.5 \\ 0 & -0.5 & 1 \end{bmatrix}}^{K}{}^{-1}$$

$$|K| = 1(1-0.25) + 0.5(-0.5) + 0$$
$$= 0.75 - 0.25$$
$$= \boxed{0.5}.$$

$$|K|^{-1} = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} = \begin{bmatrix} 0.75 & 0.5 & 0.25 \\ 0.5 & 1 & 0.5 \\ 0.25 & 0.5 & 0.75 \end{bmatrix} \xrightarrow{\text{transpose}} = \begin{bmatrix} 0.75 & 0.5 & 0.25 \\ 0.5 & 1 & 0.5 \\ 0.25 & 0.5 & 0.75 \end{bmatrix}.$$

$$M = \frac{1}{2}\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 3/2 & 1 & \frac{1}{2} \\ 1 & 2 & 1 \\ \frac{1}{2} & 1 & 3/2 \end{bmatrix} \end{array}$$

$\mu_2 1$ $\mu_4 = 1$ Hence 1 visit.

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Q)

Q.6.)

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.4 & 0.6 & 0 \\ 0.4 & 0 & 0.6 \\ 0 & 0.4 & 0.6 \end{pmatrix} \end{array} \quad \pi = (\pi_1, \pi_2, \pi_3).$$

$$\pi = \pi P = (x, y, z) = (x, y, z) \cdot \begin{pmatrix} 0.4 & 0.6 & 0 \\ 0.4 & 0 & 0.6 \\ 0 & 0.4 & 0.6 \end{pmatrix}$$

$$(x, y, z) = (0.4x + 0.4y,\ 0.6x + 0.4z,\ 0.6y + 0.6z)$$

$$\left.\begin{array}{l} 0.6x = 0.4y \\ \underline{3x = 2y} \end{array}\right] \quad \cancel{0.6x = 0} \quad y = 0.6x + 0.4z, \quad 0.4z = 0.6y \quad 3y = 2z.$$

$$(x, y, z) = \left(\frac{2y}{3}, y, \frac{3y}{2}\right)$$

$$(x + y + z = 1) \quad \frac{2y}{3} + y + \frac{3y}{2} = 1$$

$$\frac{4y + 6y + 9y}{6} = 1$$

$$19y = 6$$

$$\left(y = \frac{6}{19}\right).$$

$$\boxed{\pi = \pi P \text{ and } \sum \pi_i = 1}$$

$$S.D = \left(\frac{4}{19}, \frac{6}{19}, \frac{9}{19}\right)$$

Q.7.) A B.

X_n = amount of money A had after the n^{th} toss
$X_0 = i$

Clearly X_n depends only on X_{n-1} hence ~~B~~ it is a DTMC, as no. of tosses are also discrete and state space are also discrete hence it is a (DTMC).

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$$\text{matrix}\cdot\quad \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & \cdots & n \\ \hline 0 & & \cancel{p} & 0 & 0 & \cdots & \\ 1 & 1-p & & 0 & & & \\ 2 & 0 & & 0 & 1-p & & \\ 3 & & & & & & \\ \vdots & & & & & & \\ n & & & & & & \end{array}$$

diagram :->

Q.8.) Y_n :->

$$\cancel{\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 2 & 0 & \frac{1}{16} & \frac{3}{18} & \frac{3}{4} & \\ 3 & & & & & \\ 4 & & & & & \\ 5 & & & & & \end{array}}$$

Y_n is the no. of hosts with message after n^{th} round. and at each round only one host will spread the message. $\therefore Y_n =$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 3 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 4 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 5 & & & & & \end{array}$$

~~Q10)~~

Q.11.) Given we have a irreducible Markov chain, with for state i $P_{ii} > 0$.

As the chain is irreducible all the states are communicating. And the no. of steps in which the system will return to the state i, starting from i are $(2, 3, 4, 5 \ldots)$.

$\therefore$ their gcd = 1 $\therefore$ state i is aperiodic.

