Data Structures & Algorithms

Subodh Kumar

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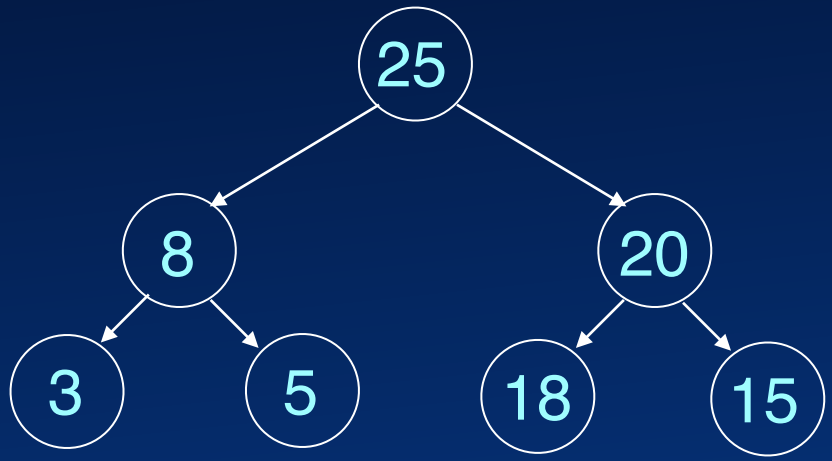
Dept of Computer Sc. & Engg.

Heap

- Left-complete tree
- Comparable keys
- "Top" key in the root
  - For every subtree



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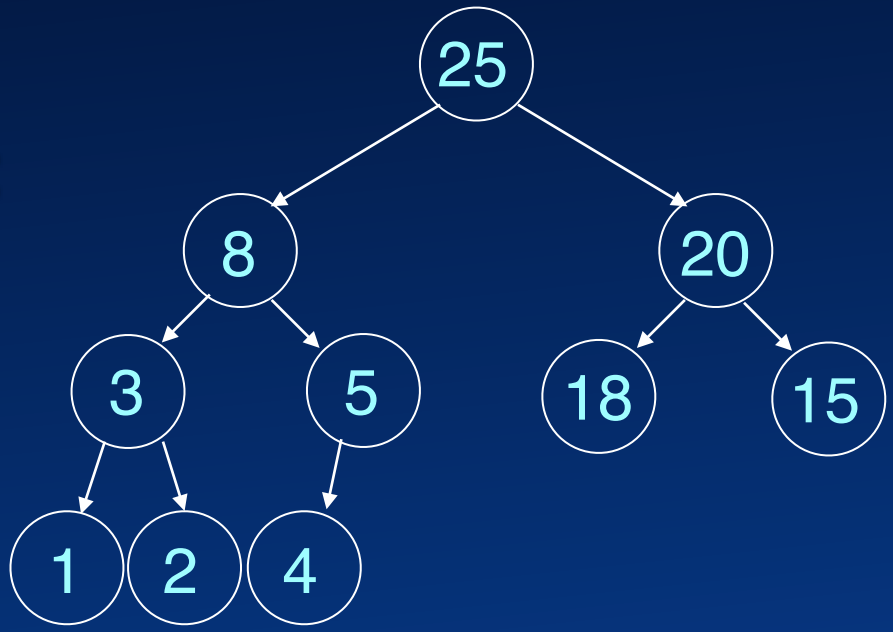

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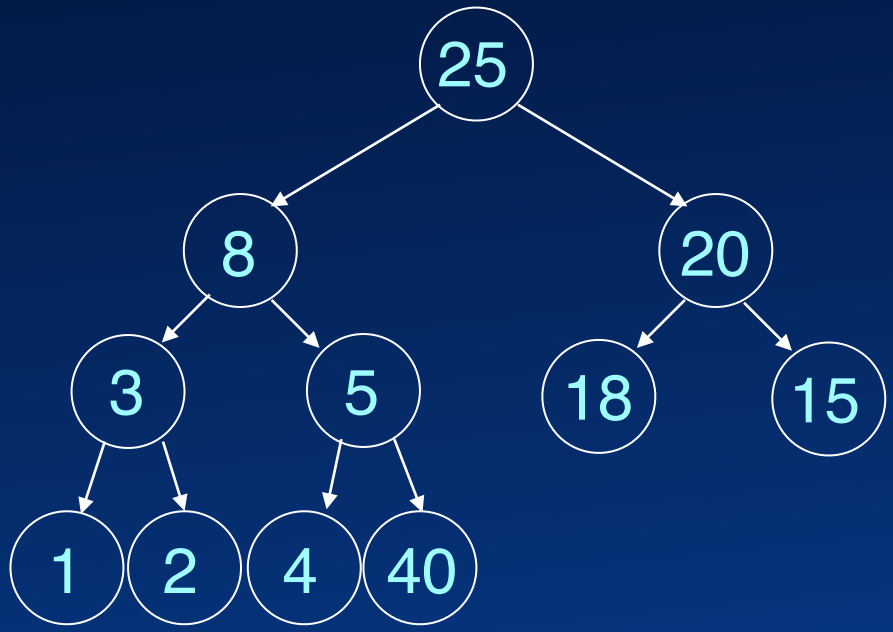

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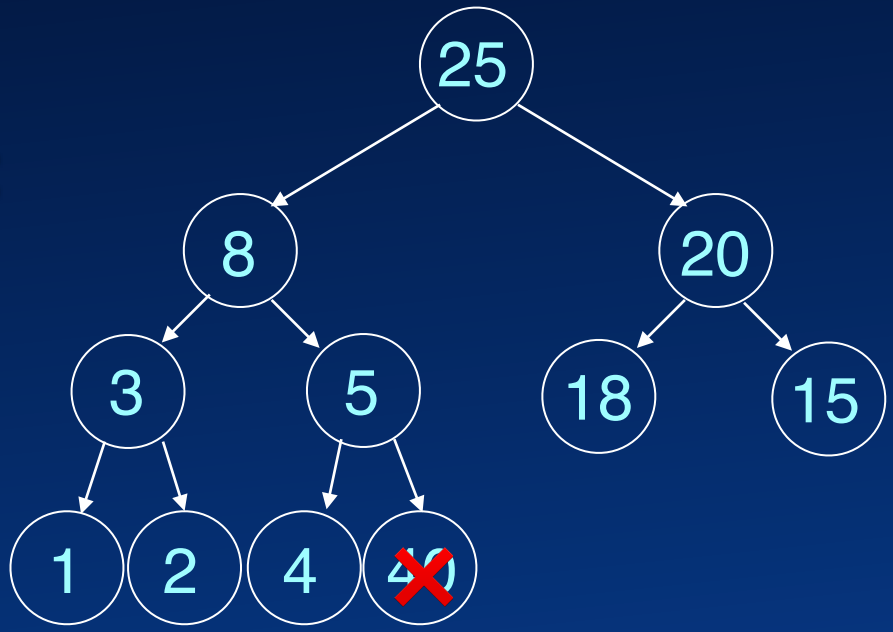

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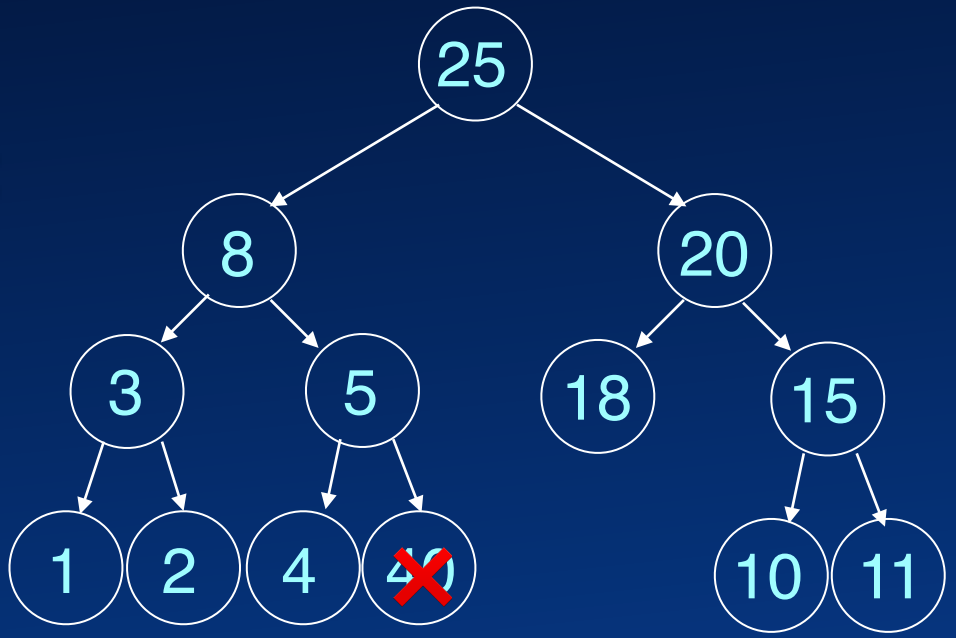

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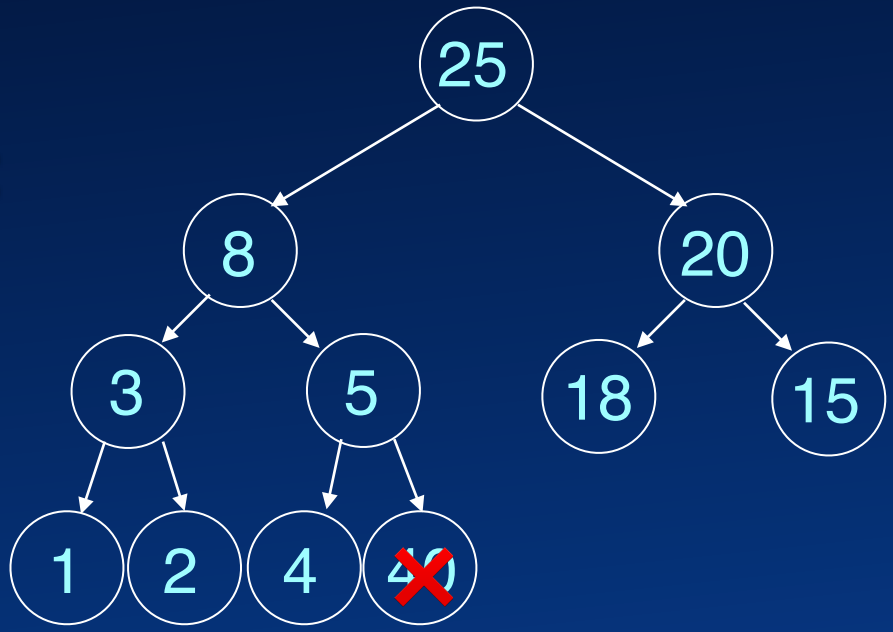

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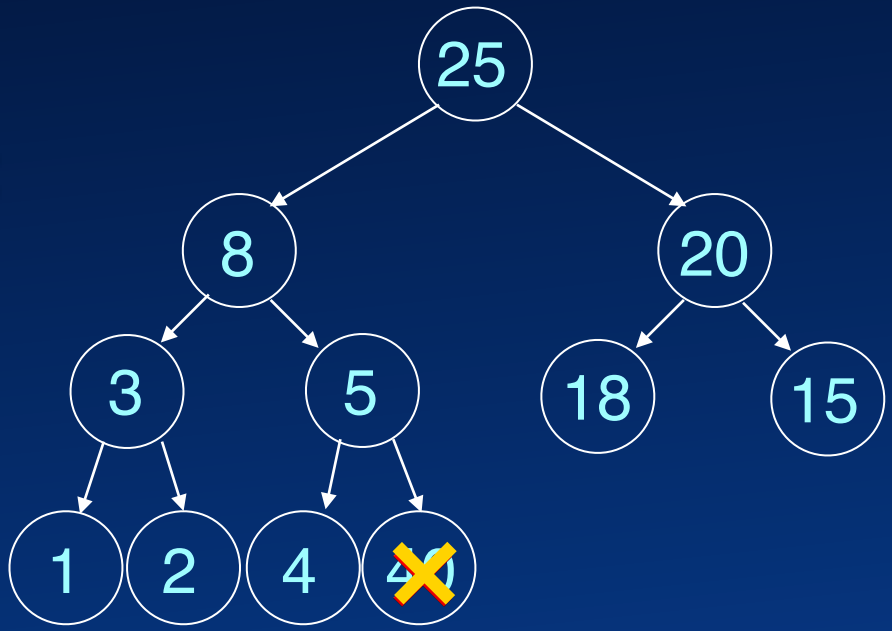

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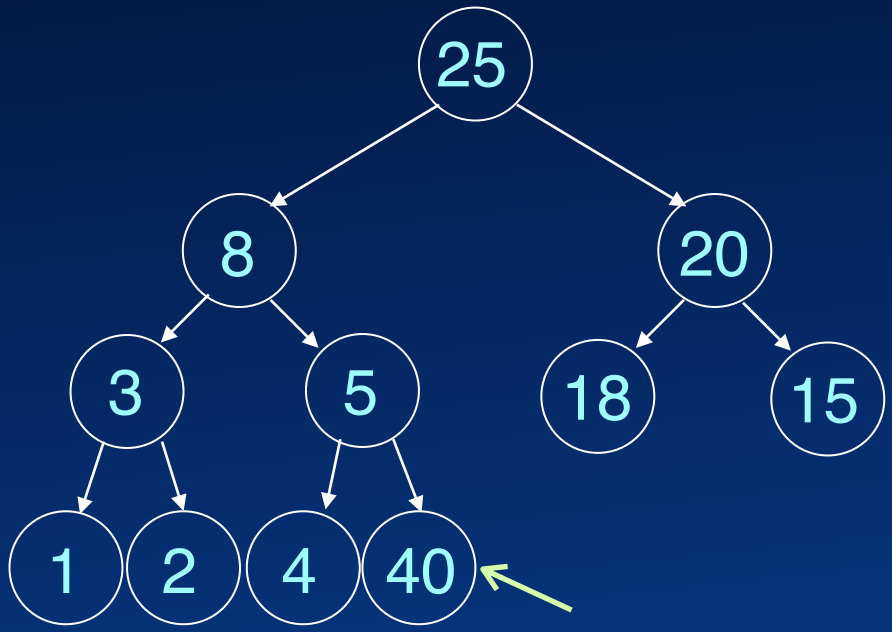

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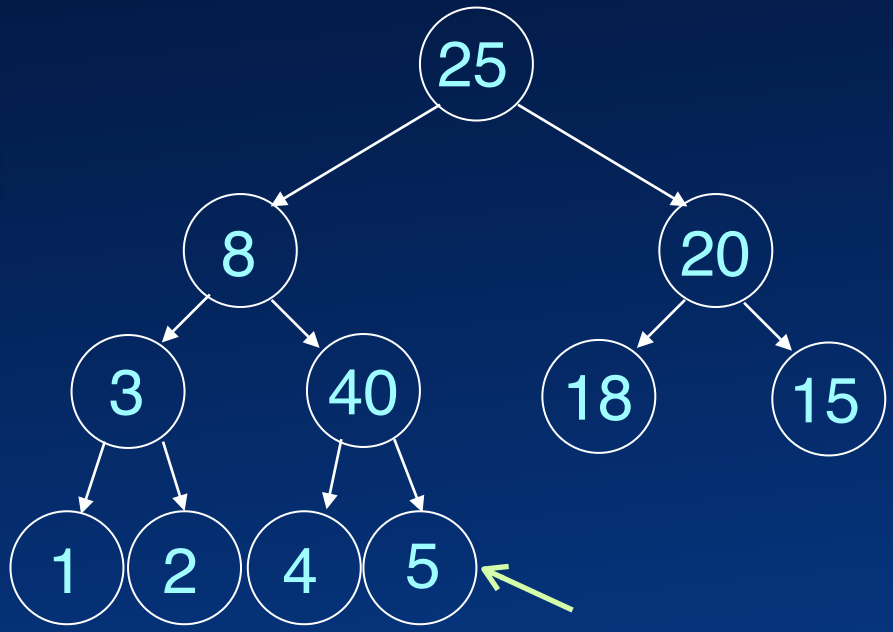

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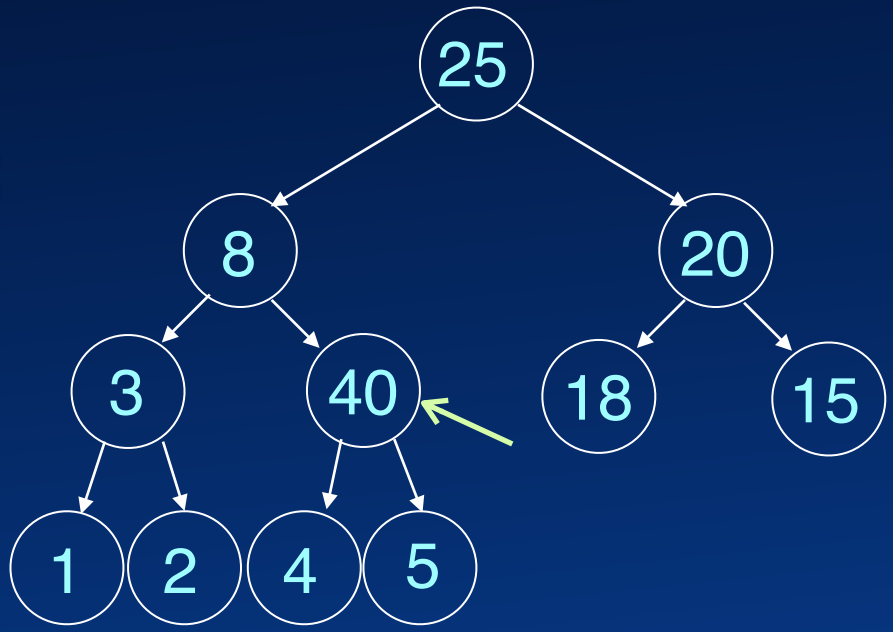

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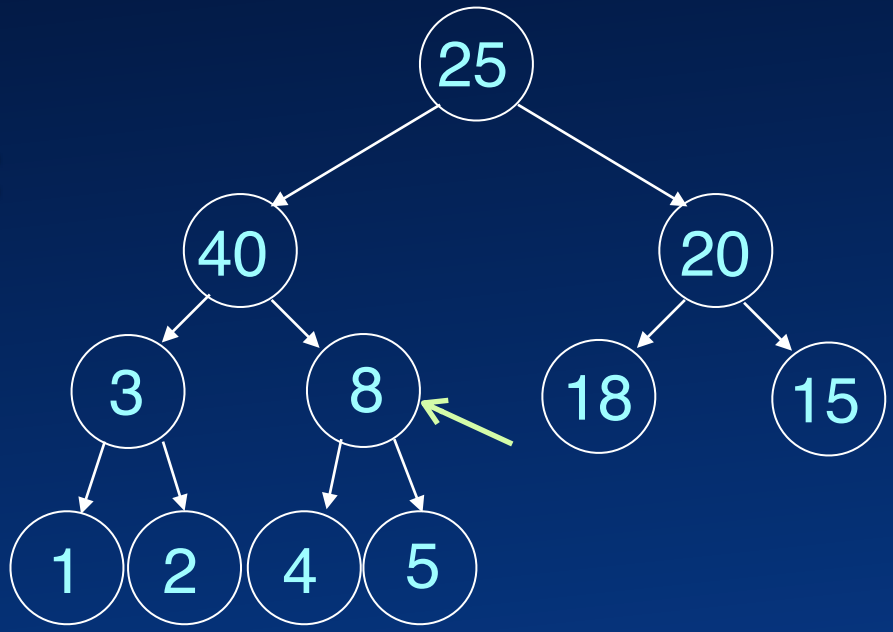

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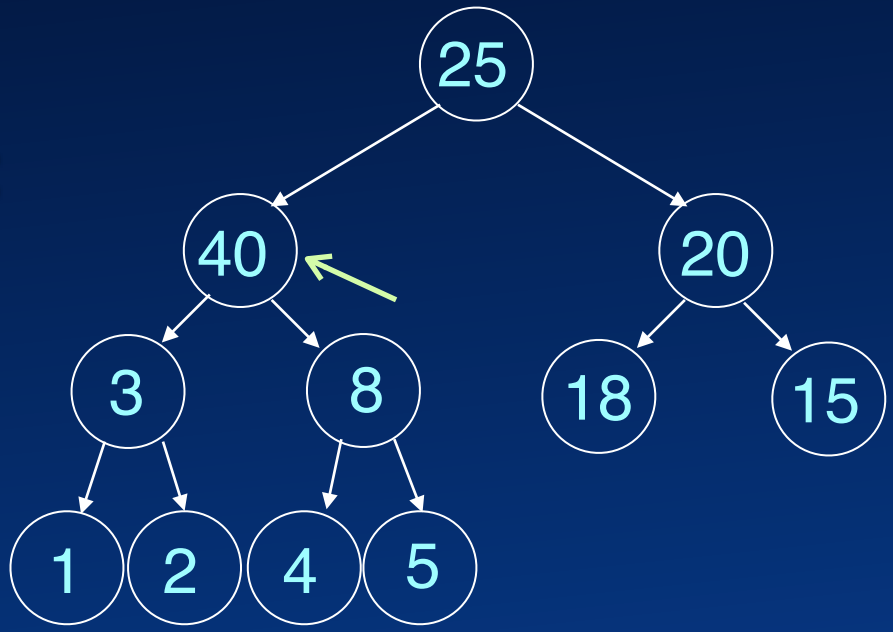

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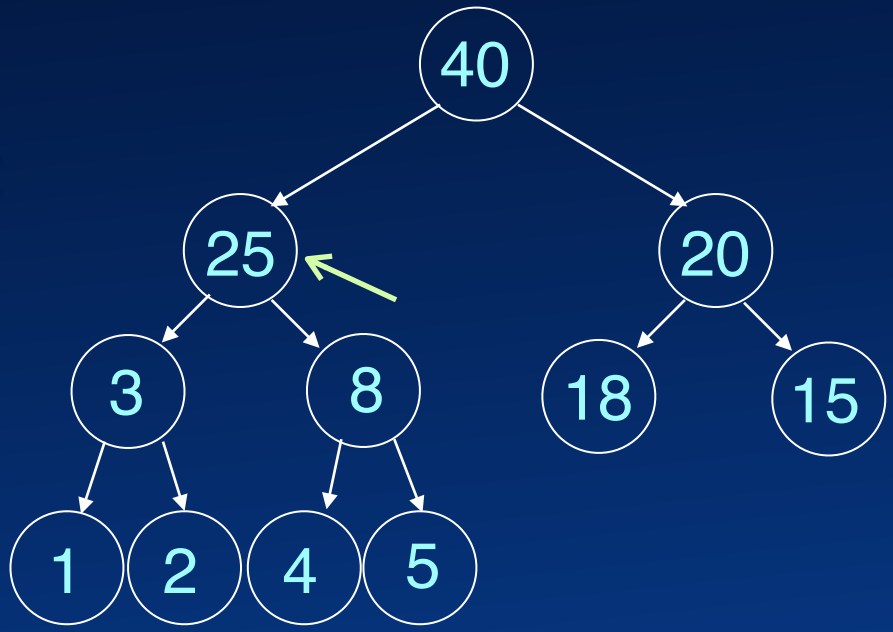

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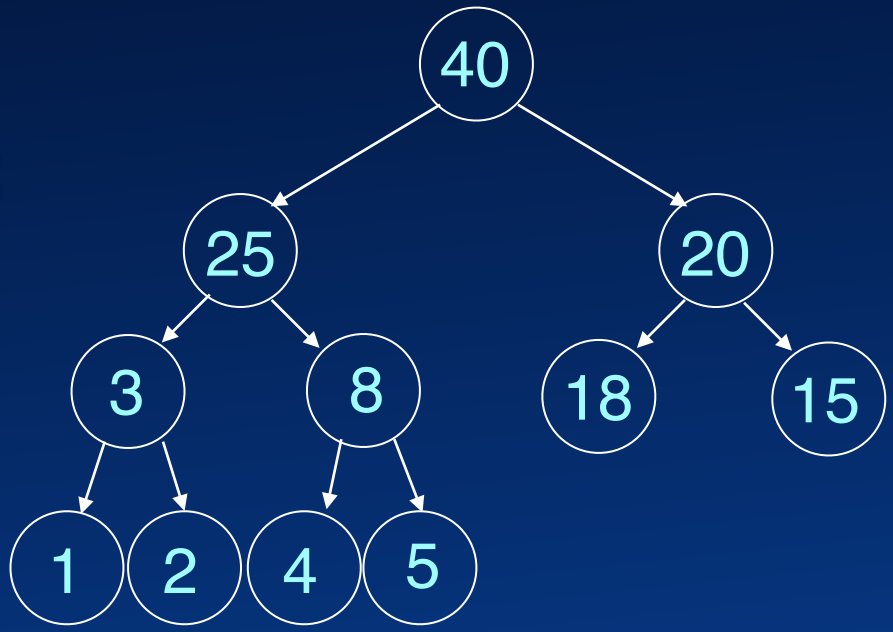

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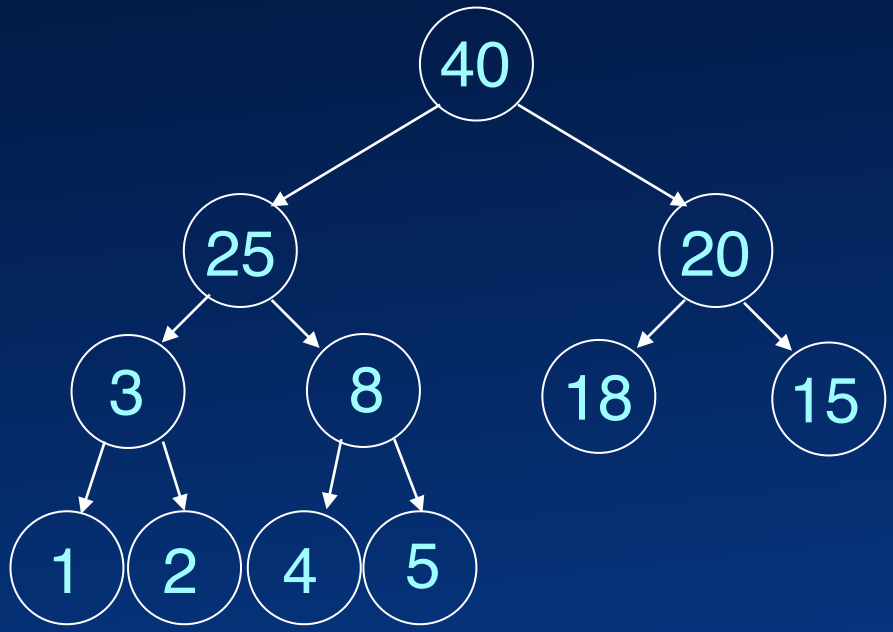
Inserted 40



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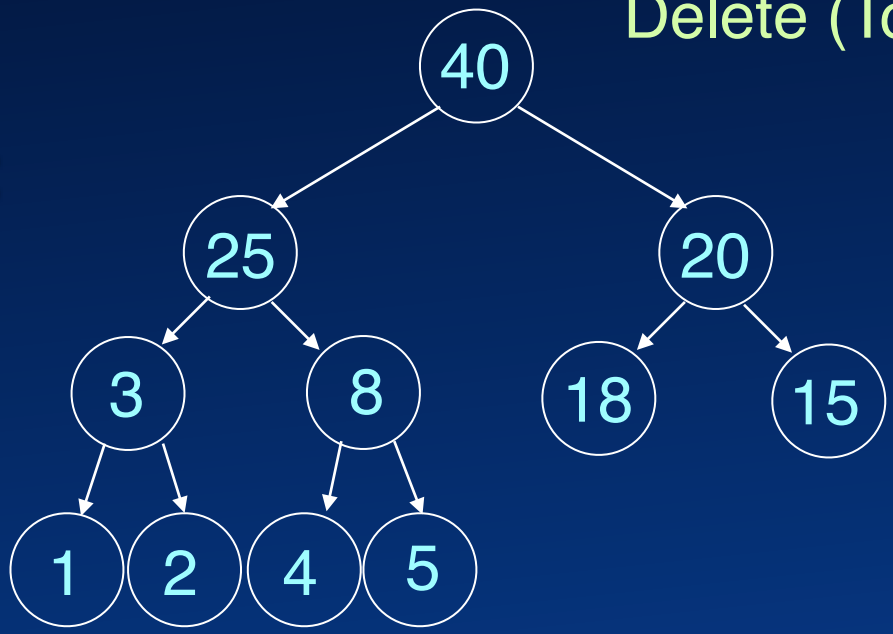
Inserted 40

Delete (Top)



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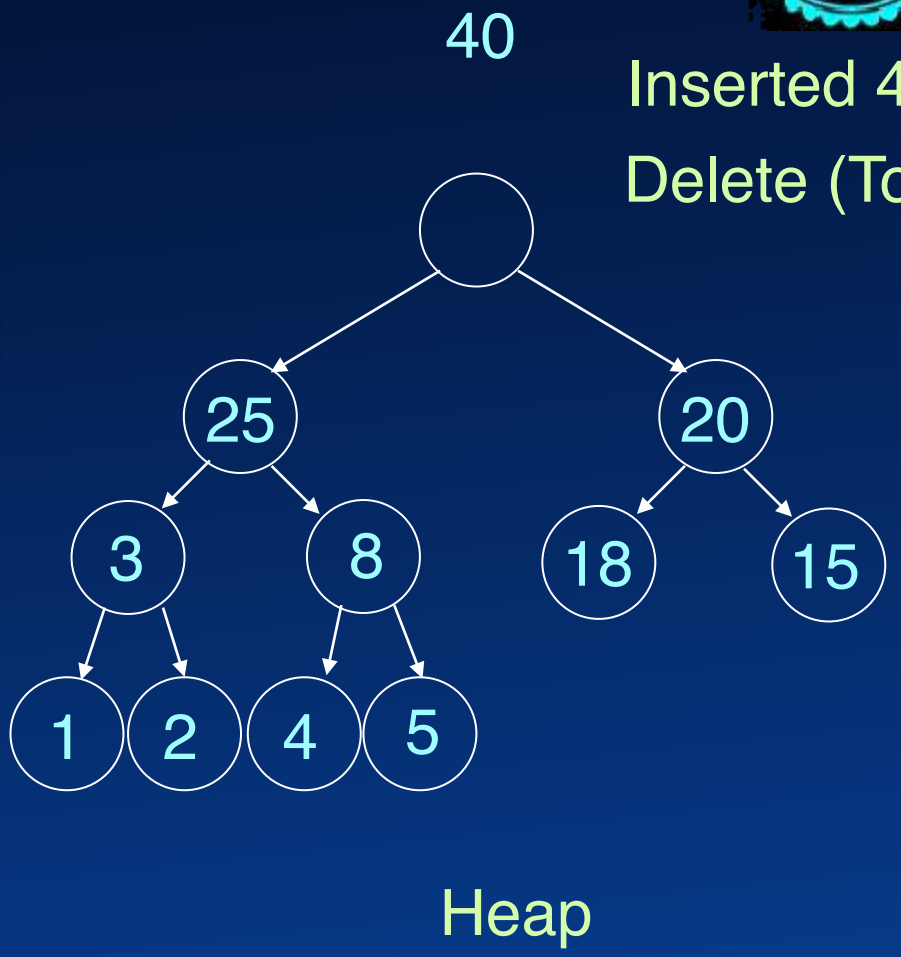

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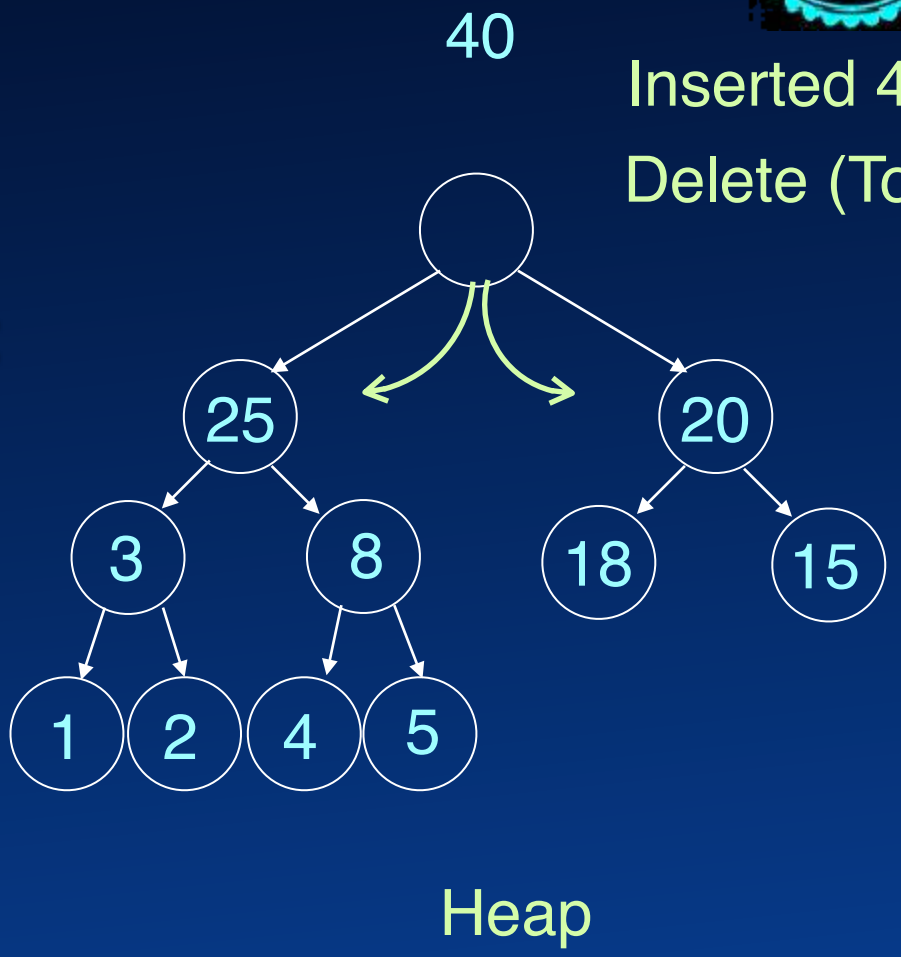

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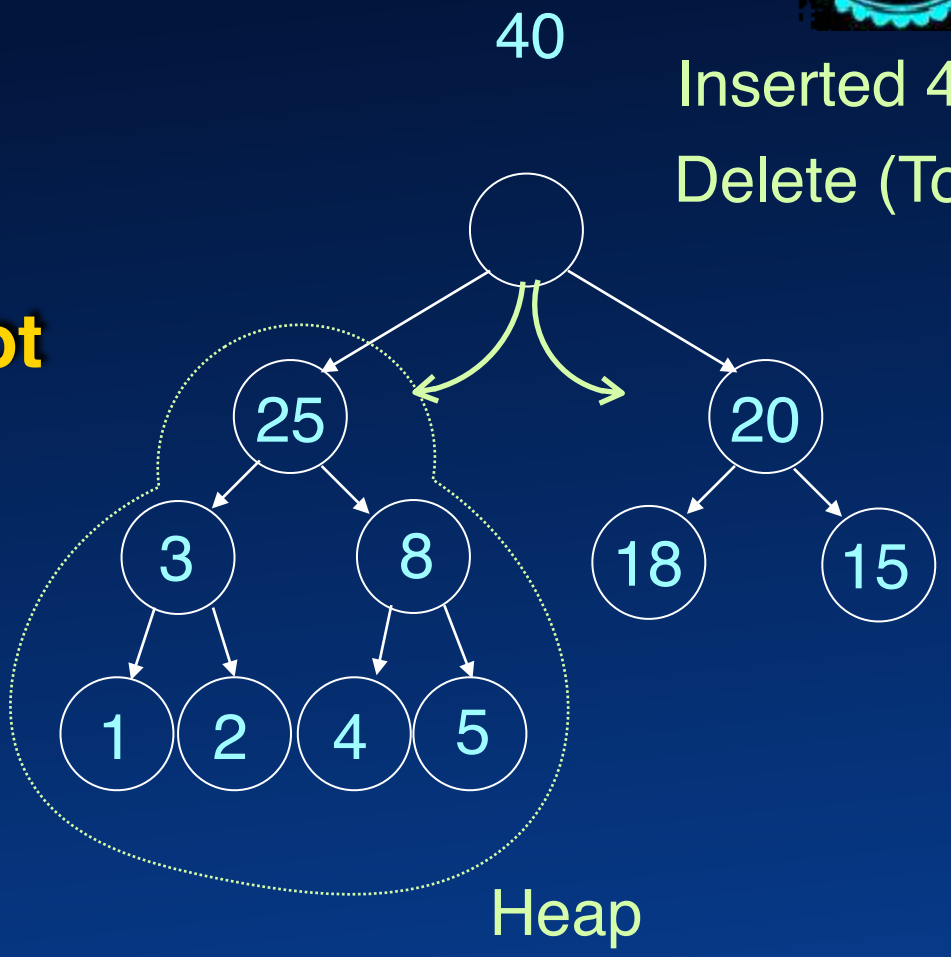

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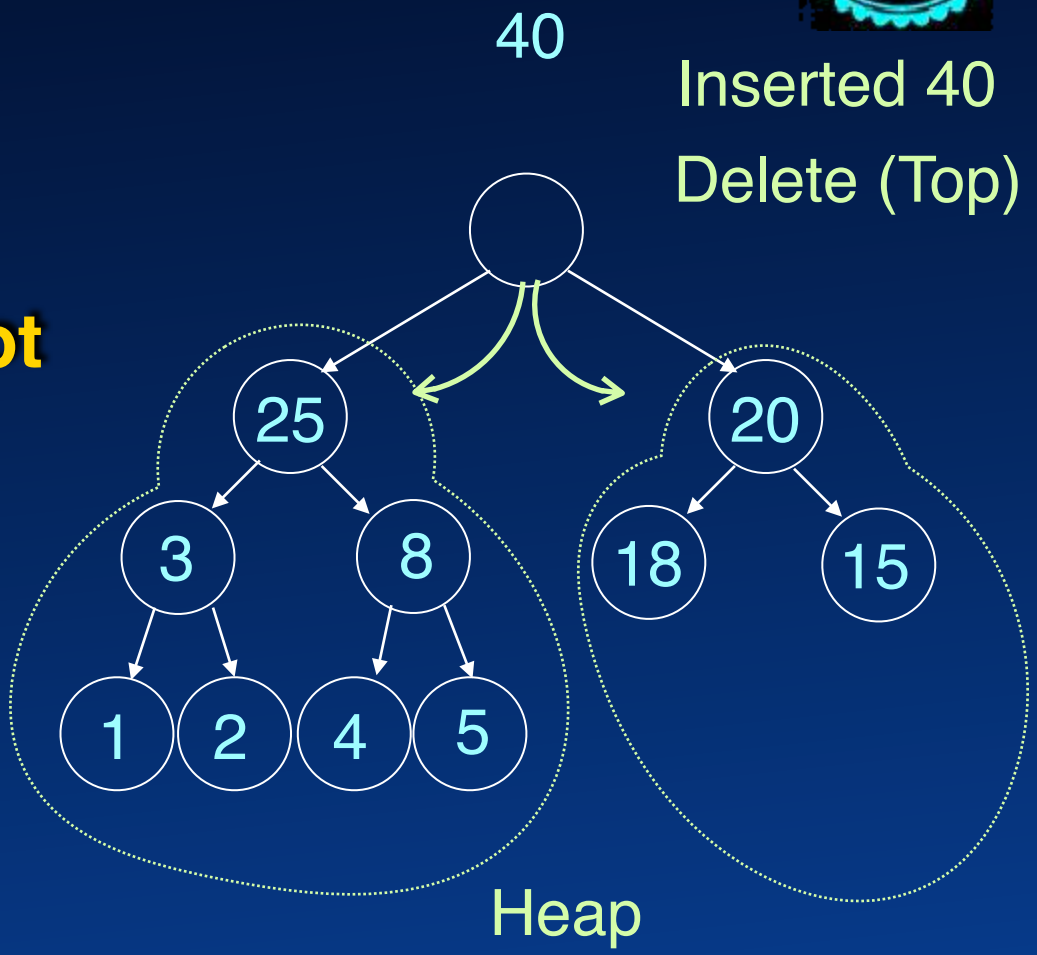

