Data Structures & Algorithms

Subodh Kumar

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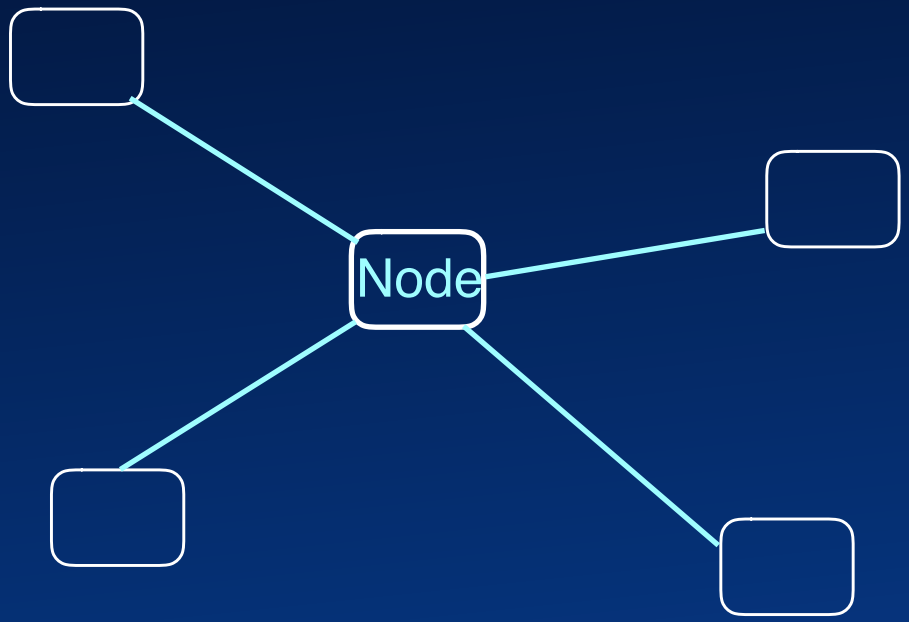
Dept of Computer Sc. & Engg.

Graph

Graph

Graph



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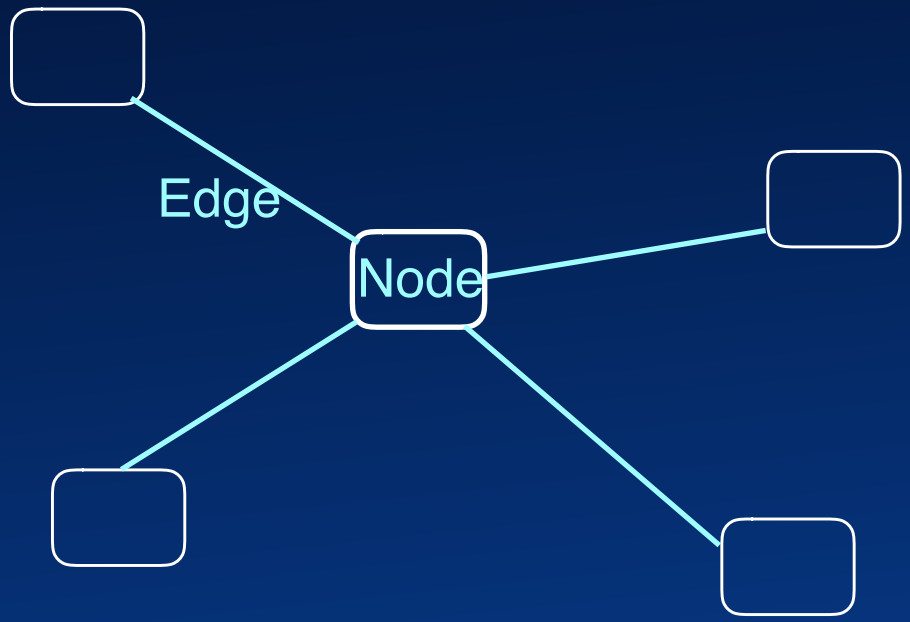

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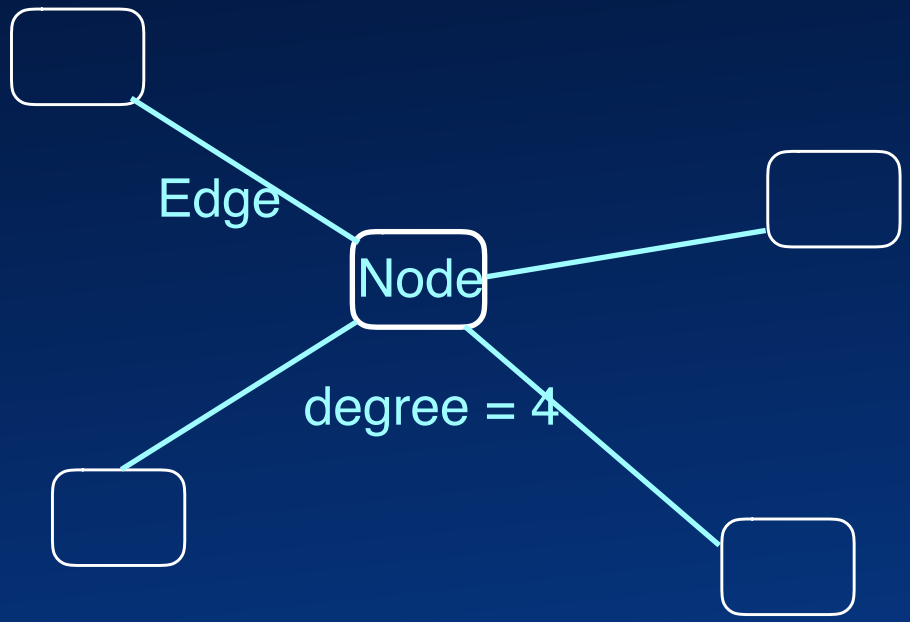
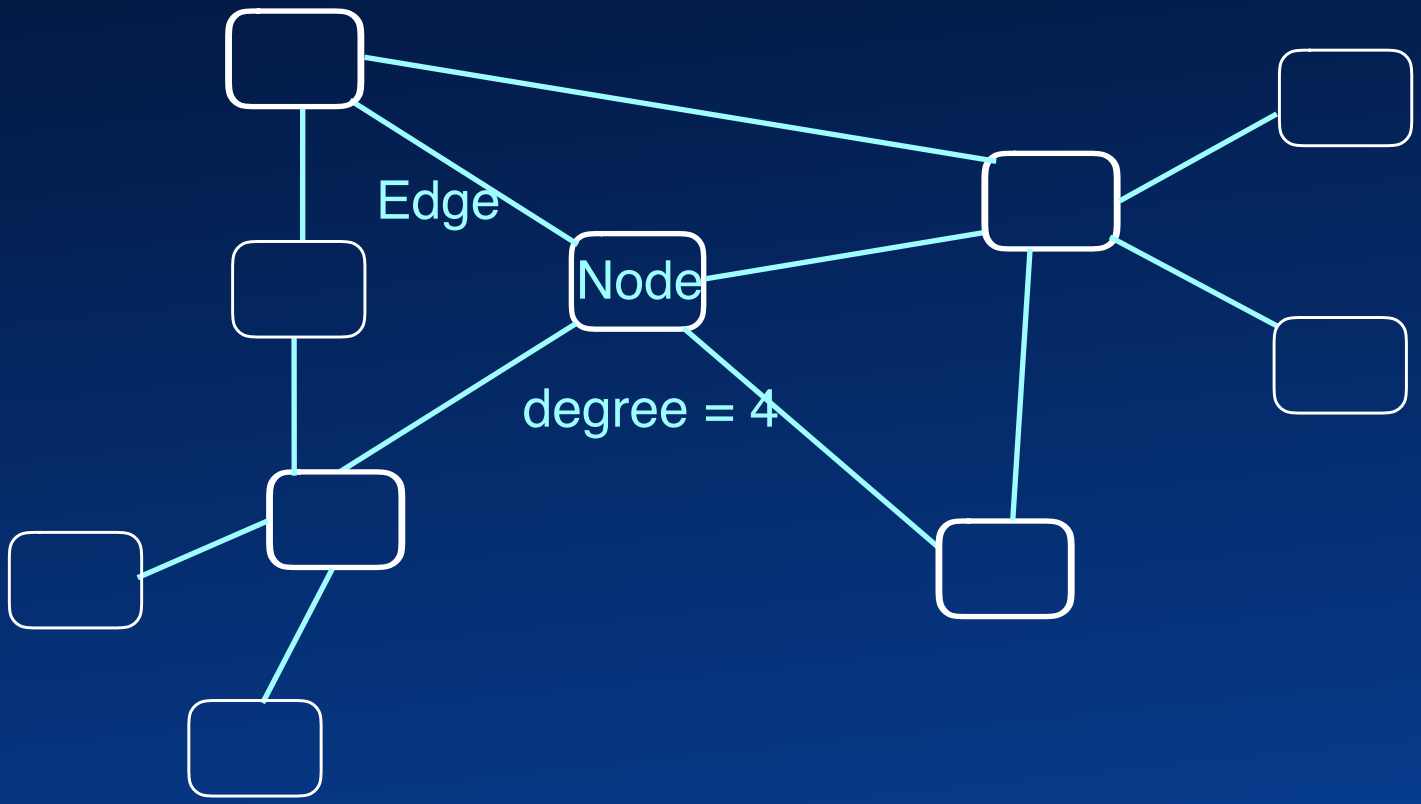
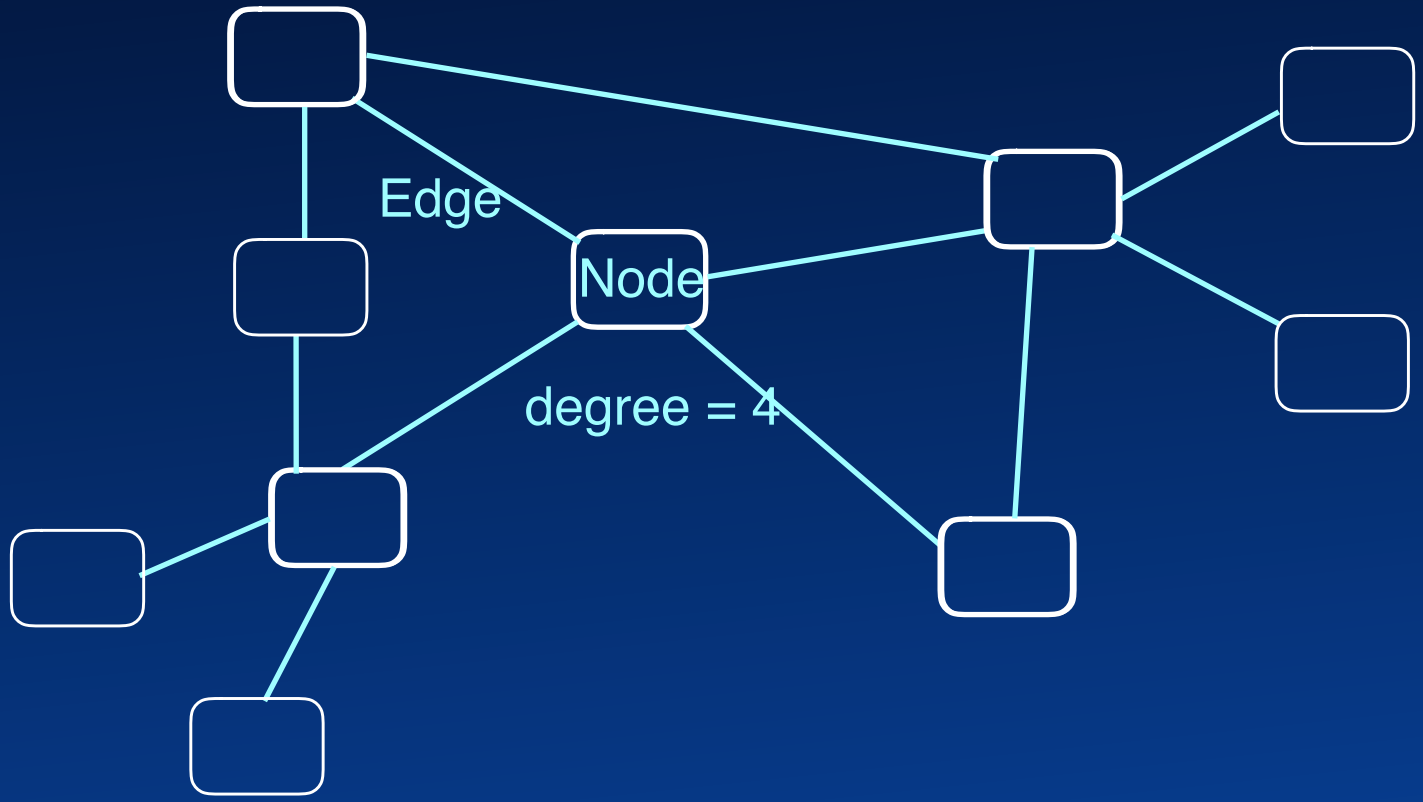


Graph



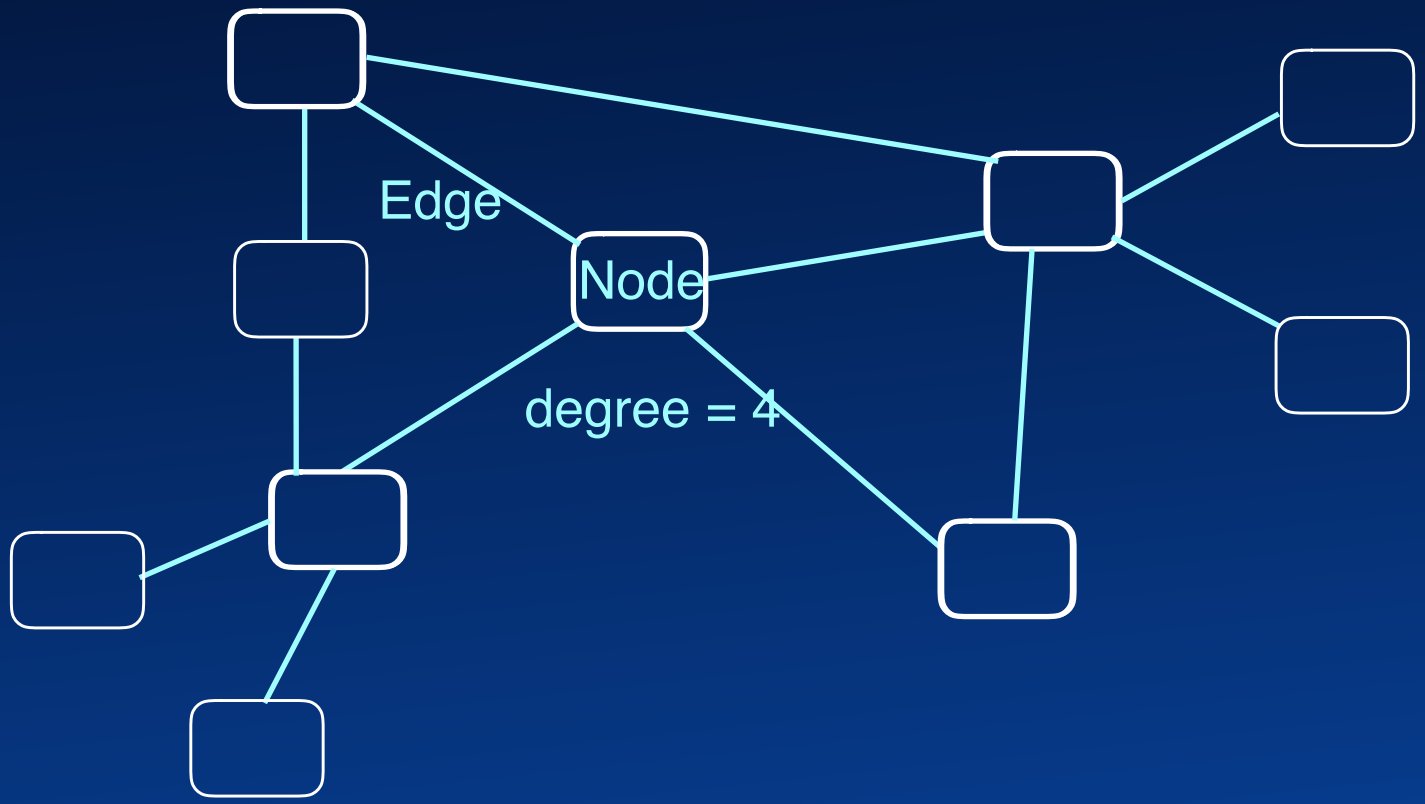
Graph

Undirected
(Symmetric)



Graph

Undirected (Symmetric) = Undirected graph



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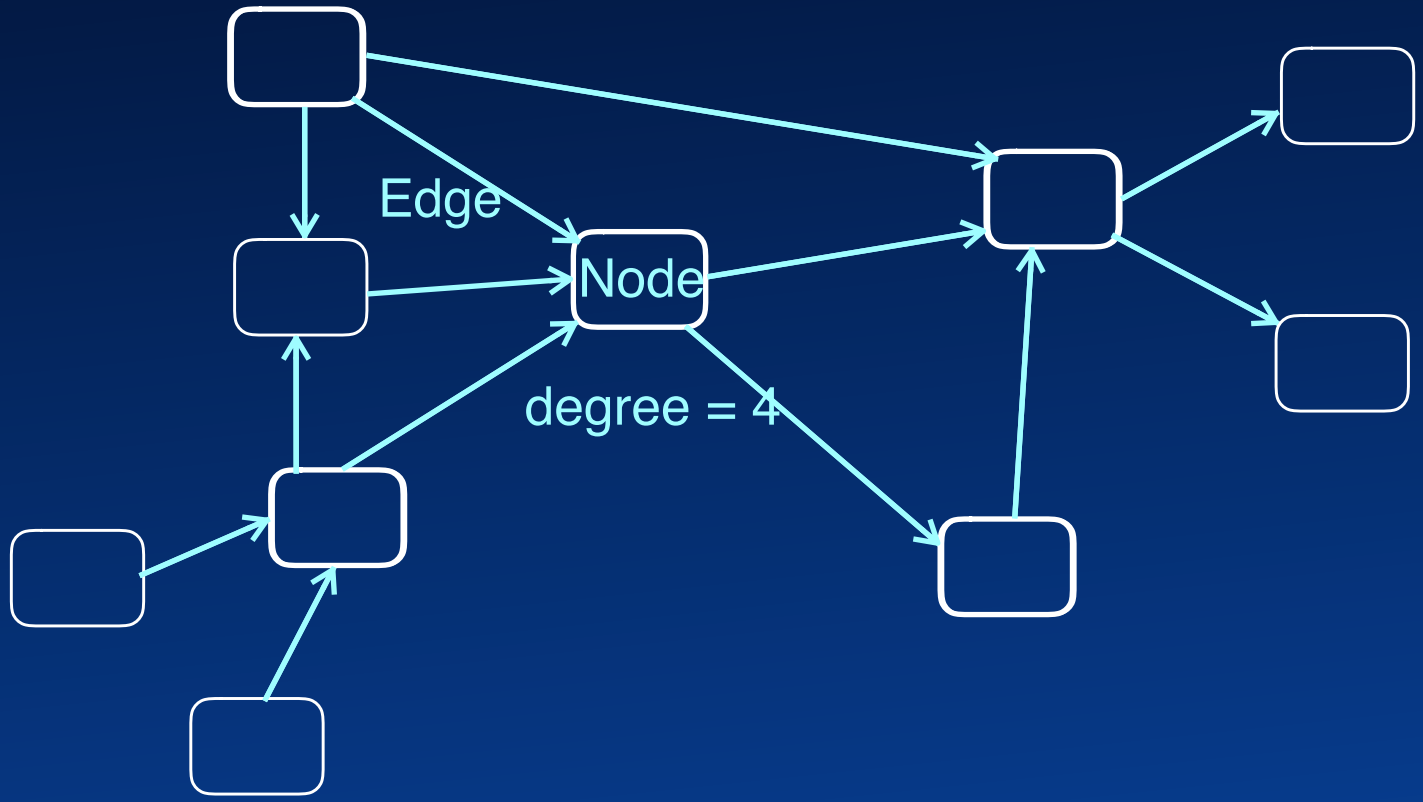

Graph

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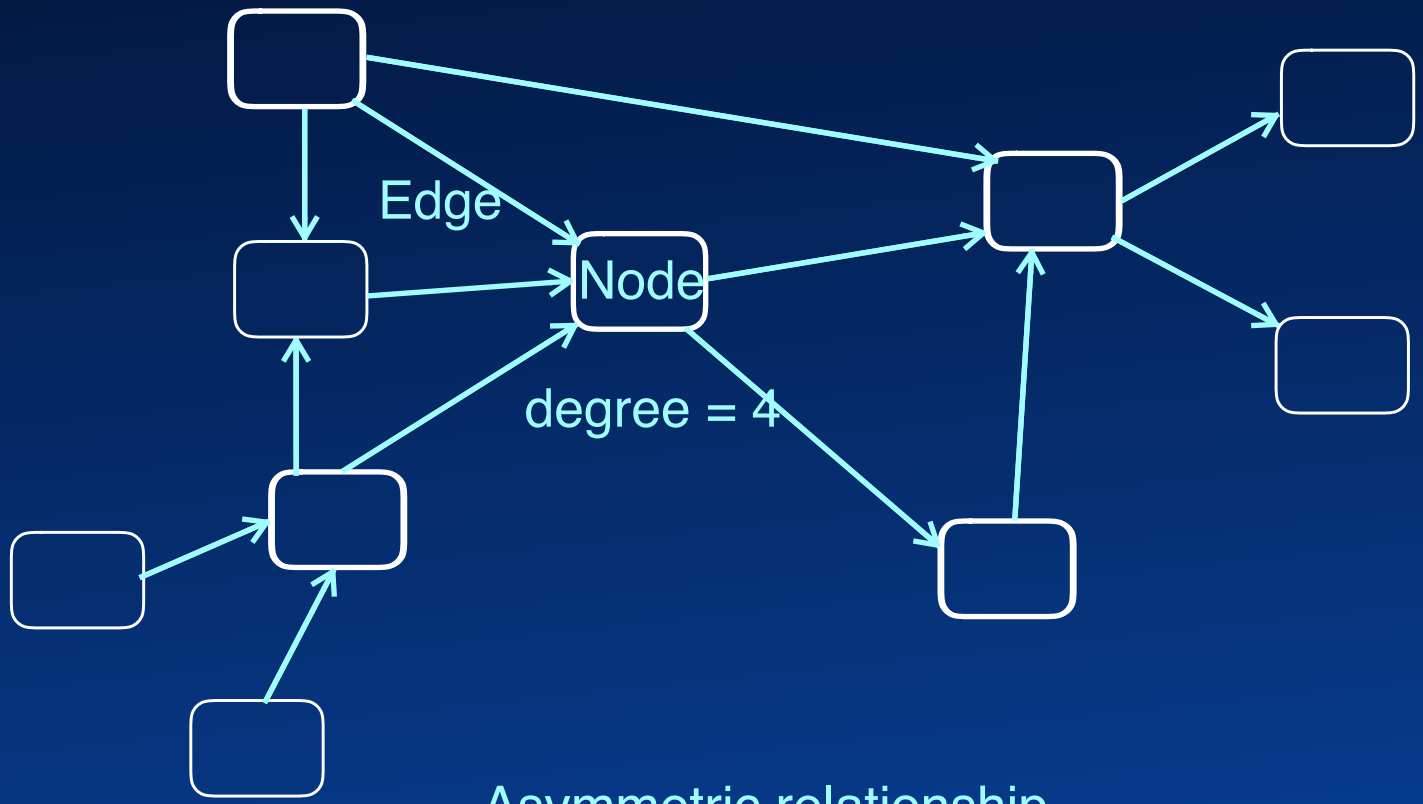
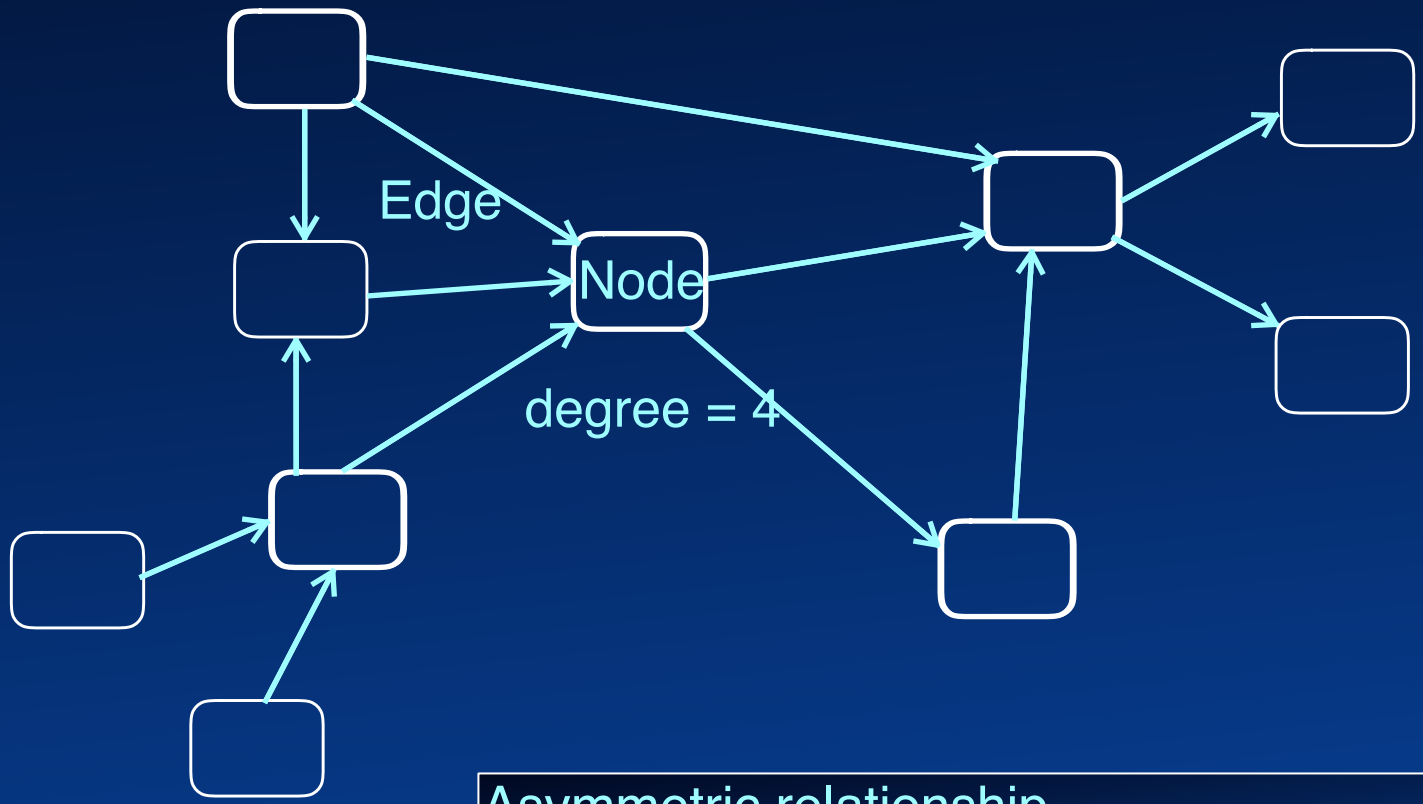


Asymmetric relationship
(Edges are directed)

Graph

Undirected (Symmetric) = Undirected graph



Asymmetric relationship
(Edges are directed) = Directed graph

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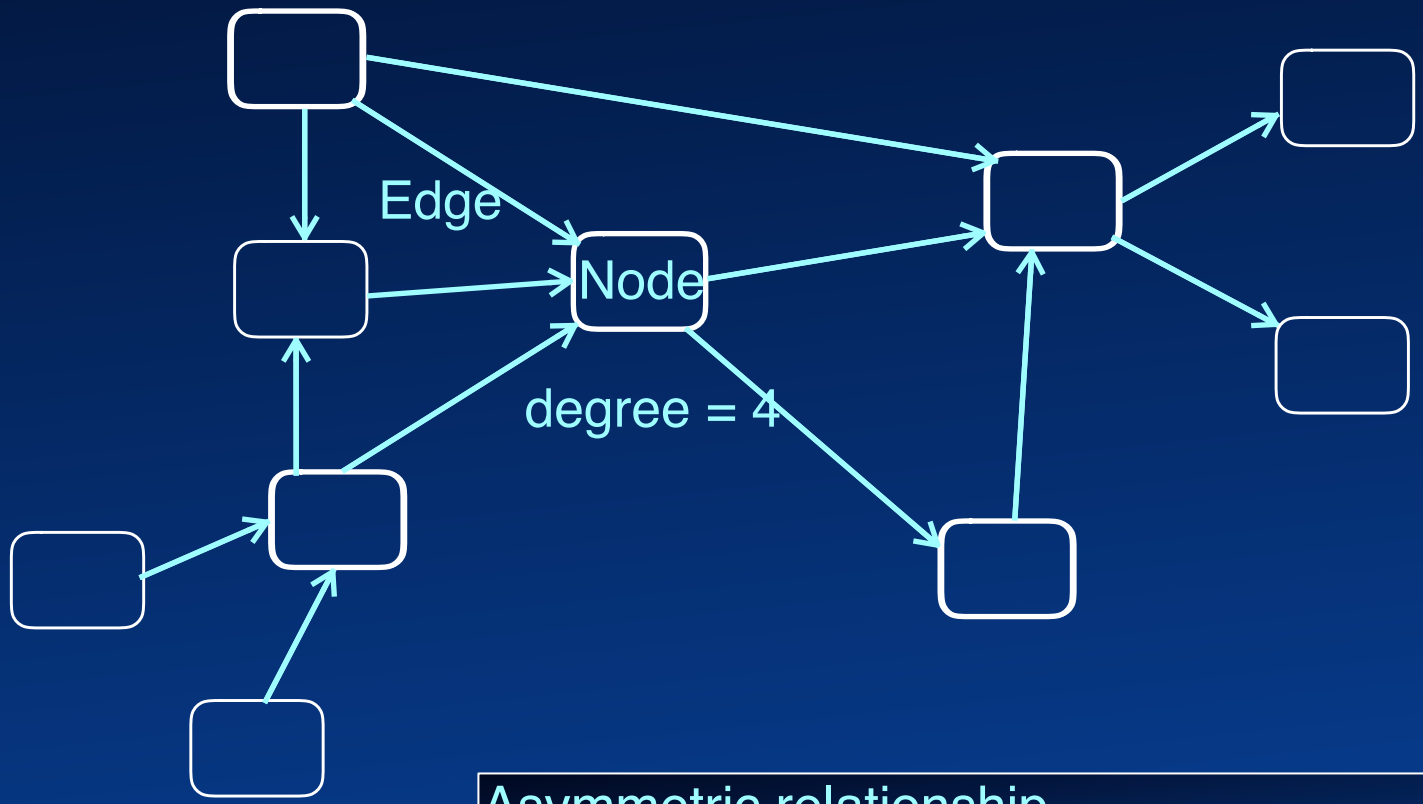

Asymmetric relationship
(Edges are directed) = Directed graph e.g., Tree

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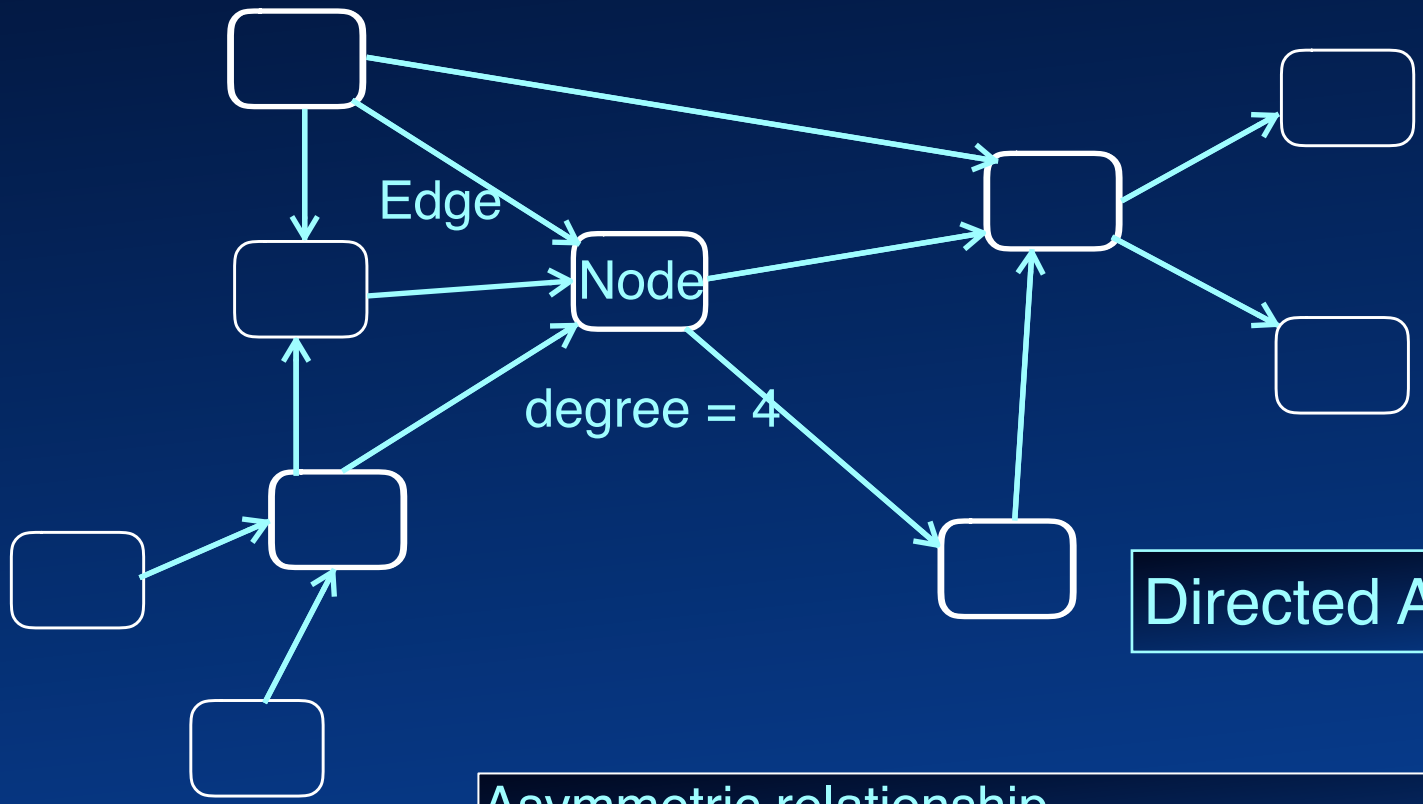

Directed Acyclic Graph

Asymmetric relationship
(Edges are directed) = Directed graph

e.g., Tree

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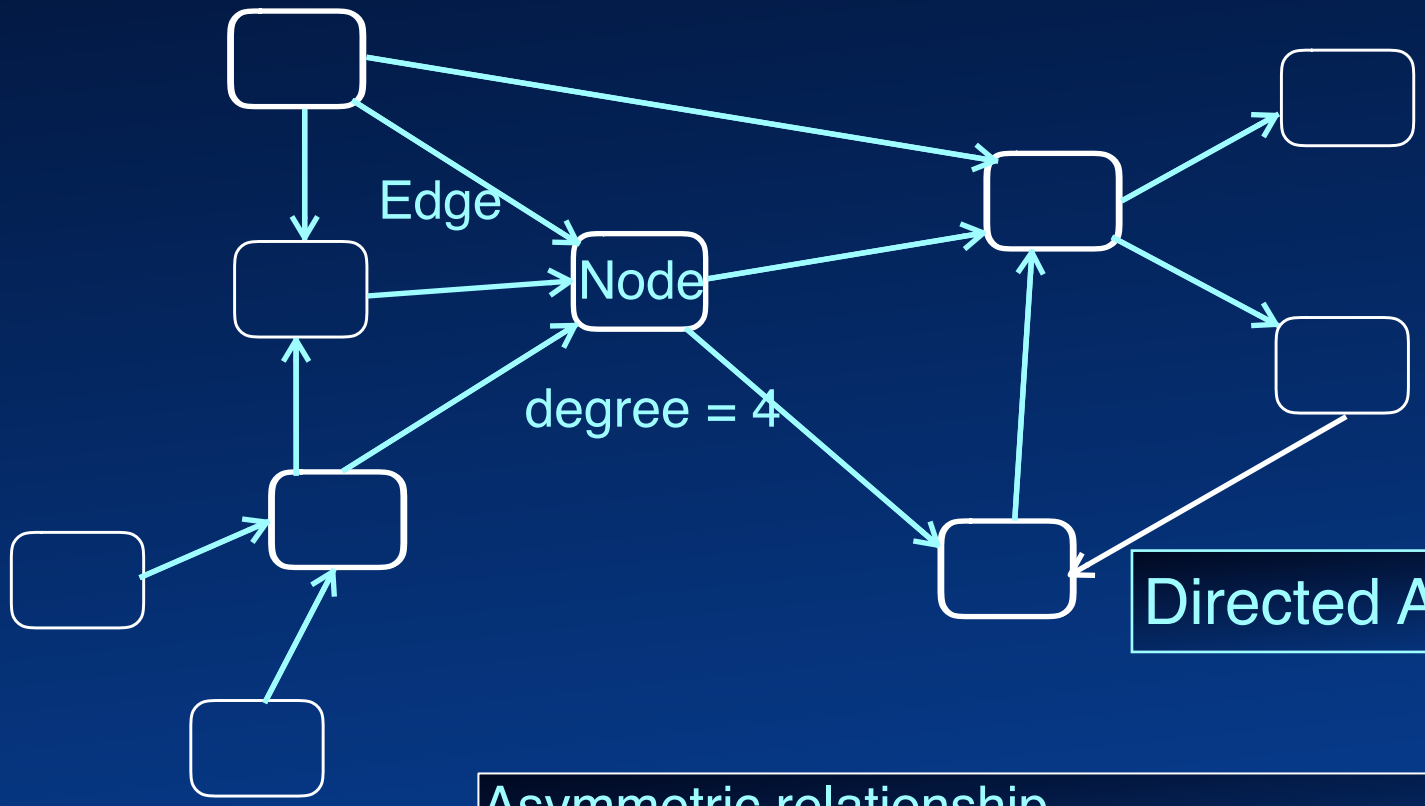