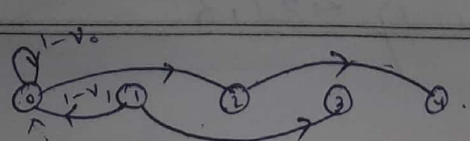
Now as chain is irreducible then all the states will have the same properties

Hence aperiodic Markov chain.

Q.9.)

$$\begin{array}{c|cccccccc} & 0 & 1 & 2 & 3 & \cdots & & & \\ \hline 0 & (1-v_0) & 0 & v_0 & 0 & 0 & 0 & \cdots & \\ 1 & 1-v_1 & 0 & 0 & v_1 & 0 & 0 & \cdots & \\ 2 & 1-v_2 & 0 & 0 & 0 & v_2 & 0 & \cdots & \\ 3 & 1-v_3 & 0 & 0 & 0 & 0 & v_3 & \cdots & \\ \vdots & & & & & & & & \end{array}$$

$P_{j\,j+2} = v_j$	$P_{j0} = (1-v_j)$
$\Rightarrow P_{02} = v_0$	$P_{00} = 1-v_0$
$\Rightarrow P_{13} = v_1$	$P_{10} = 1-v_1$
$P_{24} = v_2$	$P_{20} = 1-v_2$
$P_{35} = v_3$	$P_{30} = 1-v_3$
$P_{46} = v_4$	$P_{40} = 1-v_4$
$P_{57} = v_5$.	$P_{50} = 1-v_5$.



$F_{00} = \sum_n f_{00}(n) = f_{00}(1) + f_{00}(2) + f_{00}(3) \ldots$ ~~$= 1 - \cancel{V_0} + \cancel{V_0} + 1 - V_2 + V_0 + \ldots 1 - V_4$~~

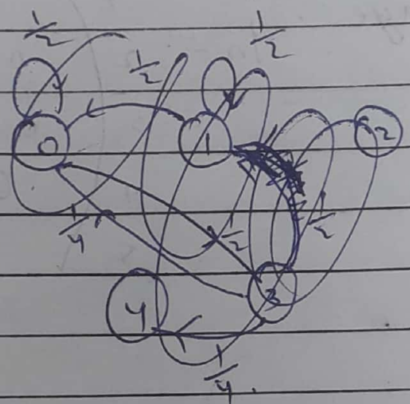
~~0 is the recurrent state.~~ ~~$\neq 1$ hence transient~~

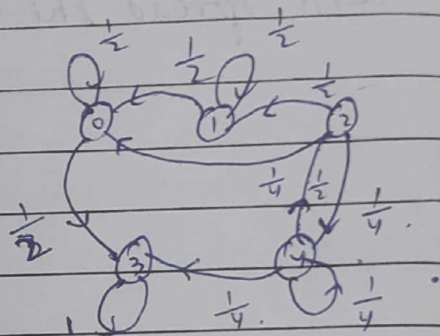
$\therefore \langle 0, 2, 4 \ldots \rangle$ forms $\boxed{C.C.C}$.

$\langle 1, 3, 5 \ldots \rangle$ are transient states.

Q.10:) a.) $\langle X_n, n = 1, 2, \ldots \rangle$ $E = \langle 0, 1, 2, 3, 4 \rangle$

One step transition diagram is :-





$0 \to F_{00} = f_{00}(1) + f_{00}(2) + f_{00}(3) \ldots = \frac{1}{2} + 0 = \frac{1}{2} < 1$

$\therefore$ transient

$1 \to F_{11} = f_{11}(1) + f_{11}(2) + f_{11}(3) \ldots = \frac{1}{2} + 0 = \frac{1}{2} < 1$ T.

$2 \to F_{22} = f_{22}(1) + f_{22}(2) + f_{22}(3) \ldots = 0 + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} + \left(\frac{1}{4}\right)^3 \cdot \frac{1}{2} \ldots$

$= \frac{1}{2} \left[\frac{\frac{1}{4}}{1 - \frac{1}{4}} \right] = \frac{1}{2} \left(\frac{1}{3} \right) = \frac{1}{2} \left[\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \ldots \right]$

$= \frac{1}{6} < 1 \therefore$ T.

