

Data Structures & Algorithms

Subodh Kumar

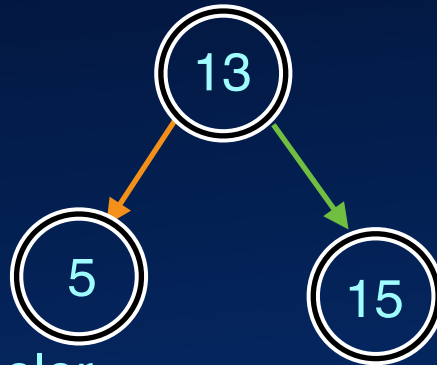
(subodh@iitd.ac.in, Bharti 422)

Dept of Computer Sc. & Engg.



Red-Black Tree

Remove 15



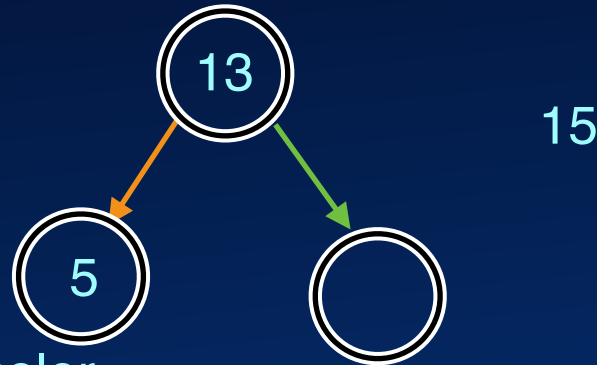
No red child: Recolor

- a) There is no red coloring
- b) 13 will have red color after 1 recoloring
- c) 5 will have red color after 1 recoloring
- d) Both 5 and 13 will have red color after 1 coloring each
- e) 13 will have black color after 2 recolorings
- f) 5 will have black color after 2 recolorings



Red-Black Tree

Remove 15



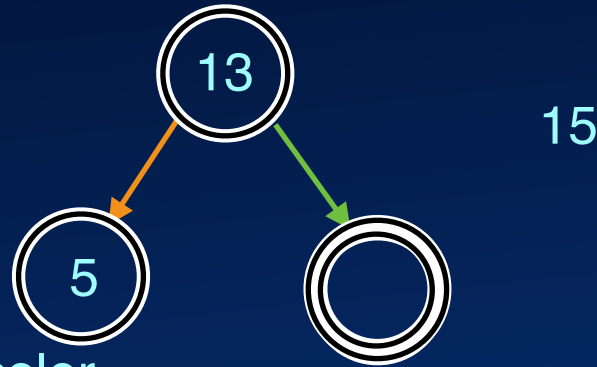
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Red-Black Tree

Remove 15



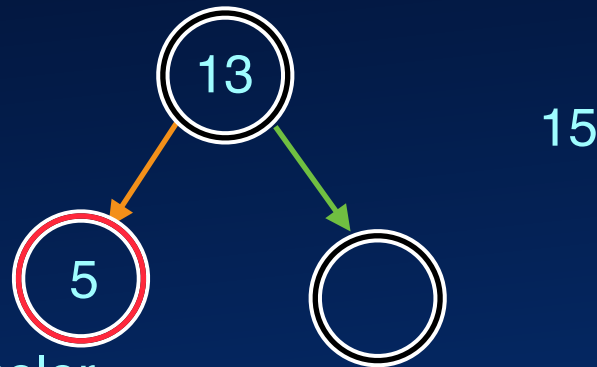
No red child: Recolor

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Red-Black Tree

Remove 15



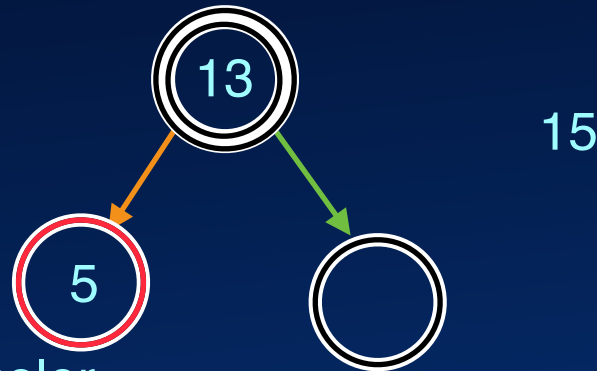
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Red-Black Tree

Remove 15



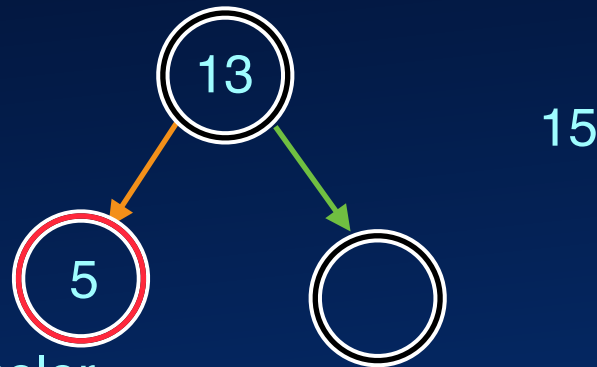
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Red-Black Tree

Remove 15



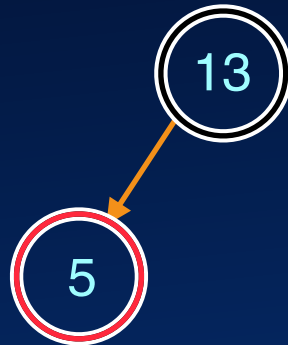
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Red-Black Tree

Remove 15



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Dictionaries

$O(1)$ expected operations, no *next* traversal

Space efficient, no references needed

Hash table

Red-black

2-3 Tree



Dictionaries

$O(1)$ expected operations, no *next* traversal

Space efficient, no references needed **Hash table**

$O(\text{length})$ operation, *next* key possible in lexicographic order

Can be space-efficient if many common prefixes **Trie**

Red-black

2-3 Tree



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$O(\text{length})$ operation, *next* key possible in lexicographic order

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$O(\log n)$ expected operation, *next* key in $O(1)$ **Skip List**

Slightly more references than binary search tree

Red-black

2-3 Tree



Dictionaries

$O(1)$ expected operations, no *next* traversal

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1 to $1.44 \log n$ in height

AVL tree

Up to $O(\log n)$ restructures for delete (but can reduce future restructures)

Good with skewed (sorted) input sequences

Red-black

2-3 Tree



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Good with skewed (sorted) input sequences

Up to $2 \log n$ high

Red-Black Tree

But only one restructure per update

Good for occasional run of sorted keys

2-3 Tree



Dictionaries

$O(1)$ expected operations, no *next* traversal

Space efficient, no references needed **Hash table**

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AVL tree

Up to $O(\log n)$ restructures for delete (but can reduce future restructures)

Good with skewed (sorted) input sequences

Up to $2 \log n$ high

Red-Black Tree

But only one restructure per update

Cache friendly

More work per node, but nodes can be cache-friendly

Works best for out of memory data structure

2-3 Tree

extended to a-b tree, Or just b-tree



Dictionaries

Unbalanced BSTs also OK

for random order of updated keys

$O(\text{length})$ operation, *next* key possible in lexicographic order

Can be space-efficient if many common prefixes **Trie**

$O(\log n)$ expected operation, *next* key in $O(1)$ **Skip List**

1 to $1.44 \log n$ in height

AVL tree

Up to $O(\log n)$ restructures for delete (but can reduce future restructures)

Good with skewed (sorted) input sequences

Up to $2 \log n$ high

Red-Black Tree

But only one restructure per update

Good for insertions, fast deletion

More work per node, but nodes can be cache-friendly

Works best for out of memory data structure

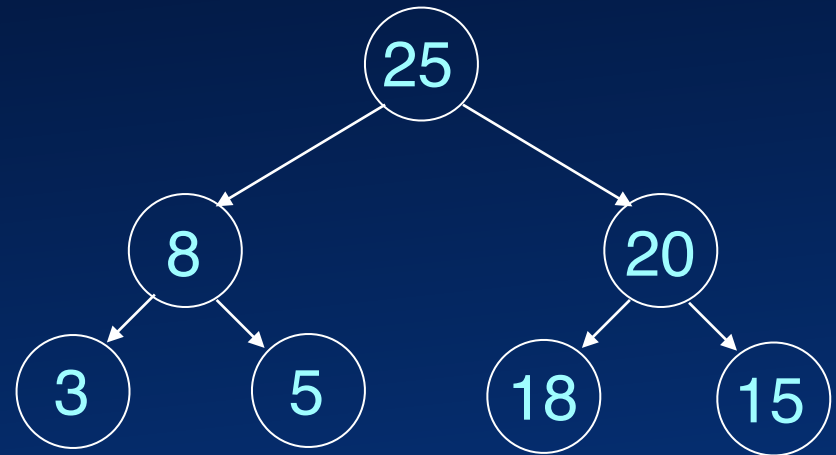
2-3 Tree

extended to a-b tree, Or just b-tree



Heap

- Left-complete tree
- Comparable keys
- “Top” key in the root
 - For every subtree

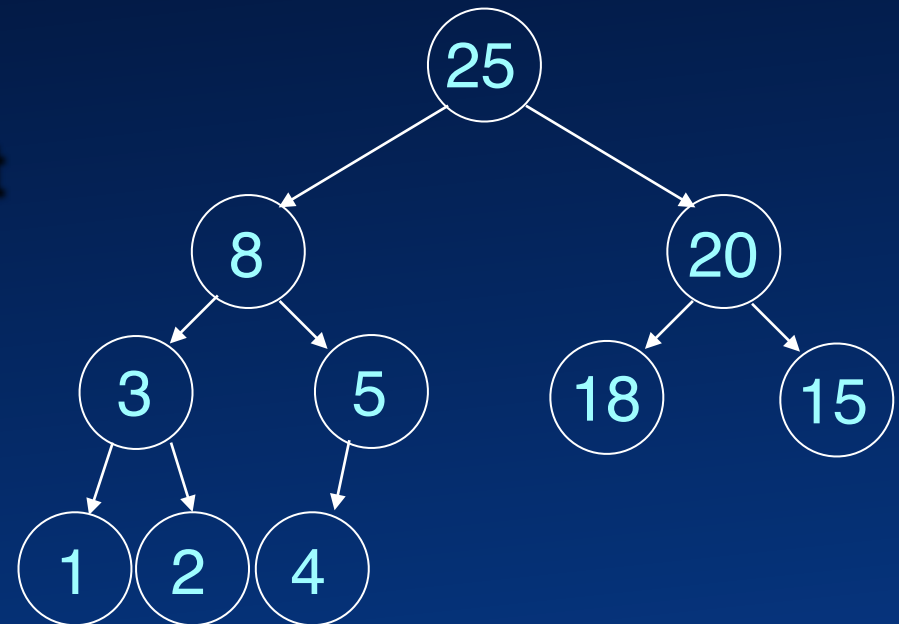


Heap



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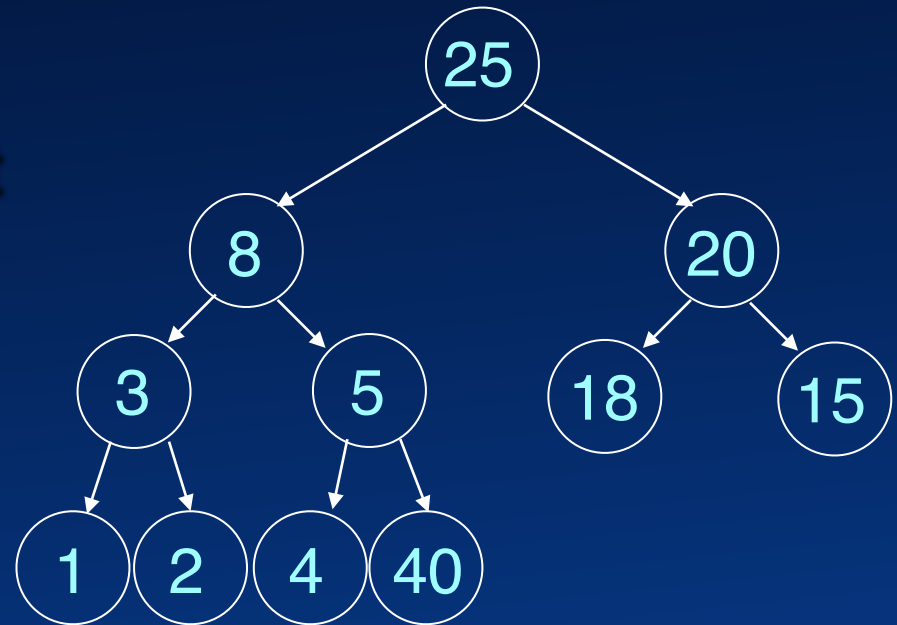


Heap



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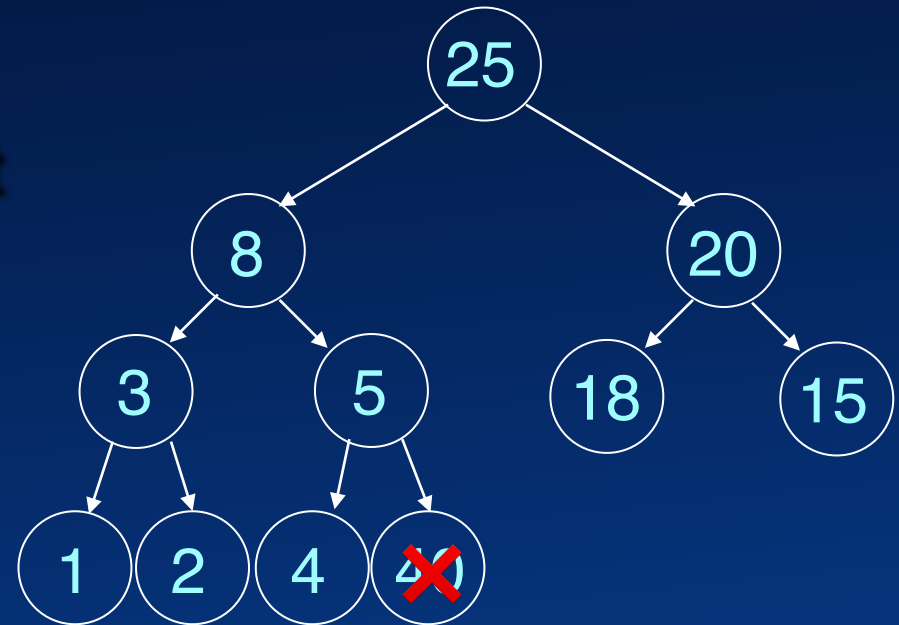


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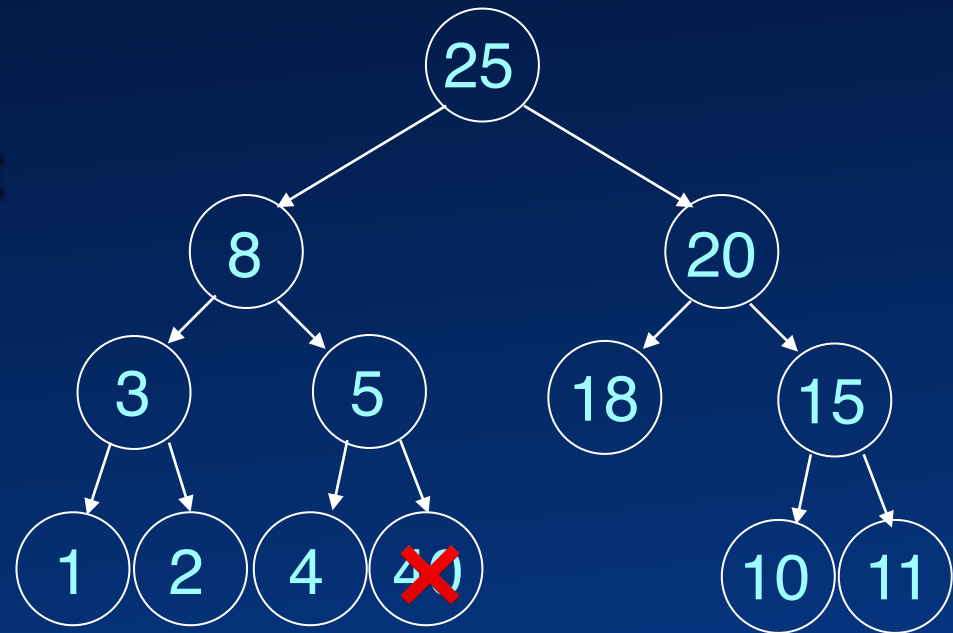