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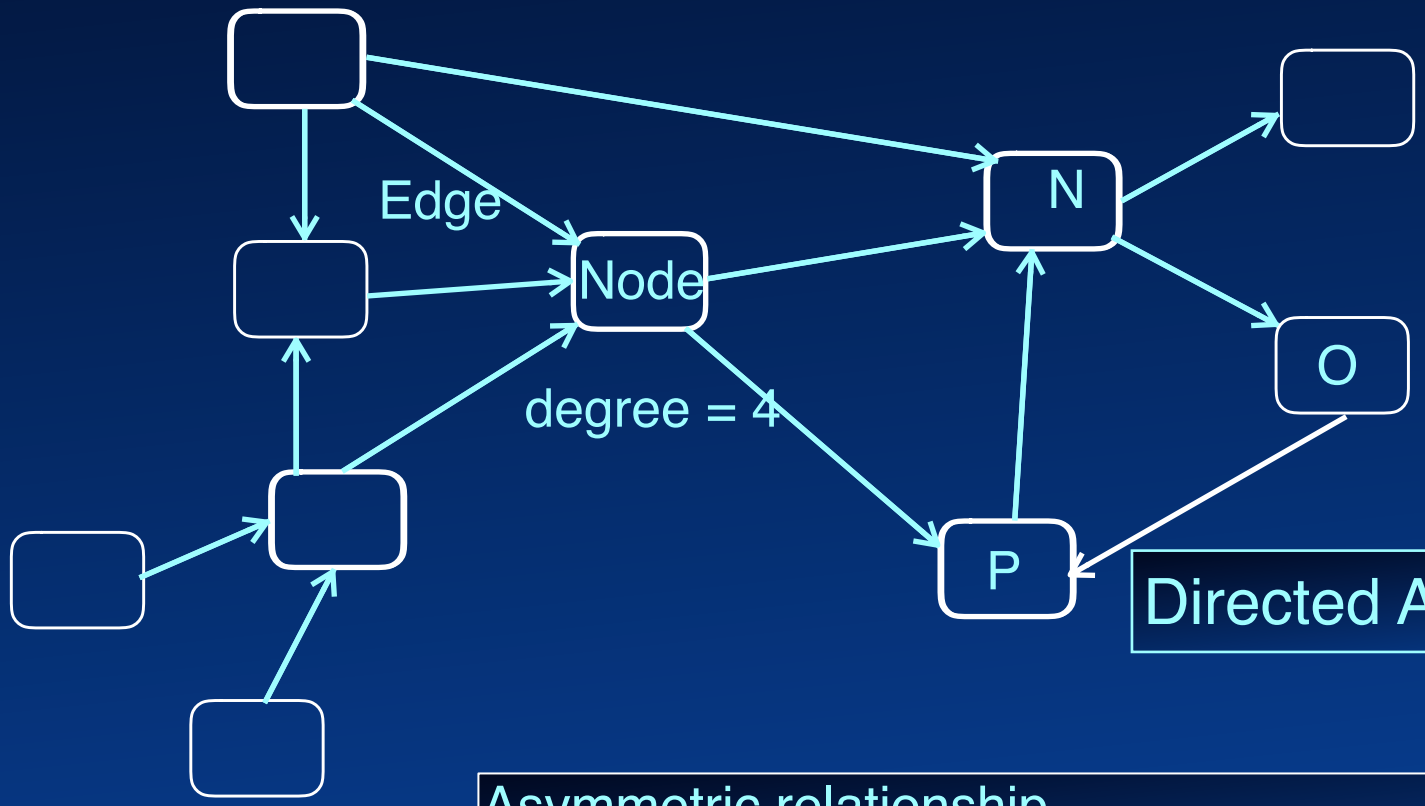

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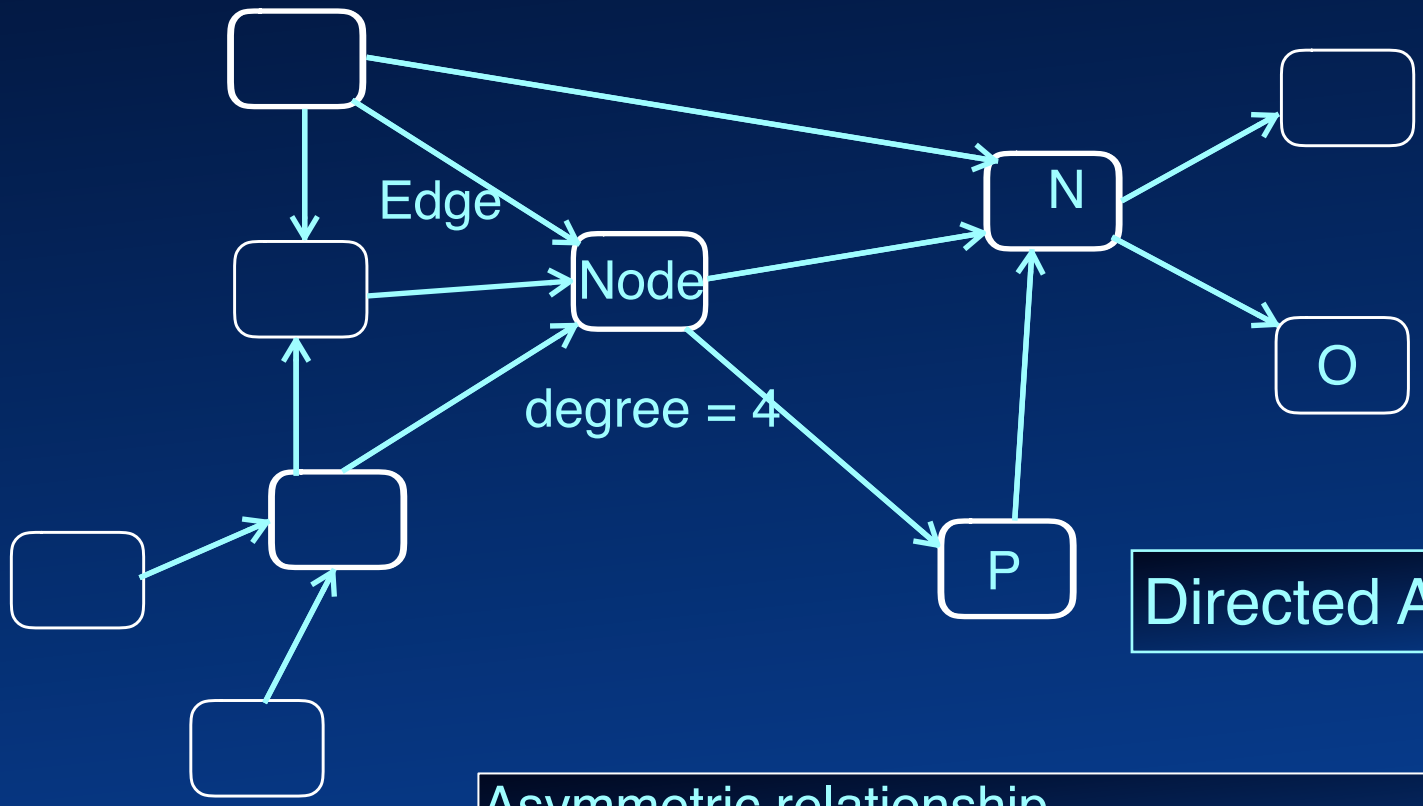

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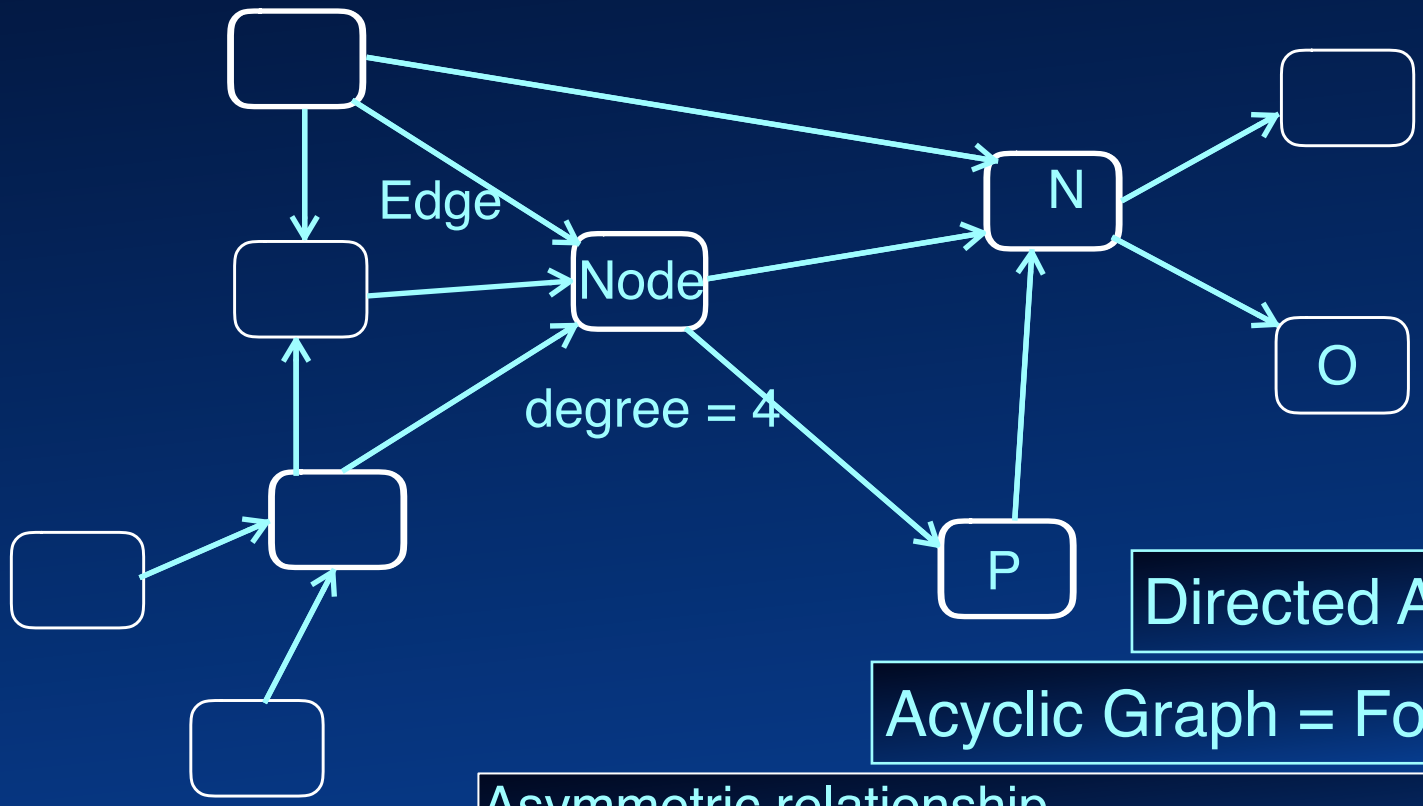

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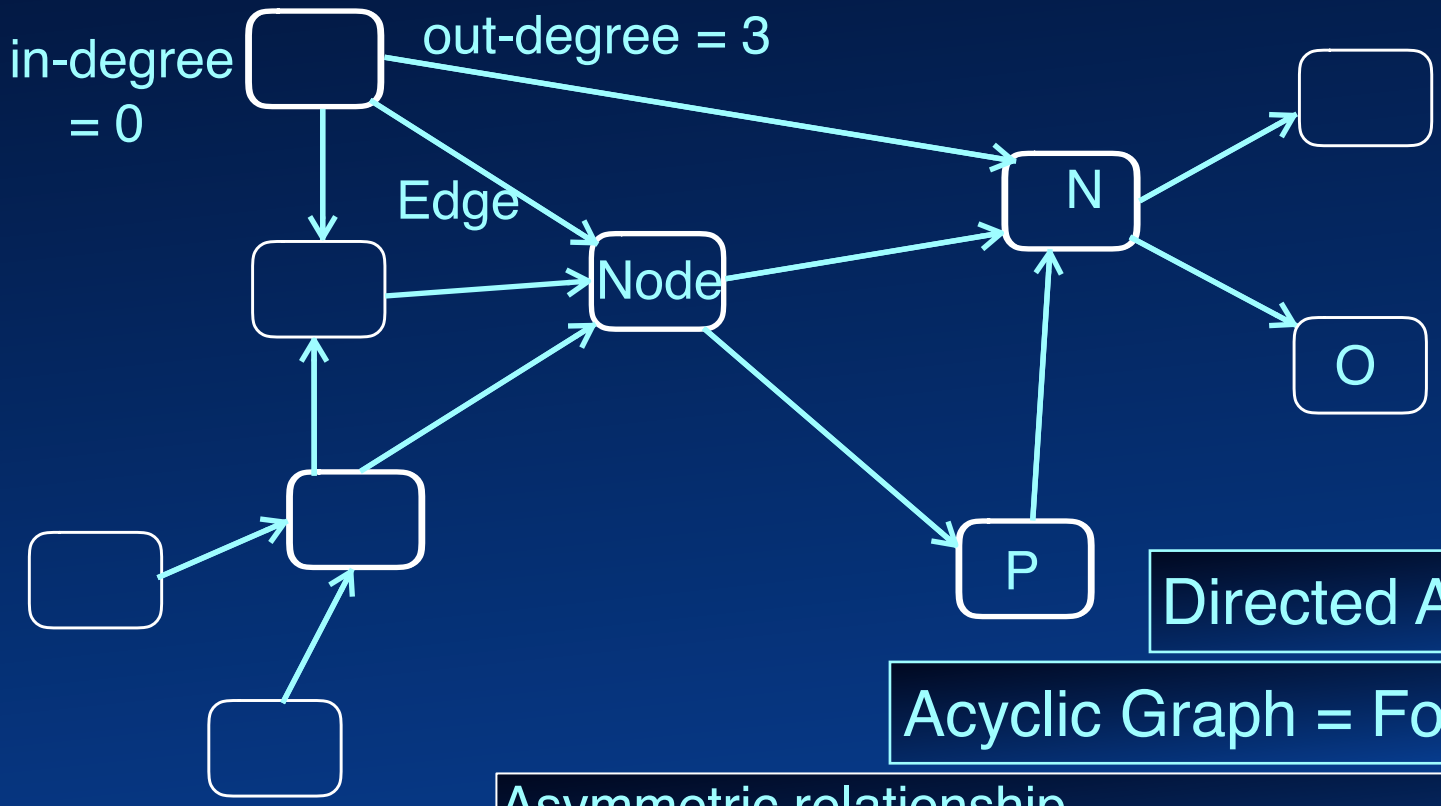

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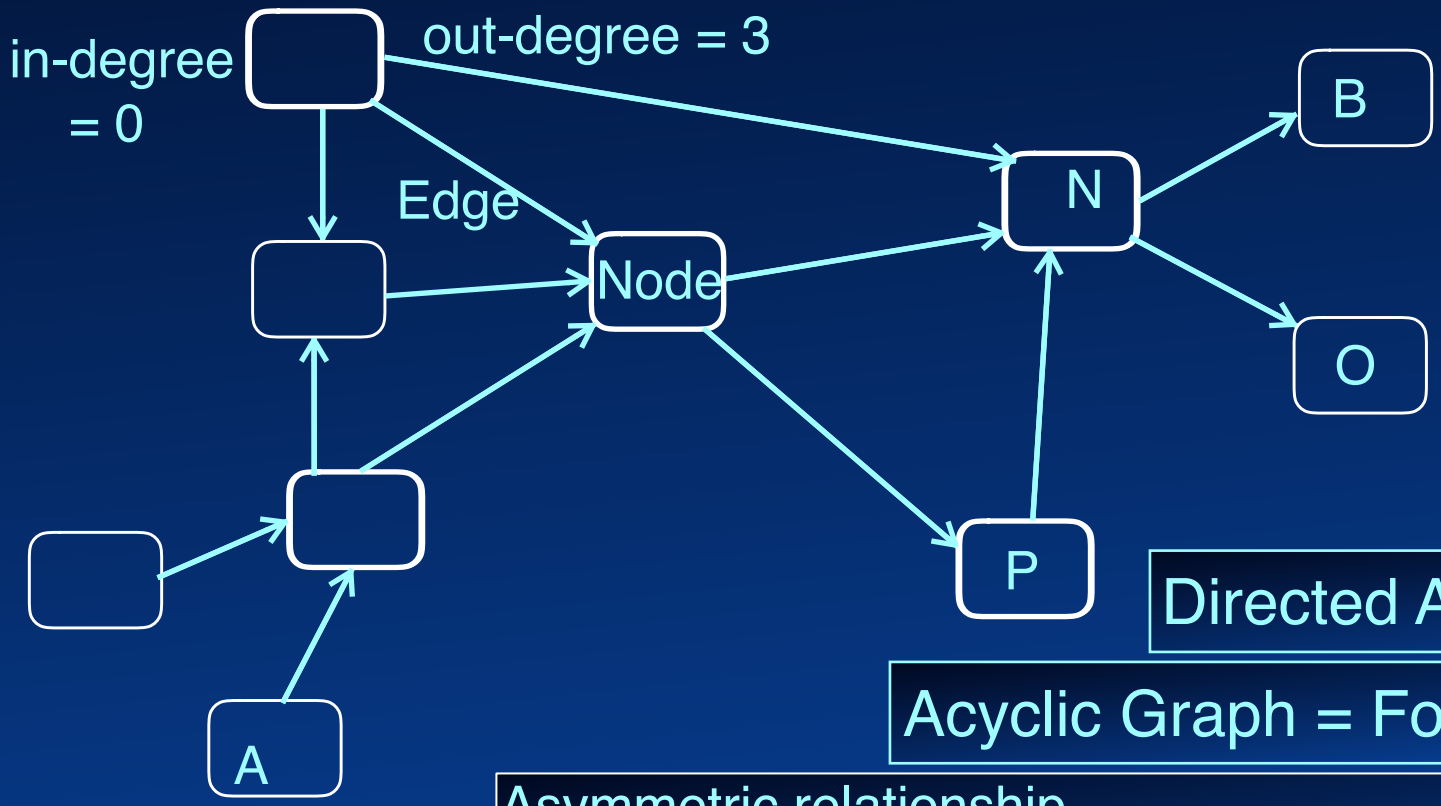

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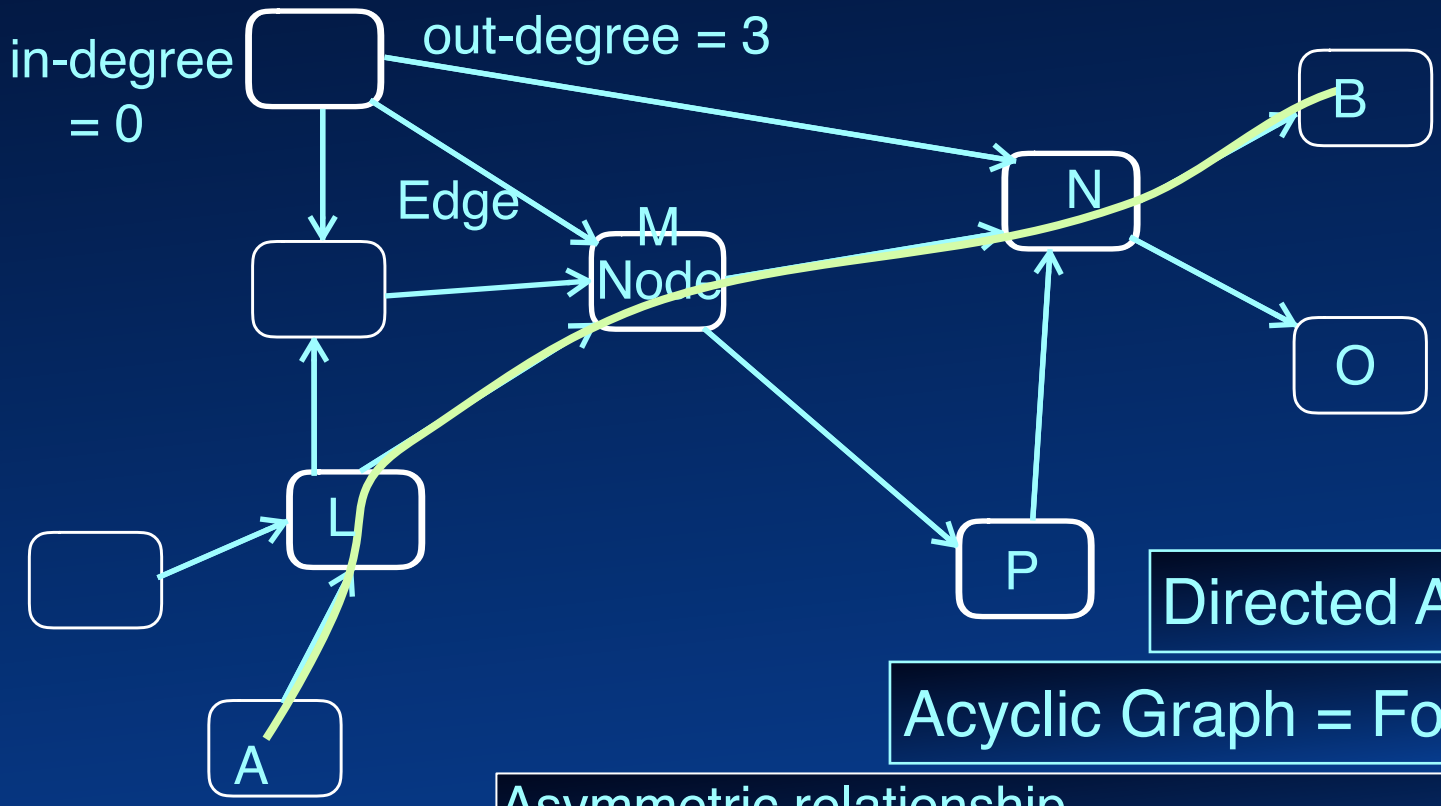

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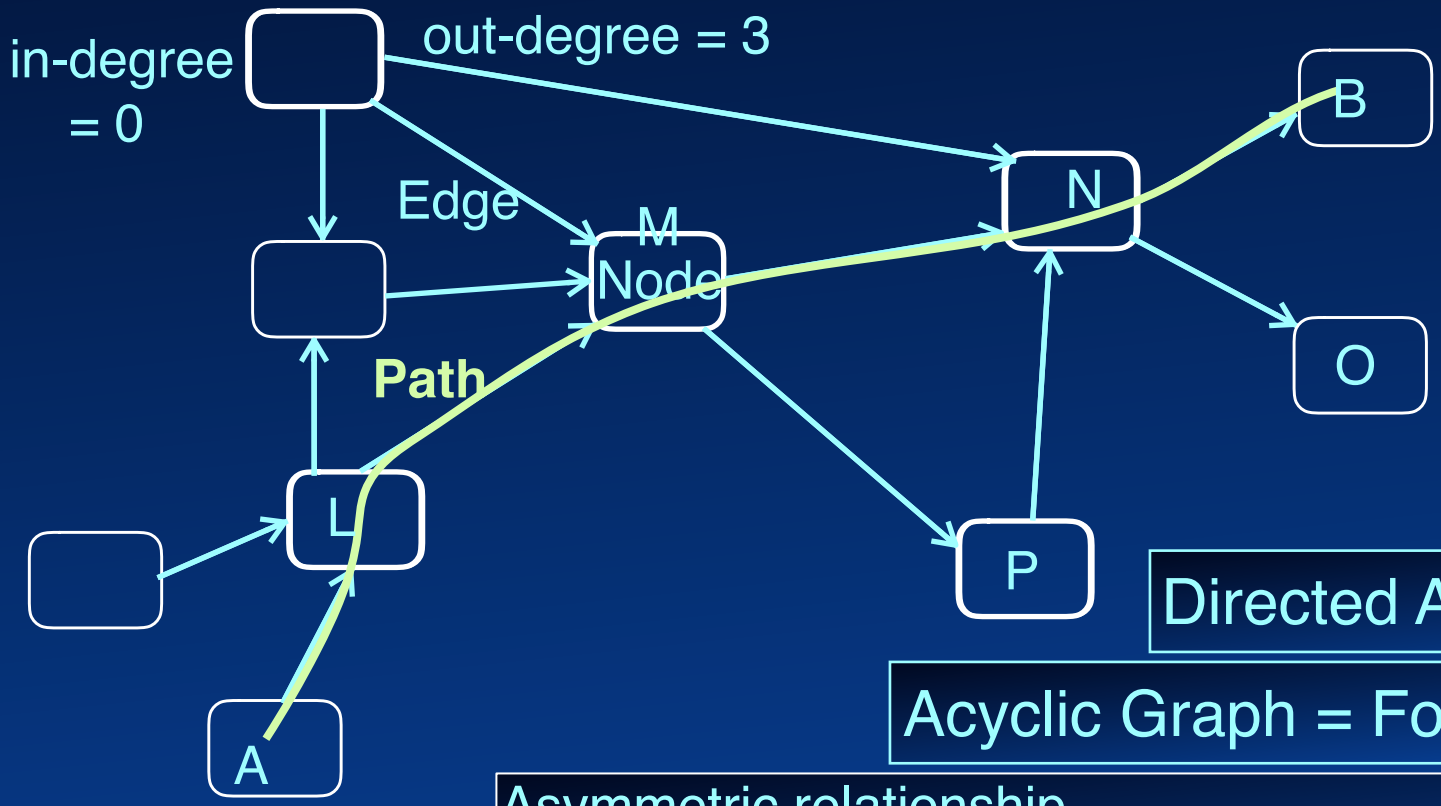

Directed Acyclic Graph

Acyclic Graph = Forest

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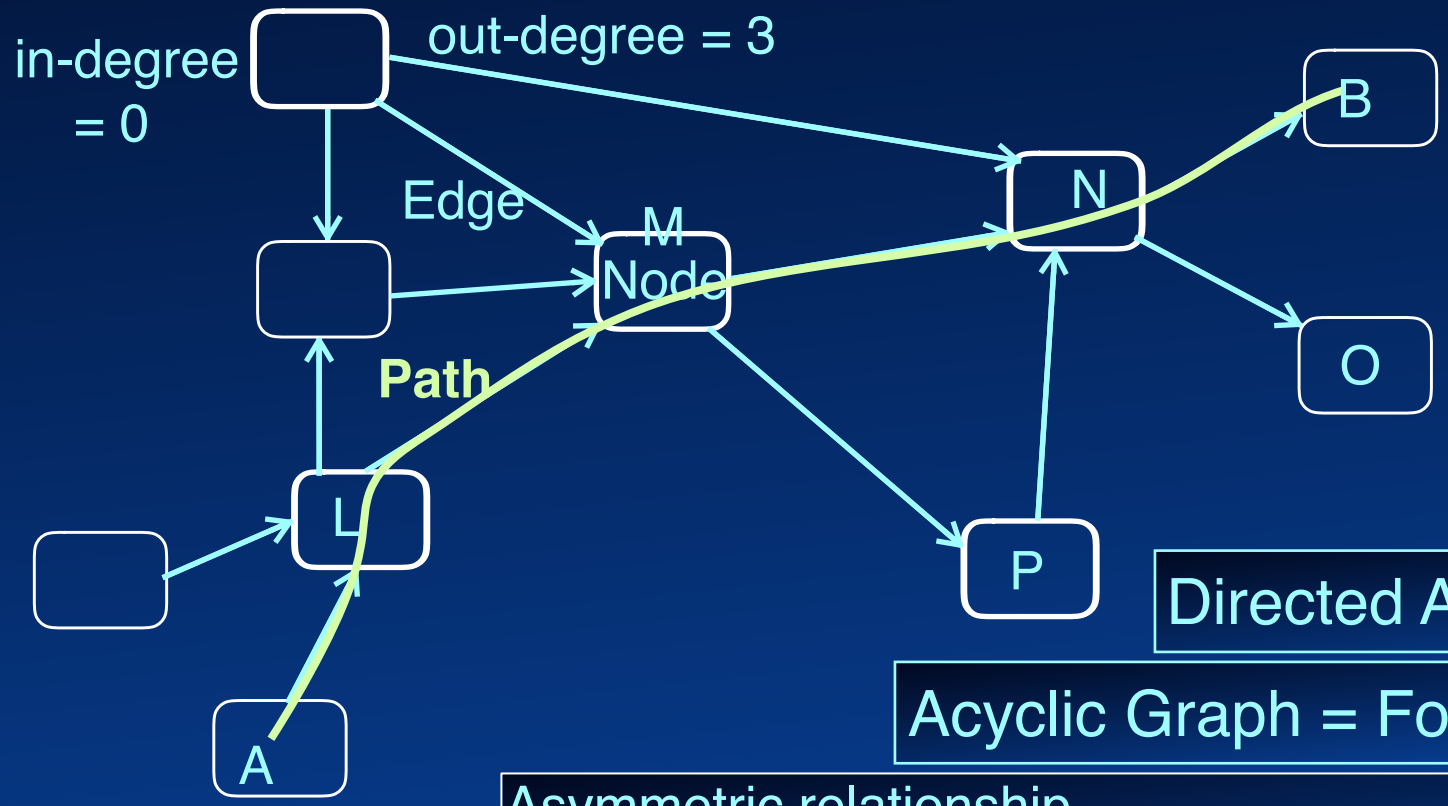
A is connected to B



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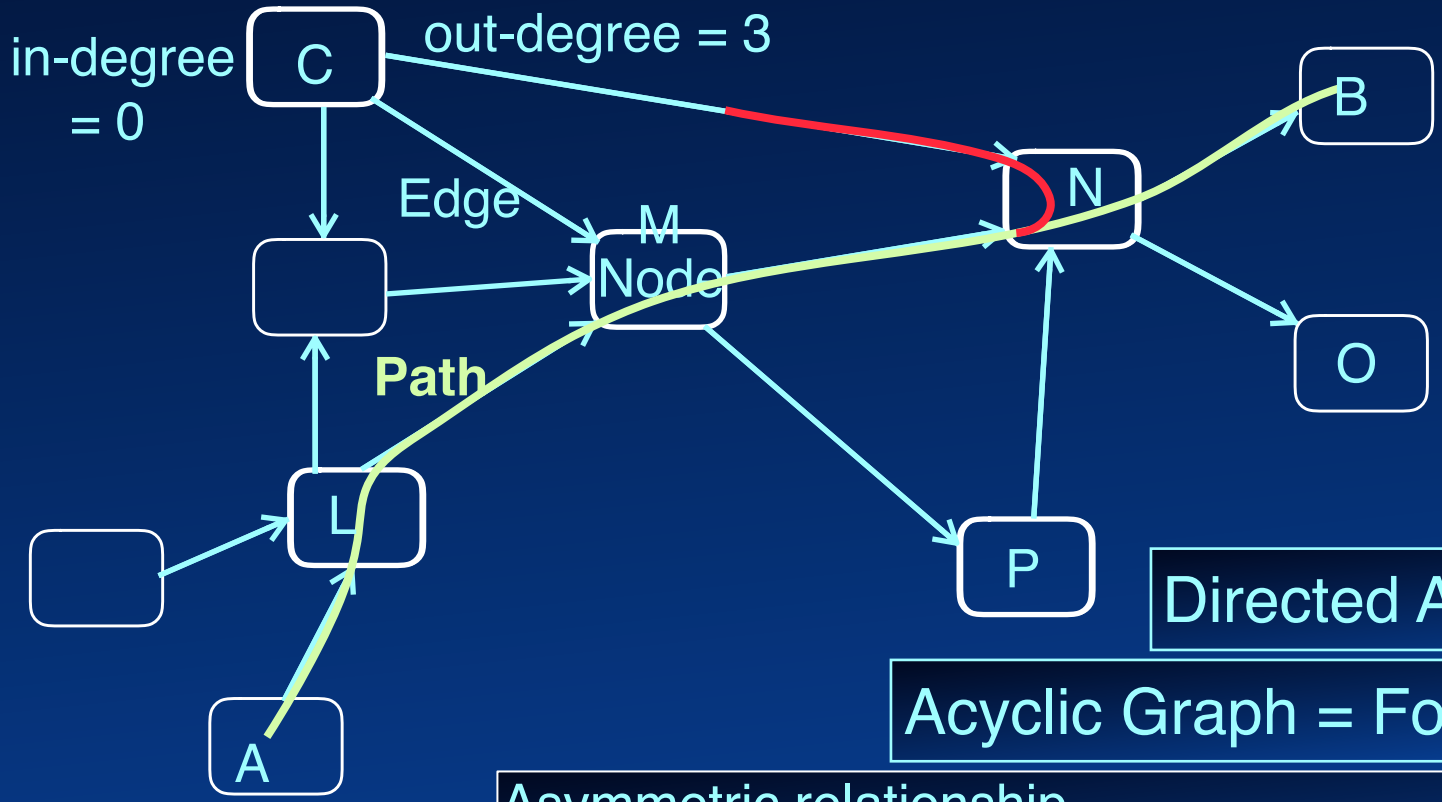

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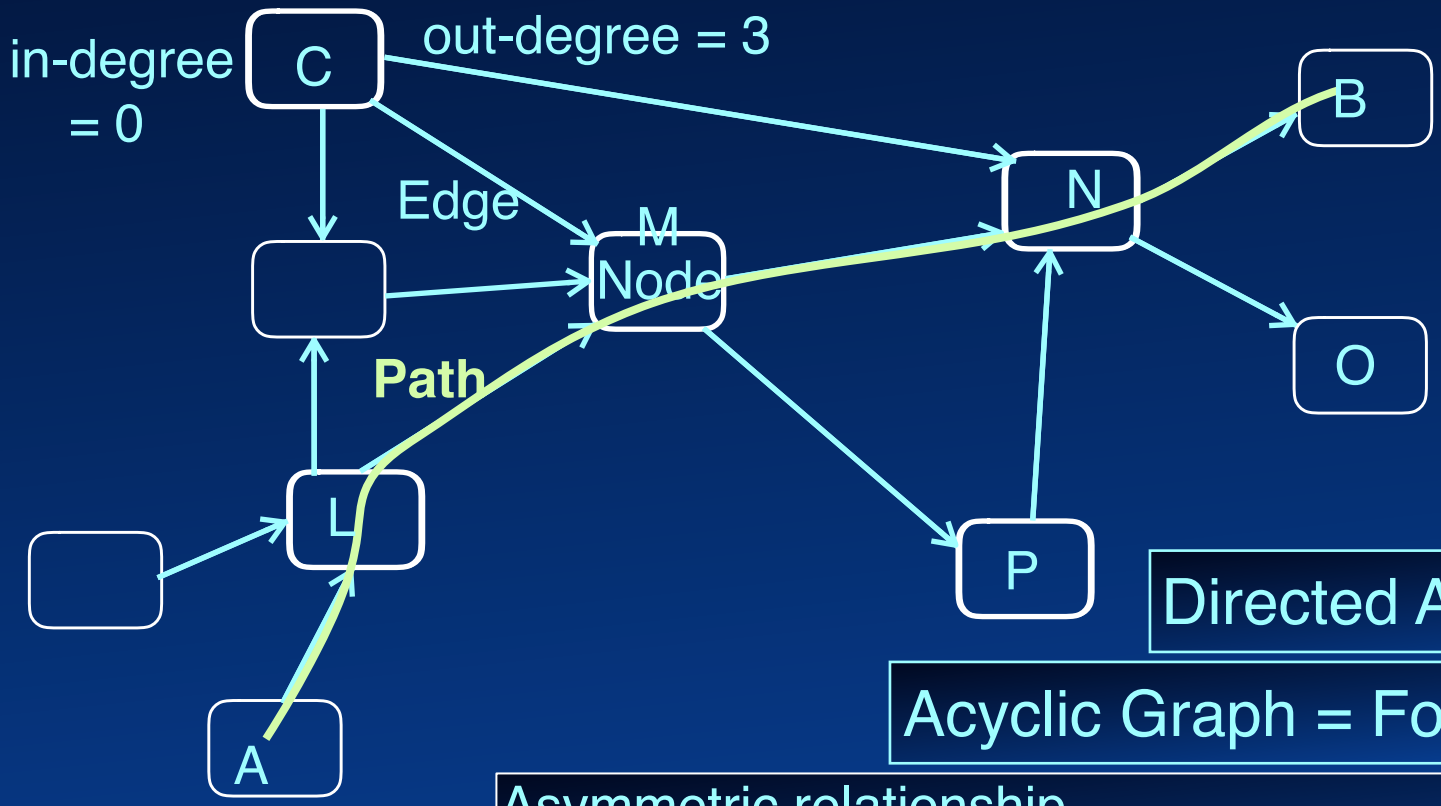