$3 \to F_{33} = f_{33}(1) + f_{33}(2) . = 1 \therefore$ Recurrent state

Note, $3 \to$ absorbing state.

$4 \to F_{44} \to f_{44}(1) + f_{44}(2) + f_{44}(3) \ldots = \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} + 0$

$= \frac{3}{8} < 1 \therefore$ T.

C.C.C = ~~{...}~~ $\langle 3 \rangle$, $\langle$

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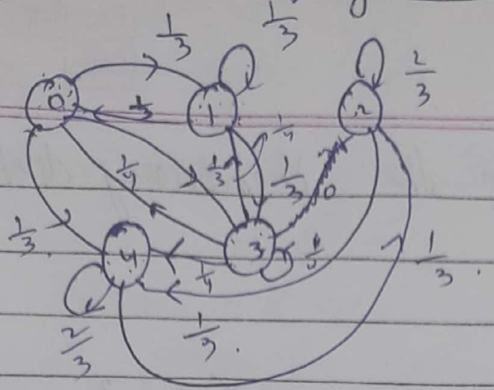
state may also be Absorbing if $F_{11} = 1$ → absorbing ~~state~~.

(b) →



0 → $F_{00} < 1$, as 0 — 4 but no backpath ∴ Transient.

1 → 1 — 0 — 4 → no return path ∴ T.

2 → 2 — 4 — 2 → Recurrent.

3 → 3 — 4 no return path ∴ T.

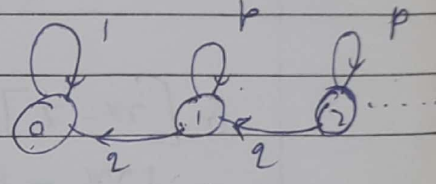
4 → 4 — 2 — 4 ∴ Recurrent.

T → ⟨0, 1, 3⟩

~~R → ⟨2, 4⟩ → are Recurrent.~~

C.C.C = ⟨2, 4⟩. we have also to specify whether the recurrence is (+ve or null).

Q.12.) $P_{00} = 1$



$f_{j0}^{(n)}$ = System reach state 0 for the 1st time on nth step starting from the state j. if j = 1] $f_{10}(1) = q$, $f_{10}(2) = pq$, $f_{10}(3) = p^2 q$, $f_{10}(4) = p^3 q$ ....

∴ Similarily $f_{20}(1) = 0$, $f_{20}(2) = q^2$, $f_{20}(3) = pq^2 + pq^2$.
$f_{20}(4) = p^2q^2$, $f_{20}(5) = p^3q^2$ ..

∴ when $n < j$ $f_{j0}(n) = 0$.
$n \geq j$ $f_{j0}(n) = p^{n-j} q^j$.

∴ $\boxed{\cancel{f_{j0}(n) = p^{n-j} \cdot q^j}}$.

Now for ex.) if we want go from 2 — 0 in three step then there are two possibilities i.e 2-2-1-0 or 2-1-1-0.
all these possibilities should also be taken into the consideration.

∴ $\boxed{f_{j0}(n) = {}^{n-1}C_{j-1} \cdot p^{n-j} \cdot q^j}$

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Q.13) Let $\pi = (x, y, z)$ be the stationary distribution and $(x+y+z=1)$.

$\therefore \pi = \pi P$

$$= (x, y, z) = (x \;\; y \;\; z) \cdot \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

$$(x, y, z) = \left(\tfrac{1}{2}(y+z), \;\; \frac{2x}{3} + \frac{z}{2}, \;\; \frac{x}{3} + \frac{y}{2} \right)$$

$2x = y + z$ ① $\qquad y = \frac{2x}{3} + \frac{z}{2}$ $\qquad z = \frac{2x + 3y}{6}$

$\Rightarrow 6y = 4x + 3z$ ② $\qquad 6z = 2x + 3y$ ③

$6 \cdot [2x - z] = 4x + 3z$

$12x - 6z = 4x + 3z$.

$8x = 9z$.

$x = \left(\frac{9z}{8}\right)$ $\qquad y = 2 \cdot \left(\frac{9z}{8}\right) - z$

$y = \frac{9z}{4} - z = \left(\frac{5z}{4}\right)$.