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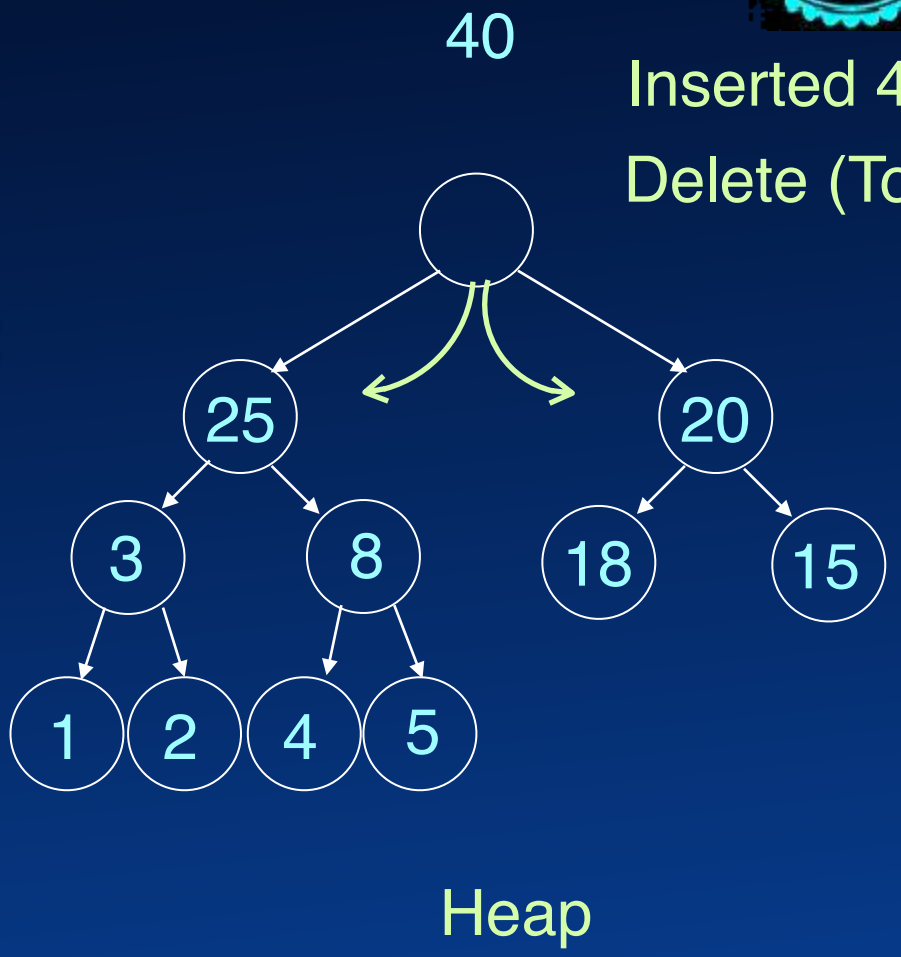

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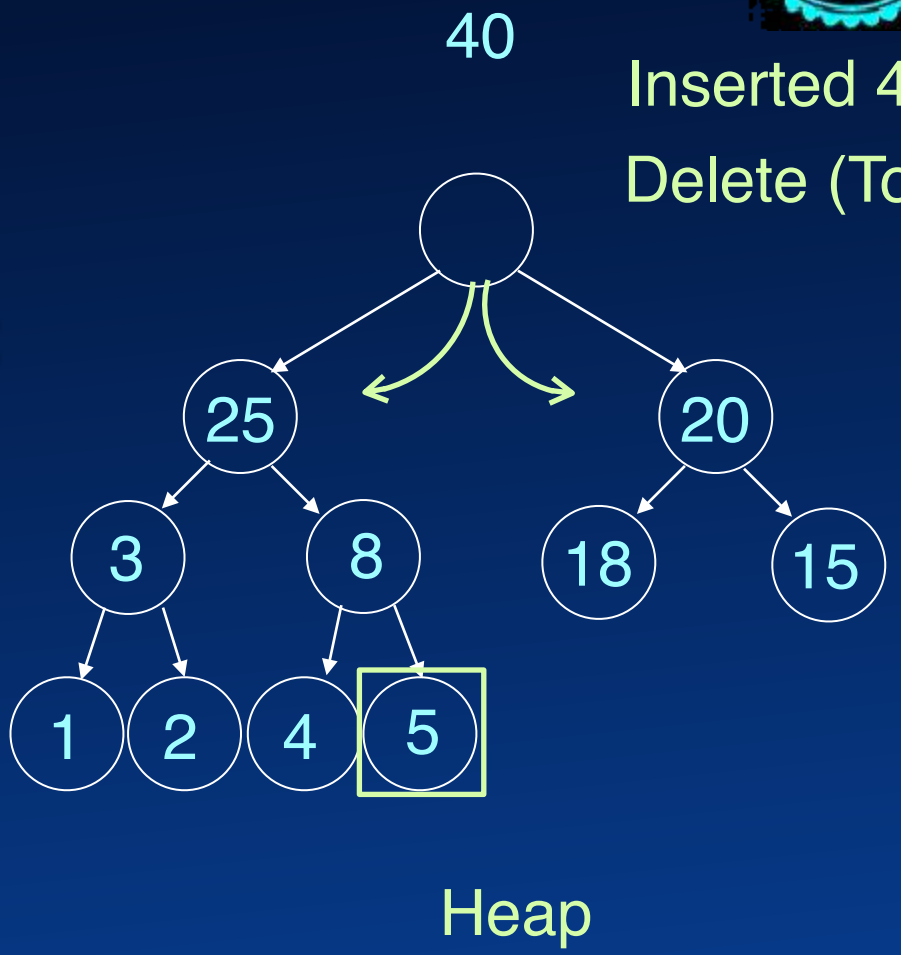

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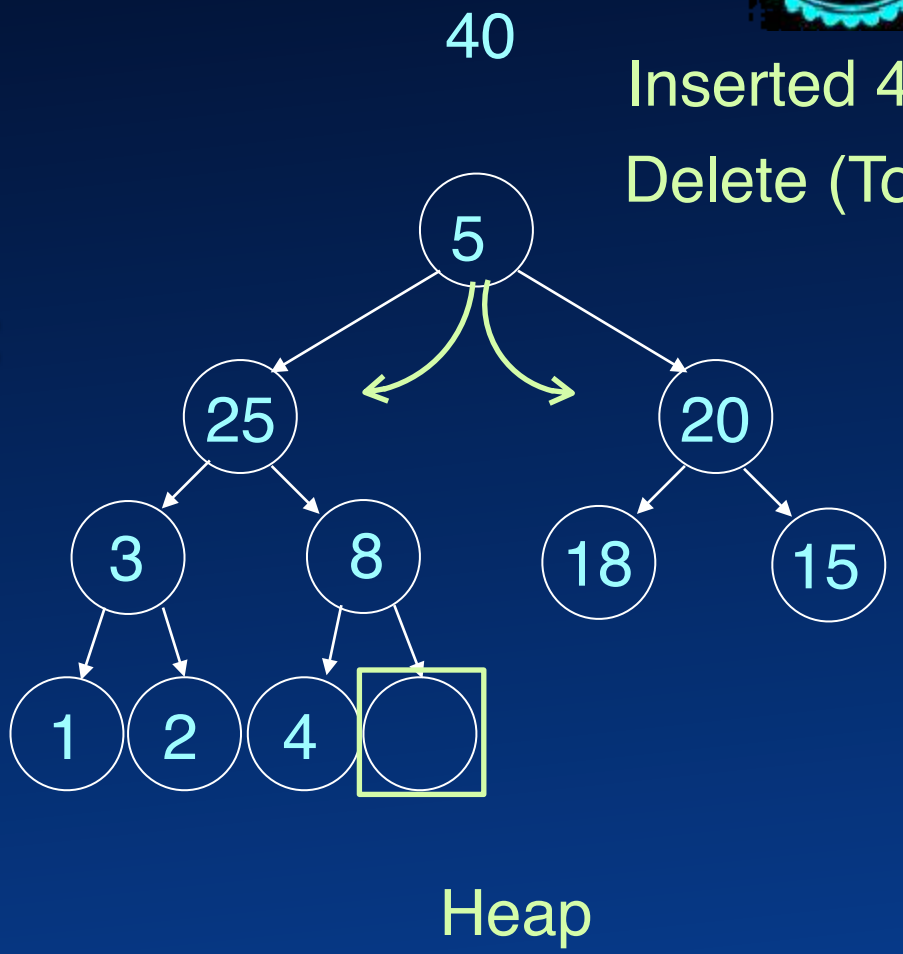

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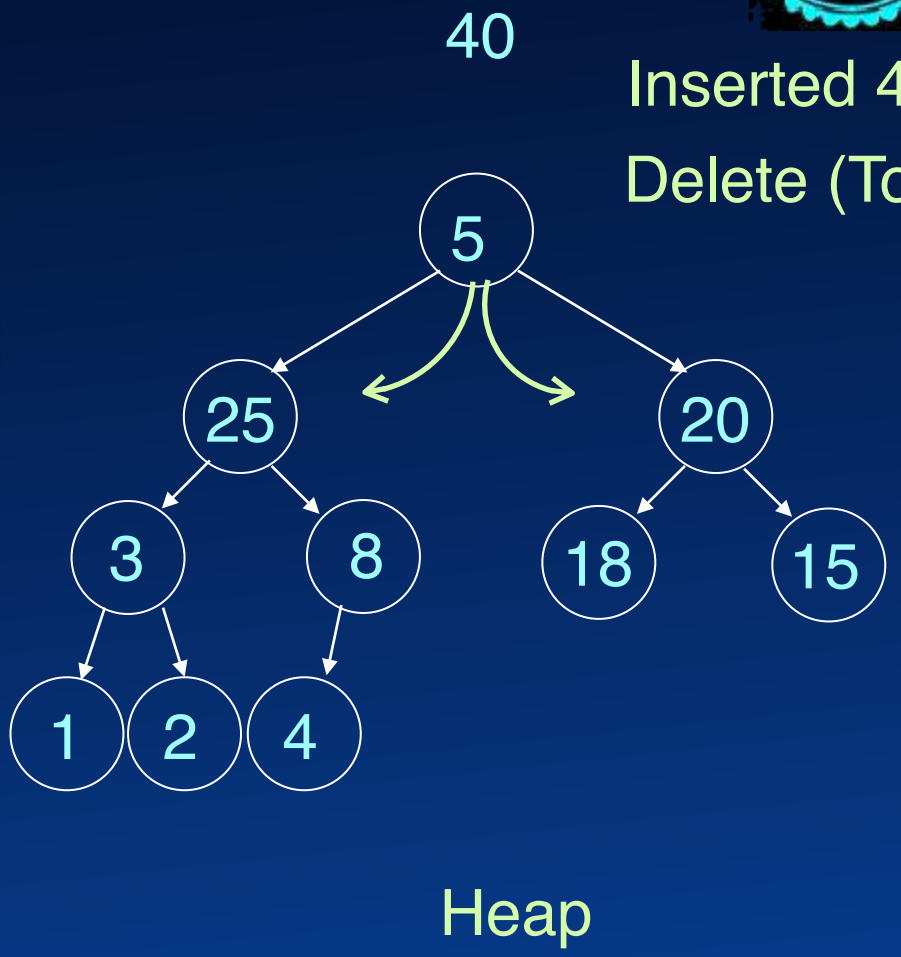

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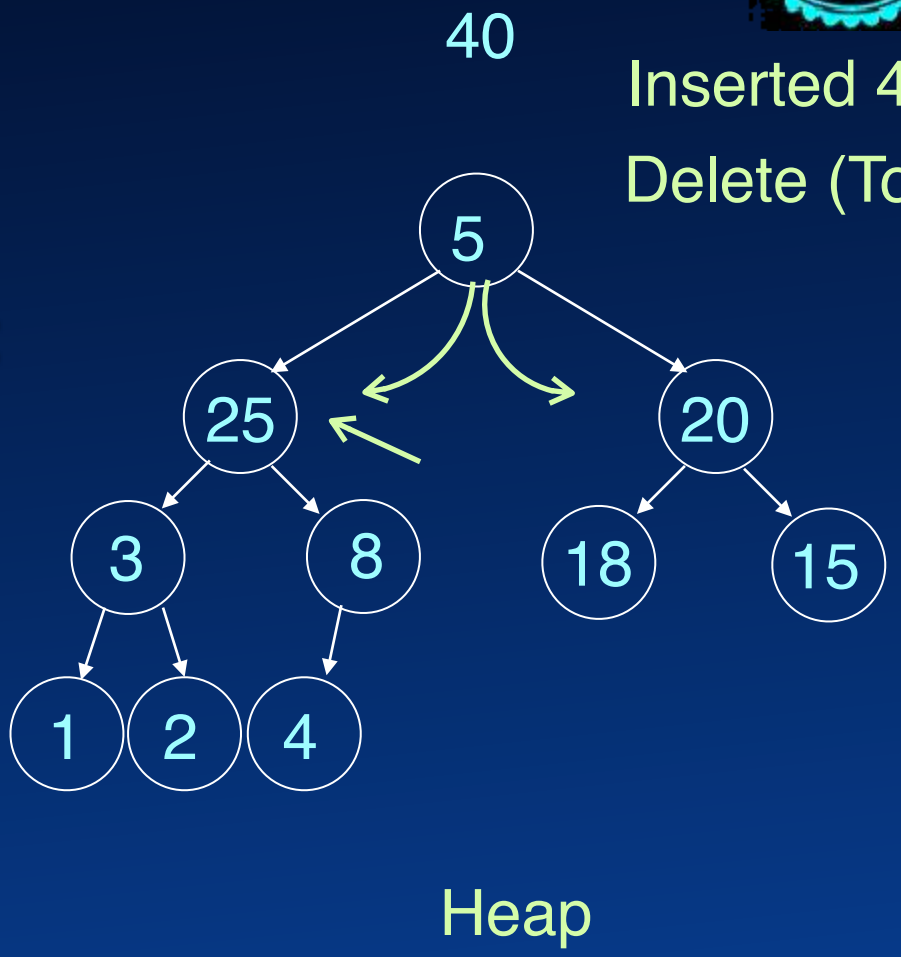

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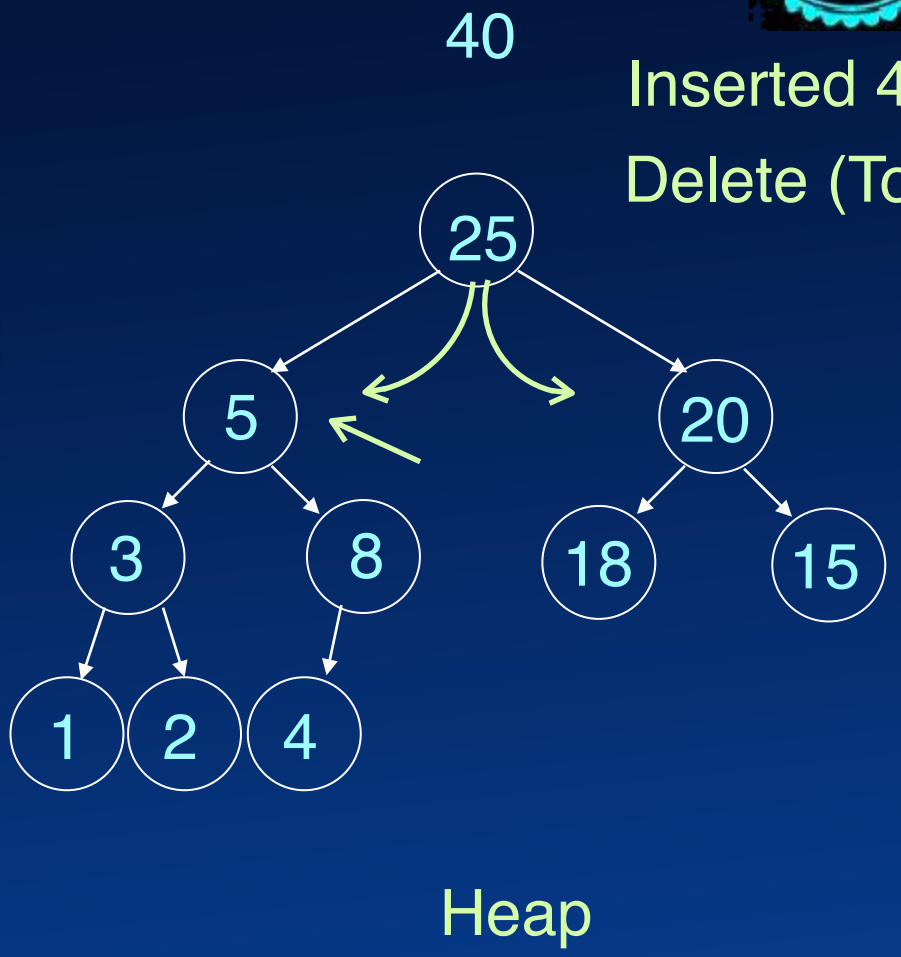

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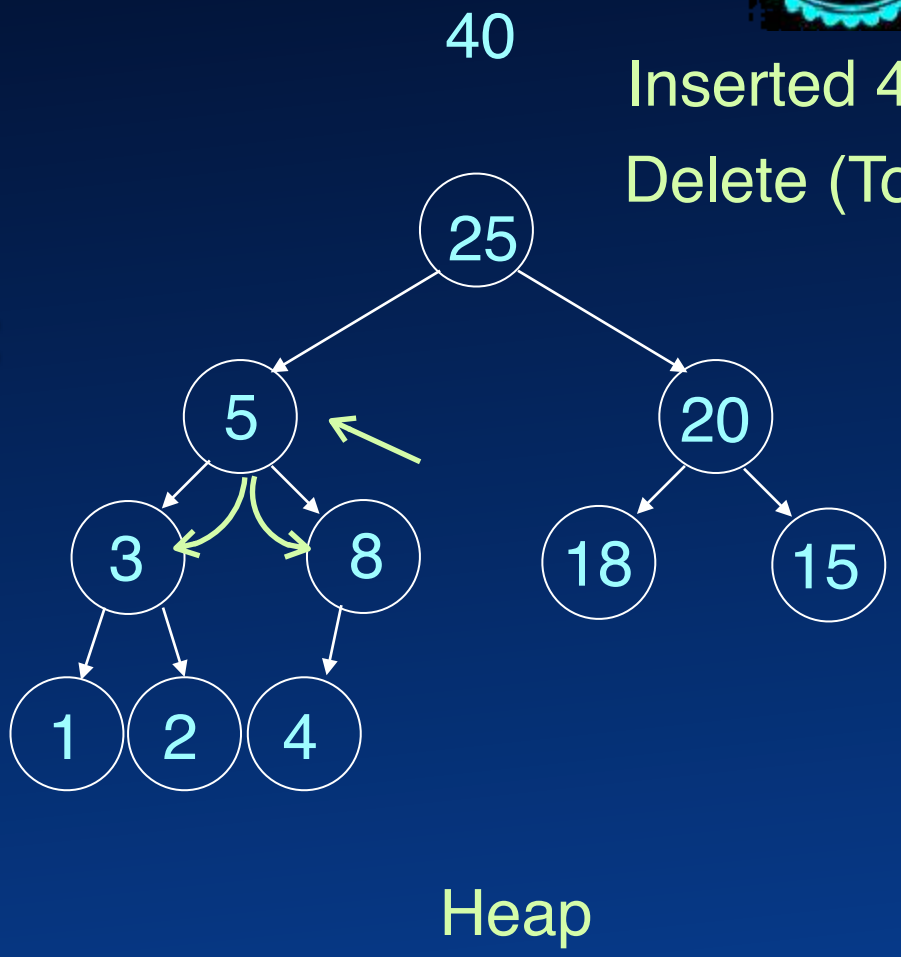

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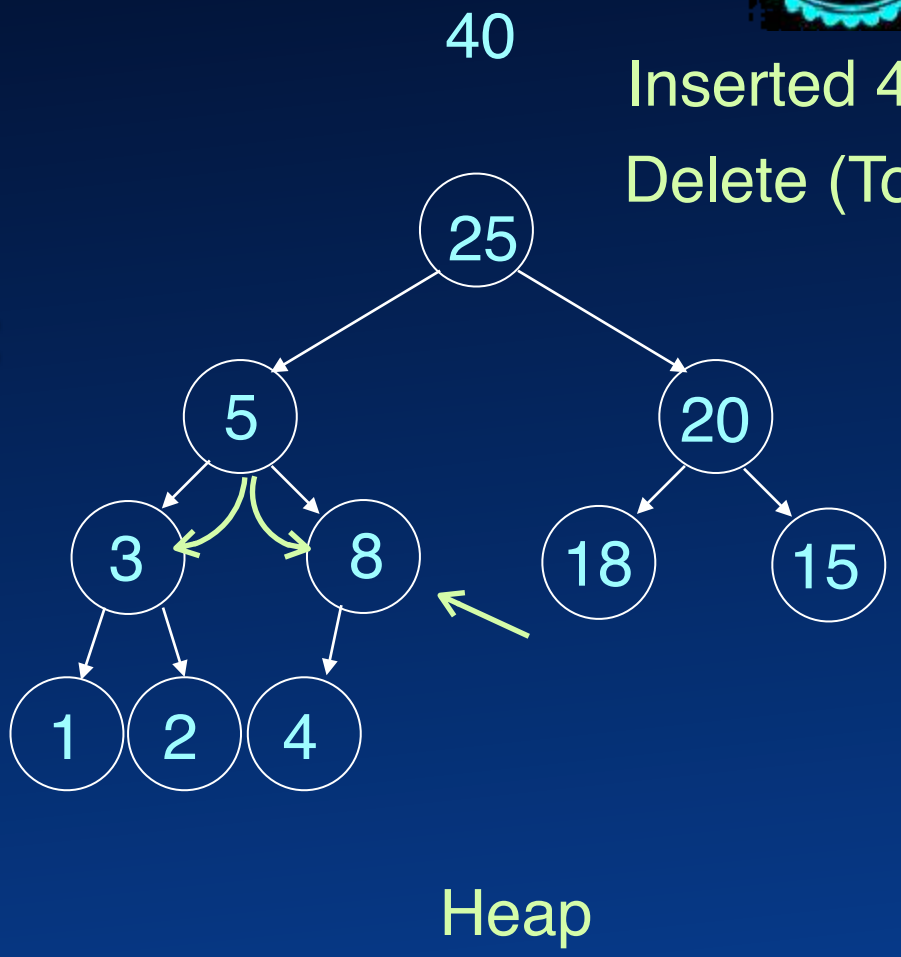

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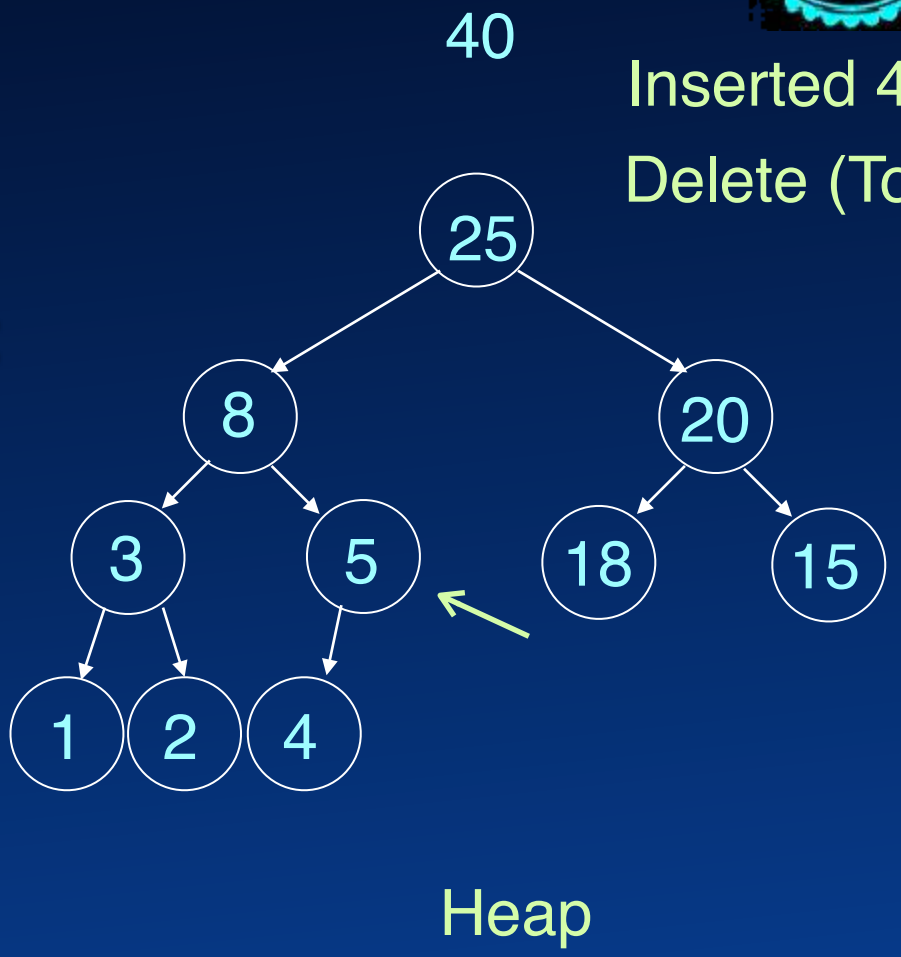

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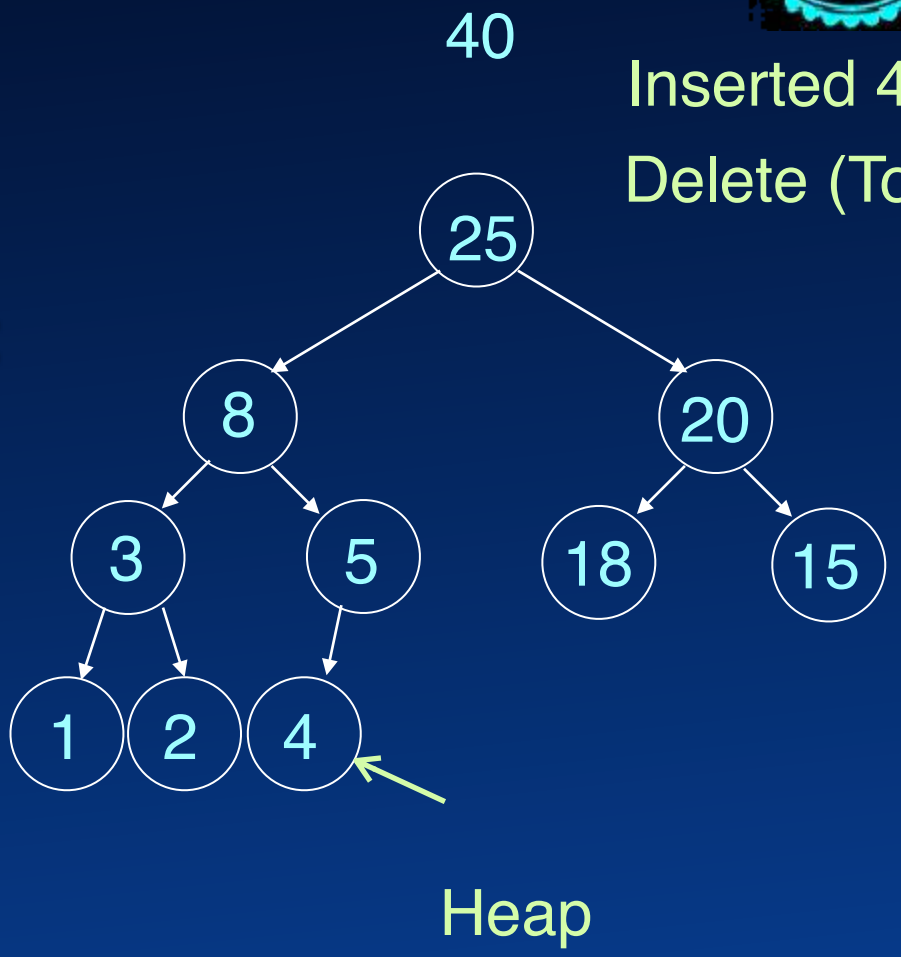

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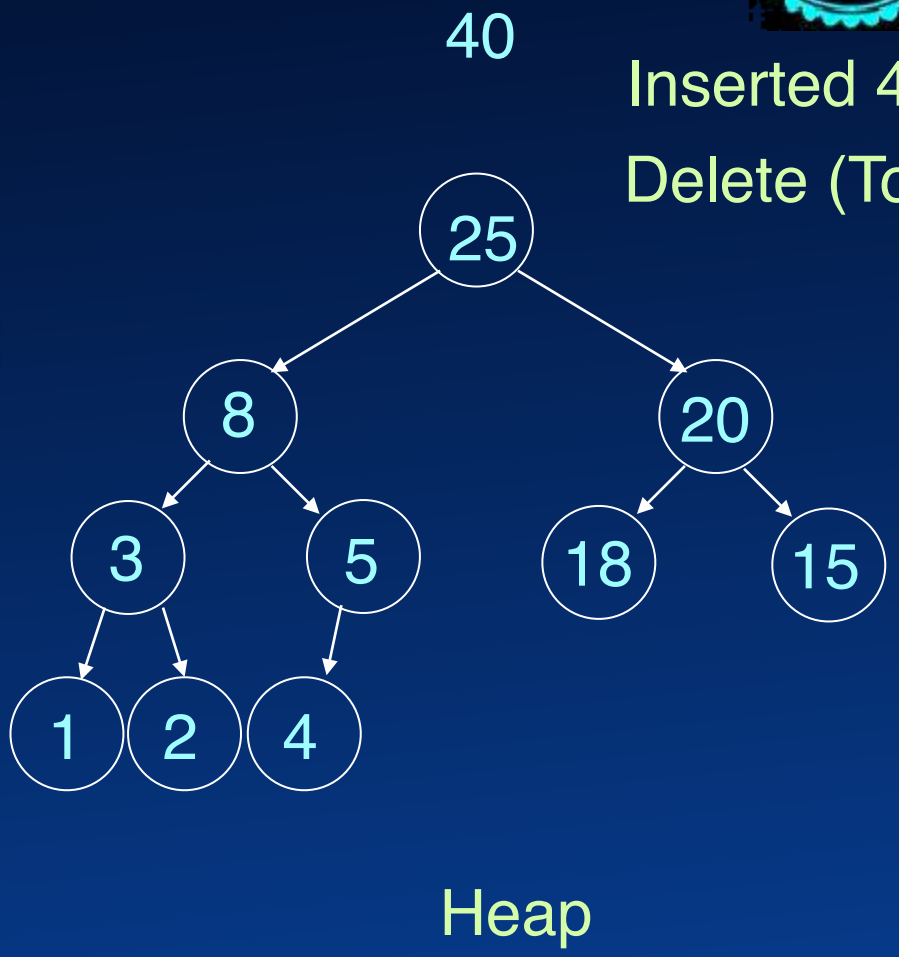

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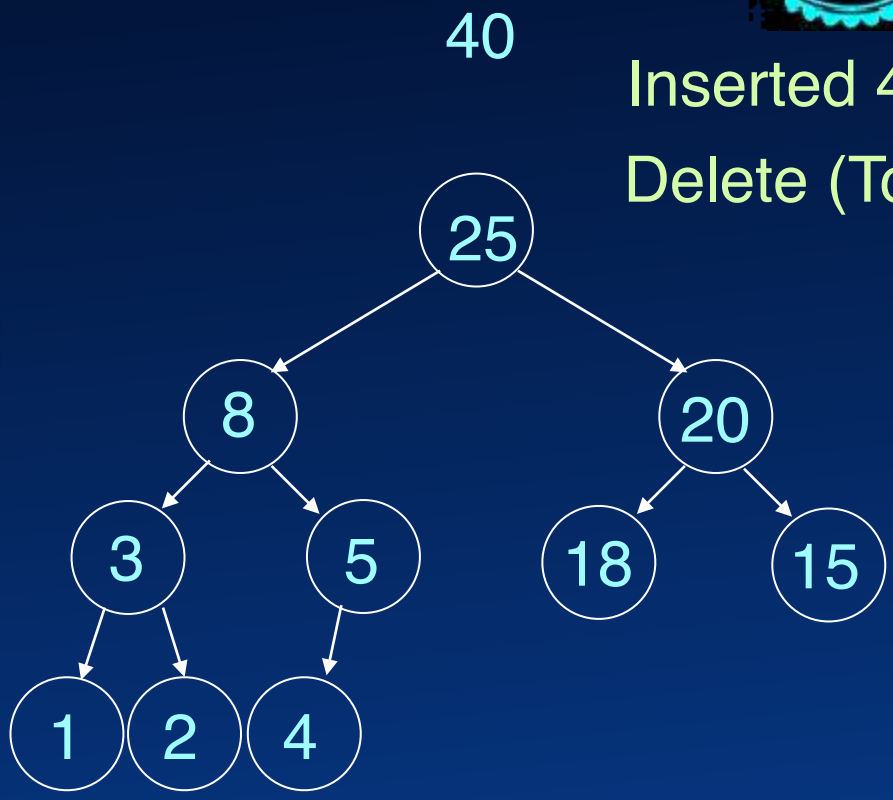

Heap

Insert:
Add node at next spot
Bubble-up
Delete:
Remove root
Replace with last spot
Bubble-down

Heap

Bubble up:
- if no parent 👍
- if(key lower-than parent.key) 👍
- swapwith(parent)
- parent.bubbleup()

Insert:
- Add node at next spot
- Bubble-up

Delete:
- Remove root
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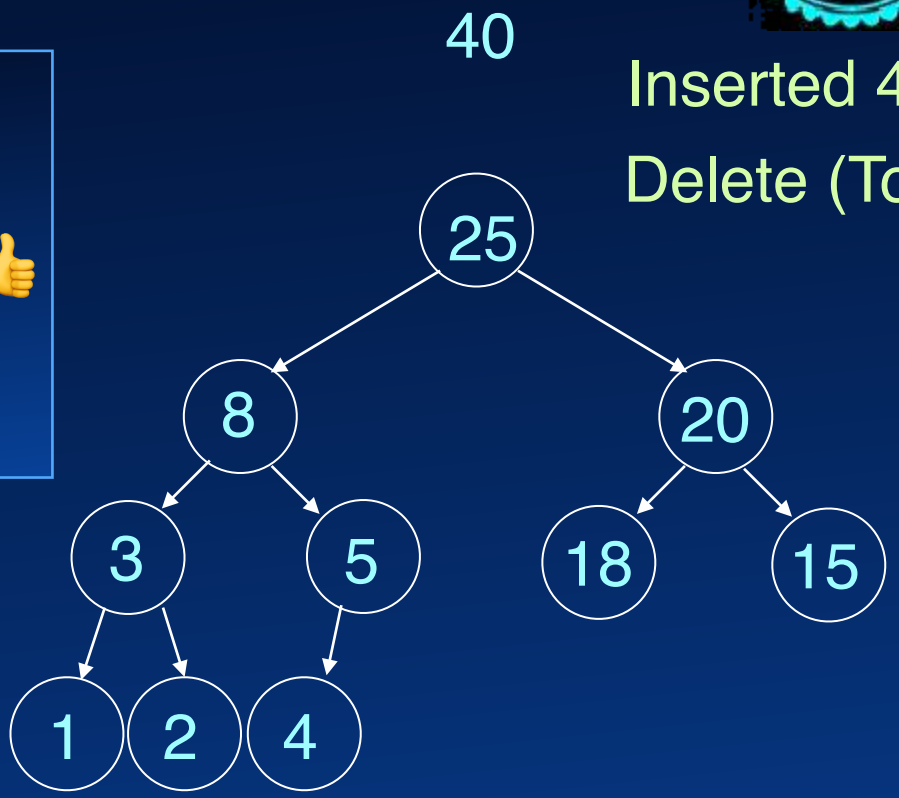

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Insert:
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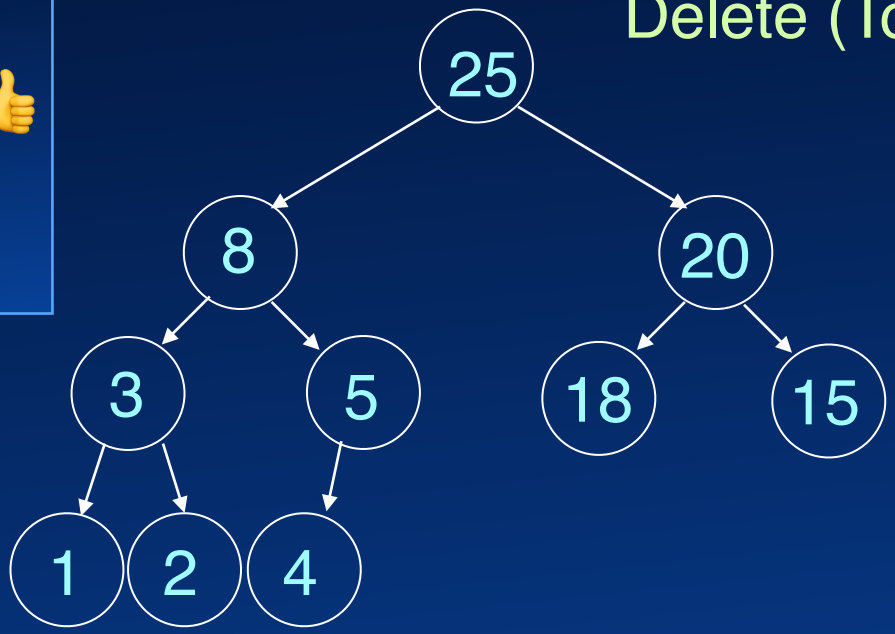
Delete:
- Remove root
- Replace with last spot
- Bubble-down

40

Inserted 40

Delete (Top)



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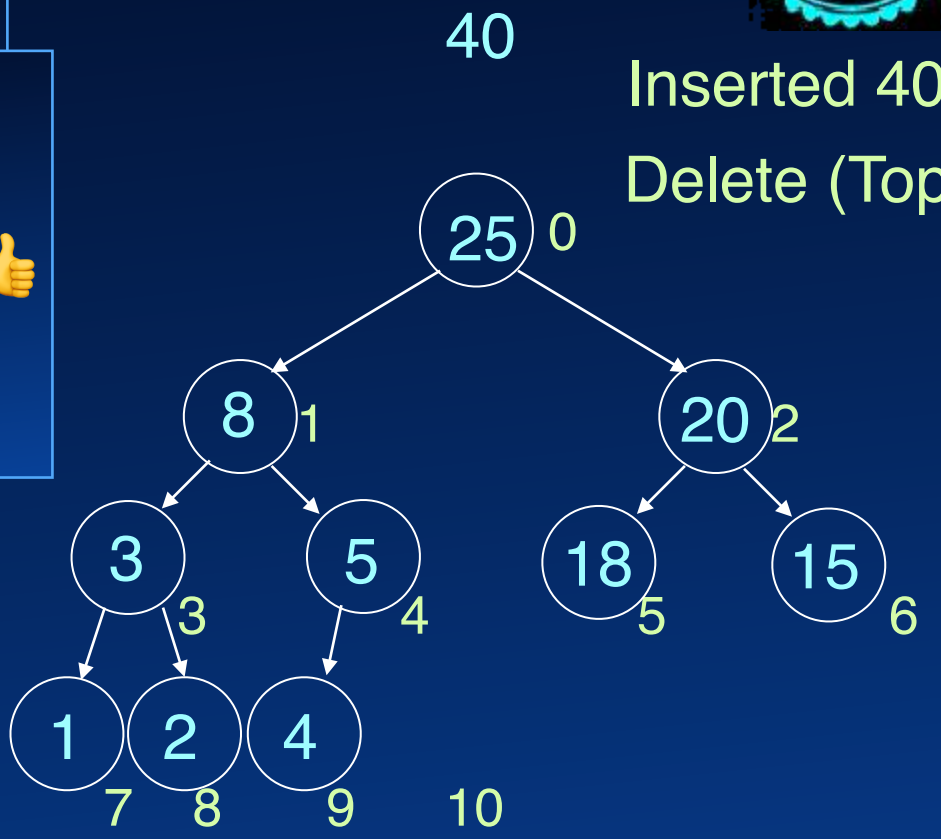

