Data Structures & Algorithms

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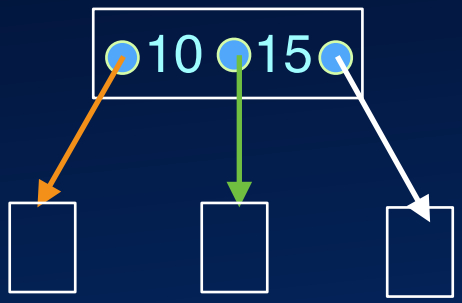
Dept of Computer Sc. & Engg.

Binar-izing a 2-4 Tree

Binar-izing a 2-4 Tree



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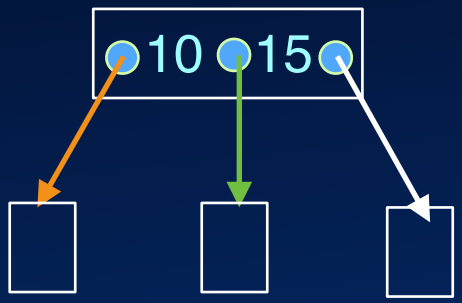

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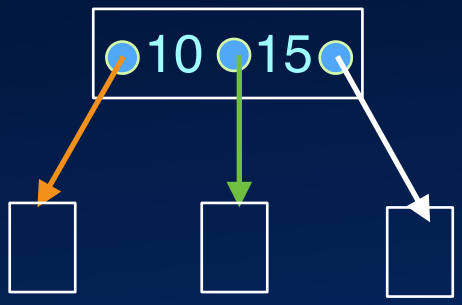

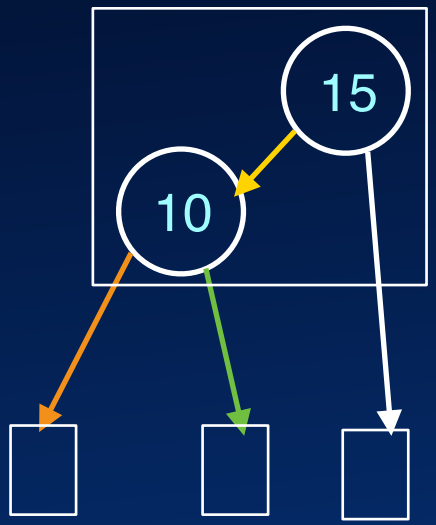

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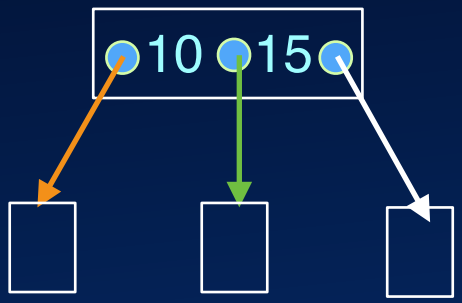

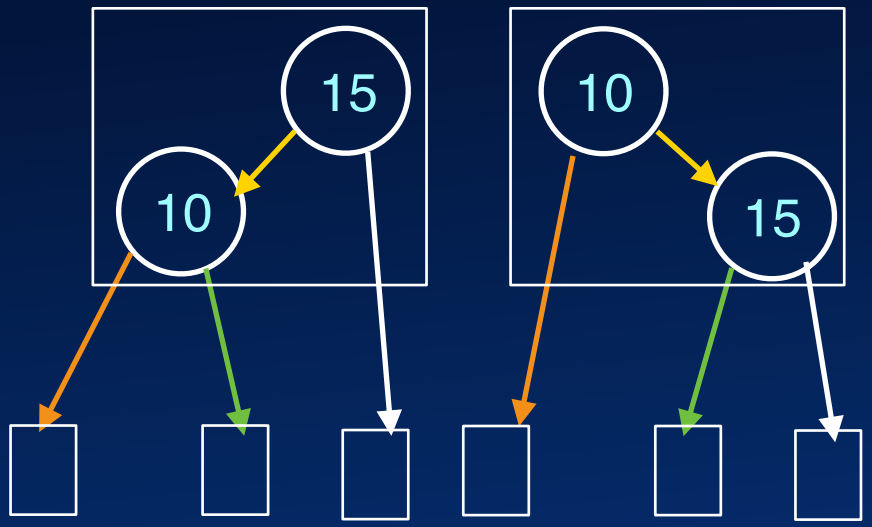

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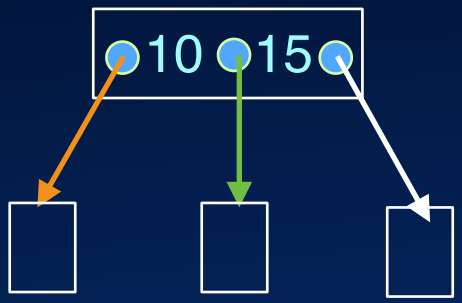

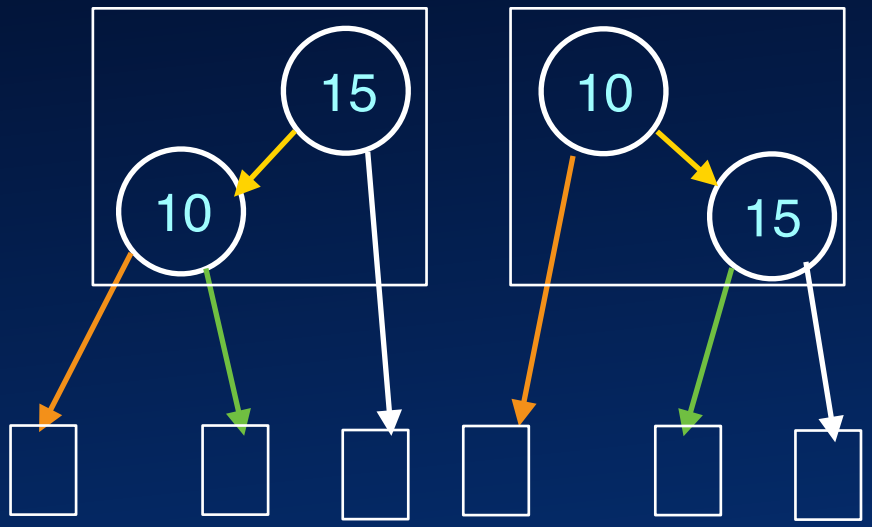

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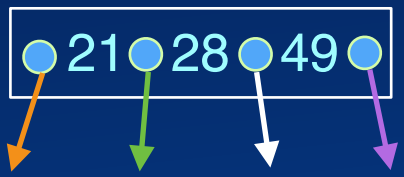

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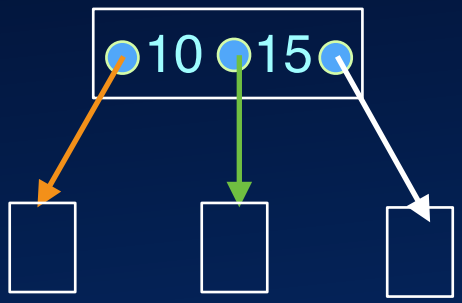

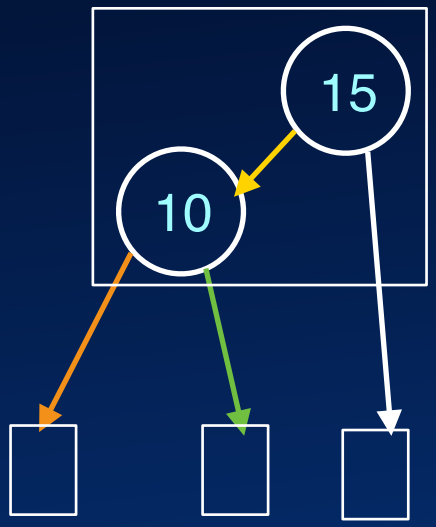

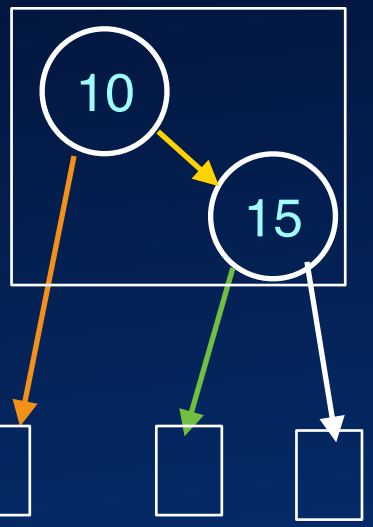

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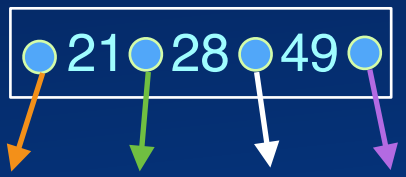

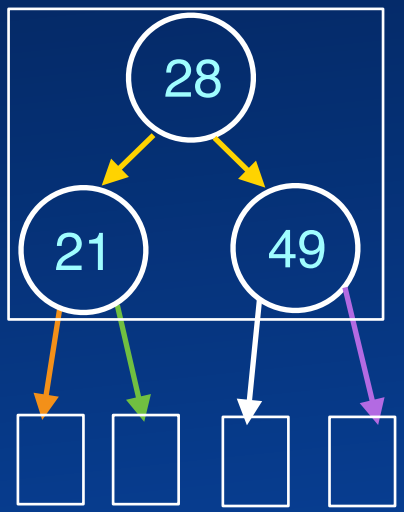

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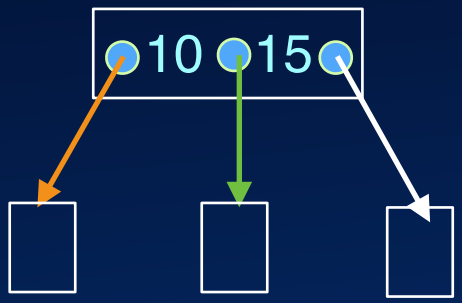

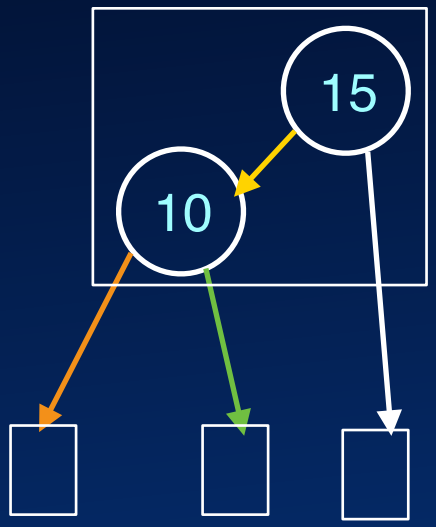

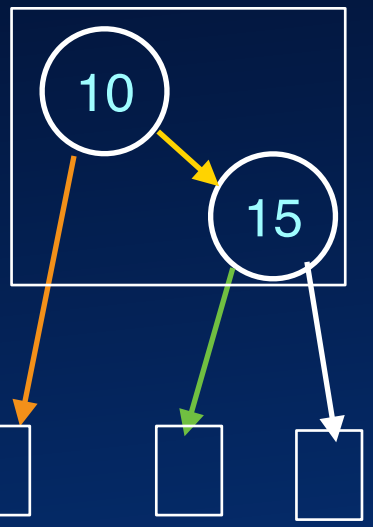

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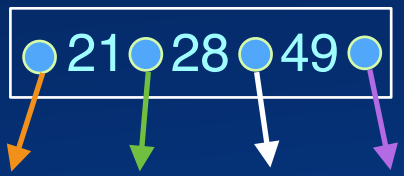

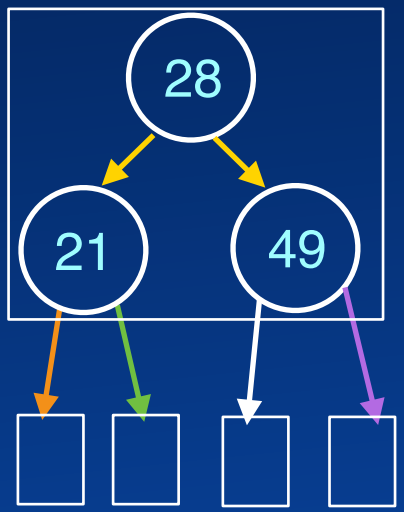

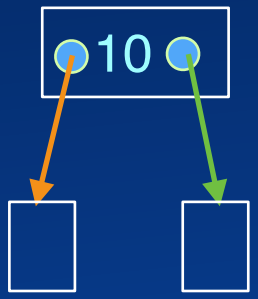

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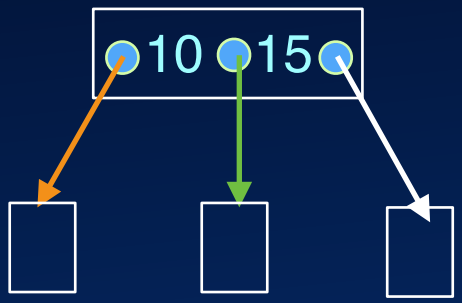

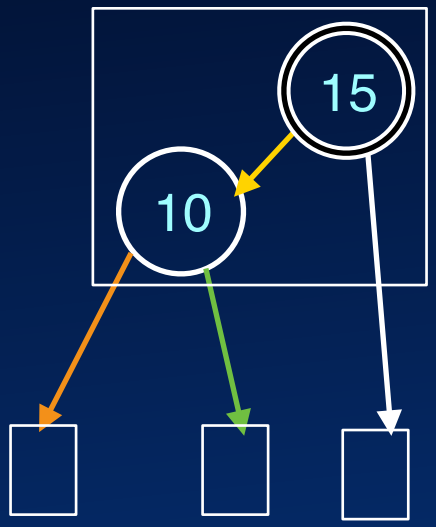

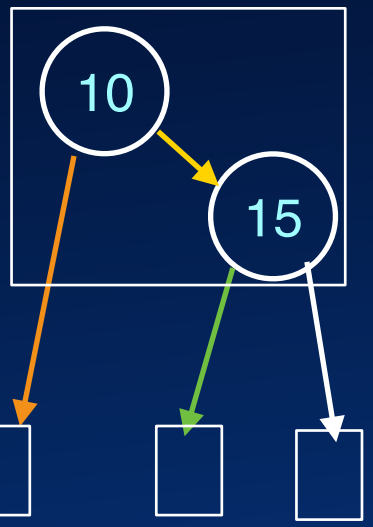

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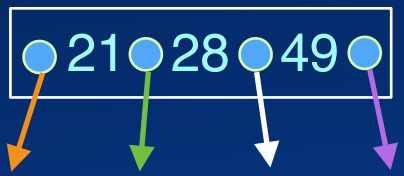