~~$x+y+z=1$~~ $\Rightarrow \frac{9z}{8} + \frac{5z}{4} + z = 1$

$\Rightarrow 9z + 10z + 8z = 8 \quad \Rightarrow 27z = 8$

$z = \left(\frac{8}{27}\right)$.

$x = \frac{9}{8} \times \frac{8}{27} = \frac{1}{3}$ $\qquad y = \frac{5}{4} \times \frac{8}{27} = \left(\frac{10}{27}\right)$.

$$(\pi) = \boxed{\left(\frac{1}{3}, \frac{10}{27}, \frac{8}{27}\right)}$$

(3W – 3B). ~~$X_n = \{0, 1, 2, 3\}$ when~~

Q.14.) 1 $\qquad$ 2 $\qquad$ State of System ~~means~~ i means i white balls in 1.

initially each has 3 balls.

At each stage 1 ball is drawn from both and interchanged. (G=m p) (9,i).

X_n :→ state of the system after the n^{th} draw.

clearly X_n i.e. no. of white balls after n^{th} draw depends only on X_{n-1} i.e. no. of " " " $(n-1)^{th}$ draw.

∴ X_t is a Markov chain.

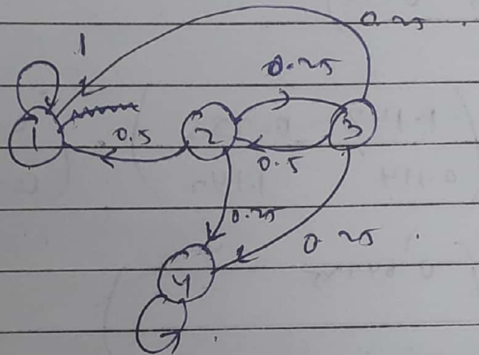
also the parameter space i.e. the no. of draws are discreate ∴ it is a (DTMC).

Now Matrix :→

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 1 & \frac{1}{9} & \frac{4}{9} & \frac{4}{9} & 0 \\ 2 & 0 & \frac{4}{9} & \frac{4}{9} & \frac{1}{9} \\ 3 & 0 & 0 & 1 & 0 \end{array}$$

Q.23.)

a.)



b.) $\underline{X_0 = 2}$.

Given $P =$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0.5 & 0 & 0.25 & 0.25 \\ 3 & 0.25 & 0.5 & 0 & 0.25 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

$T = \langle 2, 3 \rangle$.

$$B = \begin{array}{c|cc} & 2 & 3 \\ \hline 2 & 0 & 0.25 \\ 3 & 0.5 & 0 \end{array}$$ = Transition prob matrix for the transient states (2 and 3).

$$m = (I-B)^{-1} = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0.25 \\ 0.5 & 0 \end{pmatrix}\right]^{-1} = \begin{bmatrix} 1 & -0.25 \\ -0.5 & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 1 & -0.25 \\ -0.5 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0.25 \\ 0.5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.25 & 1 \end{bmatrix} = \text{Adj} = \begin{bmatrix} 1 & 0.25 \\ 0.5 & 1 \end{bmatrix}$$

$$m = \frac{1}{0.875}\begin{bmatrix} 1 & 0.25 \\ 0.1 & 1 \end{bmatrix} = \begin{array}{c|cc} & 2 & 3 \\ \hline 2 & 1.142 & 0.285 \\ 3 & 0.114 & 1.142 \end{array}$$

2W 0, 1B; 2W 0, 1B; 1/3 × 2/3 + 2/3 × 1/3; 2/3 × 2/3; 2W 1B ①; 2B 1W ②; 2/3 × 2/3; 1/3 × 2/3 + 2/3 × 1/3; 1/3 × 1/3; 3W ①; 3B ⓪; 1 − 0.125 = 0.875

$\therefore$ expected $M = \begin{array}{c|cc} & 2 & 3 \\ \hline 2 & \mu_{22} & \mu_{23} \\ 3 & \mu_{32} & \mu_{33} \end{array}$ no. of changes of state of mind

(ii) Student get into recurrent state starting at $x_0 = 2$ state

$= \mu_{22} + \mu_{23} = 1.142 + 0.285 = \boxed{1.42}$

c.) Prob. that student will go into the recurrent state j starting from the transient state i is (g_{ij}). where (g_{21}).