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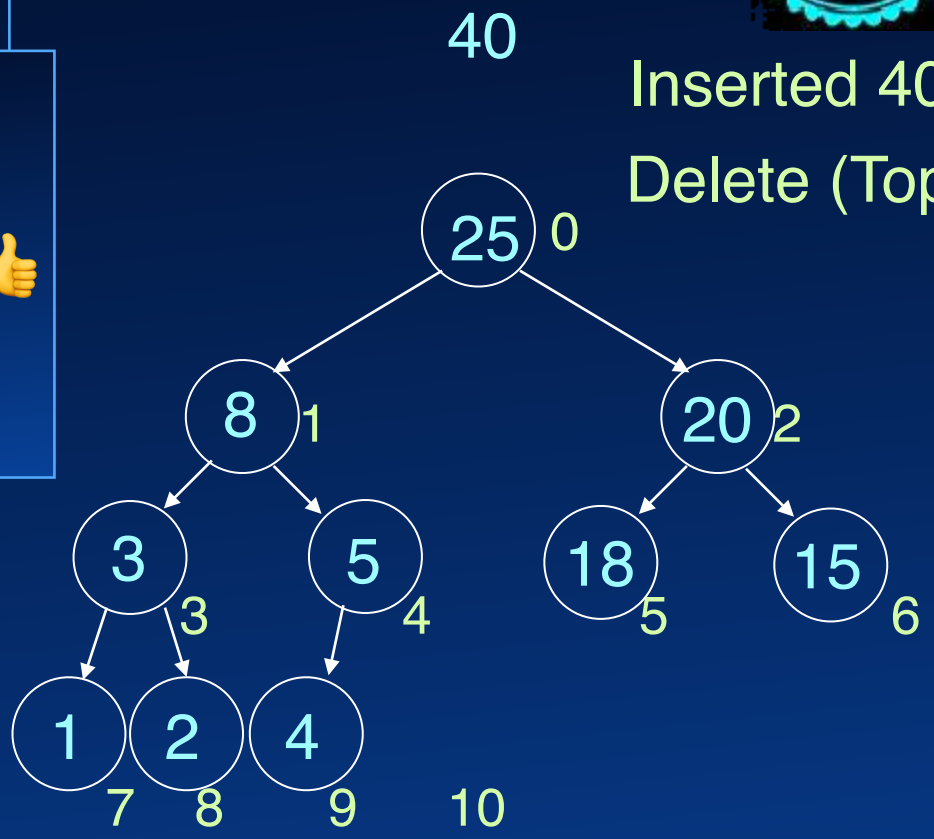

Insert:
- Add node at next spot
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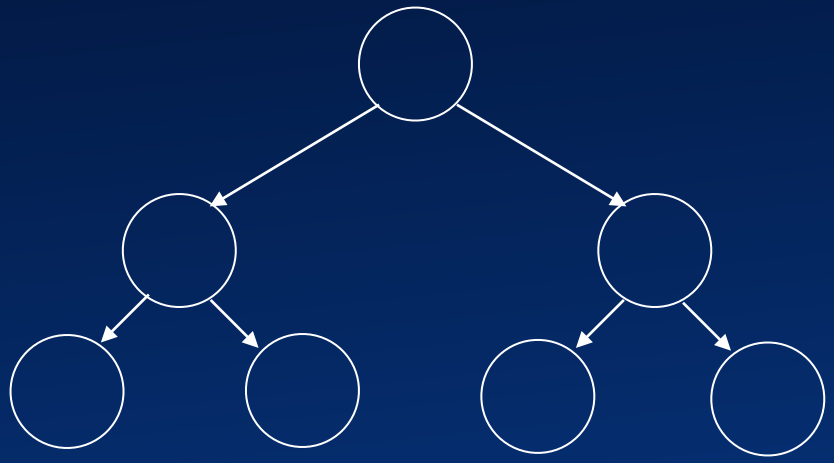
Delete:
- Remove root
- Replace with last spot
- Bubble-down

child-index = 2*parent-index + {1,2}
parent-index = (child-index-1)/2

In which node may the third largest element of a heap be?

mail: col106quiz@cse.iitd.ac.in
format: 1,2,3,4,5,6



- 1
- 2
- 3
- 4
- 5
- 6

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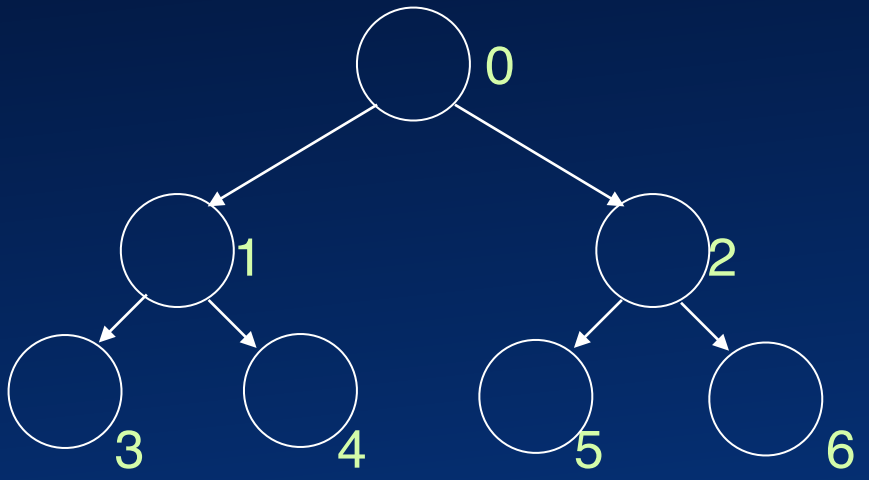

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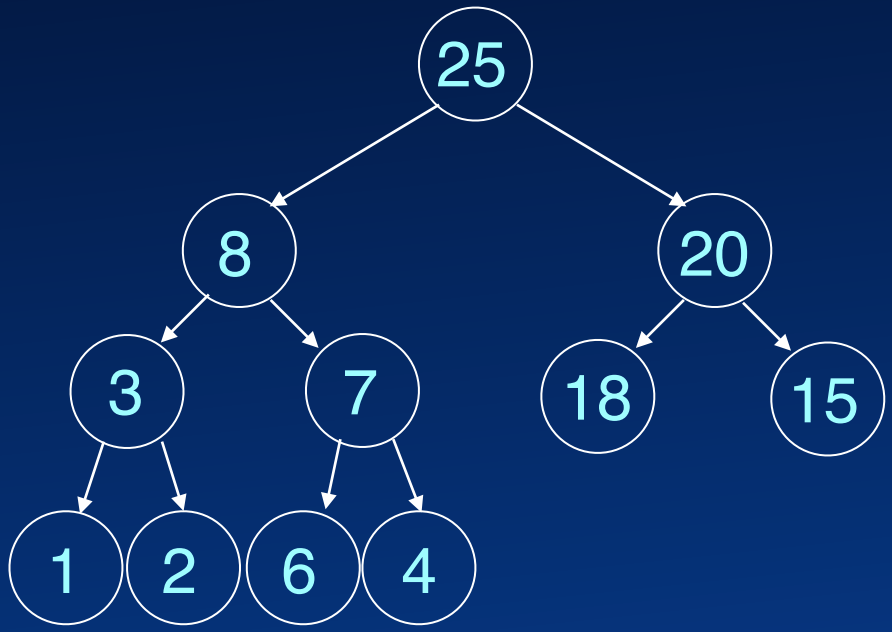

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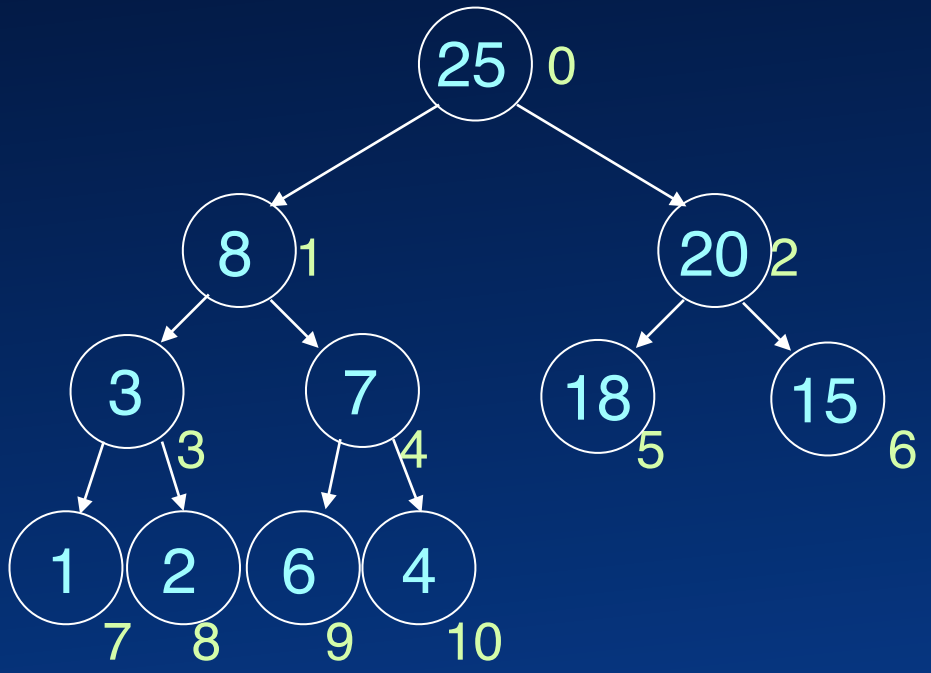

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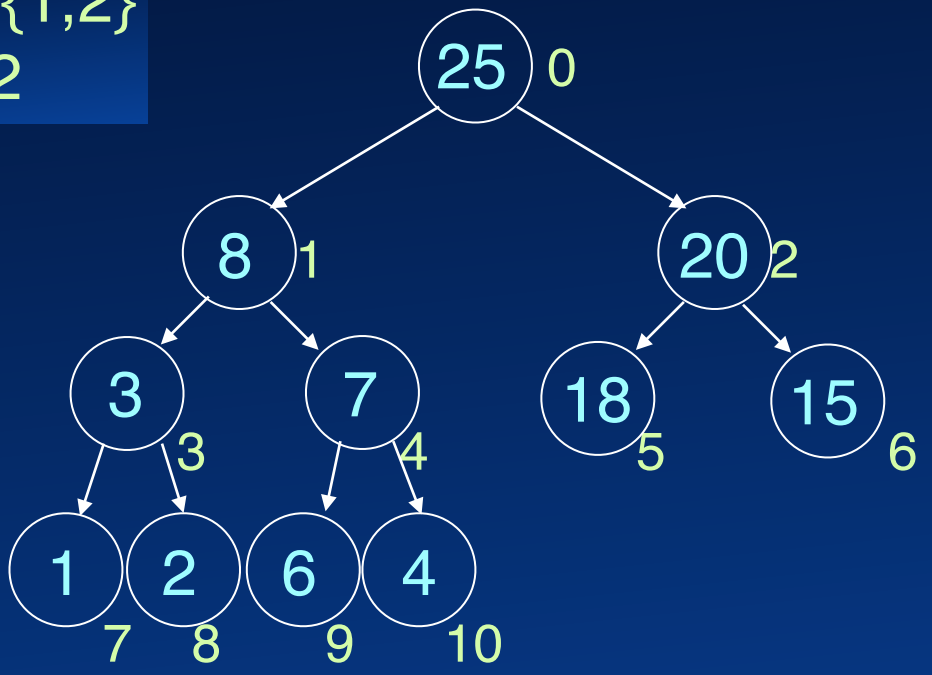

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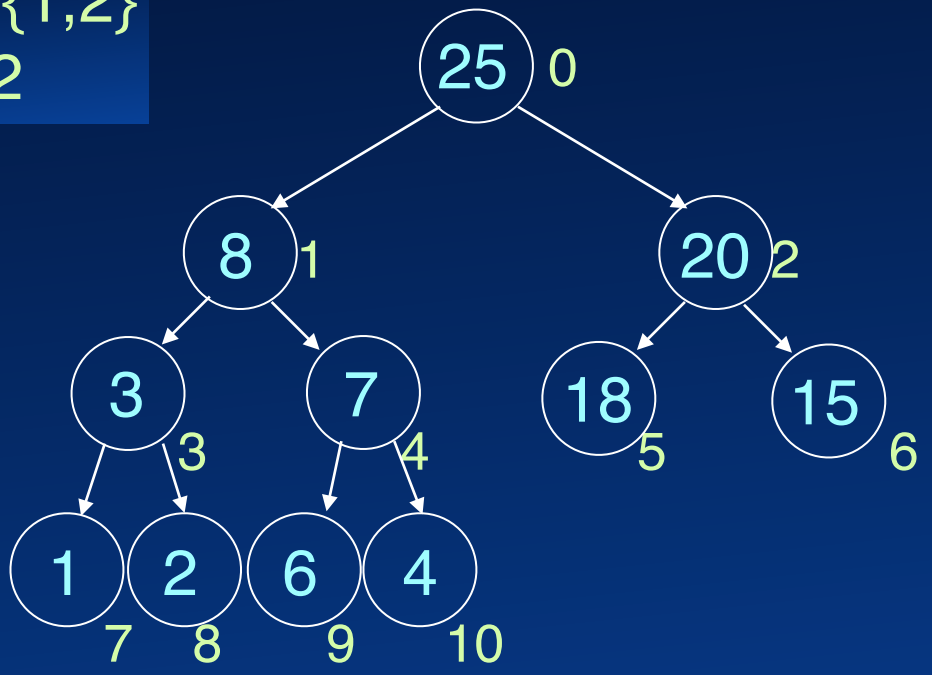

25	8	20	3	7	18	15	1	2	6	4		

Heap

Heap

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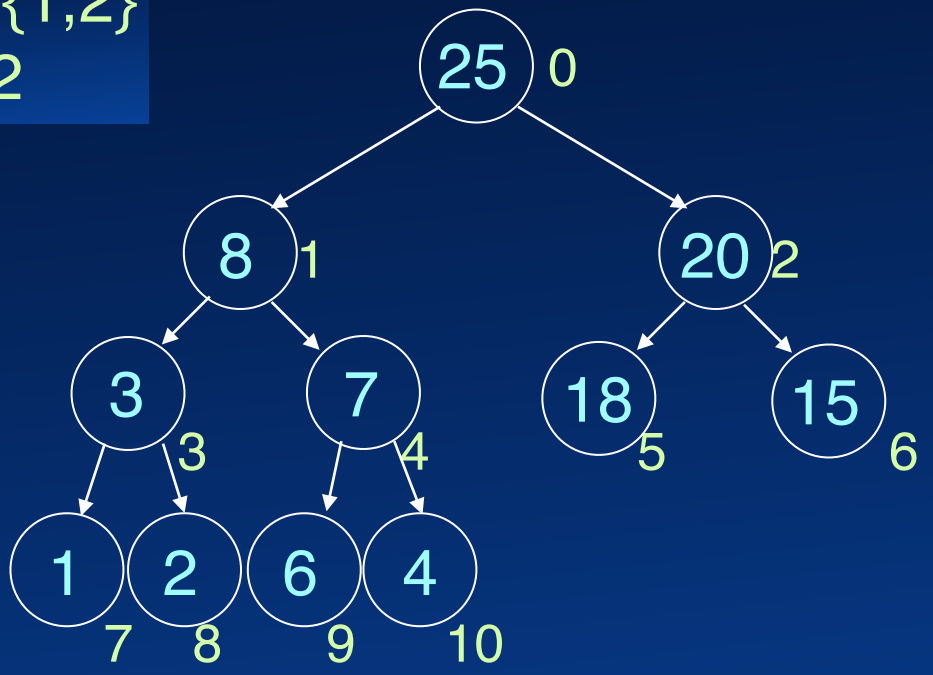

25	8	20	3	7	18	15	1	2	6	4		
0	1	2	3	4	5	6	7	8	9	10	11	

Heap

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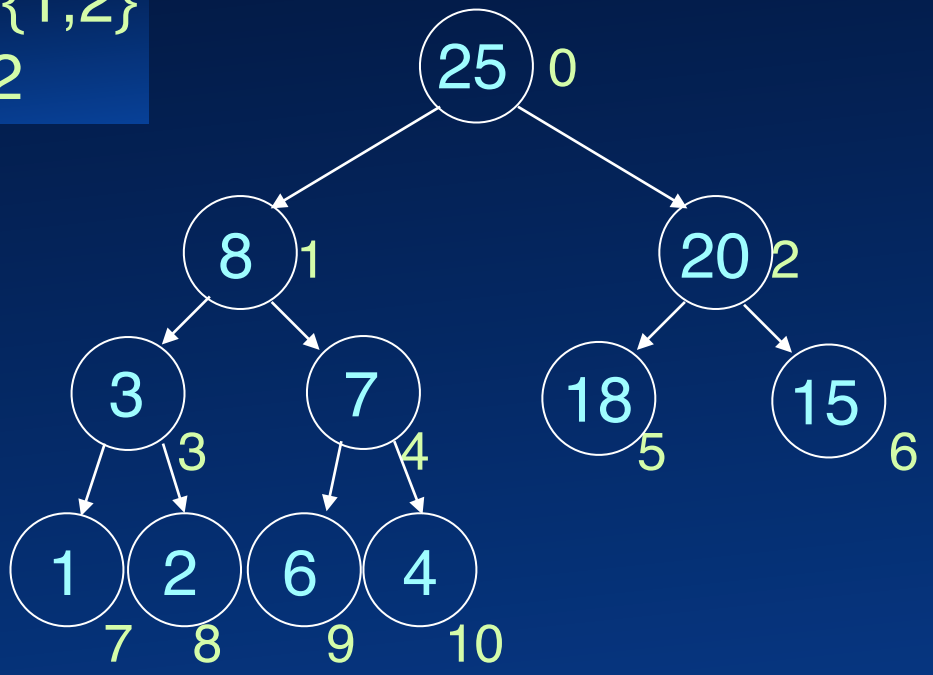

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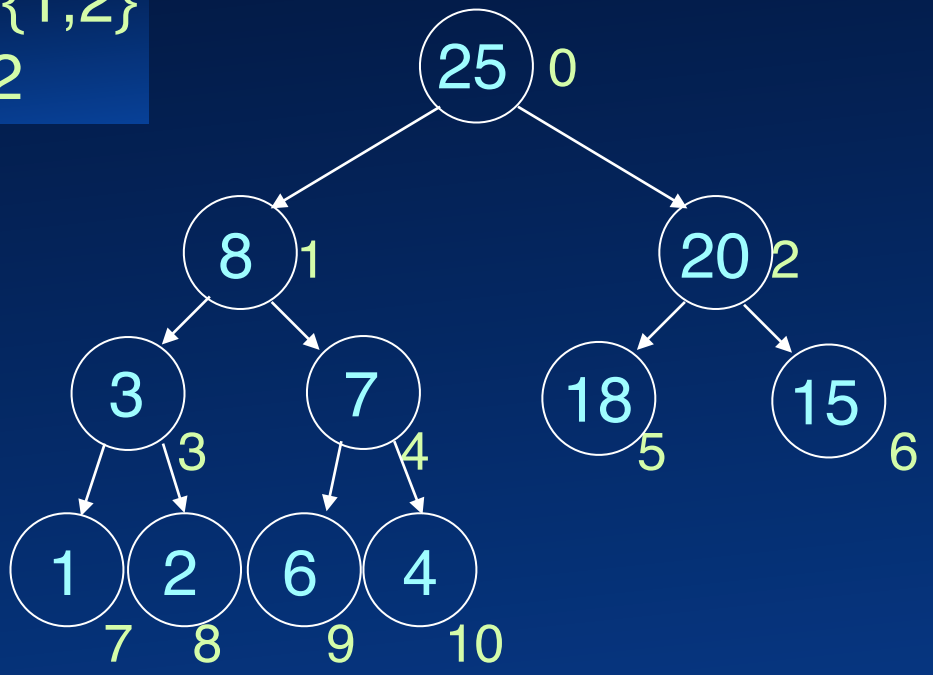

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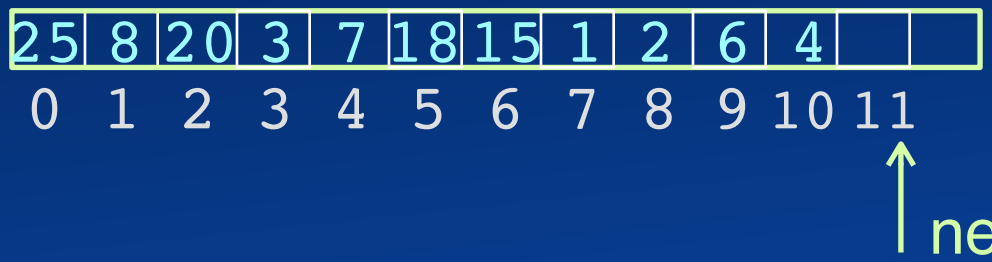

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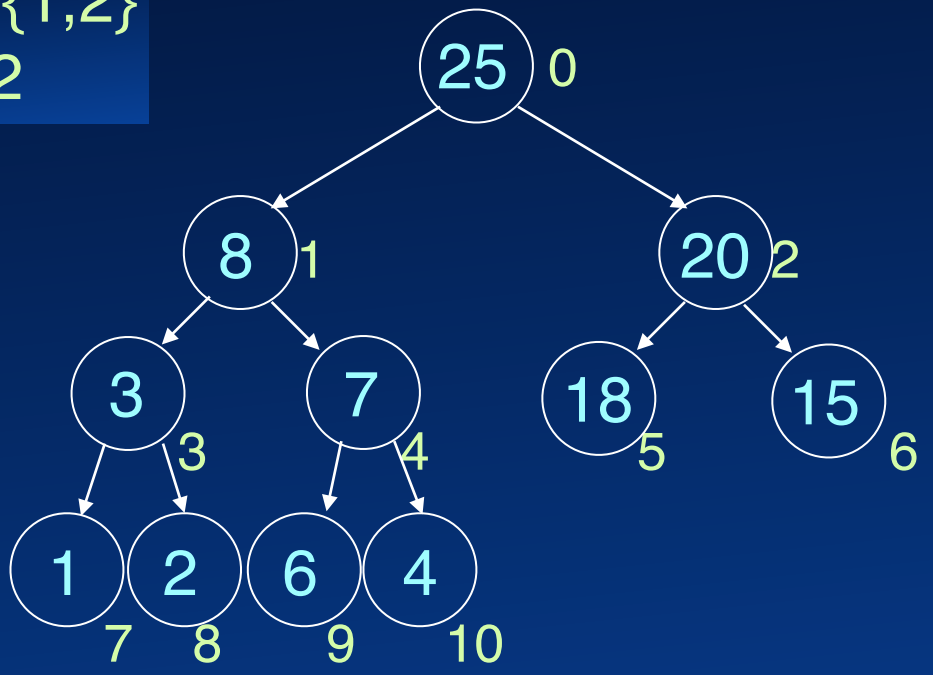

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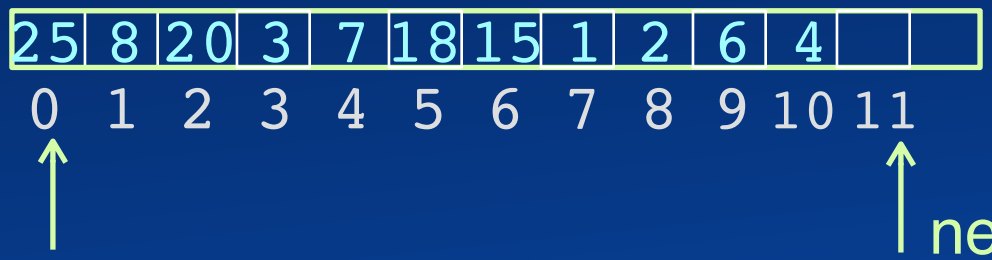

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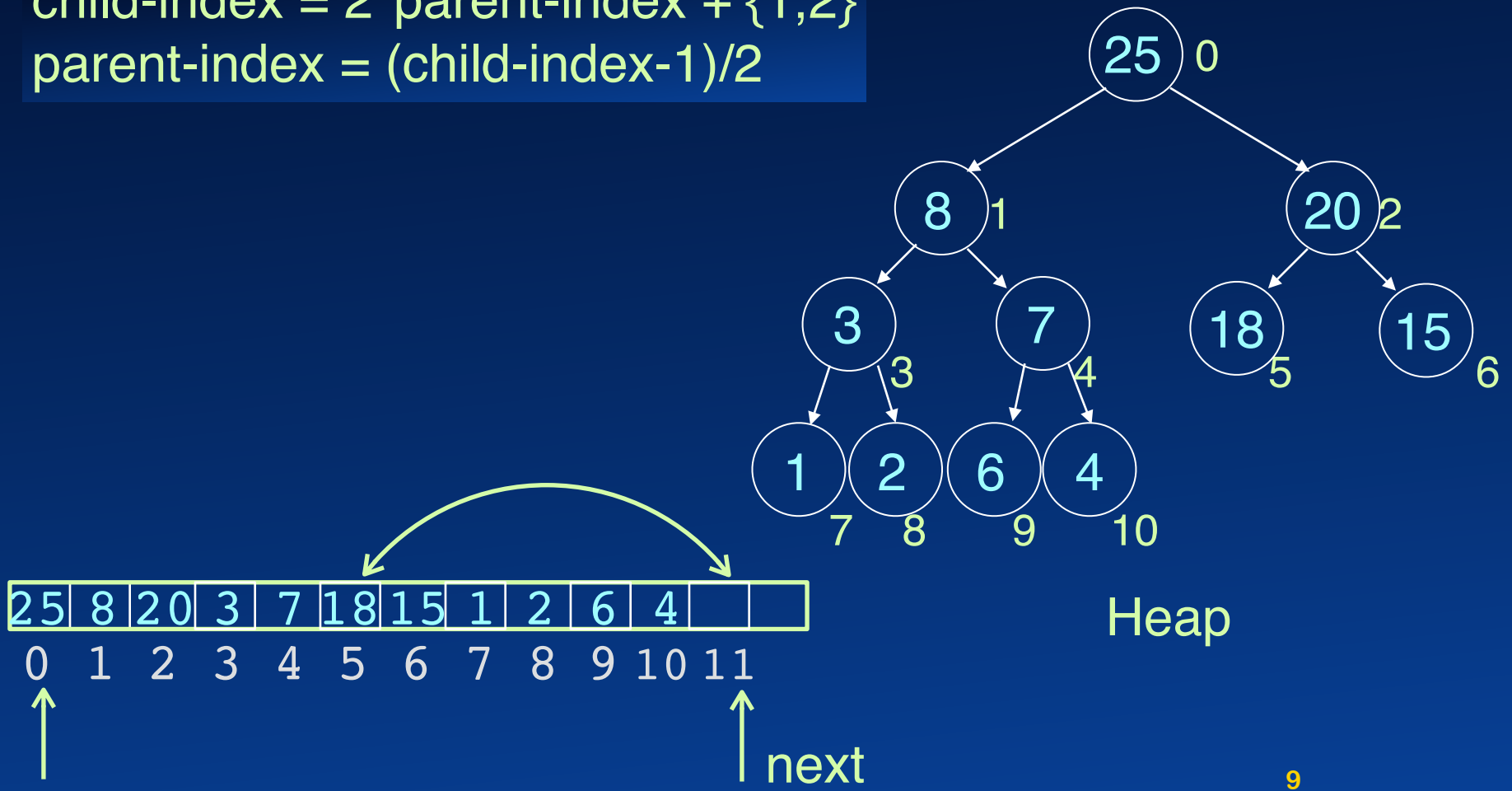

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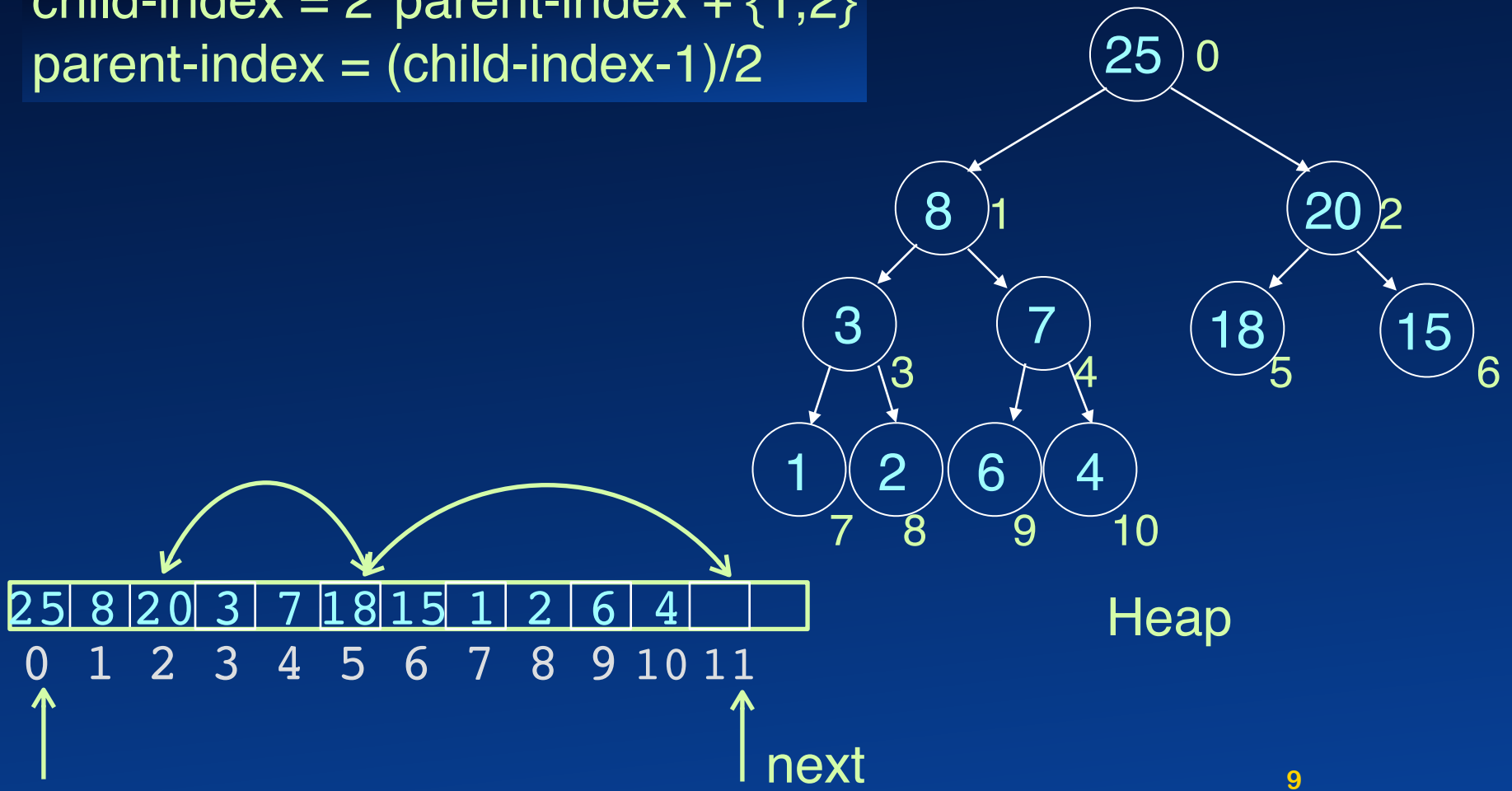

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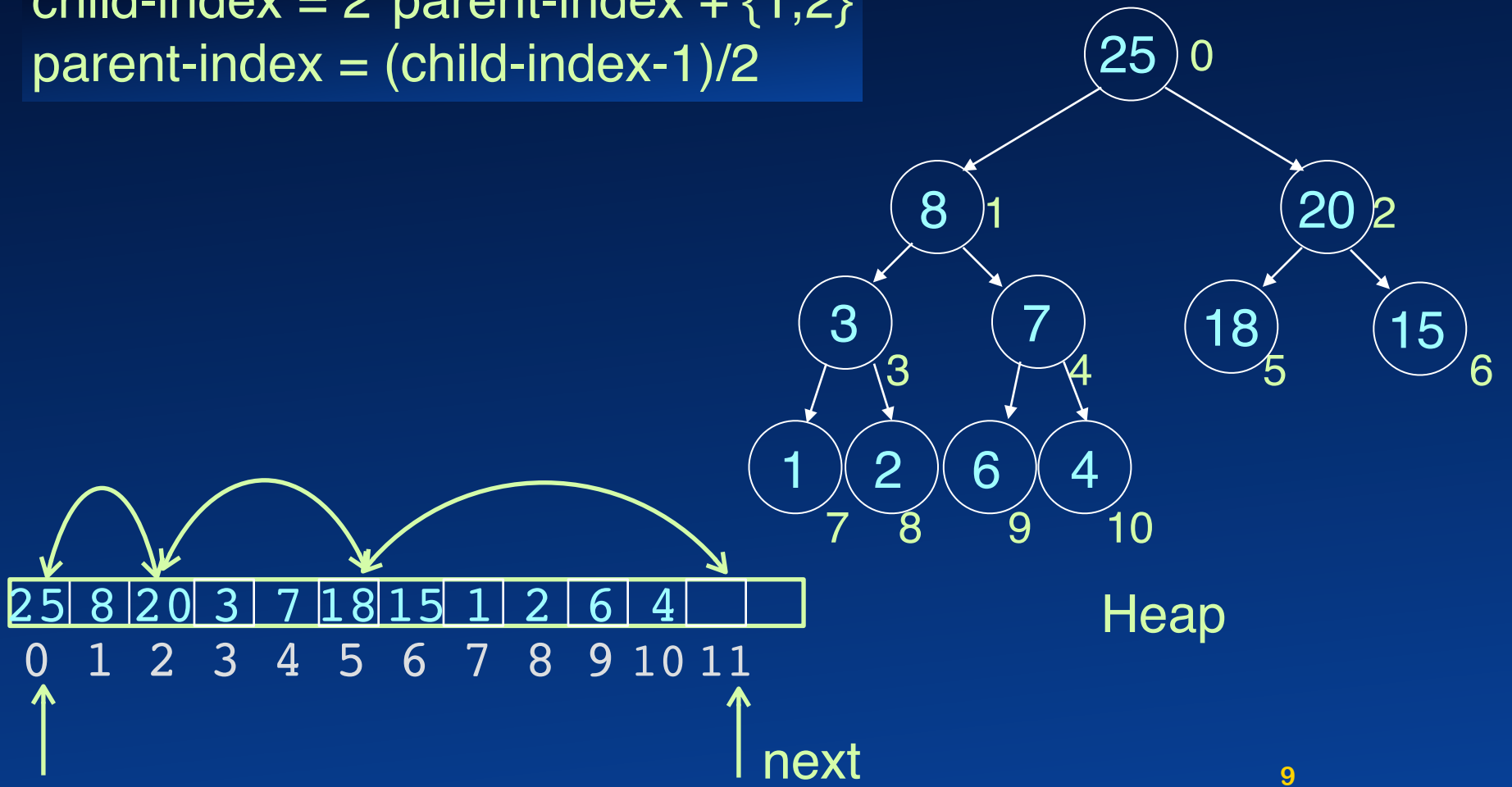

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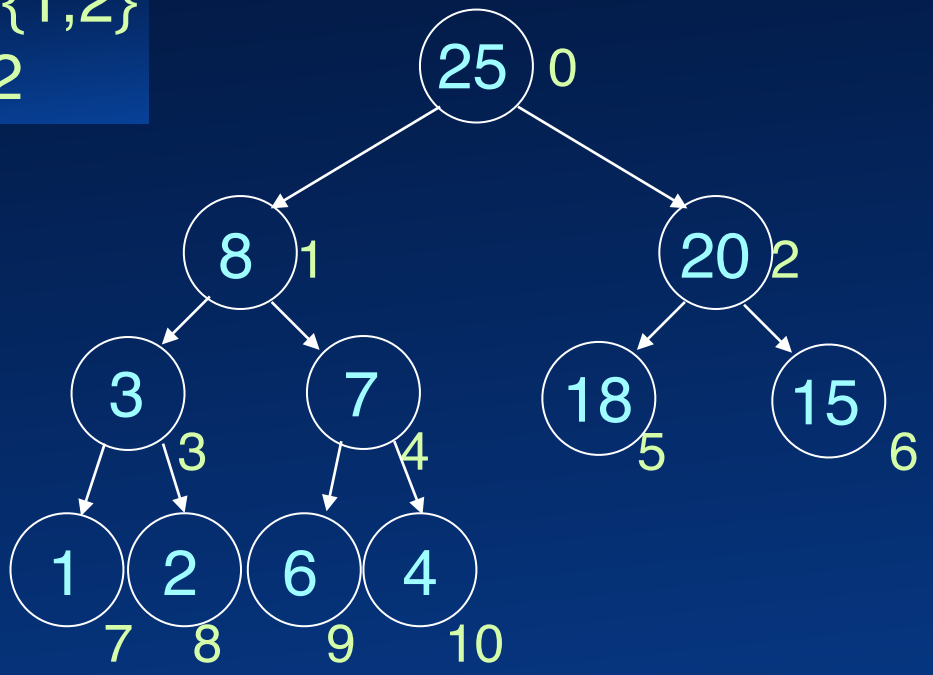
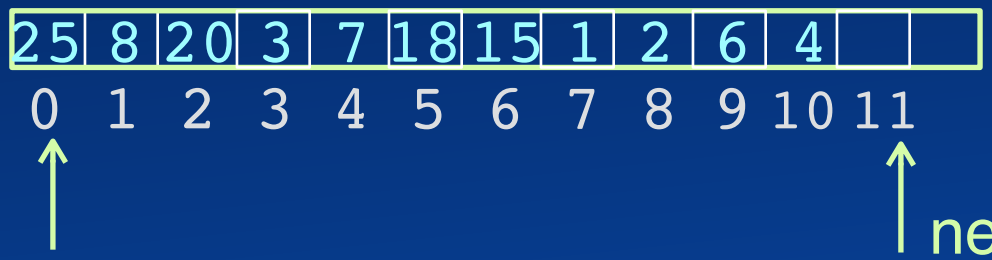


Heap



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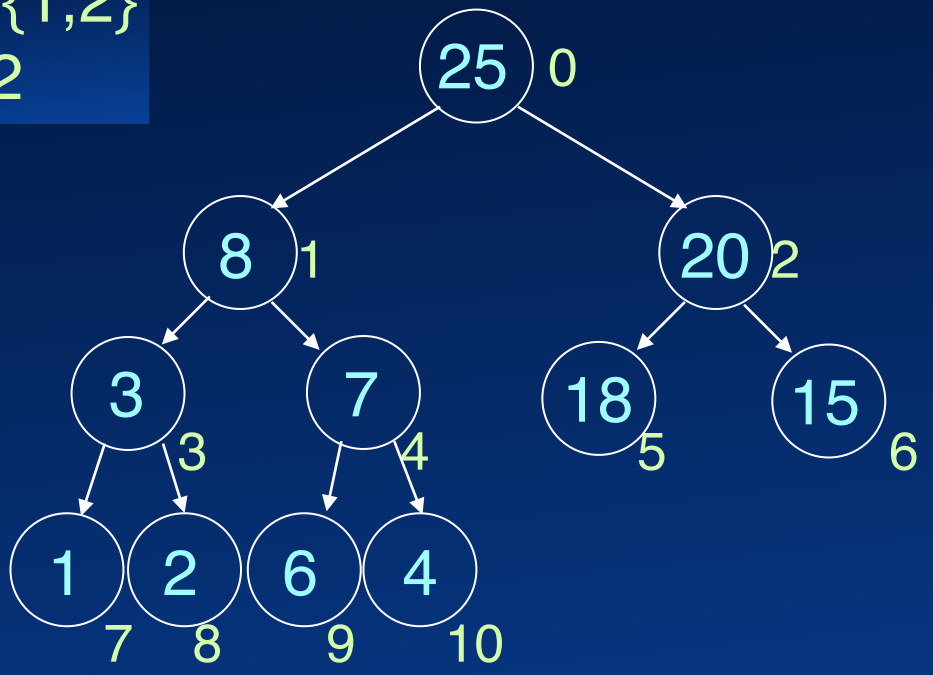