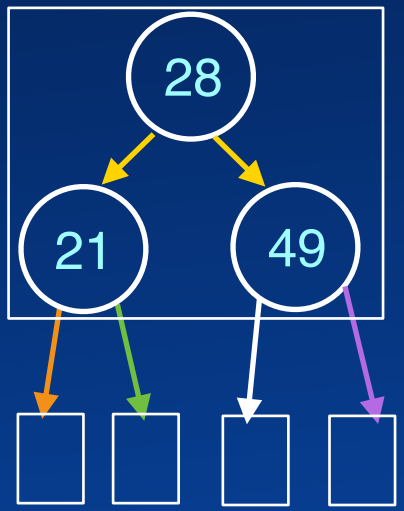

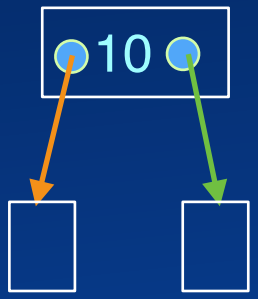

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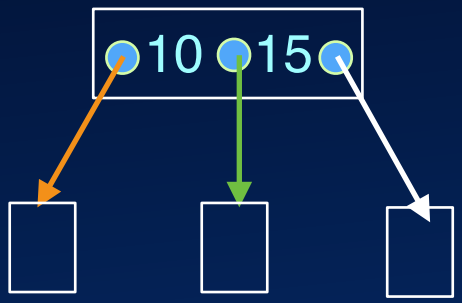

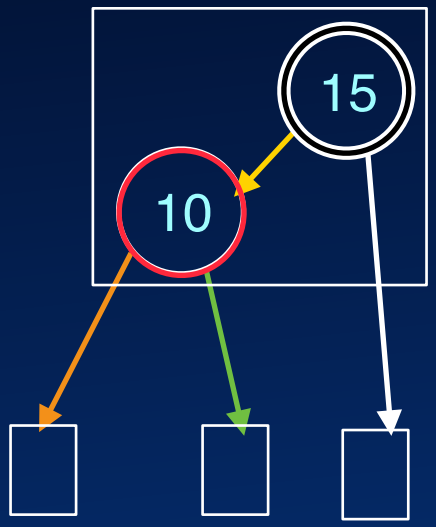

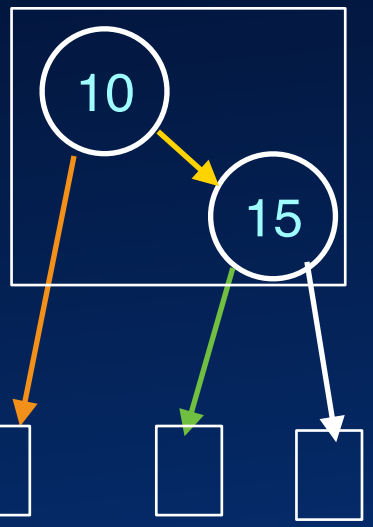

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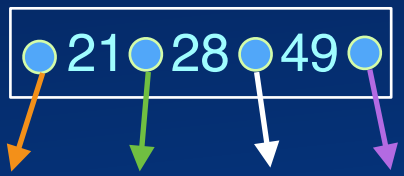

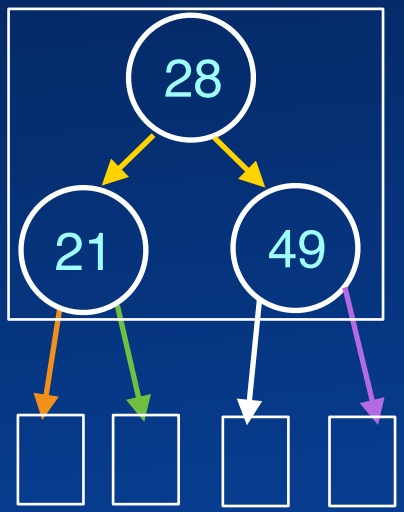

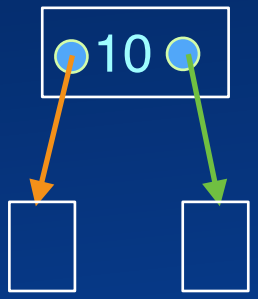

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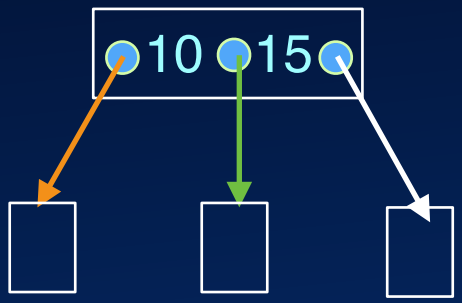

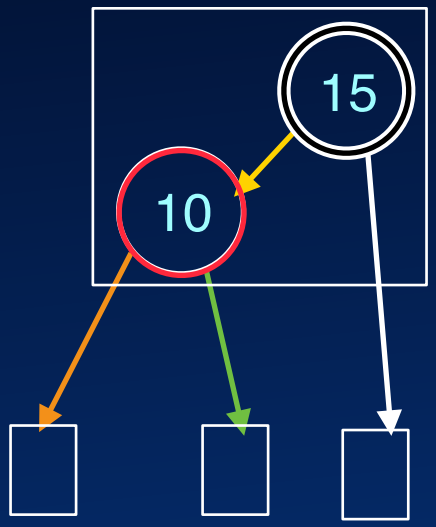

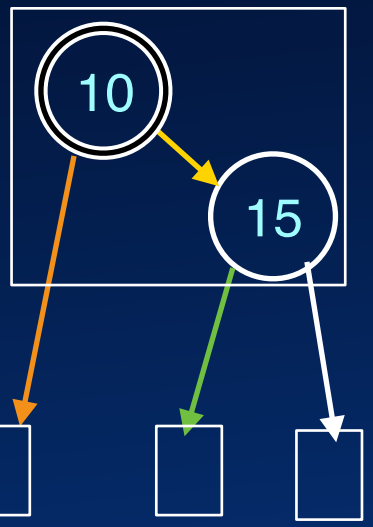

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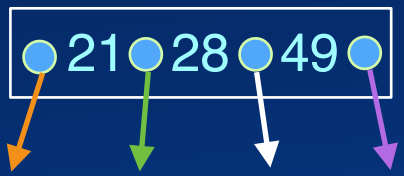

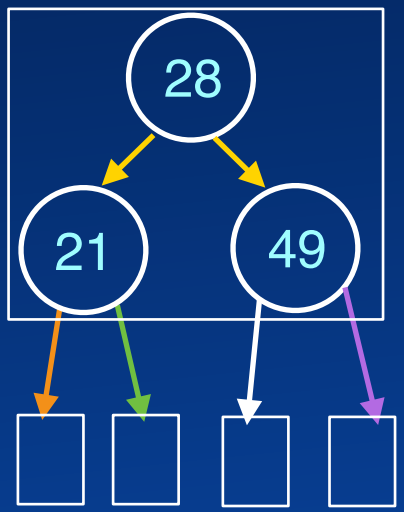

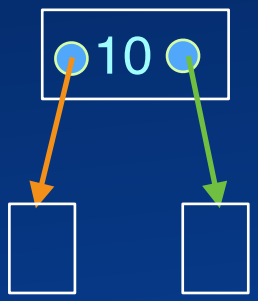

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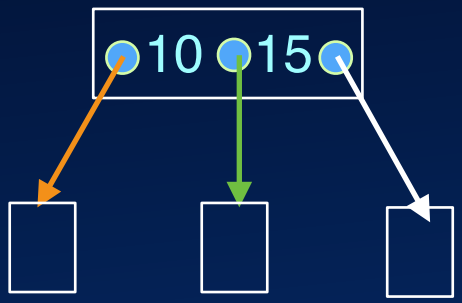

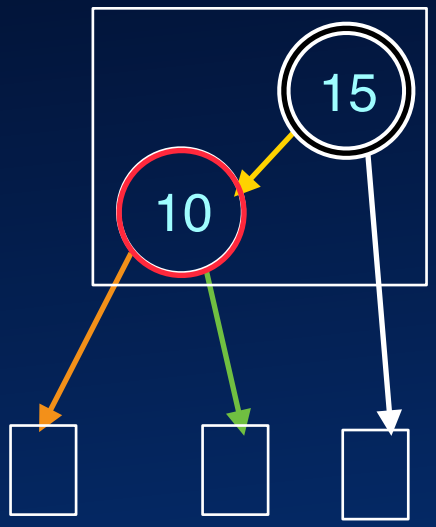

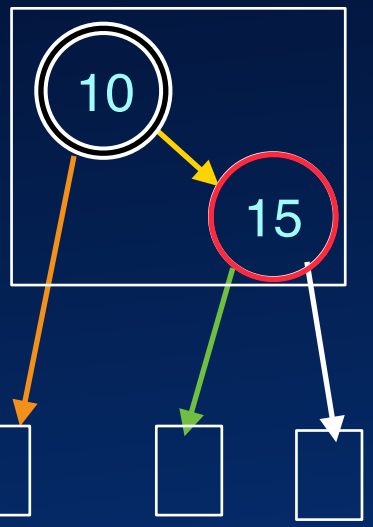

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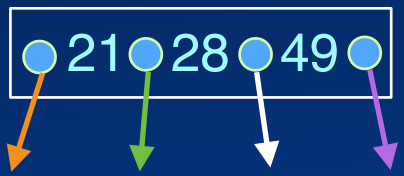

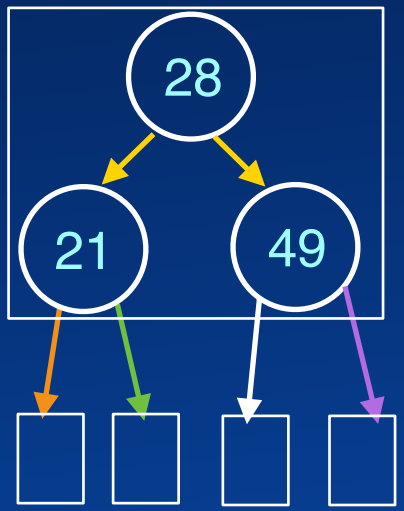

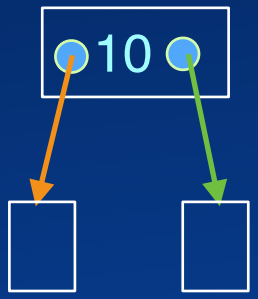

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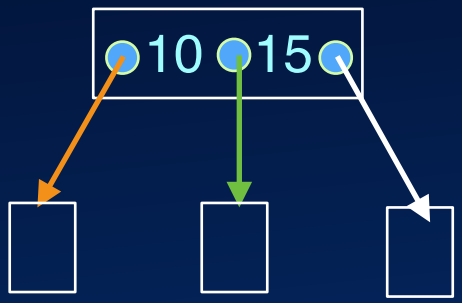

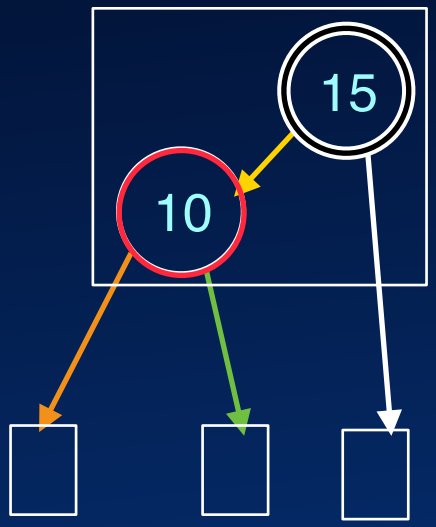

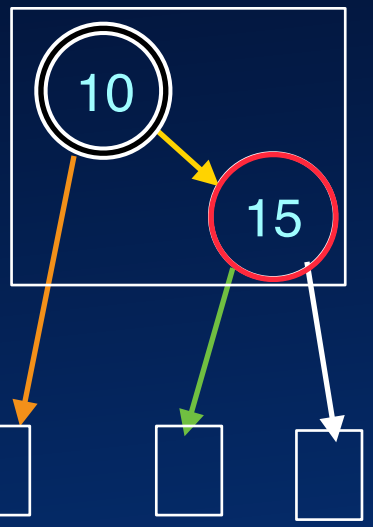

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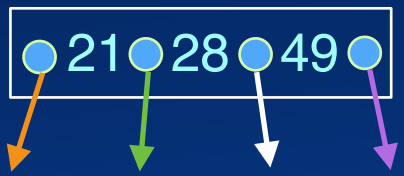