Now $G = MA$. $M = \begin{bmatrix} \cancel{1.142} & \cancel{0.285} \\ 0.114 & 1.142 \end{bmatrix}$

$A = \begin{array}{c|cc} & 1 & 4 \\ \hline 2 & 0.5 & 0.25 \\ 3 & 0.25 & 0.25 \end{array}$

$\therefore G = \begin{pmatrix} 1.142 & 0.285 \\ 0.114 & 1.142 \end{pmatrix} \cdot \begin{pmatrix} 0.5 & 0.25 \\ 0.25 & 0.25 \end{pmatrix}$

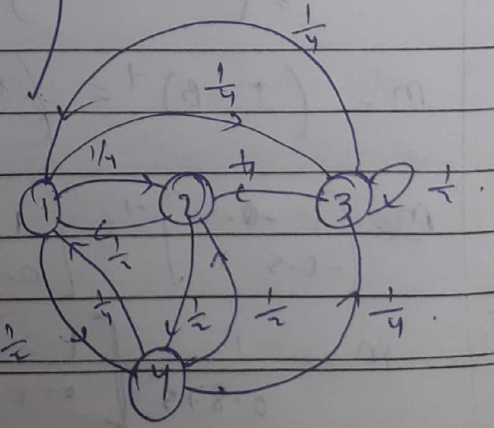
$G = \begin{array}{c|cc} & 1 & 4 \\ \hline 2 & 0.64225 & \ldots \\ 3 & \ldots & \ldots \end{array}$

$\therefore \boxed{g_{21} = 0.64225}$

Q.22.) $S = \{1, 2, 3, 4\}$.

$P = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1/4 & 1/4 & 1/2 \\ 2 & 1/2 & 0 & 0 & 1/2 \\ 3 & 1/4 & 1/4 & 1/2 & 0 \\ 4 & 1/4 & \frac{1}{2} & 1/4 & 0 \end{array}$

State transition diagram :->



$P(X_{n+1}=k \mid X_0=0, X_1=i_1, \ldots, X_n=i_n)$

$=$

a.) $1 \rightarrow$ Starting from state 1 we can go to the states (2, 3, 4) and there is always a reverse path to reach 1 hence $F_{11}=1$ $\therefore$ Recurrent.

Given M.C is irreducible with finite state space.

$\therefore$ +ve Recurrent.

$2 \rightarrow$ ~~Re~~ +ve Recurrent same explanation as above.

$3 \rightarrow$ +ve ''

$4 \rightarrow$ +ve ''.

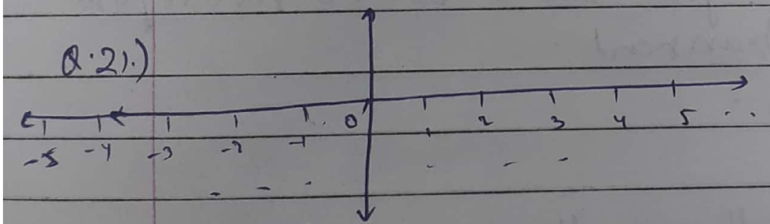
b.) $\pi = (x, y, z, w)$.

$\pi = \pi P \Rightarrow (x\ y\ z\ w) = (x\ y\ z\ w)\begin{pmatrix} 0 & 1/4 & 1/4 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 & 0 \end{pmatrix}$

$x+y+z+w=1$.

~~(x y z~~ ~~~~ solving this we get $(x, y, z, w) = \left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\right)$.

Q.21.)



~~P~~ Prob. (to go right by 1 unit) $= \frac{1}{3}$

$P(\text{left}) = \frac{1}{3}$

$P(\text{same}) = \frac{1}{3}$.

Let X_n be the R.V which represents the position of the particle after ~~n^{th} move~~ the n^{th} move.