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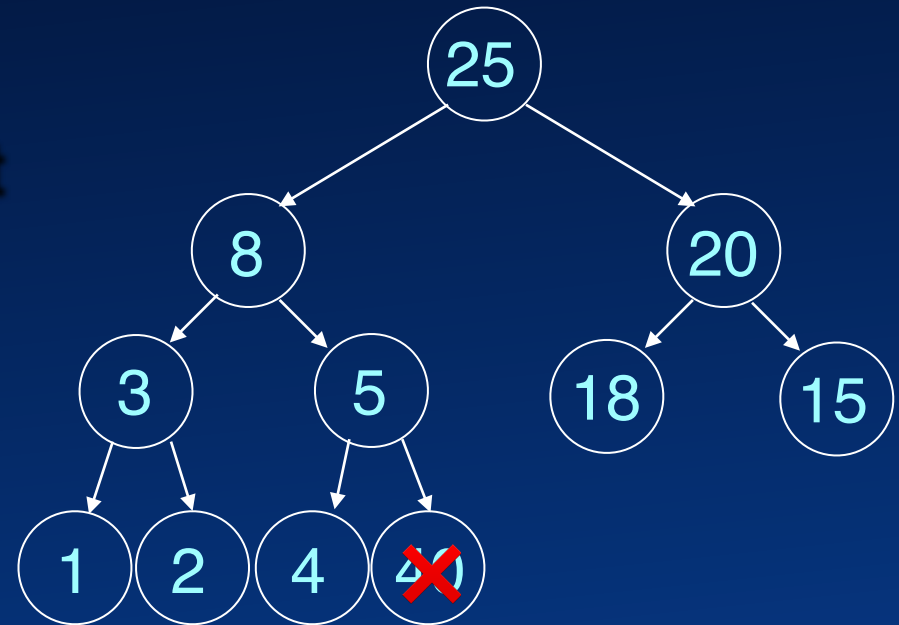


Heap



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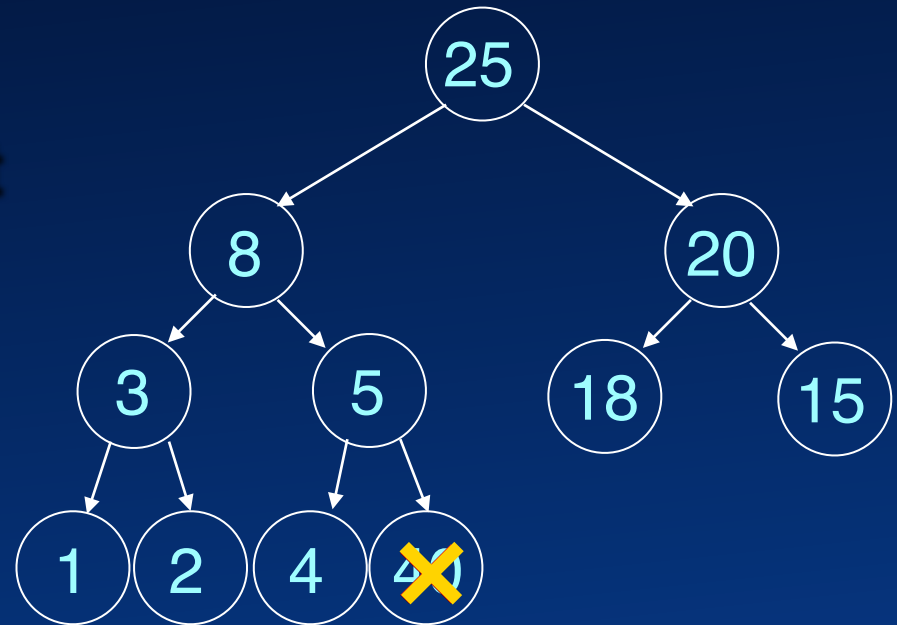


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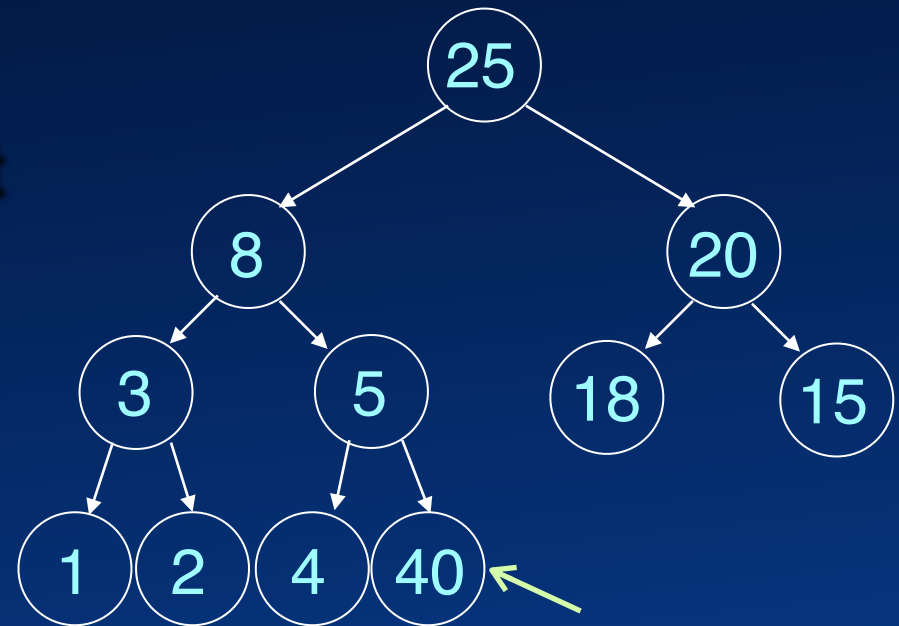


Heap



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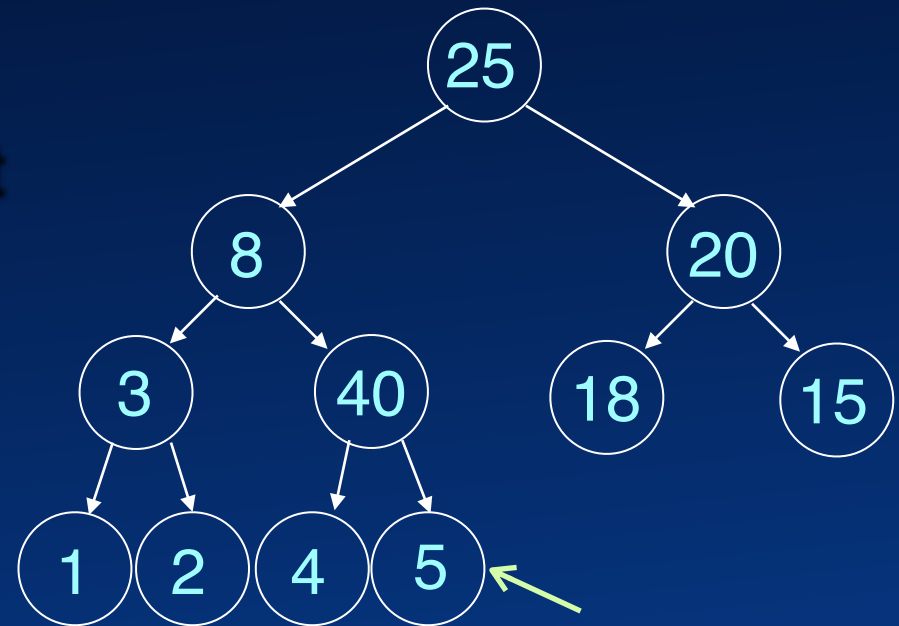


Heap



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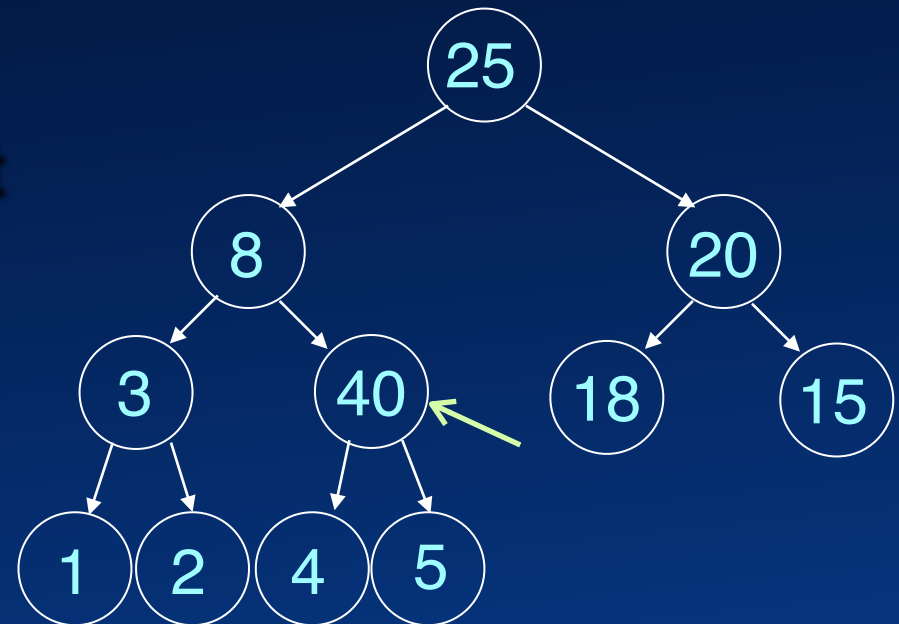


Heap



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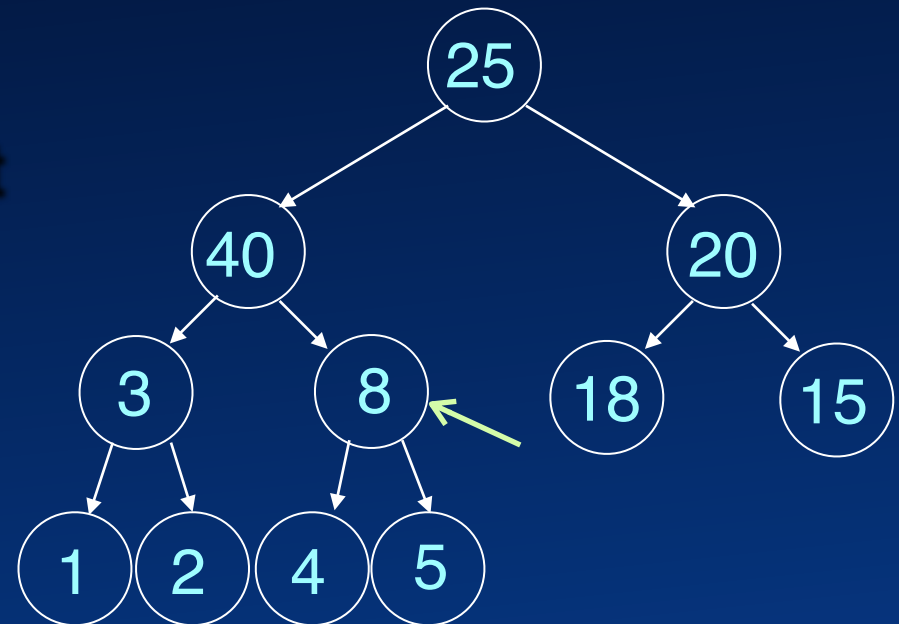


Heap



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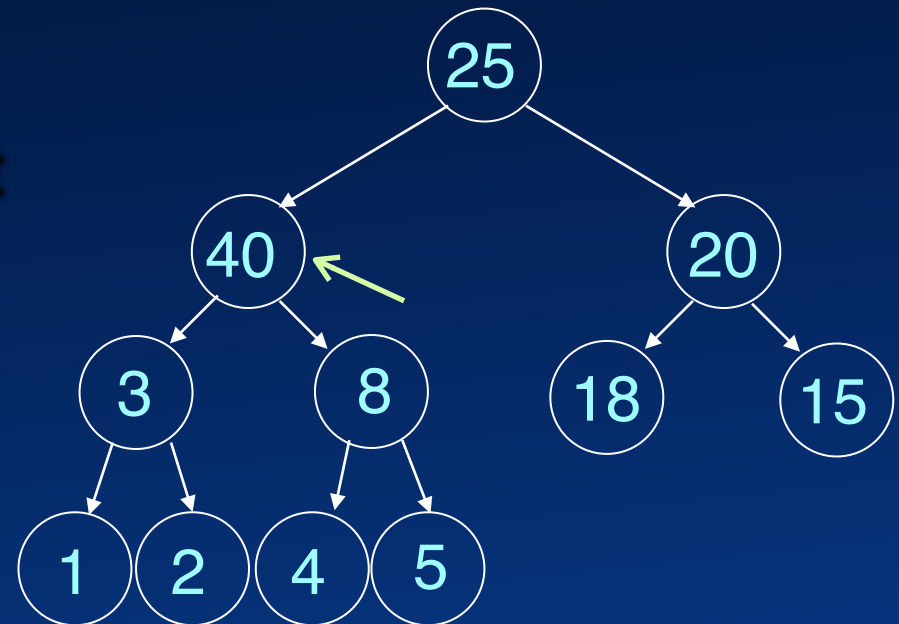


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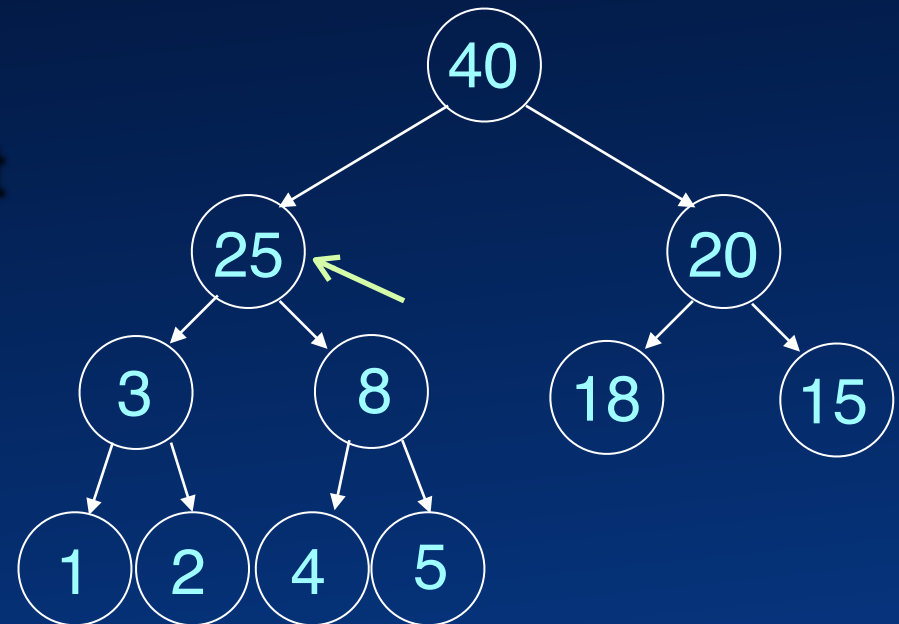