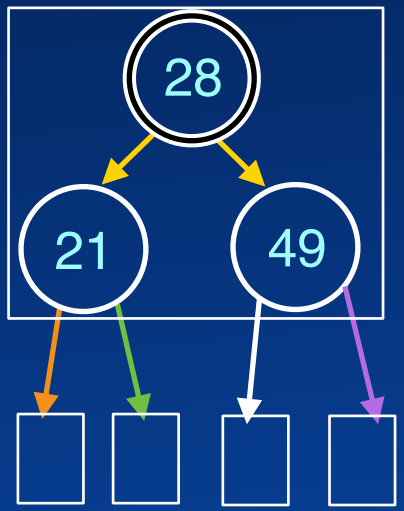

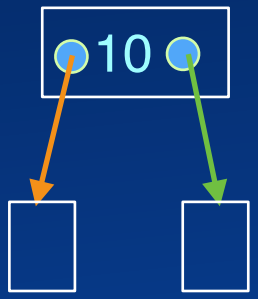

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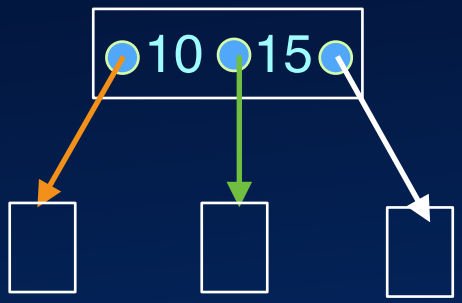
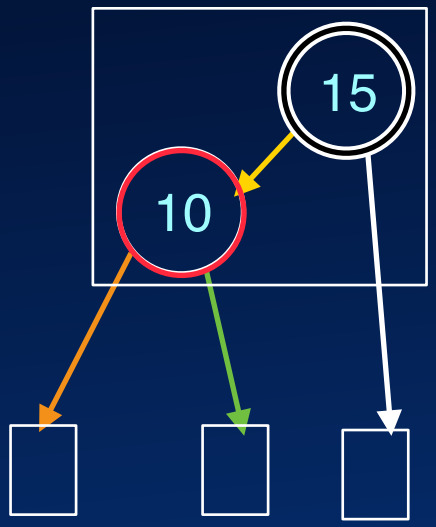
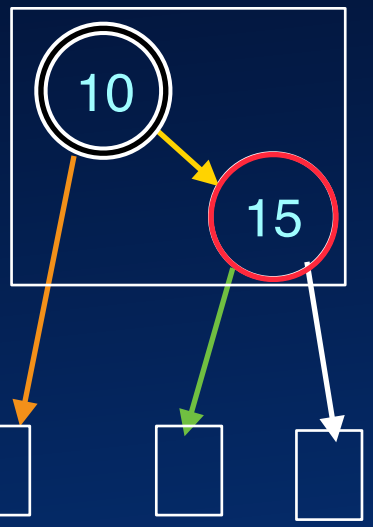
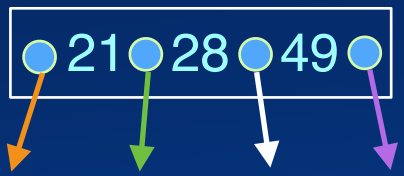
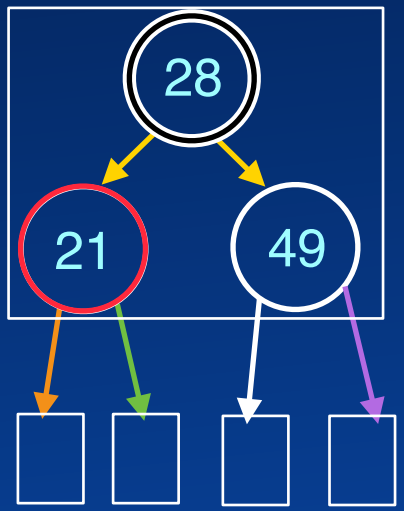


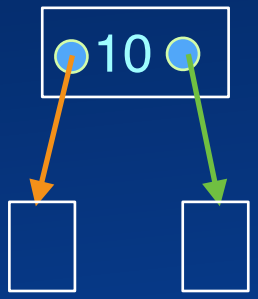




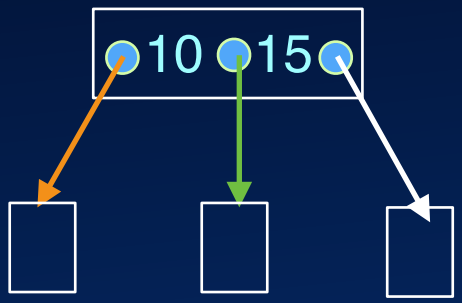
$\lg_3 n = .63 \lg_2 n$

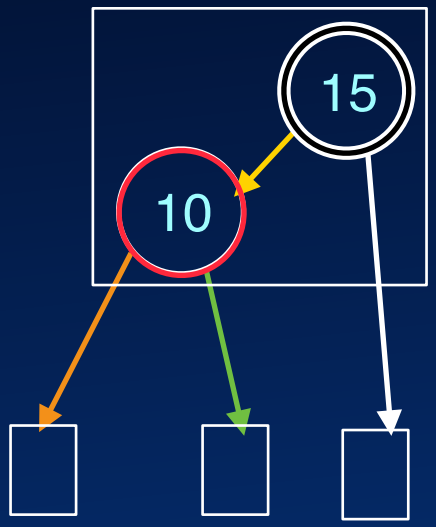


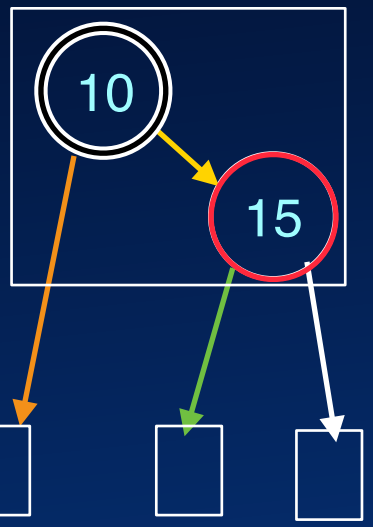




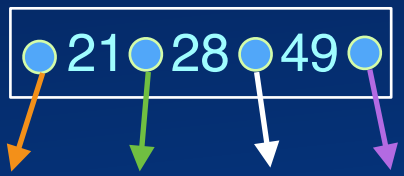
Binar-izing a 2-4 Tree

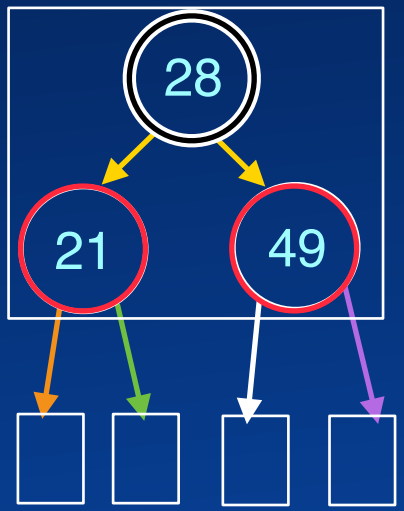


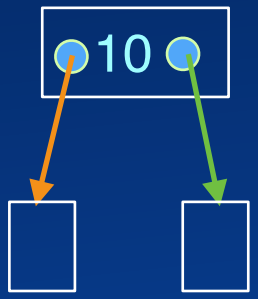




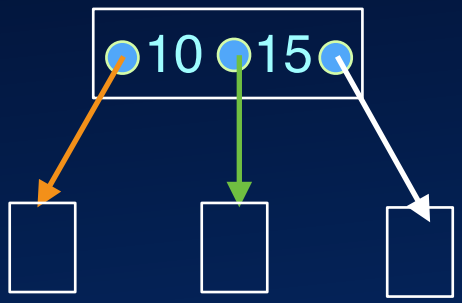
$\lg_3 n = .63 \lg_2 n$

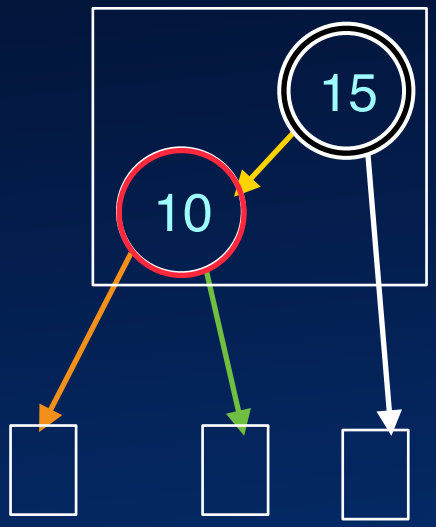


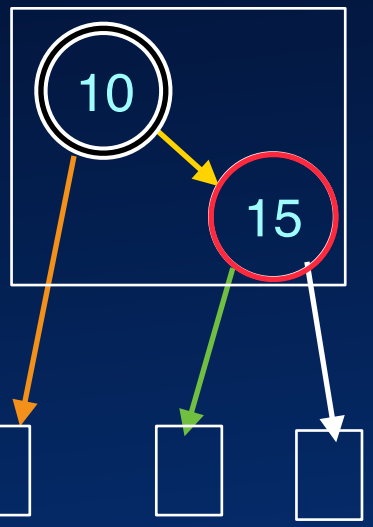




Binar-izing a 2-4 Tree







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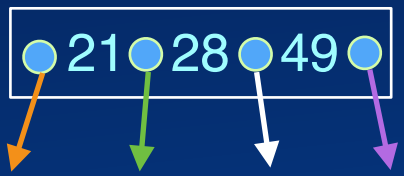
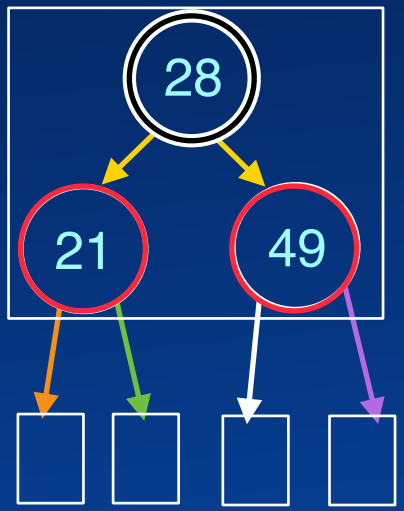
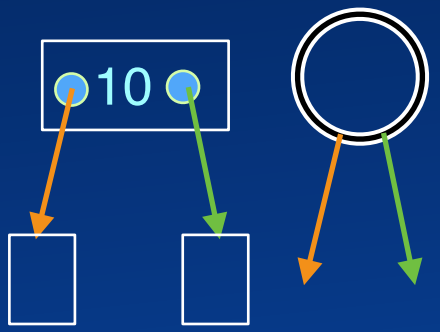
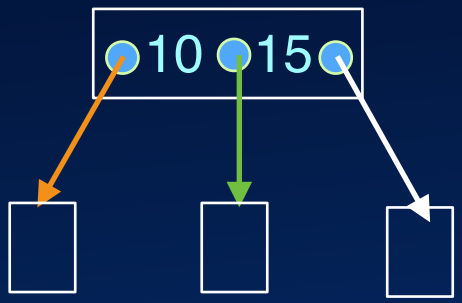
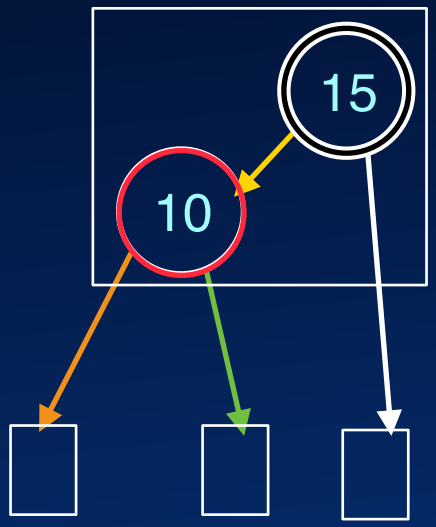


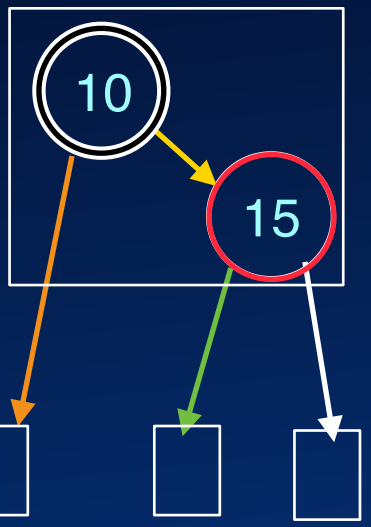




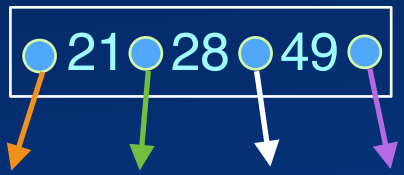
Binar-izing a 2-4 Tree



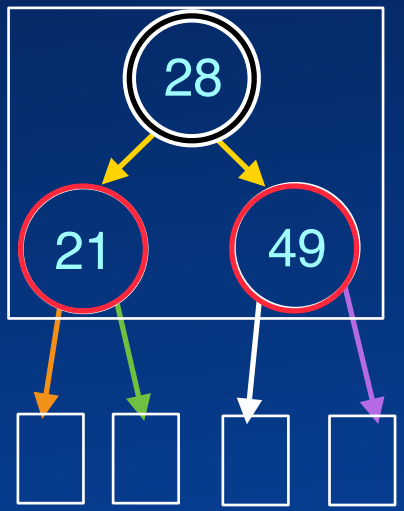


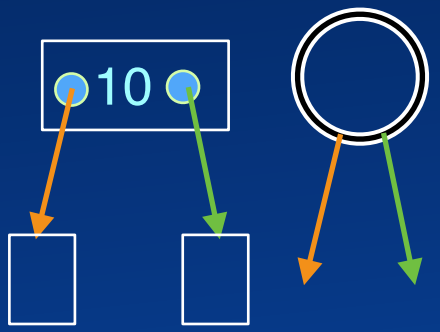


$\lg_3 n = .63 \lg_2 n$



Red Black Tree





Red Black Tree

- No red colored node has a red colored child
- The number of black colored node on the path from the root to each null reference is the same

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$$\Rightarrow \text{height} \leq 2 \log (n)$$