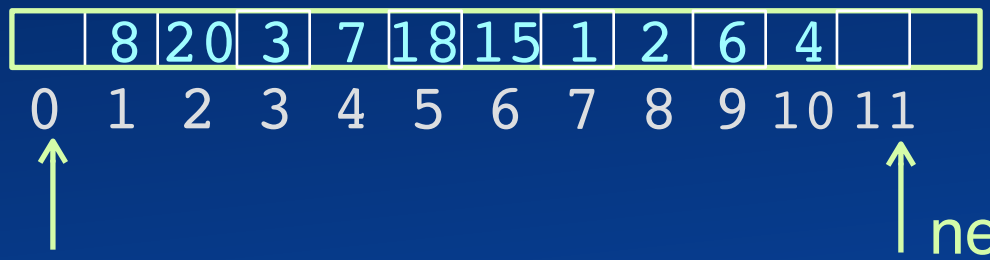

Heap

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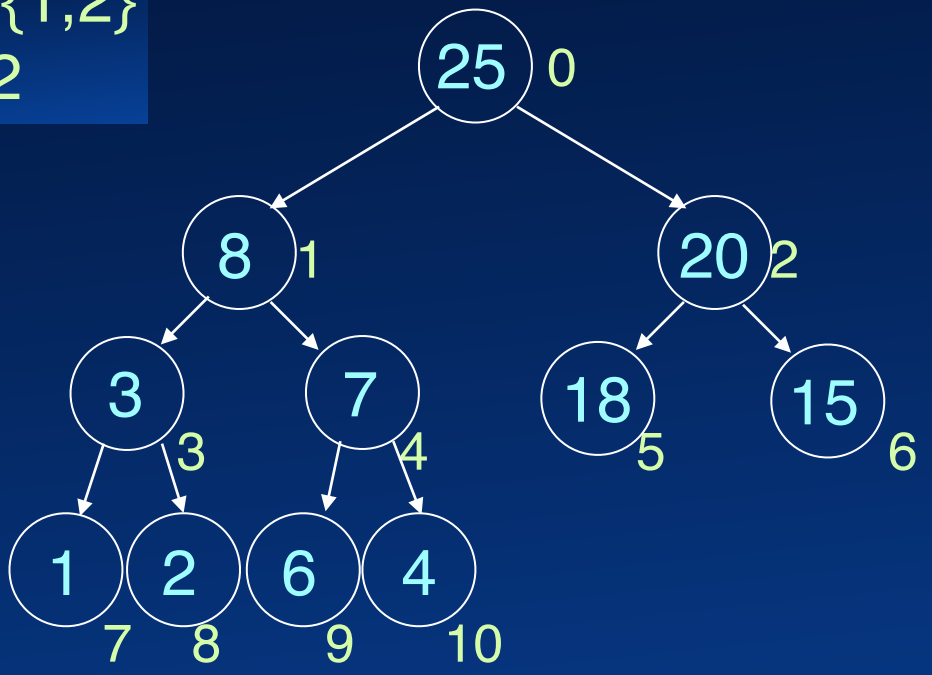
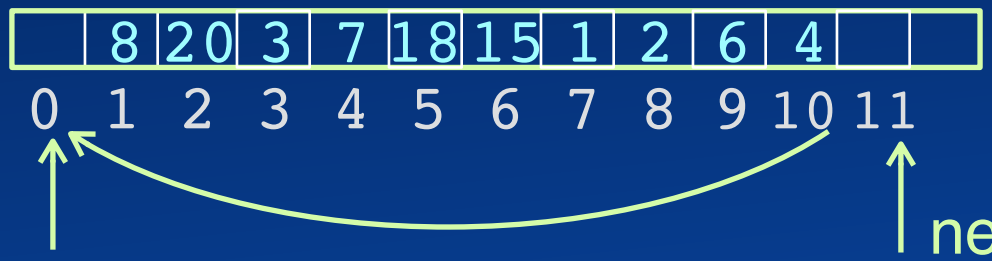


Heap



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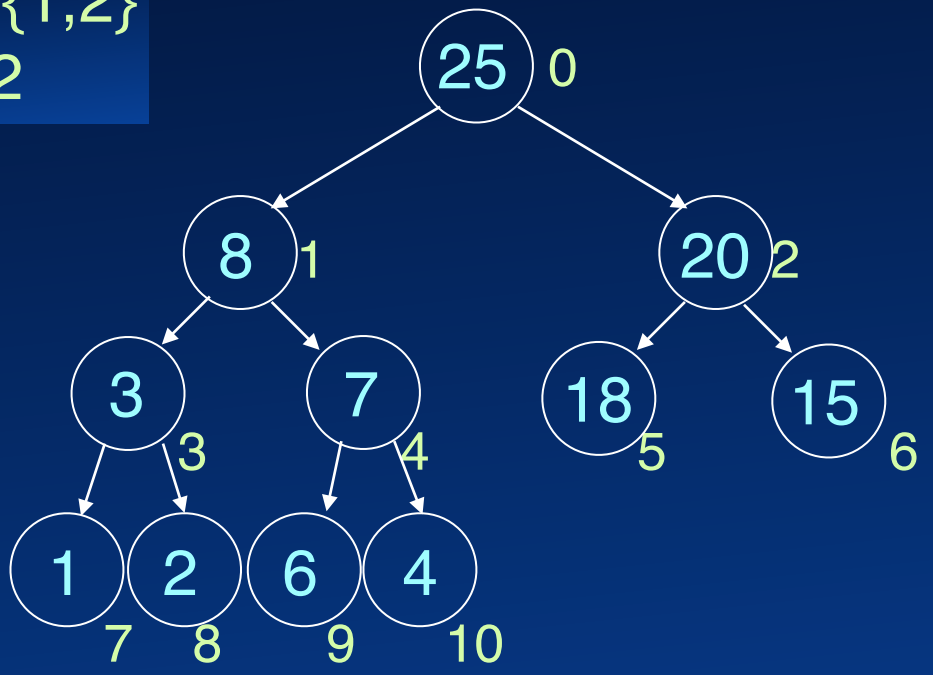
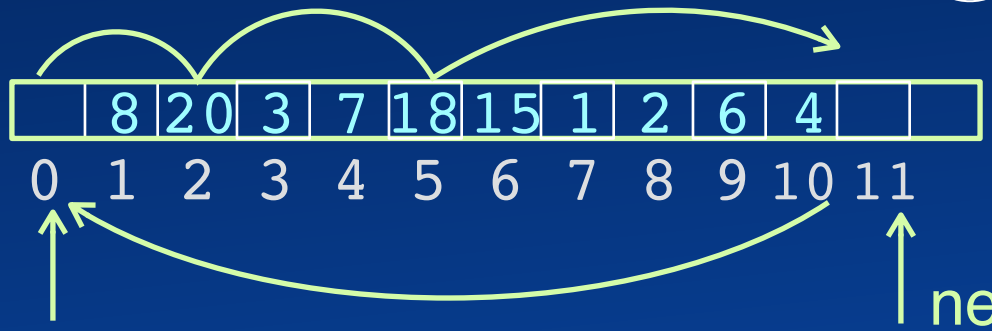
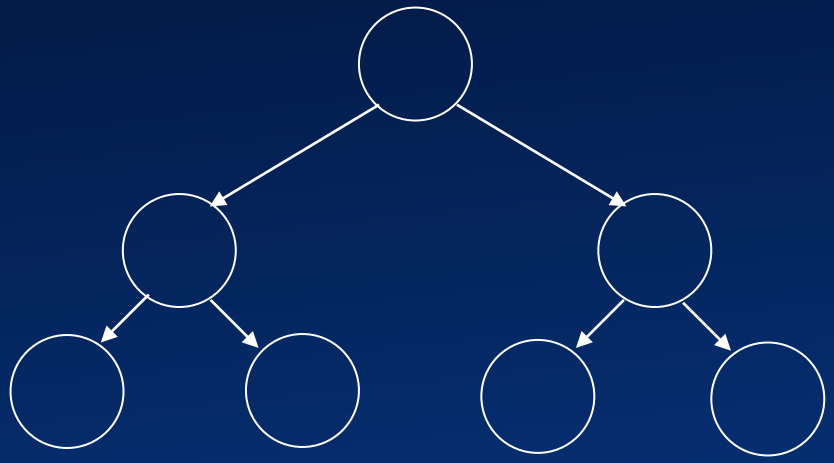


Heap



In which node may the third largest element of a heap be?

mail: col106quiz@cse.iitd.ac.in
format: 1,2,3,4,5,6



- 1
- 2
- 3
- 4
- 5
- 6

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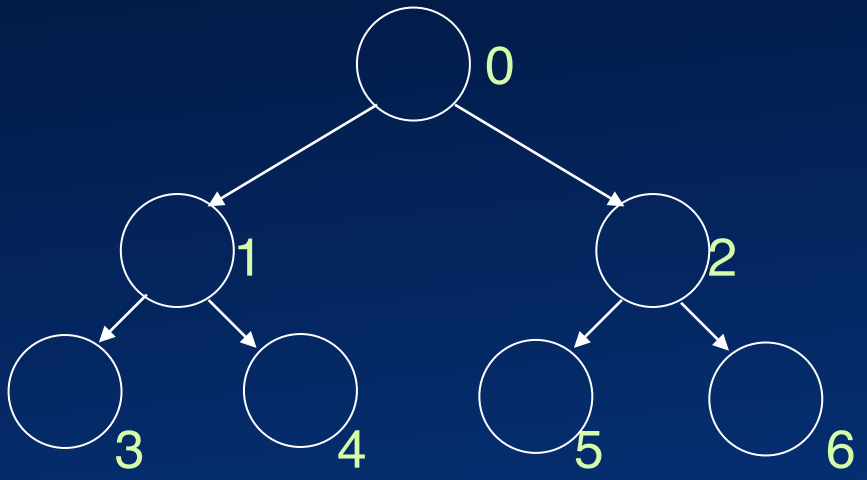
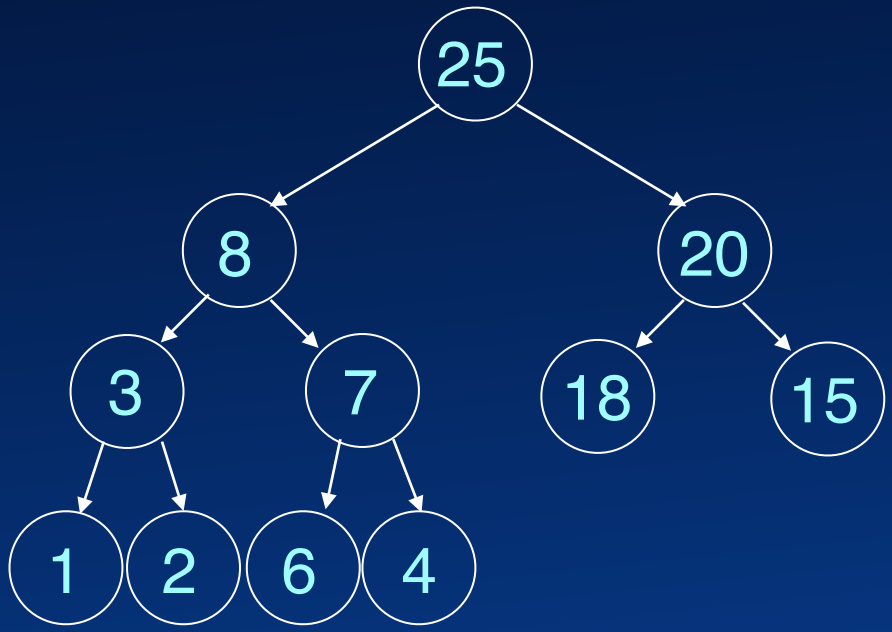


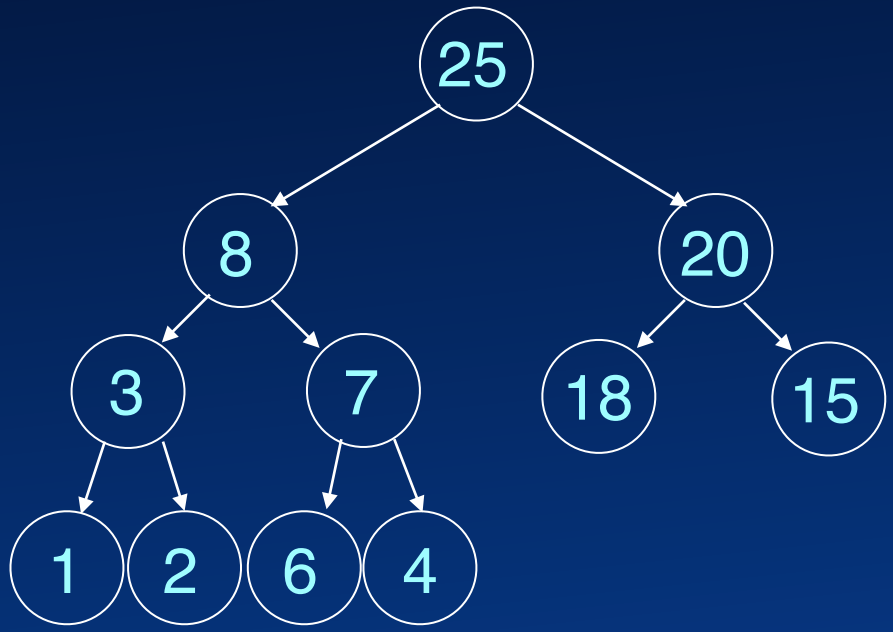
Heap Construction



Heap

Heap Construction

$? \leq \lg 1 + \lg 2 + \lg 3 + \ldots \leq ?$

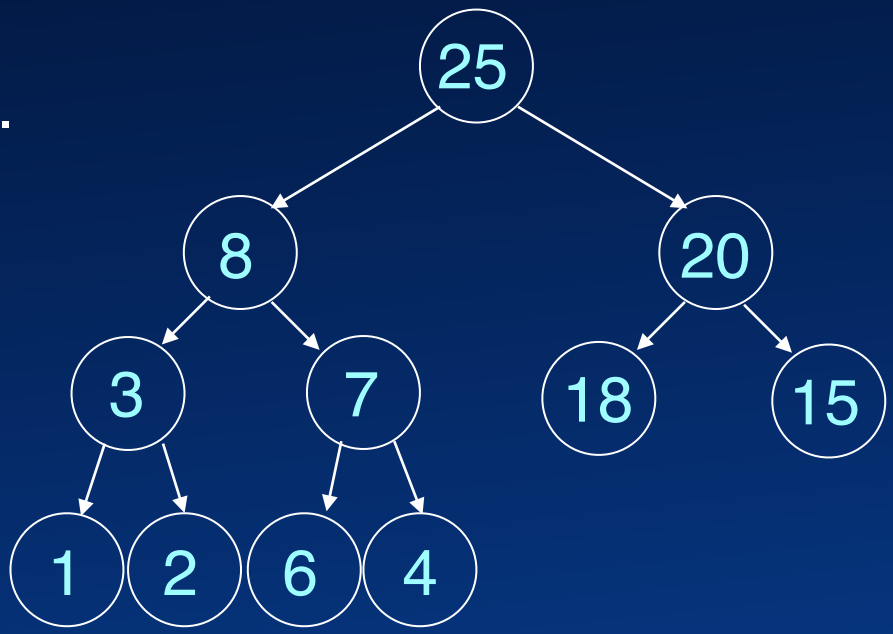


Heap

Heap Construction

$? \leq \lg 1 + \lg 2 + \lg 3 + \ldots \leq ?$

$\lg 1 + 1 + \lg 2 + 1 + \lg 3 + 1 \ldots$



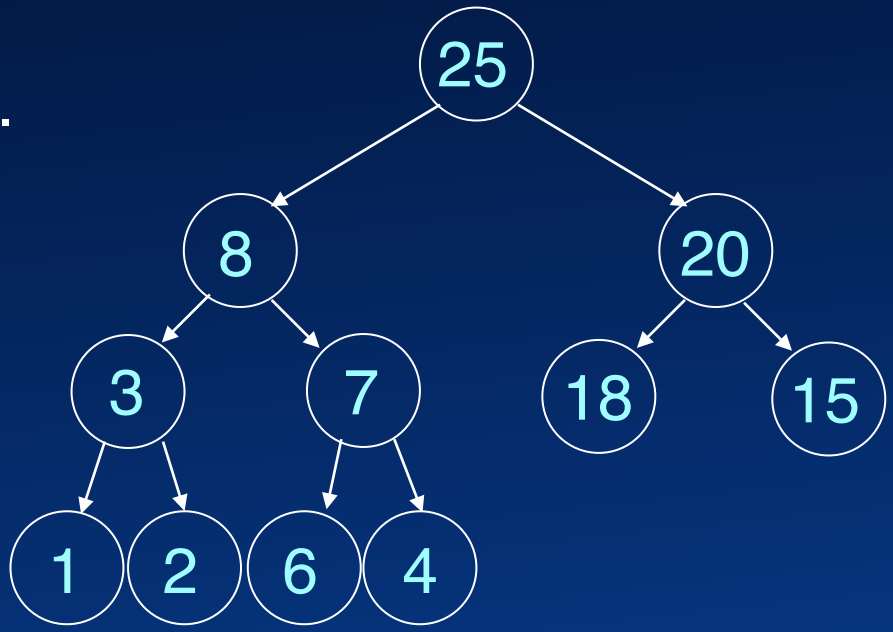
Heap

Heap Construction

$? \leq \lg 1 + \lg 2 + \lg 3 + \ldots \leq ?$

$\lg 1 + 1 + \lg 2 + 1 + \lg 3 + 1 \ldots$

$\lg (1 * 2 * 3 \ldots n\text{-}1) = \lg (n\text{-}1)!$



Heap

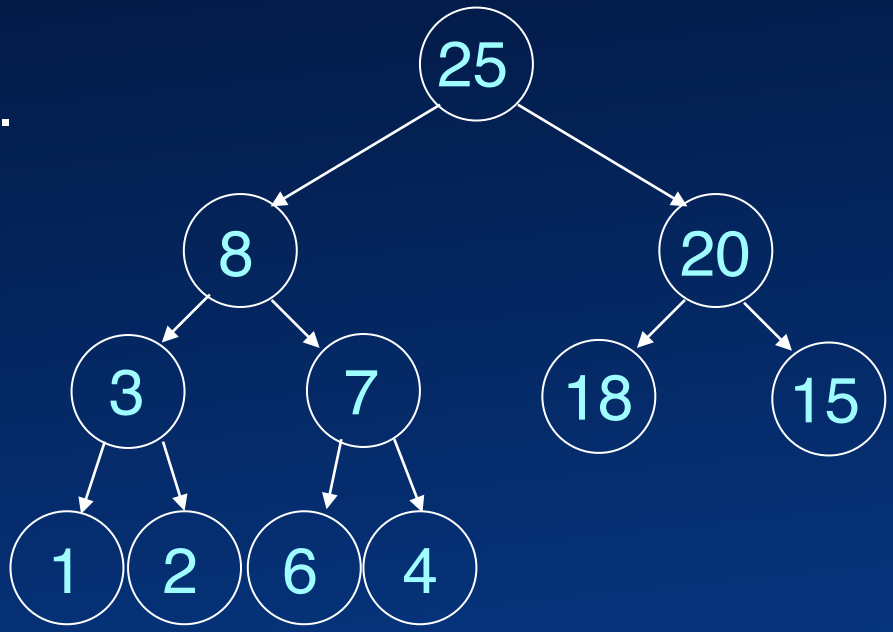
Heap Construction

? ≤ lg 1 + lg 2 + lg 3 + … ≤ ?

lg 1 + 1 + lg 2 + 1 + lg 3 + 1 ..

lg (1 * 2 * 3 … n-1) = lg (n-1)!

$\Theta(n \lg n)$



Heap

Heap Construction

? ≤ lg 1 + lg 2 + lg 3 + … ≤ ?

lg 1 + 1 + lg 2 + 1 + lg 3 + 1 ..

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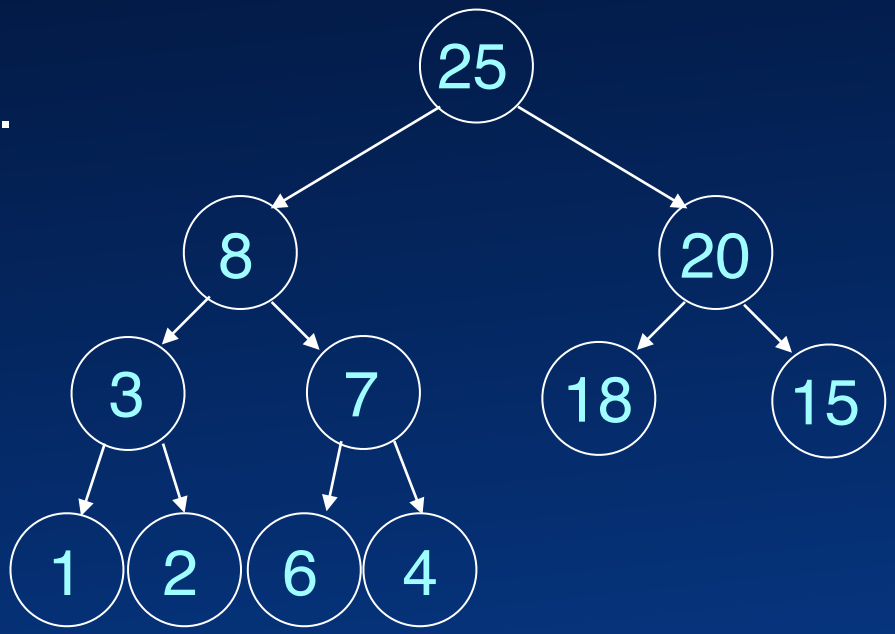
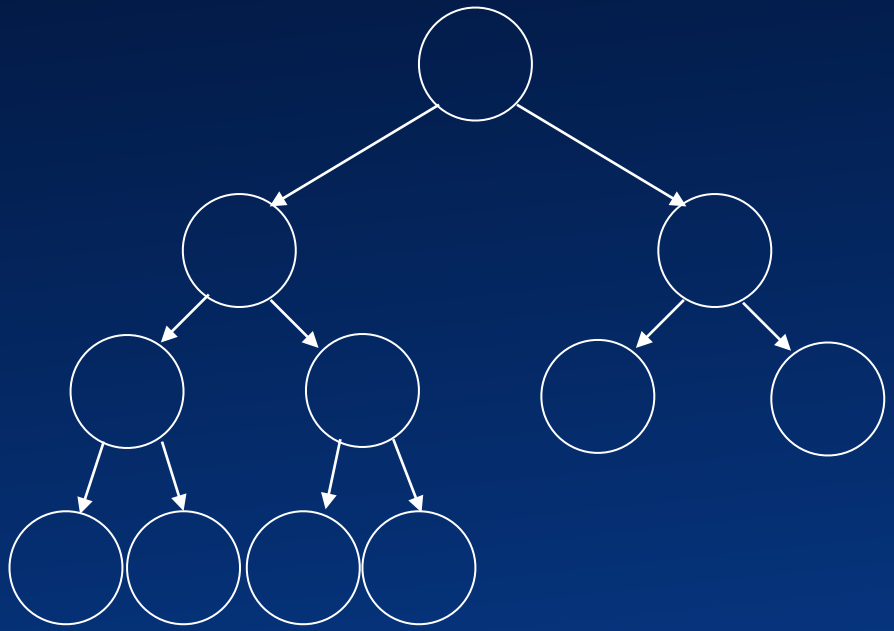


Heap

$(n/2)^{n/2} < n! < n^n, n > 1$

Heap Construction

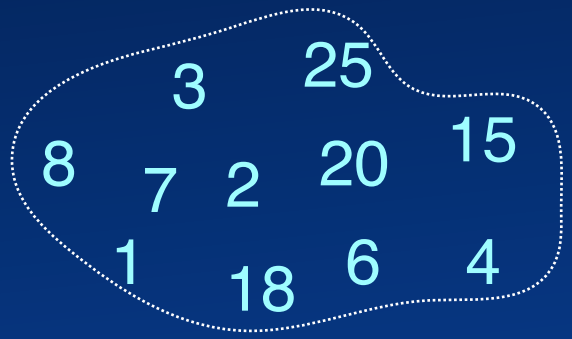


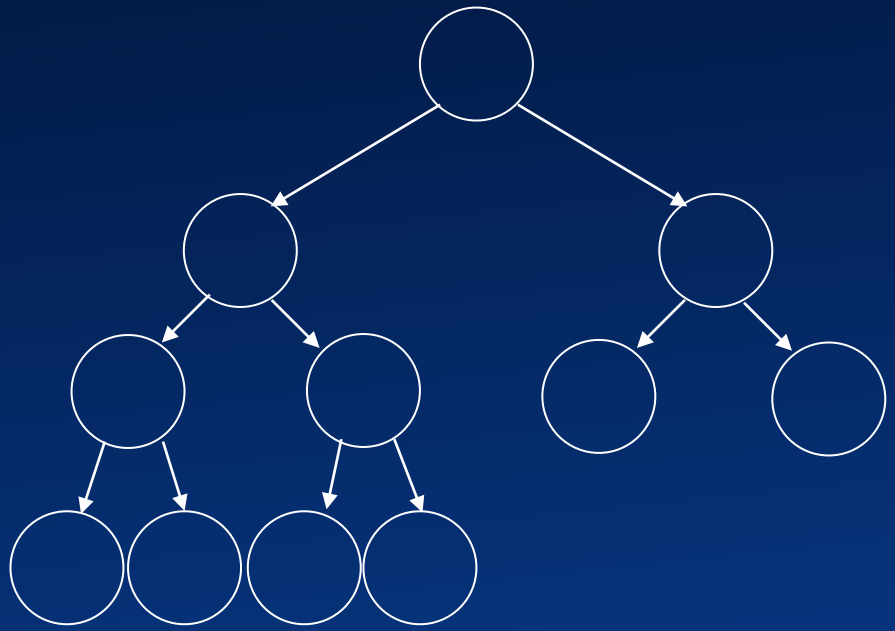
3 25

8 7 2 20 15

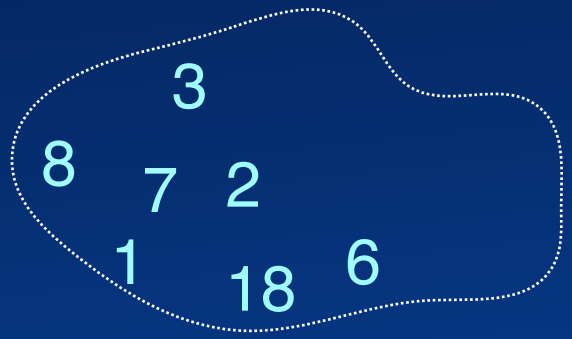
1 18 6 4

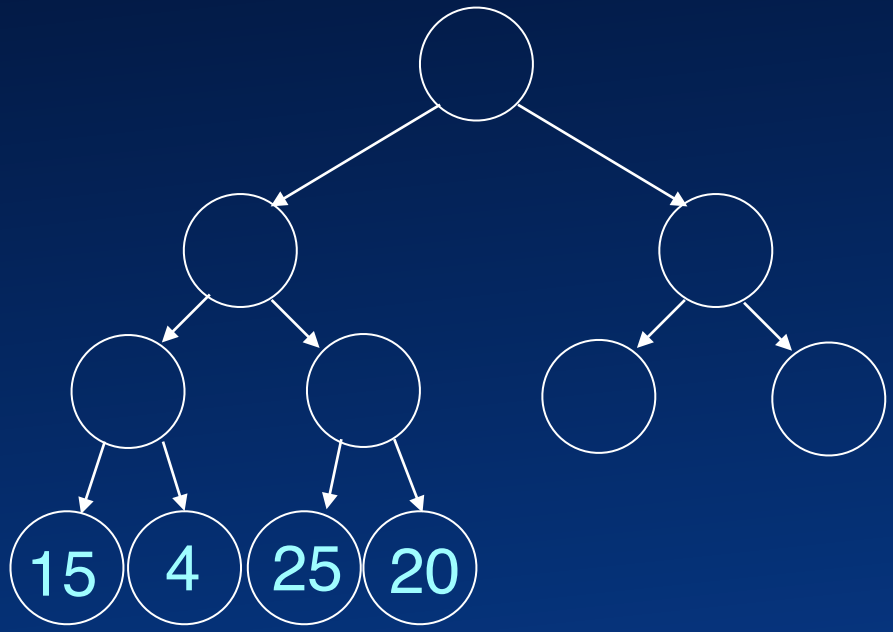
Heap Construction



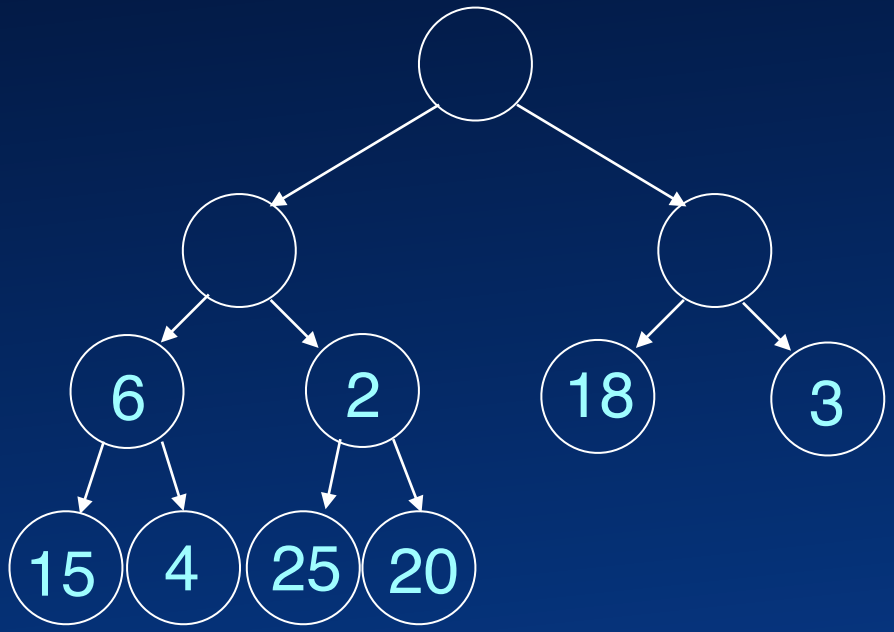


Heap Construction

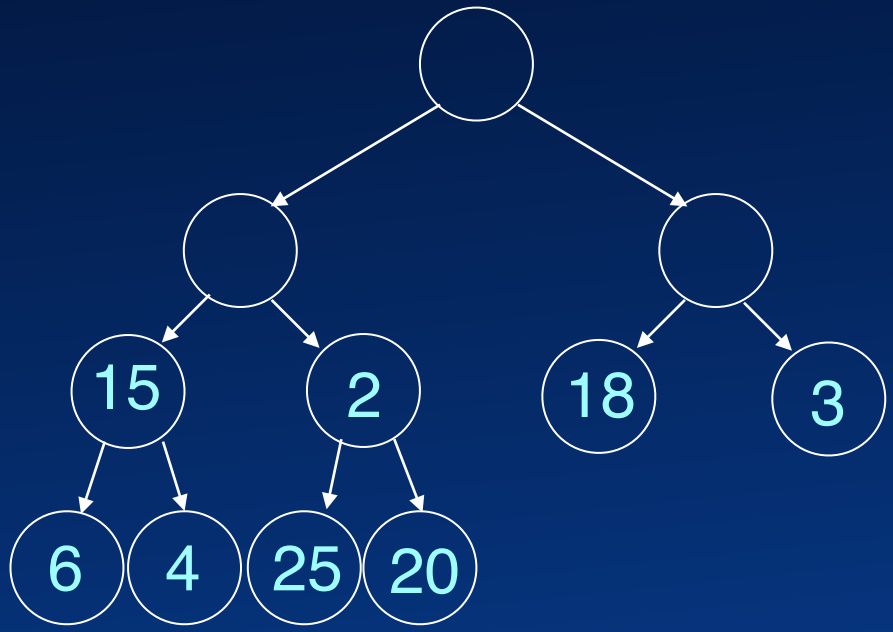




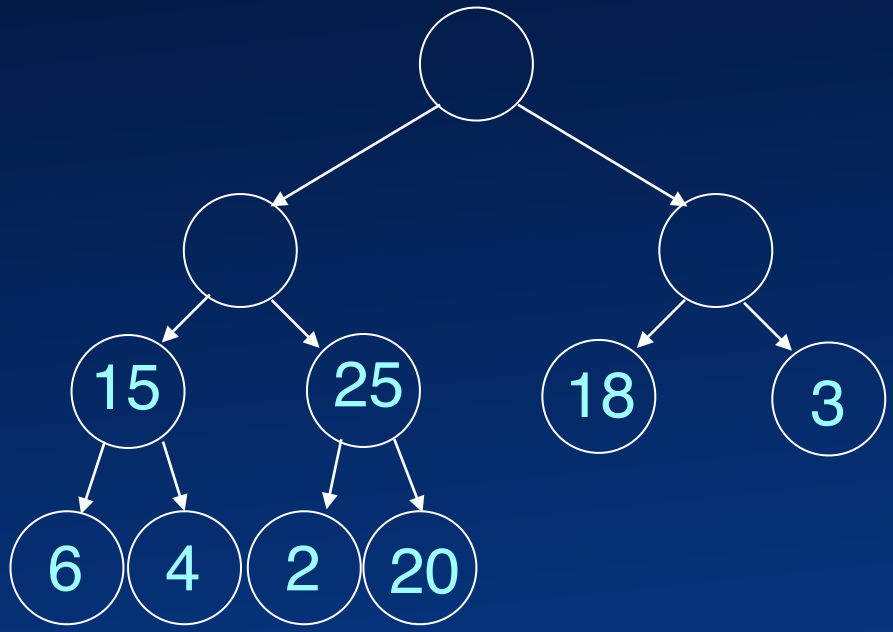
Heap Construction



Heap Construction



Heap Construction



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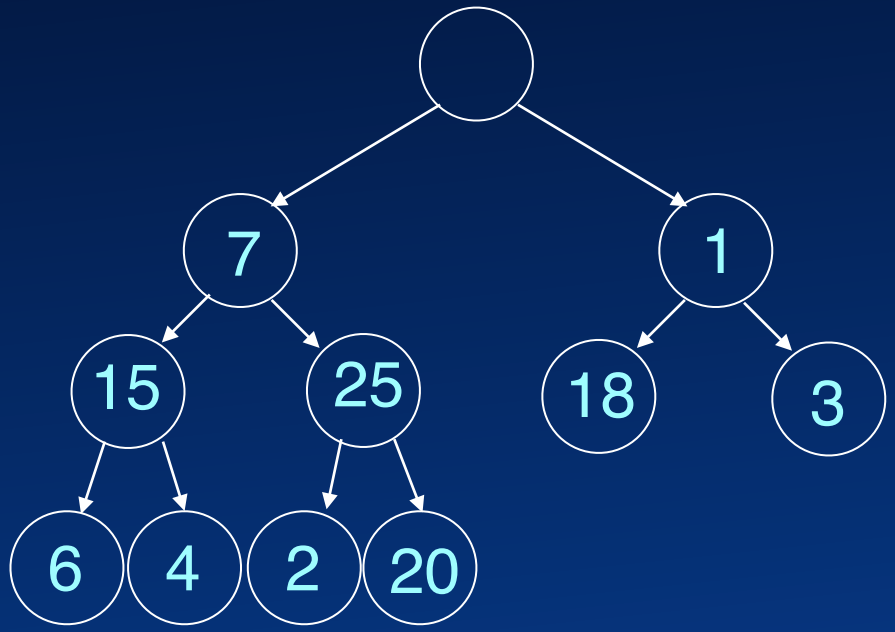
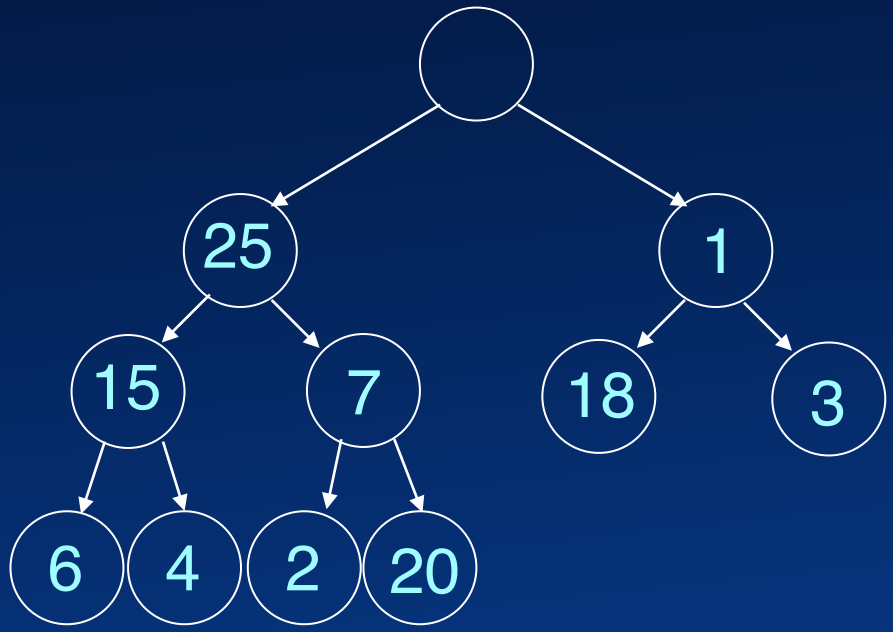
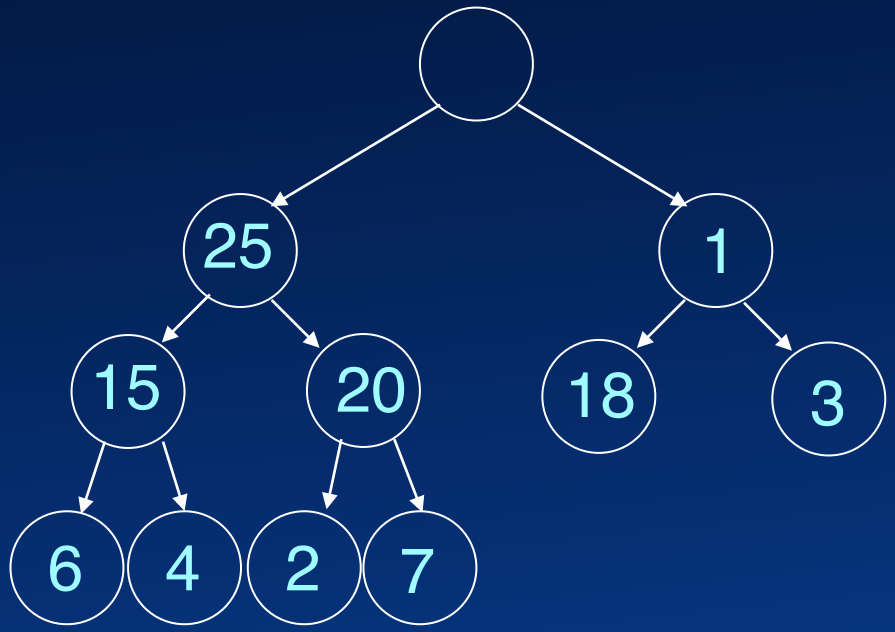