Heap



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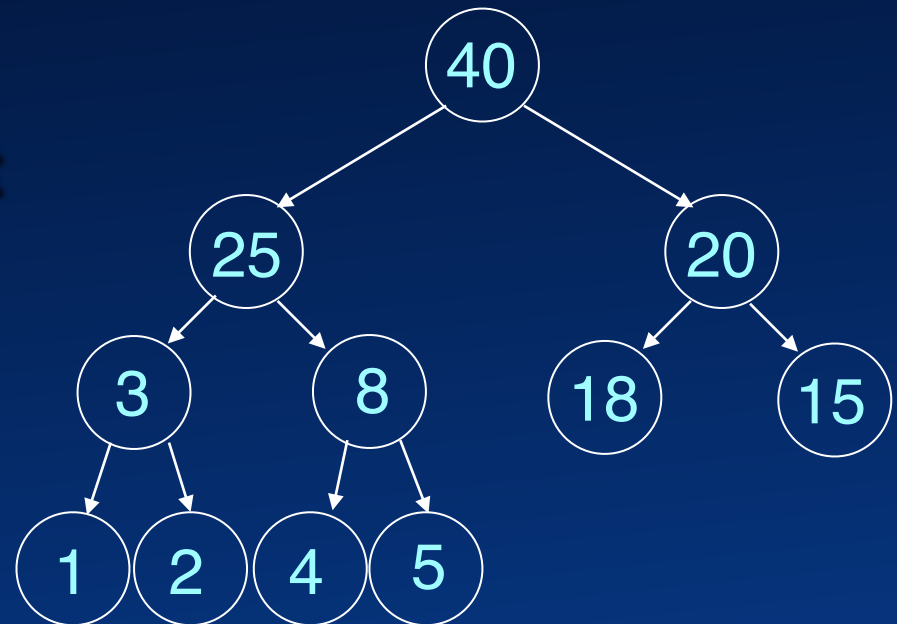


Heap



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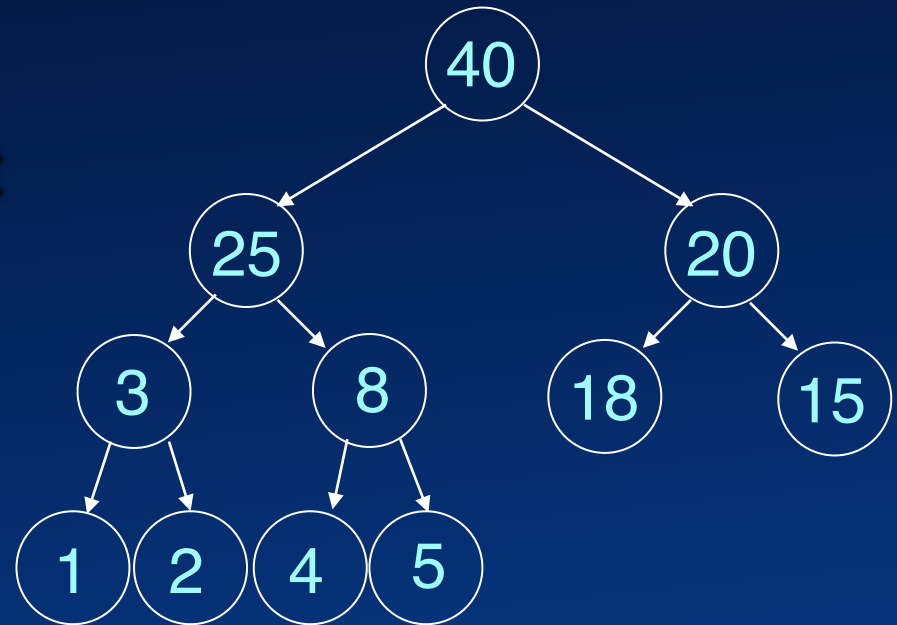
Heap



Heap

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Inserted 40



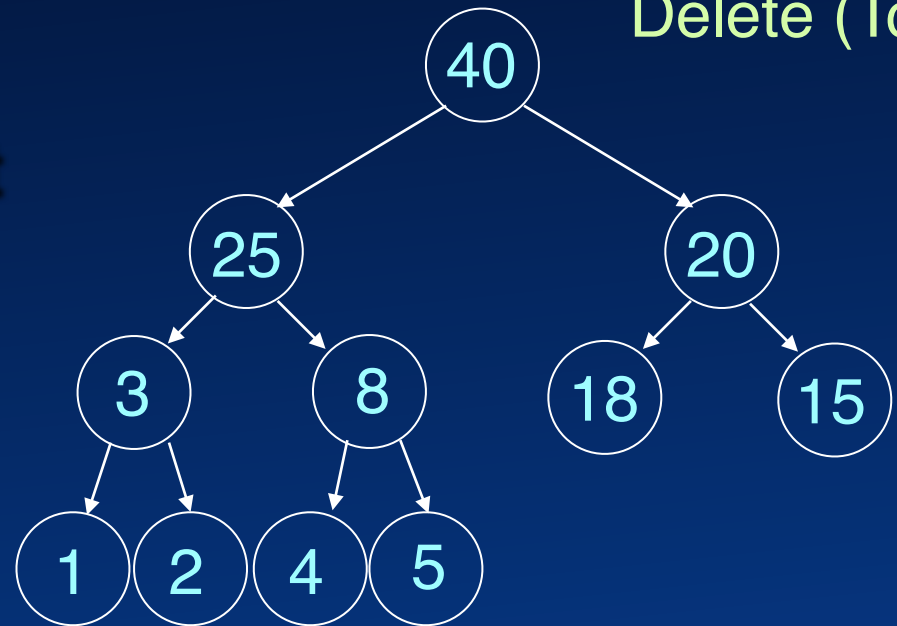
Heap



Heap

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Inserted 40
Delete (Top)

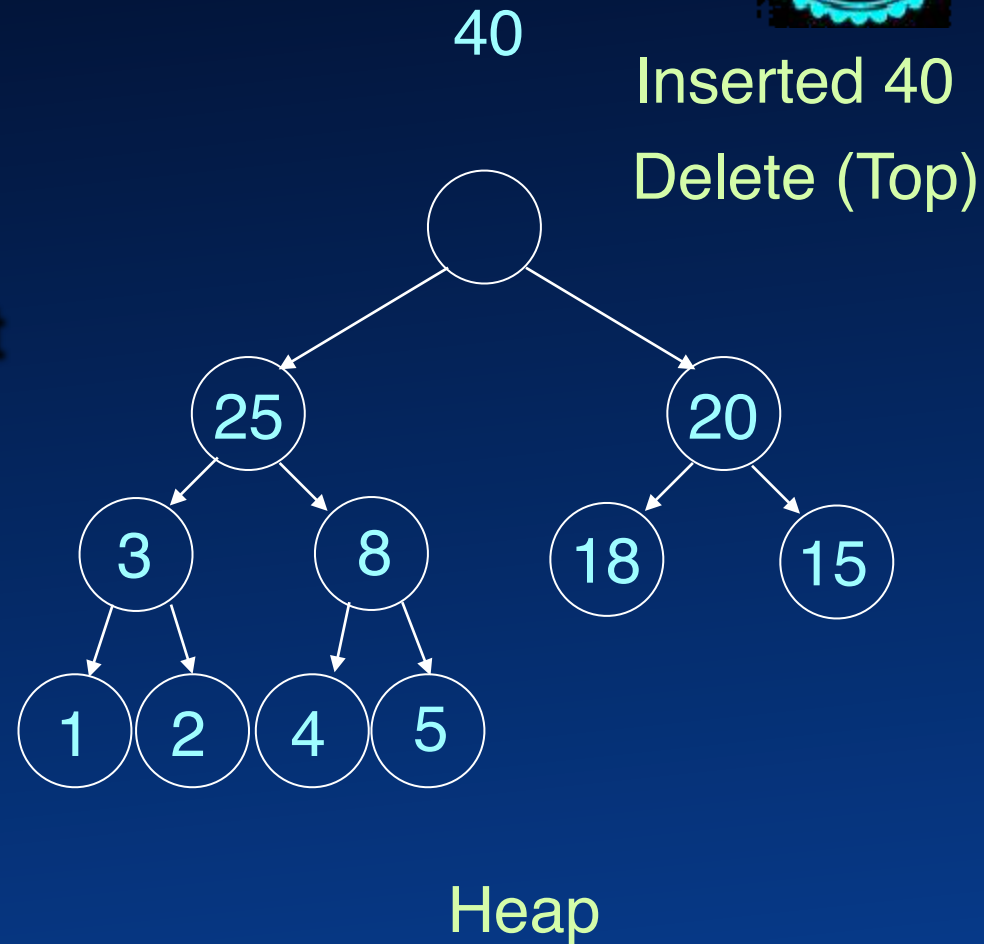


Heap



Heap

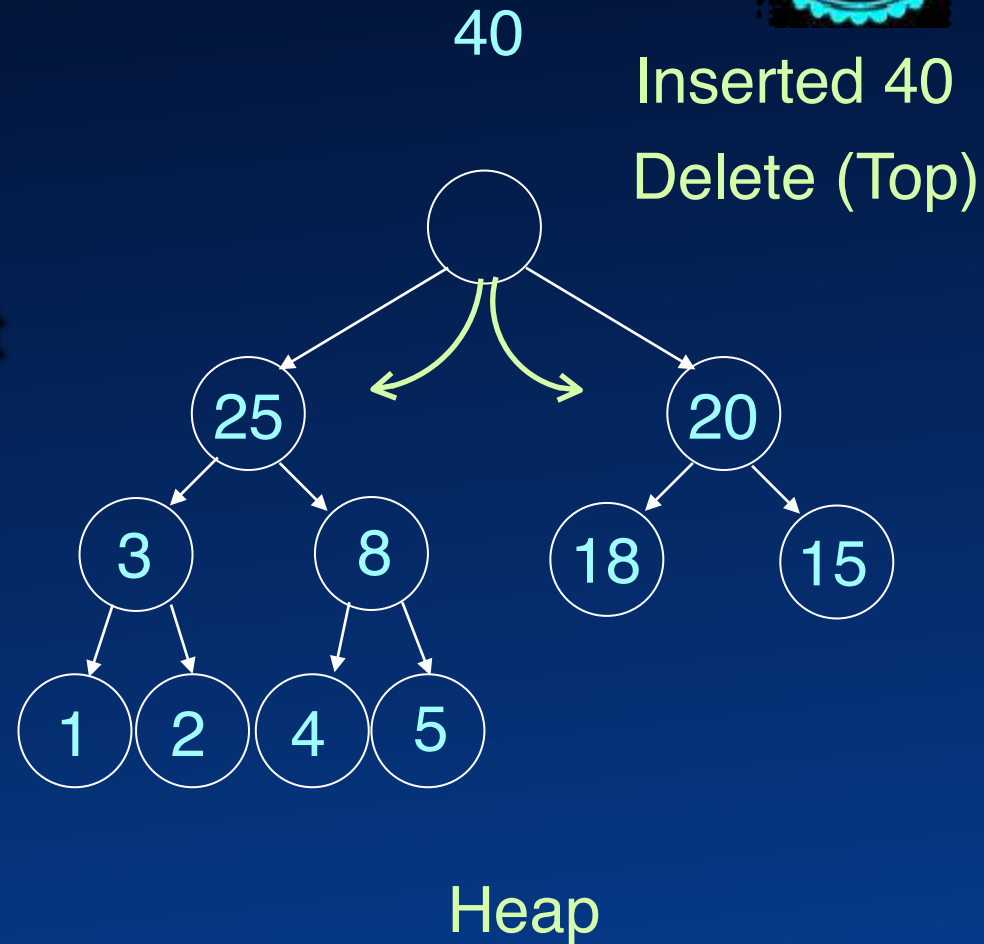
- Left-complete tree
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Heap

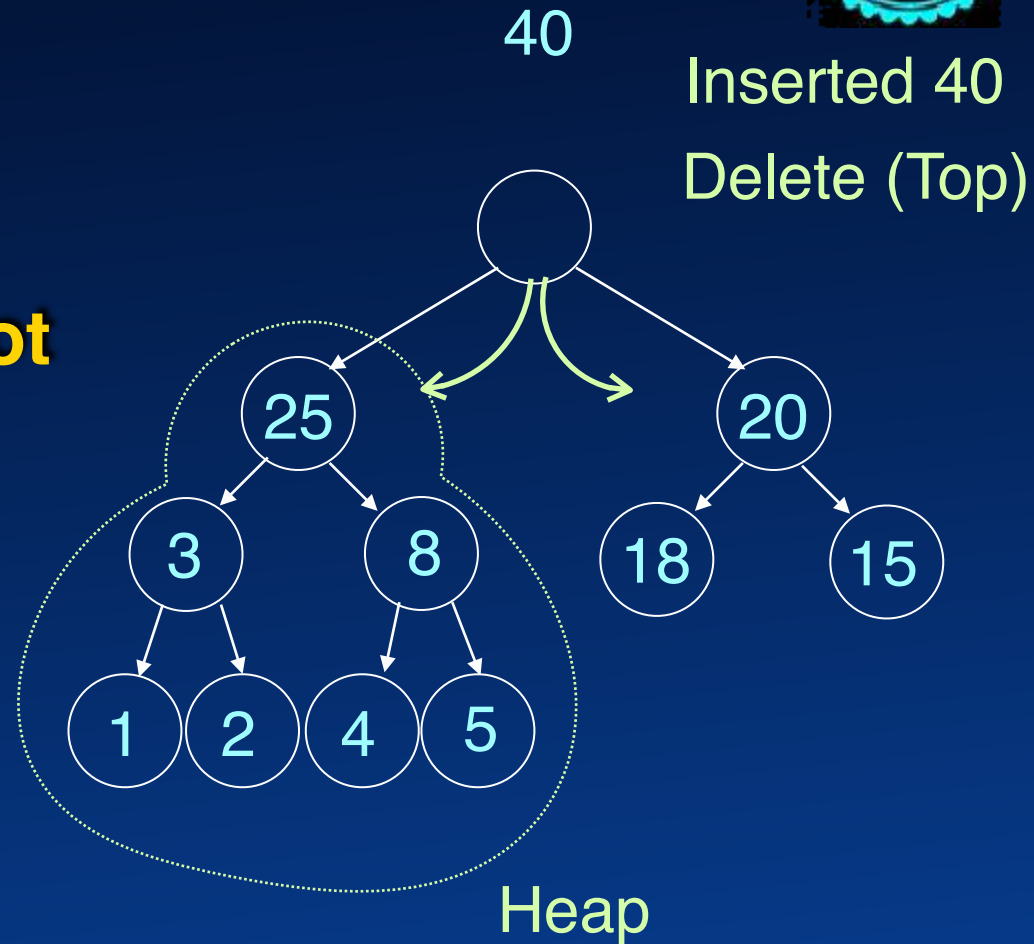
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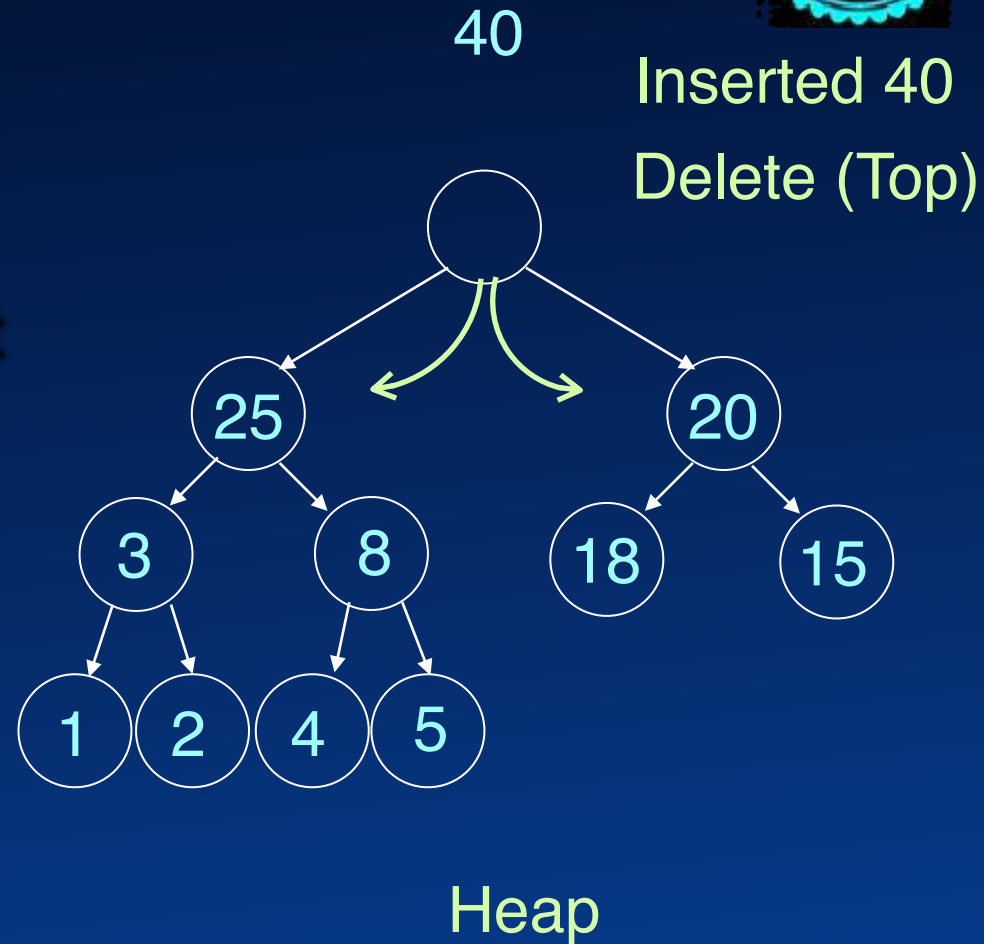
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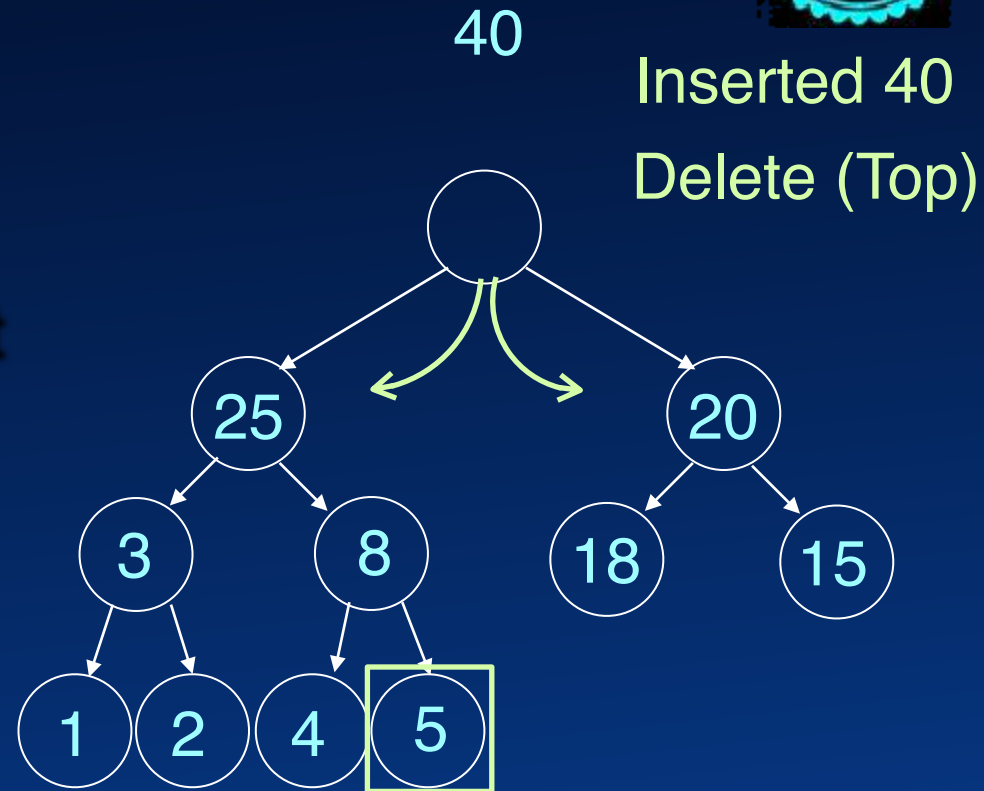
- Left-complete tree
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Heap