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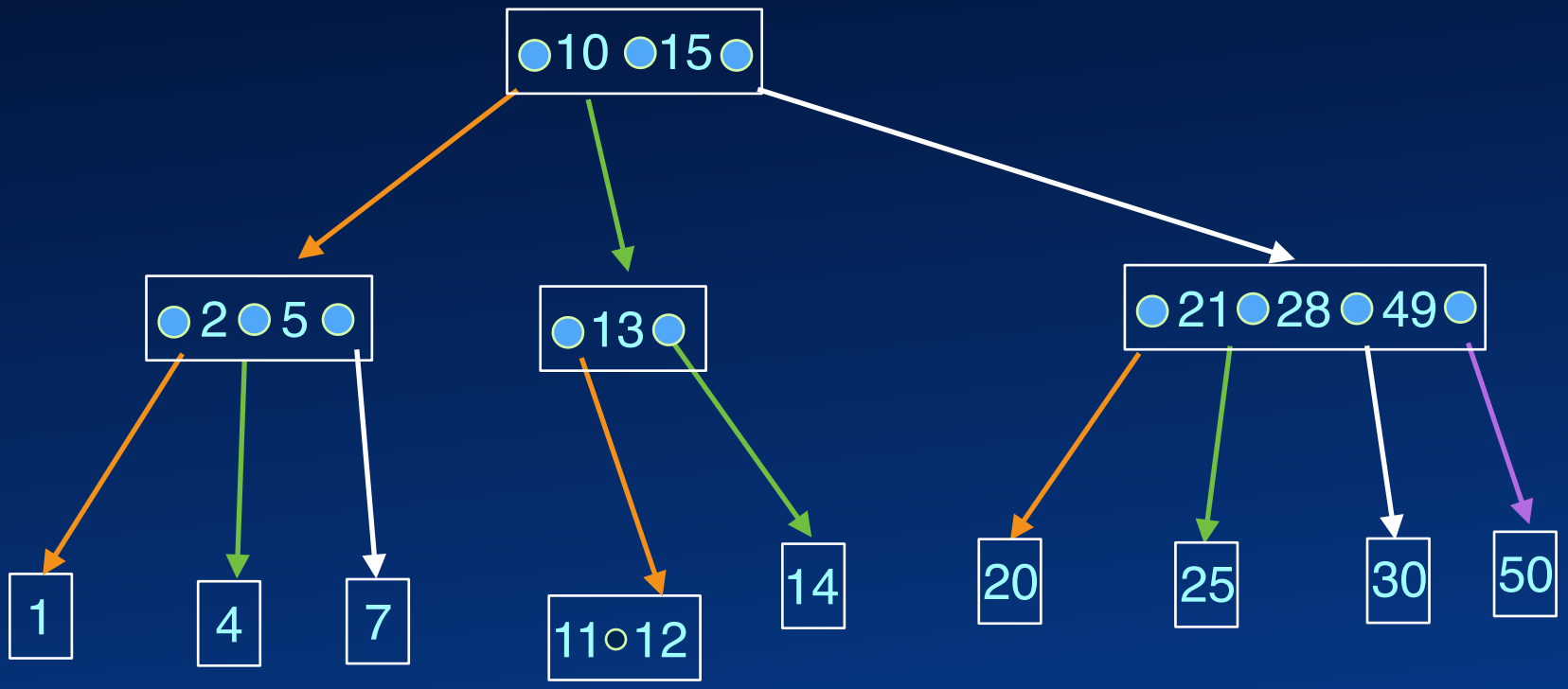
- **The number of black colored node on the path from the root to each null reference is the same**

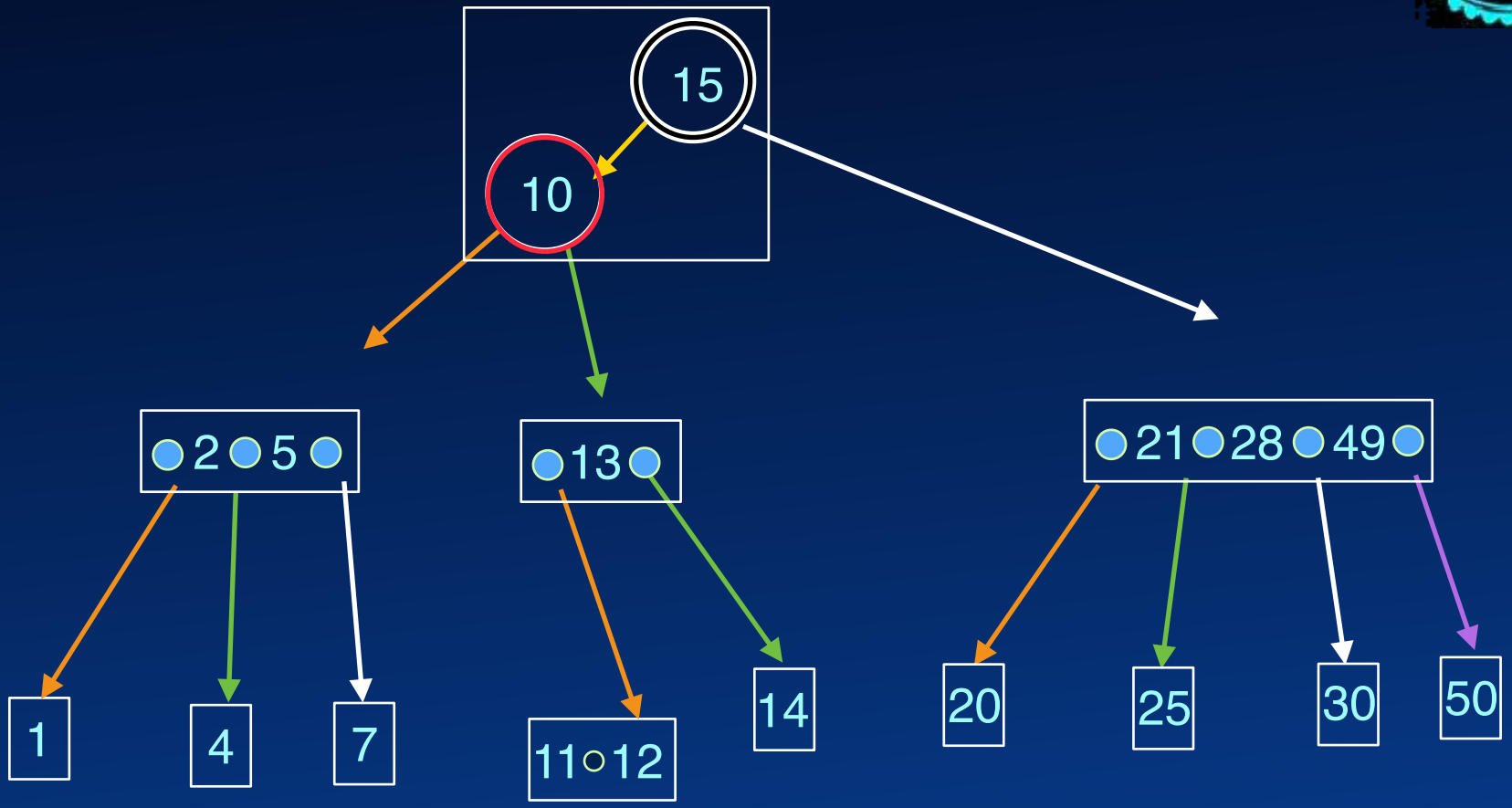
  => height ≤ 2 log (n)