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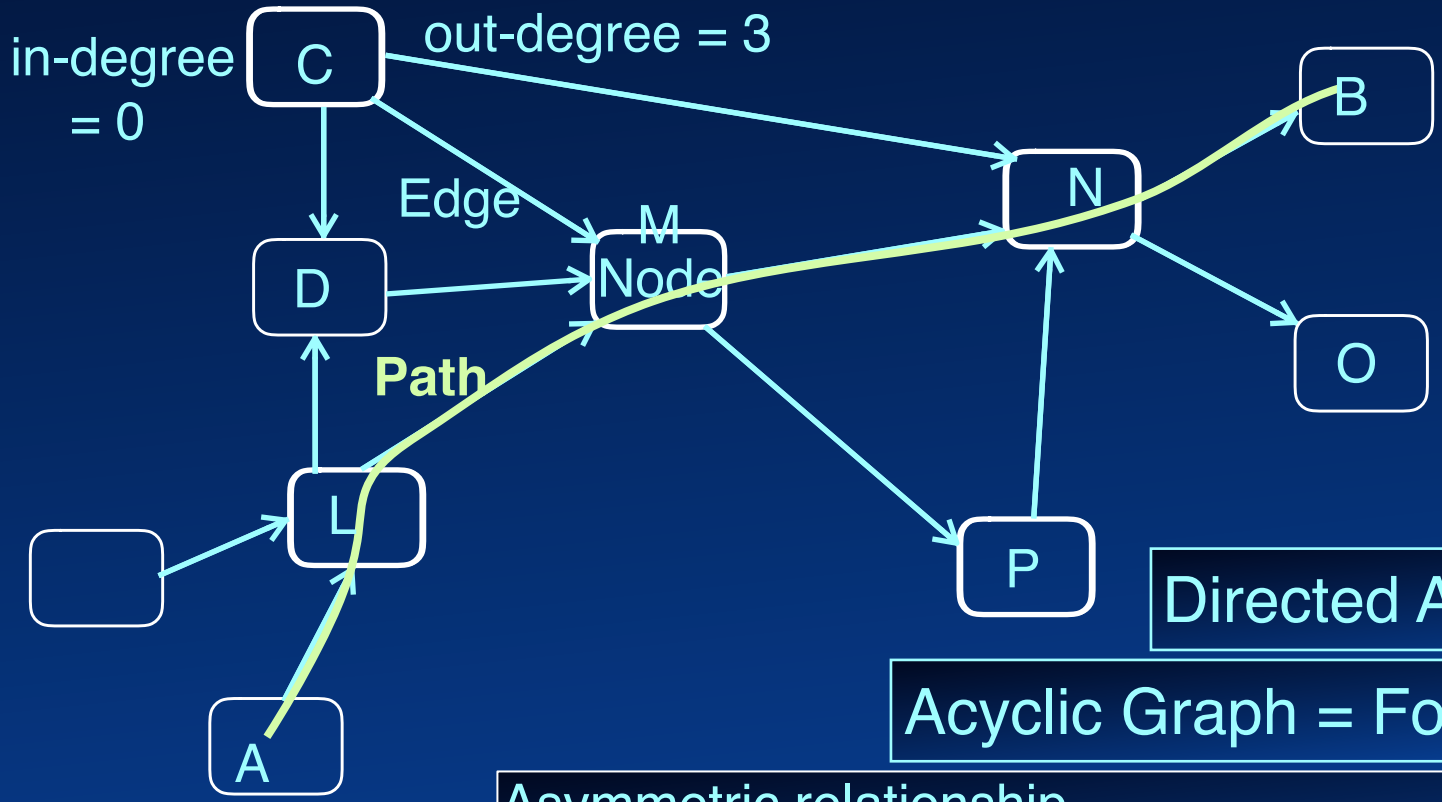

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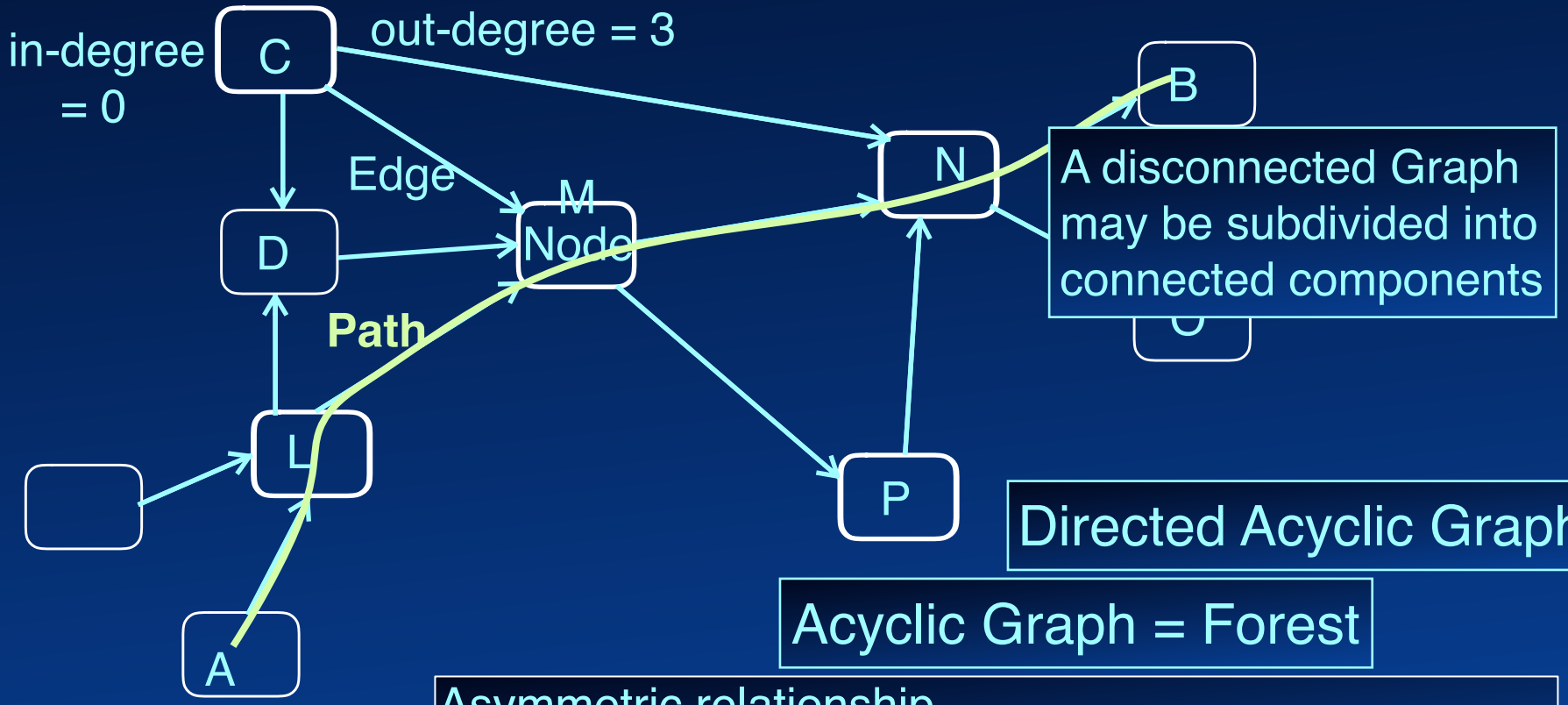

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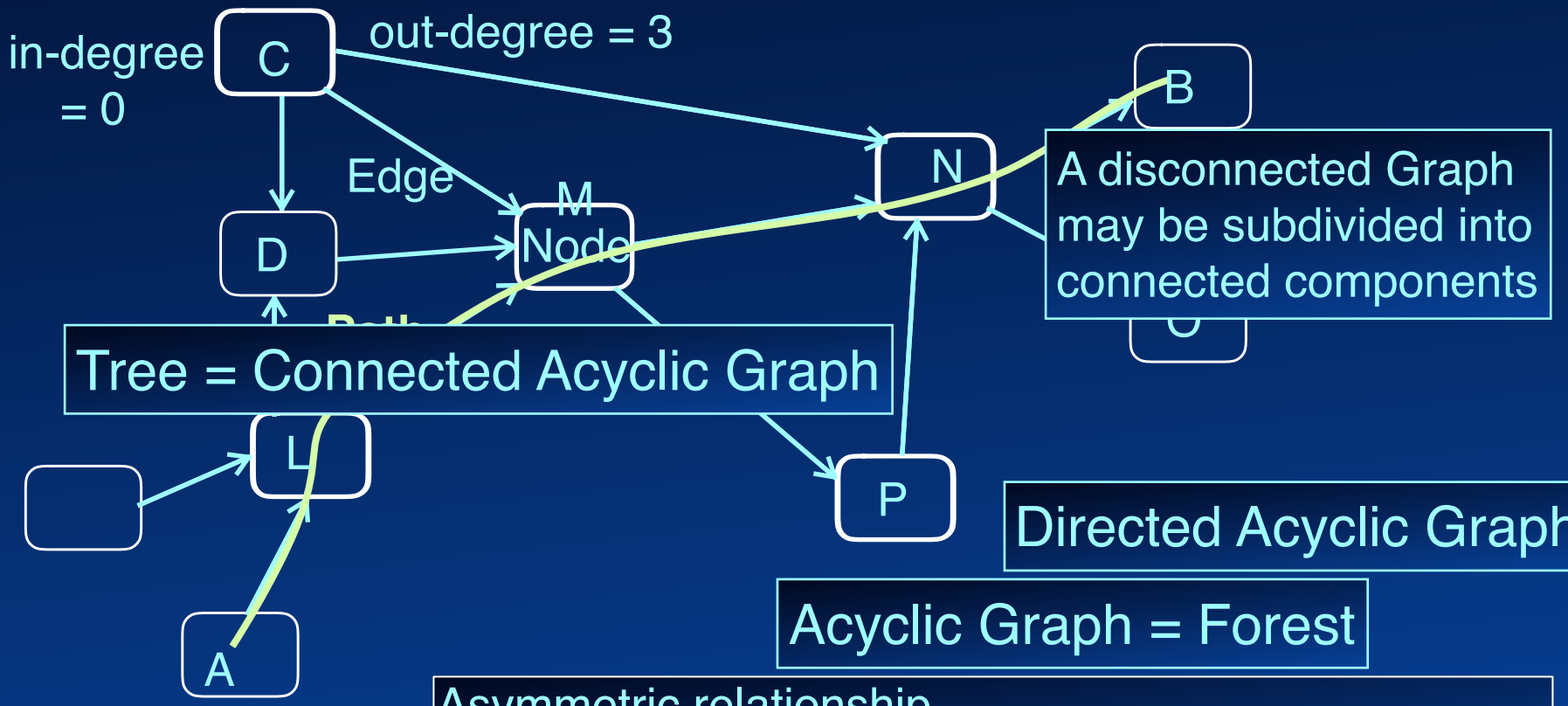
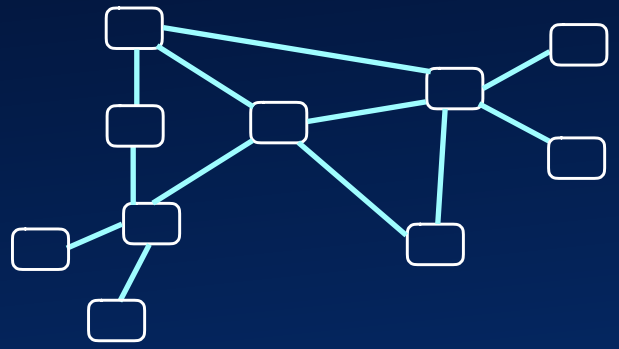
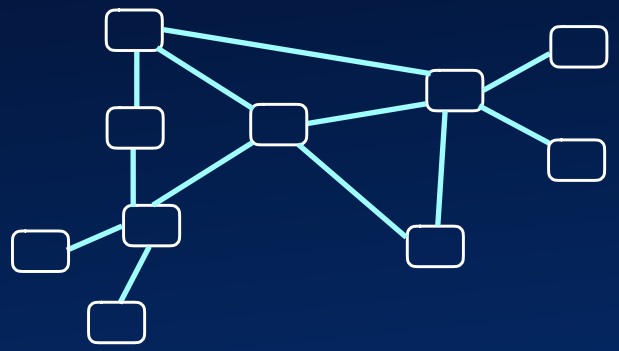

Undirected (Symmetric) = Undirected graph
A is connected to B
in-degree = 0
C
out-degree = 3
B
N
Edge
M
Node
D
A disconnected Graph may be subdivided into connected components
Tree = Connected Acyclic Graph
L
P
Directed Acyclic Graph
Acyclic Graph = Forest
A
Asymmetric relationship
(Edges are directed) = Directed graph
e.g., Tree

Graph Representations



Graph Representations

```
class Node<NV> {
    NV value;
    List<Node> adjacent;
}
```



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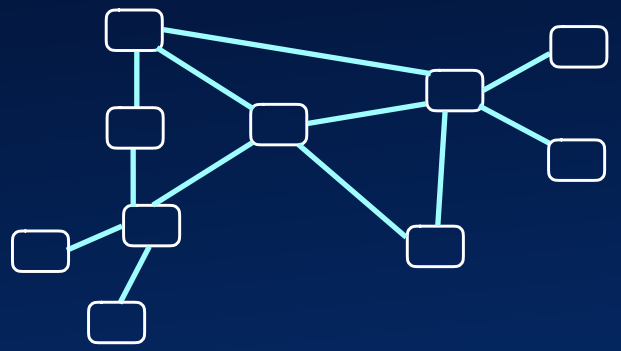

```
class NodE<NV,EV> {
    NV value;
    List<Pair<NodE,EV>> adjacent;
}
```

Graph Representations

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class Node<NV> {
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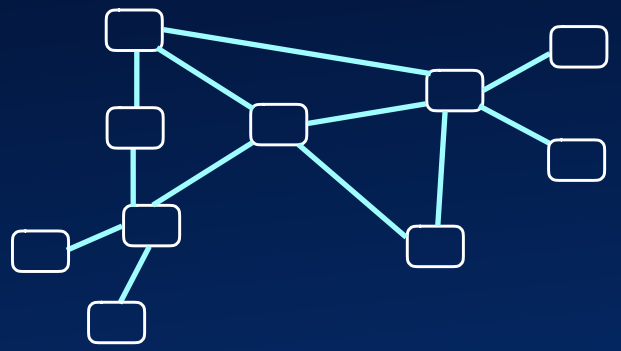

```
class NodE<NV,EV> {
    NV value;
    List<Pair<NodE,EV>> adjacent;
}
```

```
class Edge<N,EV> {
    EV value;
    N source;
    N dest;
}
```

Graph Representations

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class Node<NV> {
    NV value;
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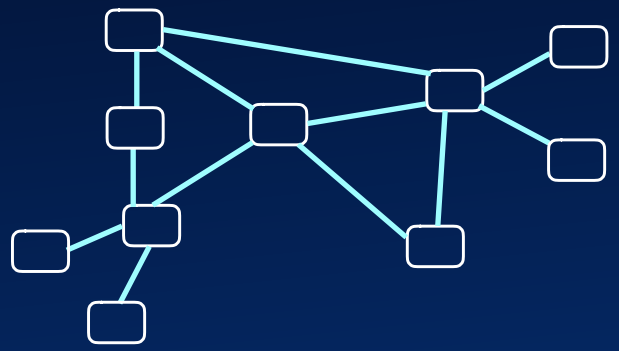
}
```



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}
```

```
class Edge<N,EV> {
    EV value;
    N source;
    N
}
```

```
class Edge<NV,EV> {
    EV value;
    Node<NV> source;
    Node<NV> dest;
}
```

Graph Representations

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class Node<NV> {
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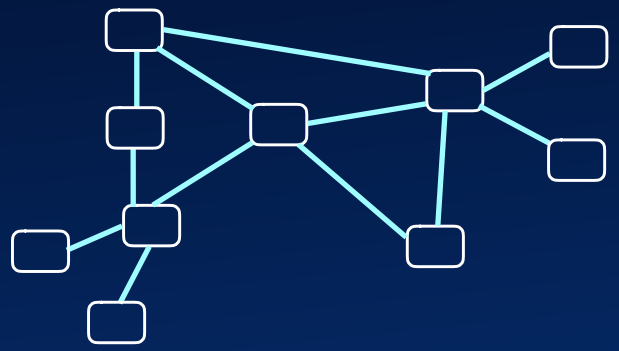

```
class NodE<NV,EV> {
    NV value;
    List<Pair<NodE,EV>> adjacent;
}
```

```
class Graph<NV,EV> {
    List<Node<NV>> nodes;
    List<Edge<NV,EV>> edges;
}
```

```
class Edge<N,EV> {
    EV value;
    N source;
    N
}
```

```
class Edge<NV,EV> {
    EV value;
    Node<NV> source;
    Node<NV> dest;
}
```

Graph Representations

```
class Node<NV> {
    NV value;
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```
class NodE<NV,EV> {
    NV value;
    List<Pair<NodE,EV>> adja
}
```

```
class Graph<NV,EV> {
    List<Node<NV>> nodes;
    List<Edge<NV,EV>> edges;
}
```

```
class Graph<NV,EV> {

    class Edge<EV> {
        int source, dest;
        EV value;
    }

    ArrayList<NV> nodes;
    List<Edge<EV>> edges;
}
```

```
EV> {
    EV value;
    Node<NV> source;
    Node<NV> dest;
}
```

Graph Representations

```
class Node<NV> {
    NV value;
    List<Node> adjacent;
}
```

```
class Graph<NV,EV> {

    class Edge<EV> {
        int source, dest;
        EV value;
    }

    ArrayList<NV> nodes;
    List<Edge<EV>> edges;
```

```
class NodE<NV,EV> {
    NV value;
    List<Pair<NodE,EV>> adja
}
```

```
class Graph<NV,EV> {
    List<Node<NV>> nodes;
    Map<Node<NV>, EV> edges;
}
```

```
class Grapl
    List<Noc
    List<Edg
}
```

```
EV> {
EV value;
Node<NV> source;
Node<NV> dest;
```

Graph Representations

```
class Node<NV> {
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class Graph<NV,EV> {

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    List<Edg
}
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    Map<Node<NV>, EV> edges;
}
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```
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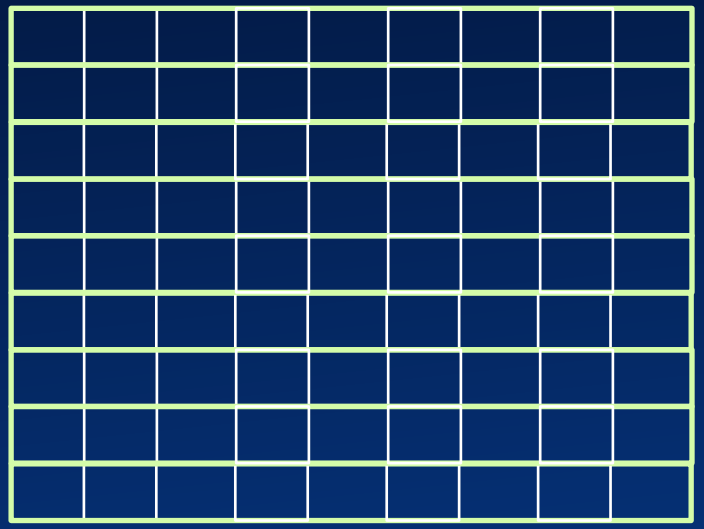
    Node<NV> dest;
}
```

Adjacency Matrix



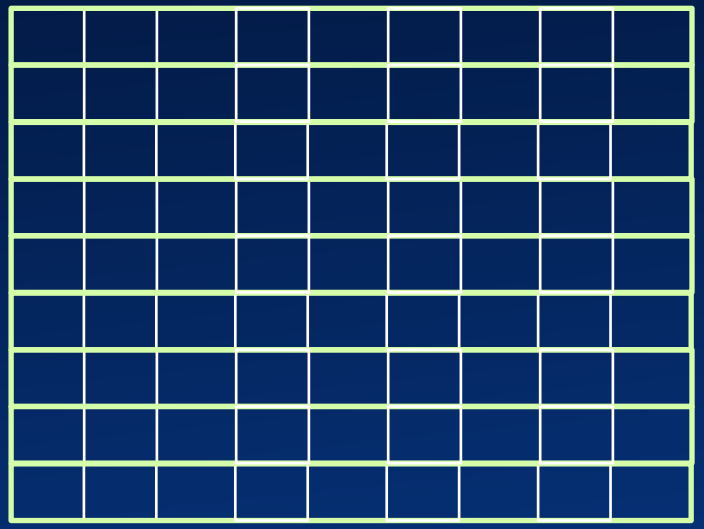
edges[i][j] contains special value
if node i and node j are not adjacent

Adjacency Matrix

```
class Graph<NV,EV> {

    class Edge<EV> {
        int source, dest;
        EV value;
    }

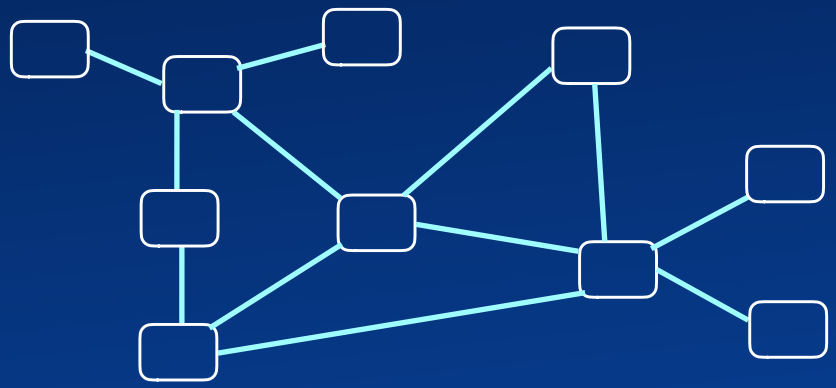
    ArrayList<NV> nodes;
    Array2D<EV> edges;
}
```



edges[i][j] contains special value
if node i and node j are not adjacent

Graph Size

- #Edges: m
- #vertices: n
- $m = O(n^2)$



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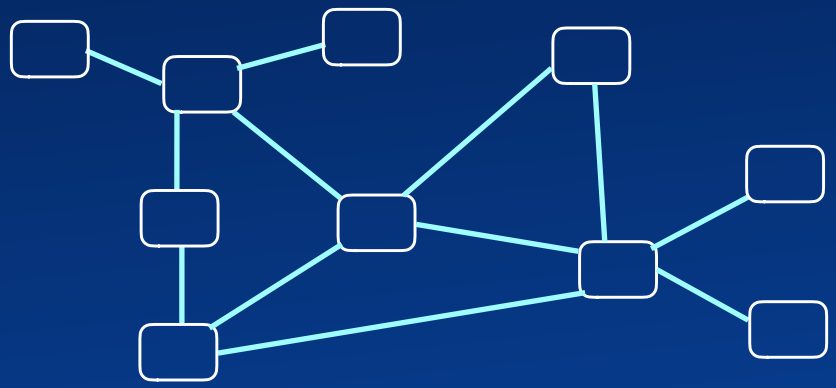
- Undirected graph

$$\sum_{v \in G} \text{degree}(v) = 2m$$



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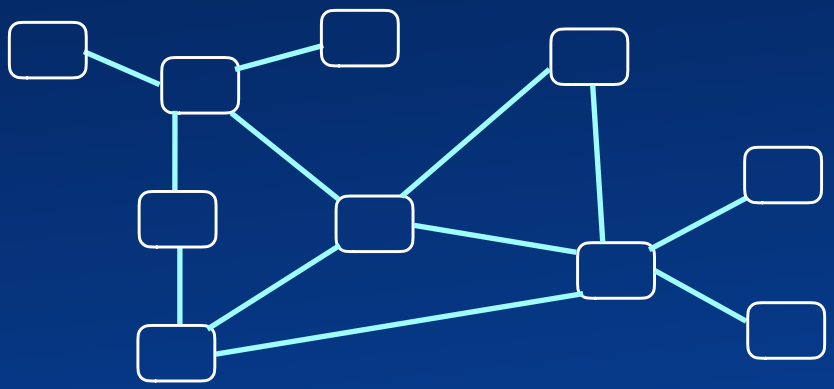

- Undirected graph

$$\sum_{v \in G} \text{degree}(v) = 2m$$

- Directed Graph:

$$\sum_{v \in G} \text{indegree}(v) = m$$

$$\sum_{v \in G} \text{outdegree}(v) = m$$

Graph Queries

- Iterator<Node> neighbor_nodes(Node n)
- Iterator<Edge> incident_edges(Node n)
- boolean isAdjacent(Node n1, Node n2)
- boolean isConnected(Node n1, Node n2)
- EV edgeValue(Node n1, Node n2)
- List<Node> apath(Node n1, Node n2)
- List<Node> shortestPath(Node n1, Node n2)
- void traverse()
- void traverse(Node n)

T/F

- An AVL tree is also a directed acyclic graph
- A directed acyclic graph is also a tree
- A graph with n vertices and n-1 edges is connected

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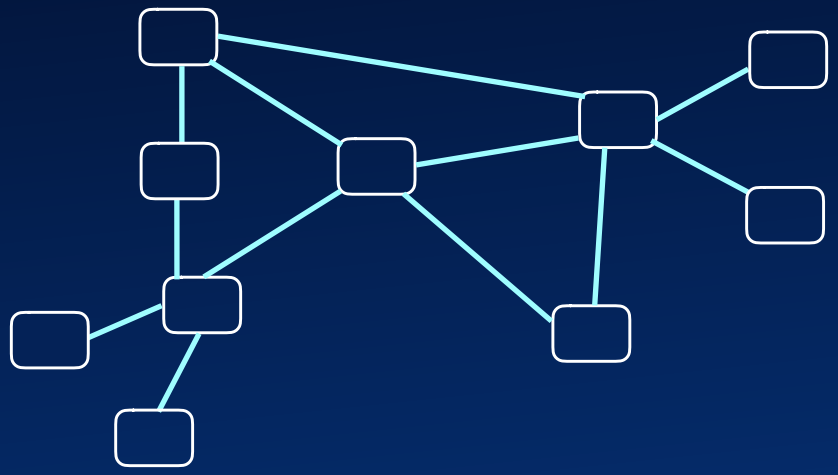
Format: t,f,f

Traversal

- **Start at any vertex**

```
traverse(v):
    process(v)
    for u in v.adjacent()
        traverse(u)
```



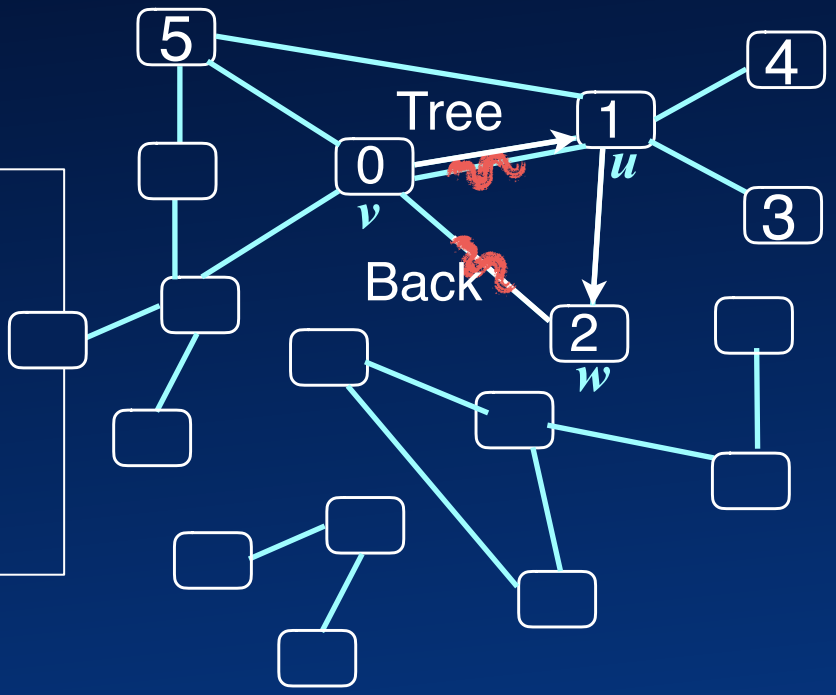
DFS Traversal

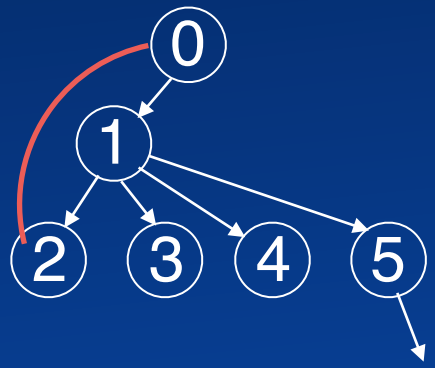
- **Start at any vertex**

```
traverse(v, parent):
    process(v) and mark(v)
    for u in v.adjacent() - parent
        visit(edge to u)
        traverse(u,v) if (! u.marked)
```

```
While ∃ unmarked v:
    traverse(v, null)
```

O(*m*+*n*)





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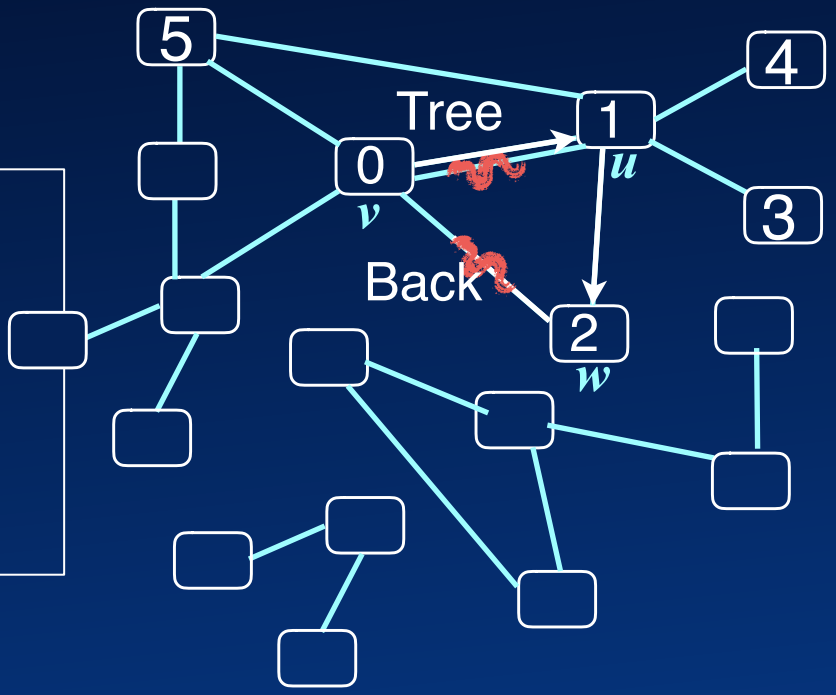

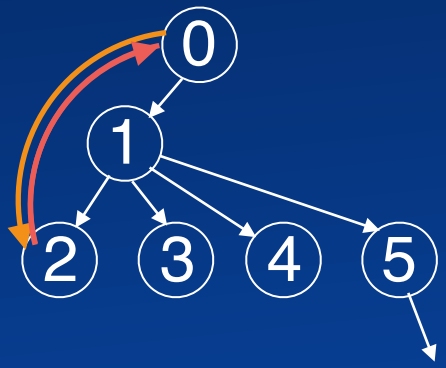

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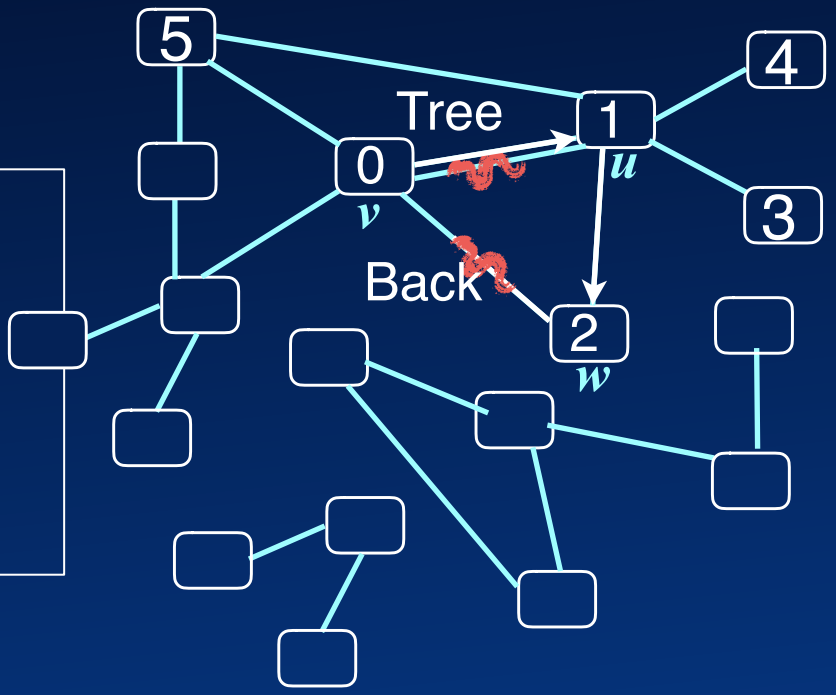

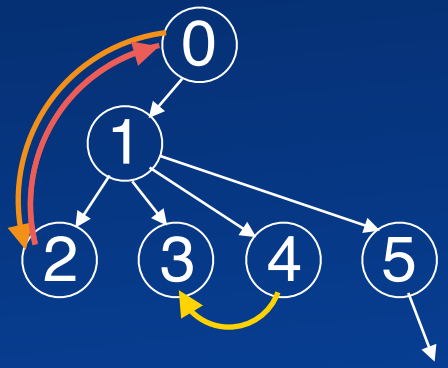

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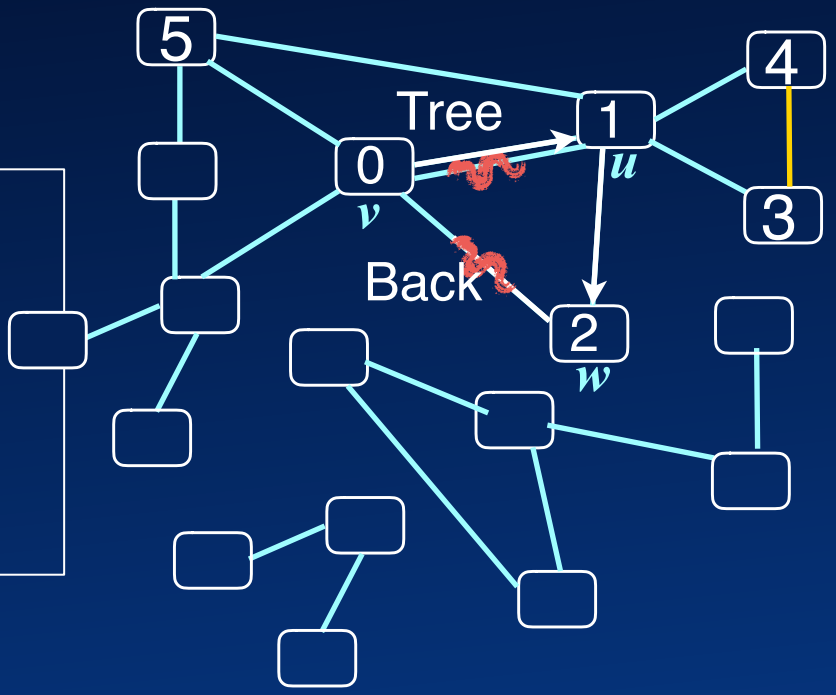