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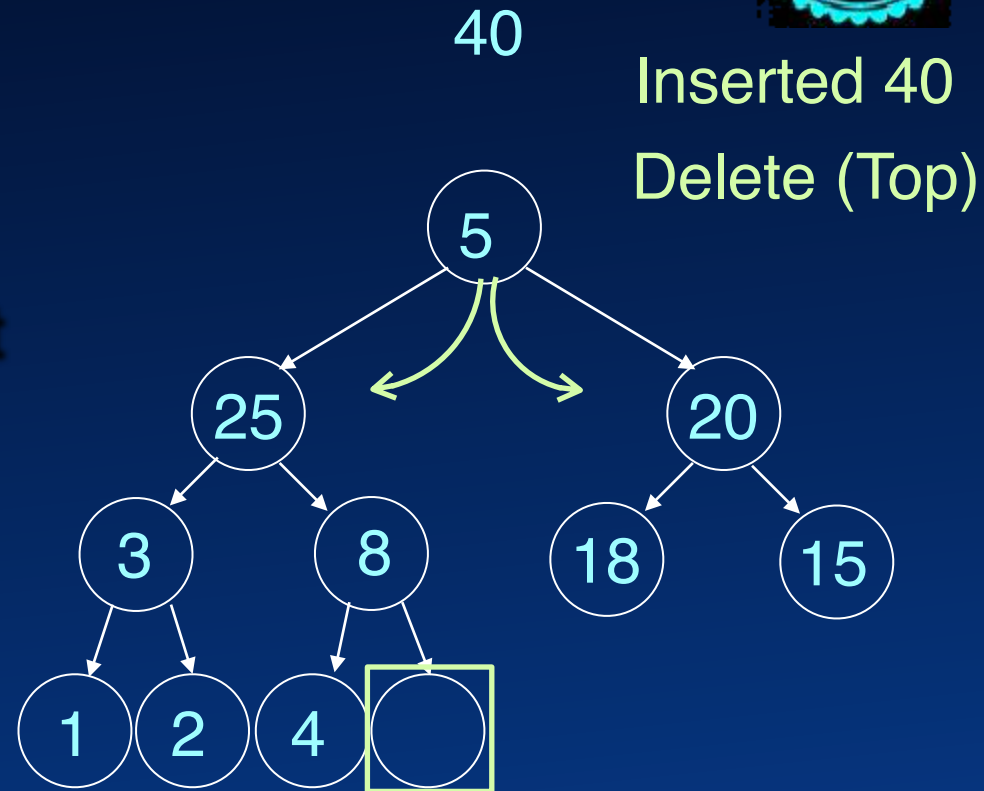


Heap



Heap

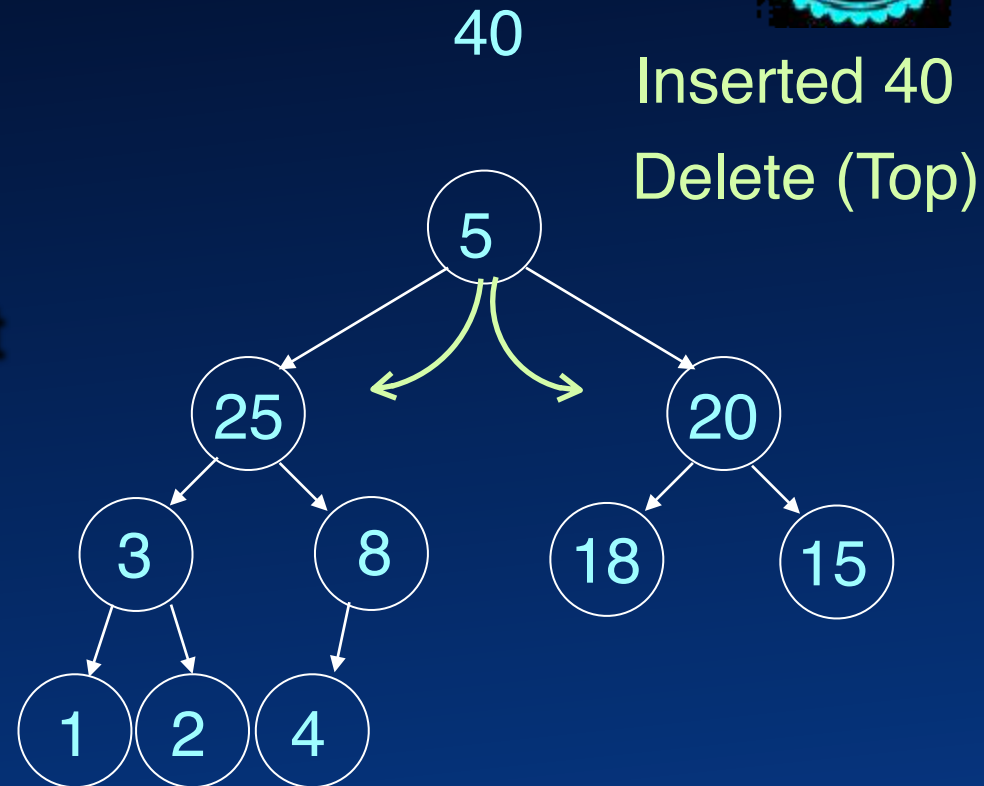
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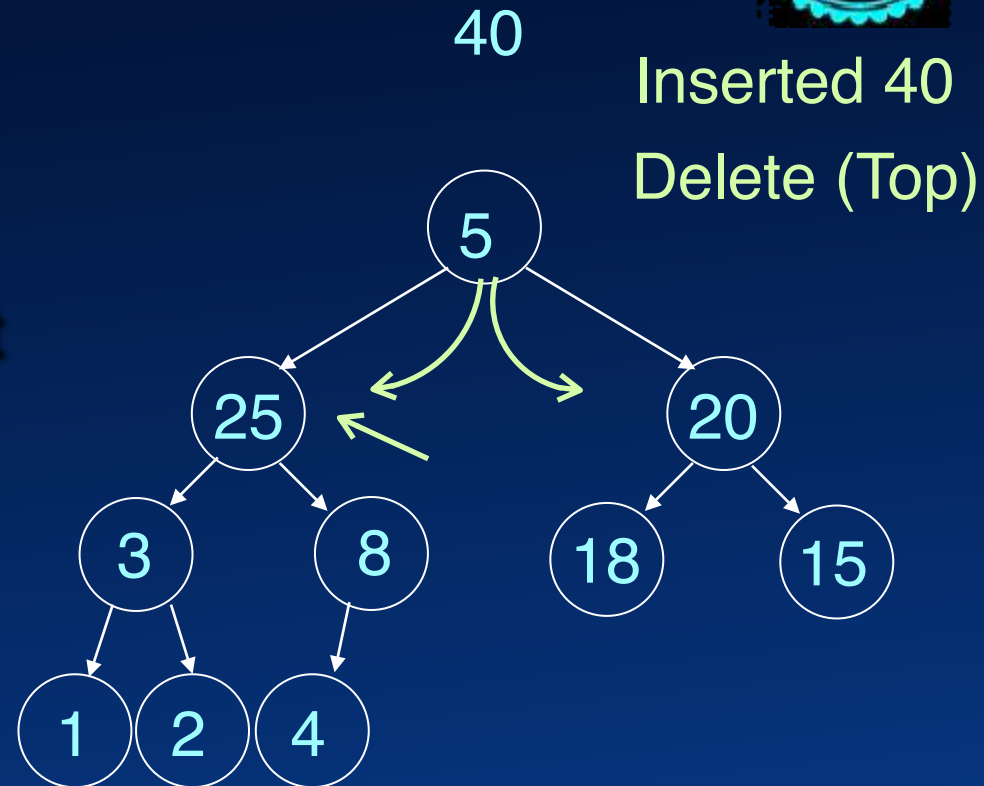


Heap



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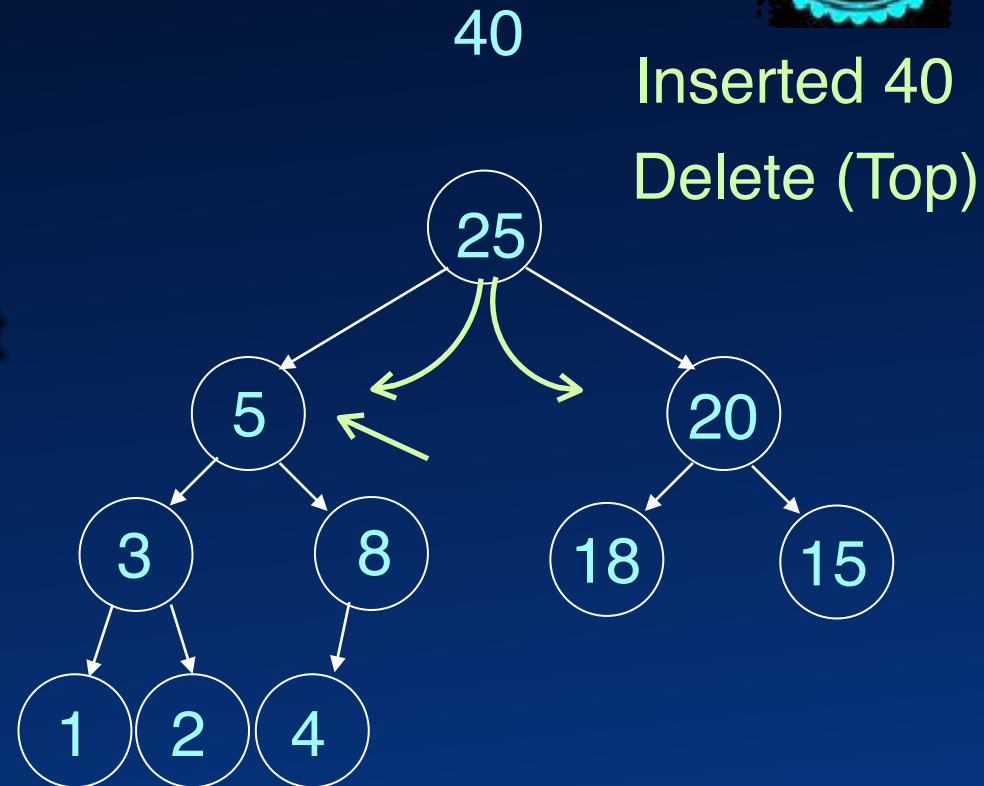


Heap



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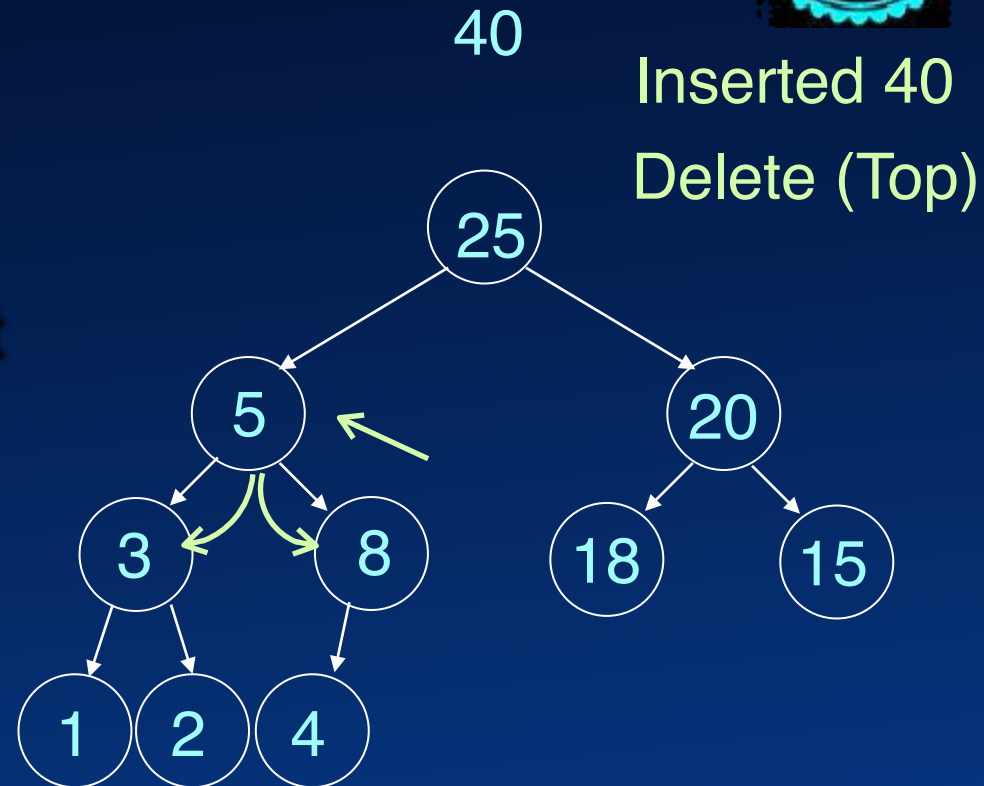