  - Null references are assumed black
  - Root is always black

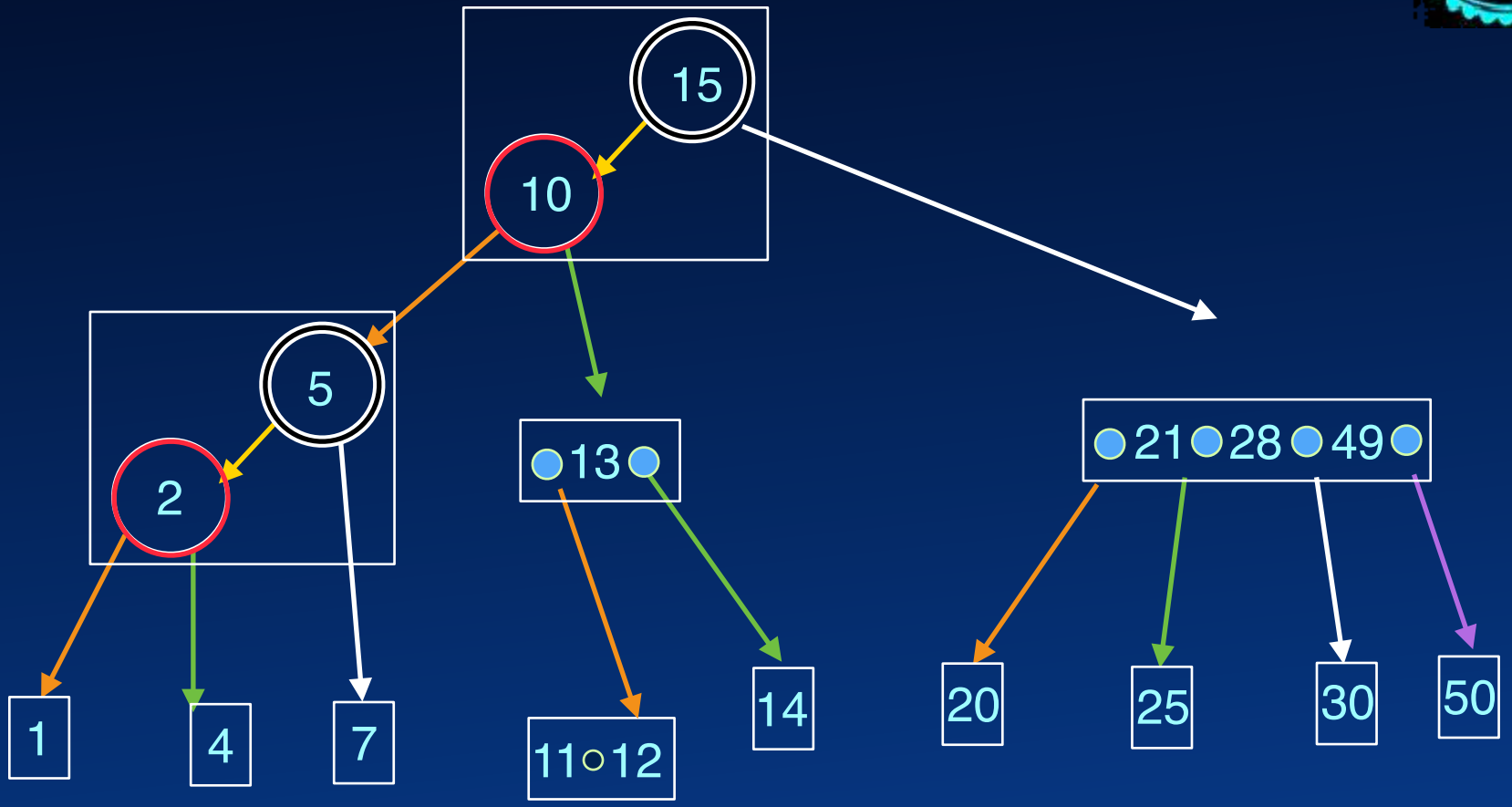
Binar-ized 2-4 Tree



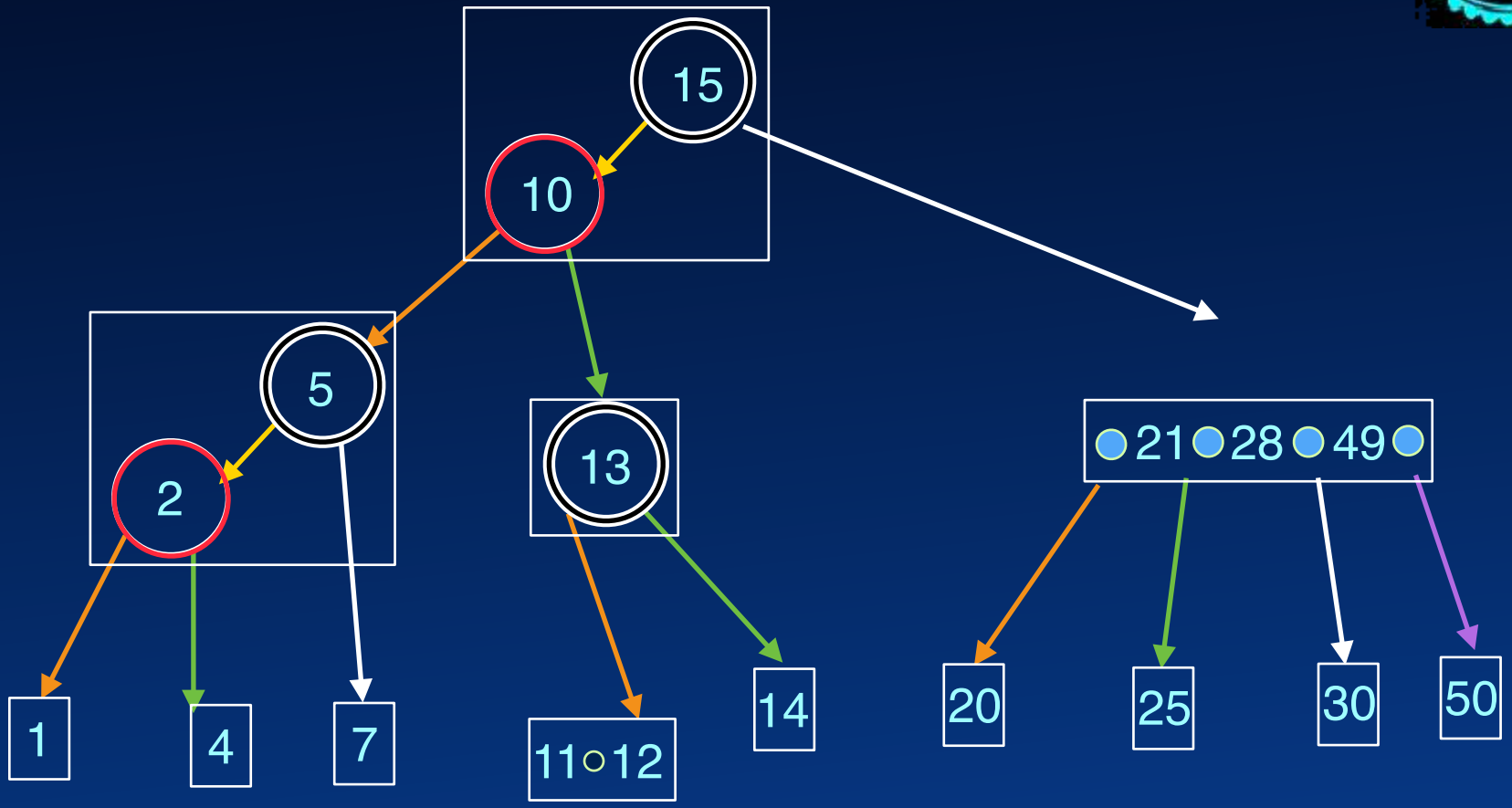
Binar-ized 2-4 Tree



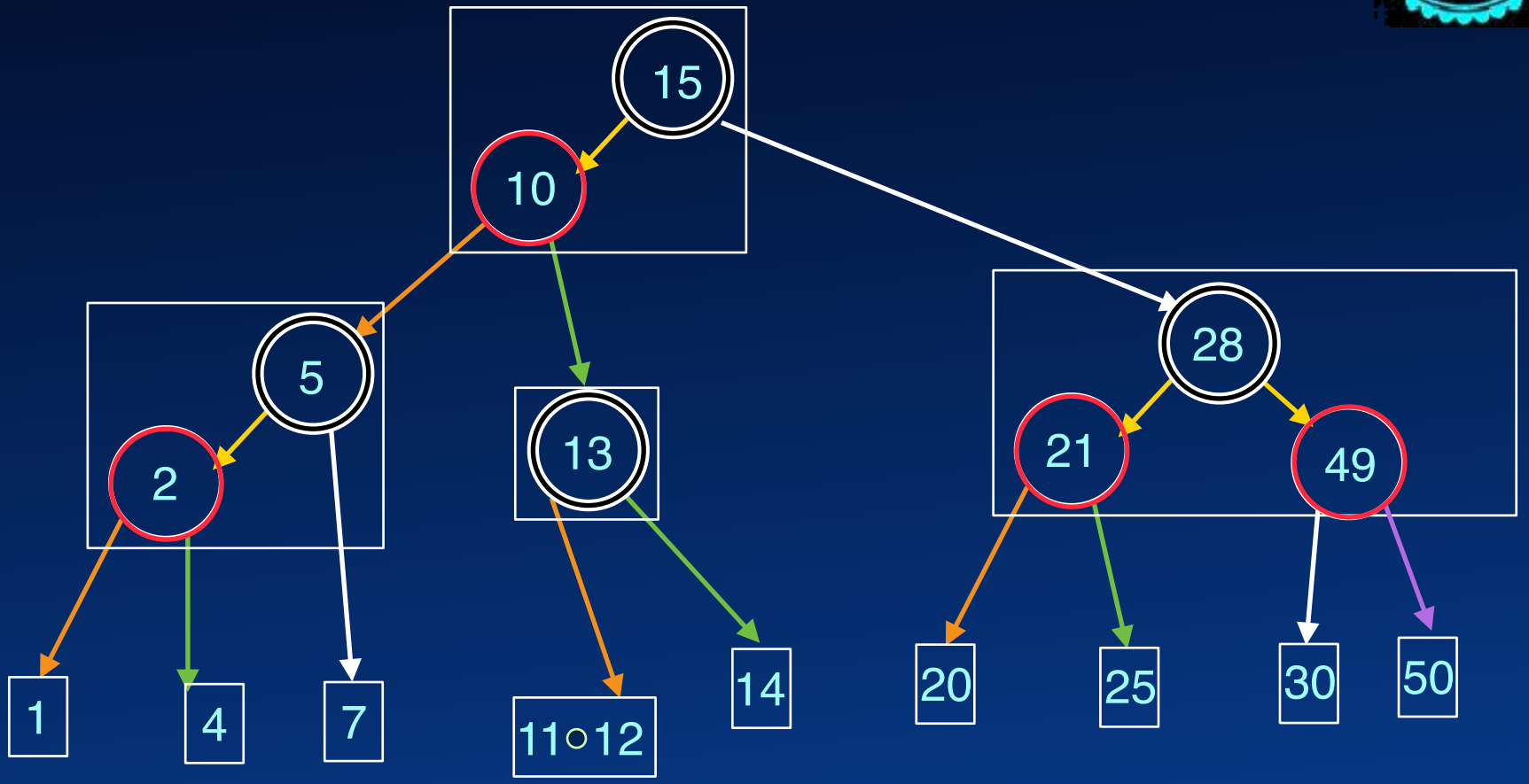
Binar-ized 2-4 Tree



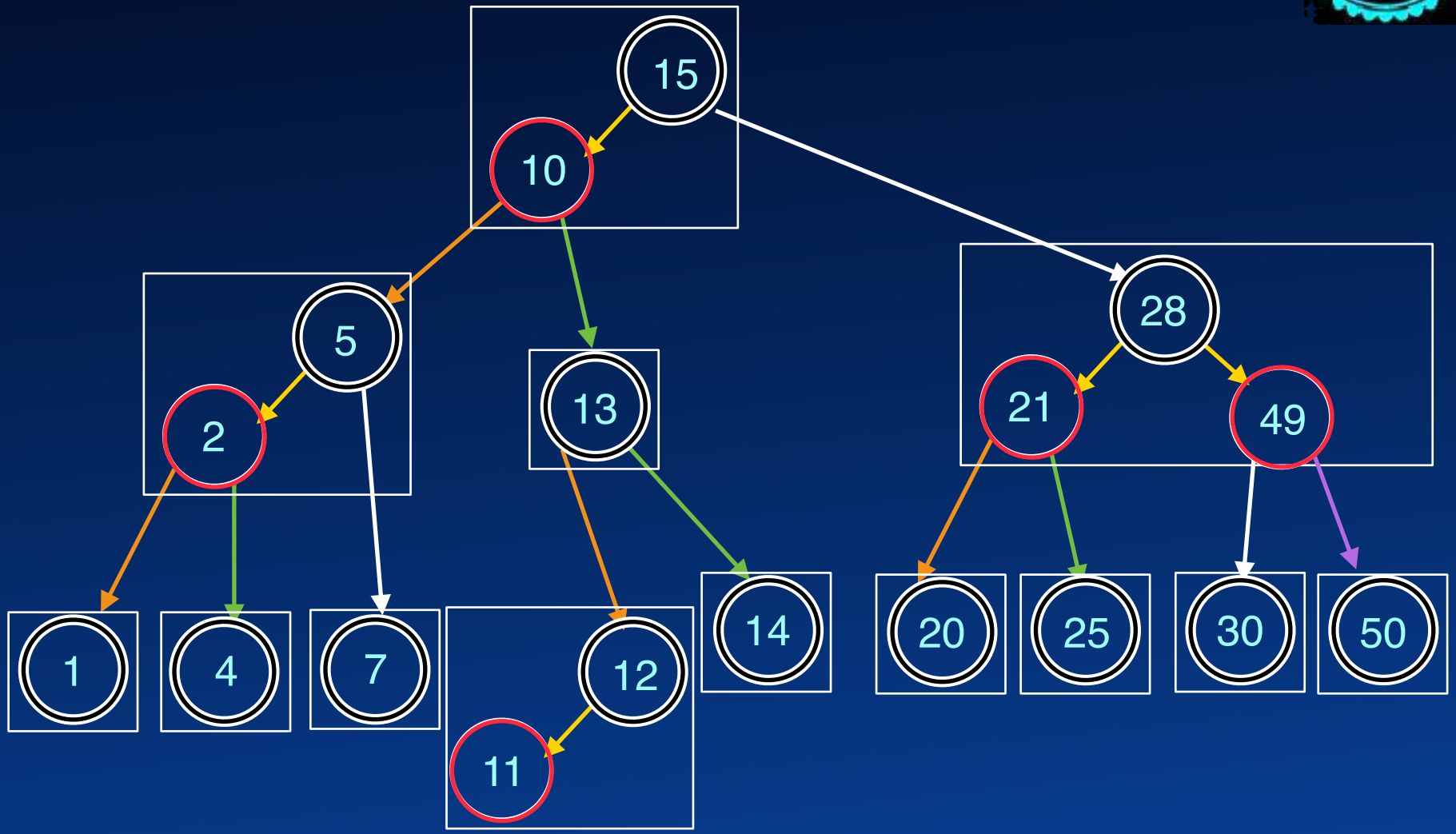
Binar-ized 2-4 Tree



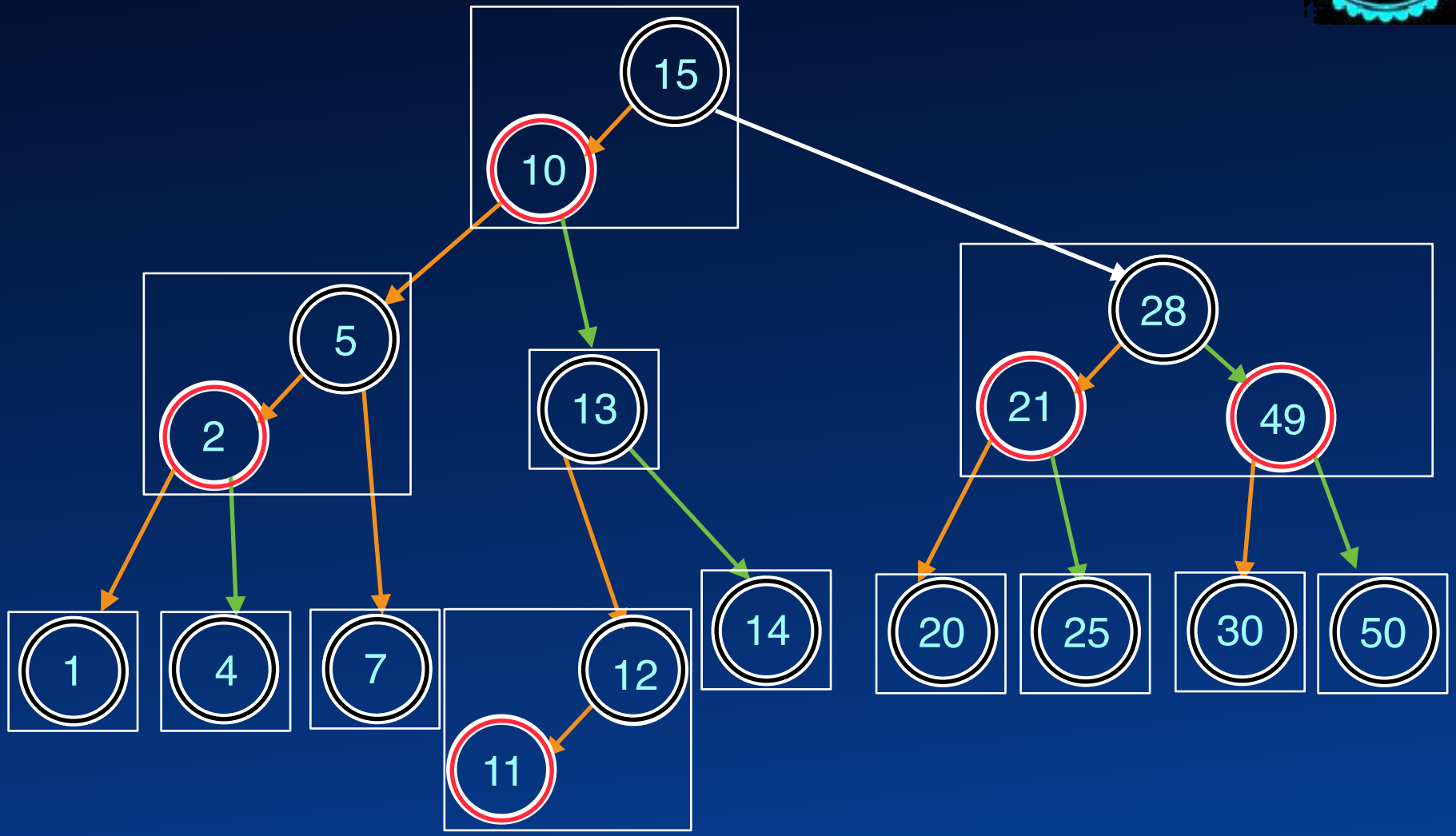
Binar-ized 2-4 Tree



Binar-ized 2-4 Tree



Red-Black Tree Insert



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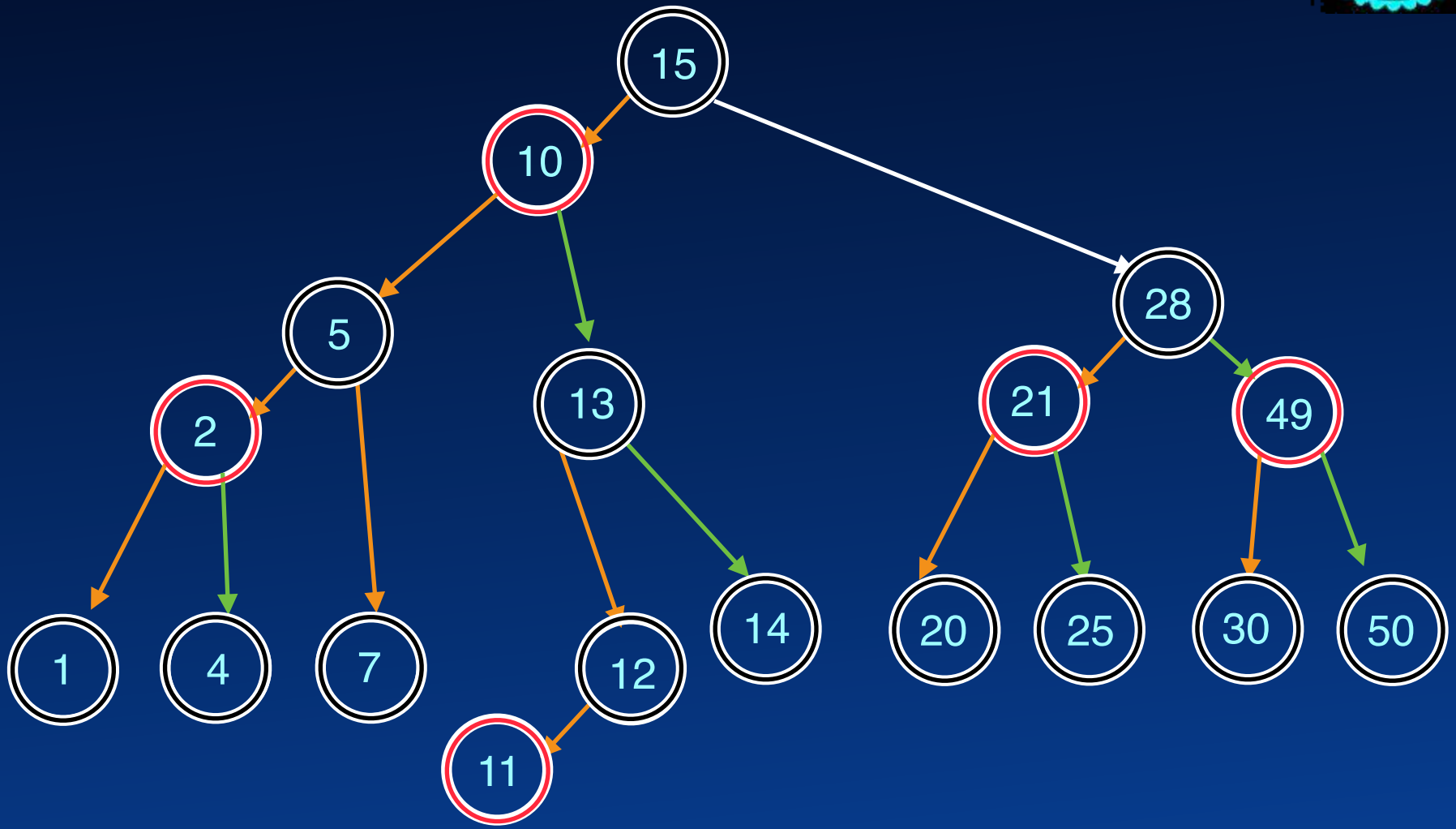