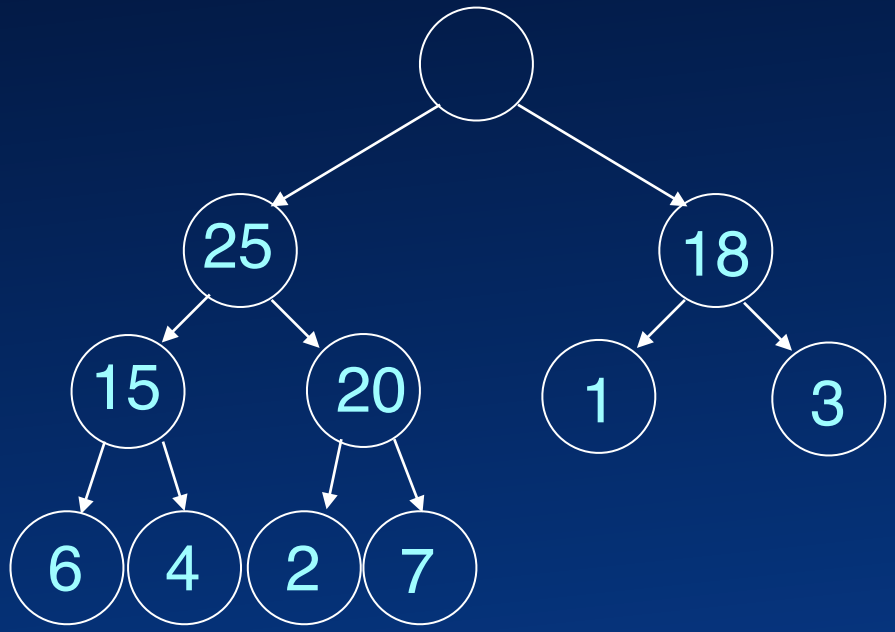
Heap Construction



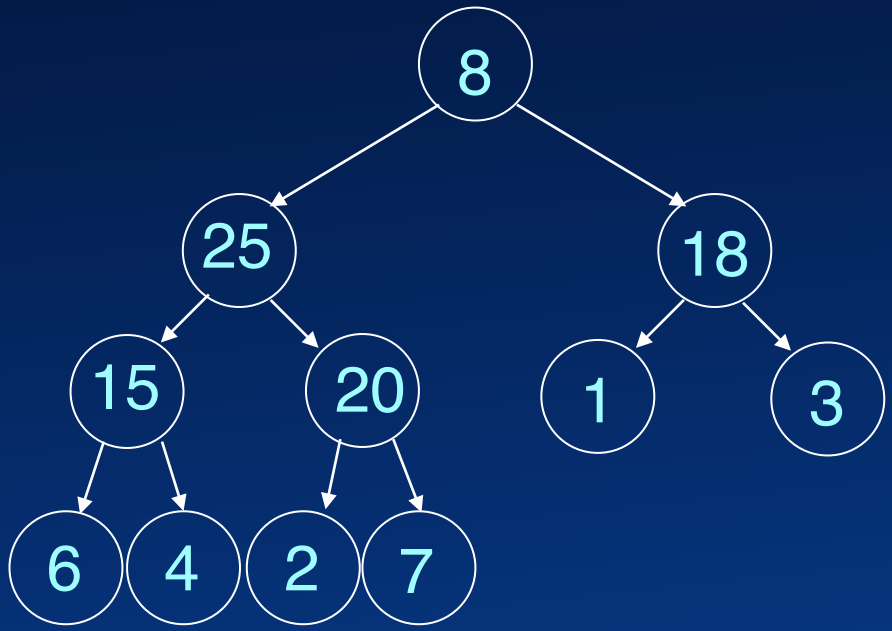
Heap Construction



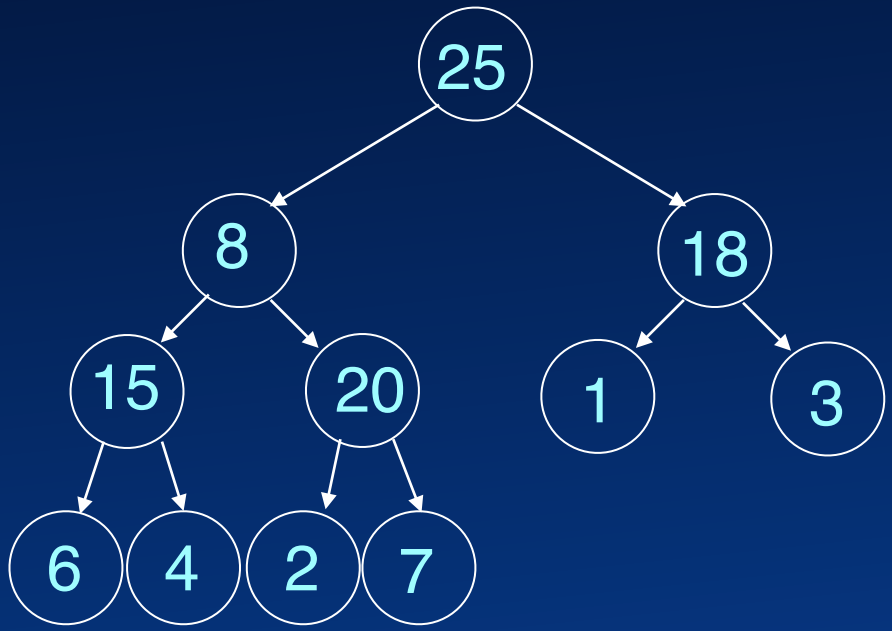
Heap Construction



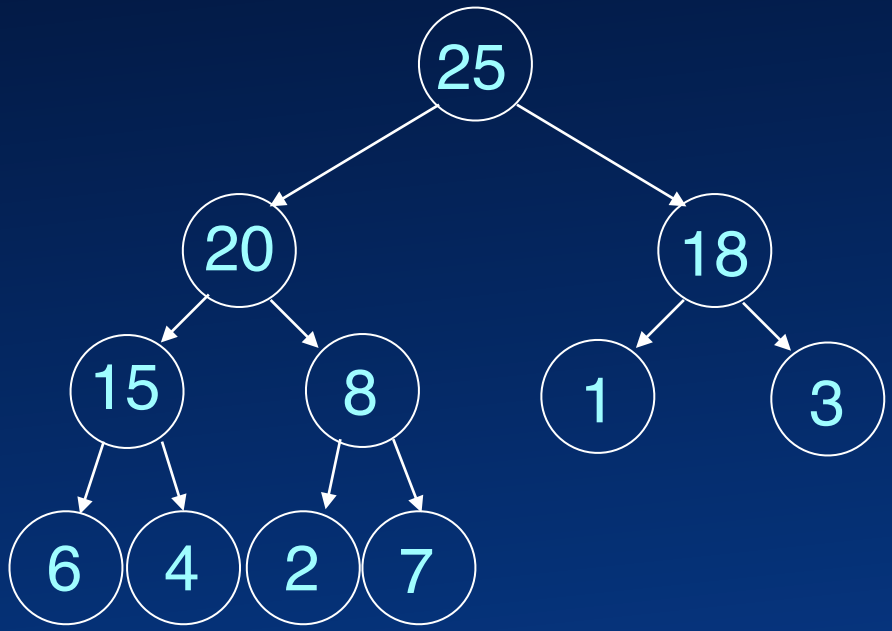
Heap Construction



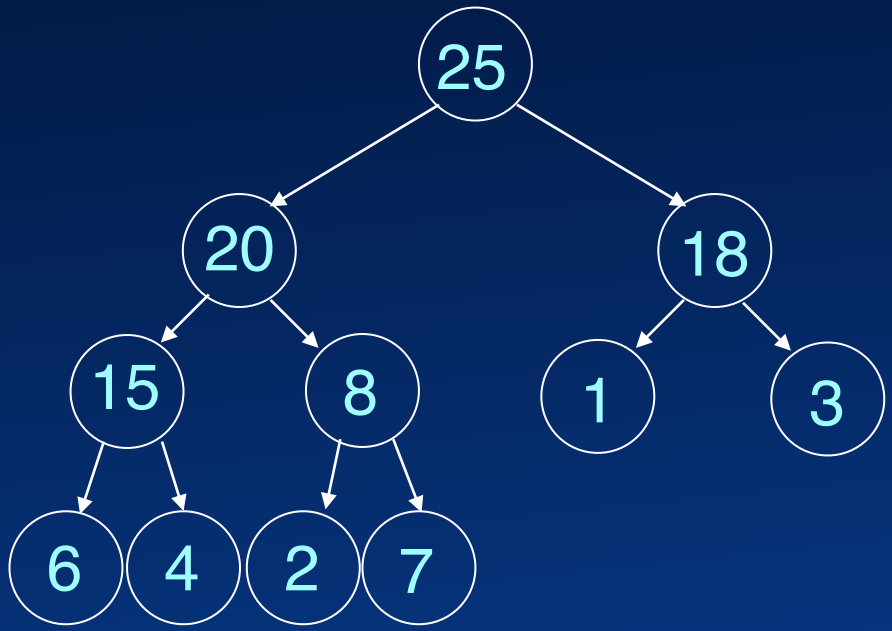
Heap Construction



Heap Construction



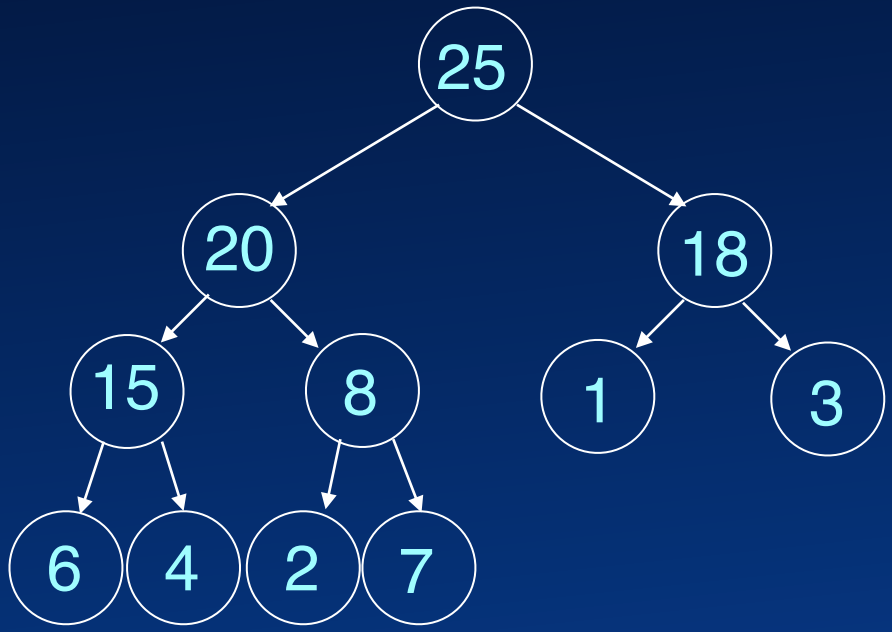
Heap Construction



Heap

Heap Construction

$$T(n) \leq (n/2) + 2(n/4) + 3(n/8) + 4(n/16)$$

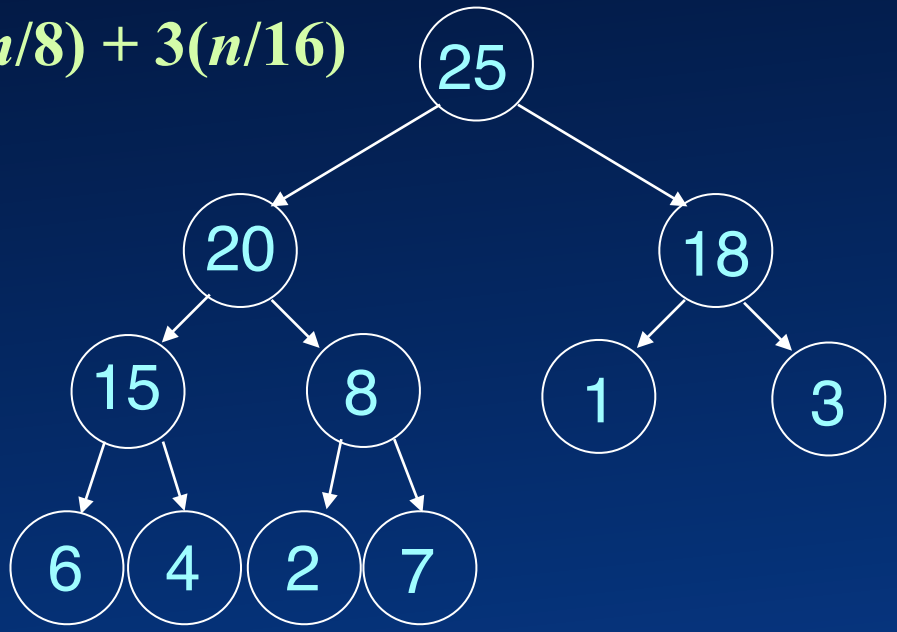


Heap

Heap Construction

$$T(n) \leq (n/2) + 2(n/4) + 3(n/8) + 4(n/16)$$

$$= n/2 + n/4 + n/8 + .. (n/4) + 2(n/8) + 3(n/16)$$

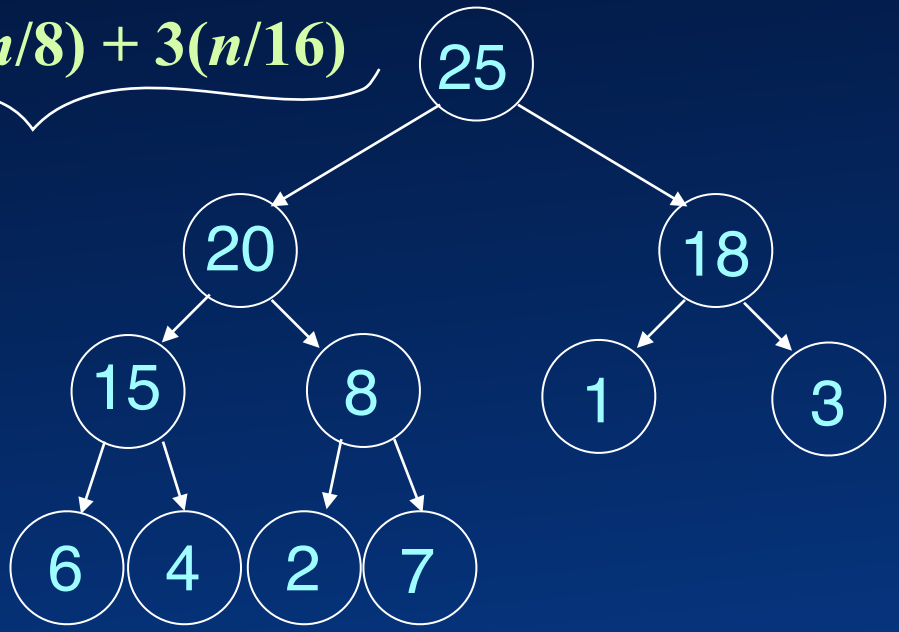


Heap

Heap Construction

$$T(n) \leq (n/2) + 2(n/4) + 3(n/8) + 4(n/16)$$

$$= n/2 + n/4 + n/8 + .. (n/4) + 2(n/8) + 3(n/16)$$



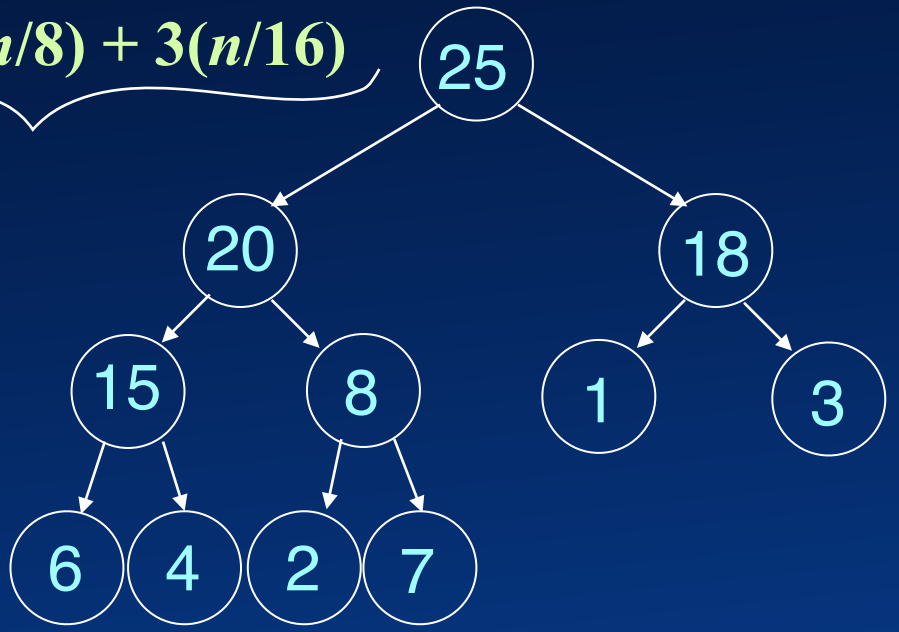
Heap

Heap Construction

$T(n) \leq (n/2) + 2(n/4) + 3(n/8) + 4(n/16)$

$= n/2 + n/4 + n/8 + .. (n/4) + 2(n/8) + 3(n/16)$

$T(n) \leq (n\text{-}1) + T(n/2)$



Heap

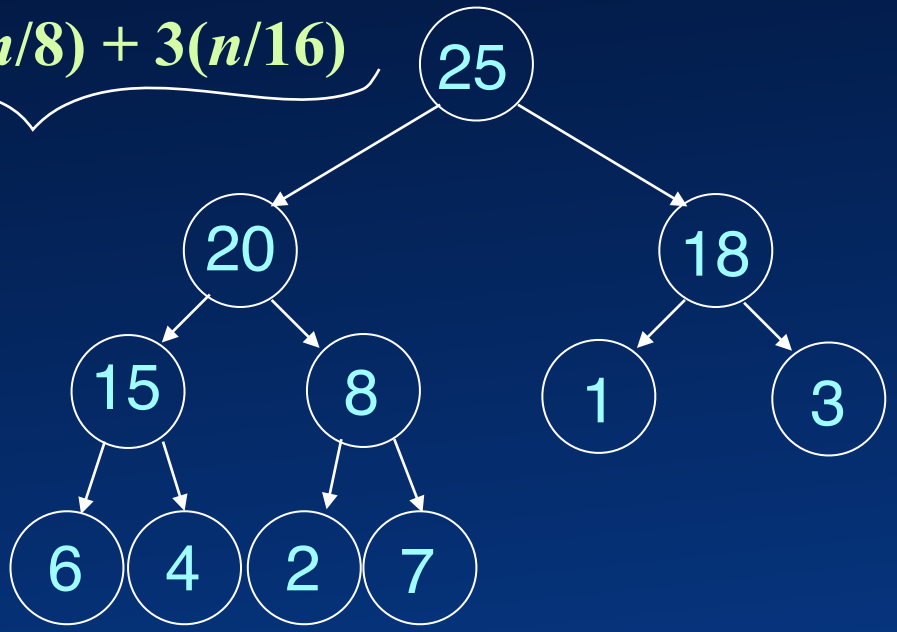
Heap Construction

$T(n) \leq (n/2) + 2(n/4) + 3(n/8) + 4(n/16)$

$= n/2 + n/4 + n/8 + .. (n/4) + 2(n/8) + 3(n/16)$

$T(n) \leq (n\text{-}1) + T(n/2)$

$= (n\text{-}1) + (n/2 - 1) ..$



Heap

Heap Construction

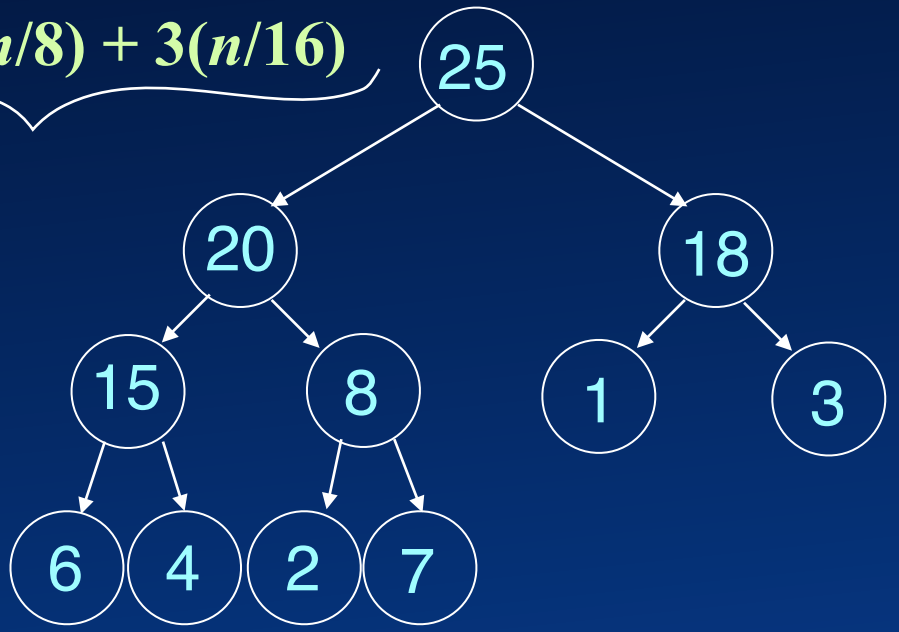
$T(n) \leq (n/2) + 2(n/4) + 3(n/8) + 4(n/16)$

$= n/2 + n/4 + n/8 + .. (n/4) + 2(n/8) + 3(n/16)$

$T(n) \leq (n\text{-}1) + T(n/2)$

$= (n\text{-}1) + (n/2 - 1) ..$

$= \Theta(n)$



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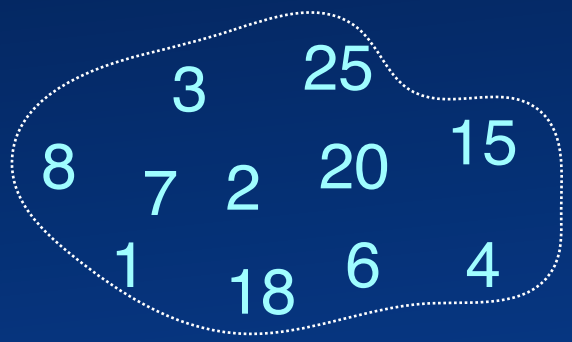
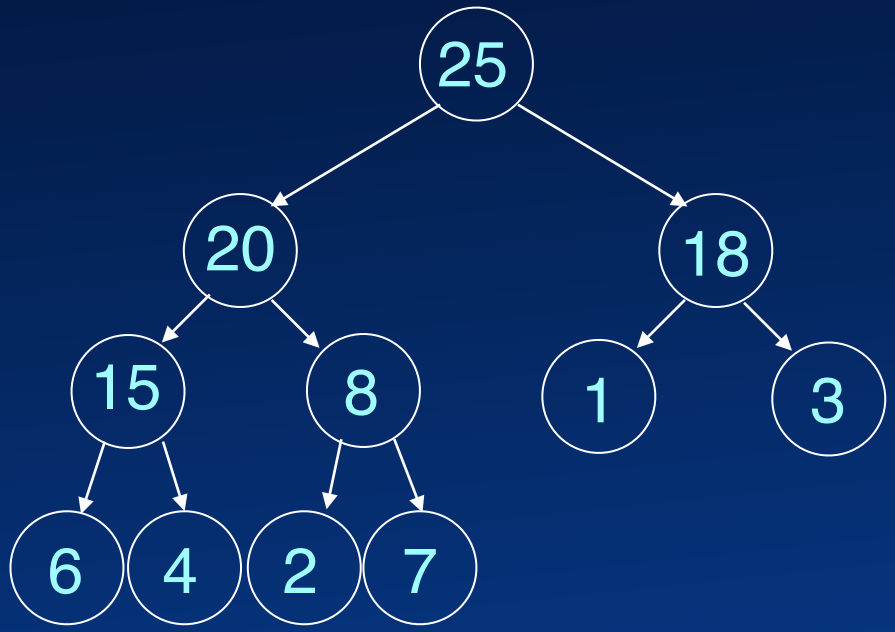
Heap Construction





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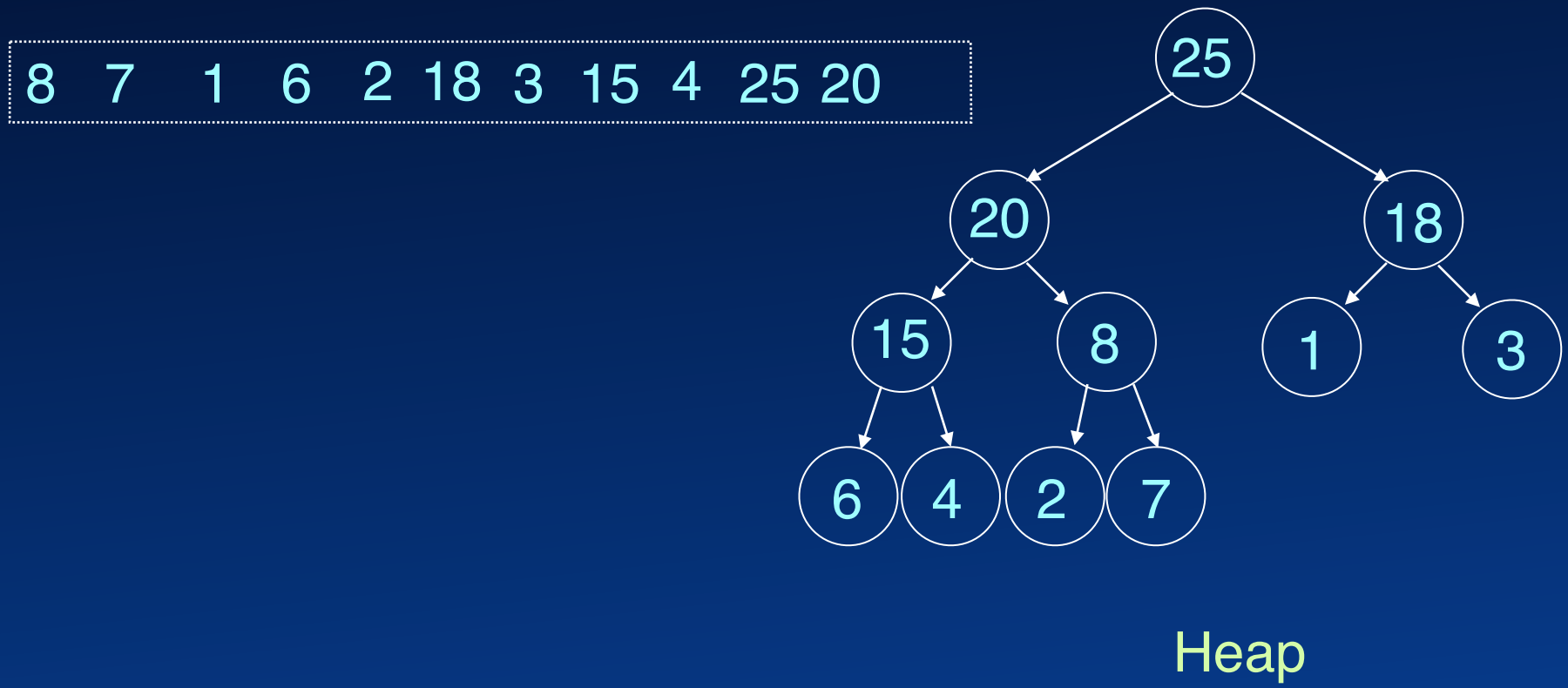

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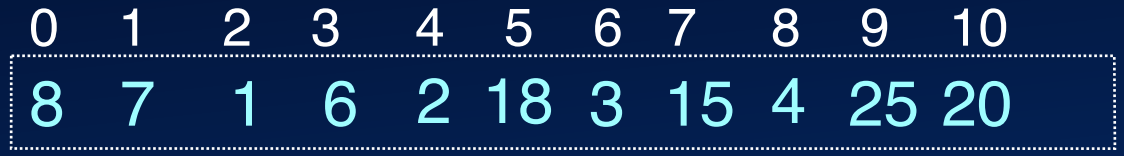

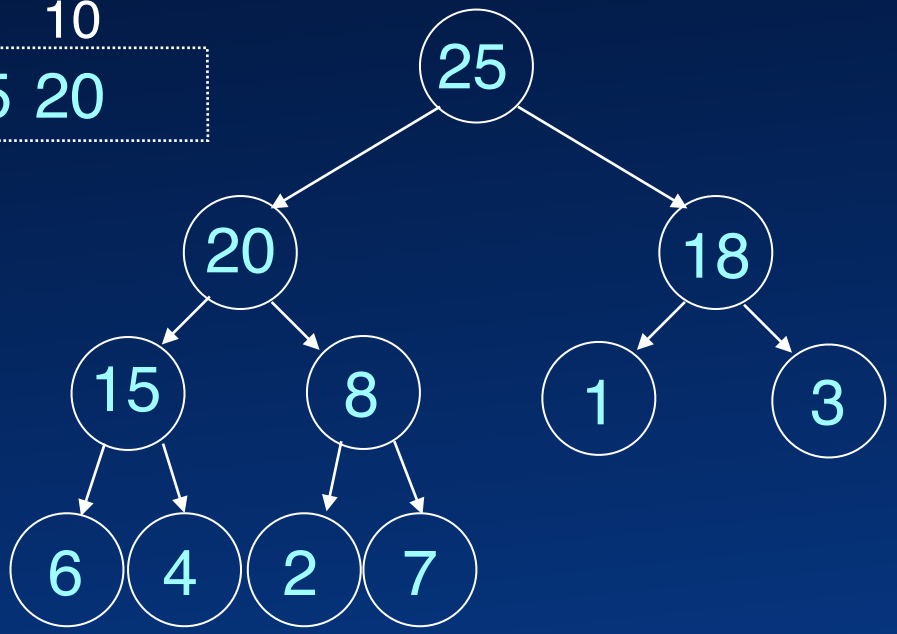

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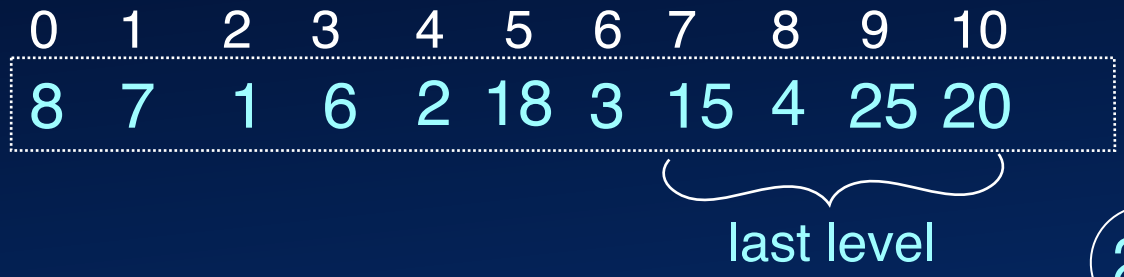

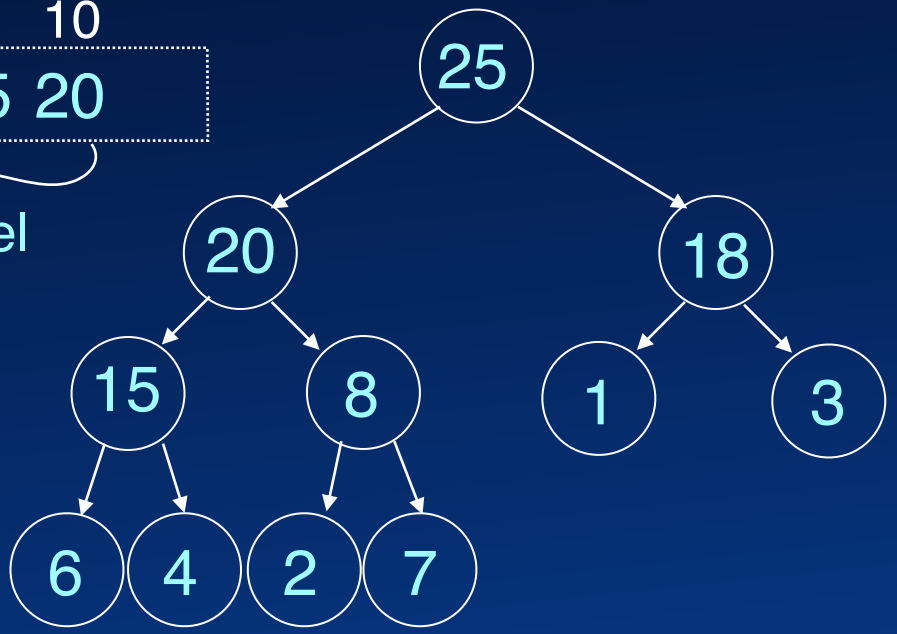

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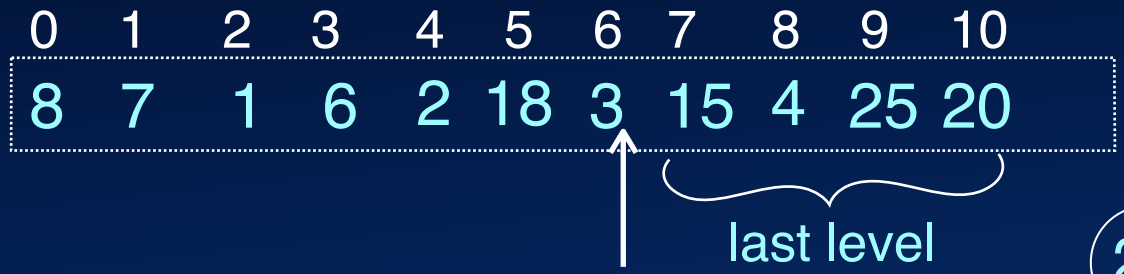

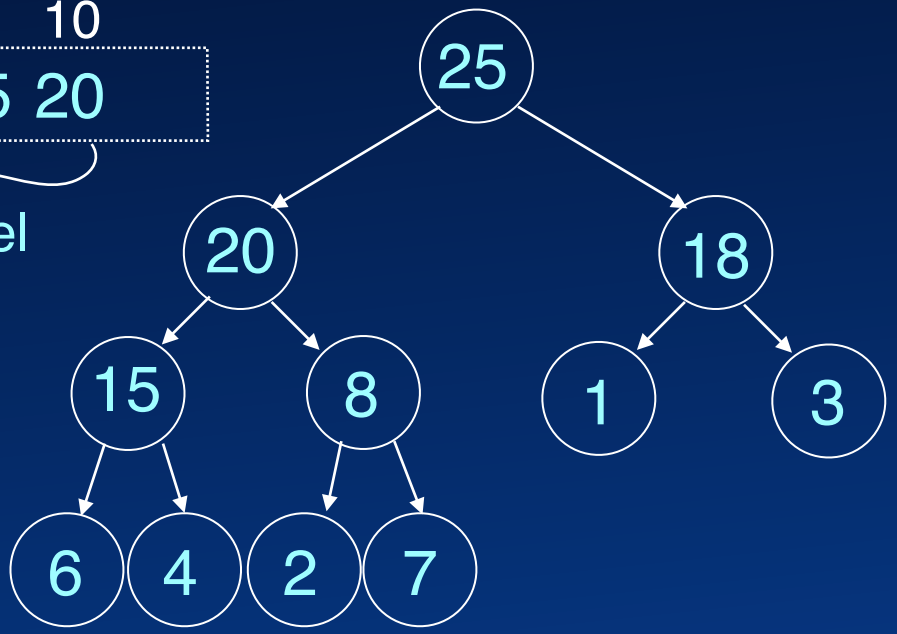

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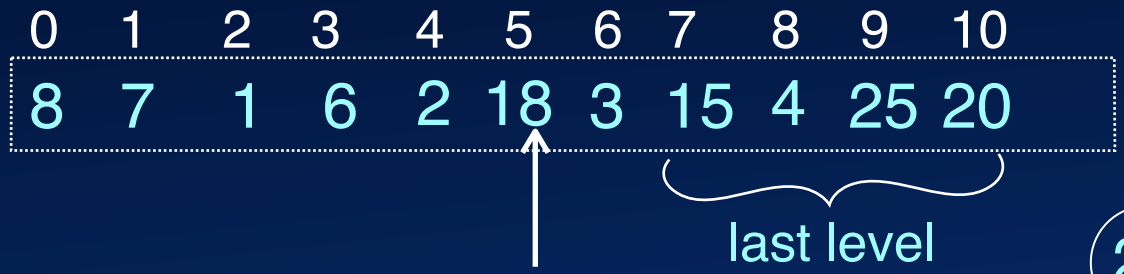

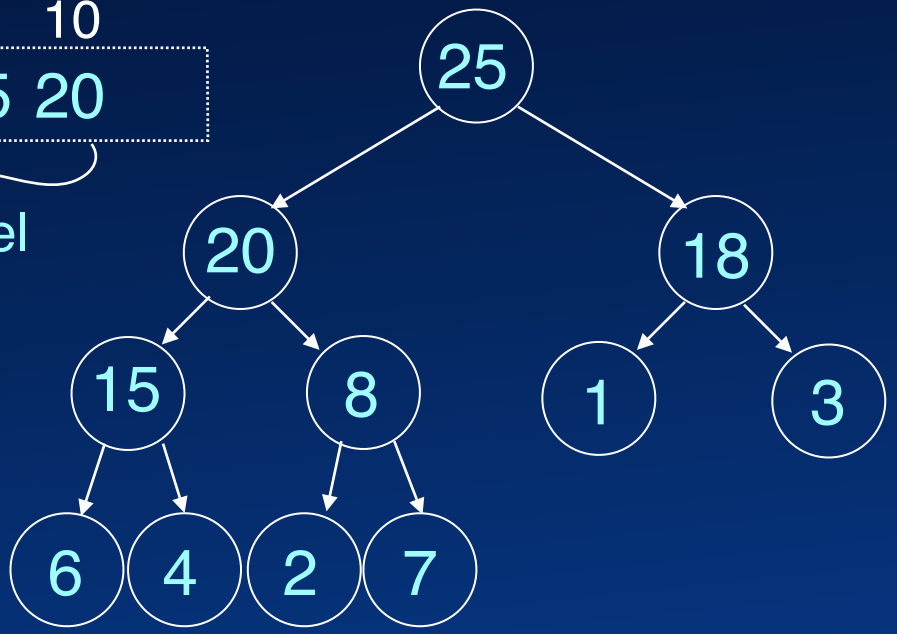

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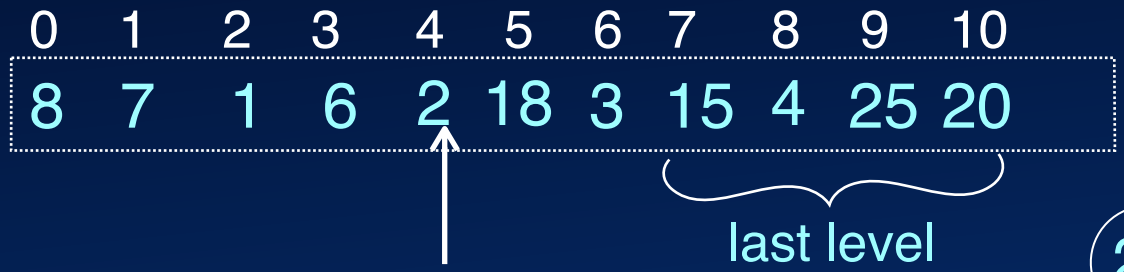