Heap



Heap

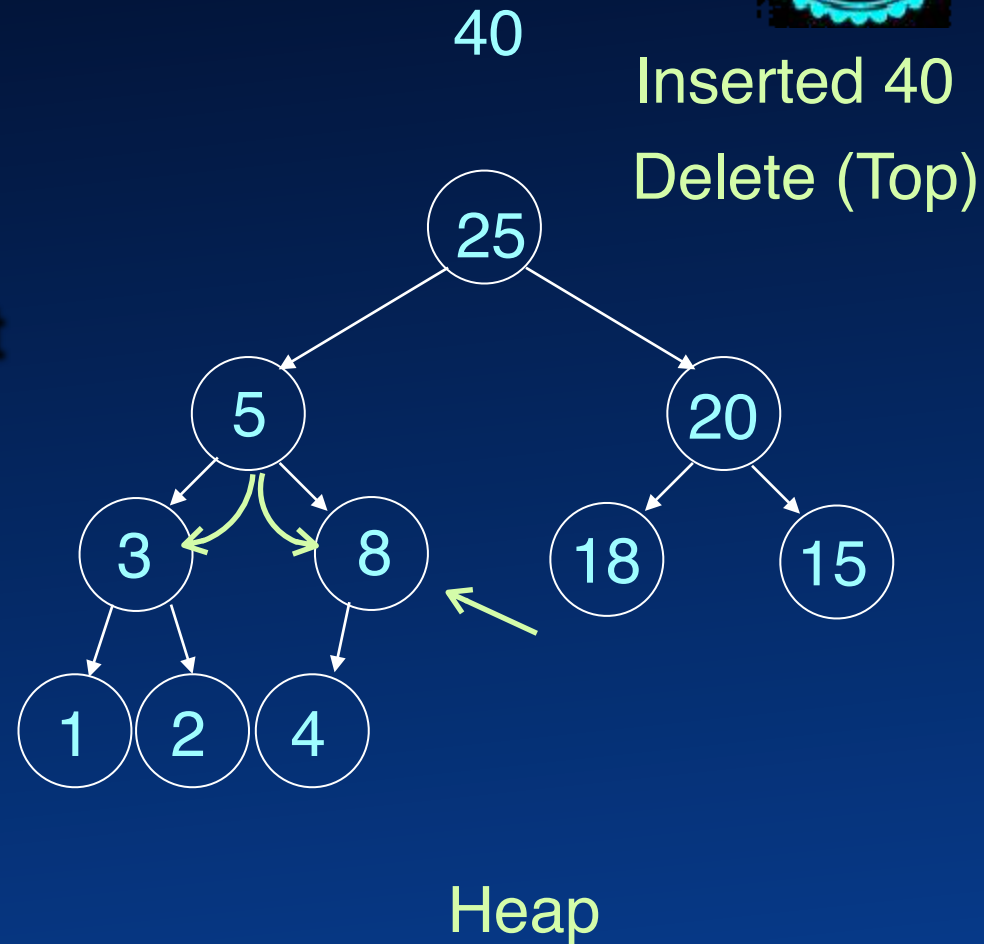
- Left-complete tree
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Heap

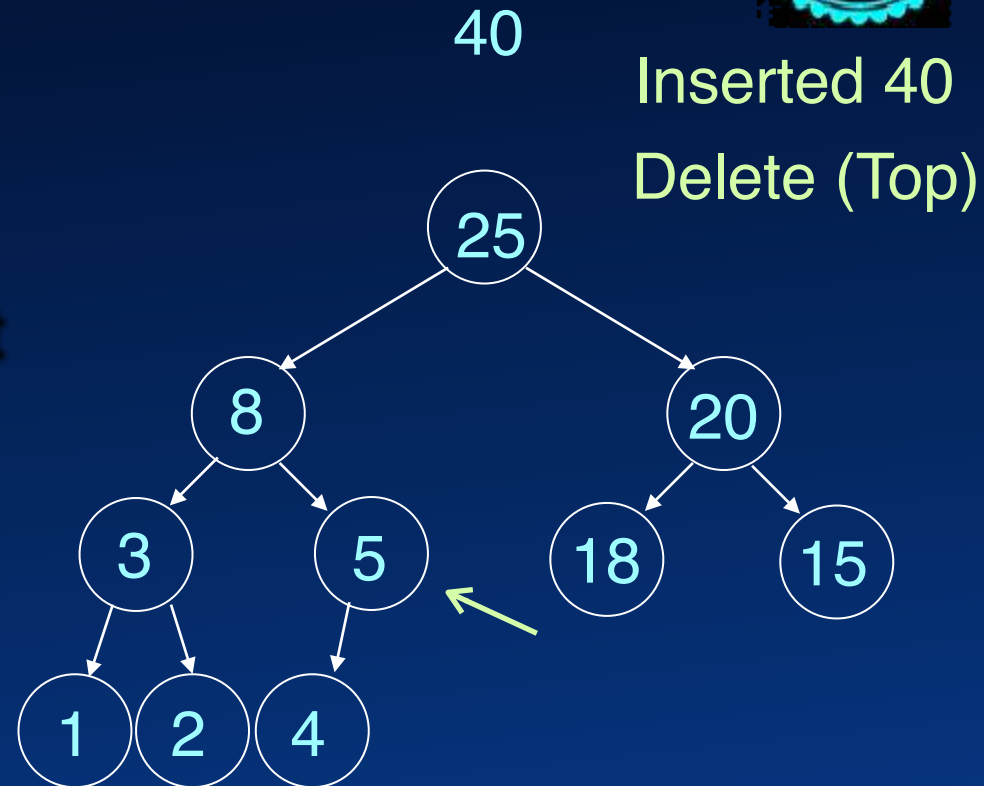
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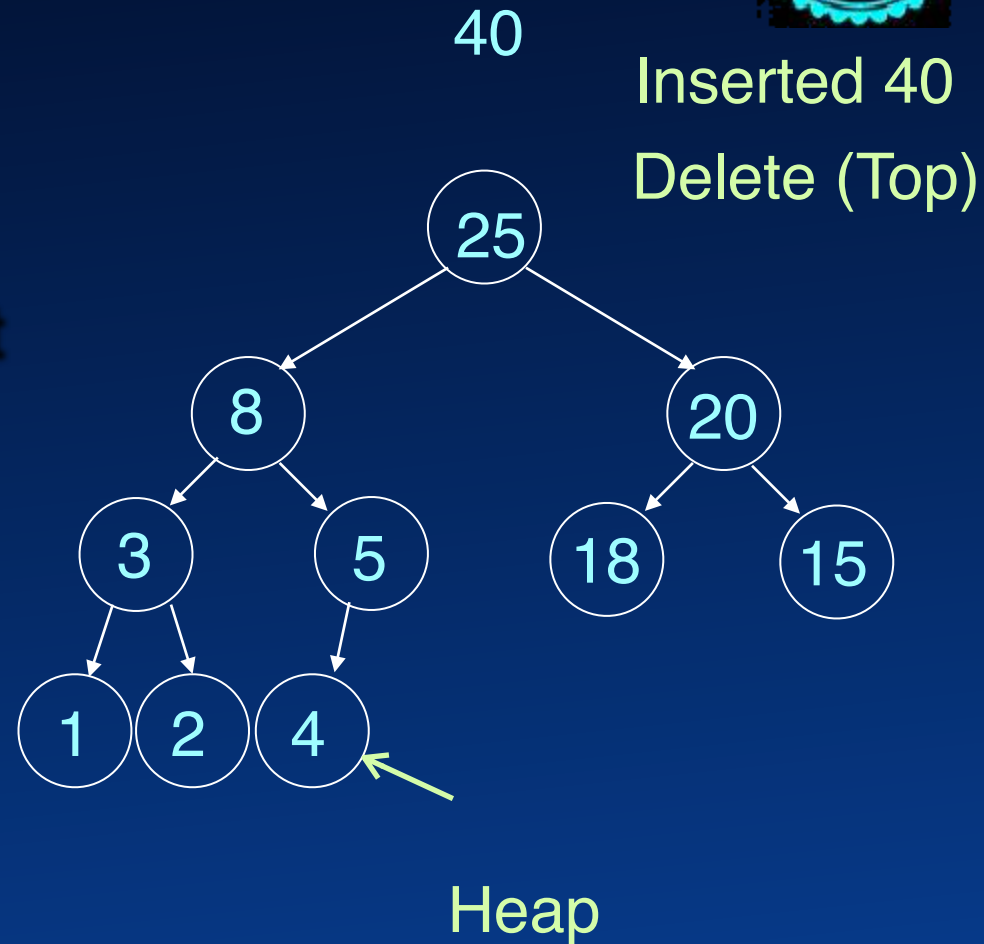
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Heap

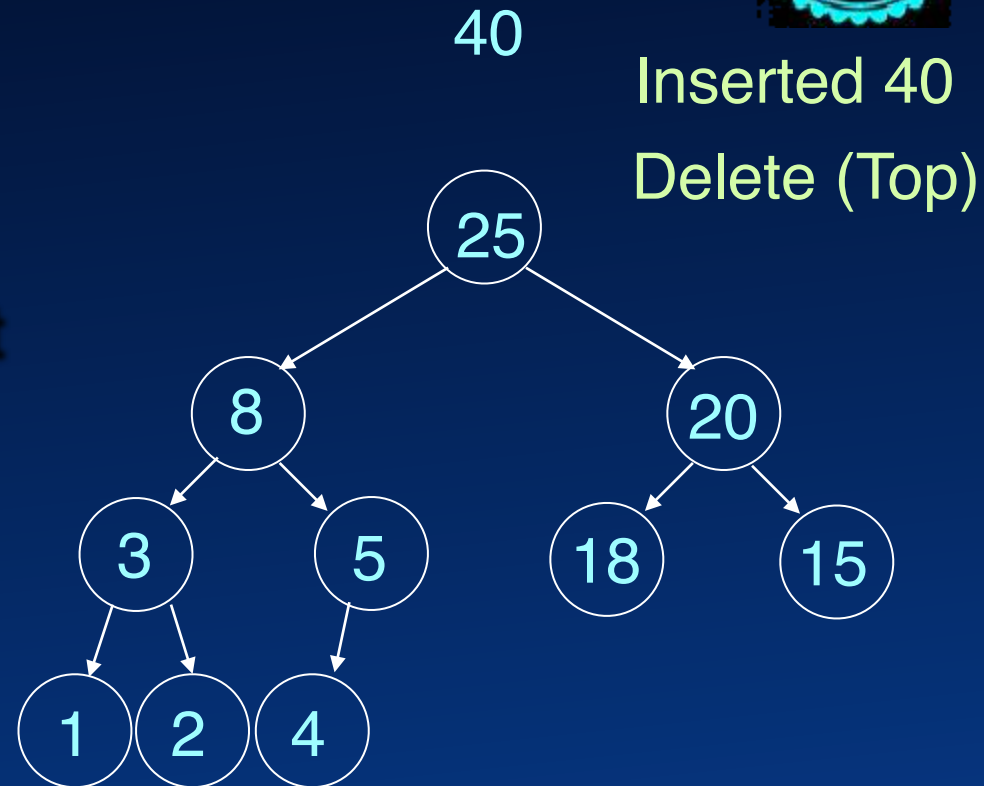
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Heap

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Heap



Heap

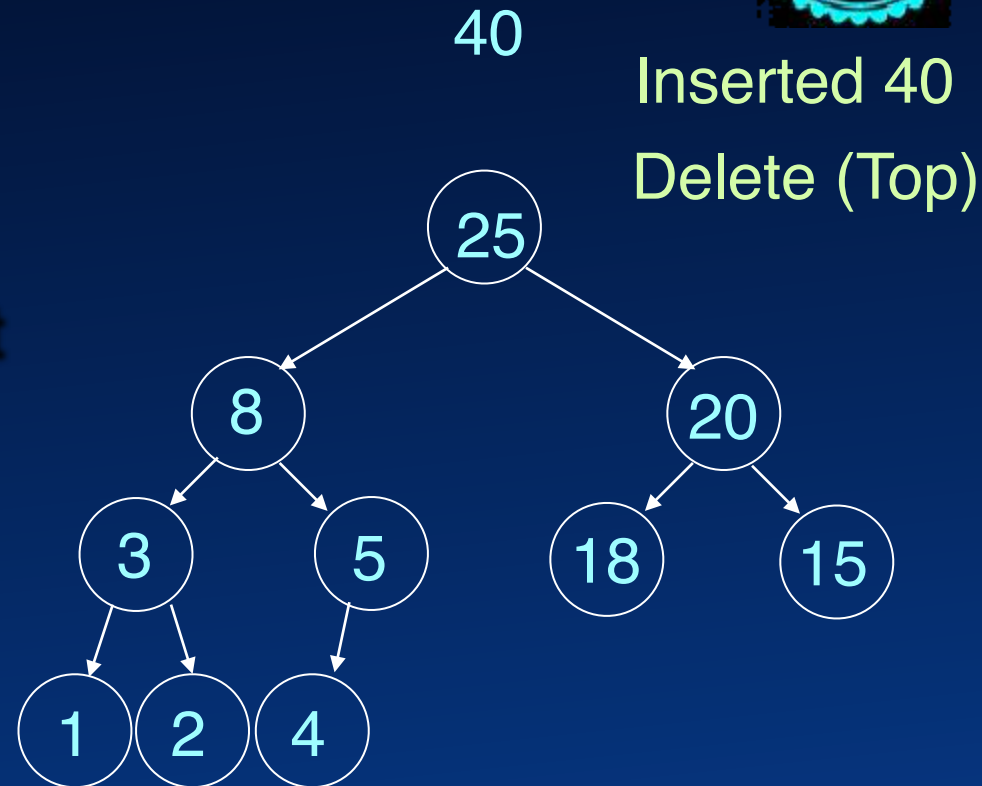
- **Left-complete tree**
- **Comparable keys**
- **“Top” key in the root**
 - **For every subtree**

Insert:

Add node at next spot
Bubble-up

Delete:

Remove root
Replace with last spot
Bubble-down



Heap



Heap

Bubble up:

if no parent 👍

if(key lower-than parent.key) 👍

swapwith(parent)

parent.bubbleup()

Insert:

Add node at next spot

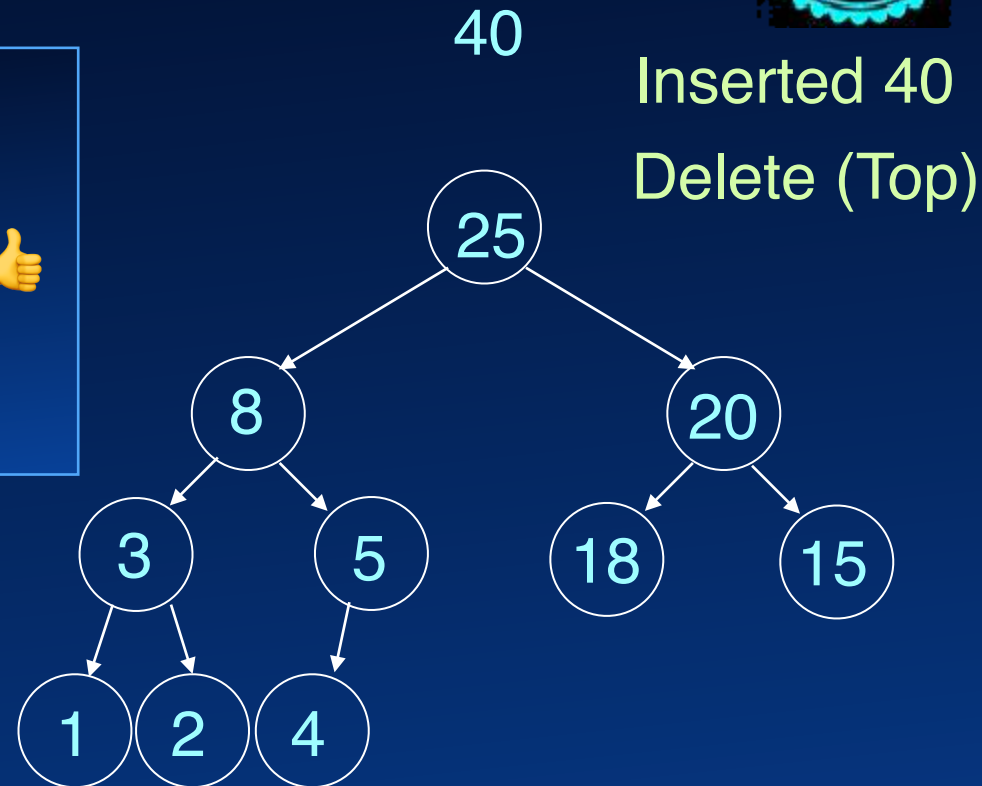
Bubble-up

Delete:

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Heap



Heap

Bubble down:

Bubble up:

if no parent 👍

if(key lower-than parent.key) 👍

swapwith(parent)

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Insert:

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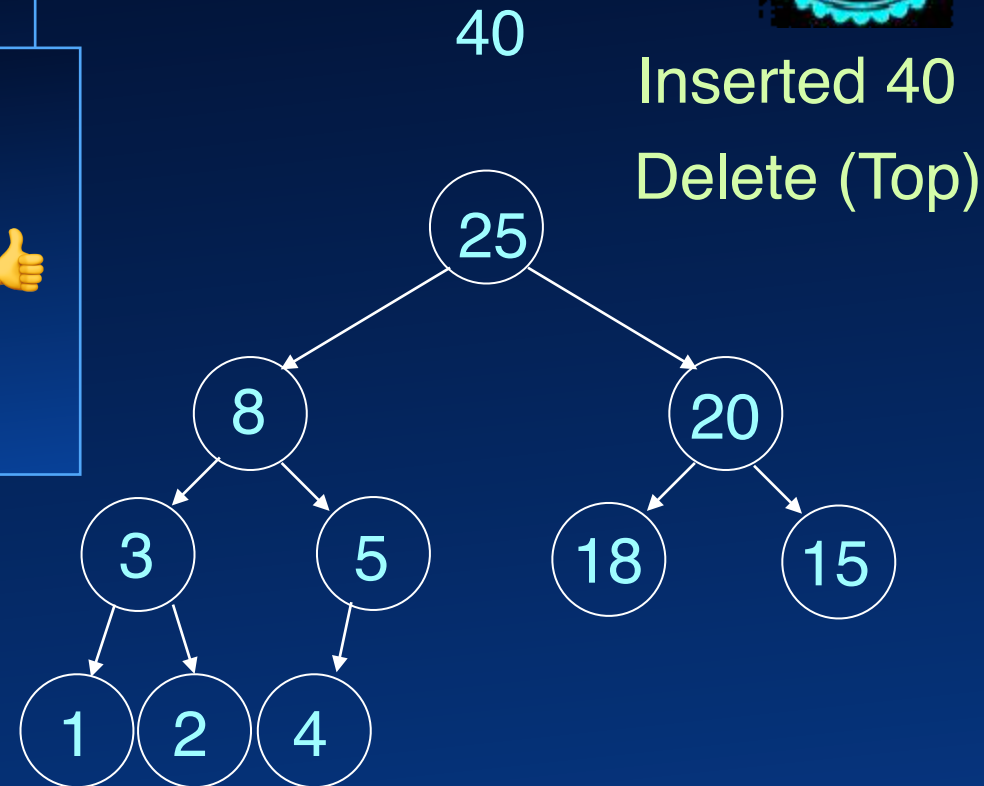
Bubble-up

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Remove root

Replace with last spot

Bubble-down



Heap



Heap

Bubble down:

Bubble up:

if no parent 👍

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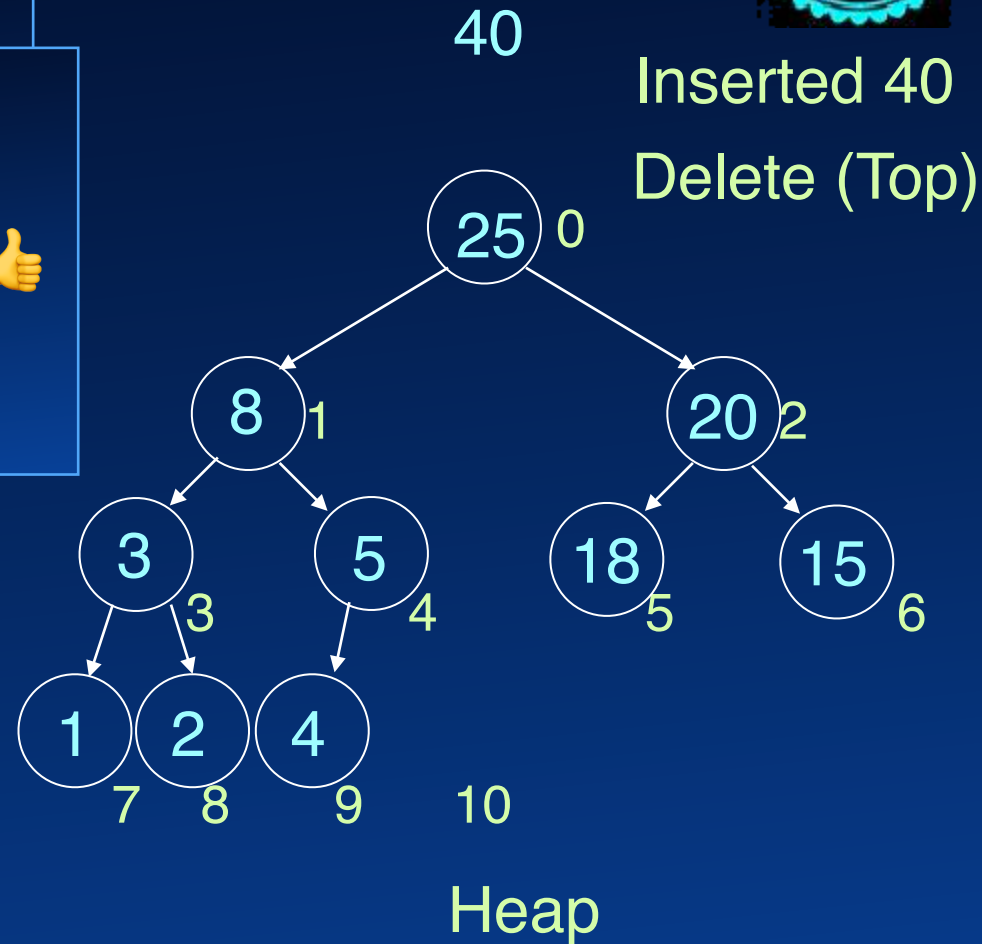
Bubble-up

Delete:

Remove root

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Heap

Bubble down:

Bubble up:

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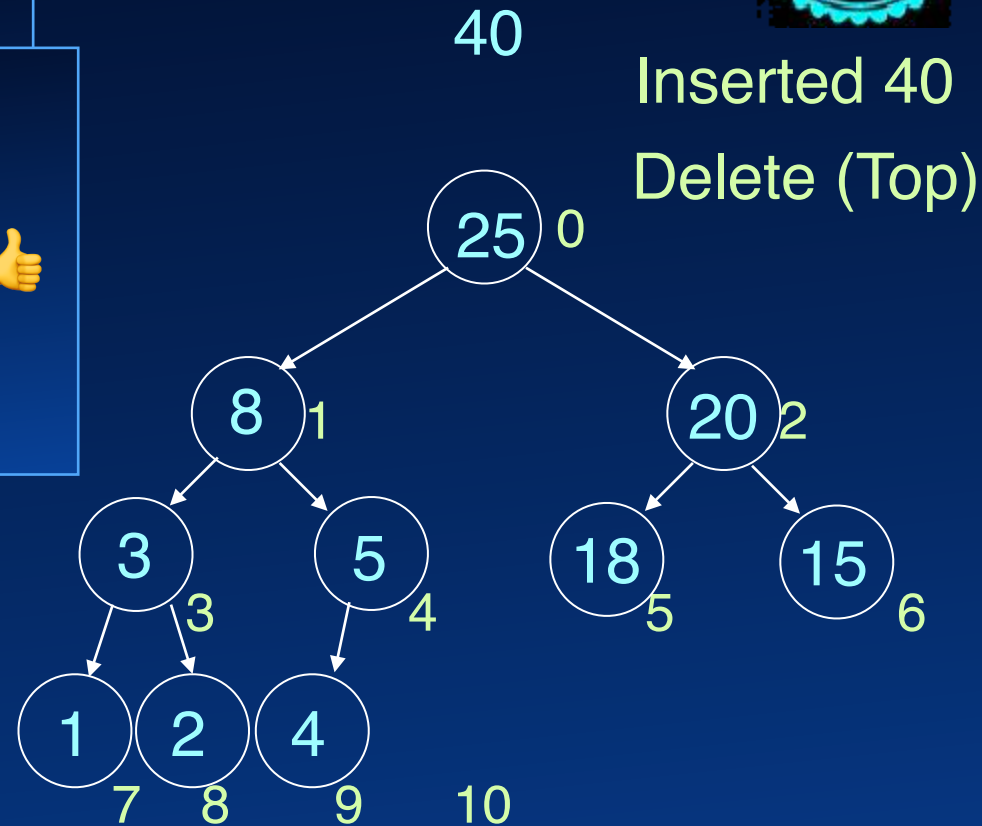
Bubble-up

Delete:

Remove root

Replace with last spot

Bubble-down



Heap

$$\text{child-index} = 2 * \text{parent-index} + \{1, 2\}$$
$$\text{parent-index} = (\text{child-index} - 1) / 2$$

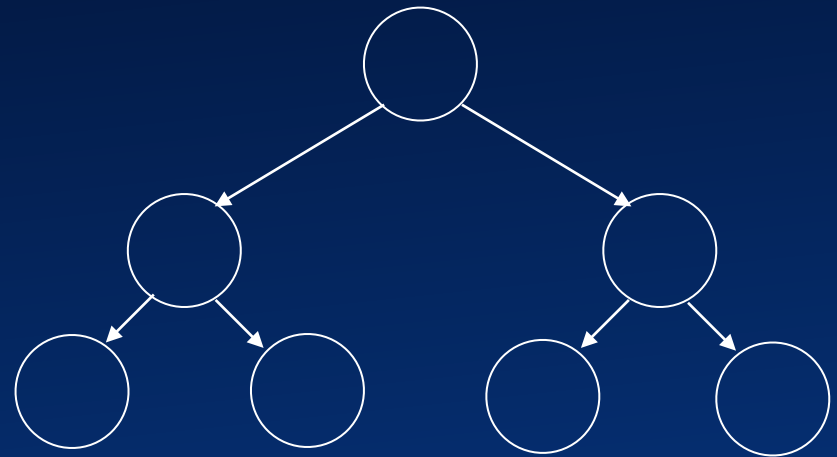
In which node may the third largest element of a heap be?



mail: col106quiz@cse.iitd.ac.in

format: 1,2,3,4,5,6

- 1
- 2
- 3
- 4
- 5
- 6



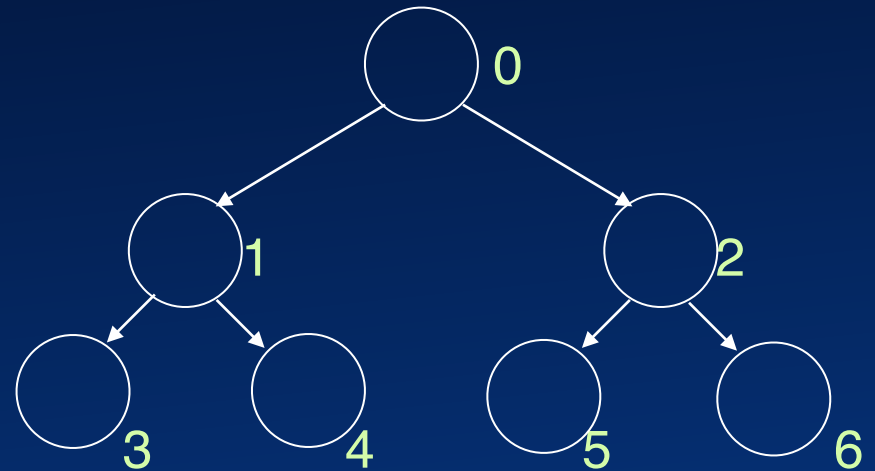
In which node may the third largest element of a heap be?