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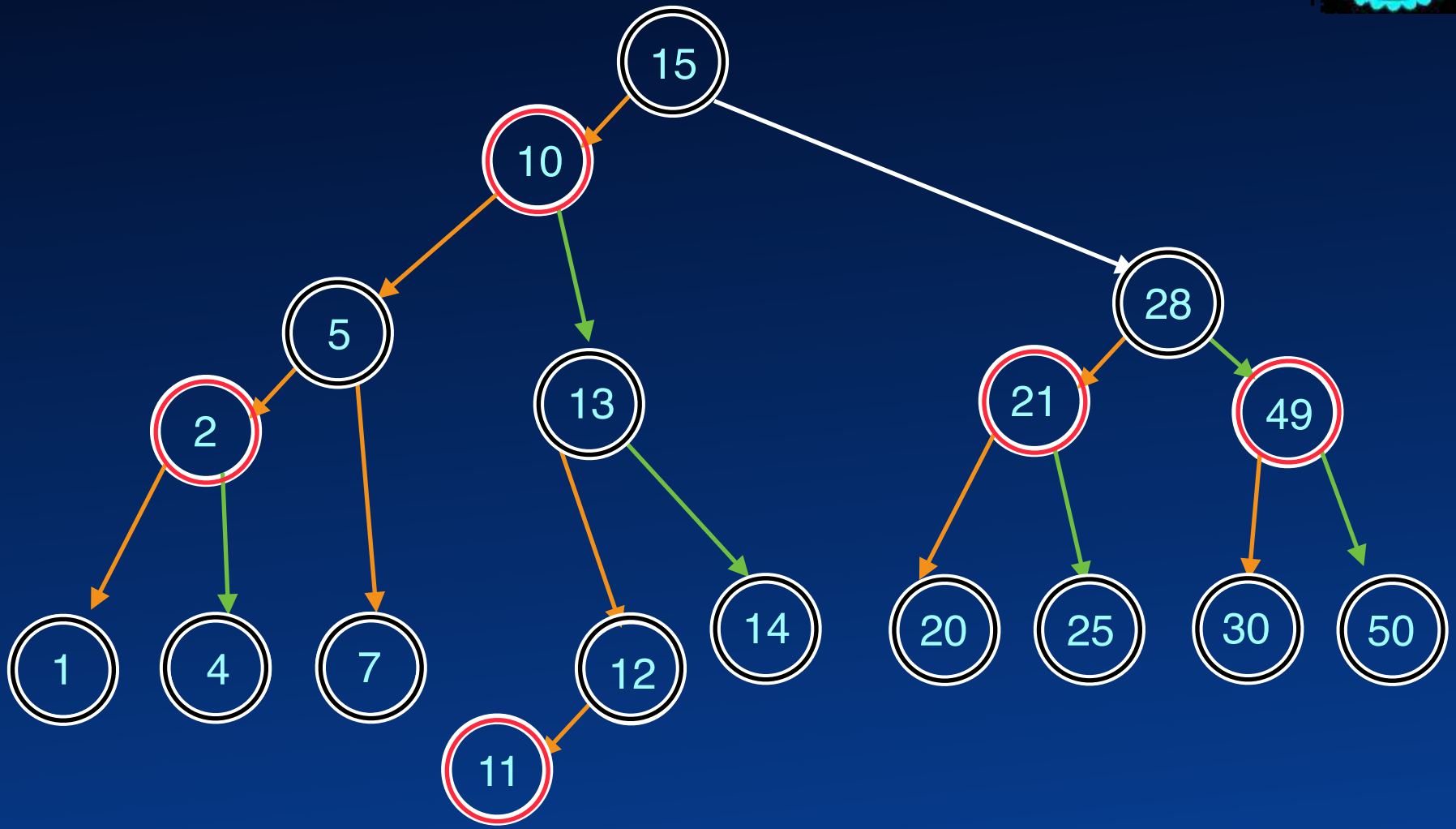
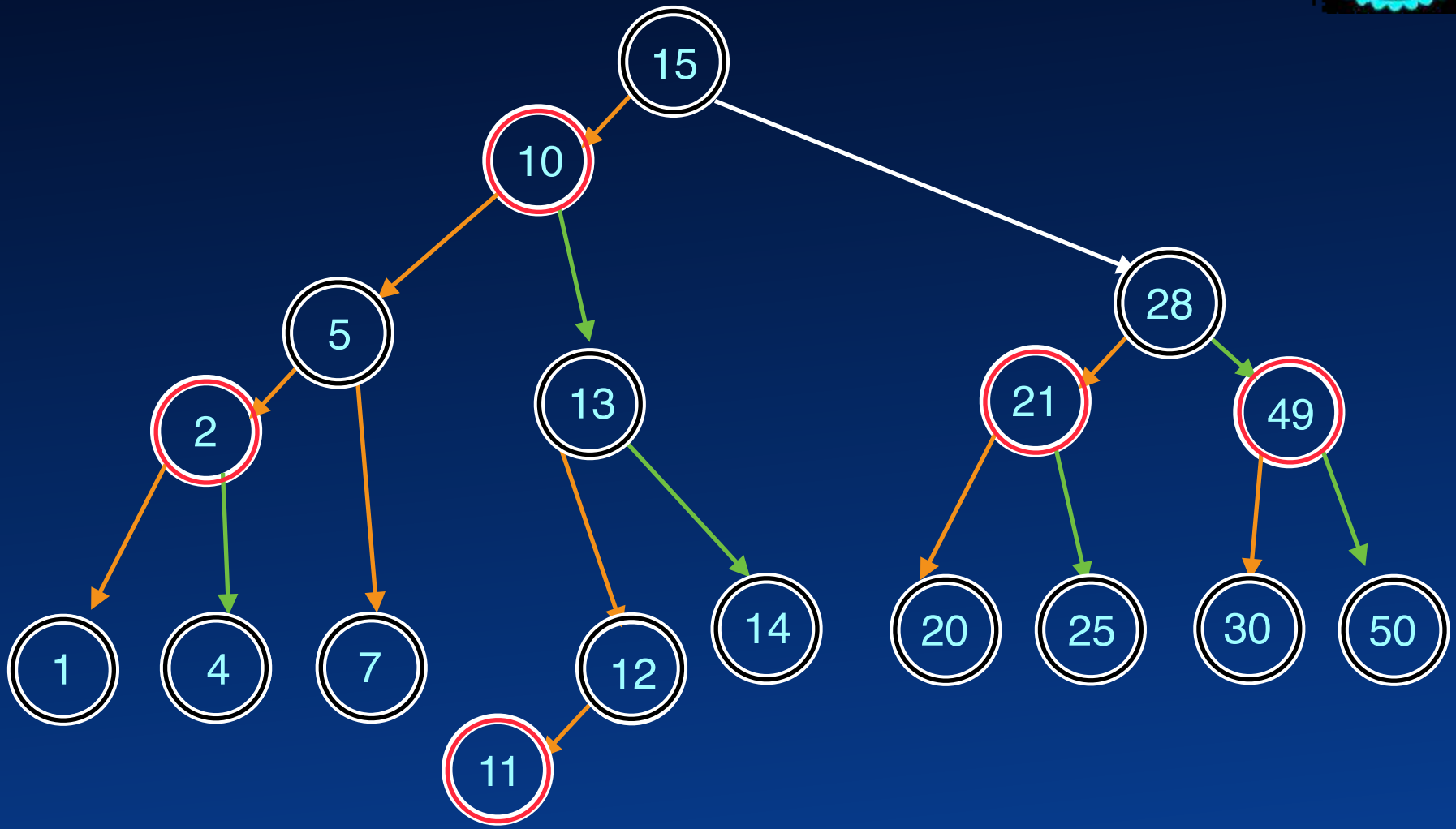


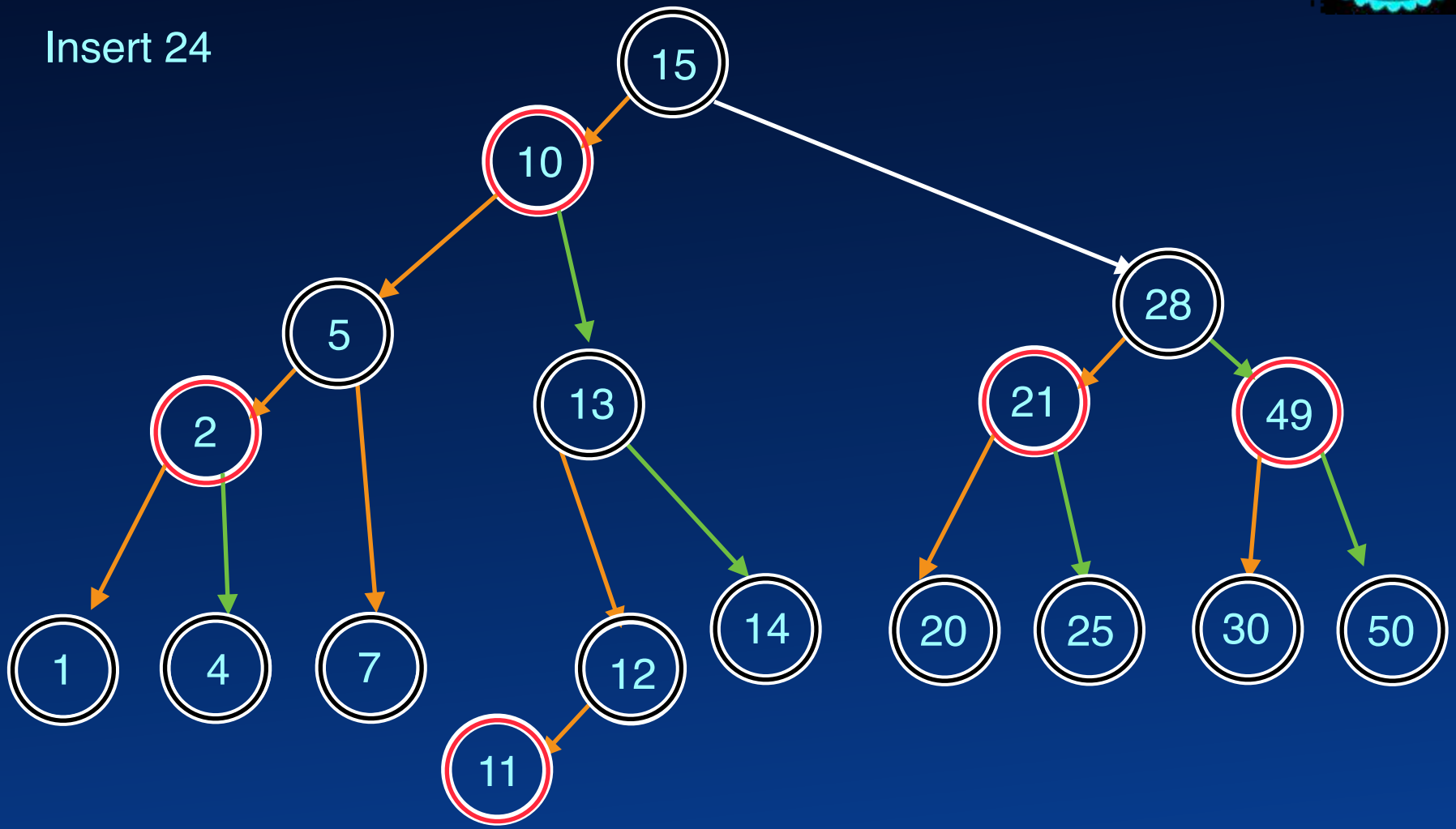
Insert

Red-Black Tree Insert



Insert @null in node ***n***

Red-Black Tree Insert



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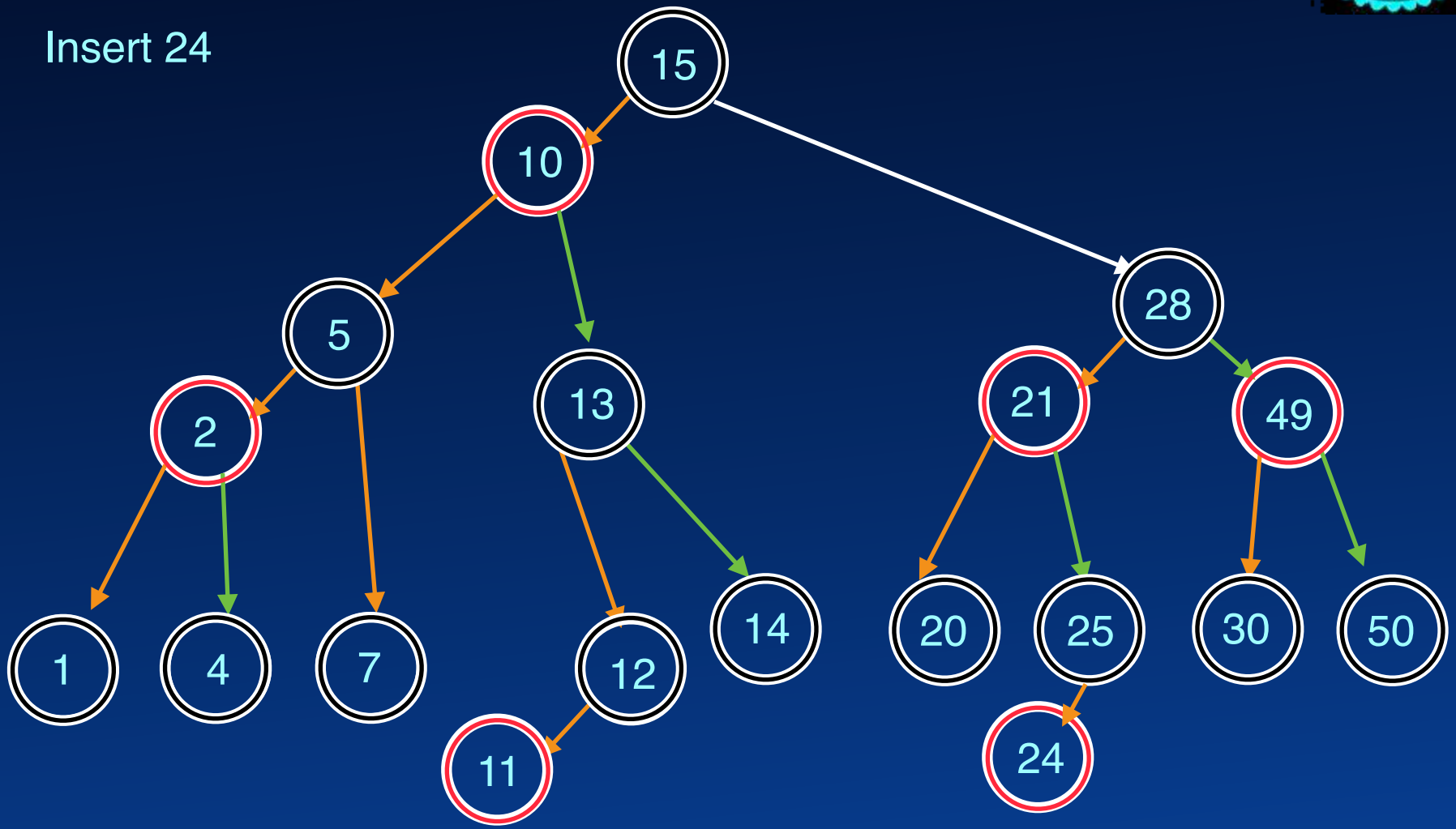
Red-Black Tree Insert