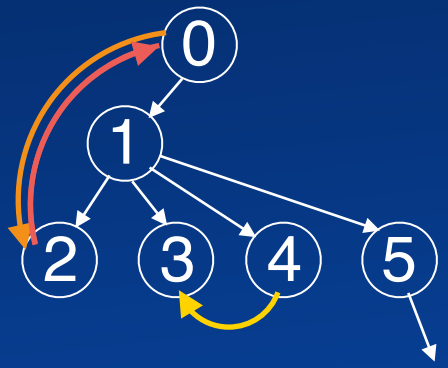

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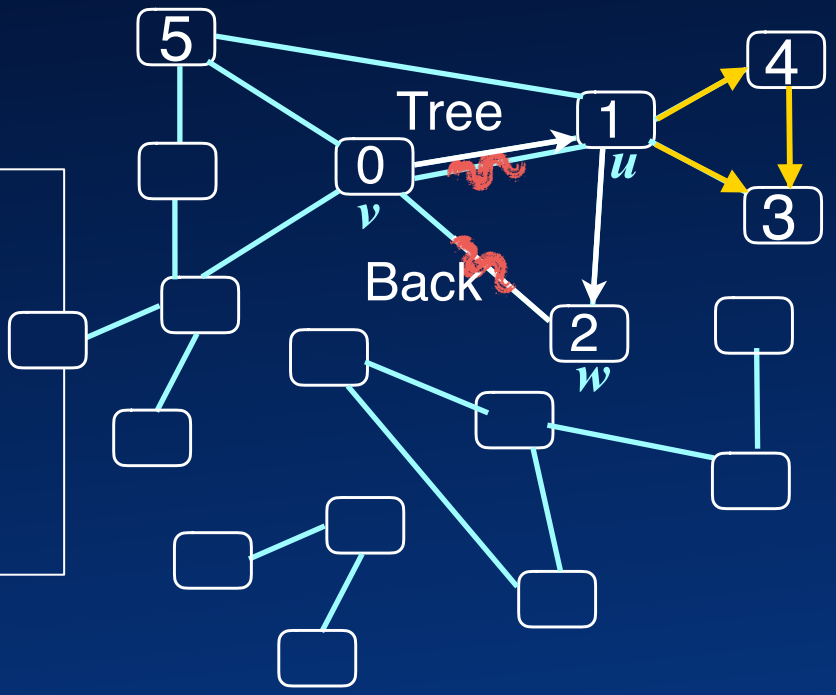
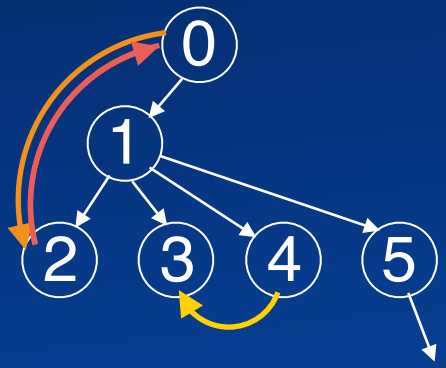
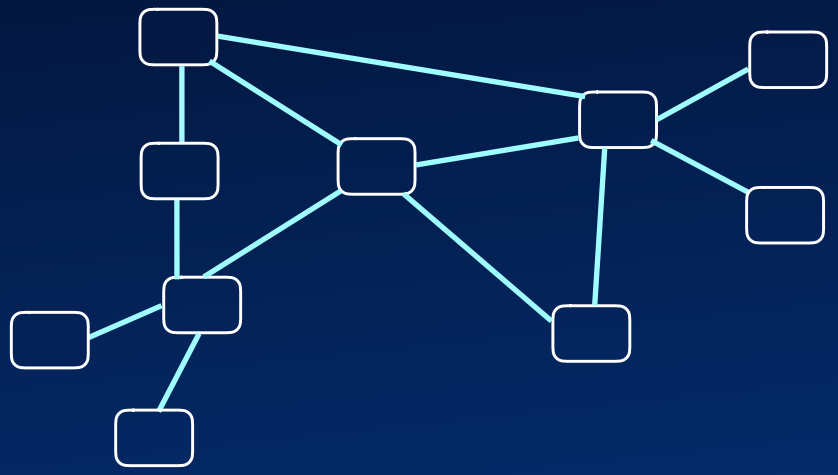




BFS Traversal



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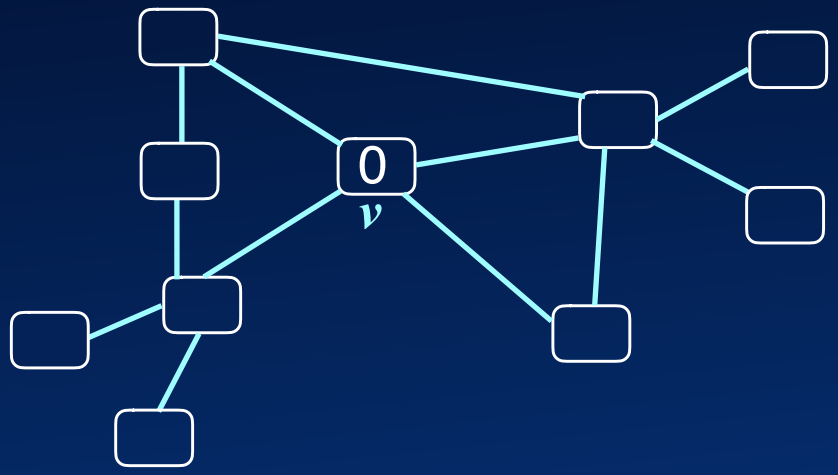

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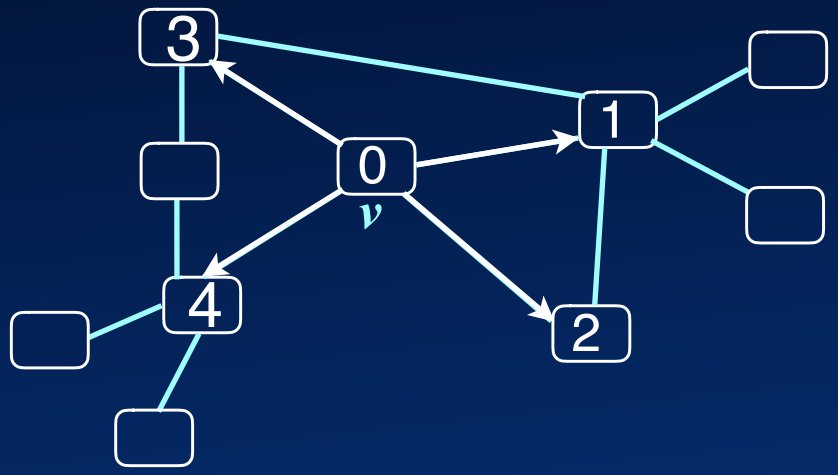

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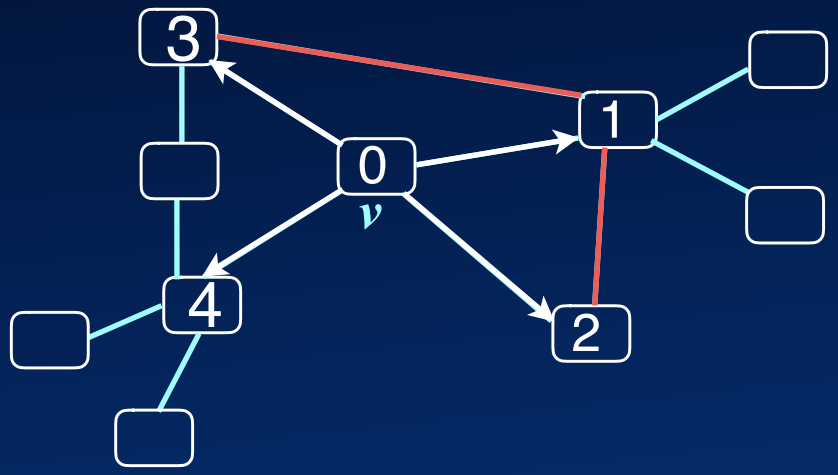

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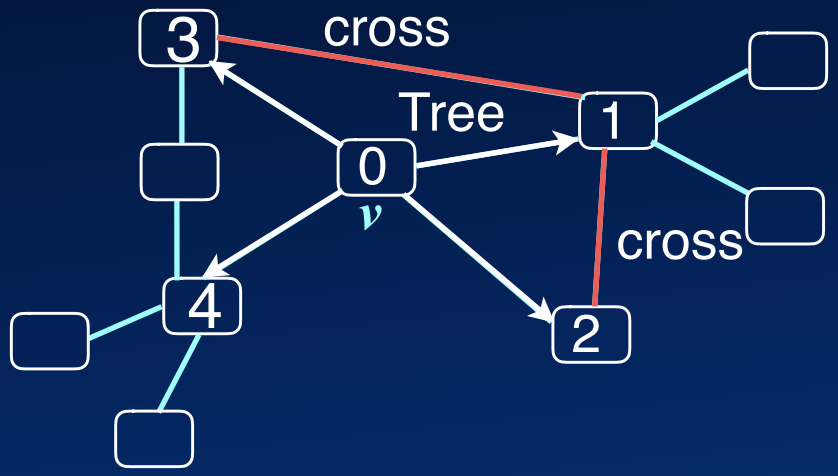

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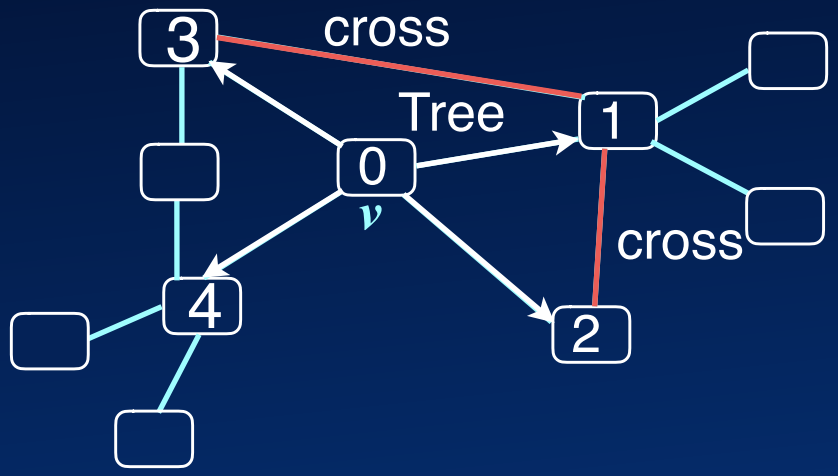

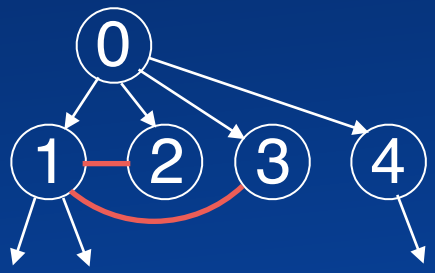

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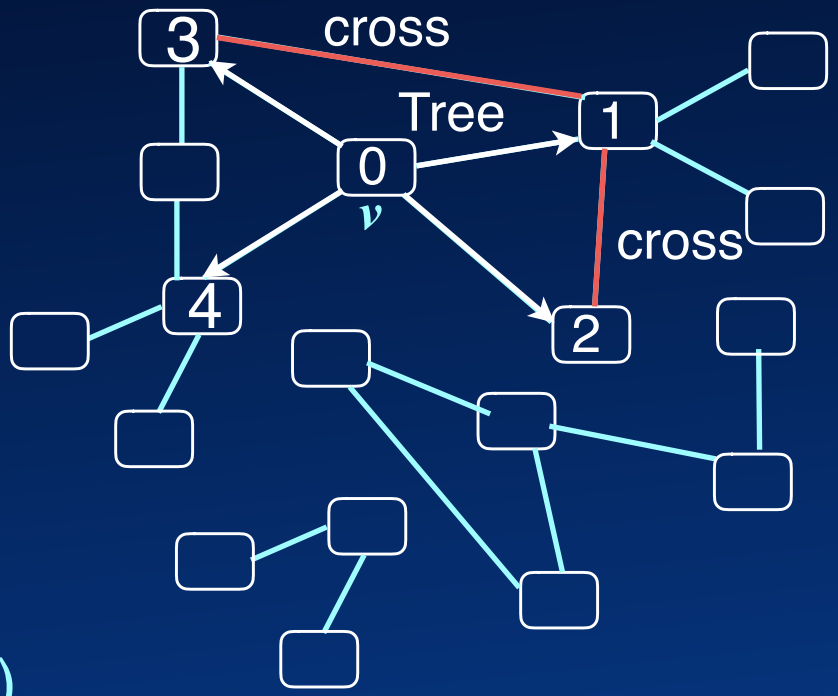

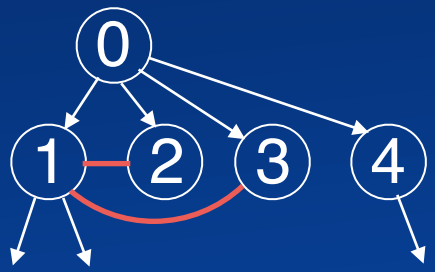

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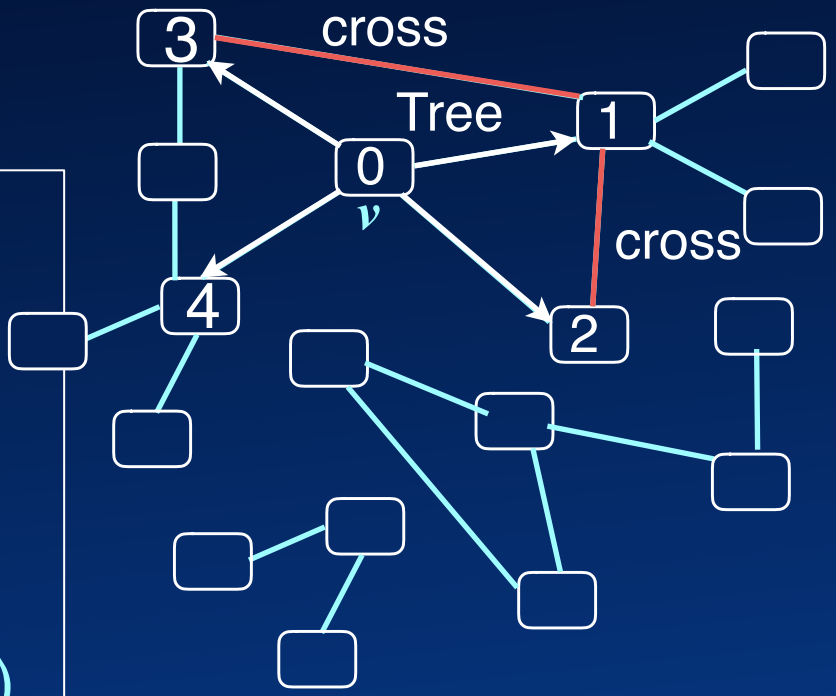

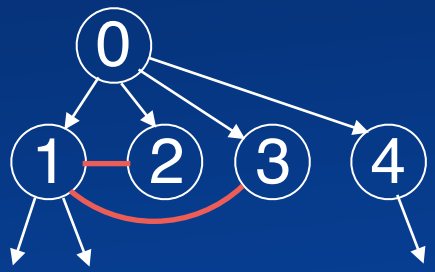

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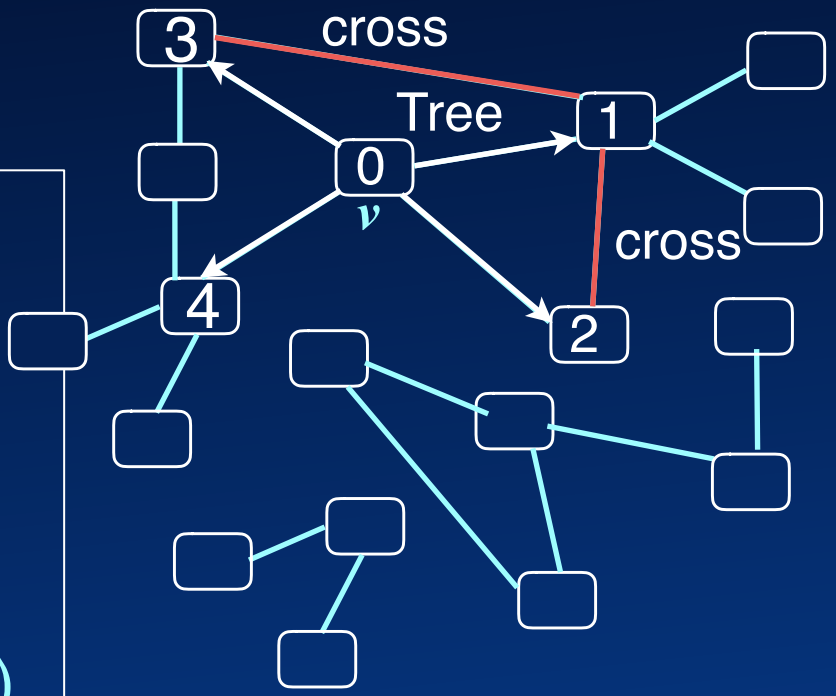
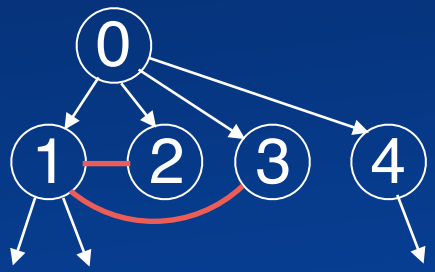



```
While ∃ unmarked v:
   enqueue(v)
   bfs()
```



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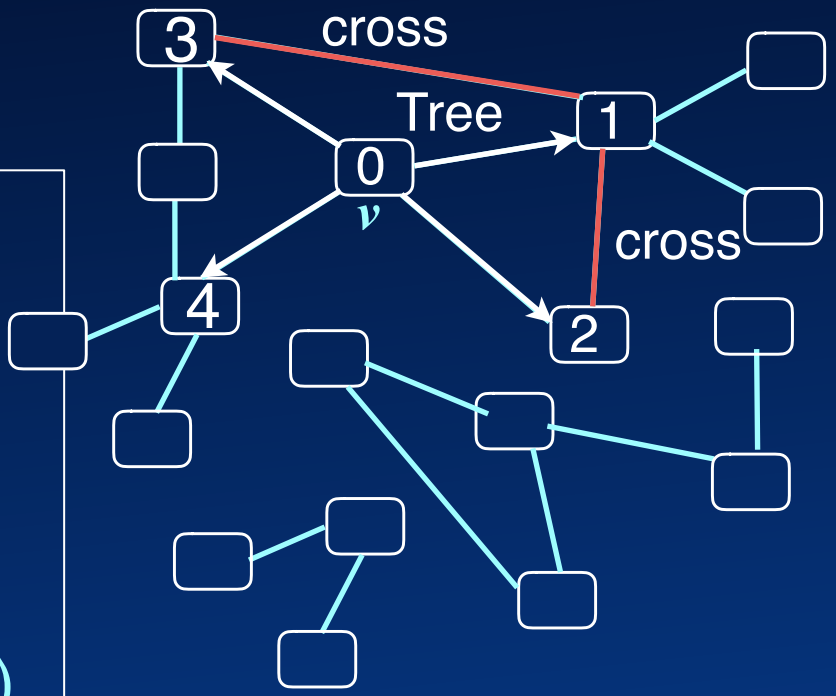


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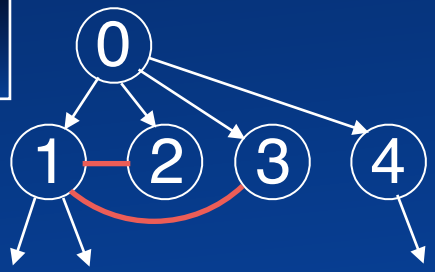
O(*m*+*n*)



T/F

- Time complexity of DFS in a graph with n nodes and m edges is O(m+n)
- Time complexity of BFS in a graph with n nodes and m edges is O(n^2)

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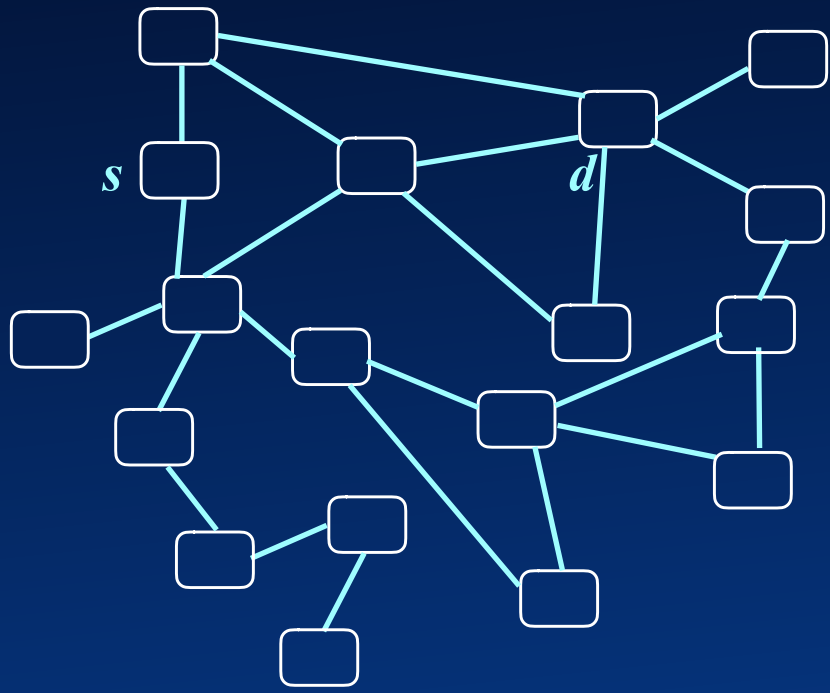
T/F

- The shortest path from node u to node v in a directed graph is equal to the shortest path from node v to node u.
- The shortest path from node u to node v in an undirected graph is equal to the shortest path from node v to node u.
- If P_1 is the shortest path from node u to node w in a directed graph, and P_2 is the shortest path from w to node v, P_1P_2 (i.e., P_1 followed by P_2) is the shortest path from u to v.

Shortest Path



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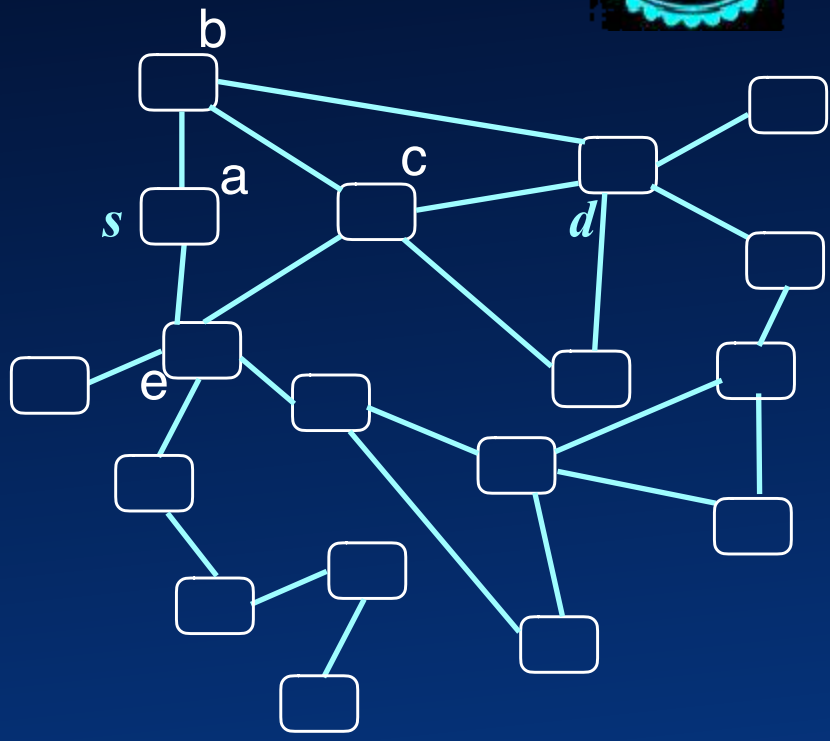

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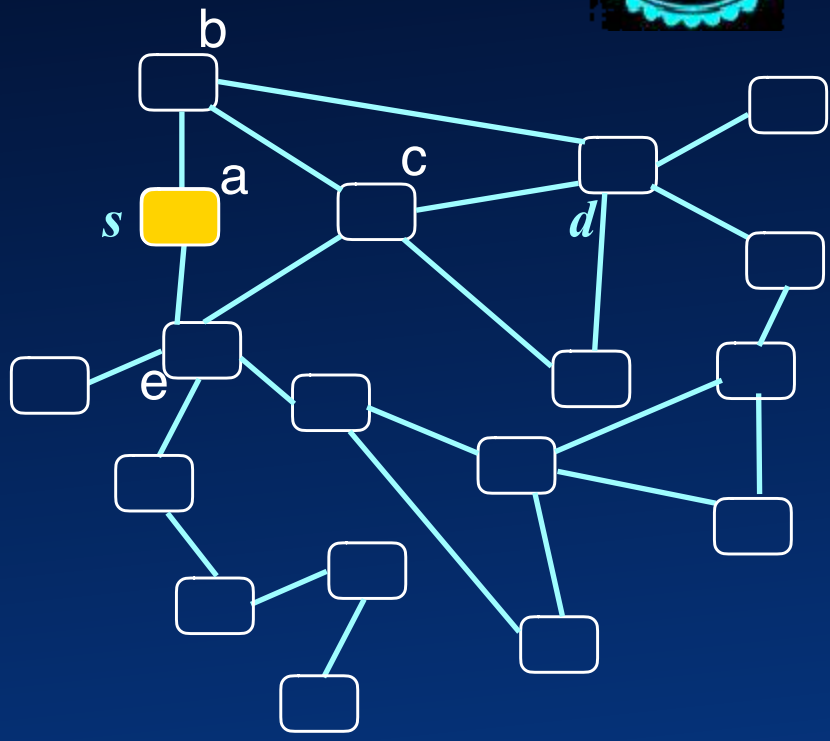

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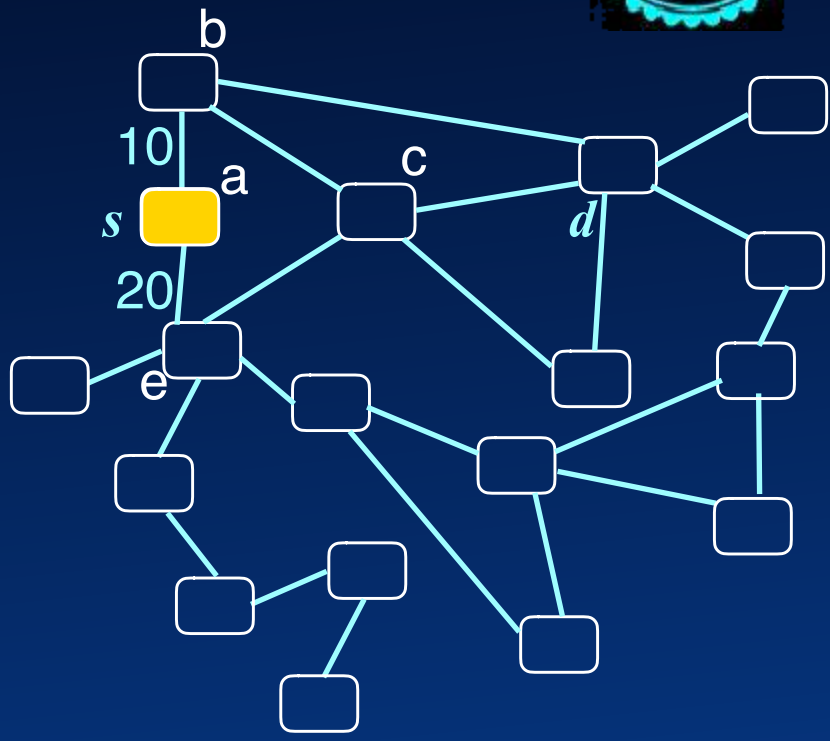

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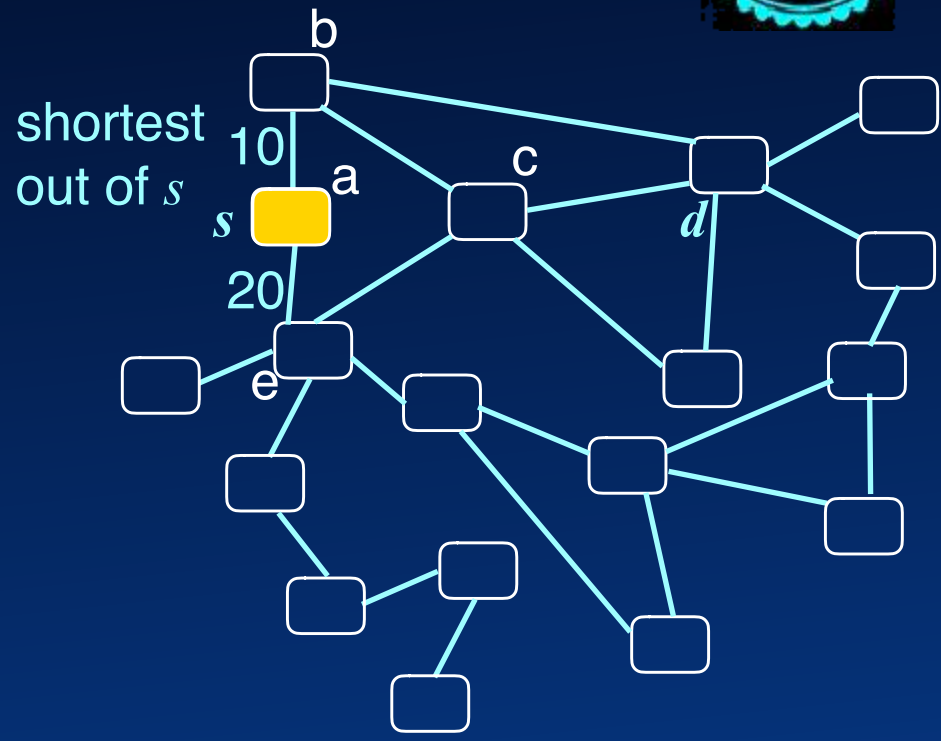

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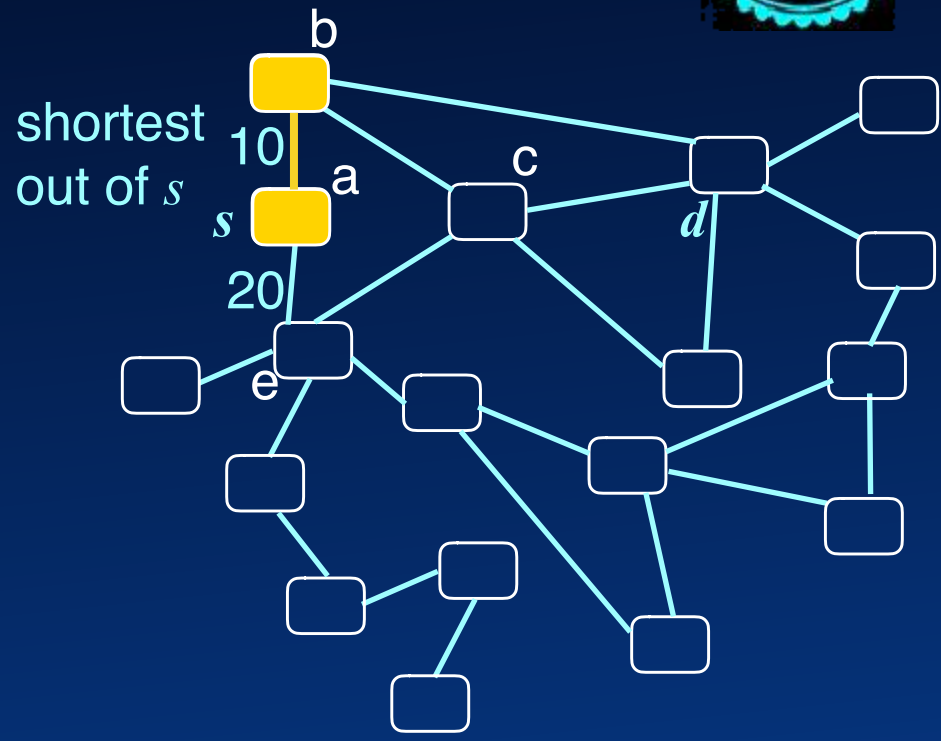

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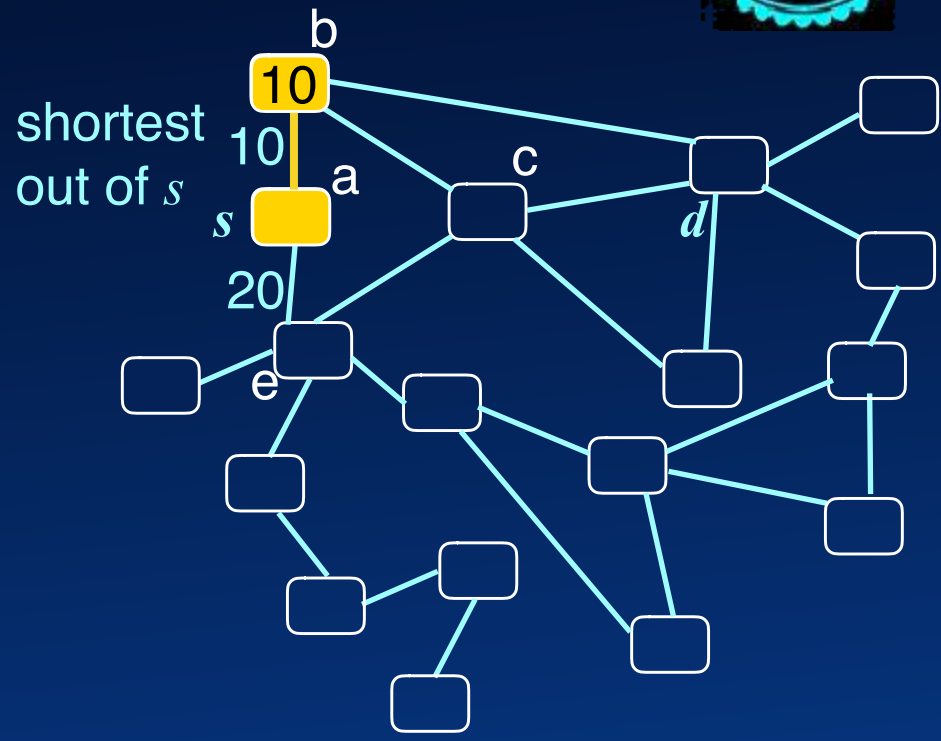

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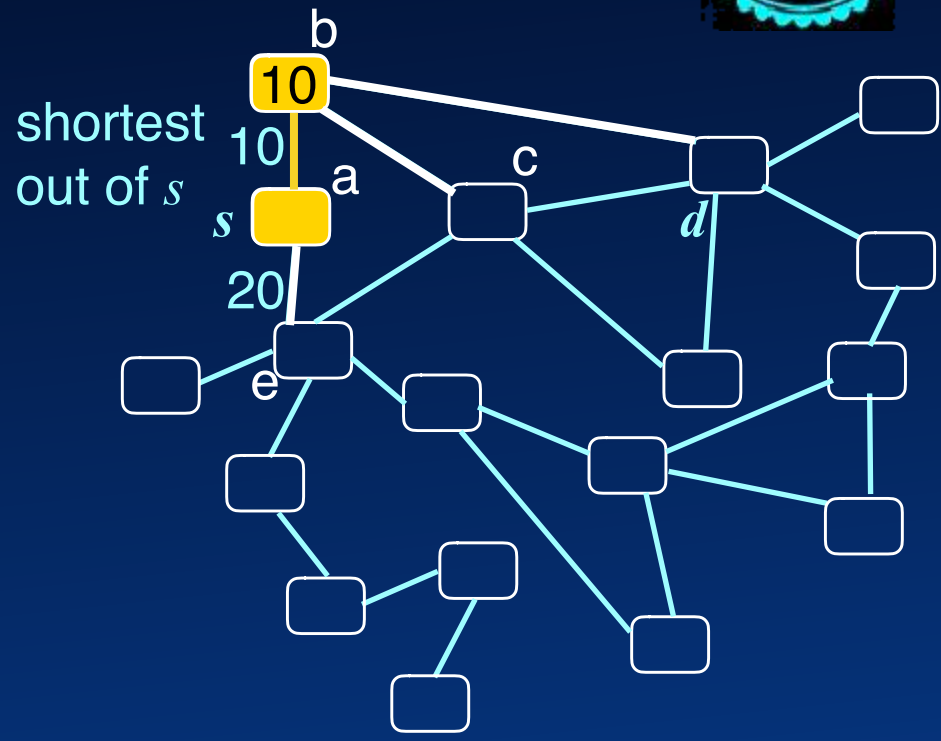

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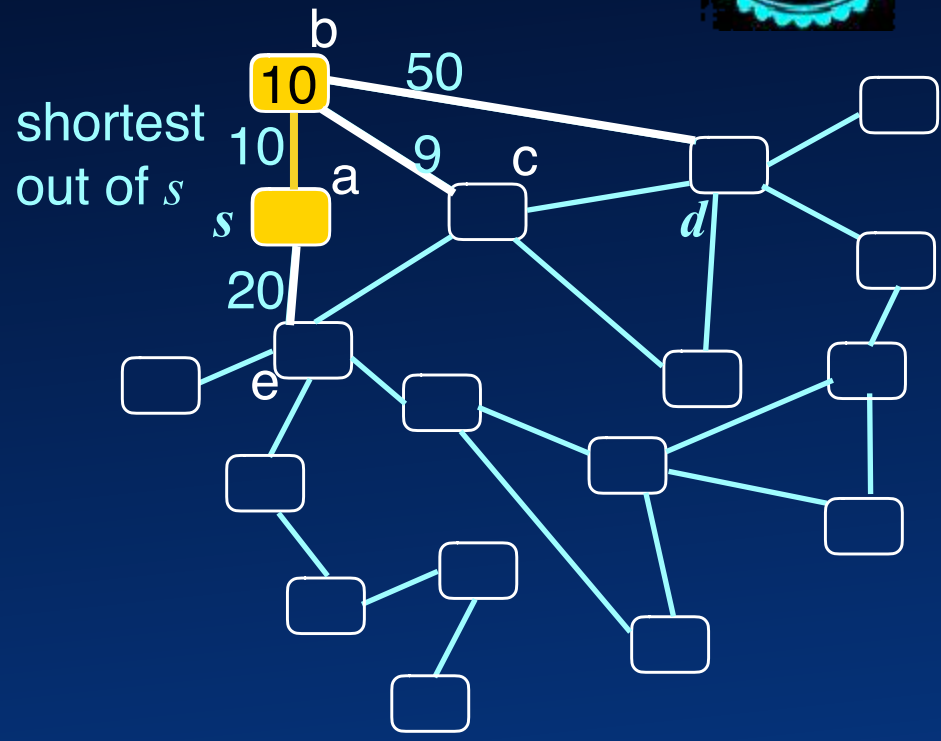