$\therefore S = \{-\infty, \ldots, -1, 0, 1, 2, \ldots \infty\}$, $T = \{0, 1, 2, 3, 4, \ldots\}$.

$X_0 = 0$.

Now clearly we can see that the position of the particle after n^{th} ~~step~~ move will only depends on the position of the particle after $(n-1)^{th}$ move hence X_n follows the markov property

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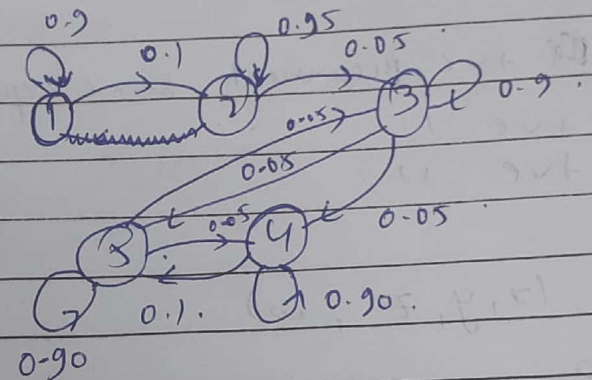
Q.20)

$$P = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0.9 & 0.1 & 0 & 0 & 0 \\ 2 & 0 & 0.95 & 0.05 & 0 & 0 \\ 3 & 0 & 0 & 0.90 & 0.05 & 0.05 \\ 4 & 0 & 0 & 0 & 0.90 & 0.10 \\ 5 & 0 & 0 & 0.05 & 0.05 & 0.90 \end{array}$$

transition diagram :⇒



State 1 → from state 1 we can go to 2 ~~but no reverse path~~.
∴ $F_{11} < 1$ ⇒ transient.

Period :→ 1.

State 2 → from state 2 we can go to state 3 but no reverse path
∴ $F_{22} < 1$ ⇒ transient.
period = 1.

State 3 → 3 → 4 – 5 And there is alway a reverse path to 3.
∴ $F_{33} = 1$ ⇒ Recurrent.

and $\mu_{33} = (1\times 0.9 + \cancel{2\times 0.05\times 0.05} + \cancel{3\times 0.05\times 0.05\times 0.05} \dots)$

∴ F < ∞ ∴ +ve Recurrent.

period → 1.

4 → 4 − 5 − 3 ∴ and always a reverse path

∴ +ve Recurrent. Period = 1

5 → +ve Recurrent $P = 1$.

∴ Chain is aperiodic

b.) Do ($\pi = \pi P$) to find the limiting distribution.

Q-19.) P(rain in afternoon) = $\left(\frac{1}{5}\right)$. total $U = 2$.

~~P (rain in morning)~~ = $\frac{1}{20}$

Whether it rains or not is independent of the previous day rain.

Initial 1 at home and 1 at office.

~~two trips each day~~.

→ Define X_n :→ State of the prof. after n^{th} trip.

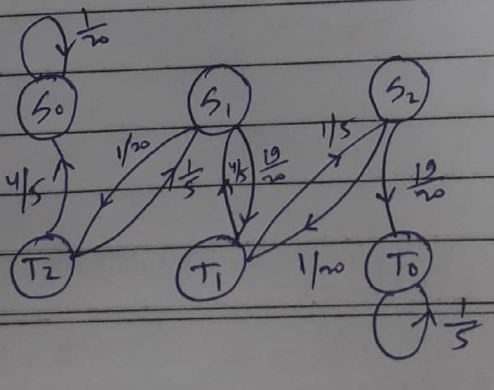
There are 6 states possible.

S_i ⇒ State when prof. at home and i umbrellas at home.

T_i → State when prof. at office and i ,, at office.

∴ ~~$S_0, S_1, S_2, T_0, T_1, T_2$~~ are possible.

Diagram:→



Transient states are

$T = \{S_1, S_2, T_1, T_2\}$.