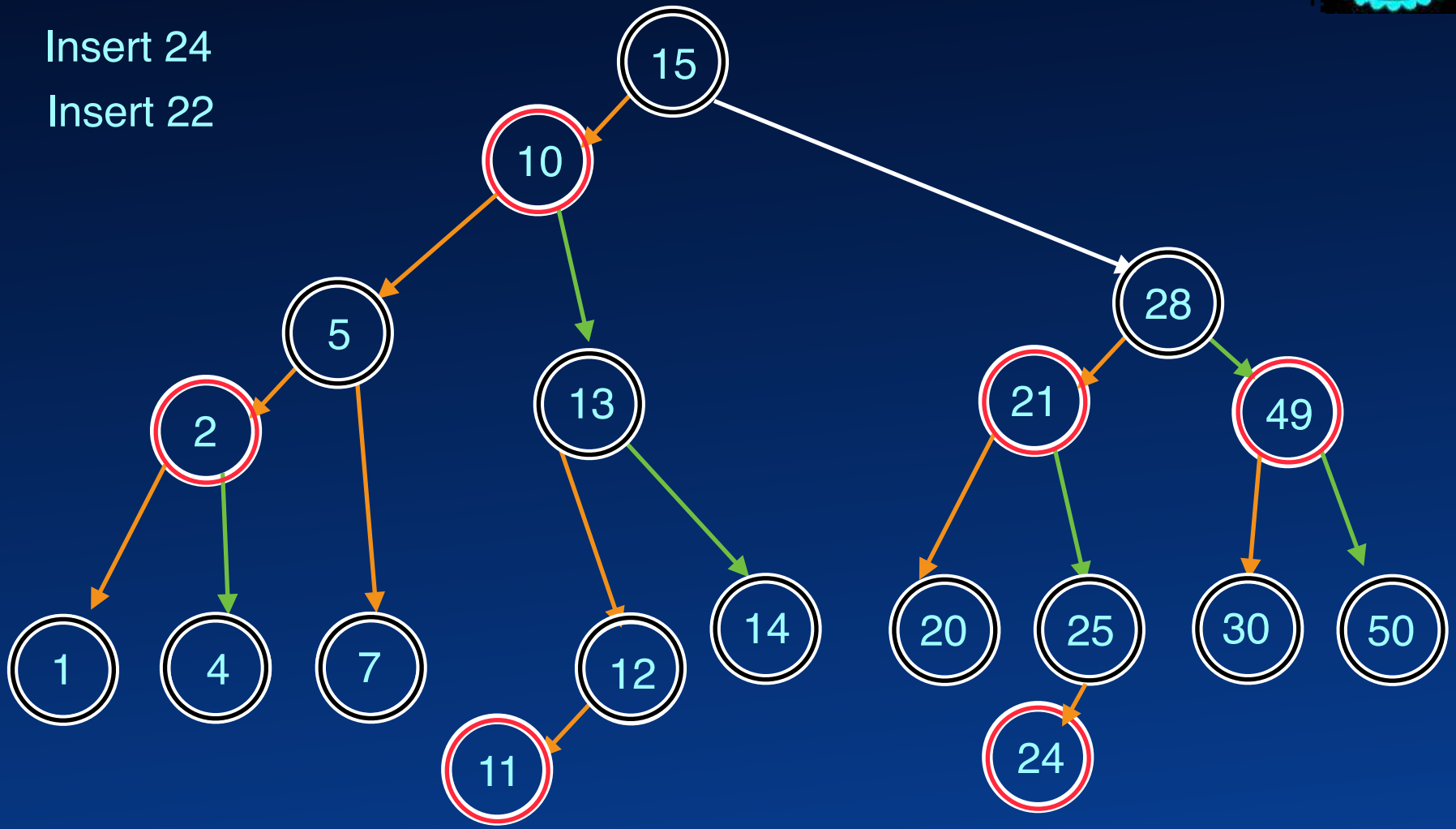


Insert @null in node ***n***

Red-Black Tree Insert

Insert 24

Insert 22



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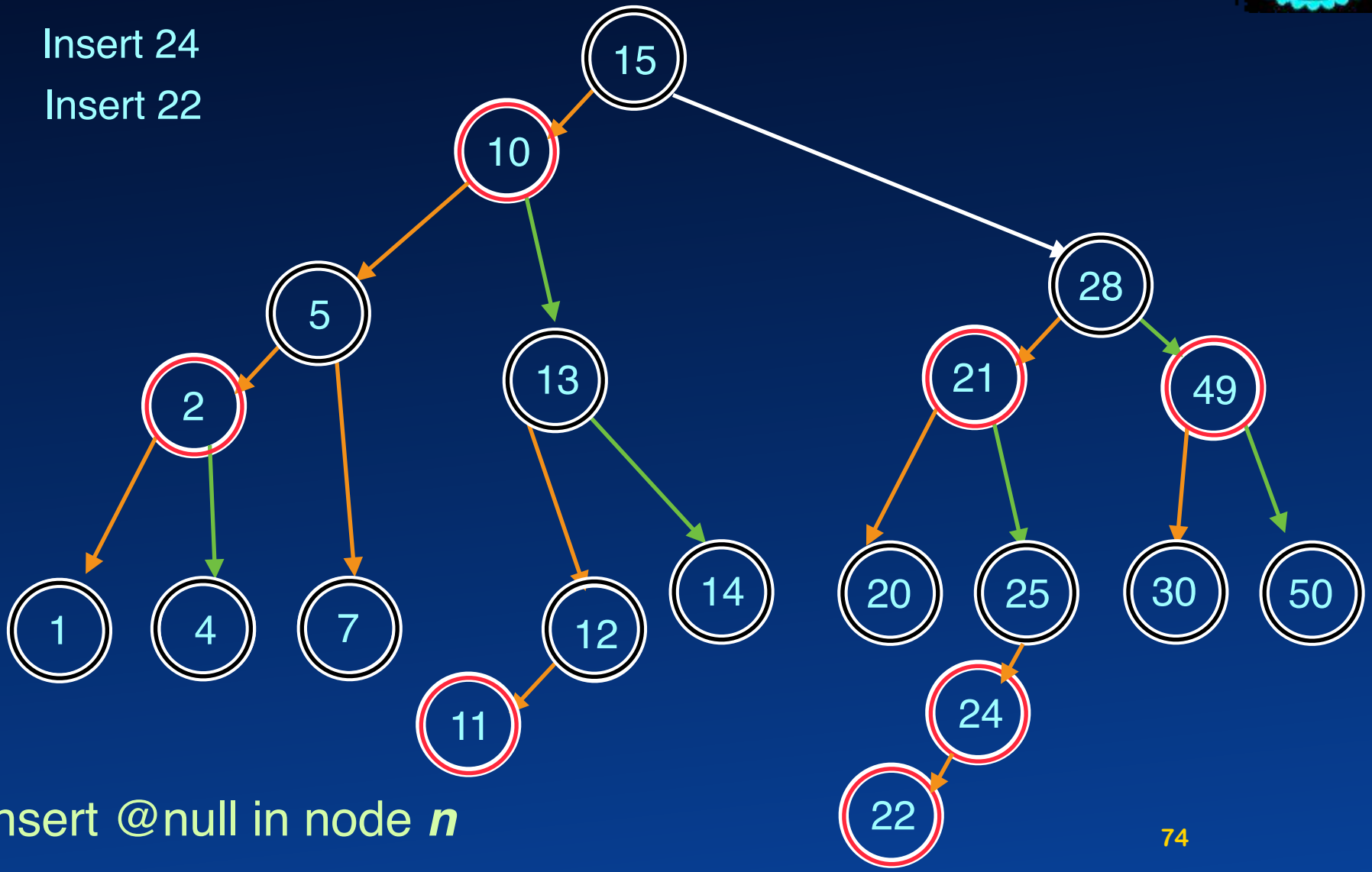

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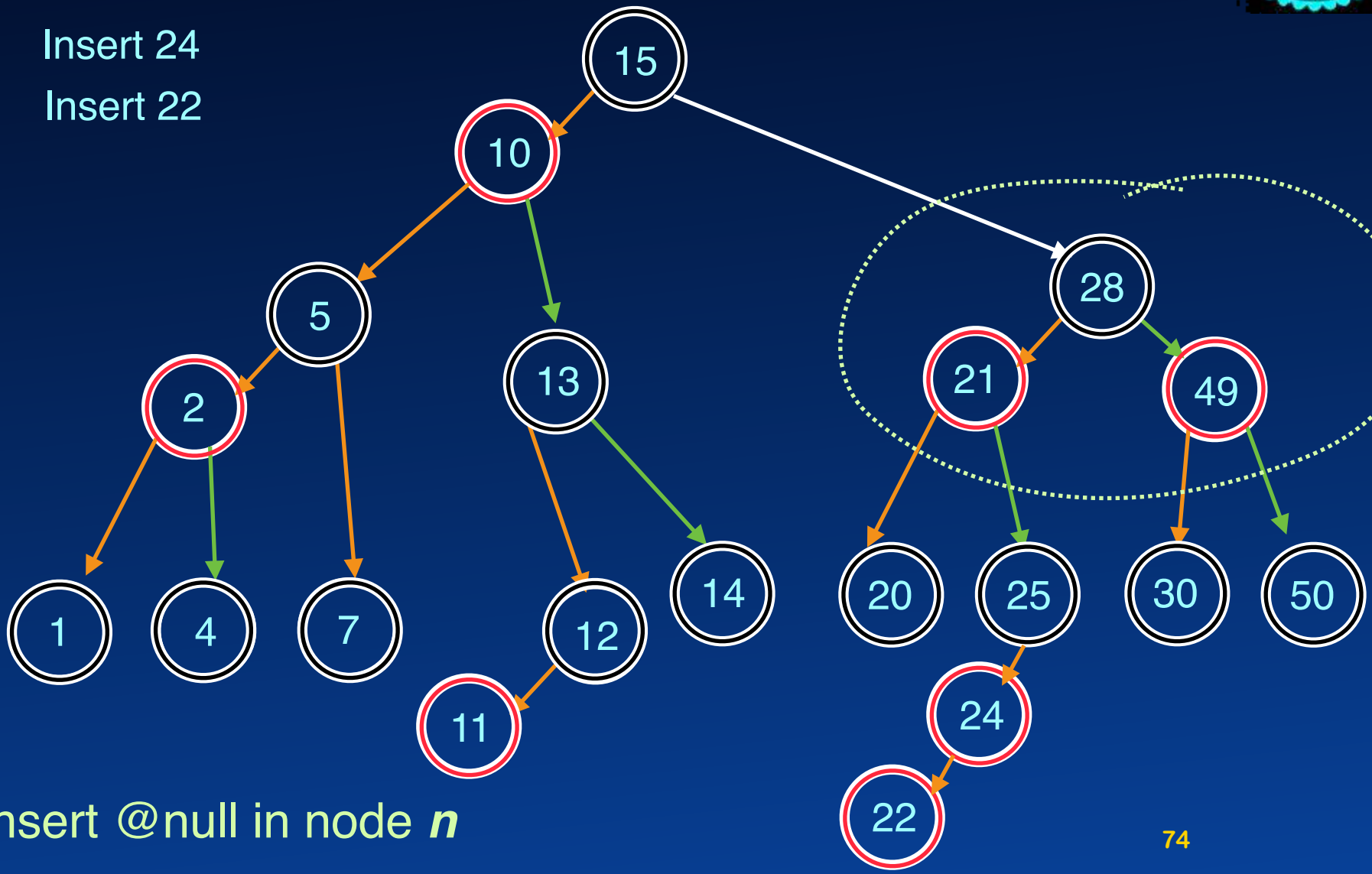