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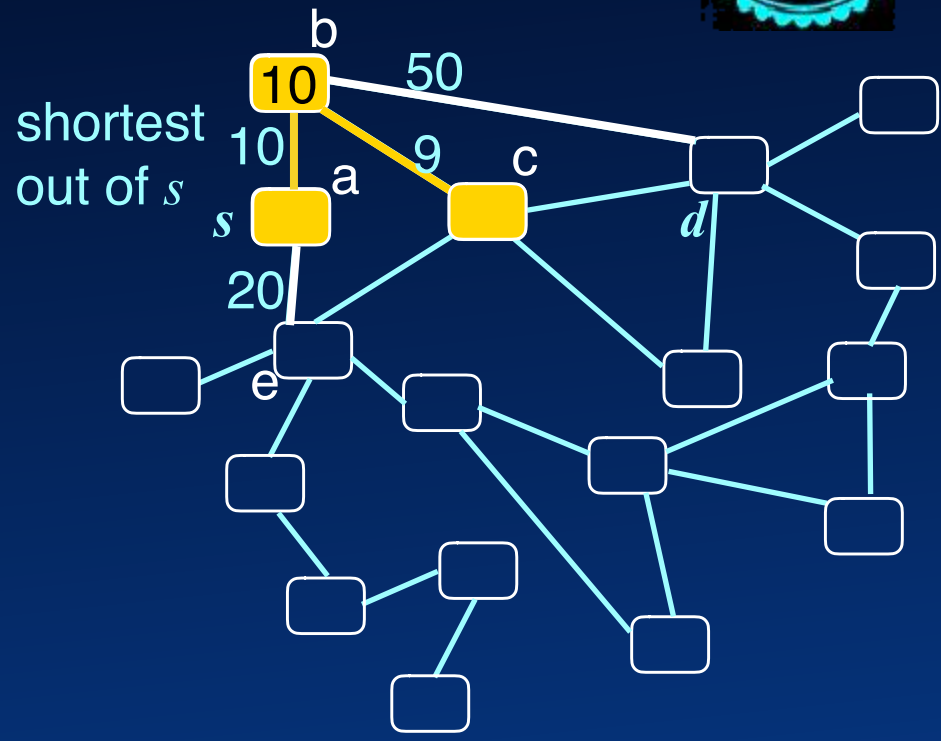

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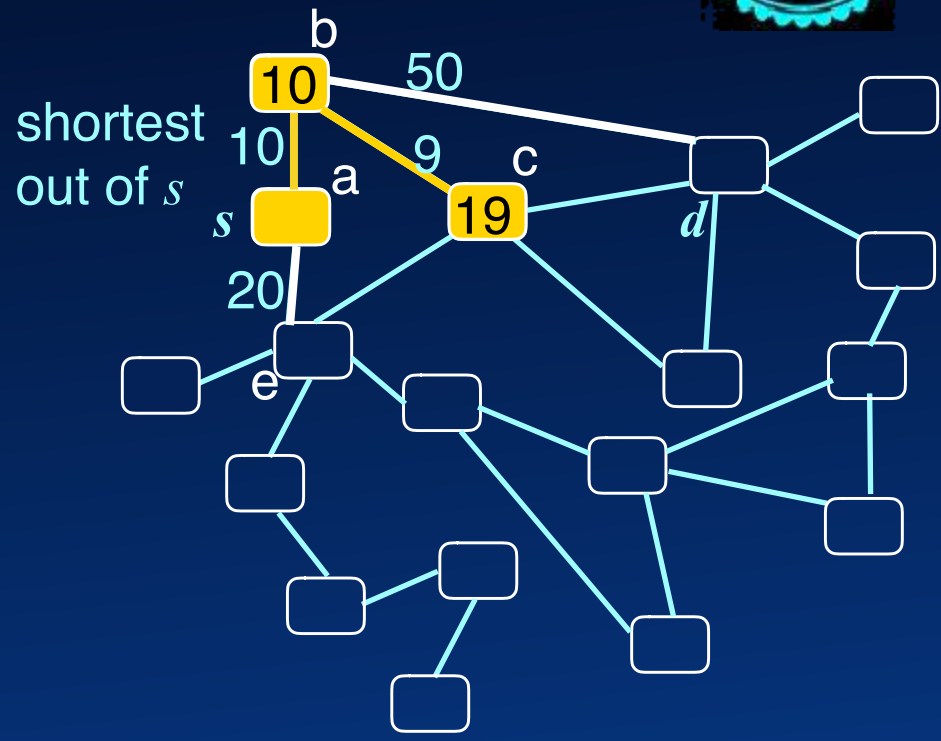

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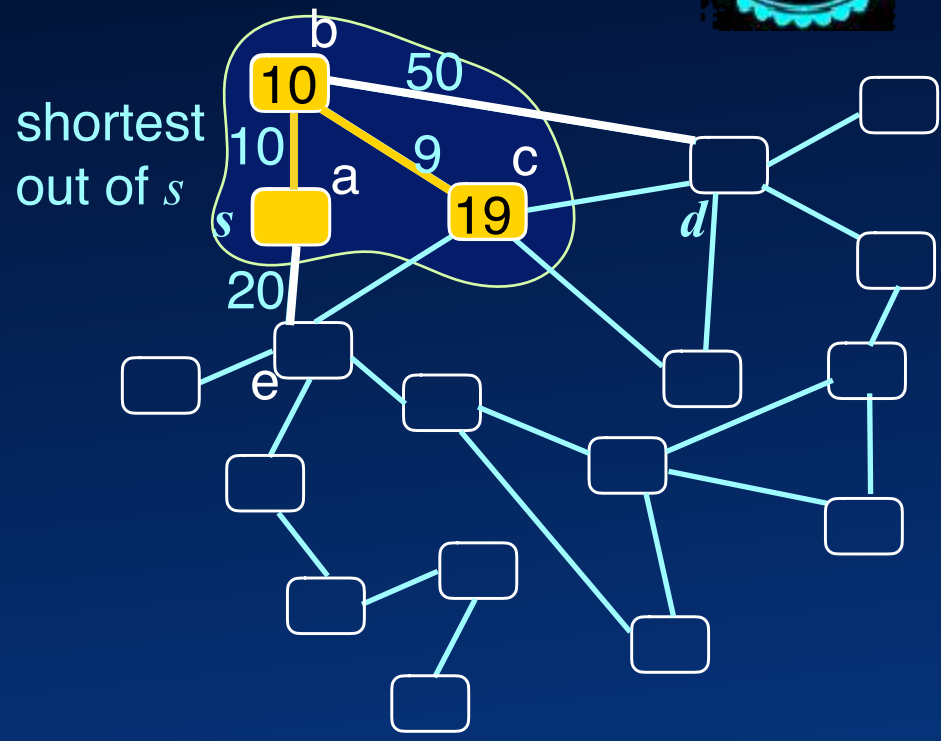

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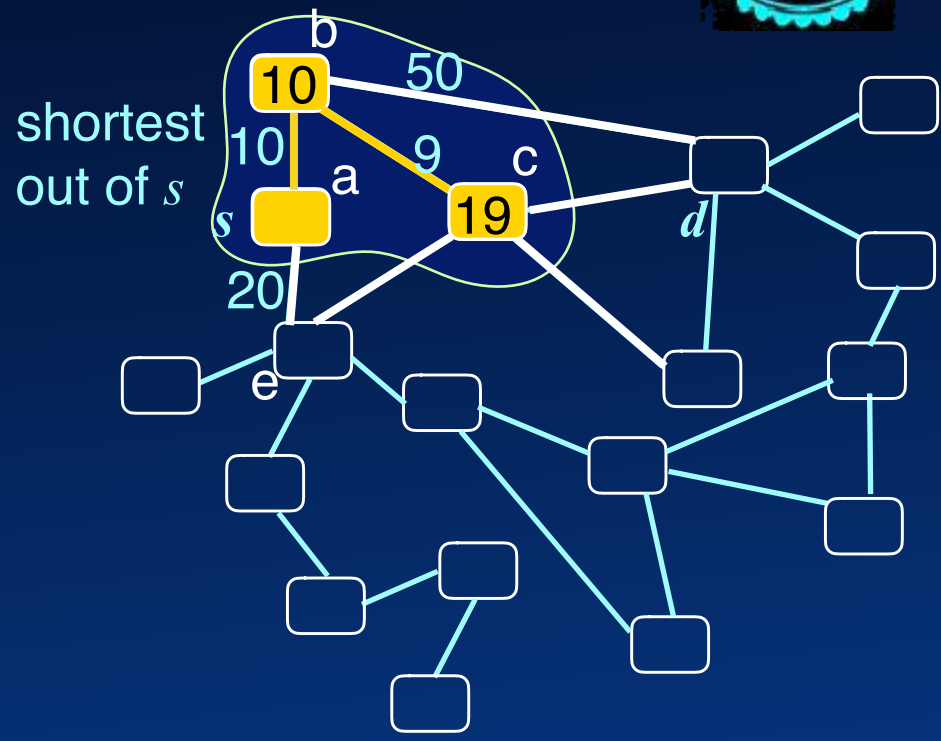

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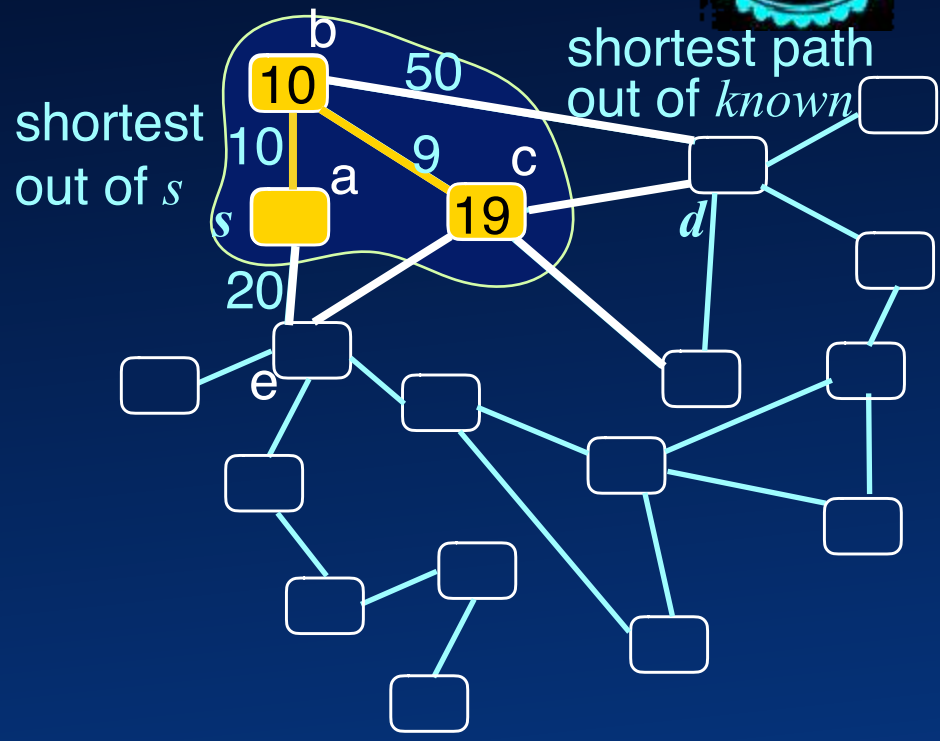

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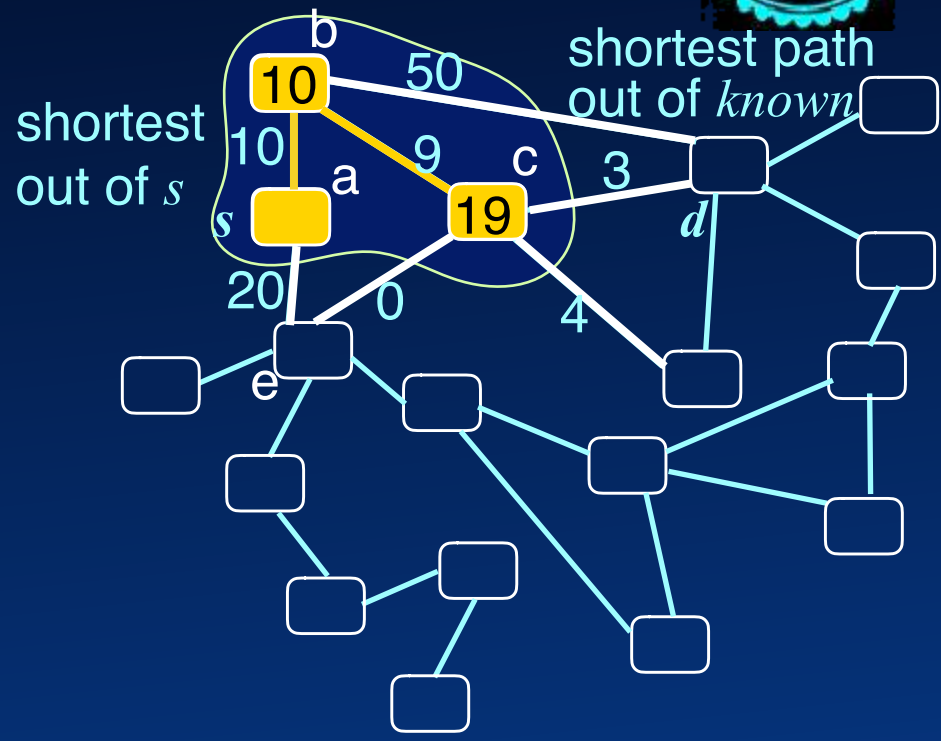

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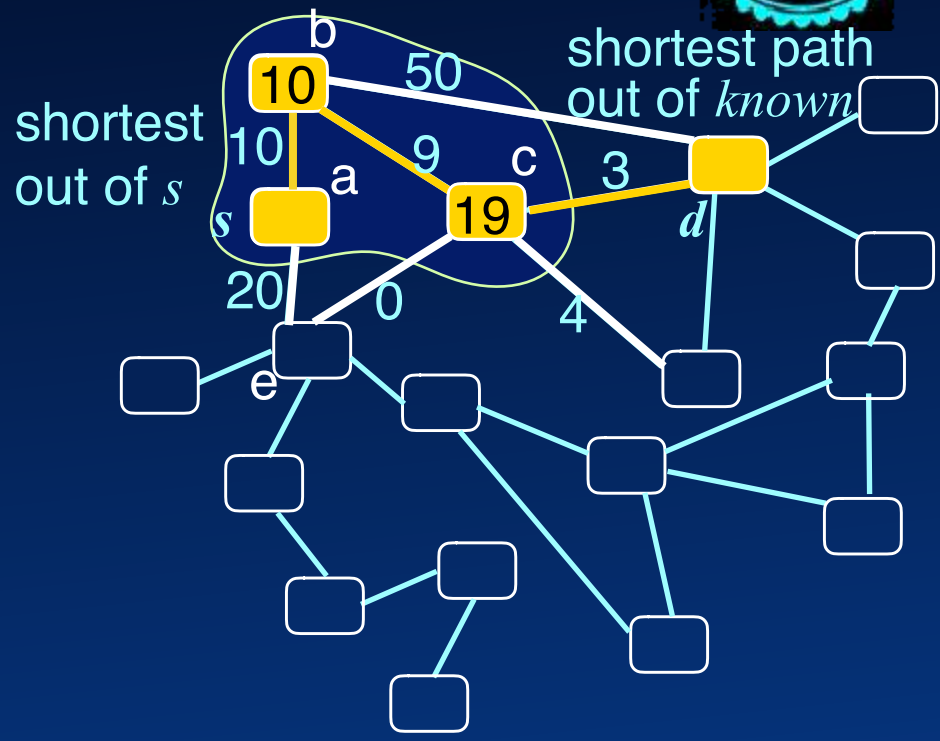

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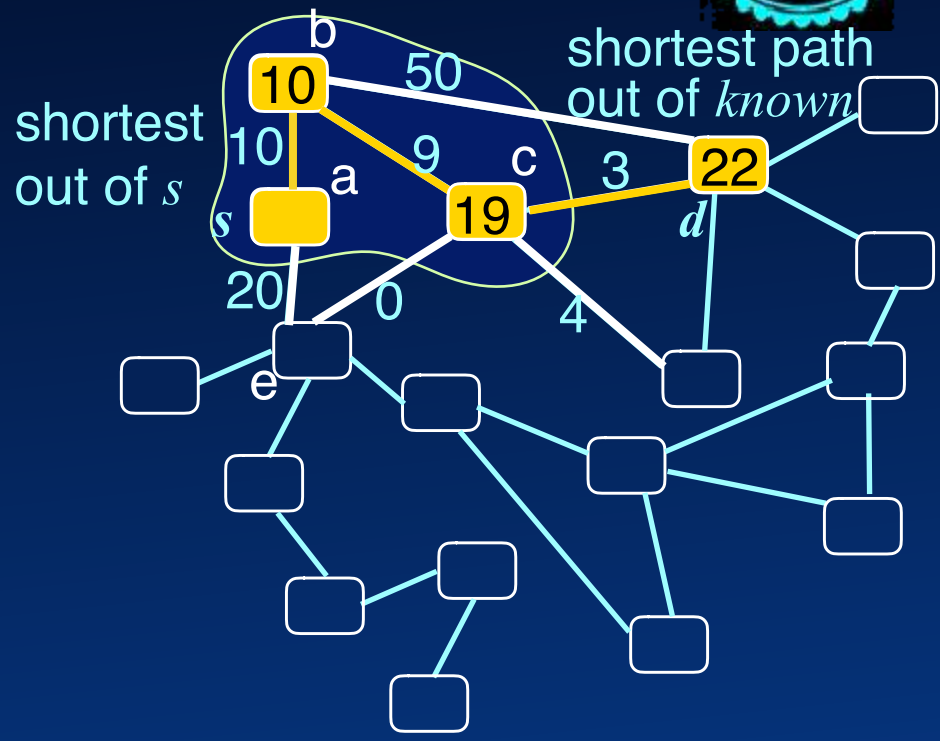

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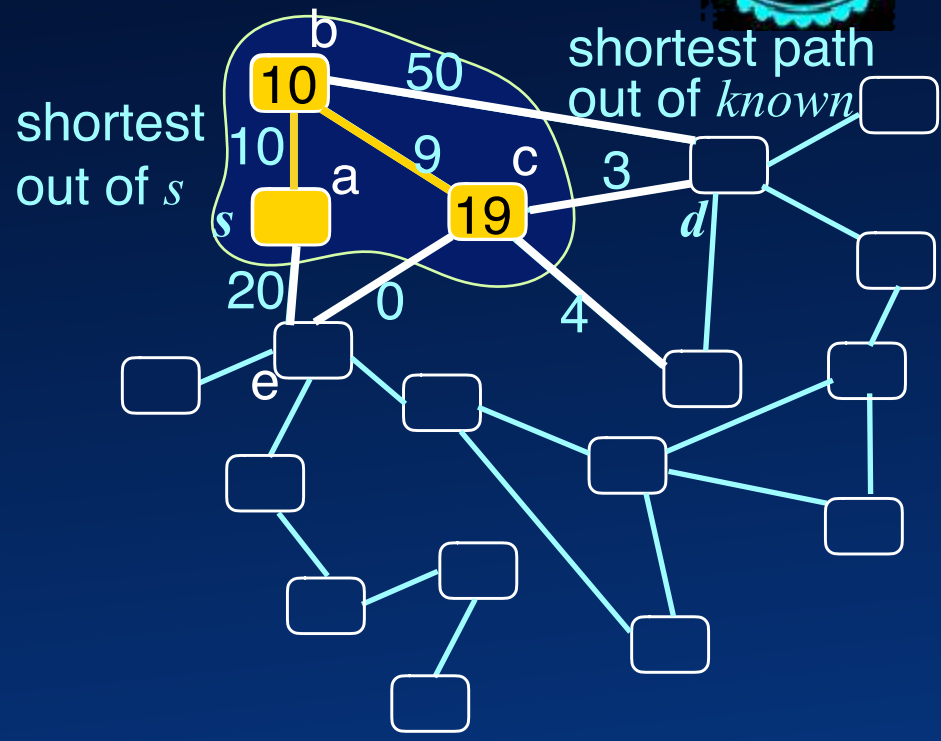

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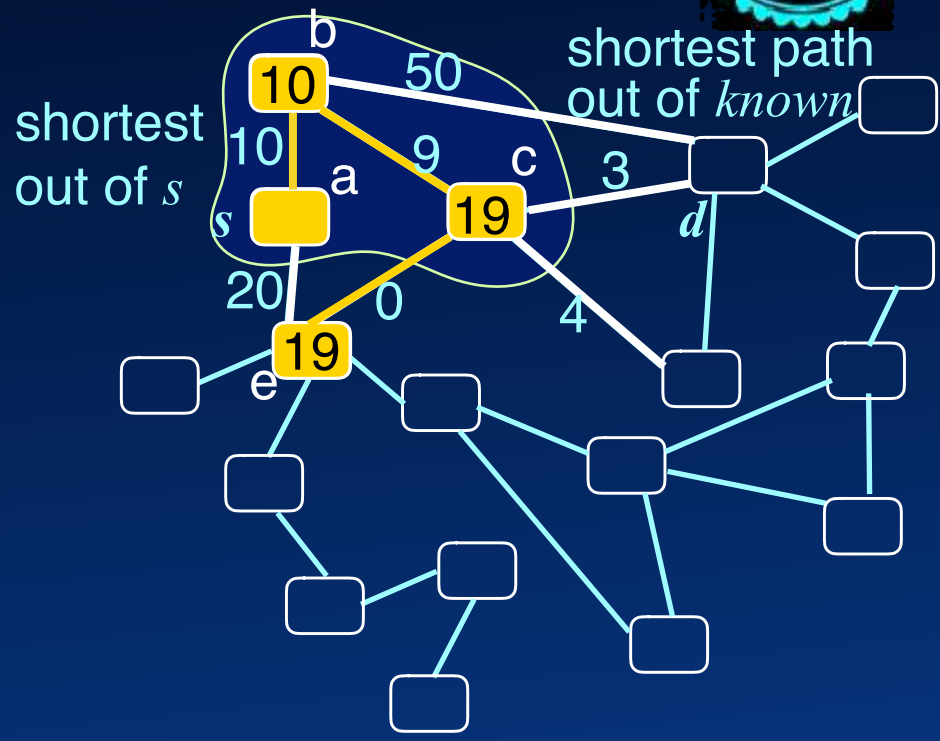

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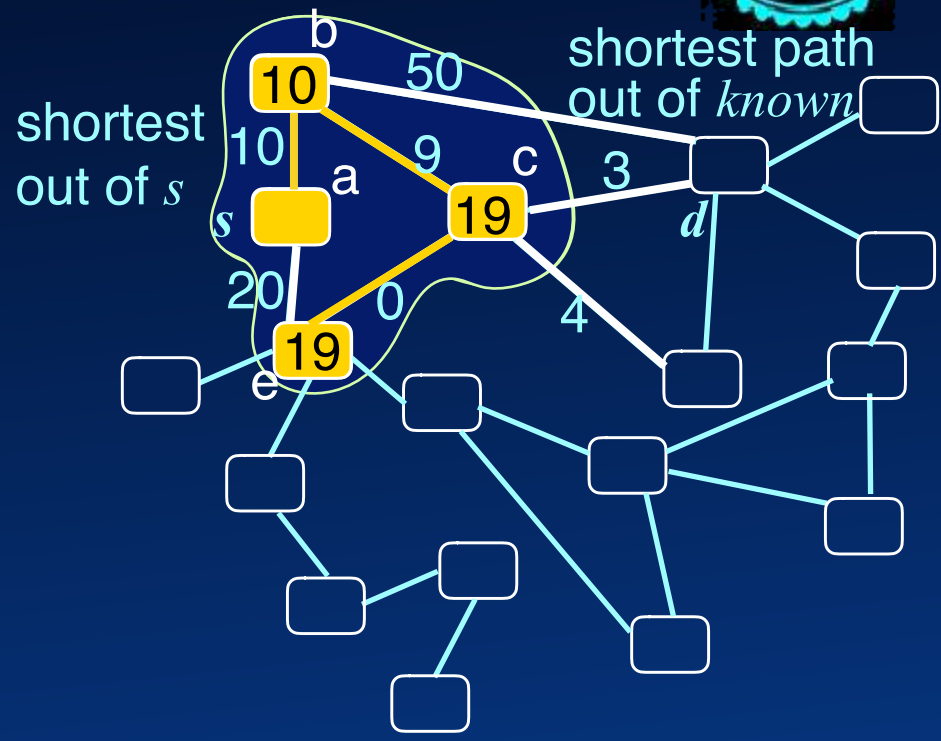

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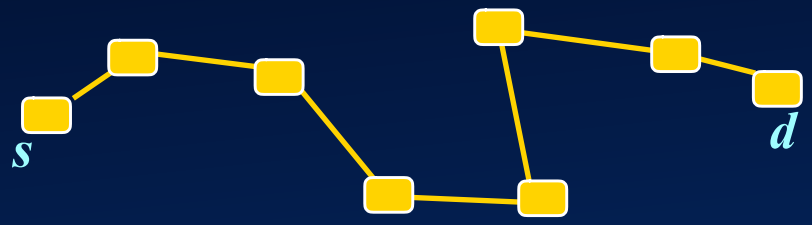

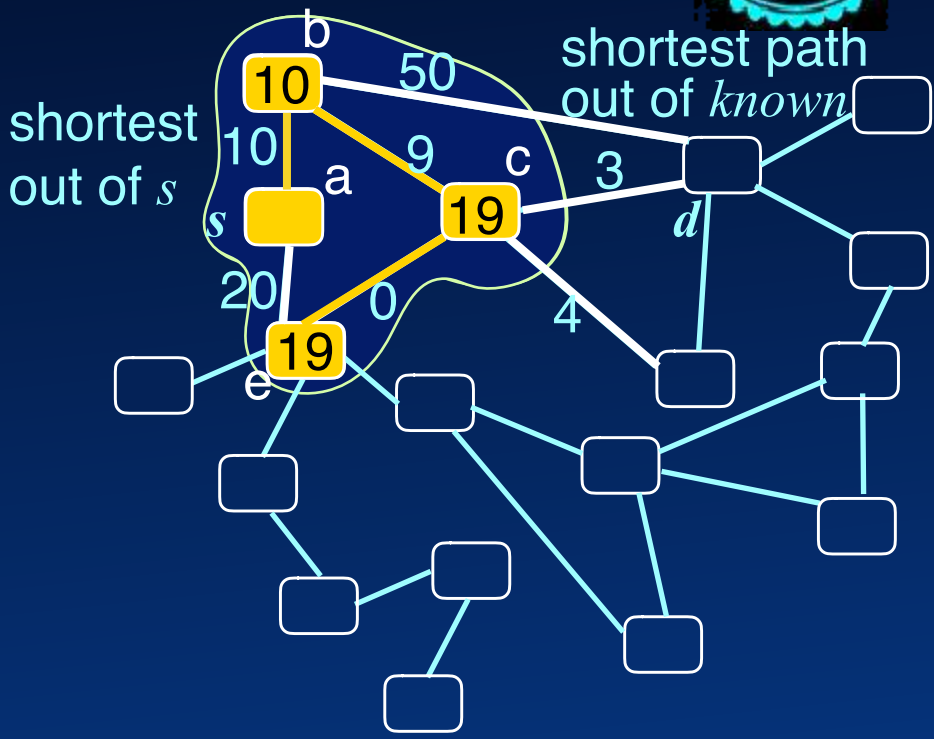

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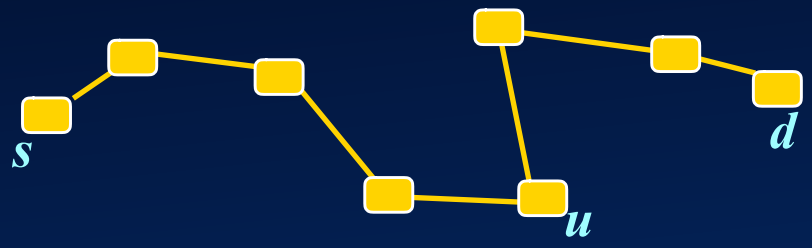

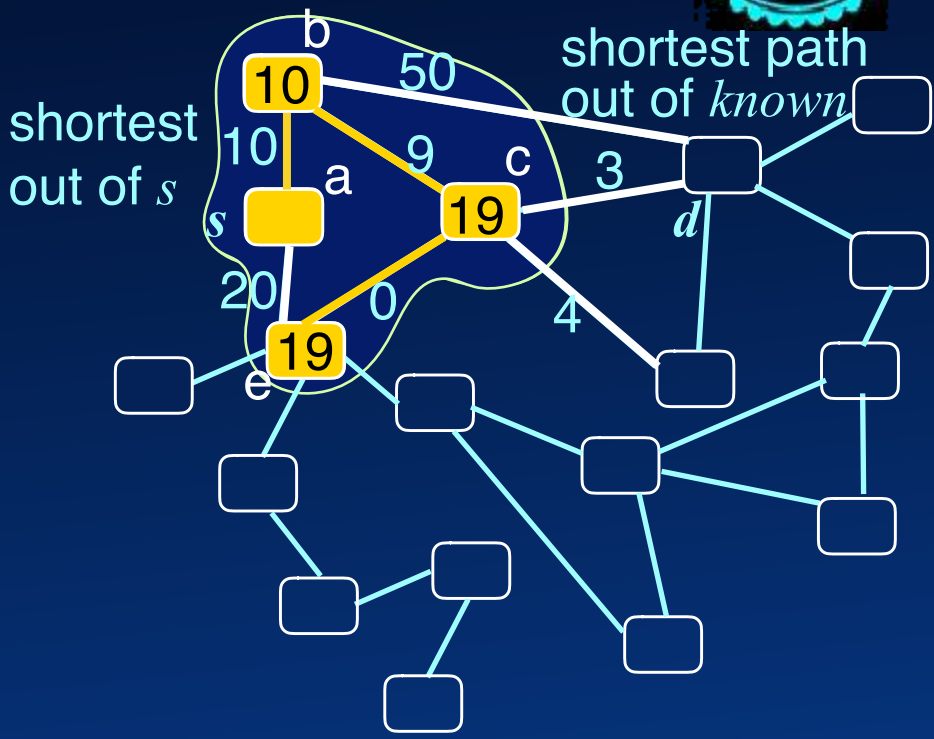

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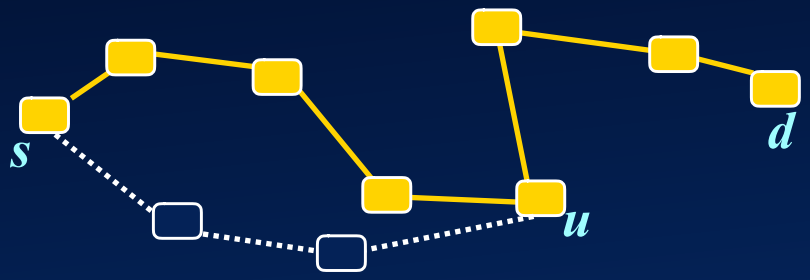

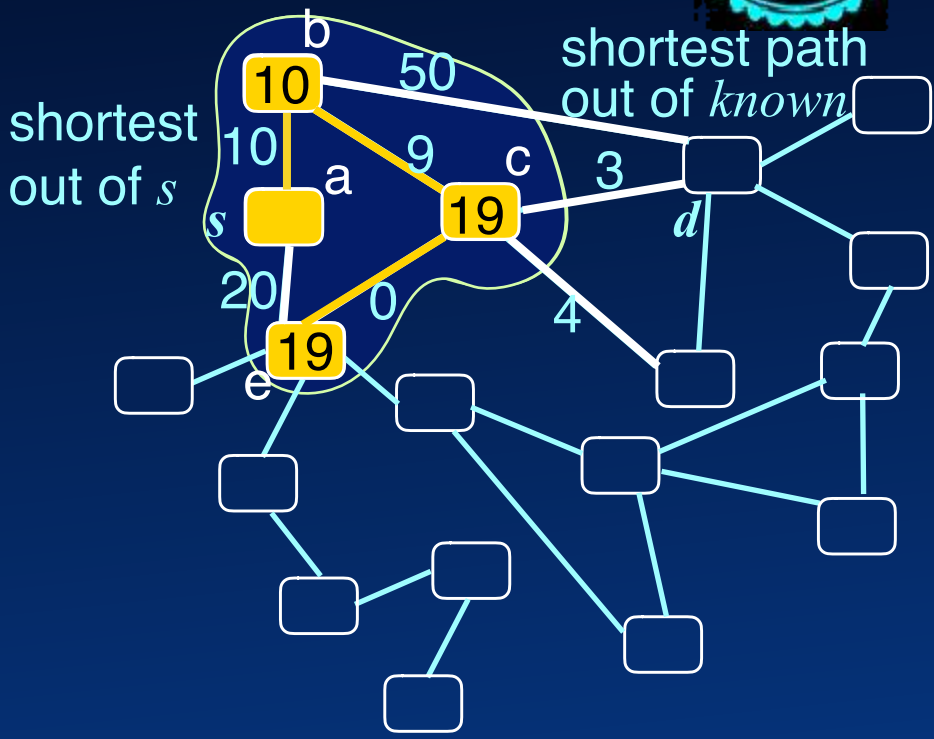

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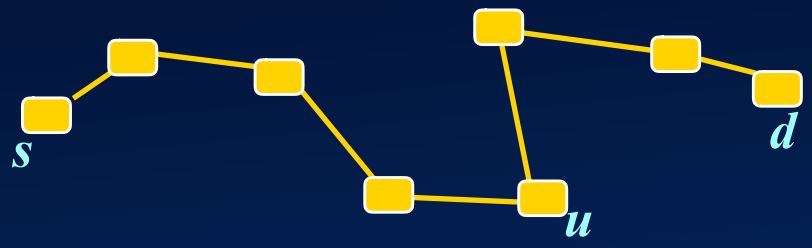

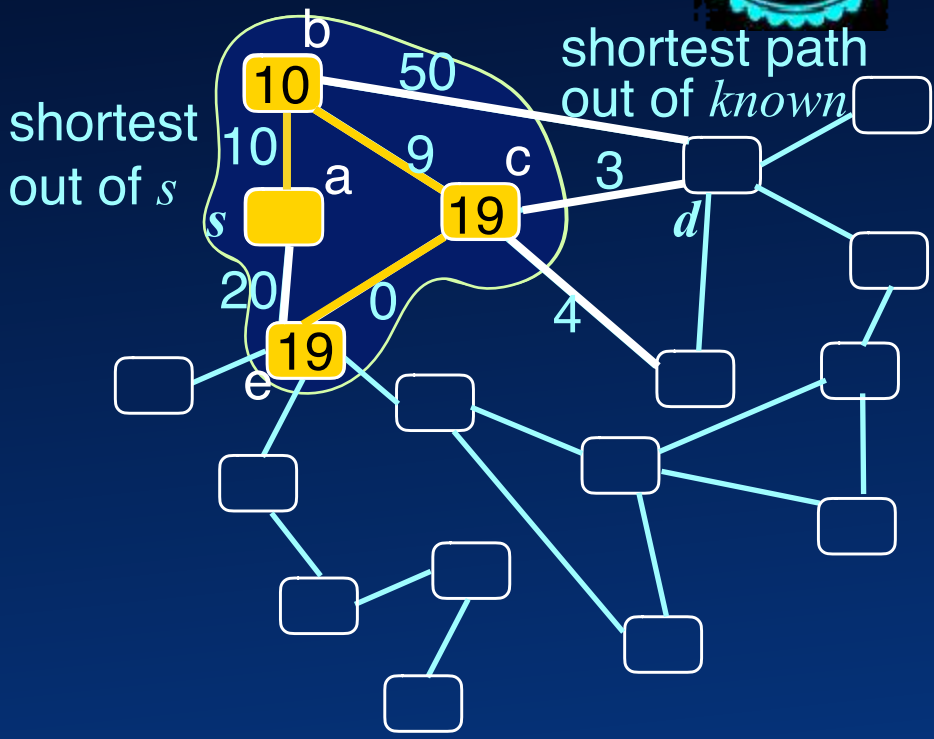

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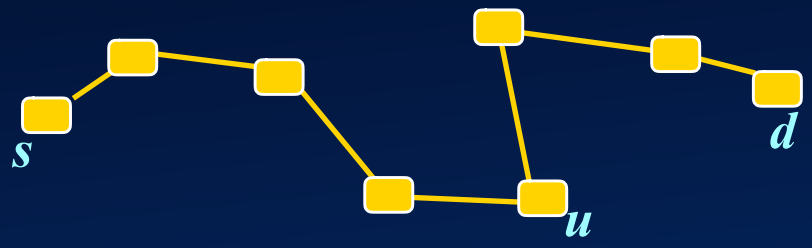