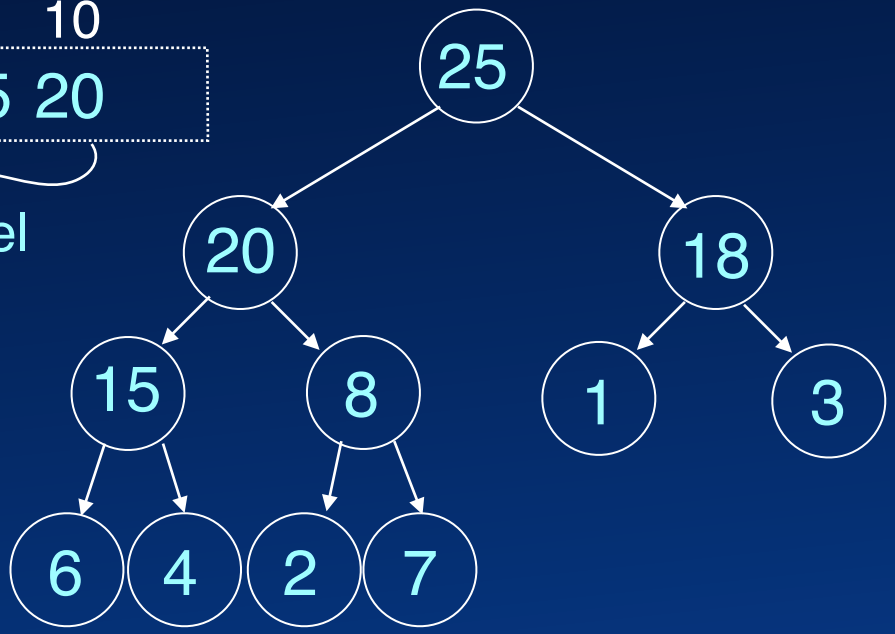

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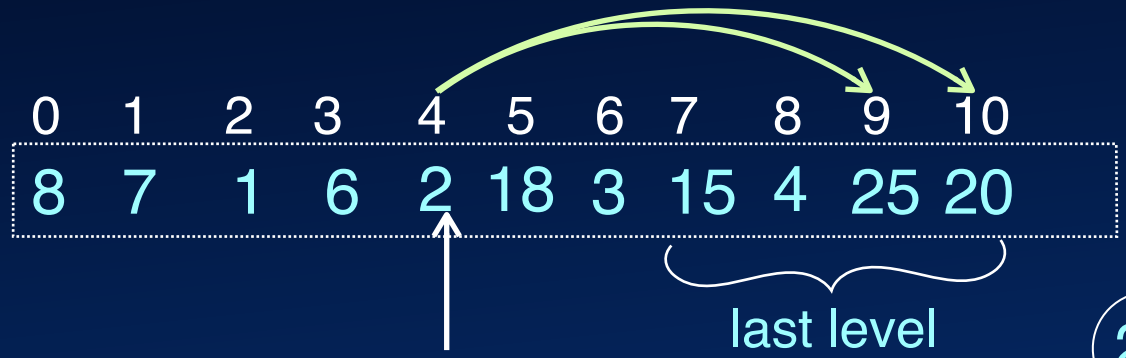

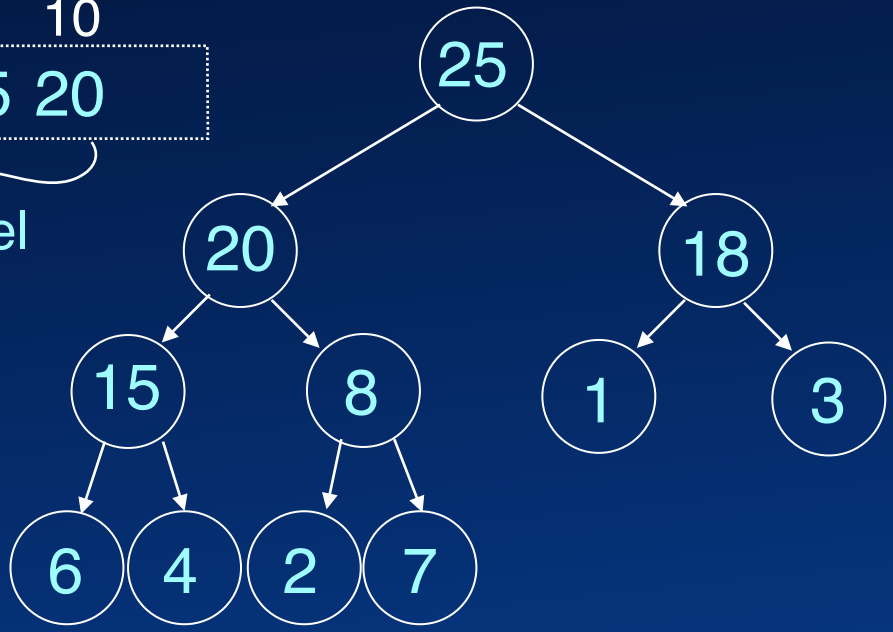

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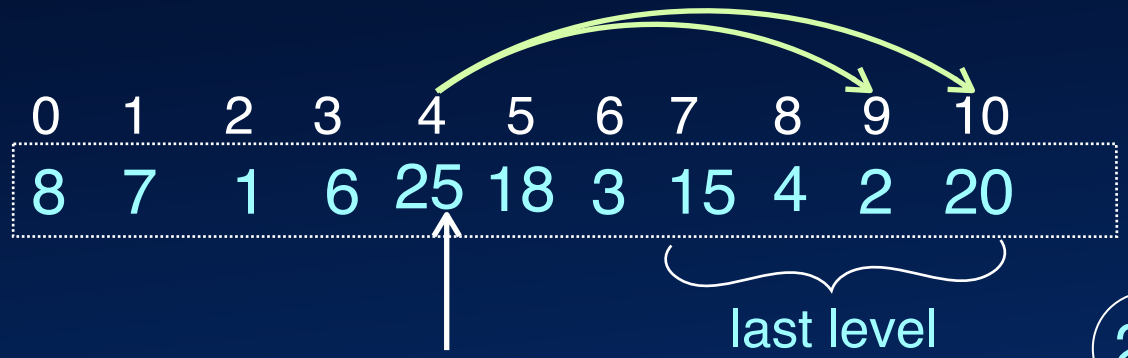

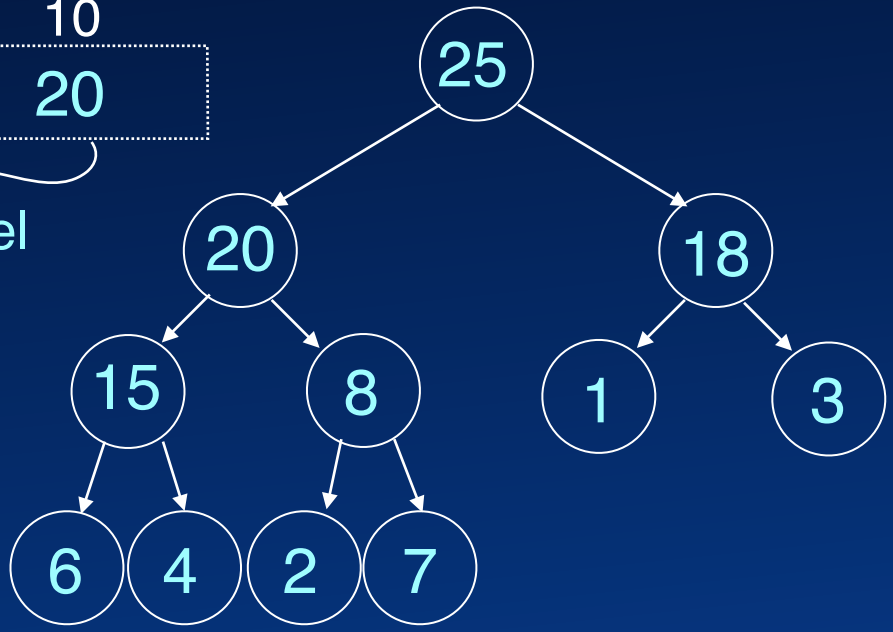

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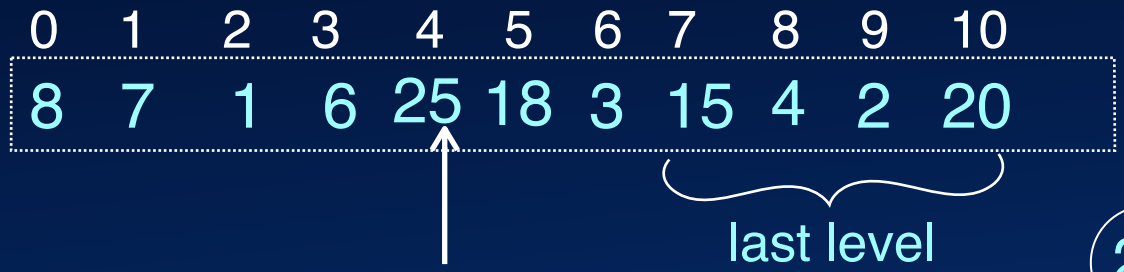

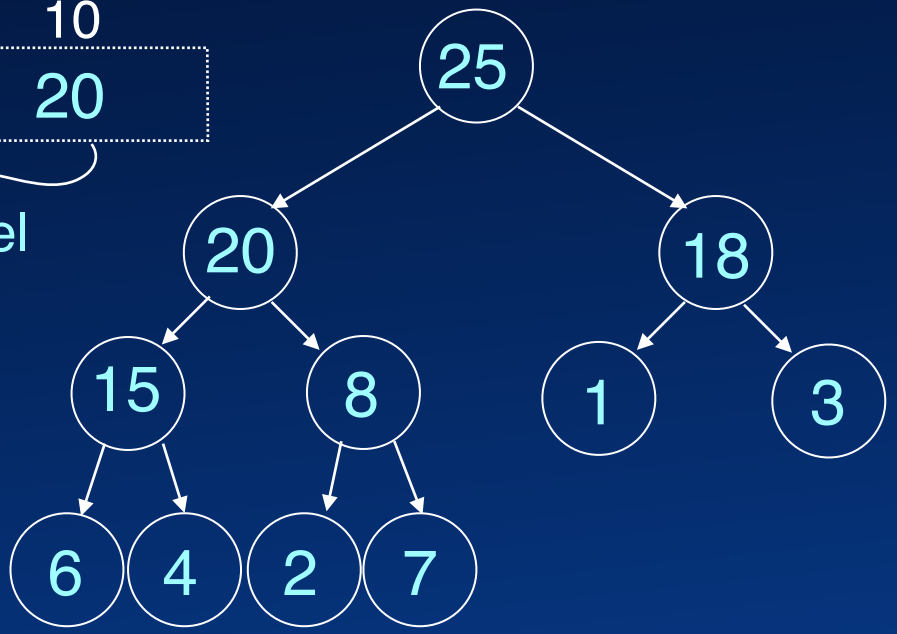

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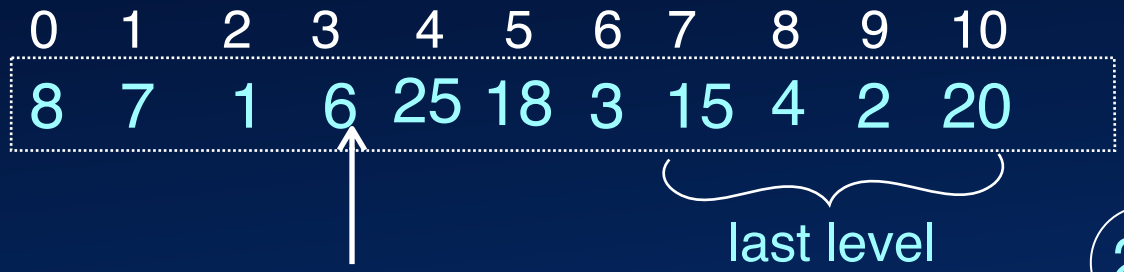

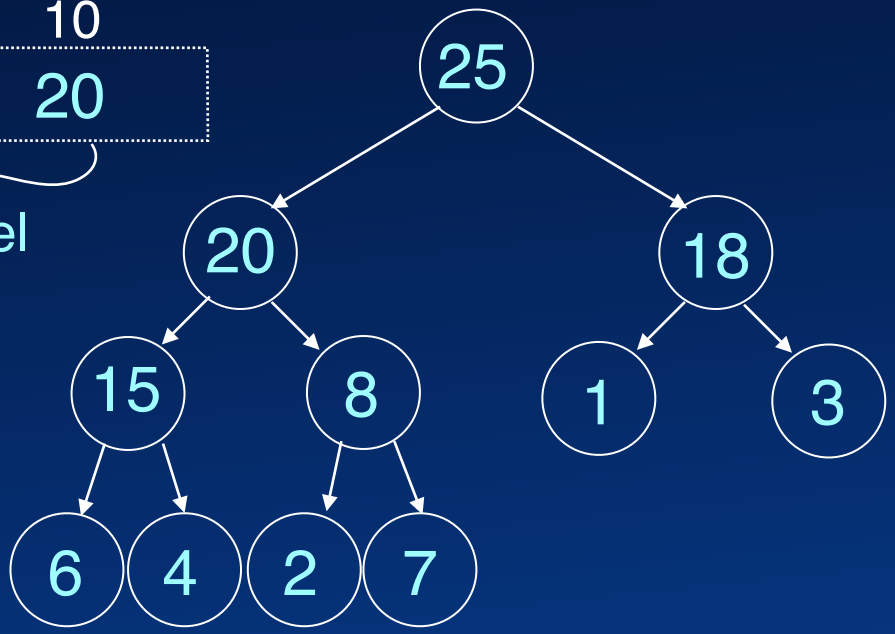

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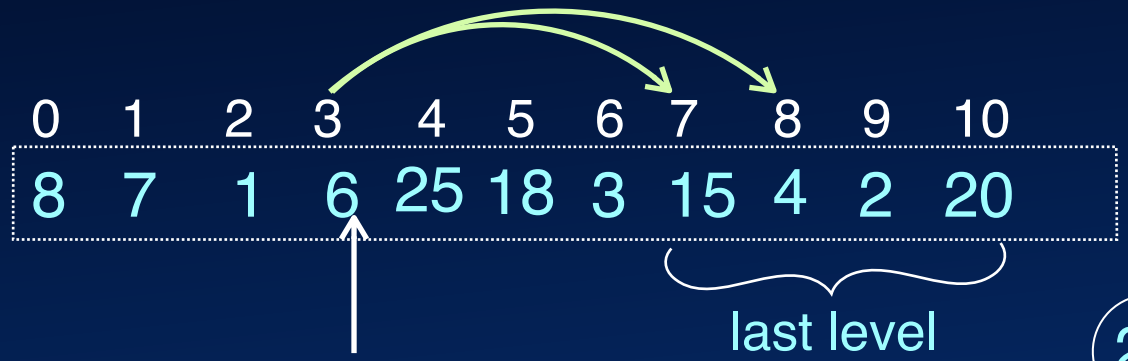

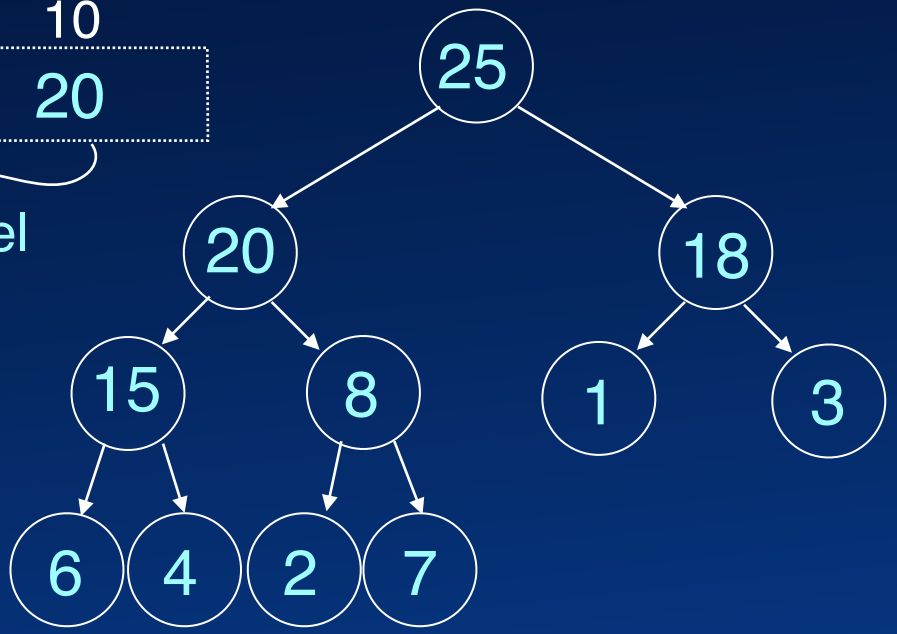

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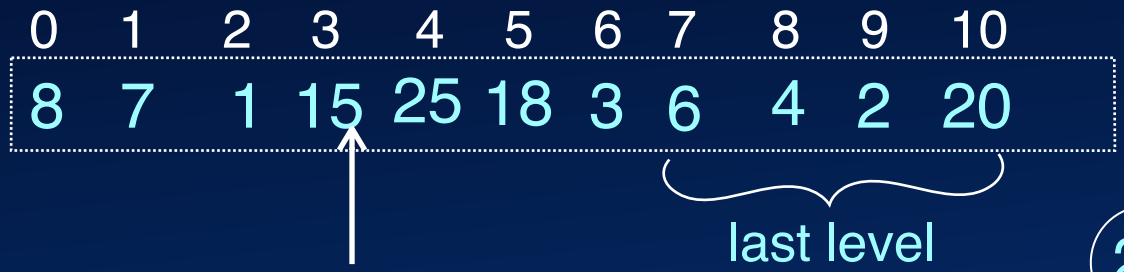

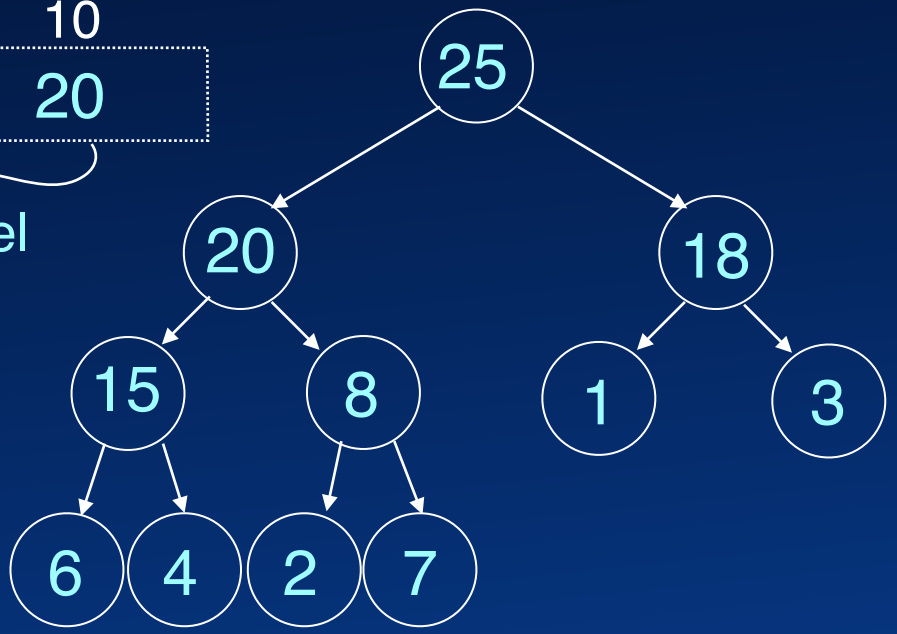

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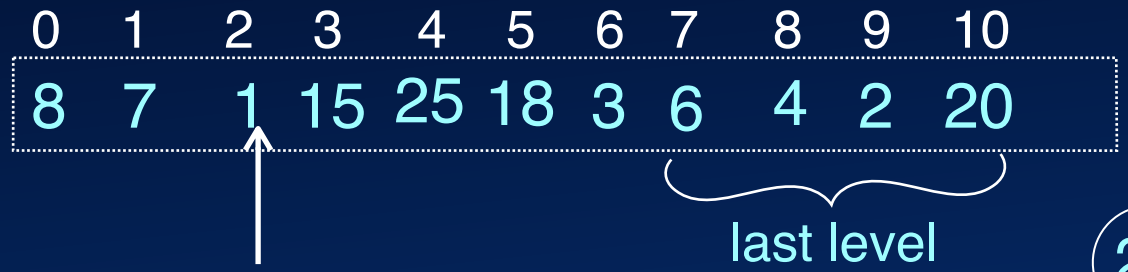

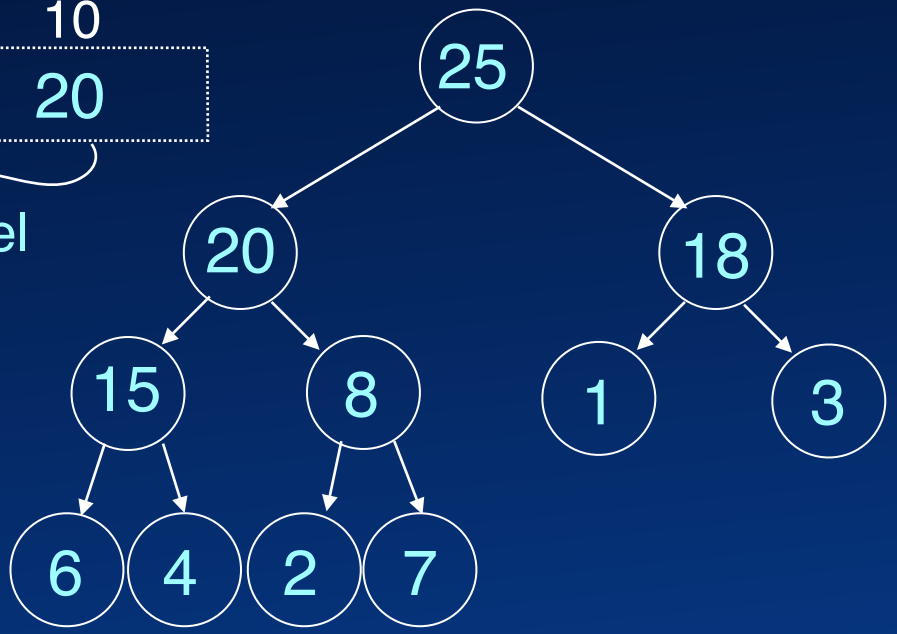

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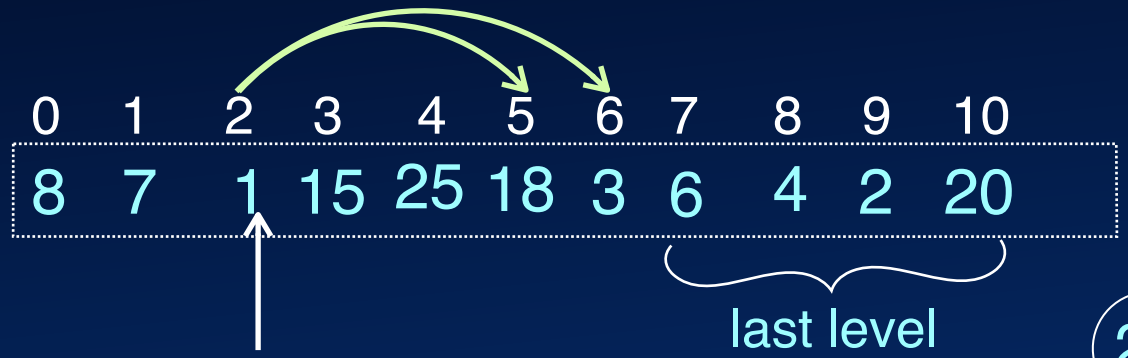

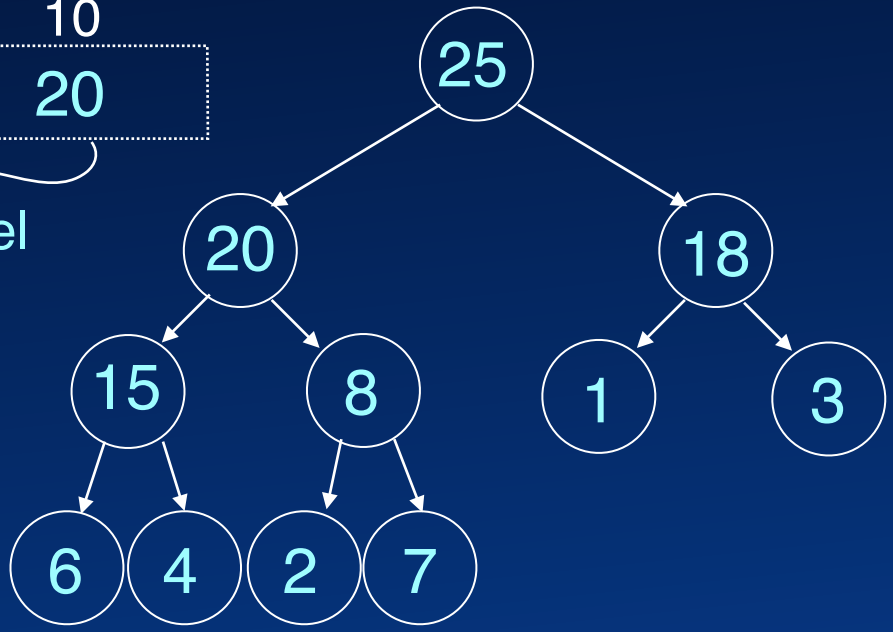

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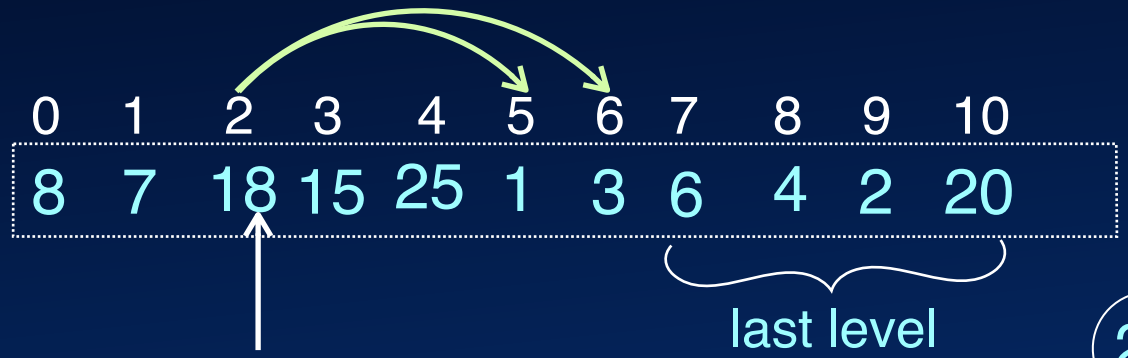

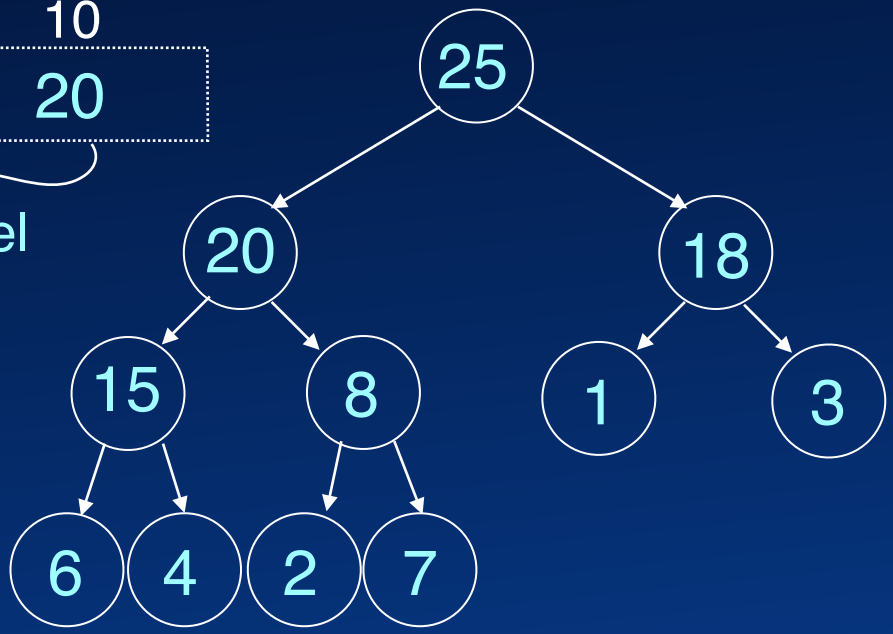

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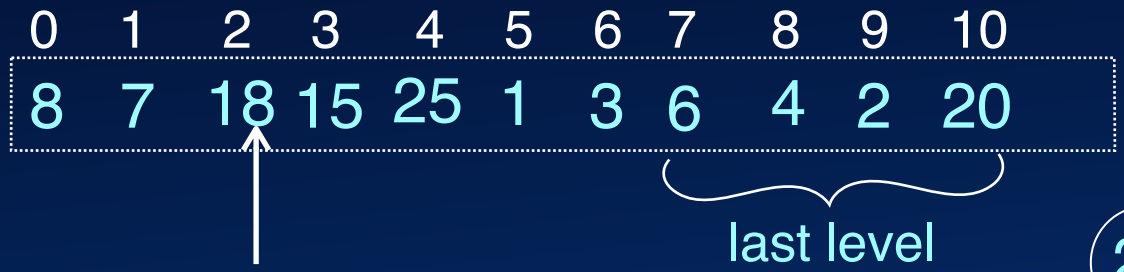

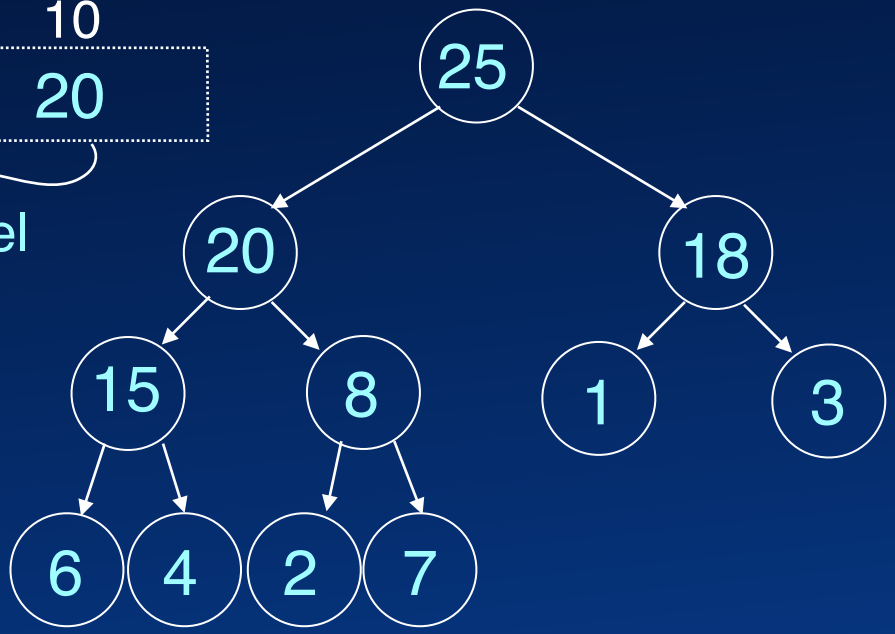

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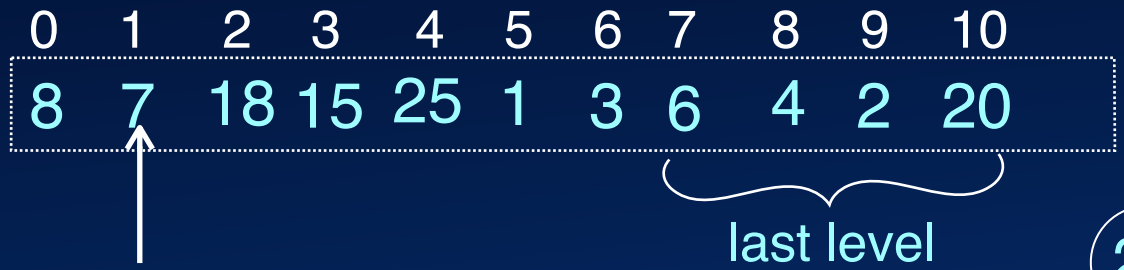

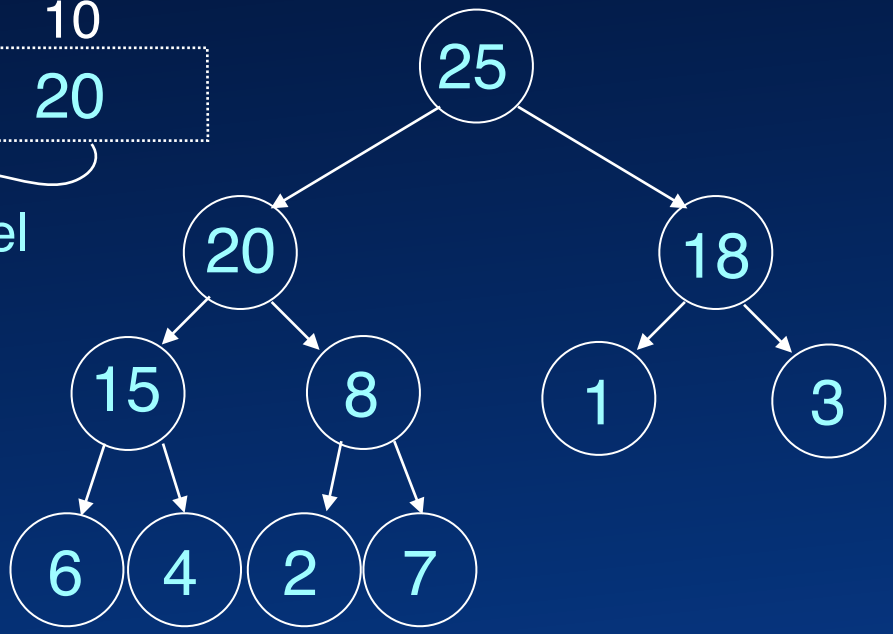

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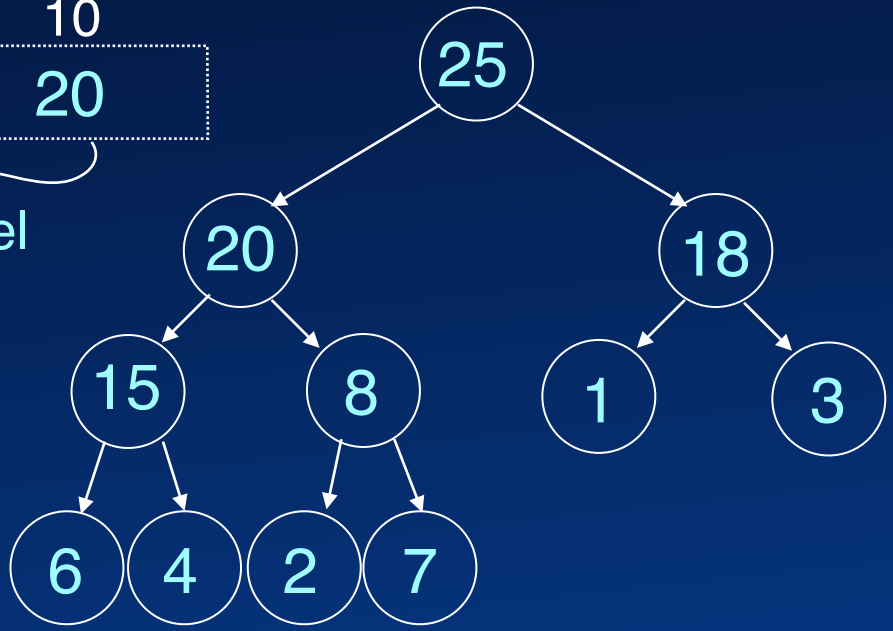

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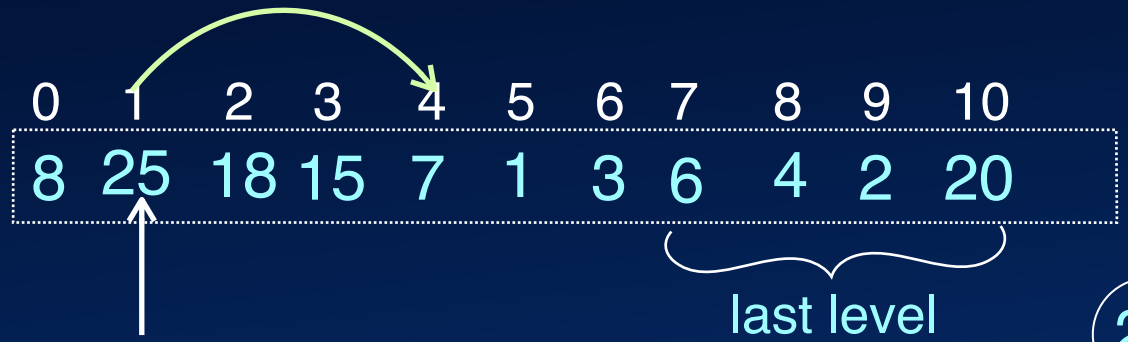

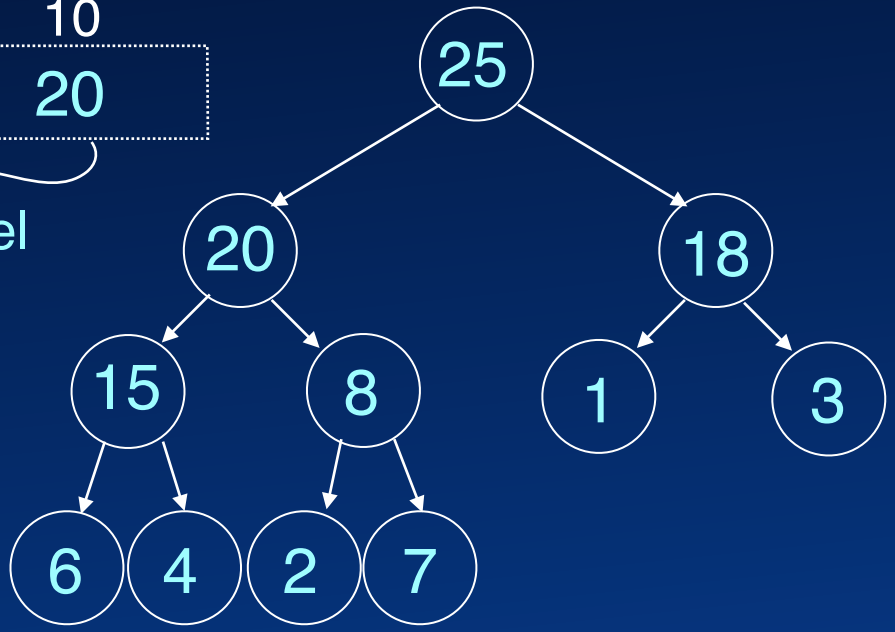

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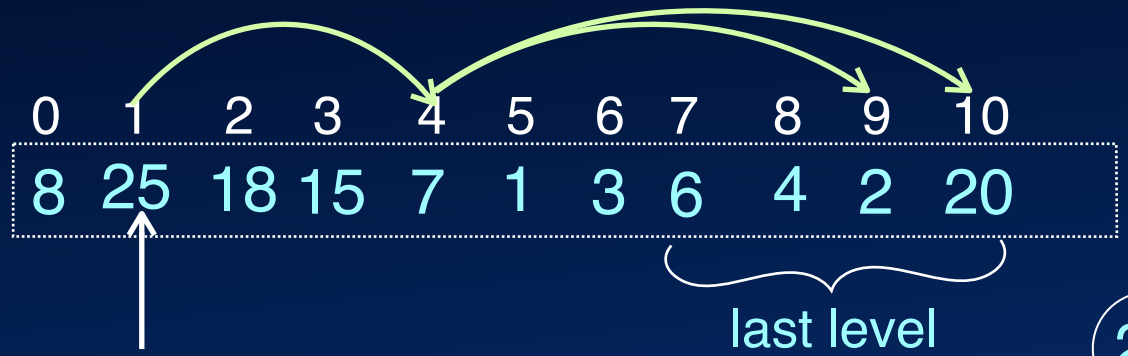