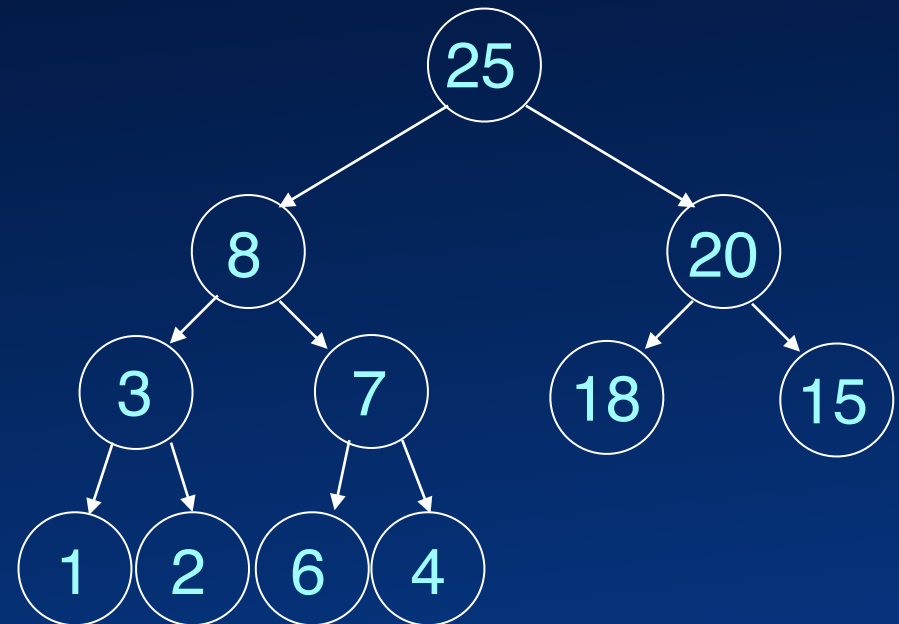
mail: col106quiz@cse.iitd.ac.in

format: 1,2,3,4,5,6

- 1
- 2
- 3
- 4
- 5
- 6

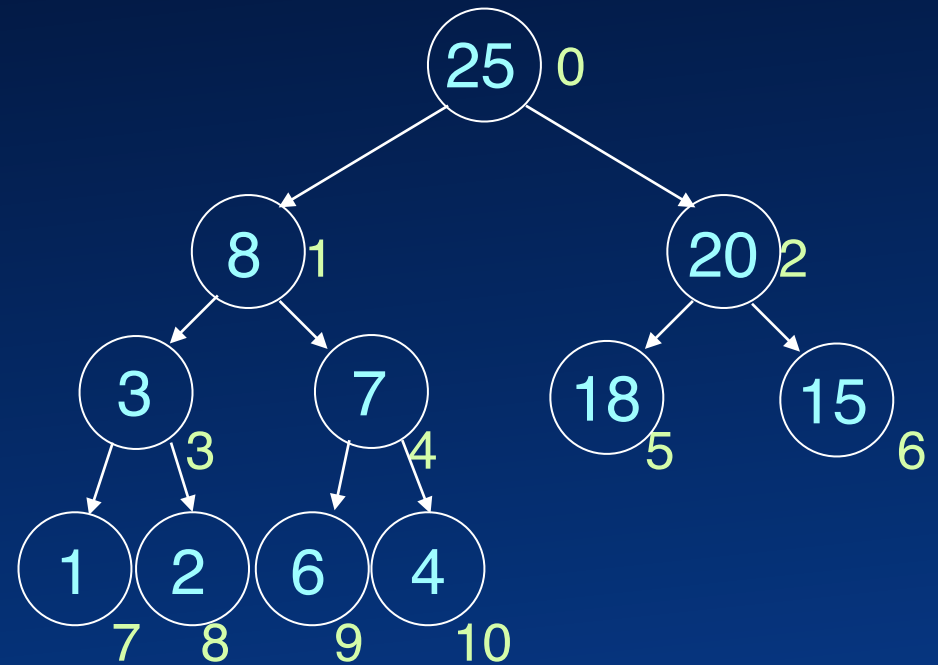


Heap



Heap

Heap

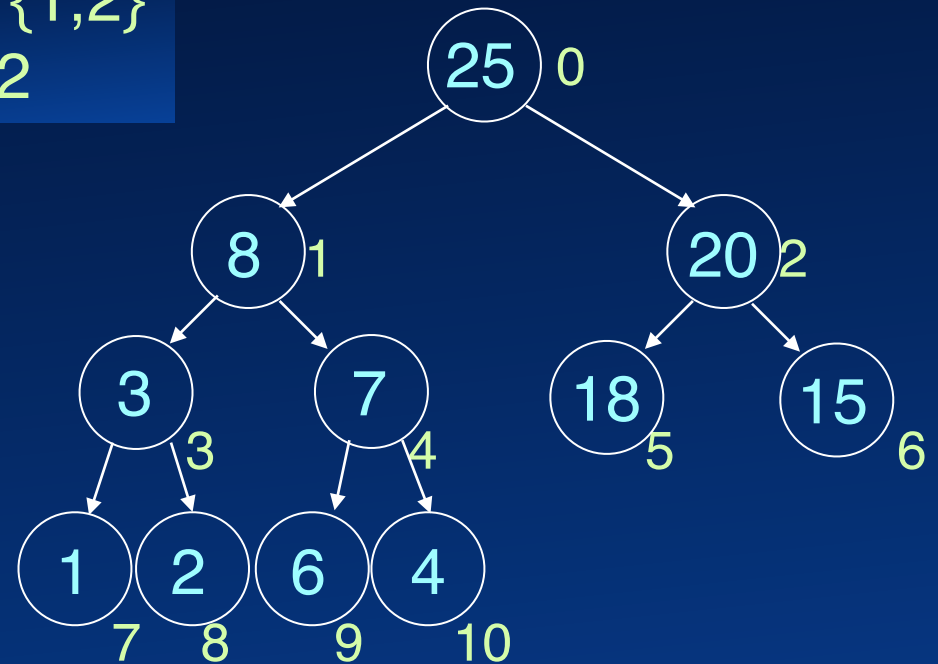


Heap



Heap

$\text{child-index} = 2 * \text{parent-index} + \{1, 2\}$
 $\text{parent-index} = (\text{child-index} - 1) / 2$

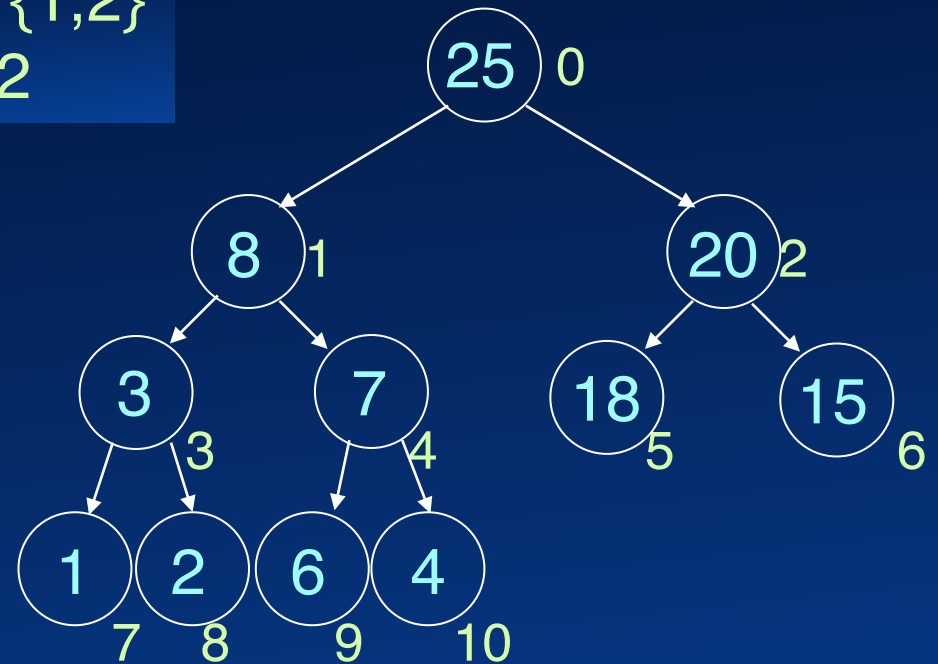


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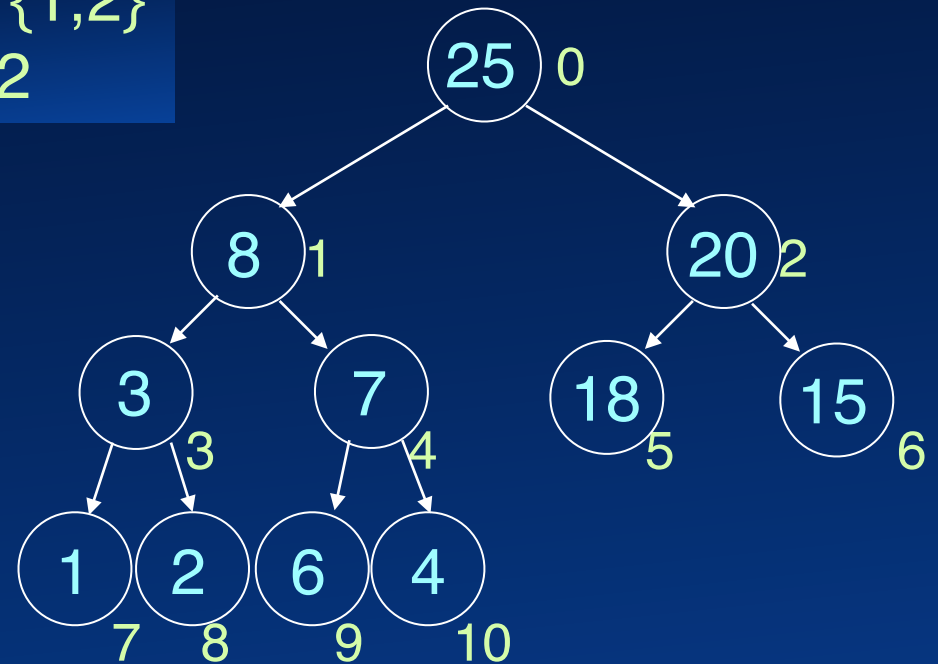
25	8	20	3	7	18	15	1	2	6	4		
----	---	----	---	---	----	----	---	---	---	---	--	--

Heap



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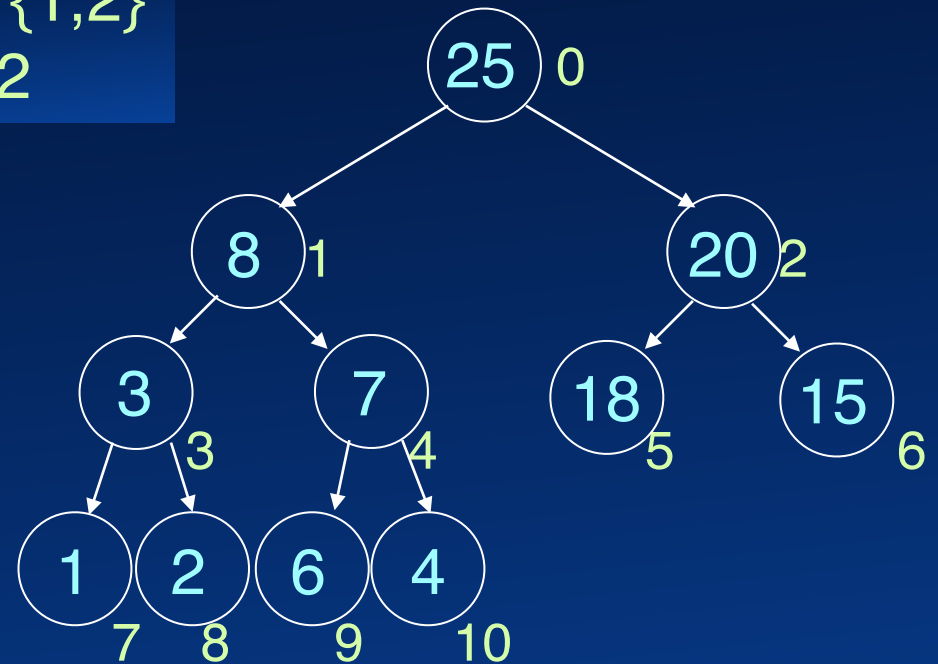
25	8	20	3	7	18	15	1	2	6	4		
0	1	2	3	4	5	6	7	8	9	10	11	

Heap



Heap

$\text{child-index} = 2 * \text{parent-index} + \{1, 2\}$
 $\text{parent-index} = (\text{child-index} - 1) / 2$



25	8	20	3	7	18	15	1	2	6	4		
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0 1 2 3 4 5 6 7 8 9 10 11

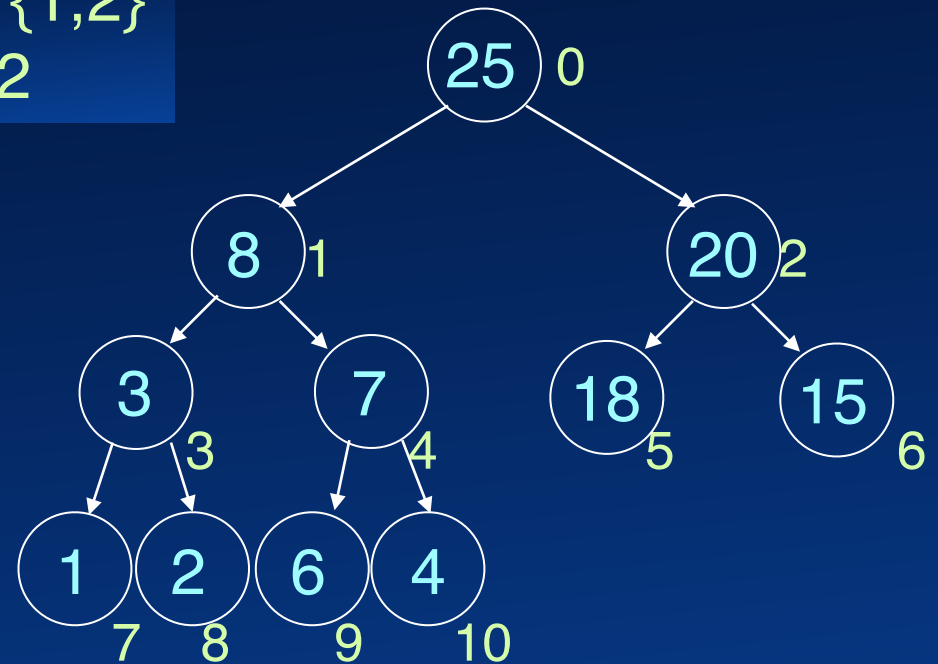


Heap



Heap

$\text{child-index} = 2 * \text{parent-index} + \{1, 2\}$
 $\text{parent-index} = (\text{child-index} - 1) / 2$



25	8	20	3	7	18	15	1	2	6	4		
----	---	----	---	---	----	----	---	---	---	---	--	--

0 1 2 3 4 5 6 7 8 9 10 11

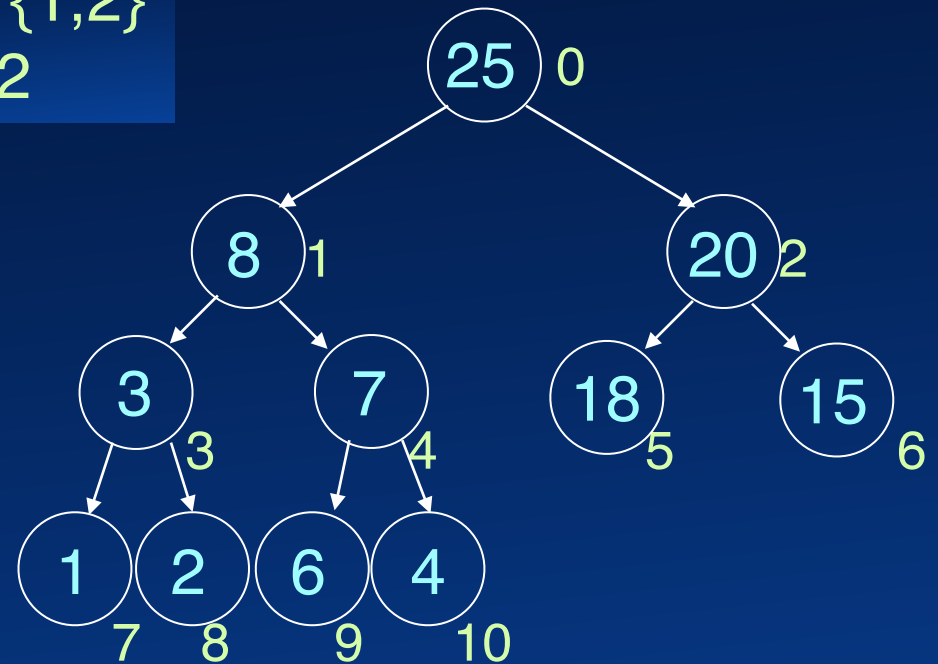
↑
next

Heap



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 $\text{parent-index} = (\text{child-index} - 1) / 2$



0 1 2 3 4 5 6 7 8 9 10 11

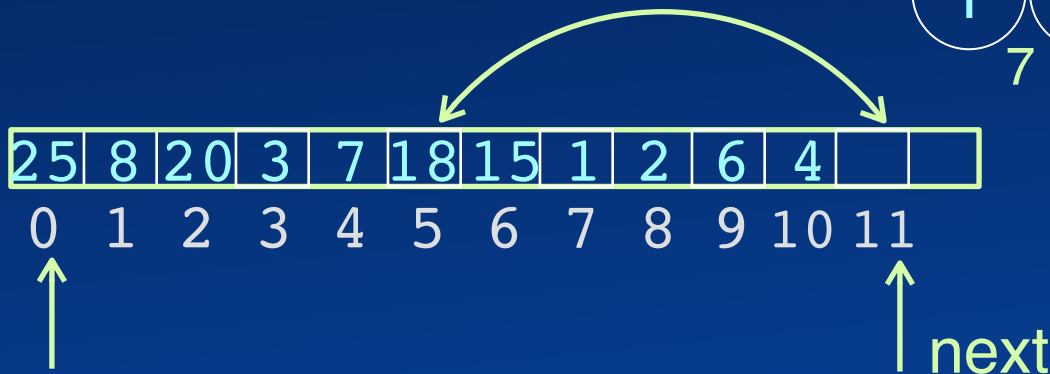
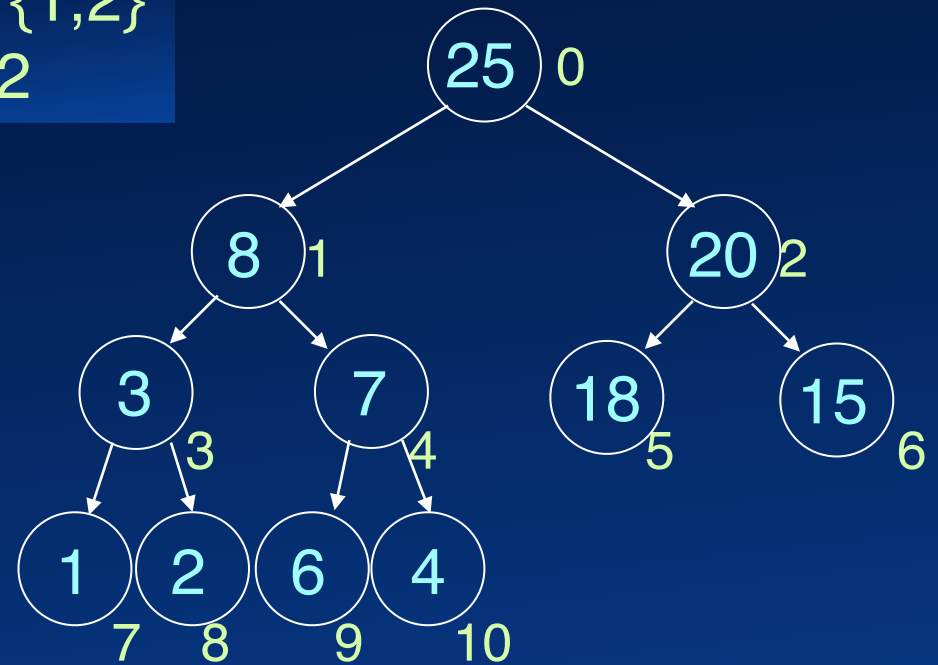
↑ next