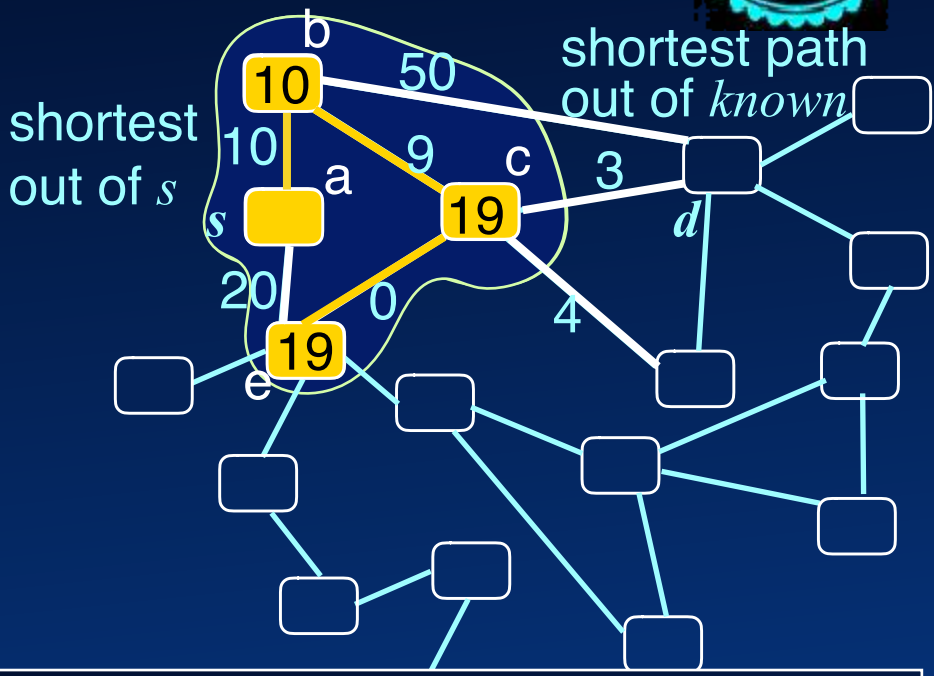

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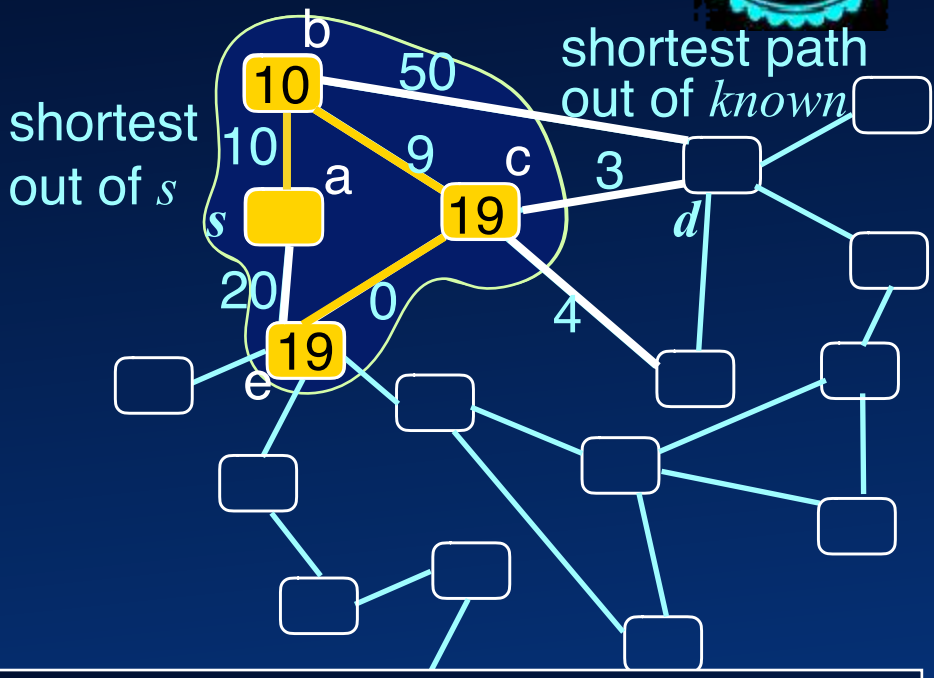

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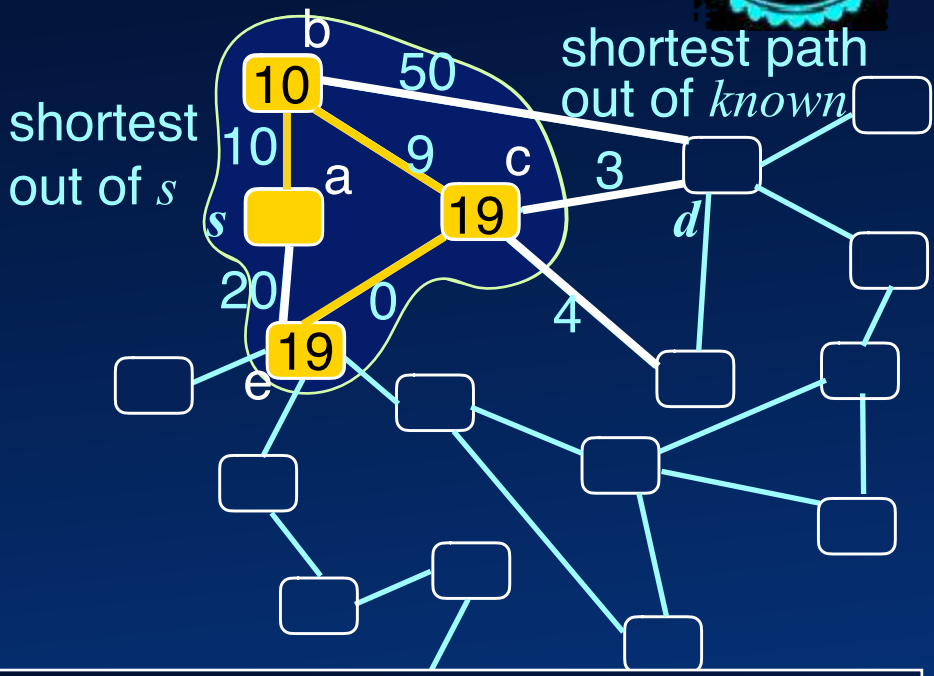

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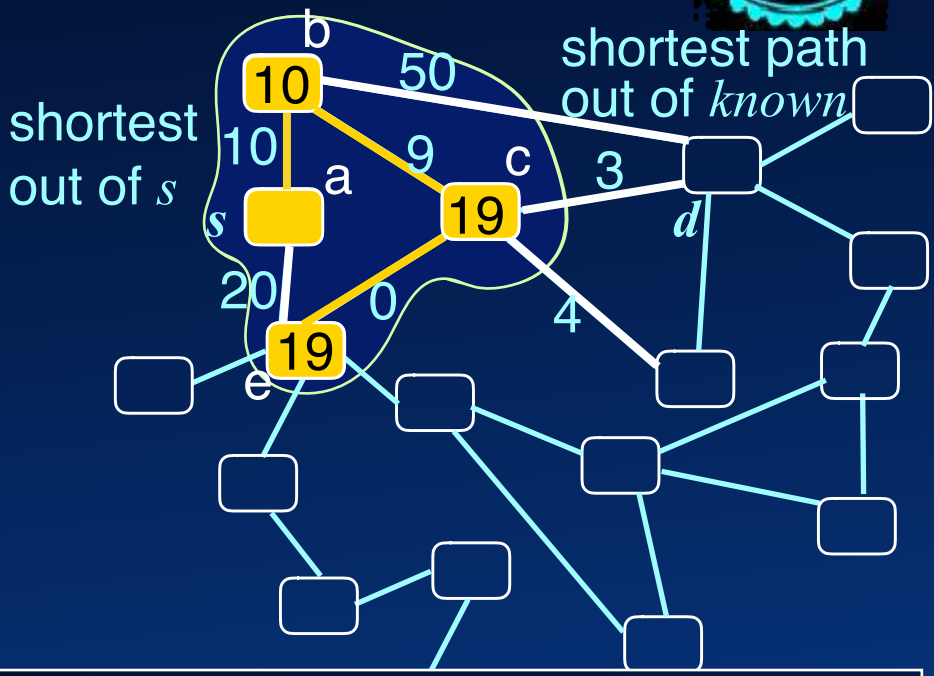

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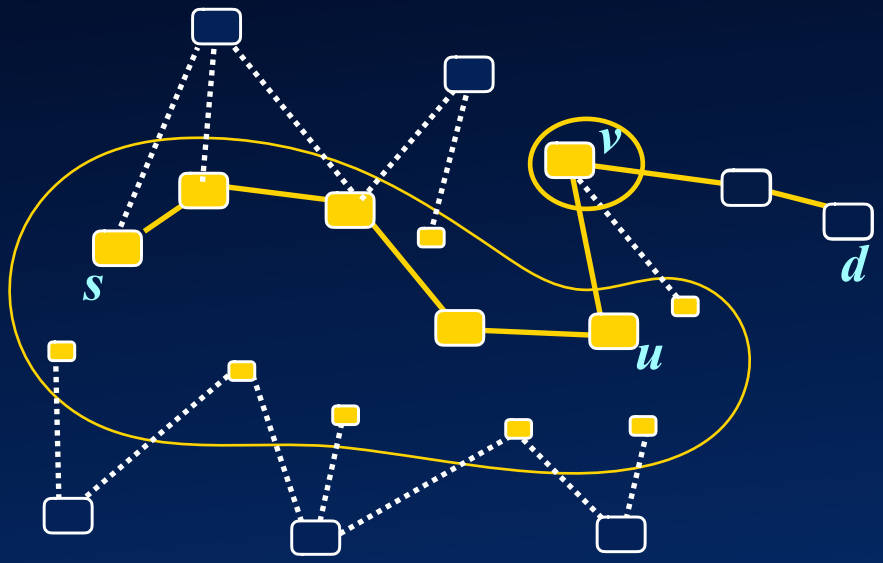

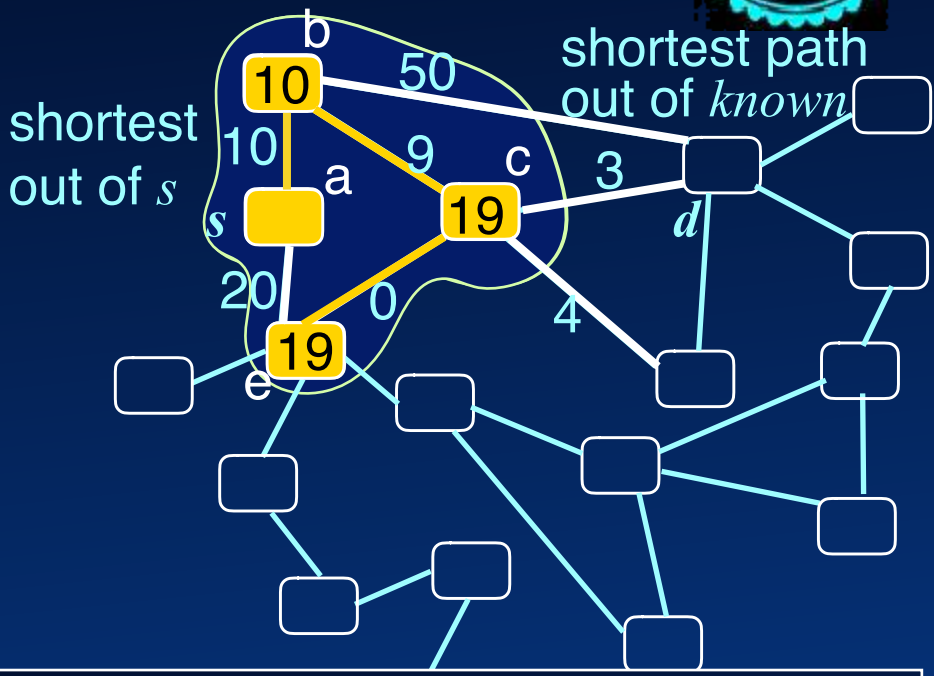

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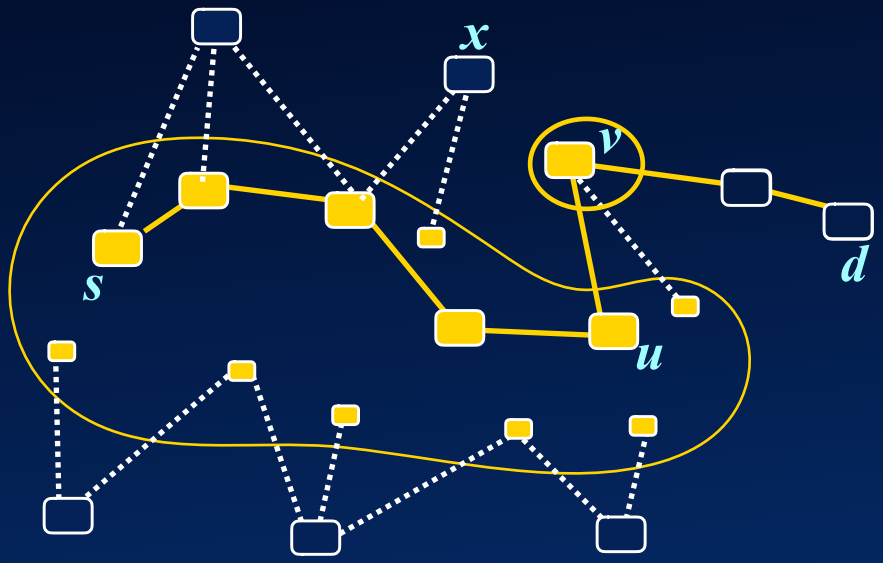

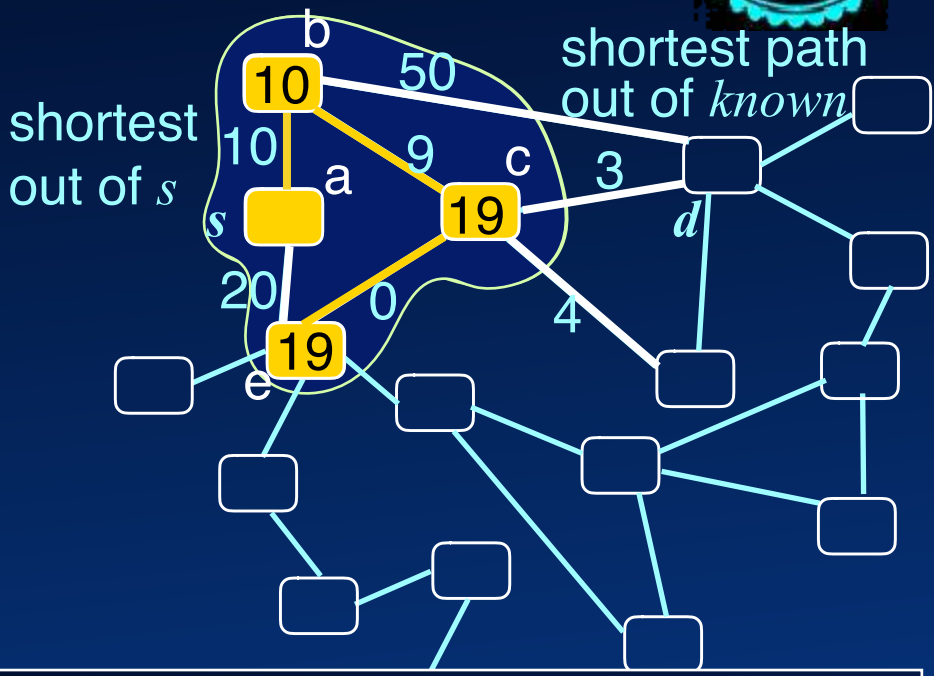

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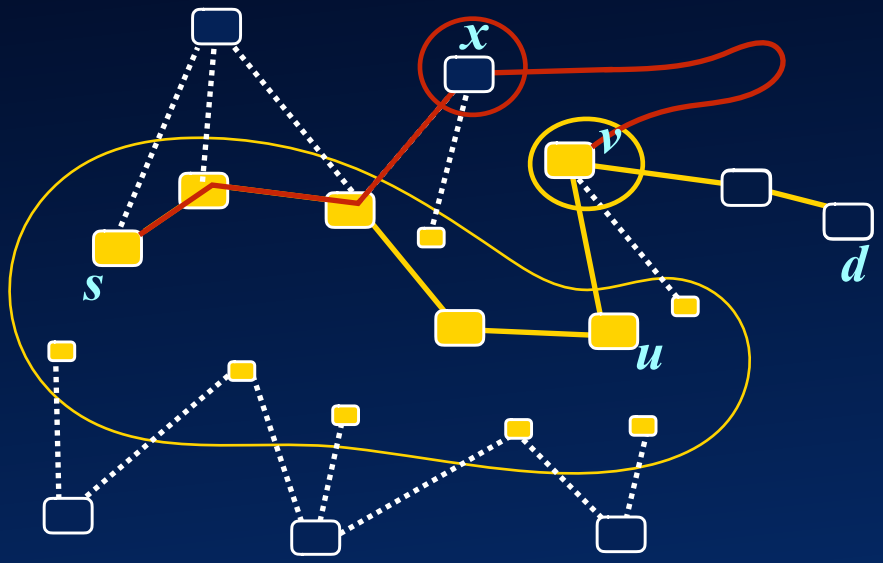

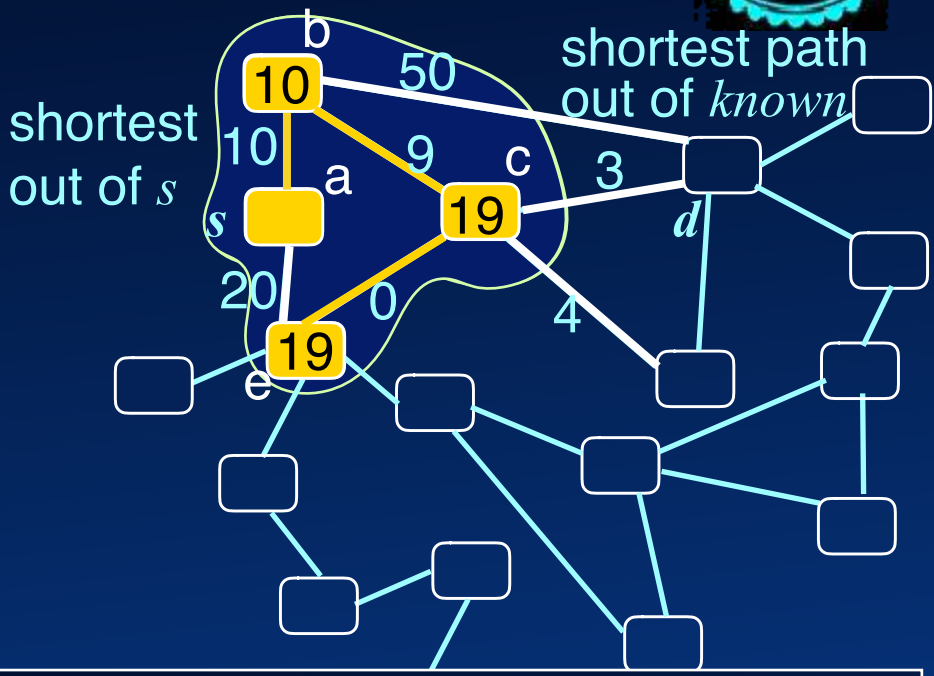

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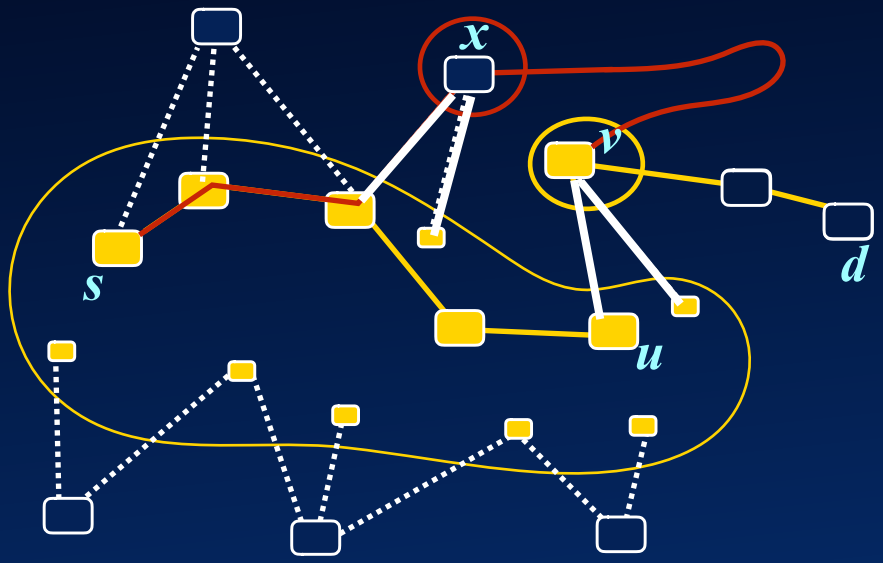

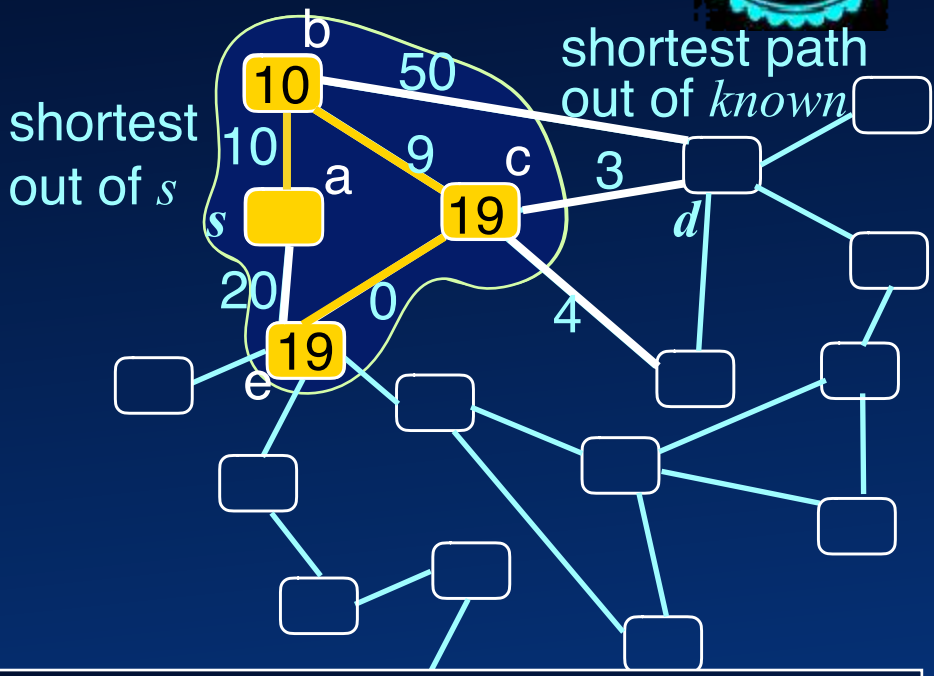

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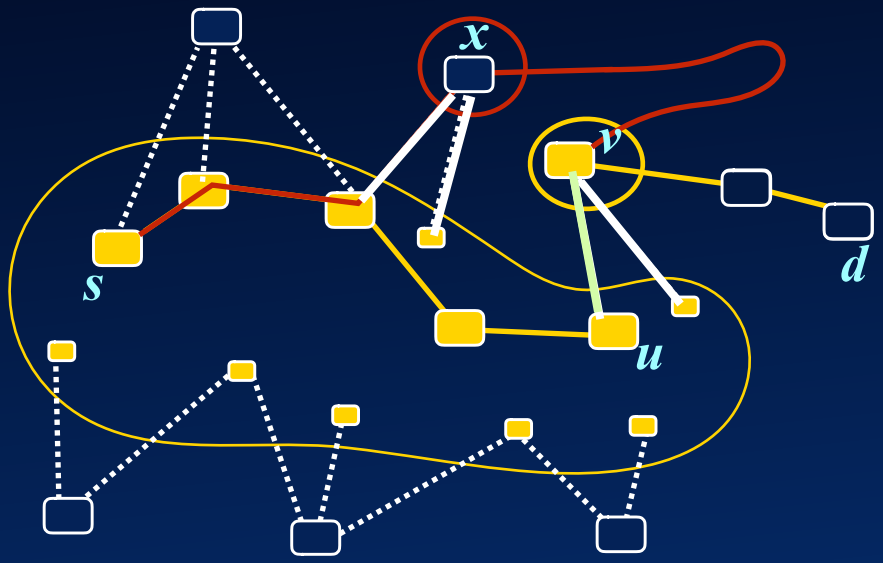

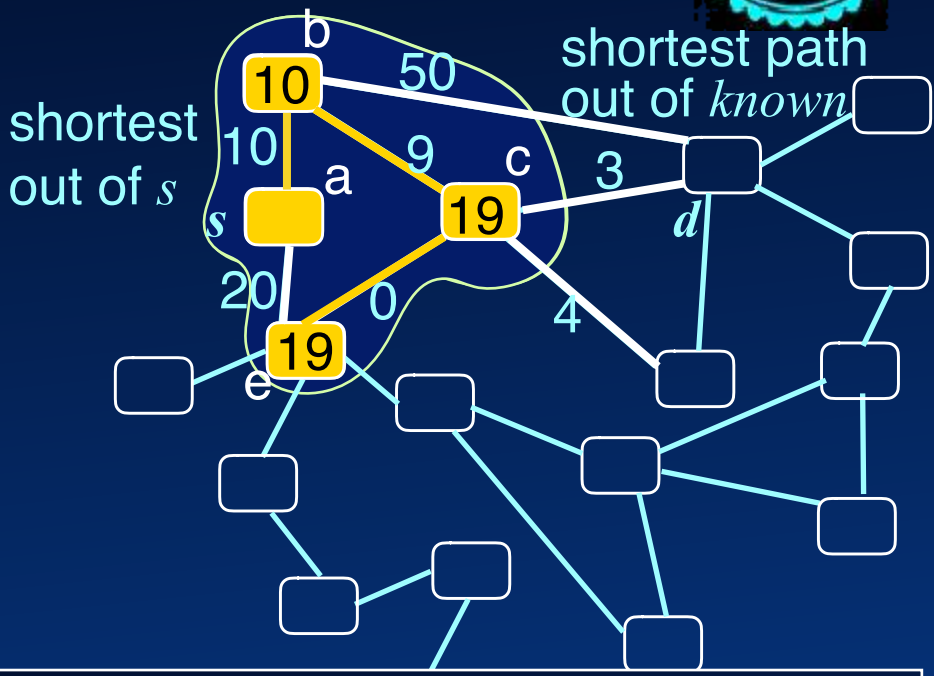

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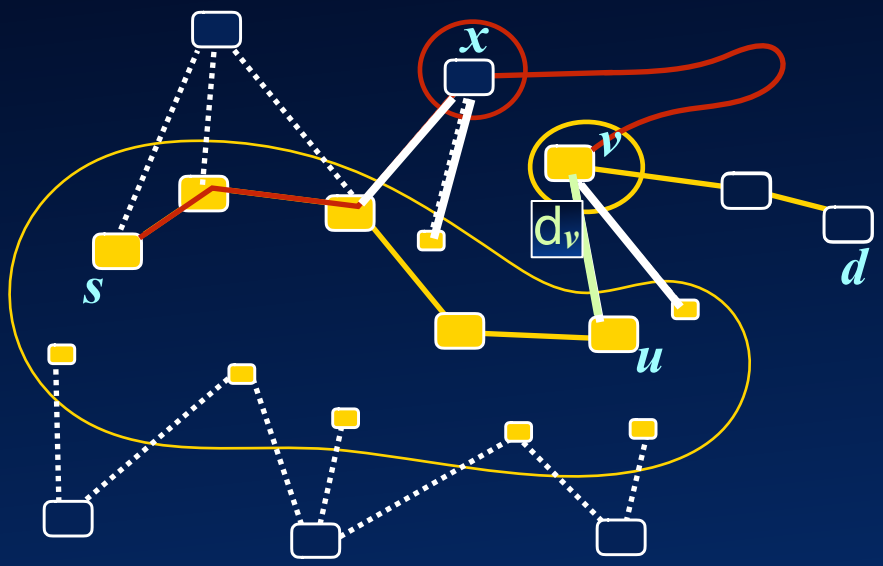

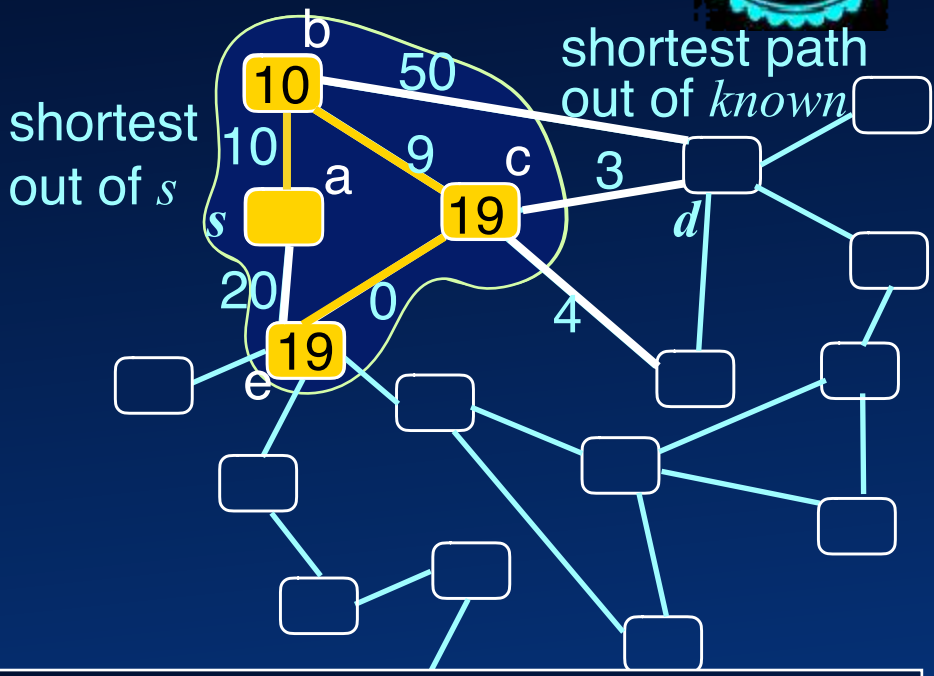

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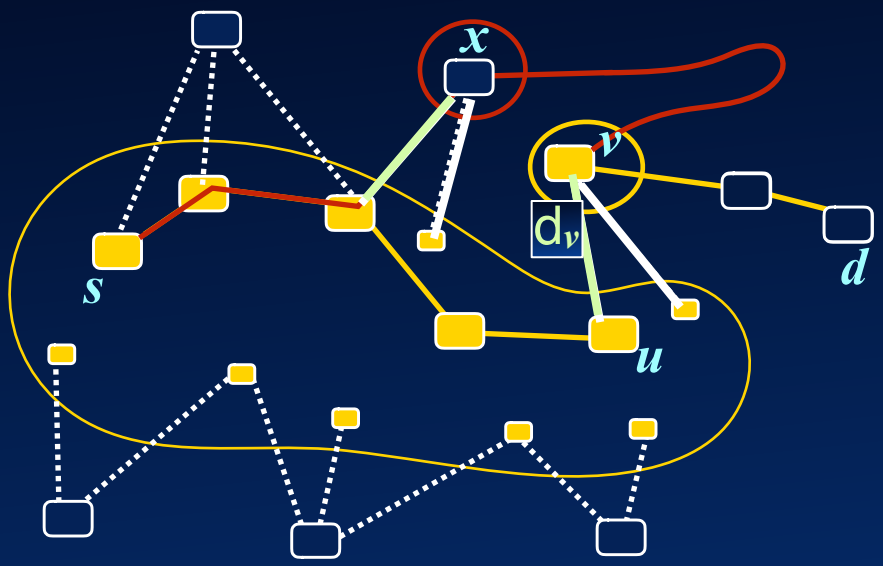

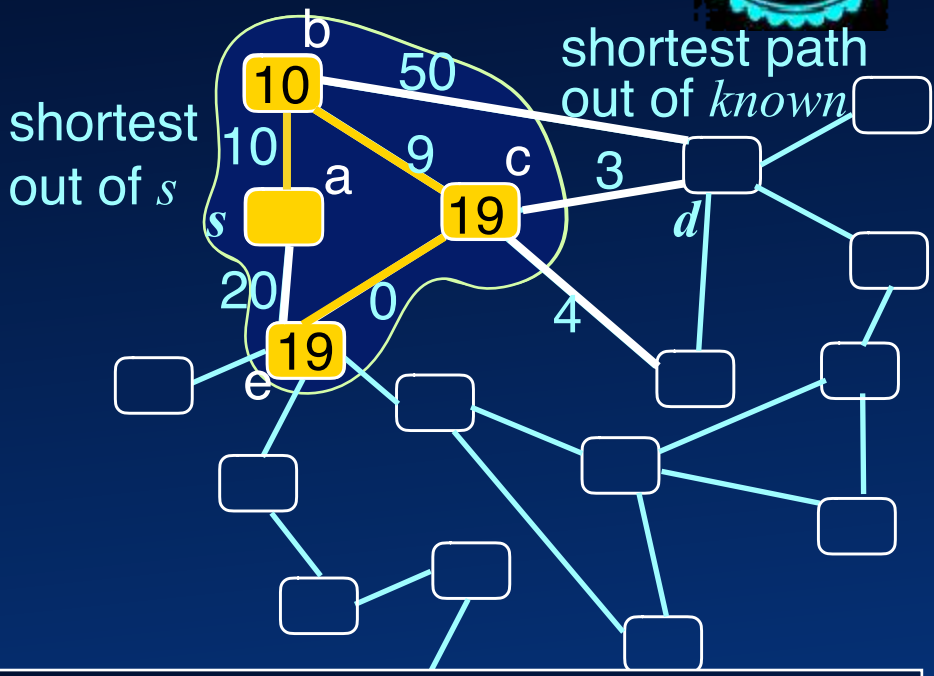

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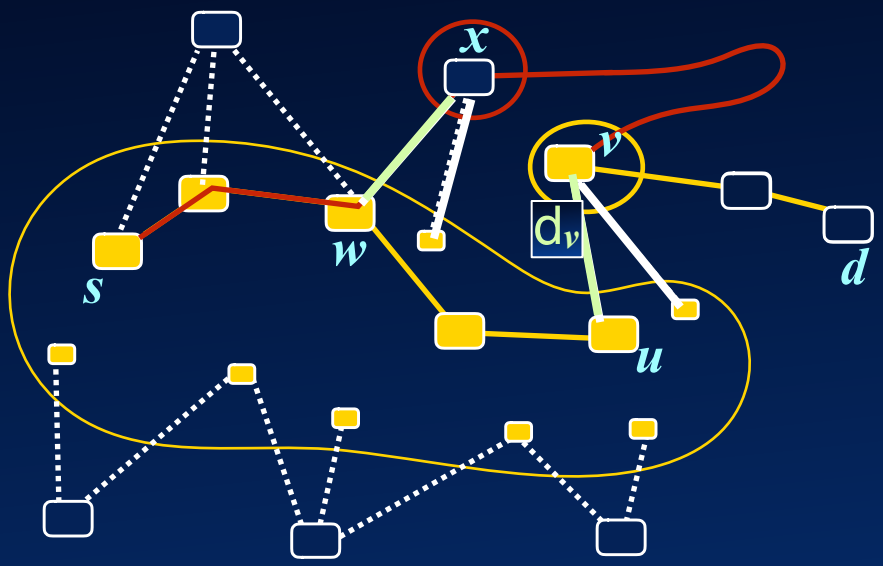

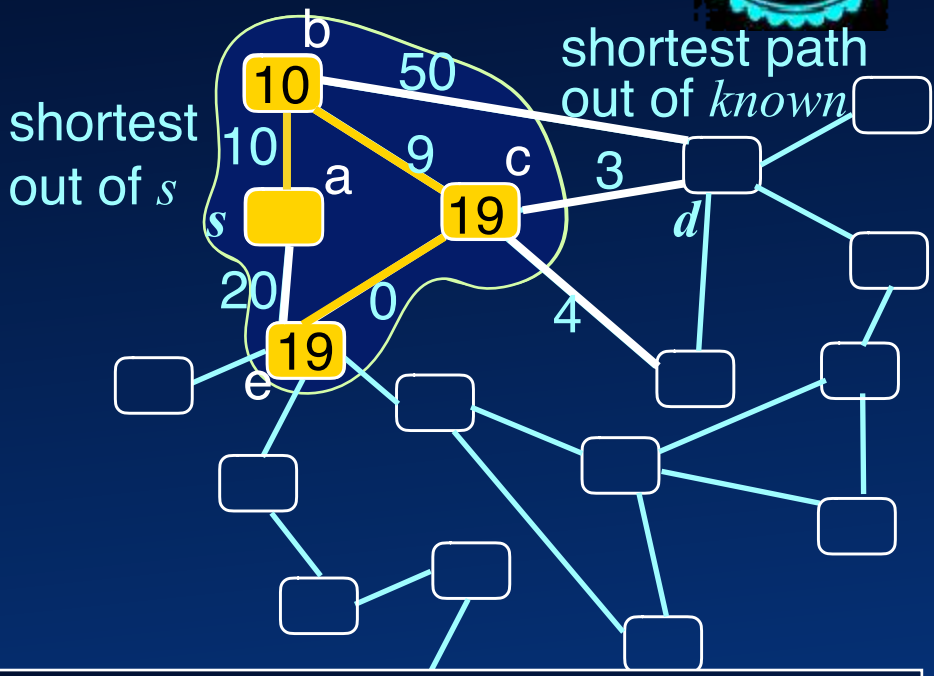

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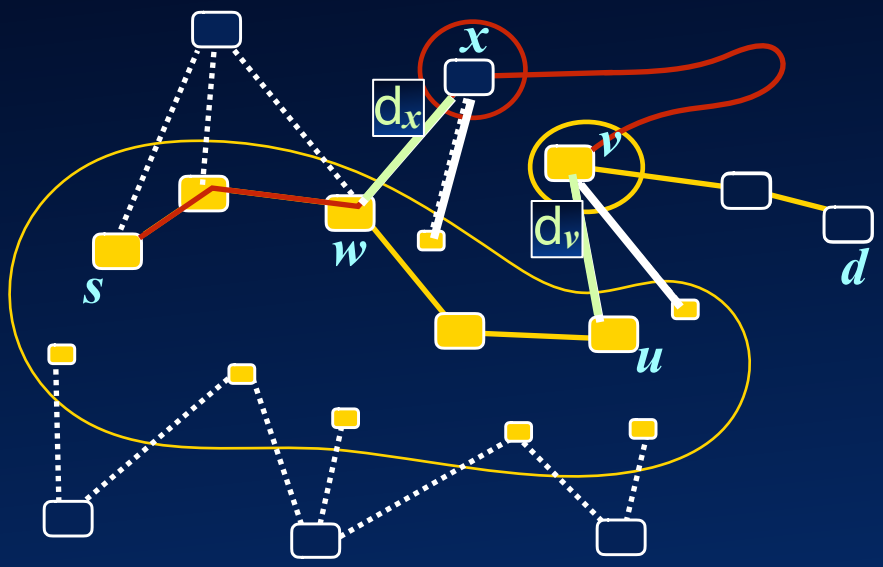