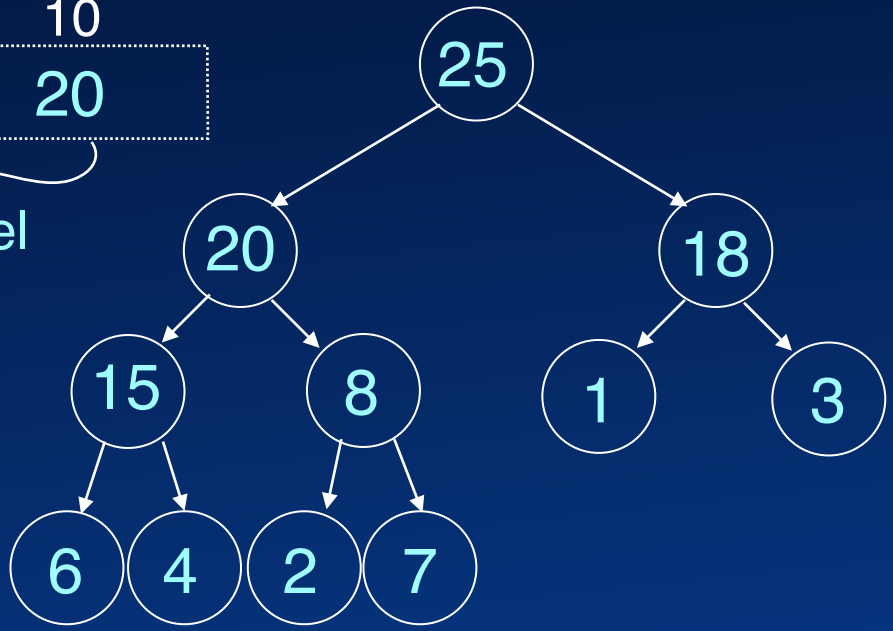

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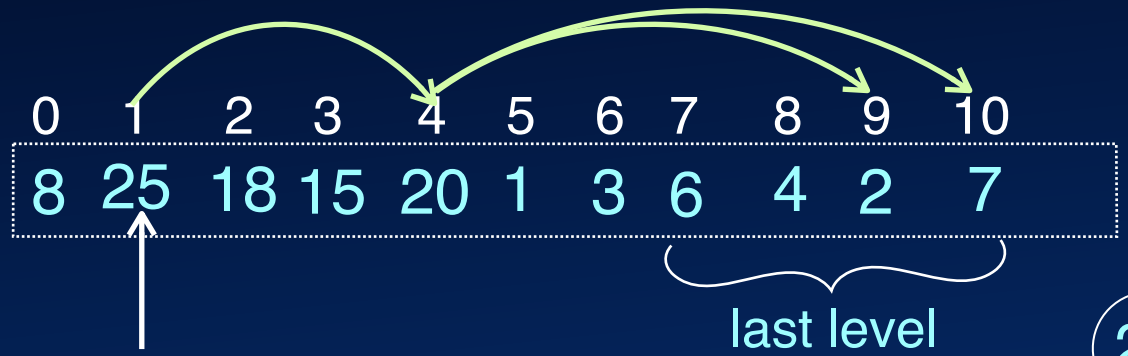

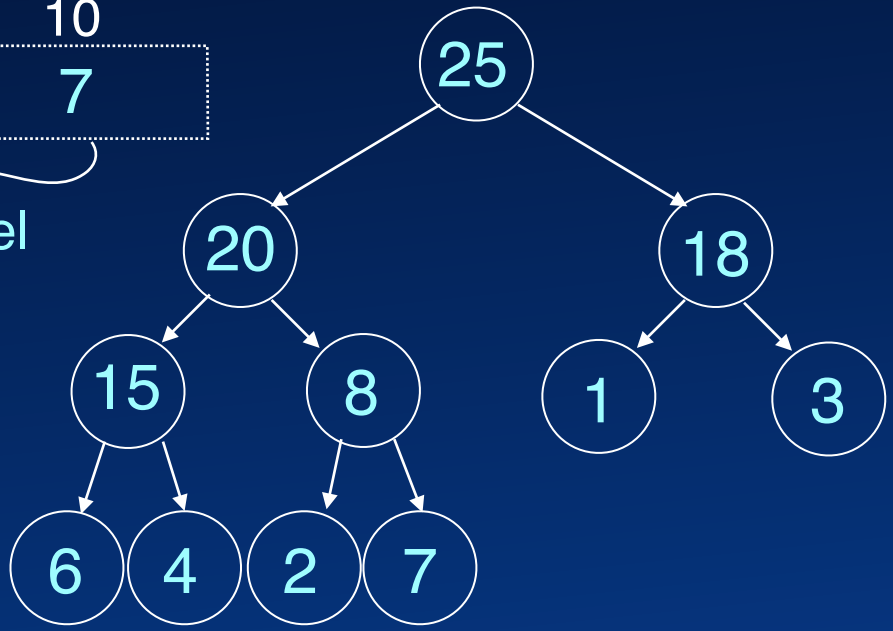

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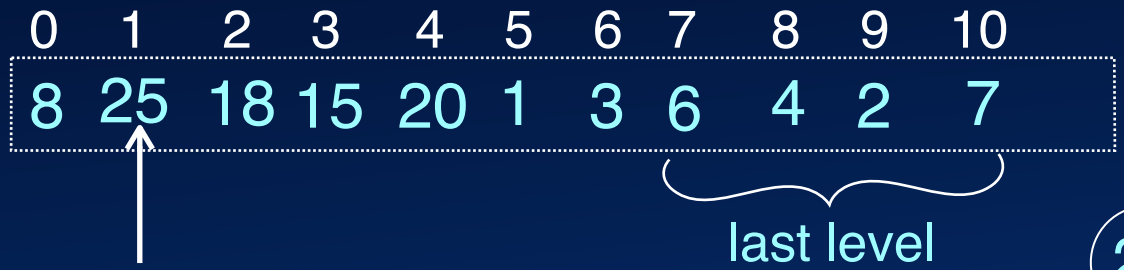

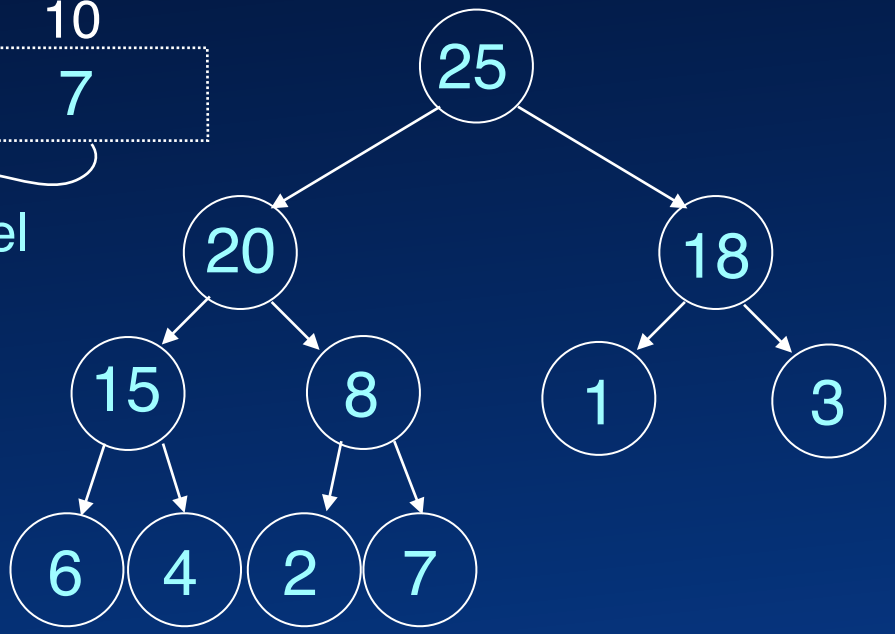

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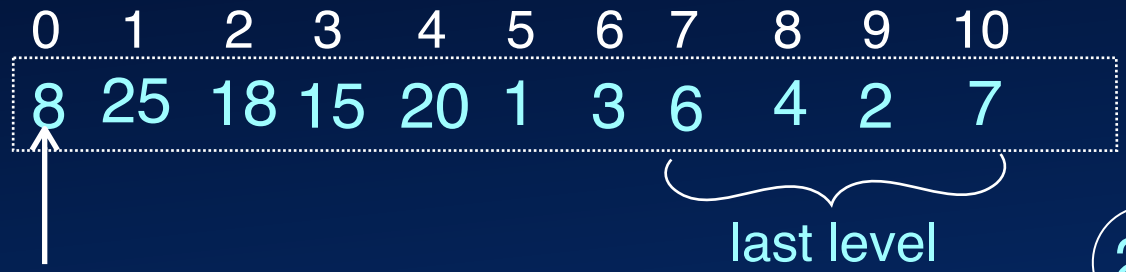

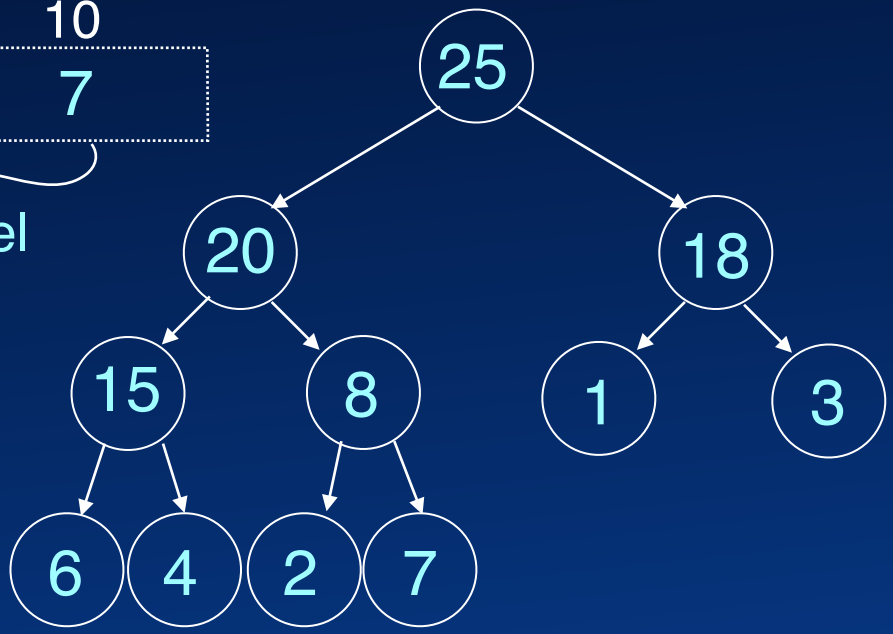

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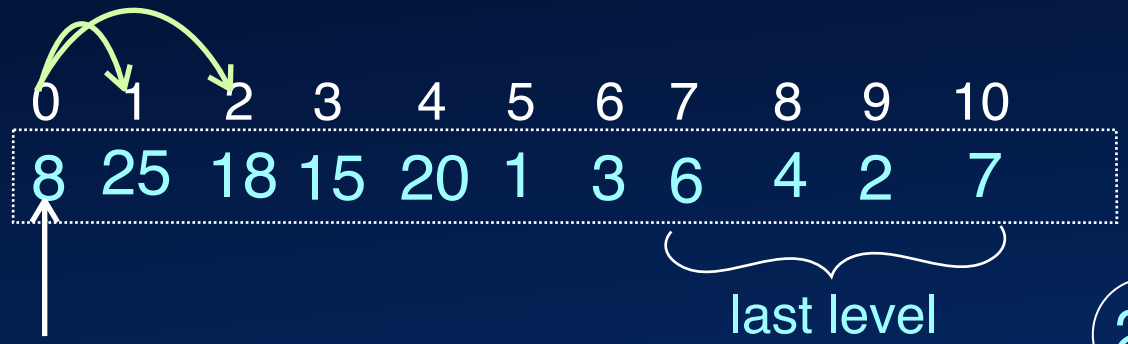
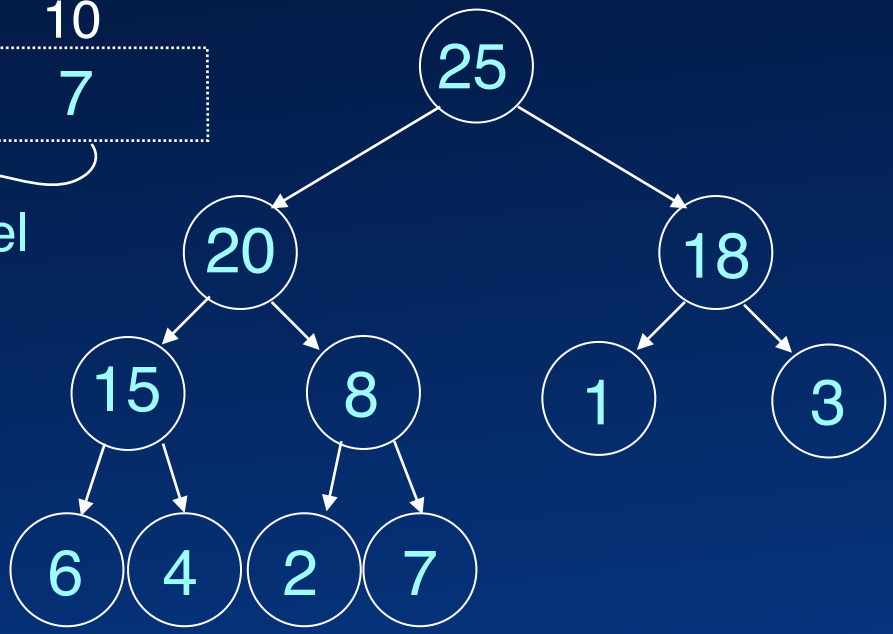
Heap Construction





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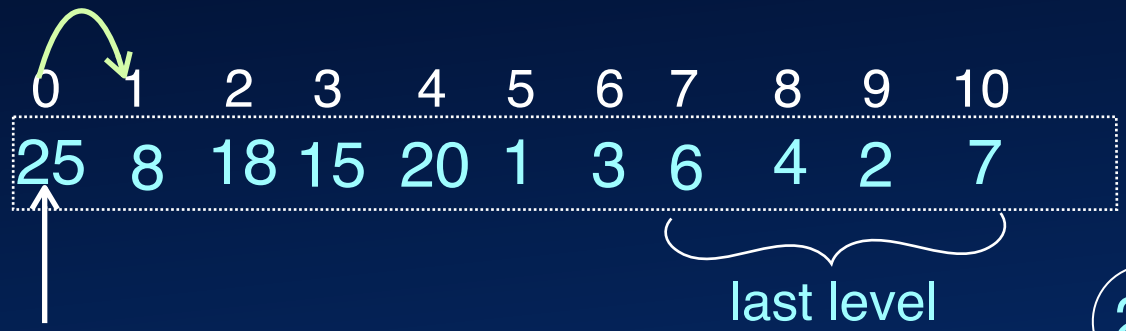

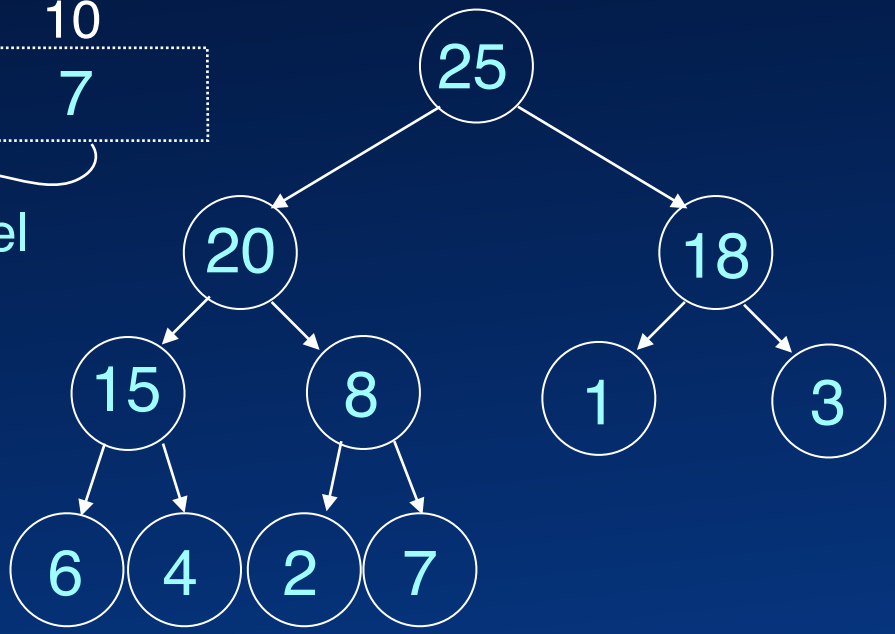

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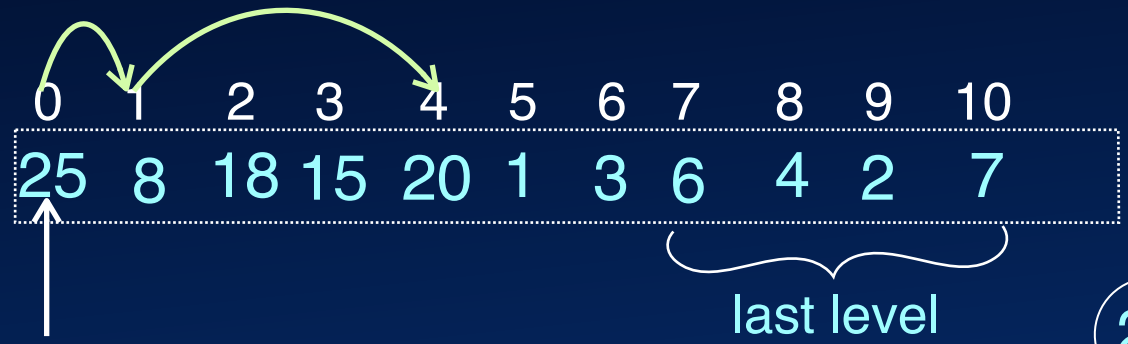

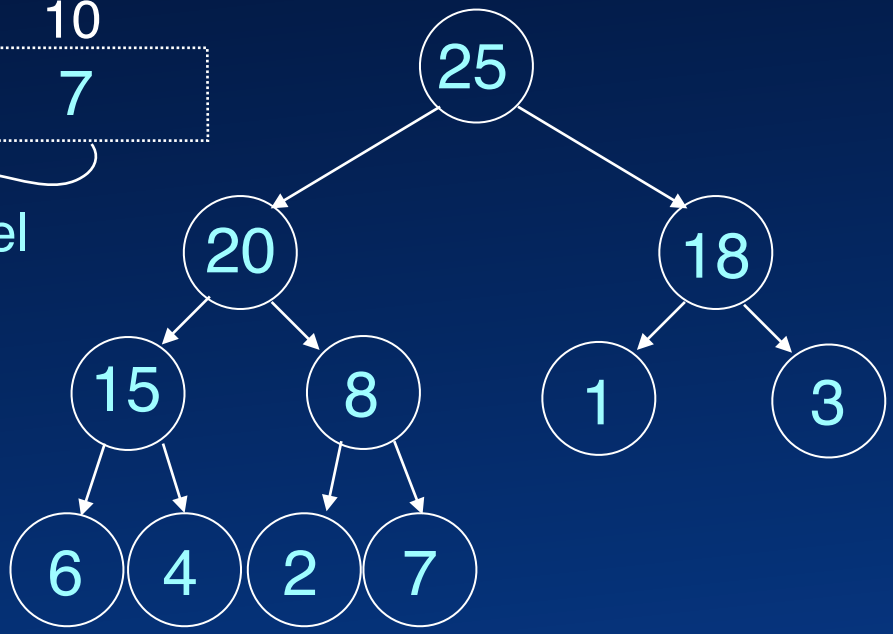

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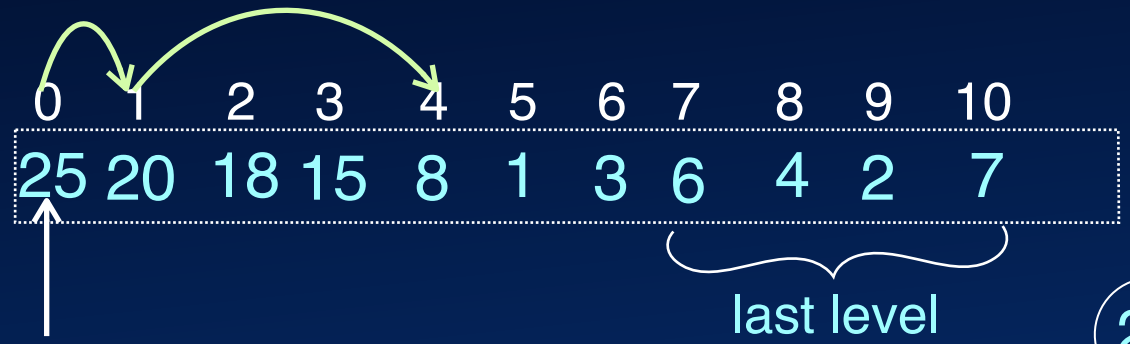

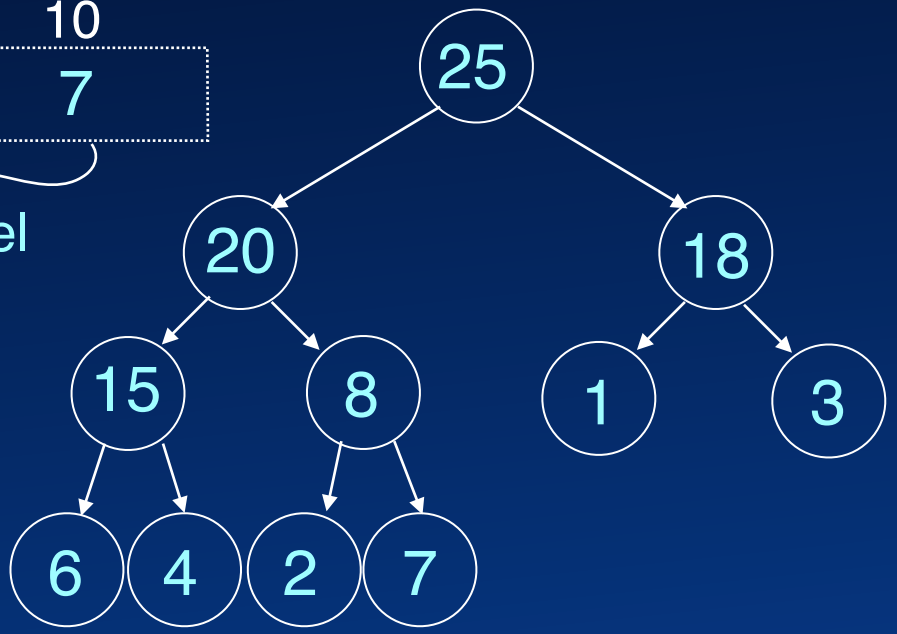

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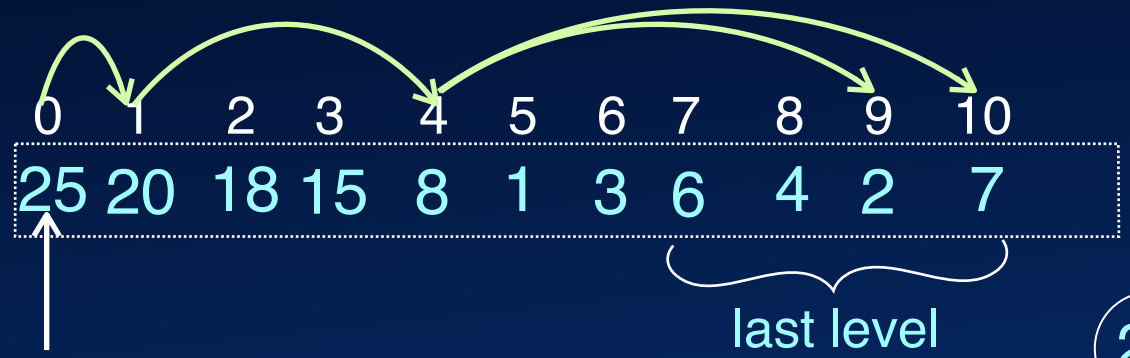

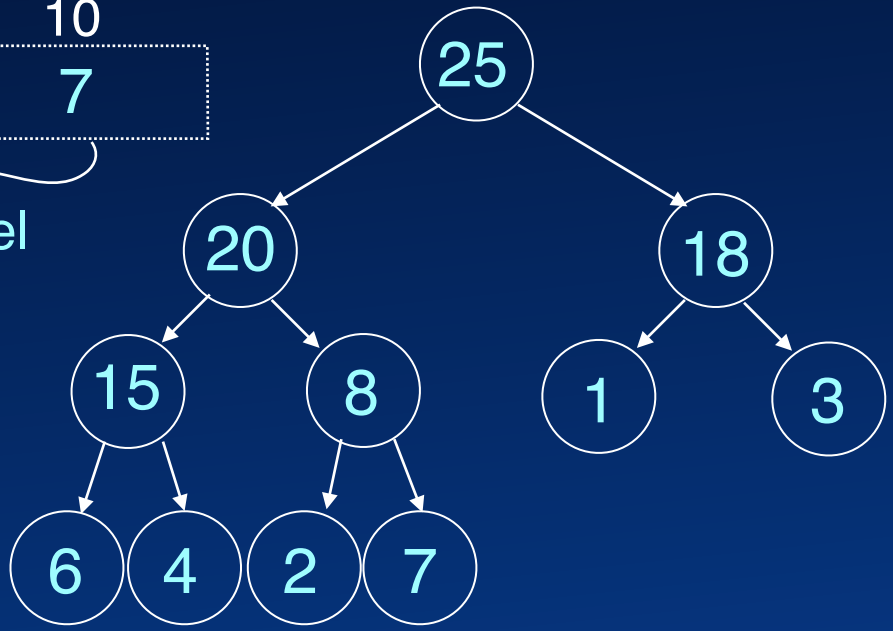

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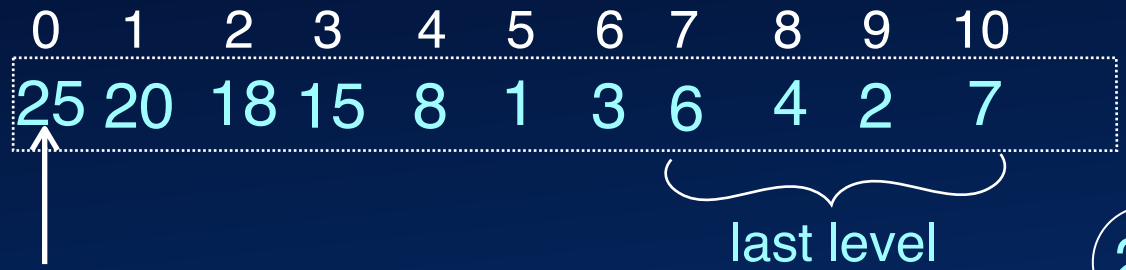

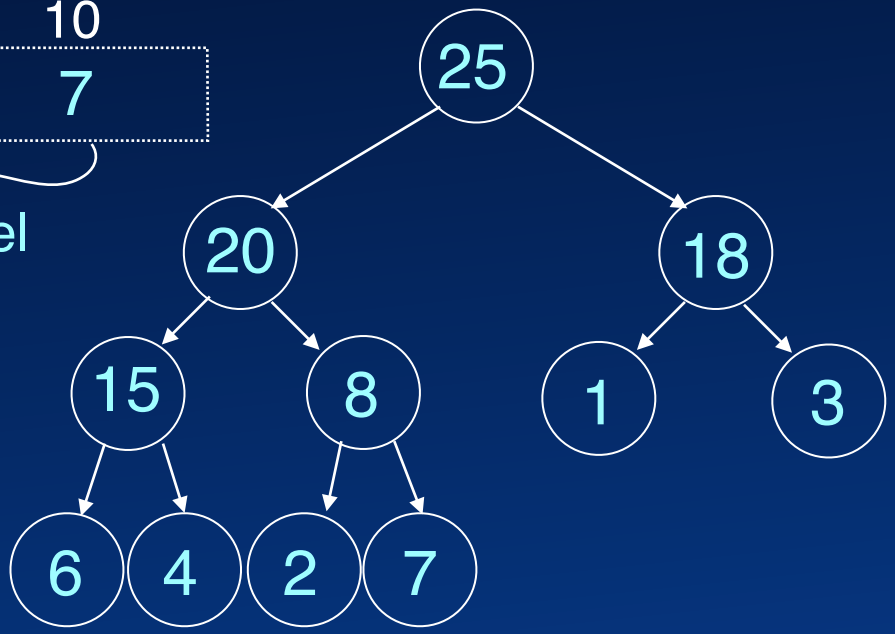

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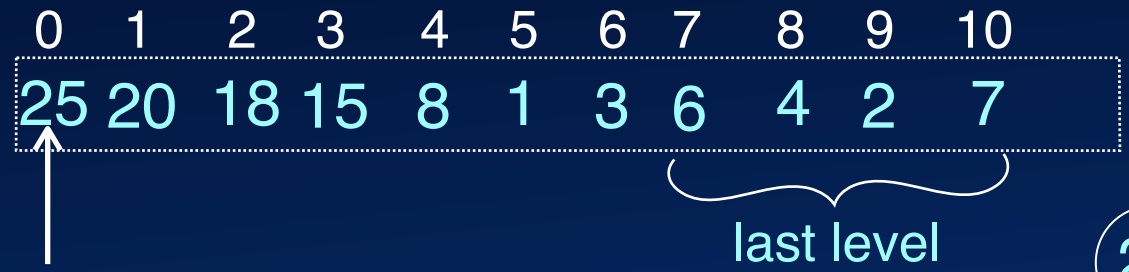

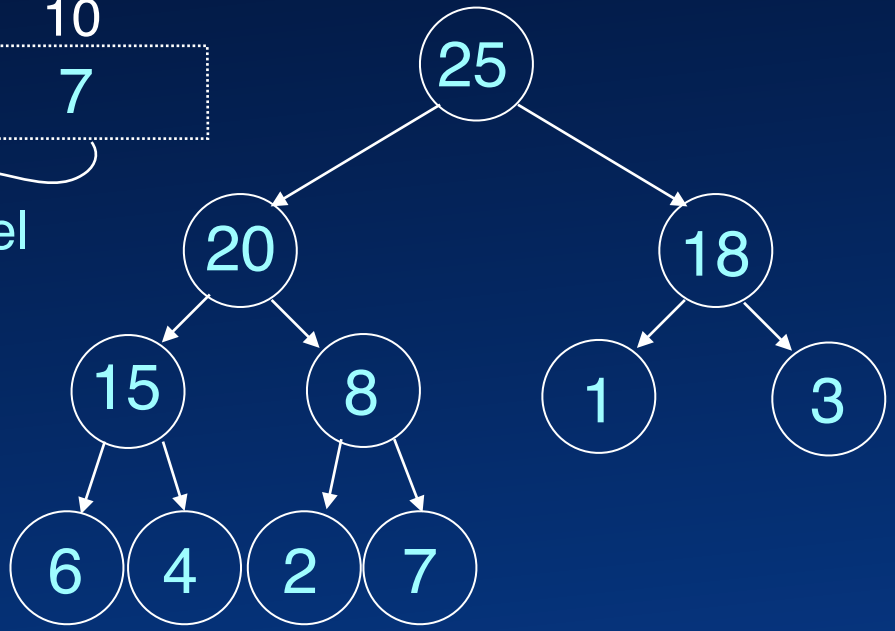

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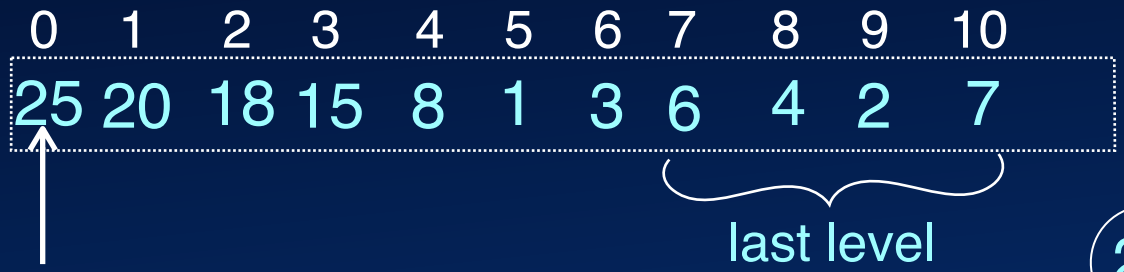
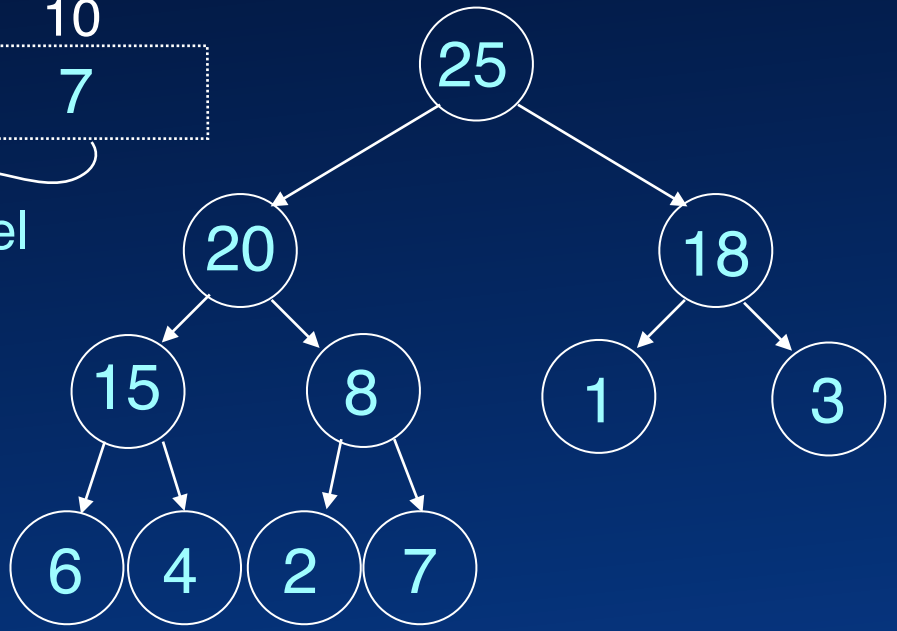


height $h = \lceil \lg n+1 \rceil - 1$



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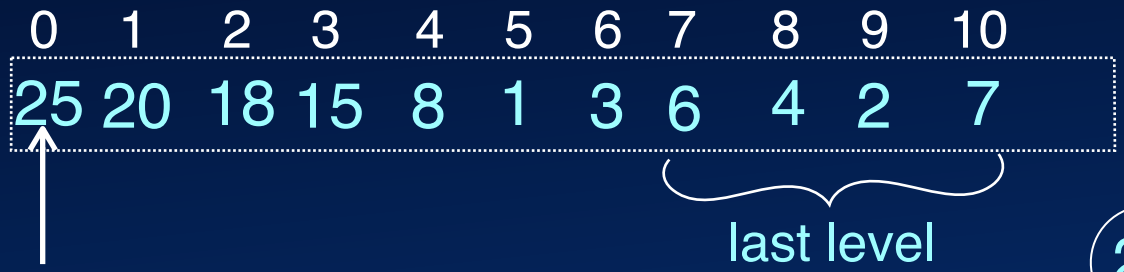

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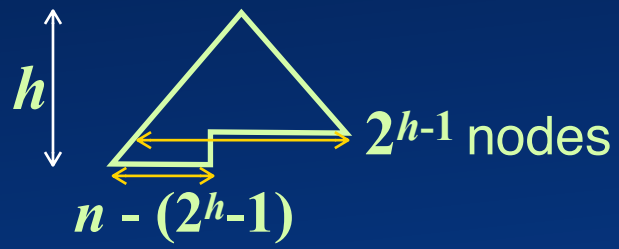

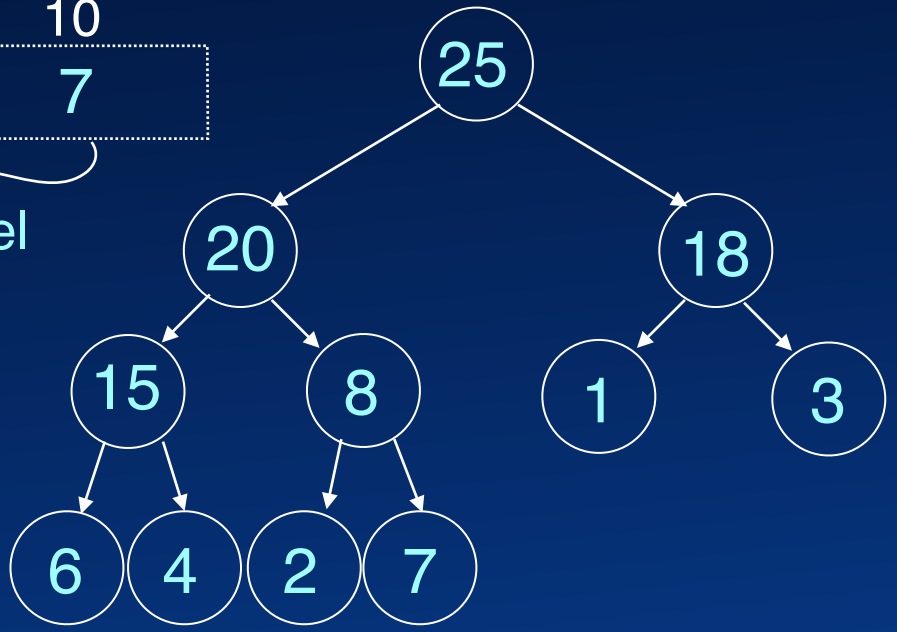

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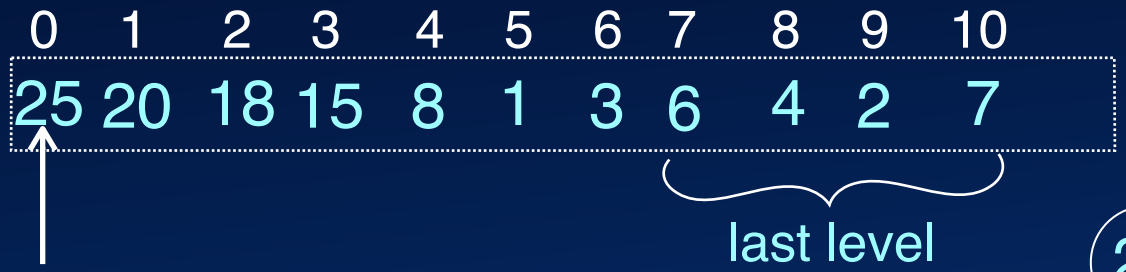