Heap



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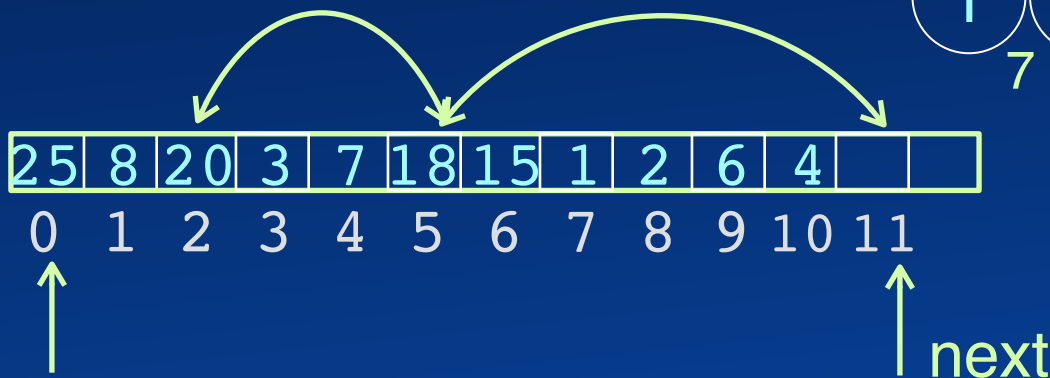
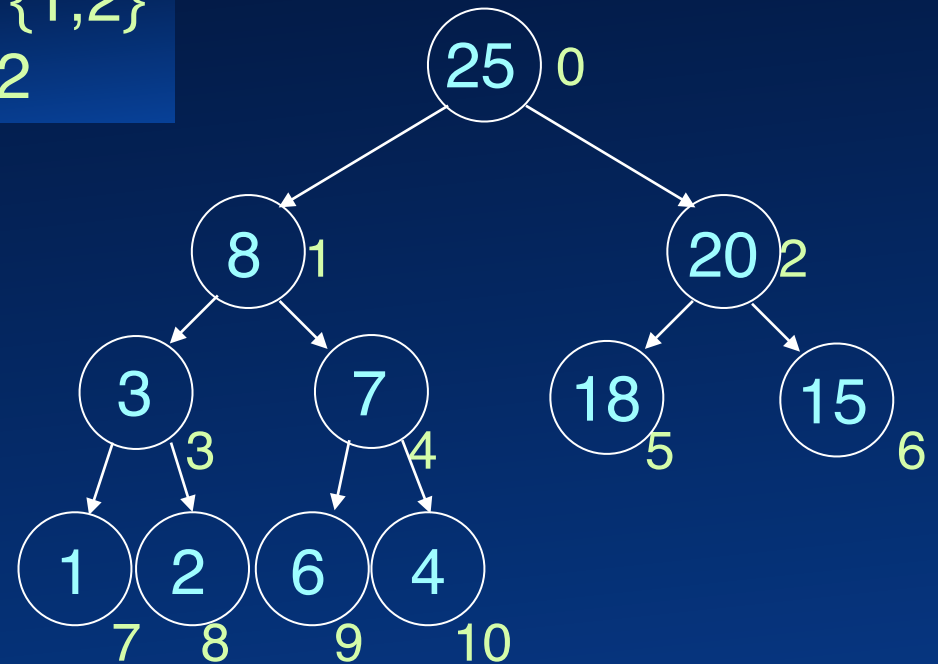


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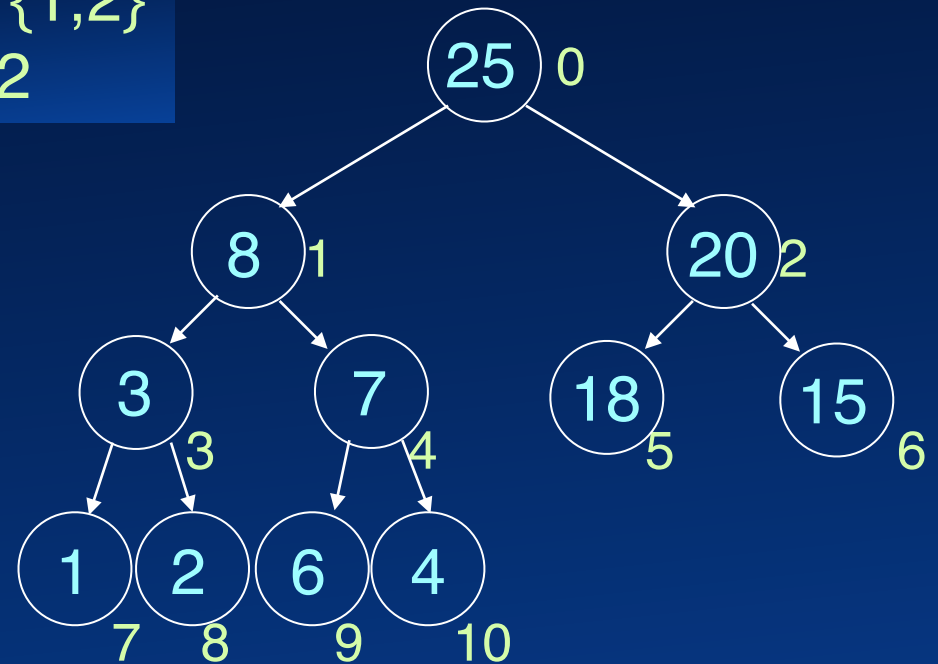
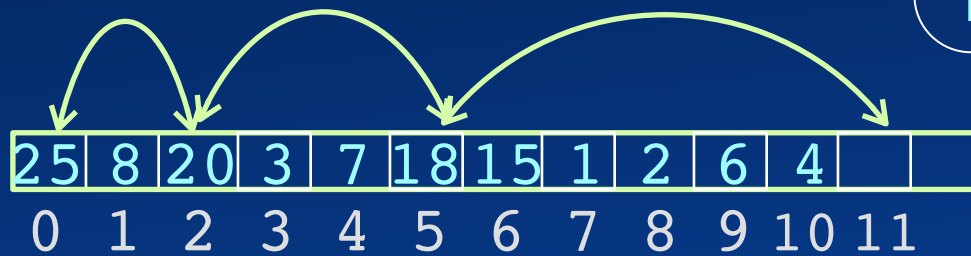


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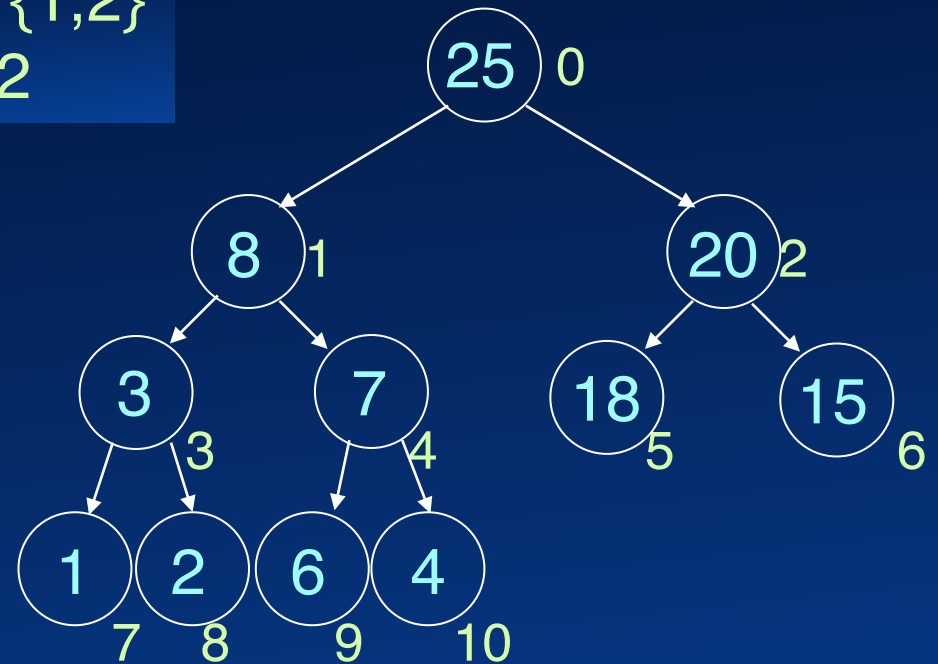


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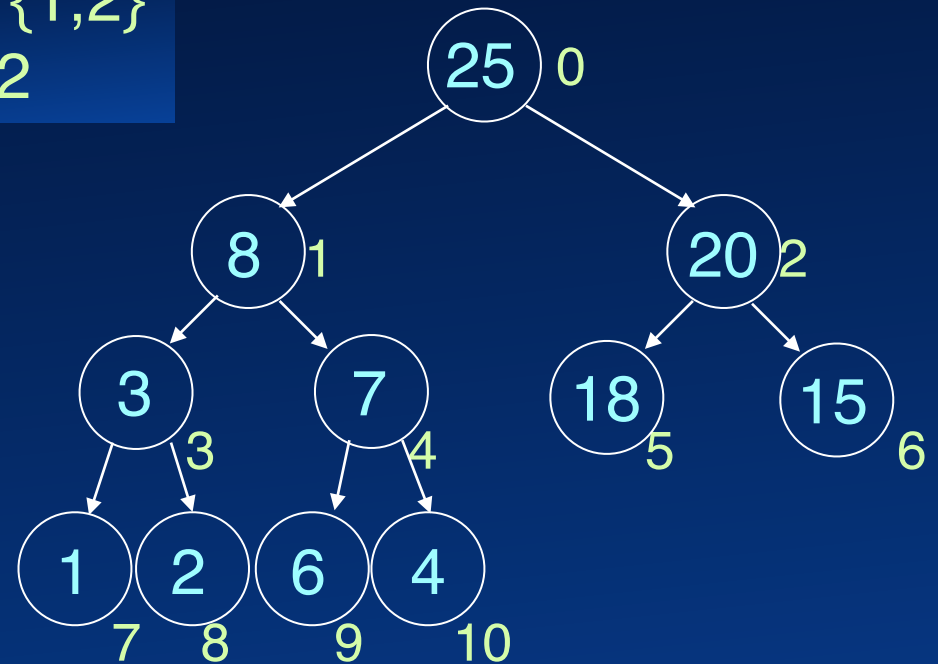
0 1 2 3 4 5 6 7 8 9 10 11 12
↑ next

Heap



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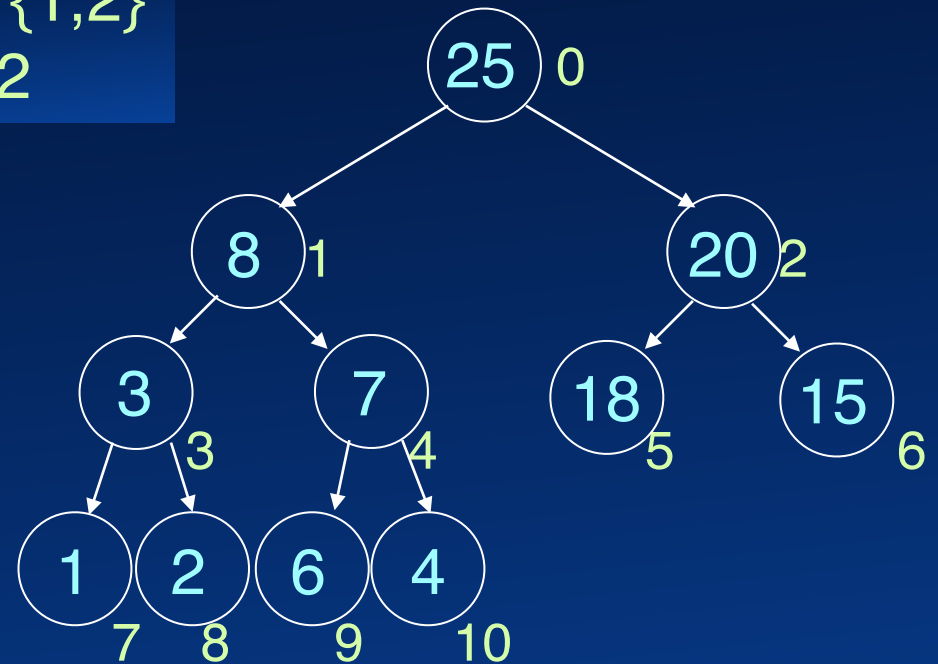
0 1 2 3 4 5 6 7 8 9 10 11 12
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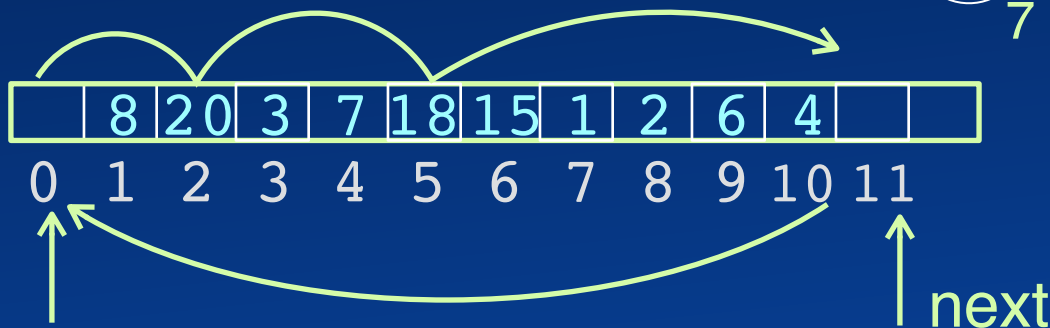
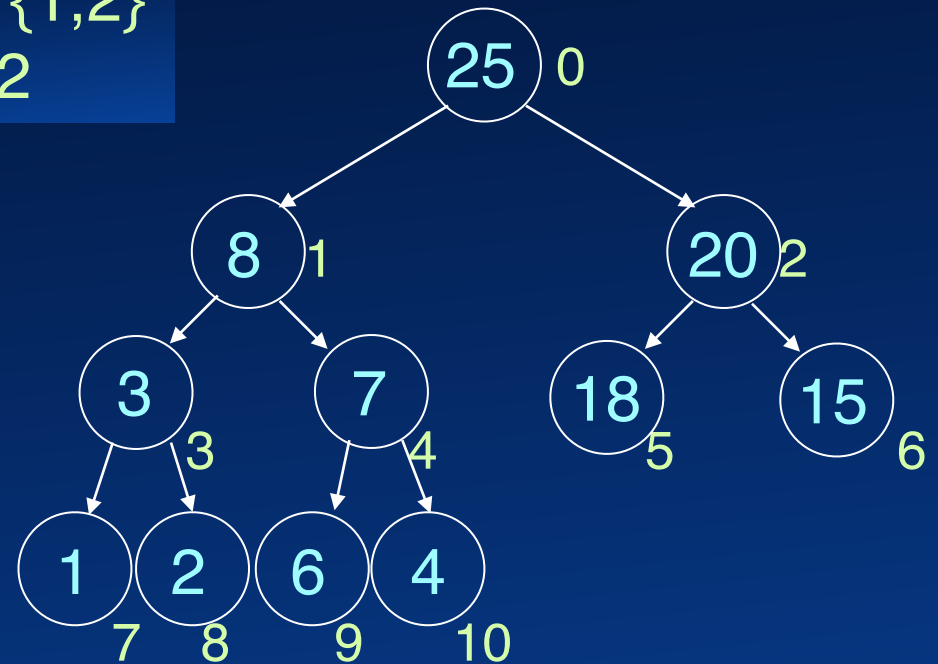


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Heap