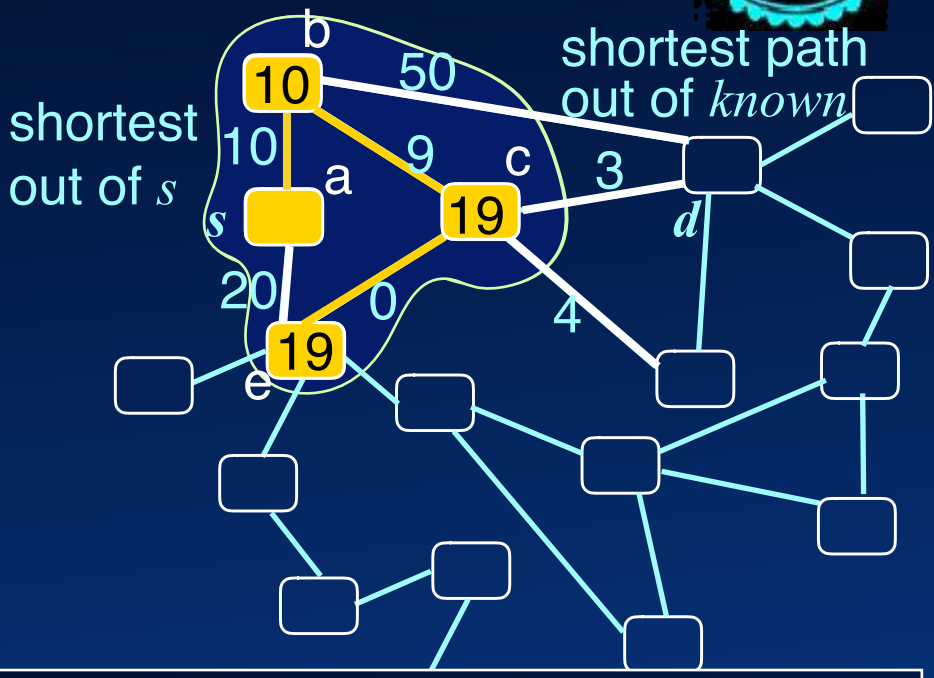

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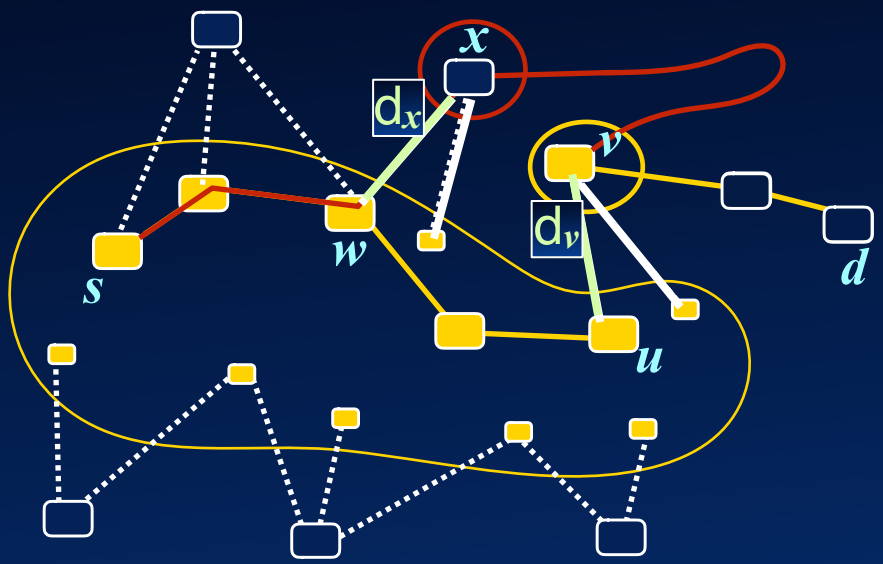

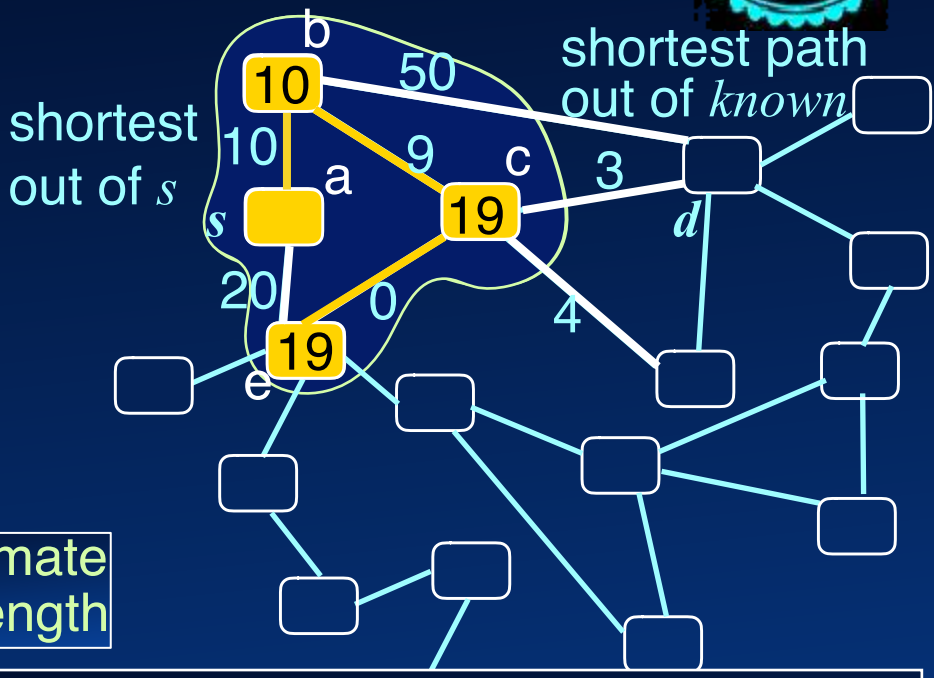

d_v is the smallest estimate
$\Rightarrow$ d_v is the shortest length

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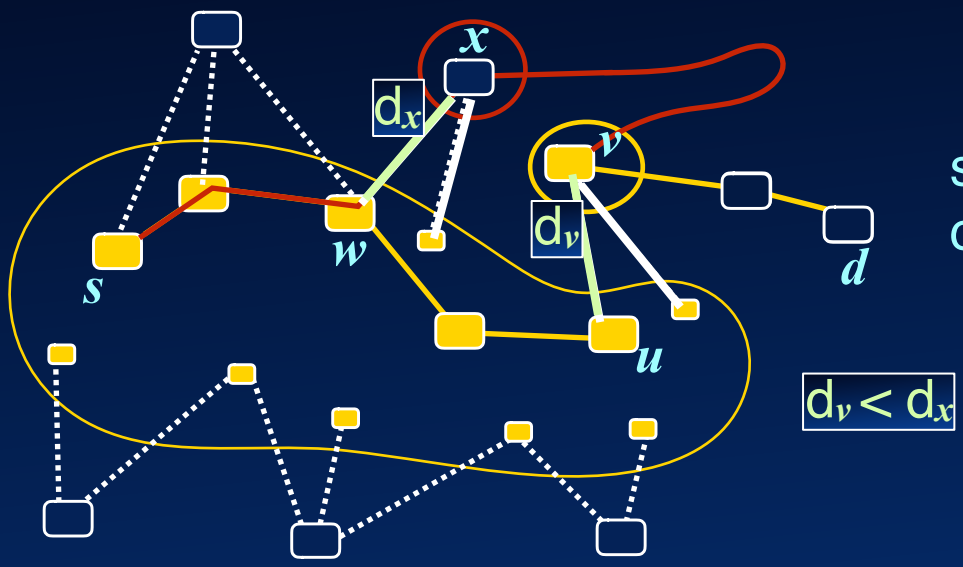

$d_v < d_x$

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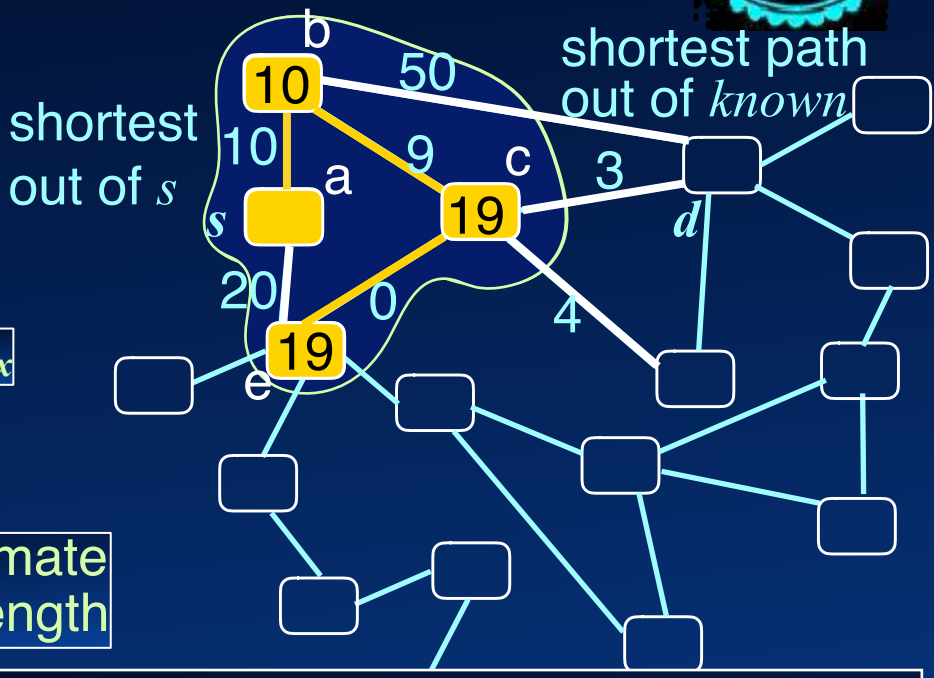

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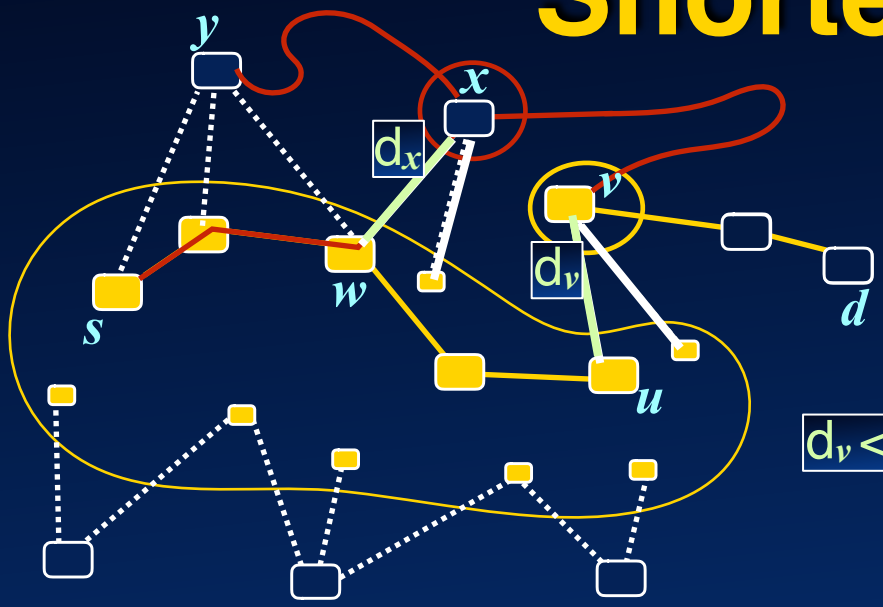

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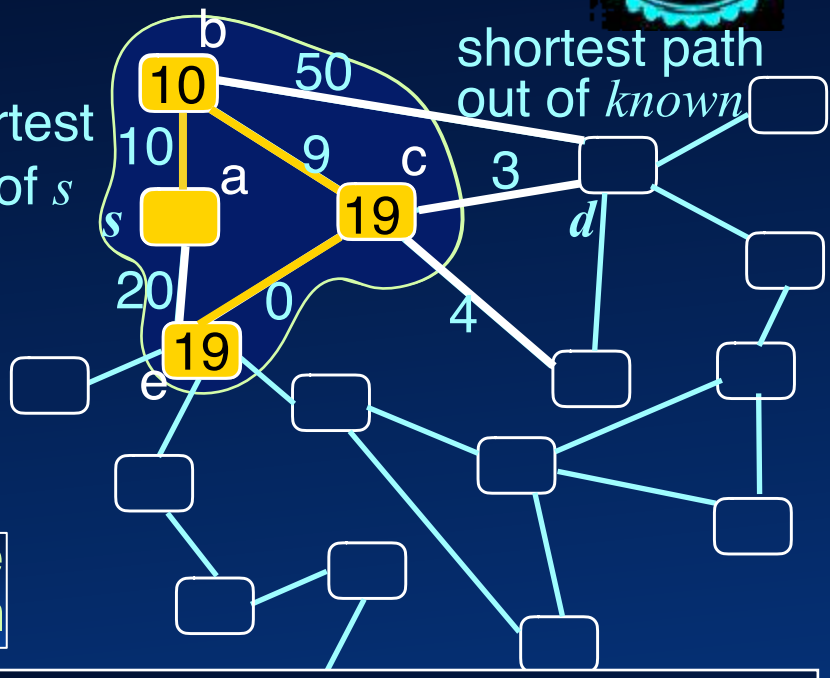

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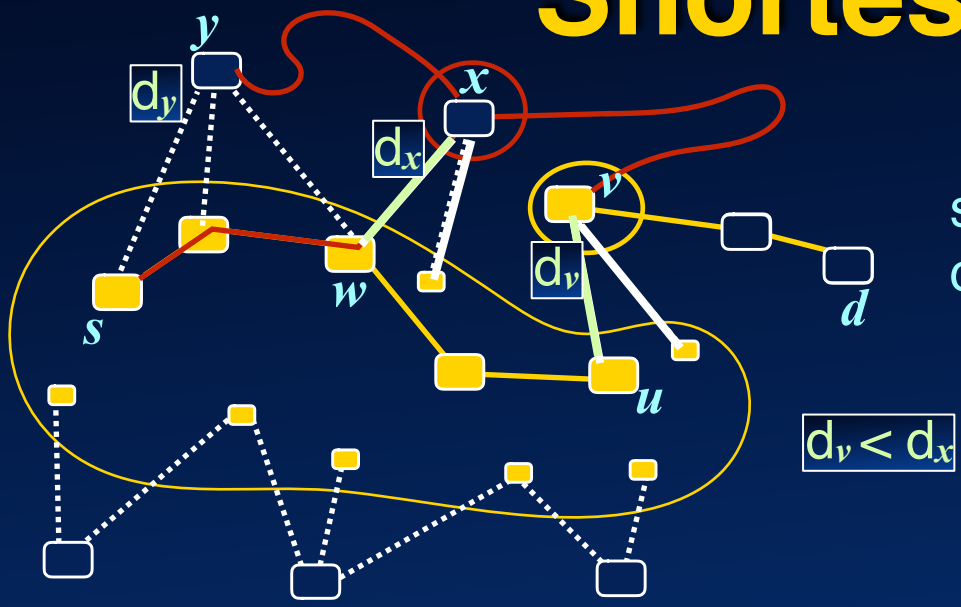

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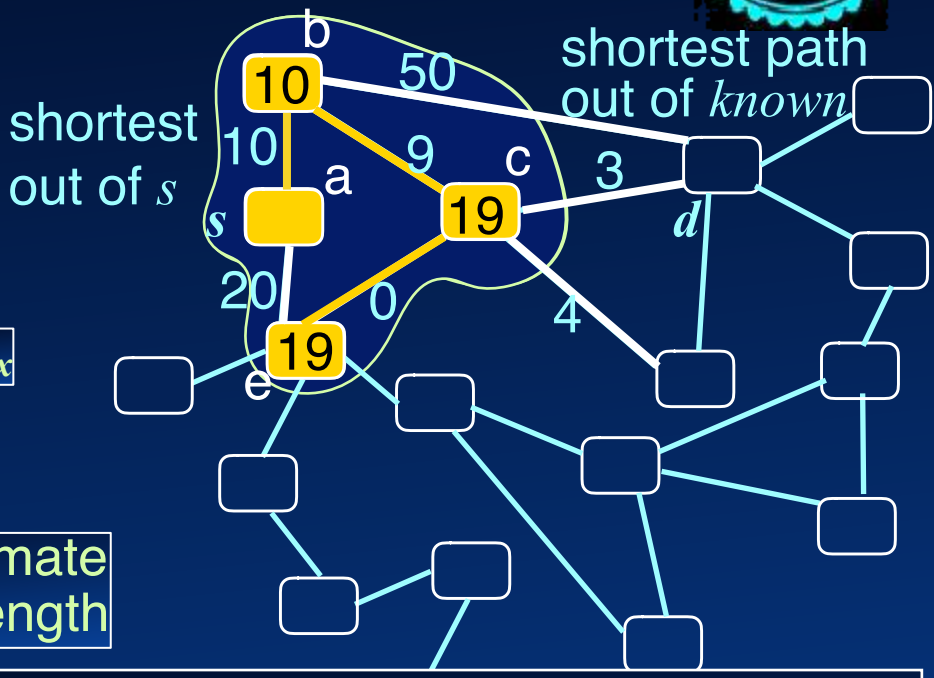

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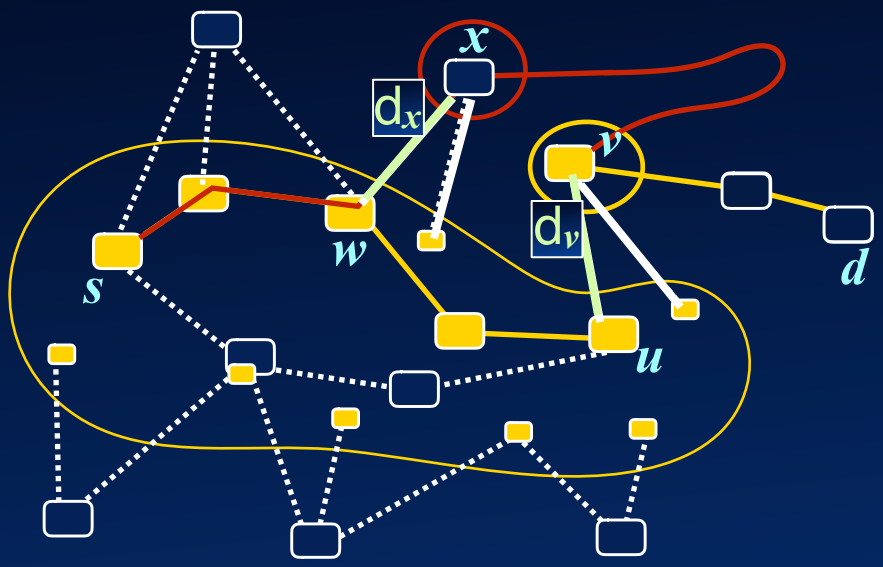
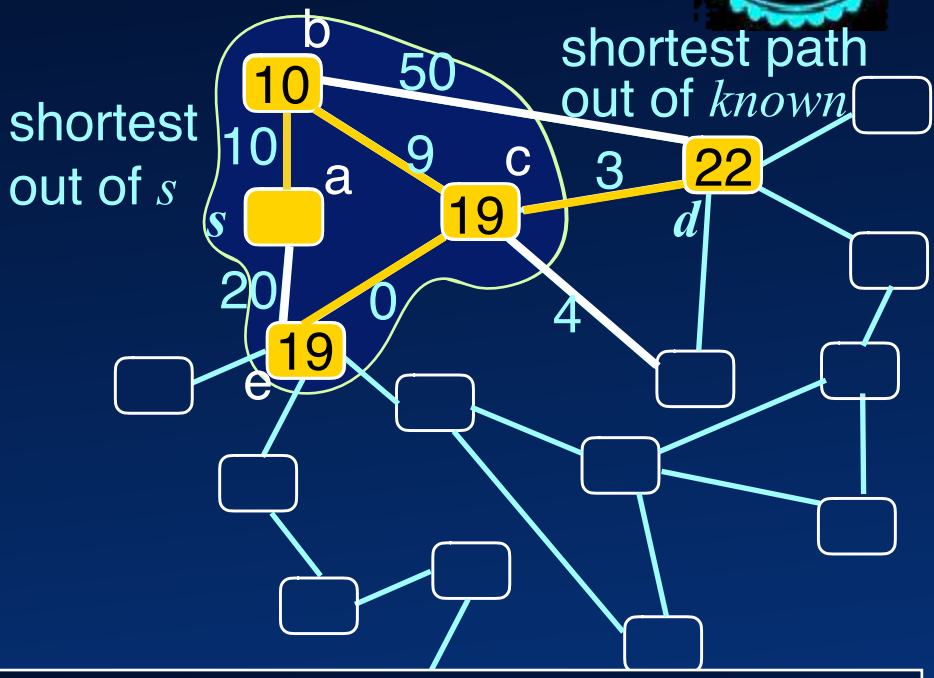
Mail: col106quiz@cse.iitd.ac.in
Format: f,t,t/f

T/F

- The shortest path from node u to node v in a directed graph is equal to the shortest path from node v to node u.
- The shortest path from node u to node v in an undirected graph is equal to the shortest path from node v to node u.
- If P_1 is the shortest path from node u to node w in a directed graph, and P_2 is the shortest path from w to node v, P_1P_2 (i.e., P_1 followed by P_2) is the shortest path from u to v.

Shortest Path





Induction Hypothesis:

Shortest path from s to all nodes in cluster is given

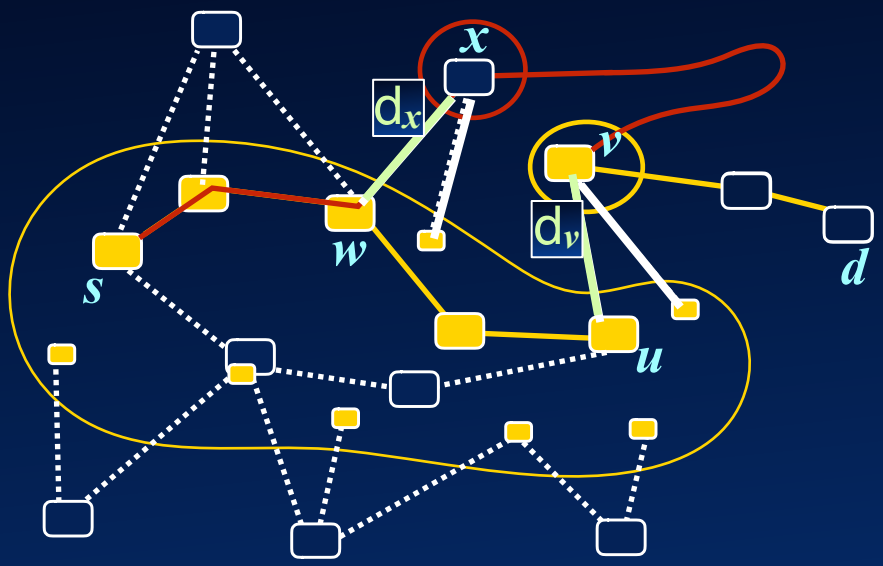
Induction step:

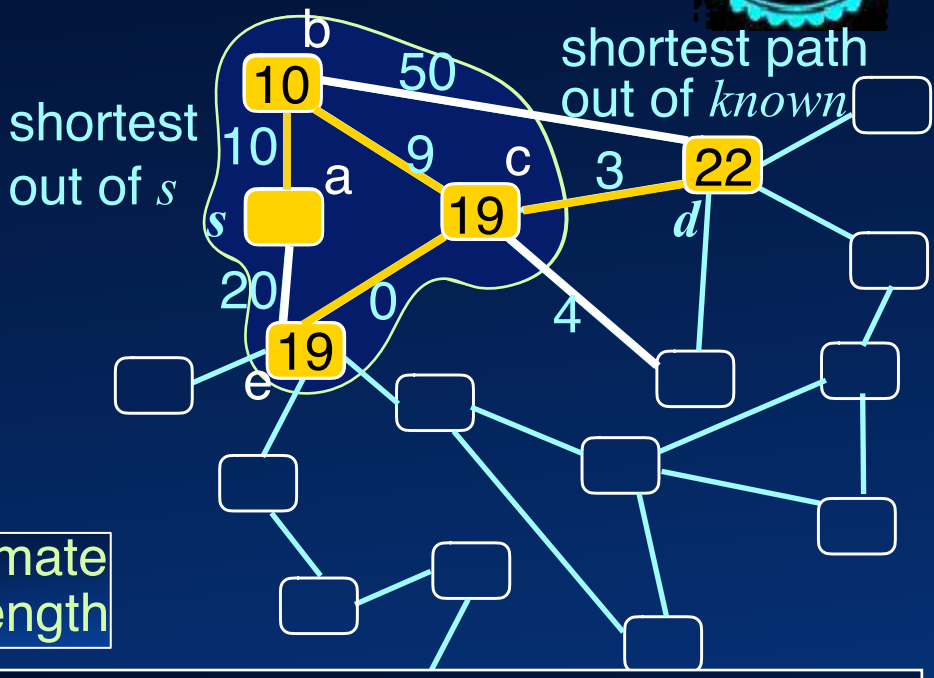
Find the shortest path from s to the next node added to the cluster

Base:

Cluster = {s}

Shortest Path





d_v is the smallest estimate
$\Rightarrow$ d_v is the shortest length

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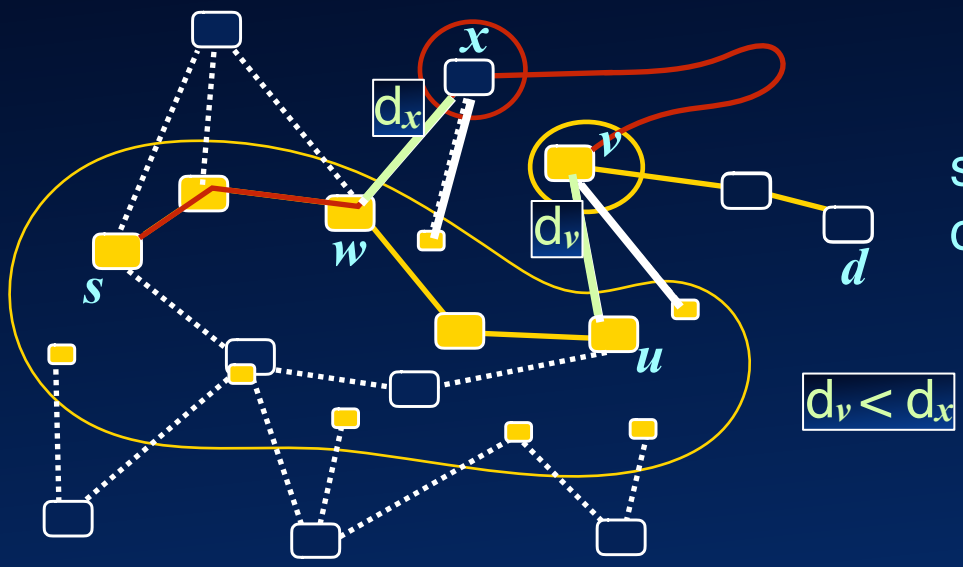

$d_v < d_x$

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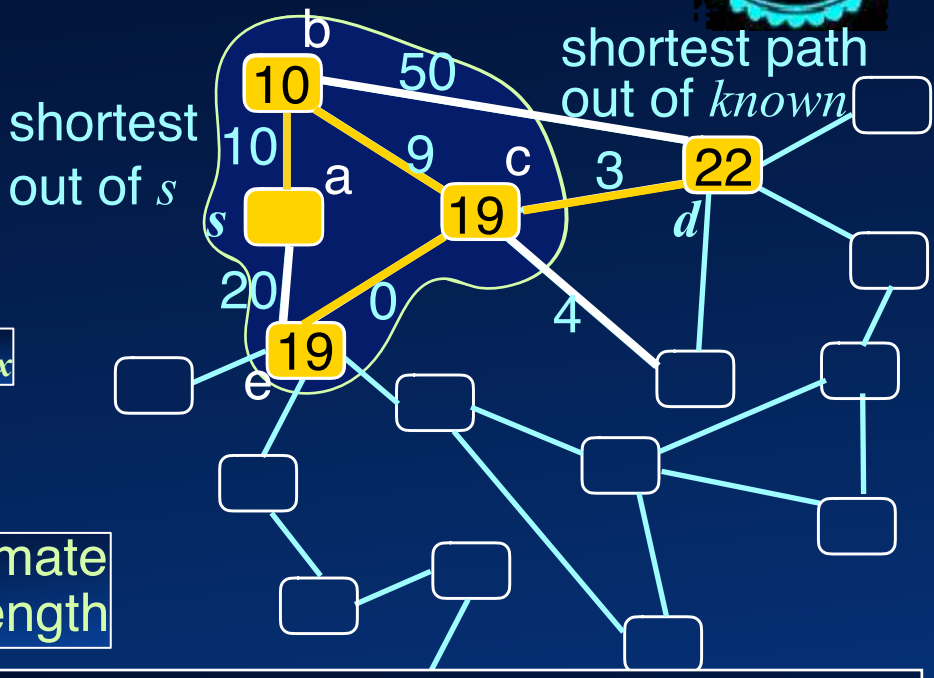

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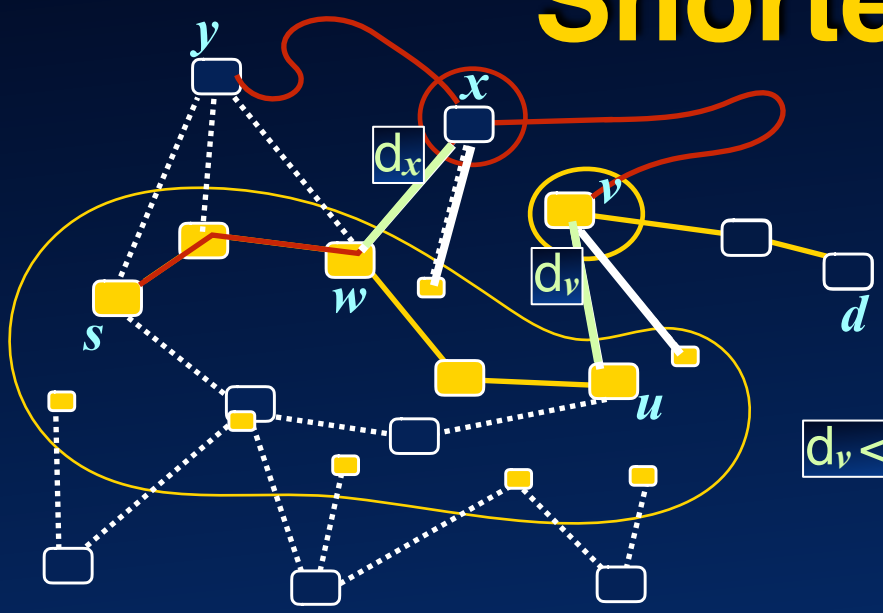

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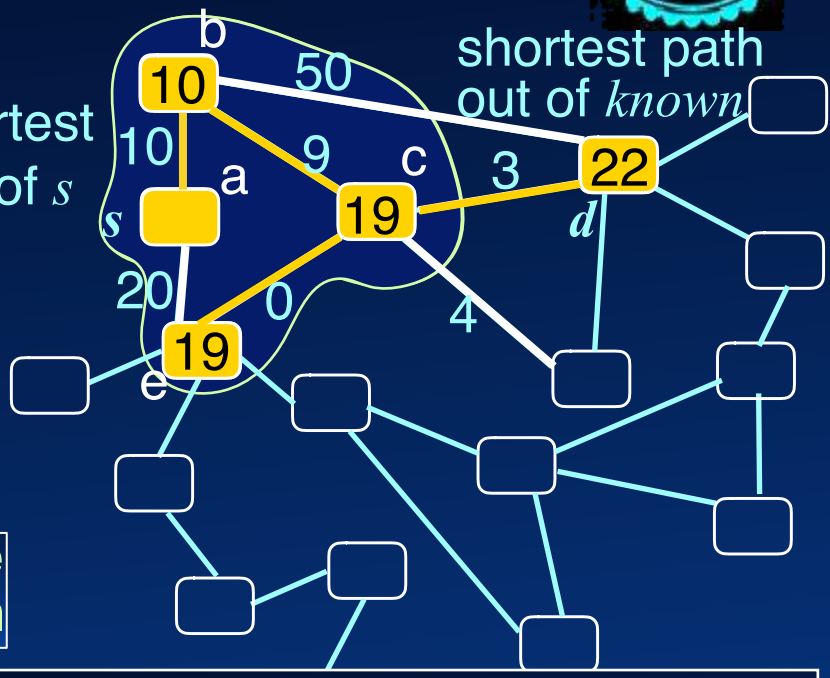

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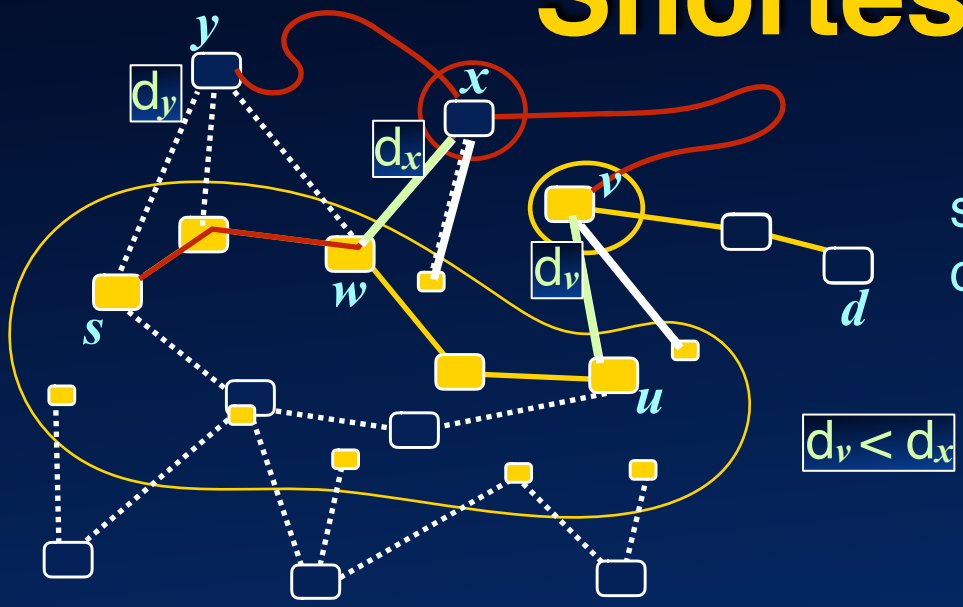

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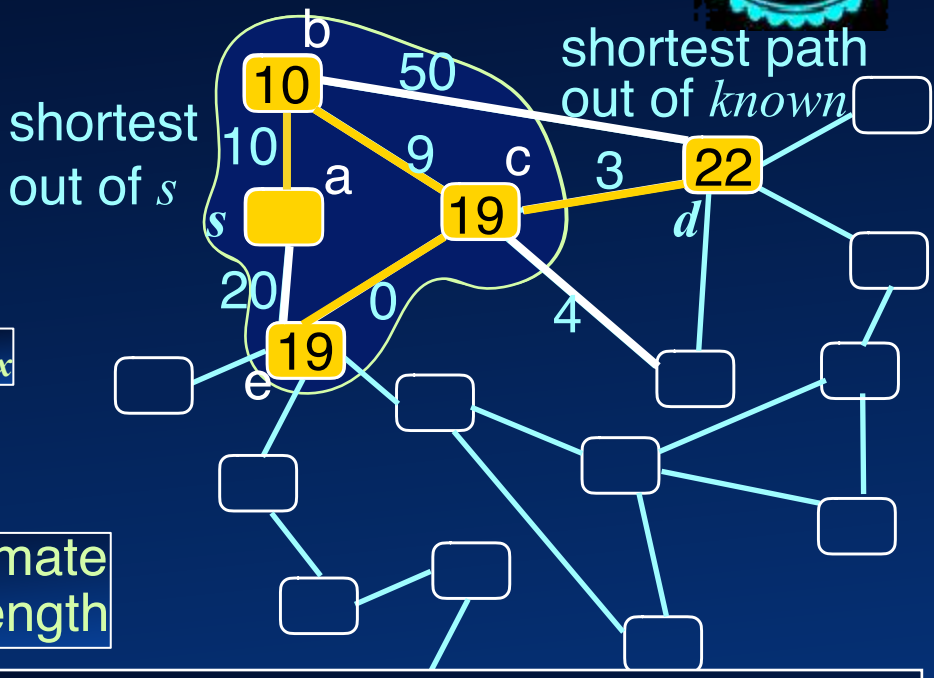