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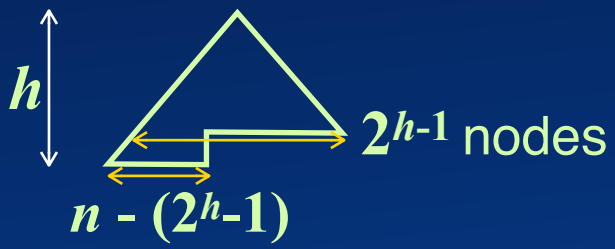
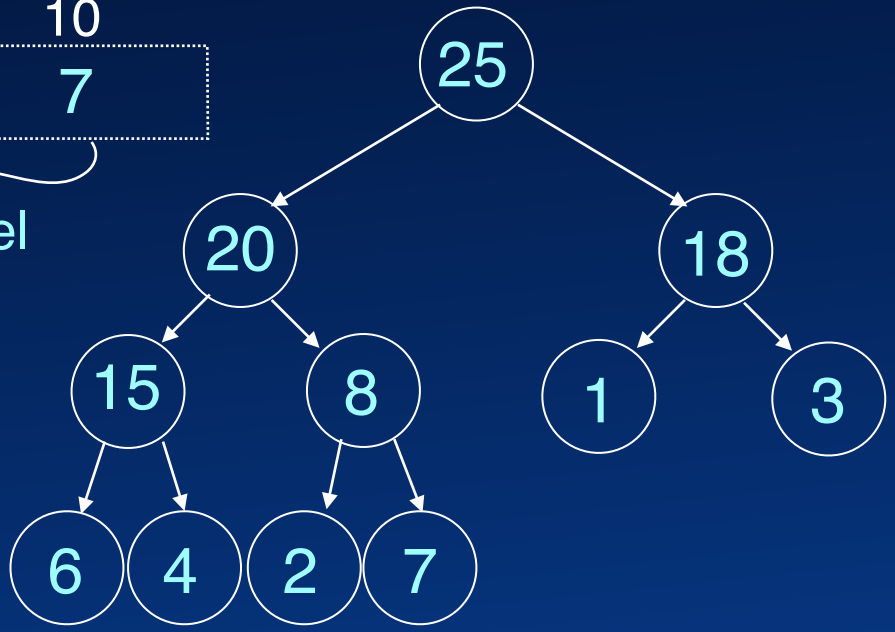
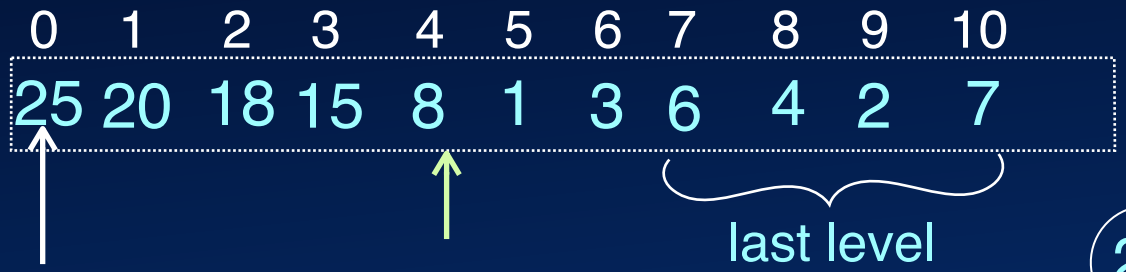




Heap

Heap Construction



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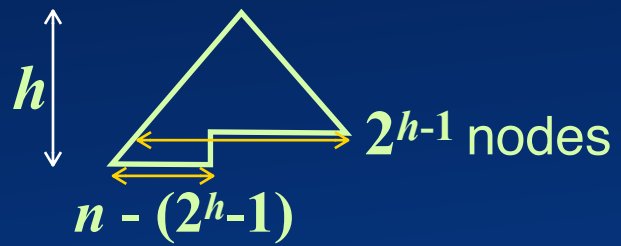

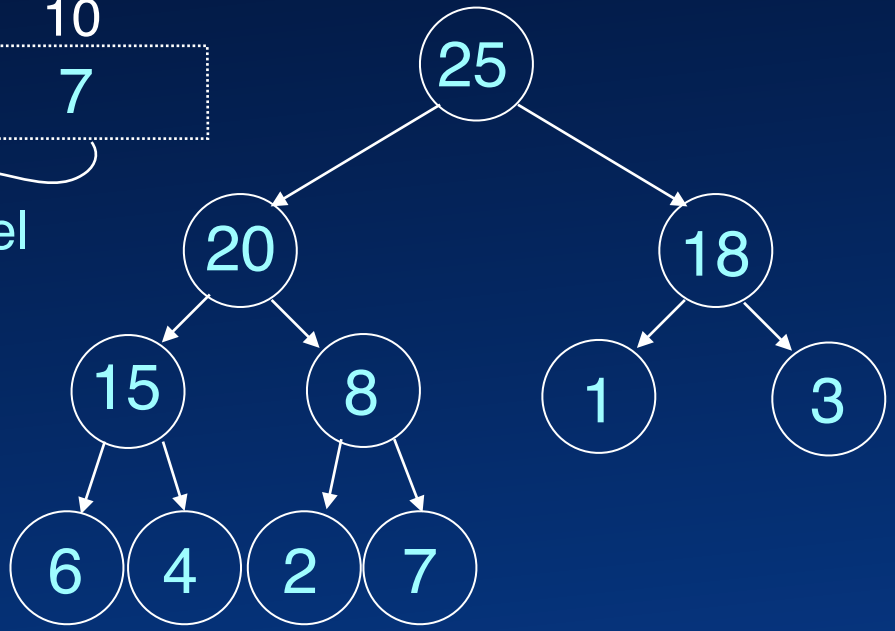

Heap

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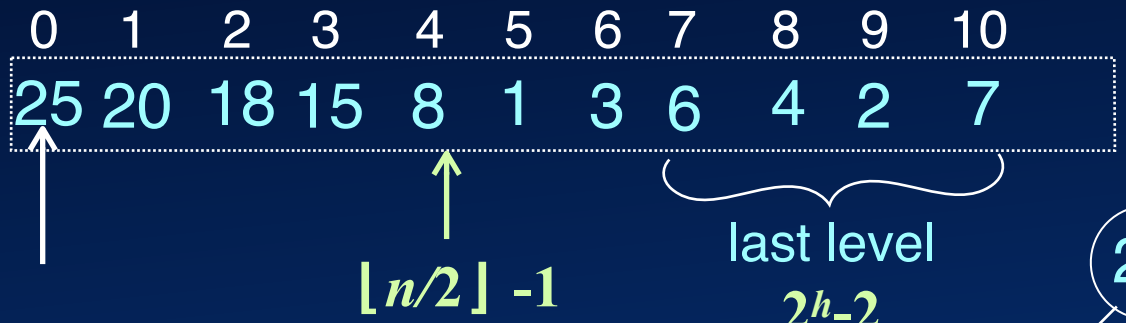

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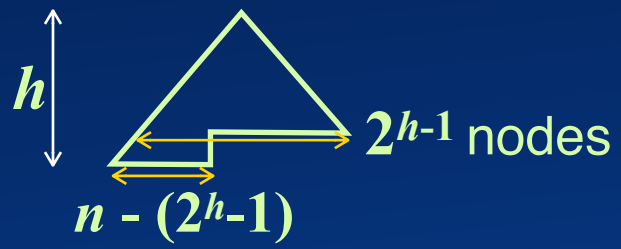

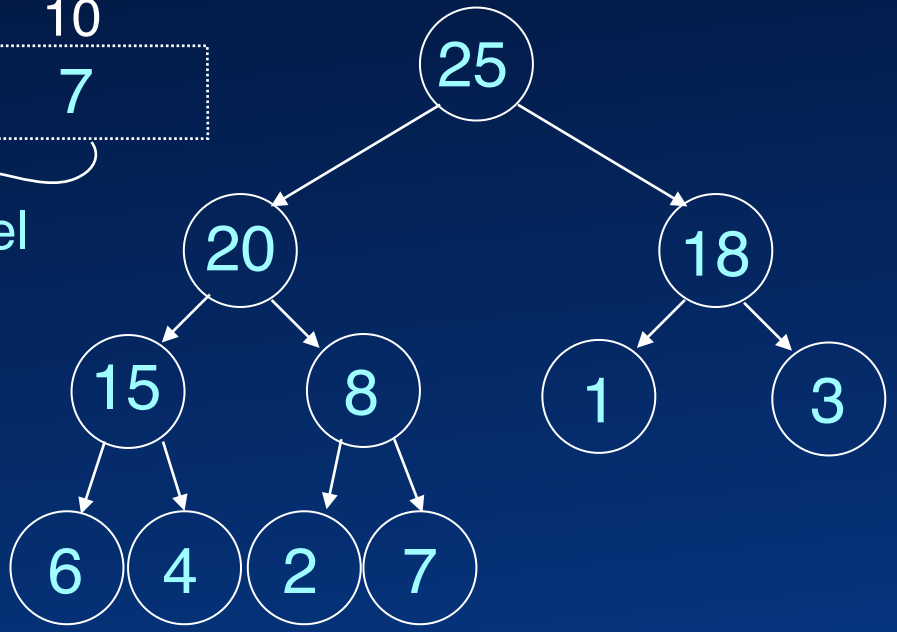

Heap

Quiz

Time required to turn an array of n comparable objects into a heap is:

a. $\Theta(1)$

b. $\Theta(n)$

c. $O(\log n)$

d. $\Theta(n \log n)$

e. None of the above

mailto: col106quiz@cse.iitd.ac.in
format: b

Priority Queue

- Heap
- Balanced tree
- Skip list

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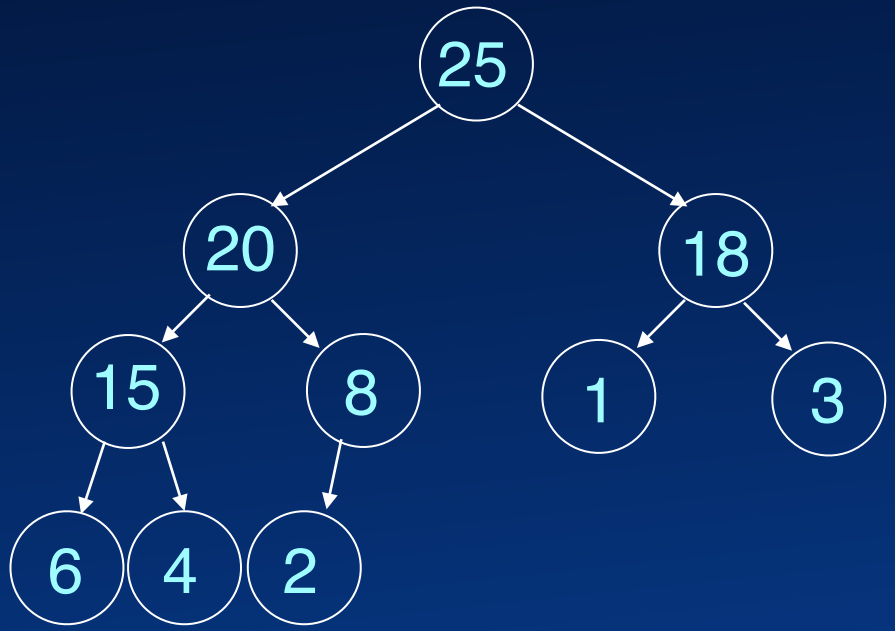

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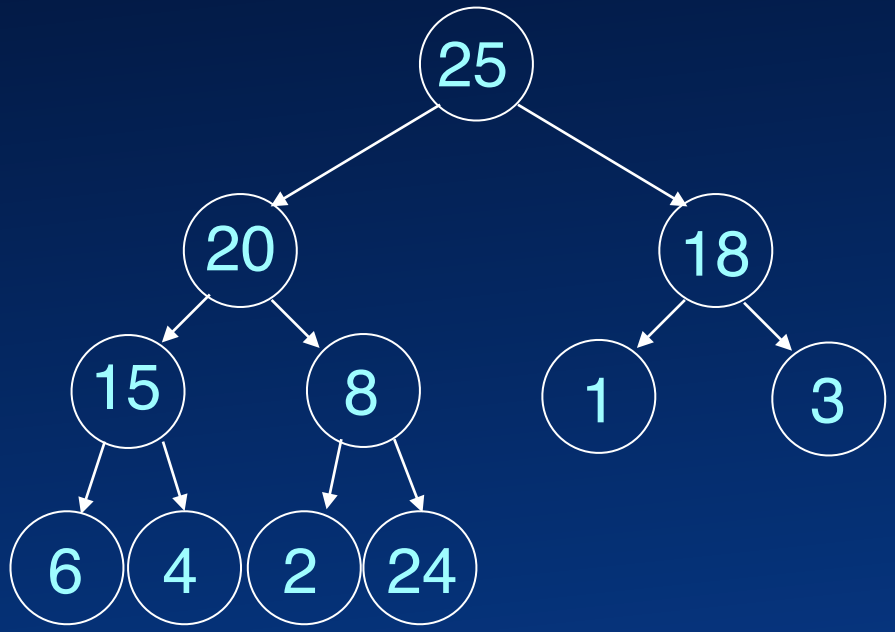

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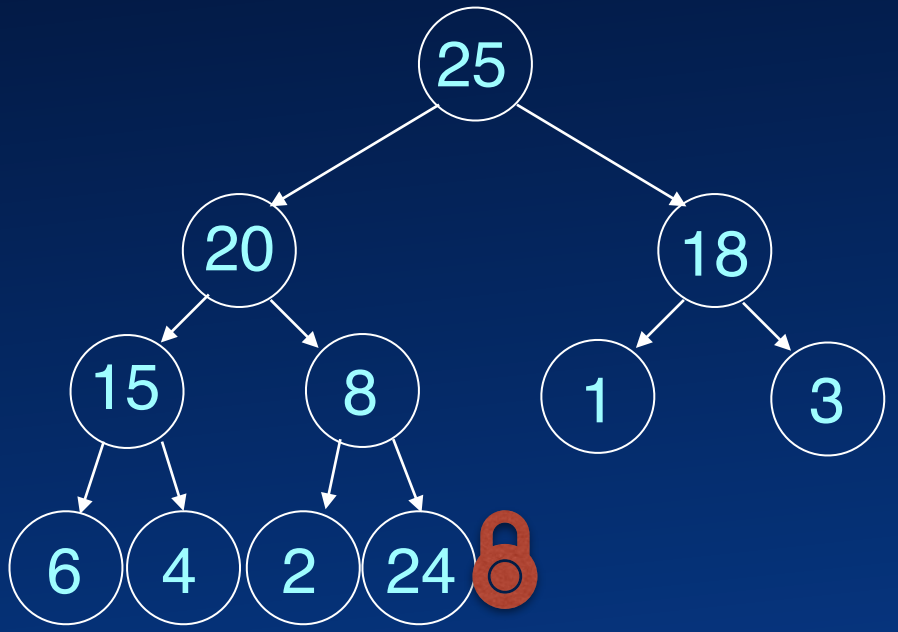

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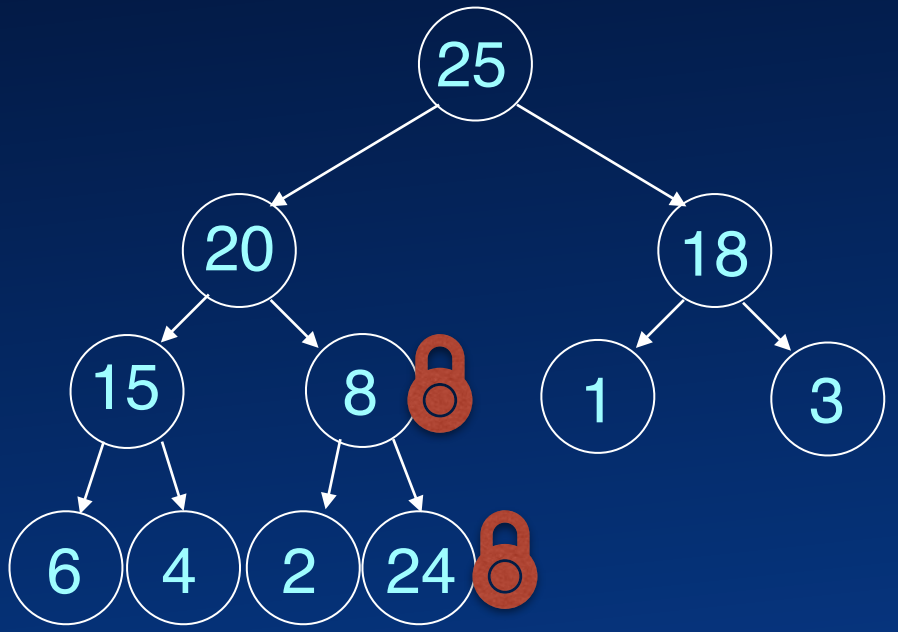

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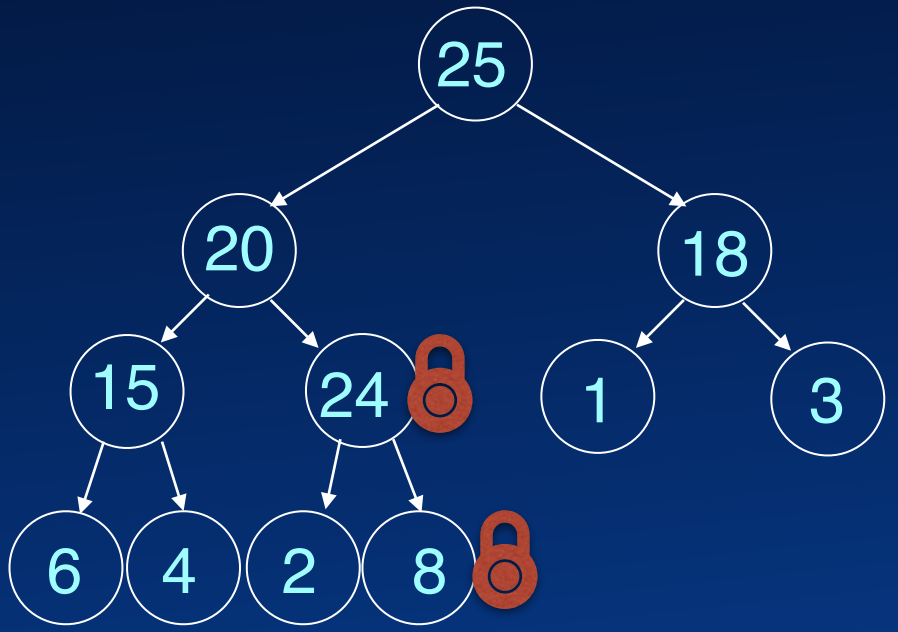

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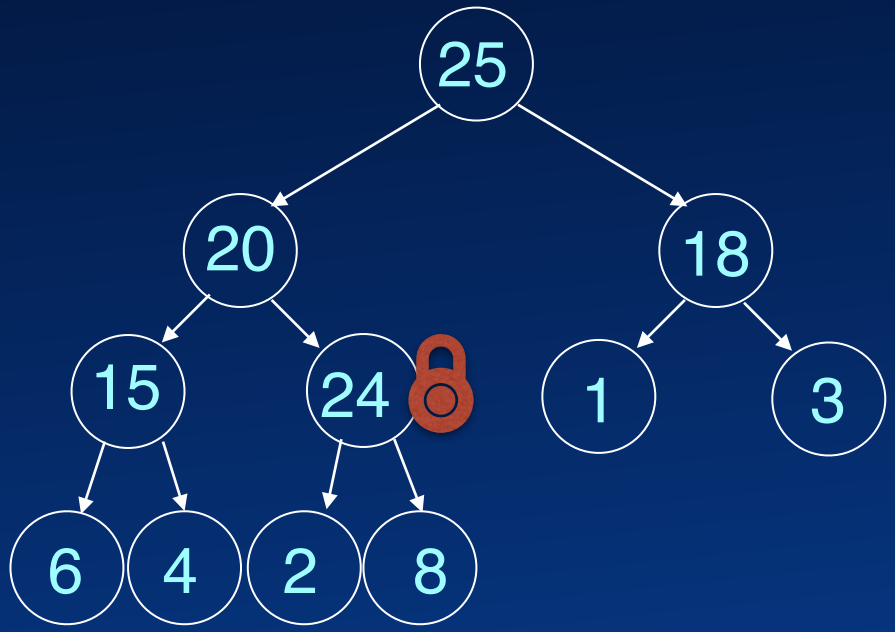

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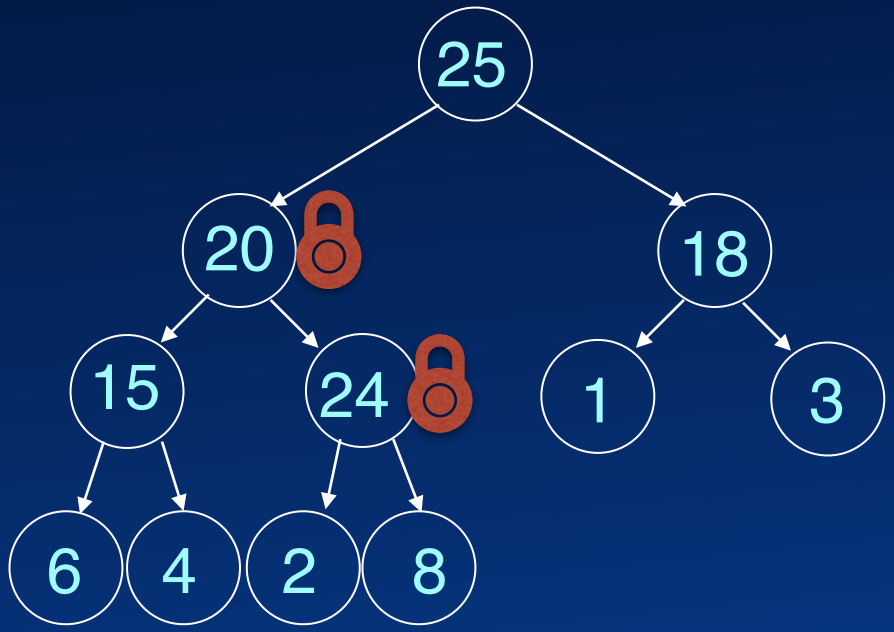

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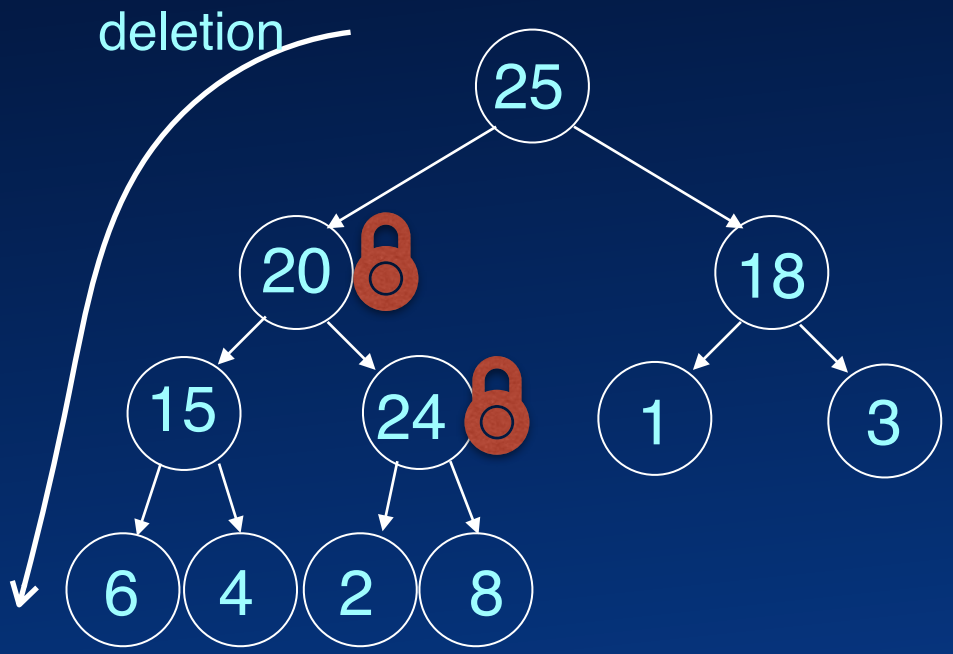

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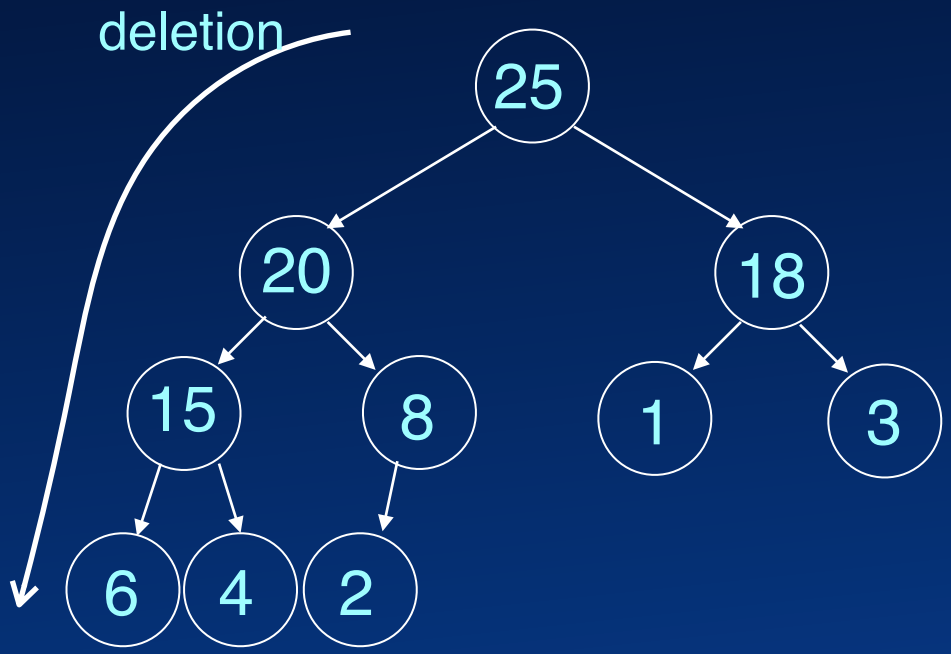

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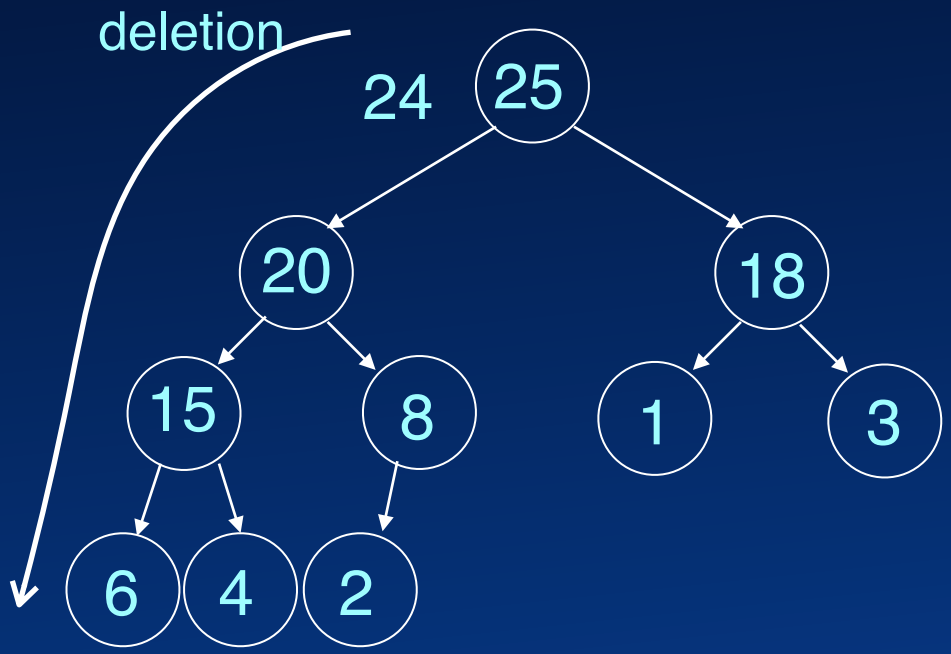

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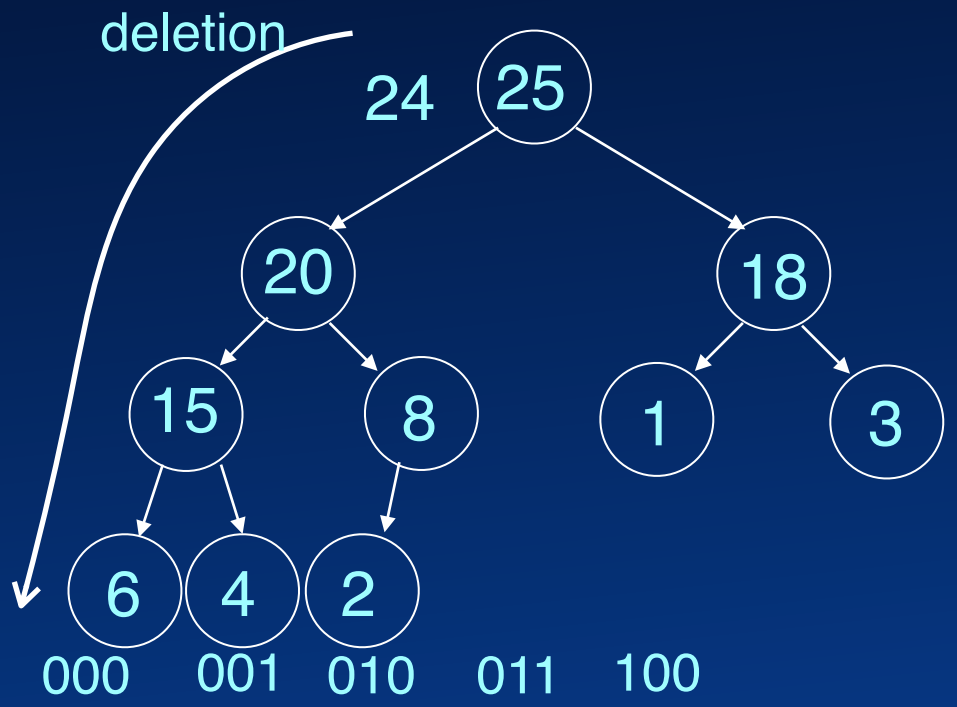

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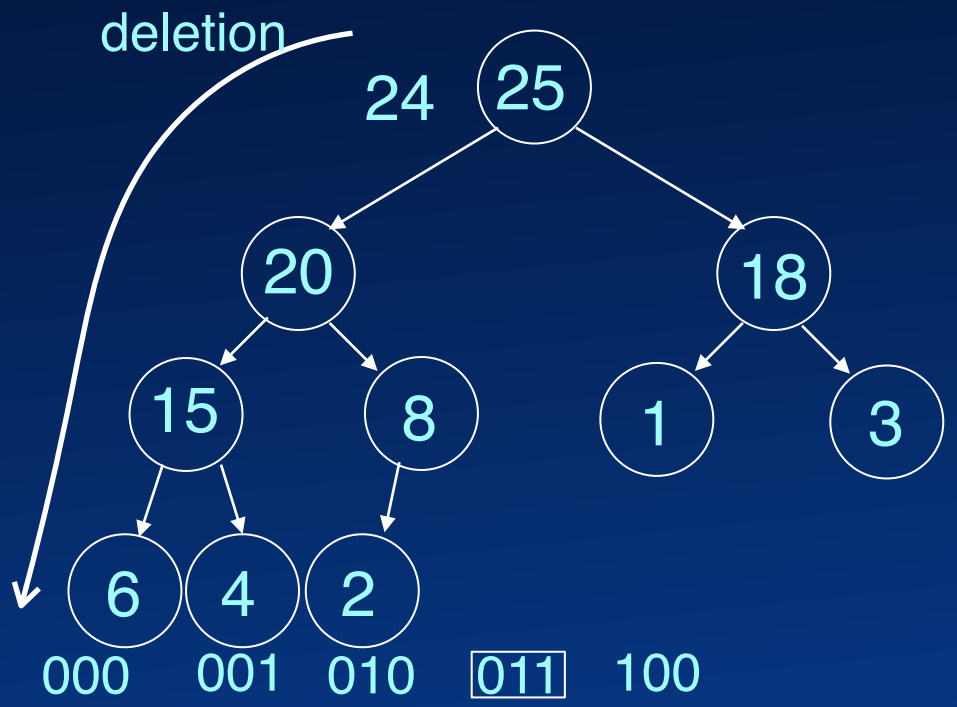

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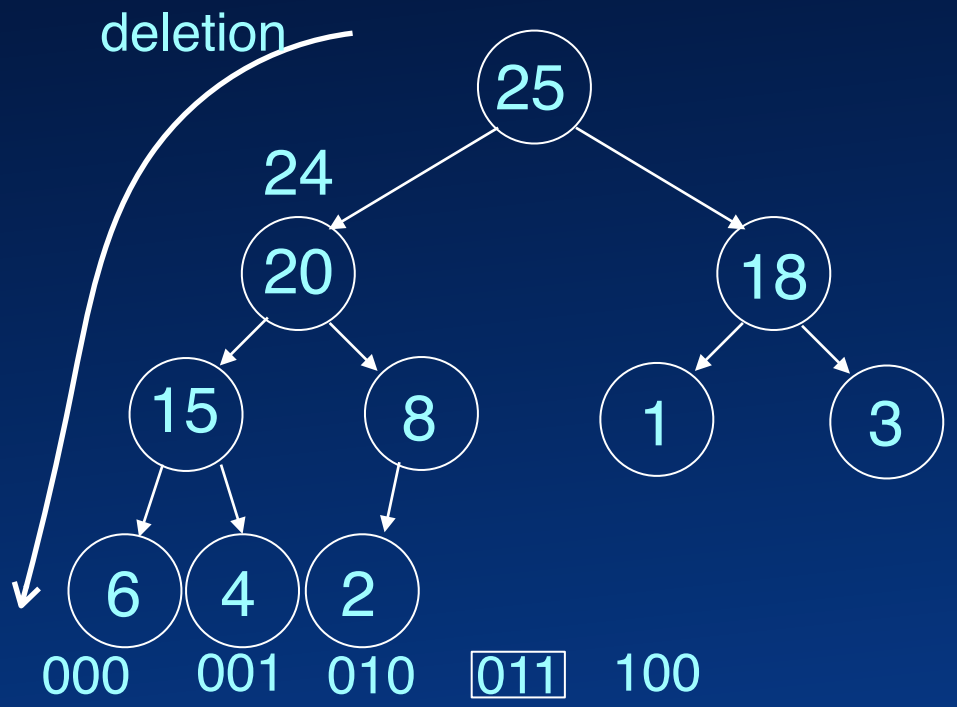

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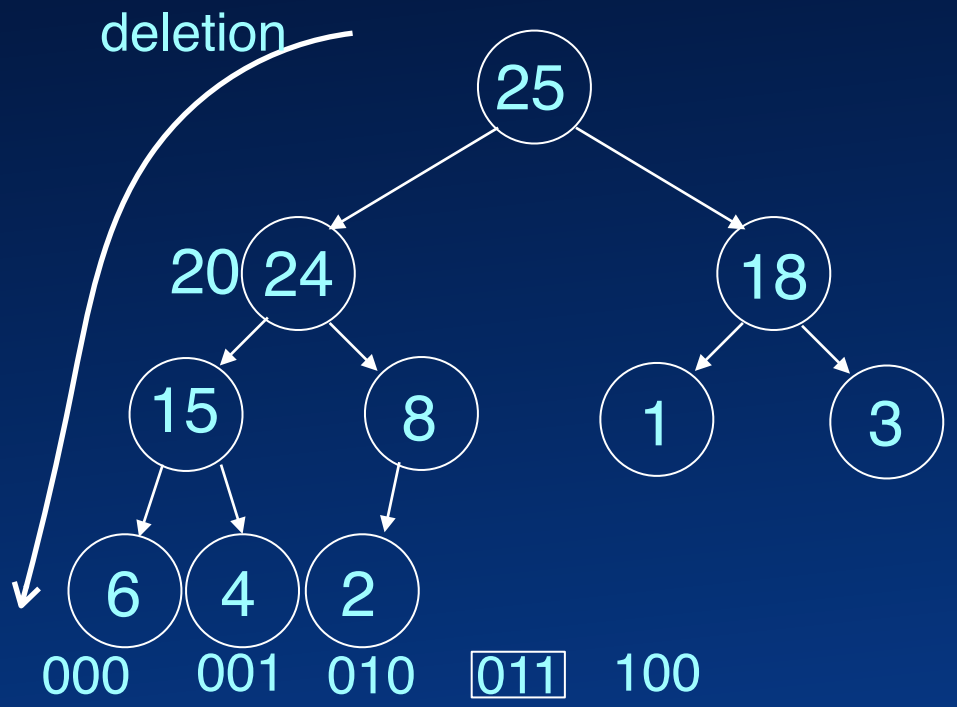

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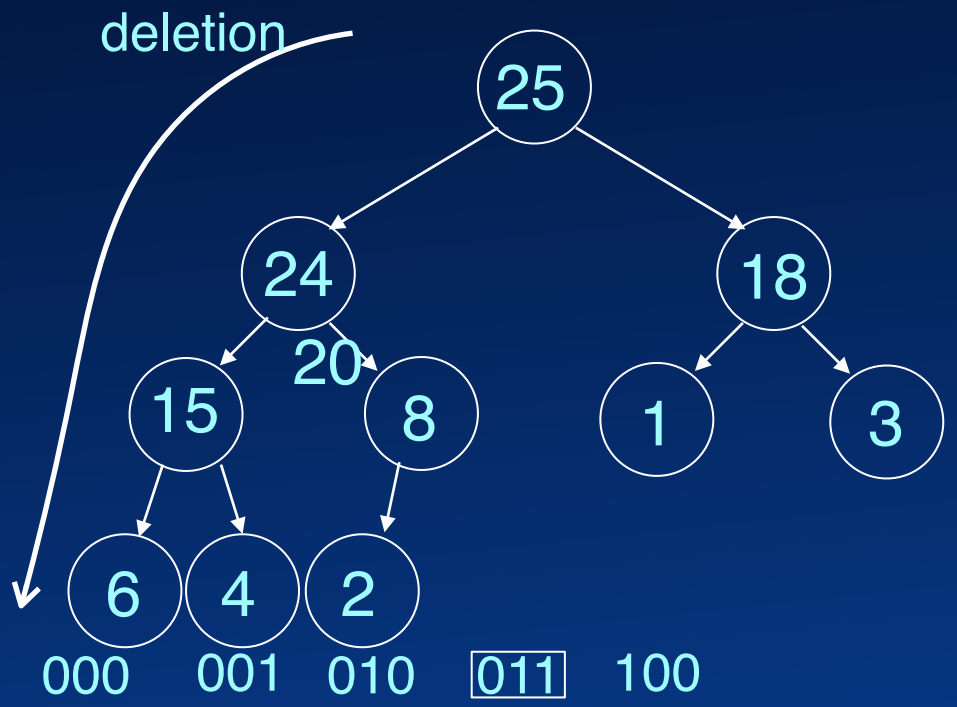

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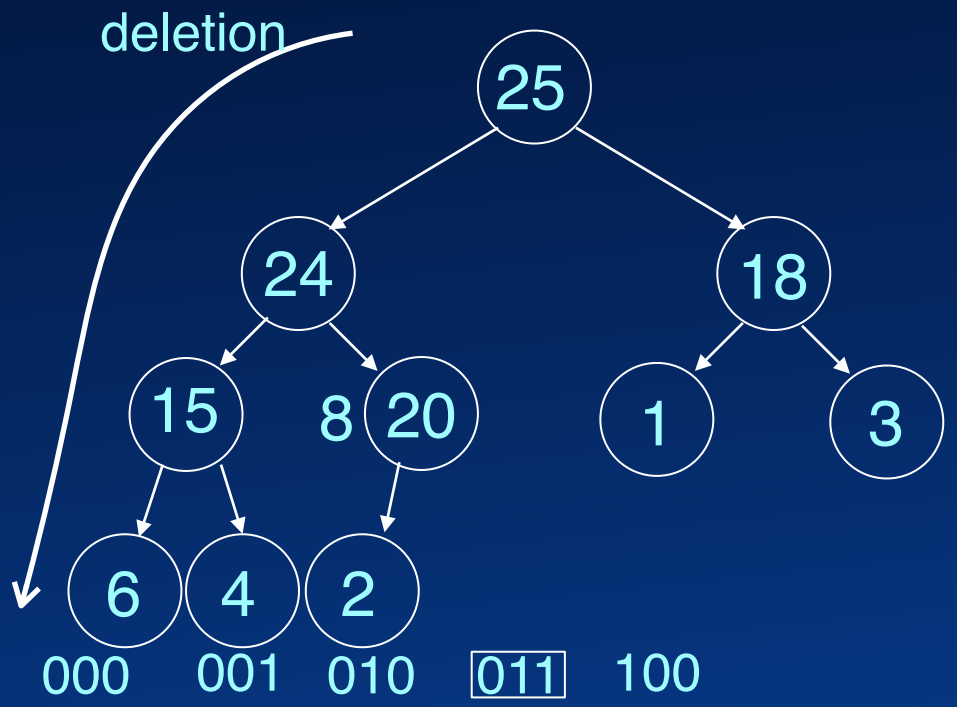

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