$\therefore$ transient states matrix

$$B = \begin{array}{c} \\ S_1 \\ S_2 \\ T_1 \\ T_2 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & T_1 & T_2 \end{array} \\ \begin{pmatrix} 0 & 0 & 19/20 & 1/20 \\ 0 & 0 & 1/20 & 0 \\ 4/5 & 1/5 & 0 & 0 \\ 1/5 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$m = (I-B)^{-1}$.

and Expected no. of days $= \left(\dfrac{\text{sum of } 1^{st} \text{ row of } m}{2} \right)$

$= \dfrac{9.88}{2} = (4.94)$ days.

b.) ~~Asked prob. that he wets in the state T_0~~

~~g_{ij} = prob. that system starts from the state~~ T_S i and goes into the recurrent state j.

we need $g(S_1 T_0)$.

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ T_1 \\ T_2 \end{array} \begin{array}{c} \begin{array}{cc} S_0 & T_0 \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 19/20 \\ 0 & 0 \\ 4/5 & 0 \end{pmatrix} \end{array}$$

4×2

$\therefore G = mA$

$= (4 \times 4) \times (4 \times 2)$

$= (4 \times 2)$.

$$G = \begin{array}{c} \\ S_1 \\ S_2 \\ \\ \end{array} \begin{array}{c} \begin{array}{cc} S_0 & T_0 \end{array} \\ \begin{pmatrix} 0.1799 & 0.82 \\ \vdots & \vdots \\ \vdots & \vdots \end{pmatrix} \end{array}$$

we want $g(S_1 T_0) = \boxed{0.82}$

(Q.18.).

Q.17.) Given $\{X_n, n = 0, 1, \ldots\}$ is a DTMC.

and $\sum_i P_{ij} = \sum_j P_{ij} = 1$.

To prove $\pi_i = \frac{1}{|m|}$ $|m|$ = no. of states.

~~proof~~ :→ $\sum \pi_j = 1$. Let $\pi_i = \frac{1}{|m|}$

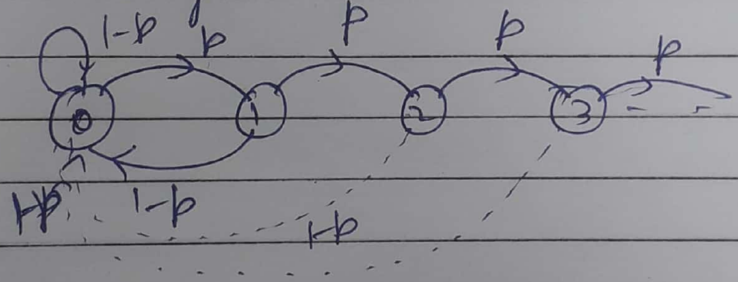
And $\pi_j = \sum_i \pi_i \cdot P_{ij} = \frac{1}{|m|} \sum P_{ij} = \frac{1}{|m|}$

$\therefore \pi_j = \frac{1}{|m|}$.

Δ $\sum \pi_i = \sum_i \frac{1}{|m|} = \frac{|m|}{|m|} = 1$ which is true.

hence $\pi_i = \frac{1}{|m|}$ is the sol^n.

Q.16.) Transition diagram :→



$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & \cdots & \end{array} \\ \begin{pmatrix} 1-p & p & 0 & 0 & 0 & 0 & \cdots & \\ 1-p & 0 & p & 0 & 0 & \cdots & & \\ 1-p & 0 & 0 & p & 0 & & & \\ 1-p & 0 & 0 & 0 & p & 0 & & \\ 1-p & 0 & 0 & 0 & 0 & p & \cdots & \\ 1-p & 0 & 0 & 0 & 0 & 0 & p & \end{pmatrix} \end{array}$$

$(\pi_0, \ldots) = (\pi_0, \pi_1, \pi_2 \ldots) \cdot (P)$

$(\pi_0, \pi_1, \pi_2, \pi_3 \ldots) = \left((1-p) \cdot (\pi_0 + \pi_1 + \ldots), \pi_0 p, \pi_1 p, \pi_2 p \ldots\right)$

$\pi_0 = 1-p$, $\pi_1 = p(1-p)$, $\pi_2 = p^2(1-p)$, $\pi_3 = p^3(1-p) \ldots$

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