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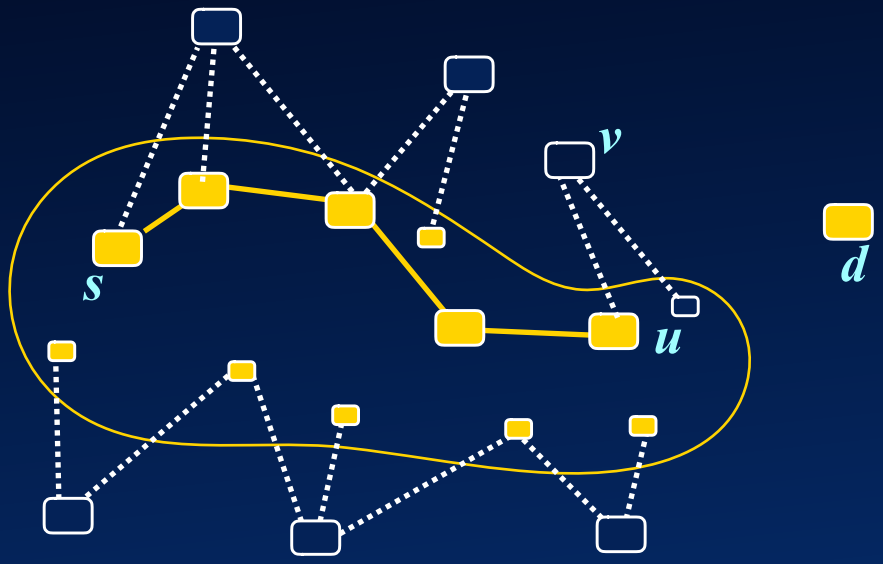

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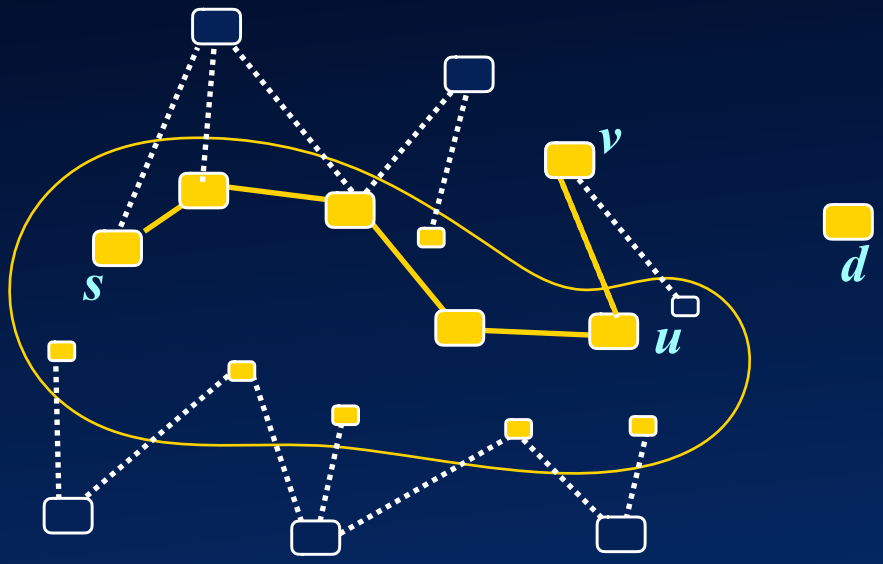

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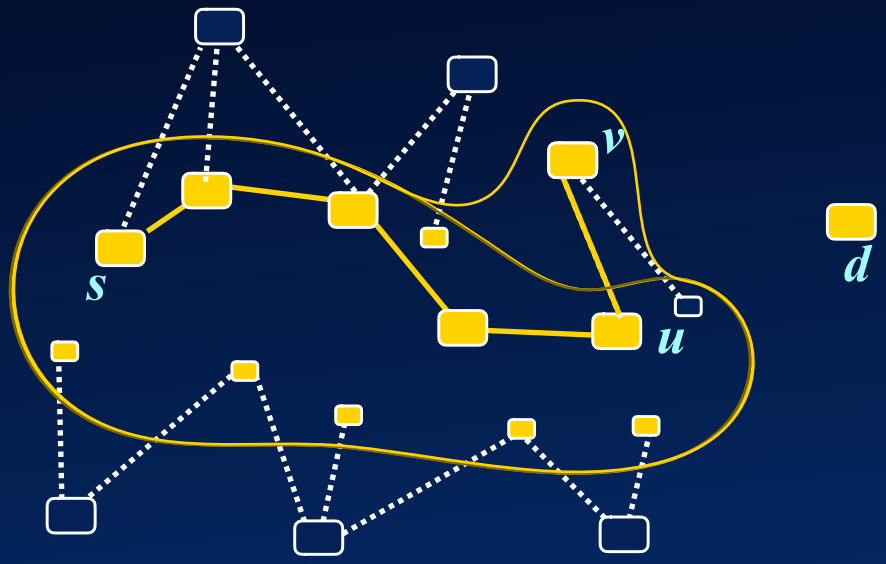

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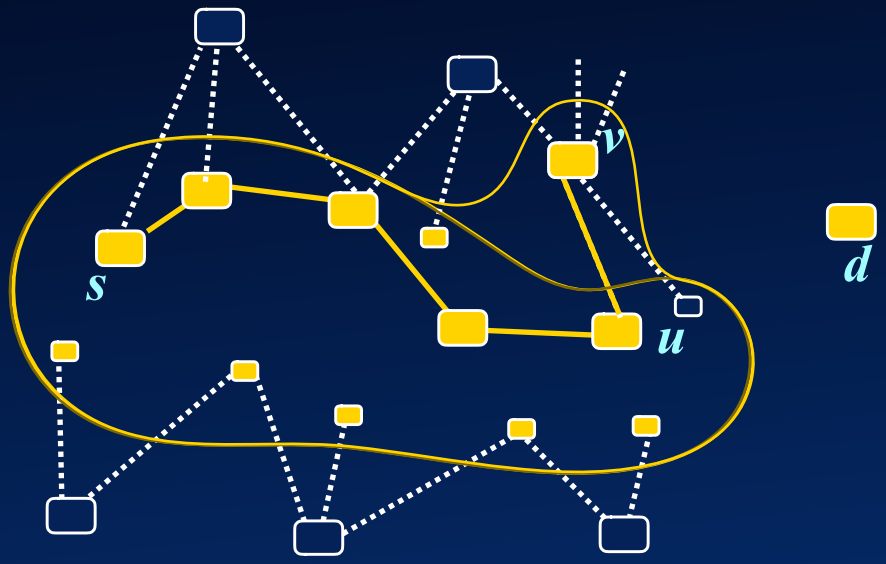

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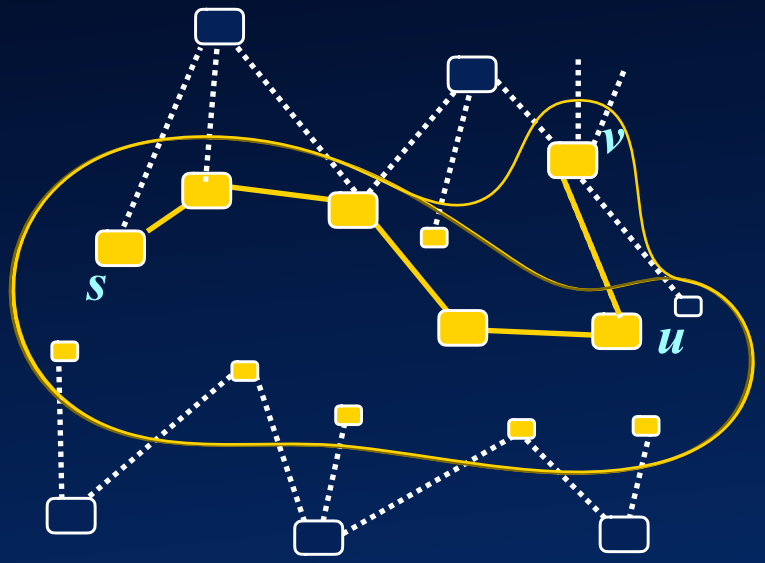

Dijkstra's Algorithm

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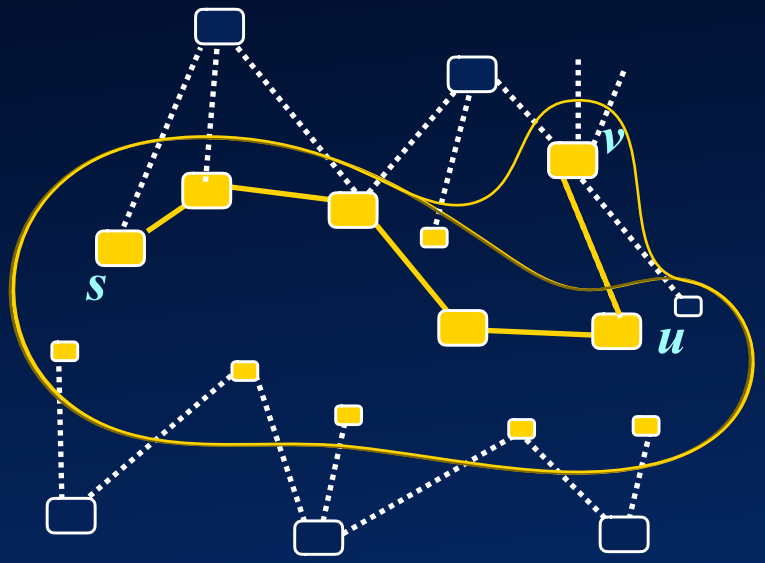
Shortest Path



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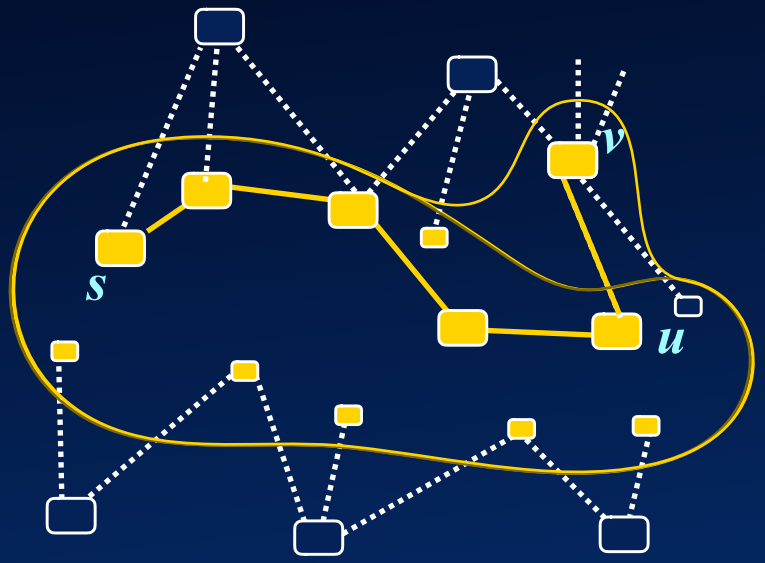

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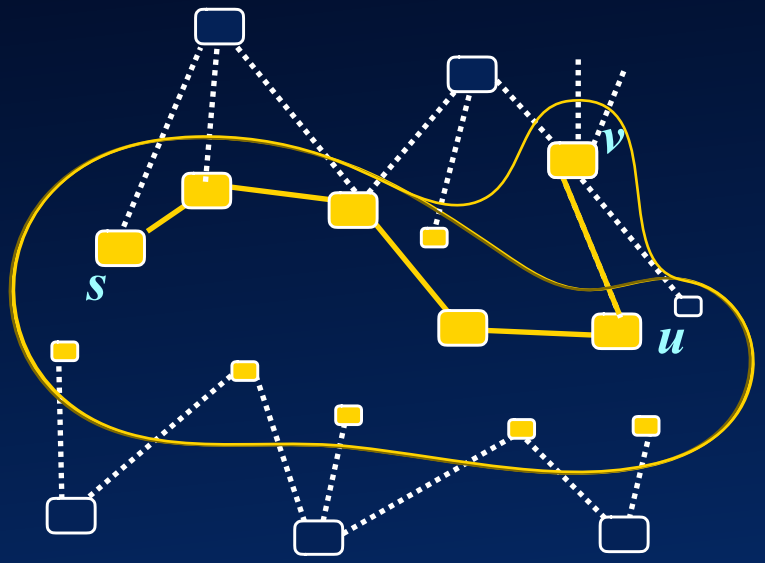

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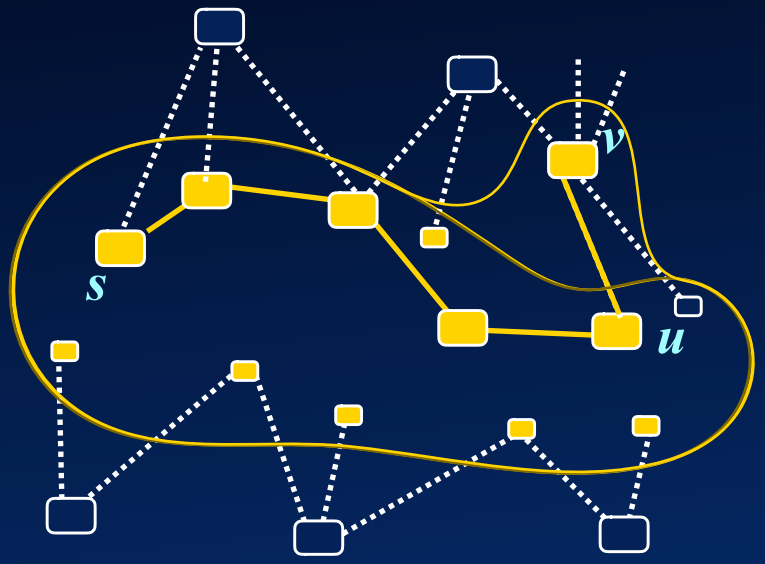

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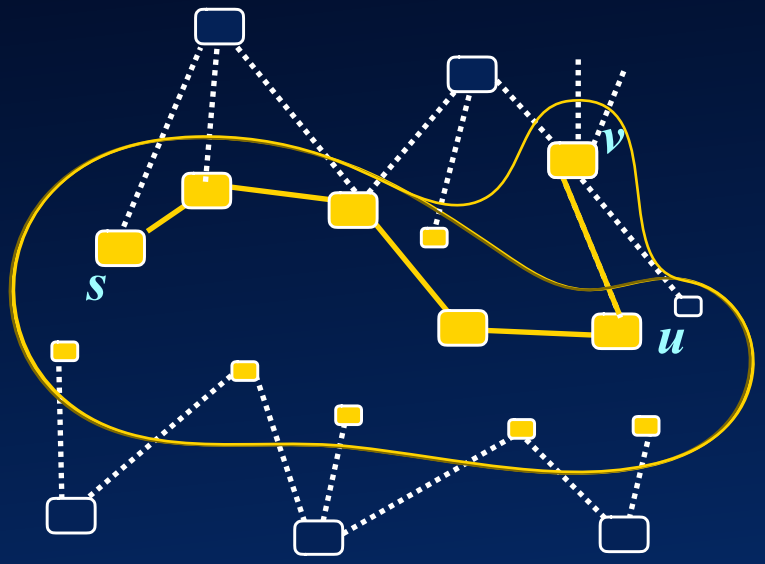

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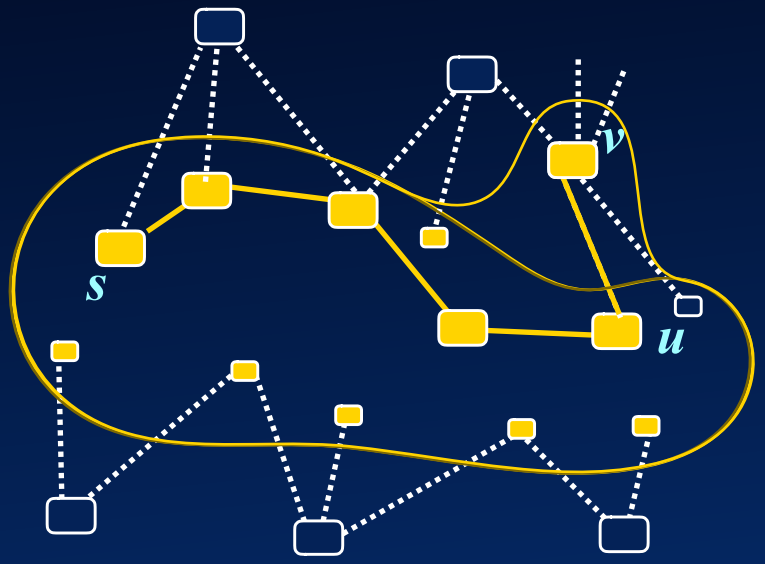

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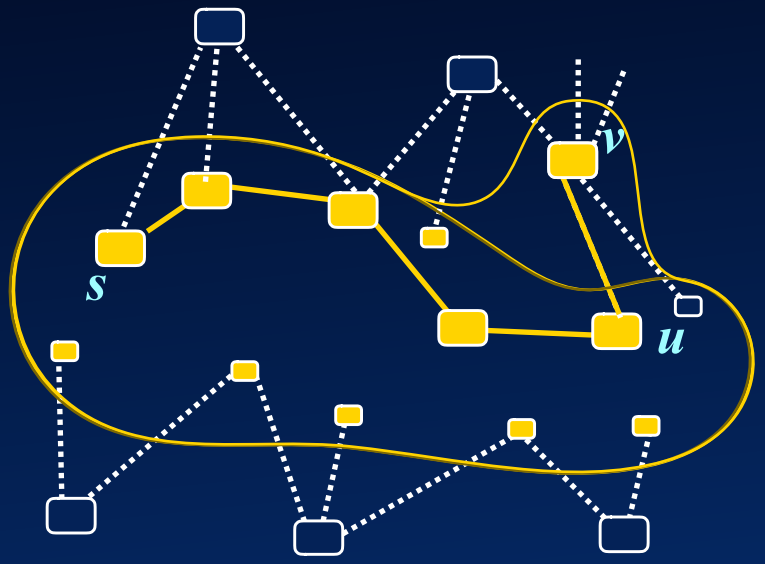
Shortest Path



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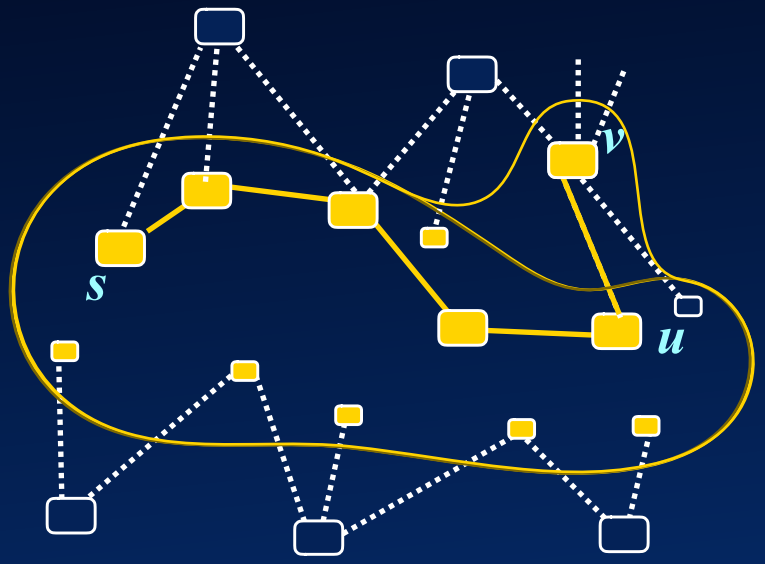
Shortest Path



d

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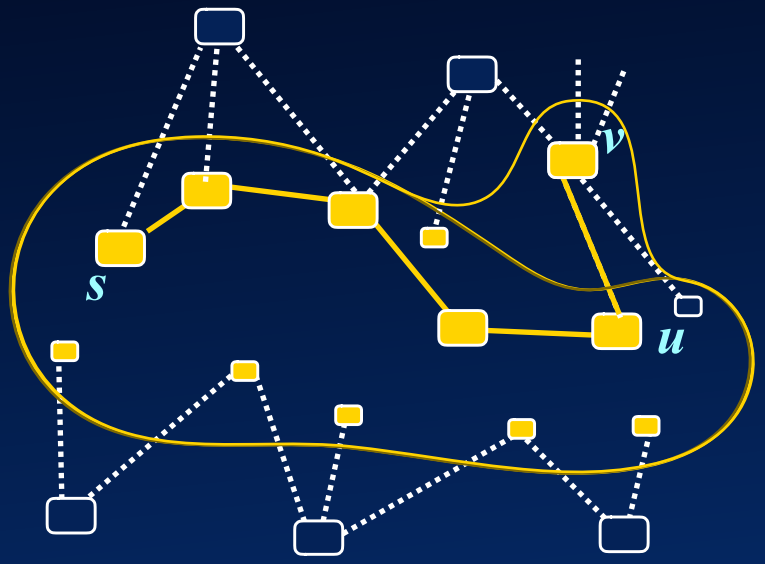

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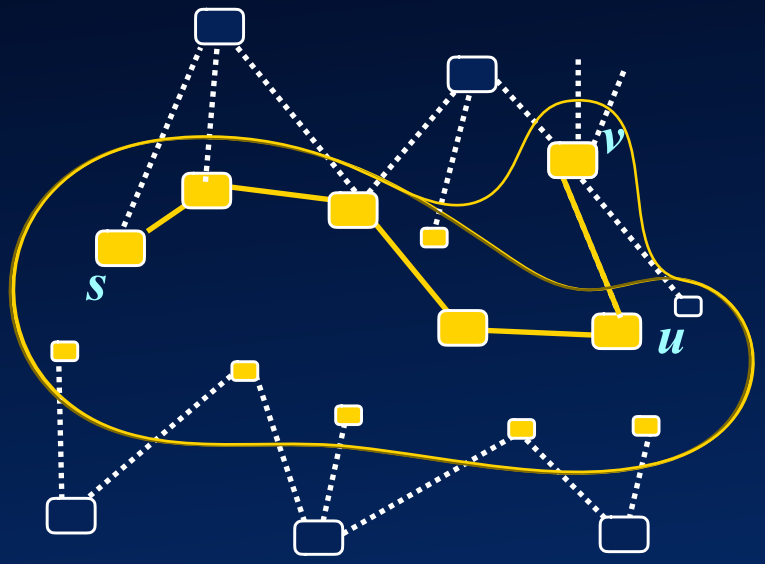

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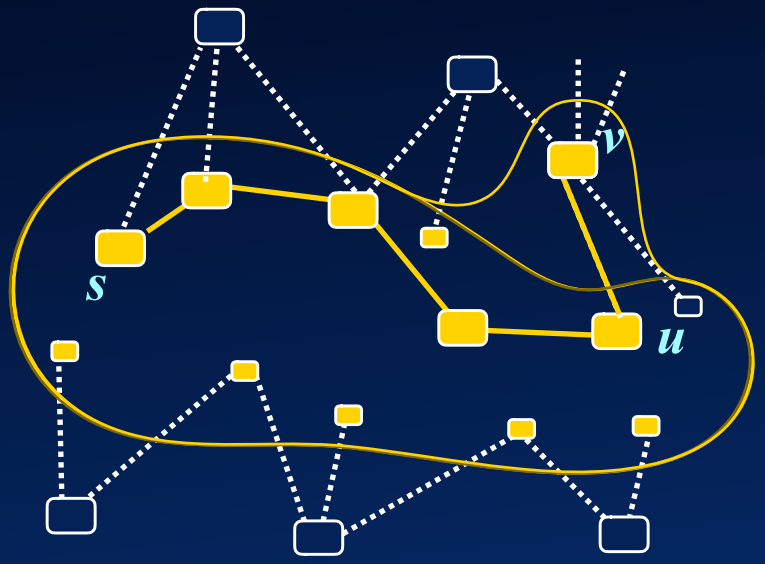

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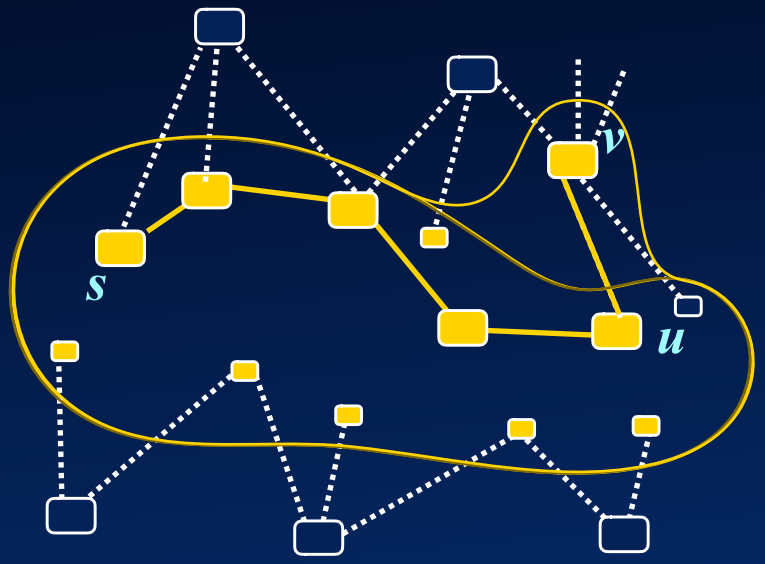

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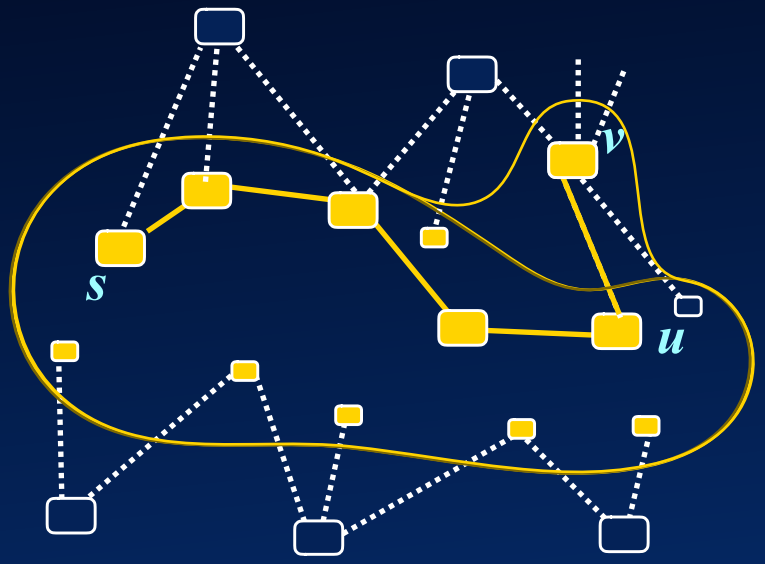

d

∑degree(u) = *m*

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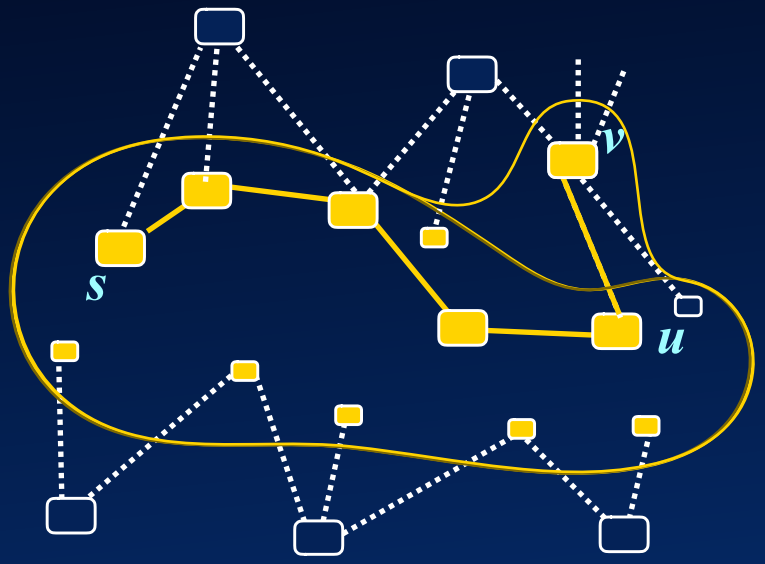

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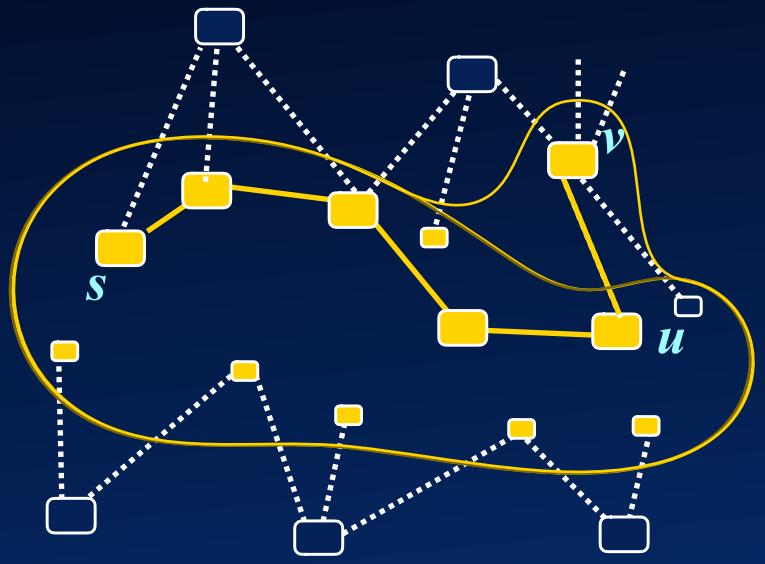
$\sum$degree(u) = m

n * O(log n): PQ.next()

m * O(log n): PQ.edit

Shortest Path



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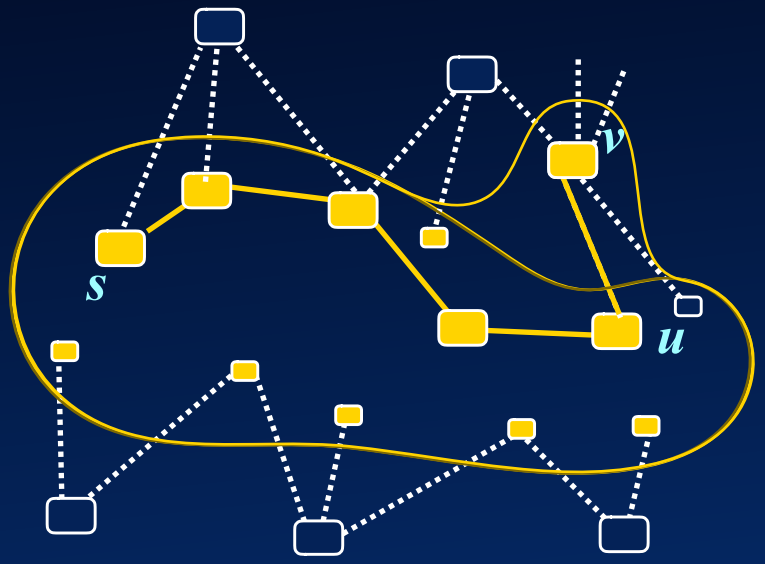
$O(m+n)\log n$ if PQ is a heap

Shortest Path



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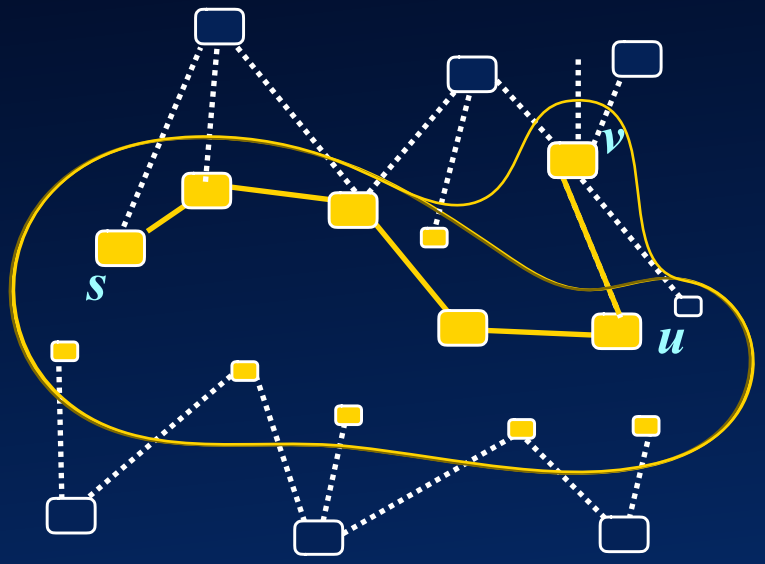
$O(m+n)\log n$ if PQ is a heap

Shortest Path



Heap

d

Dijkstra's Algorithm

```
Shortest(s,d):
  s.dist = 0
  u.dist = ∞ ∀ u ∈ G-s                    ← n
  PQ.insert(G.nodes)                      ← n
  while PQ.hasNext():                     ← n
    v = PQ.next()   if v == d
    for w in v.adjacent():                ← degree
      if(w.dist > v.dist + length(v,w))
        w.dist = v.dist + length(v,w)
        ~~if(w.equals(d)) return w.dist~~
        PQ.edit(w)
```

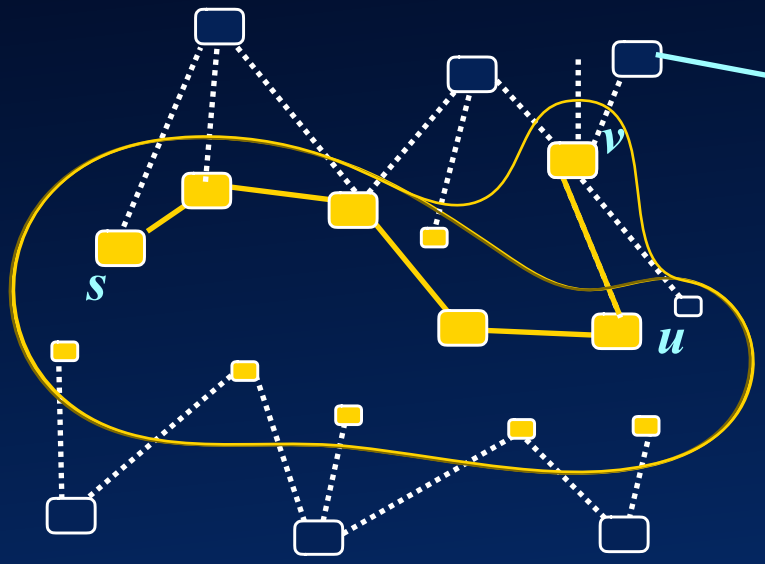
$\sum$degree(u) = m

n * O(log n): PQ.next()

m * O(log n): PQ.edit

$O(m+n)\log n$ if PQ is a heap

Shortest Path



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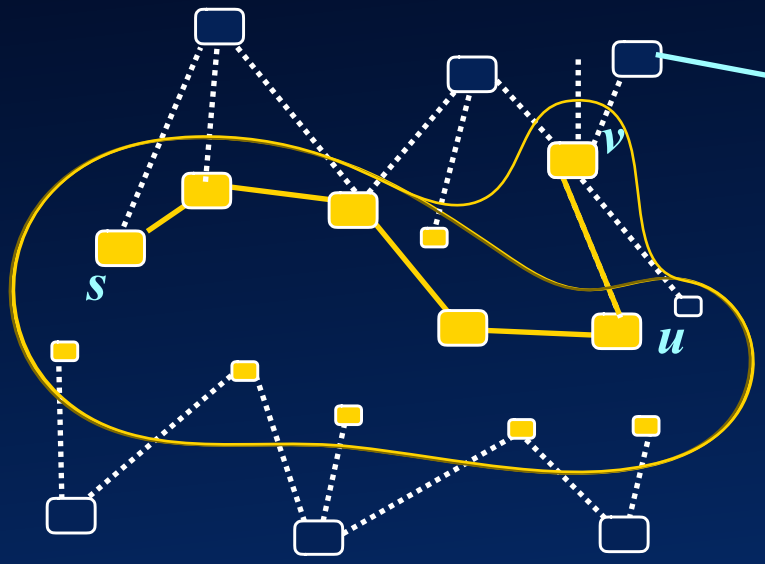
$O(m+n)\log n$ if PQ is a heap

Shortest Path



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```

$\sum$degree(u) = m

n * O(log n): PQ.next()

m * O(log n): PQ.edit

$O(m+n)\log n$ if PQ is a heap

$O(n^2)$ if PQ is a list