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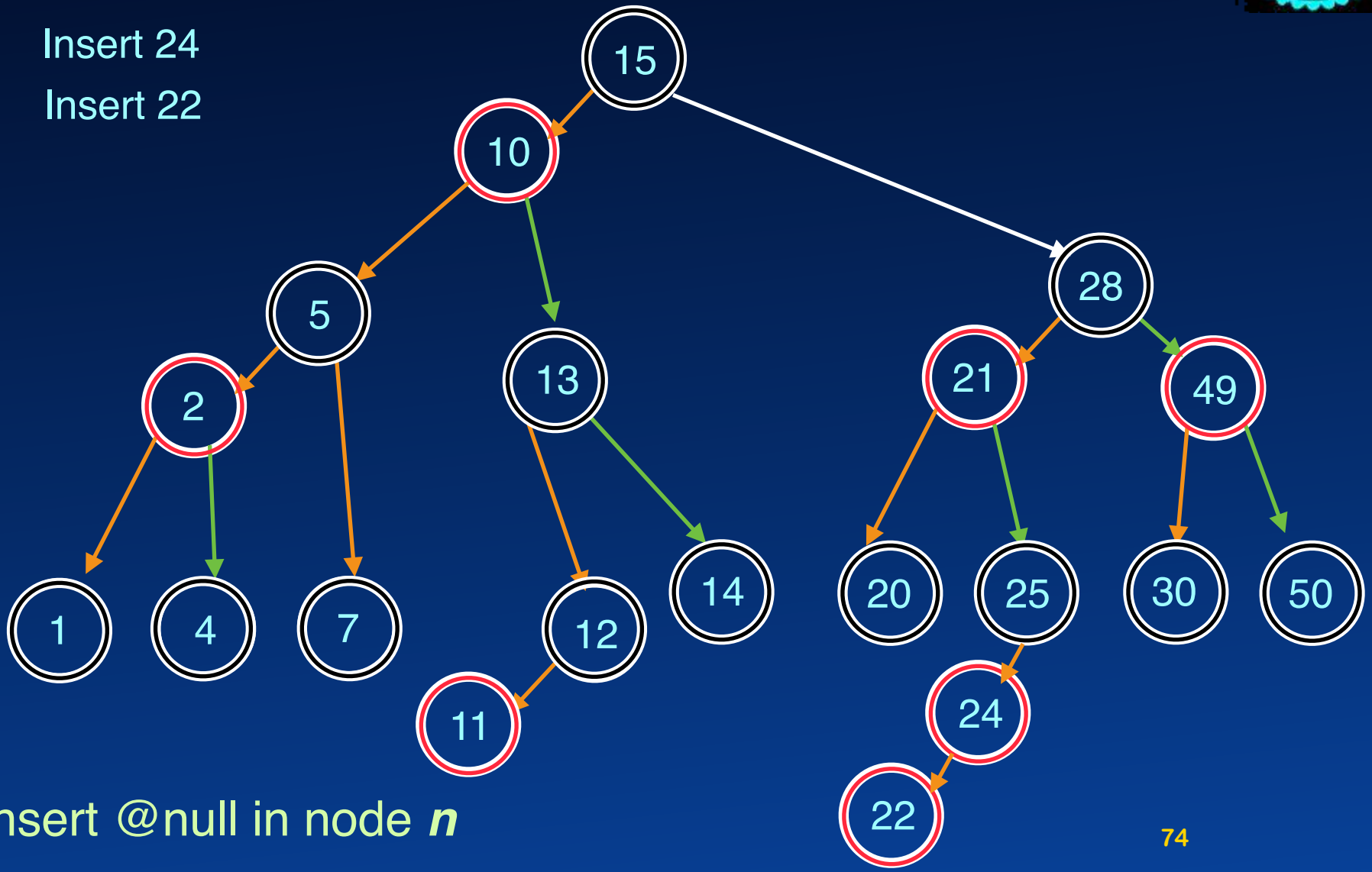

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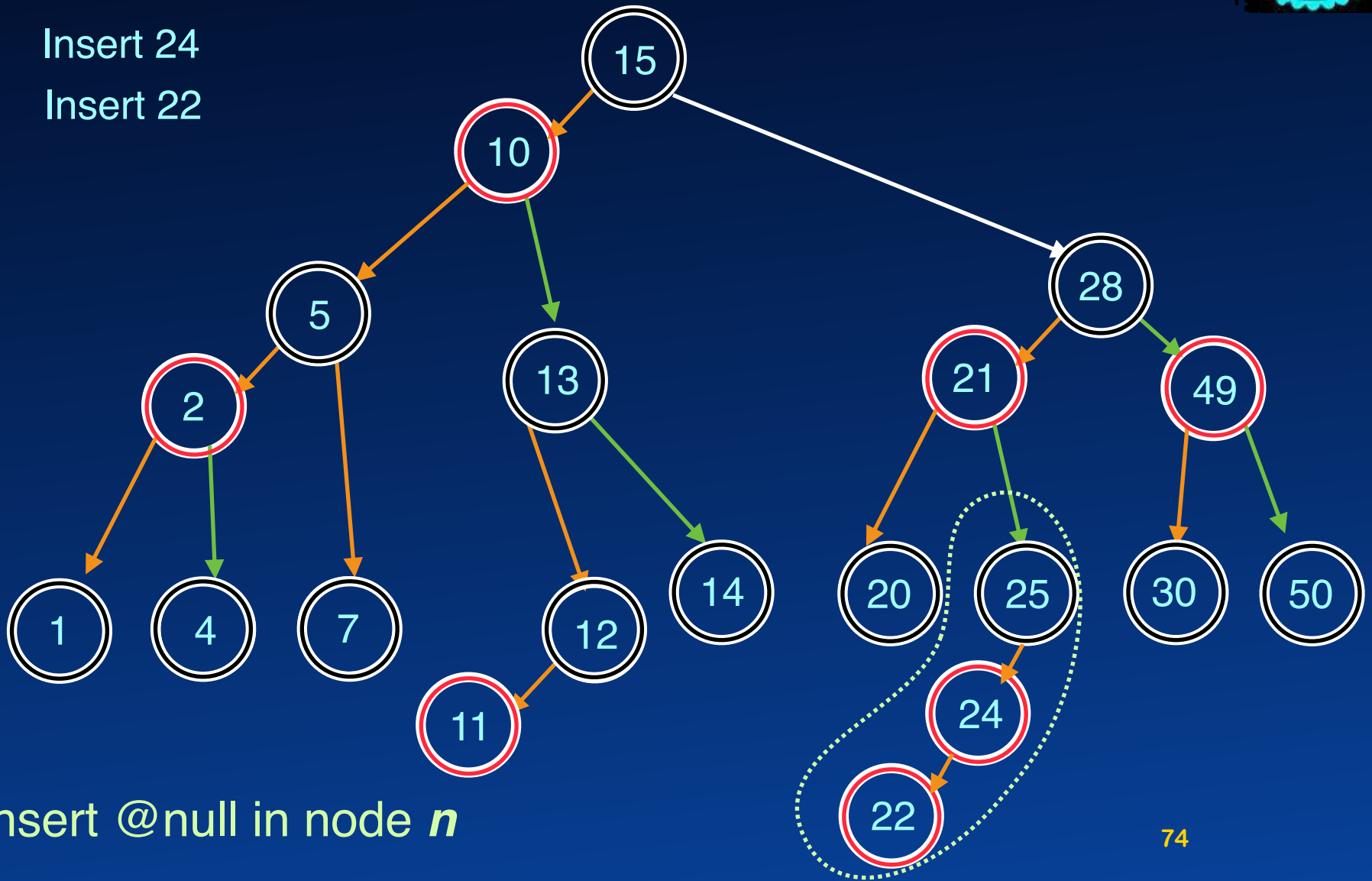

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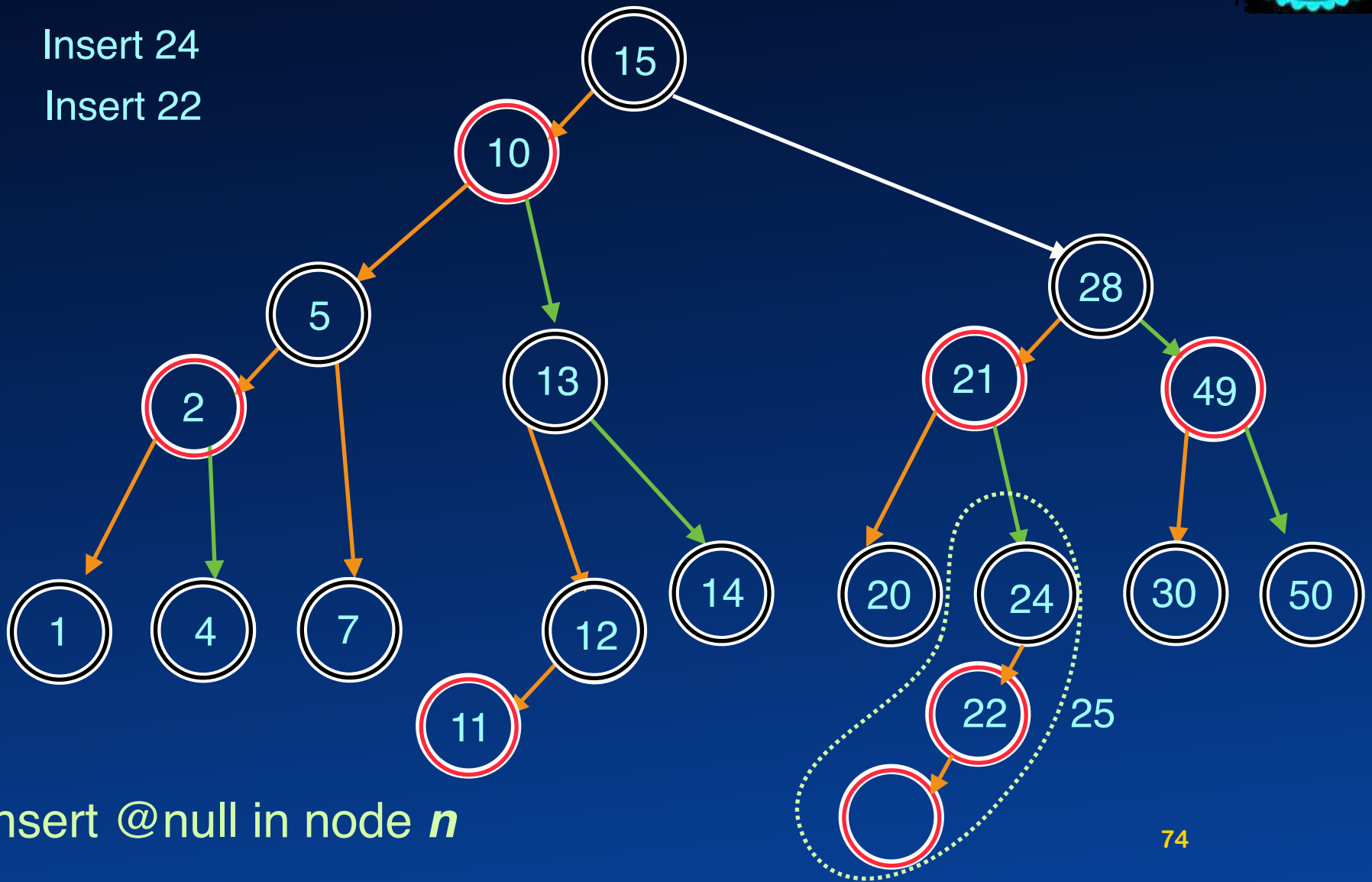

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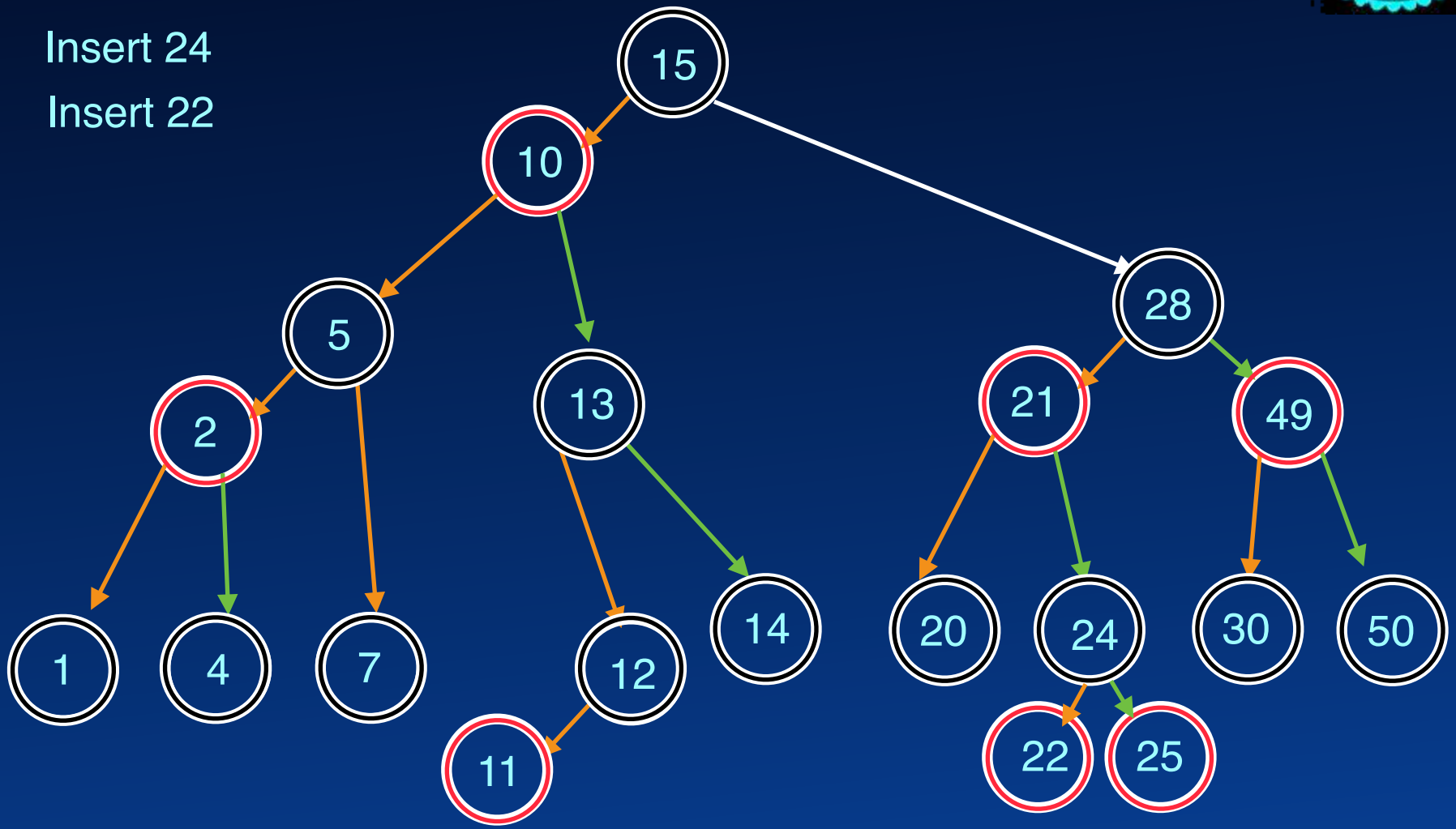

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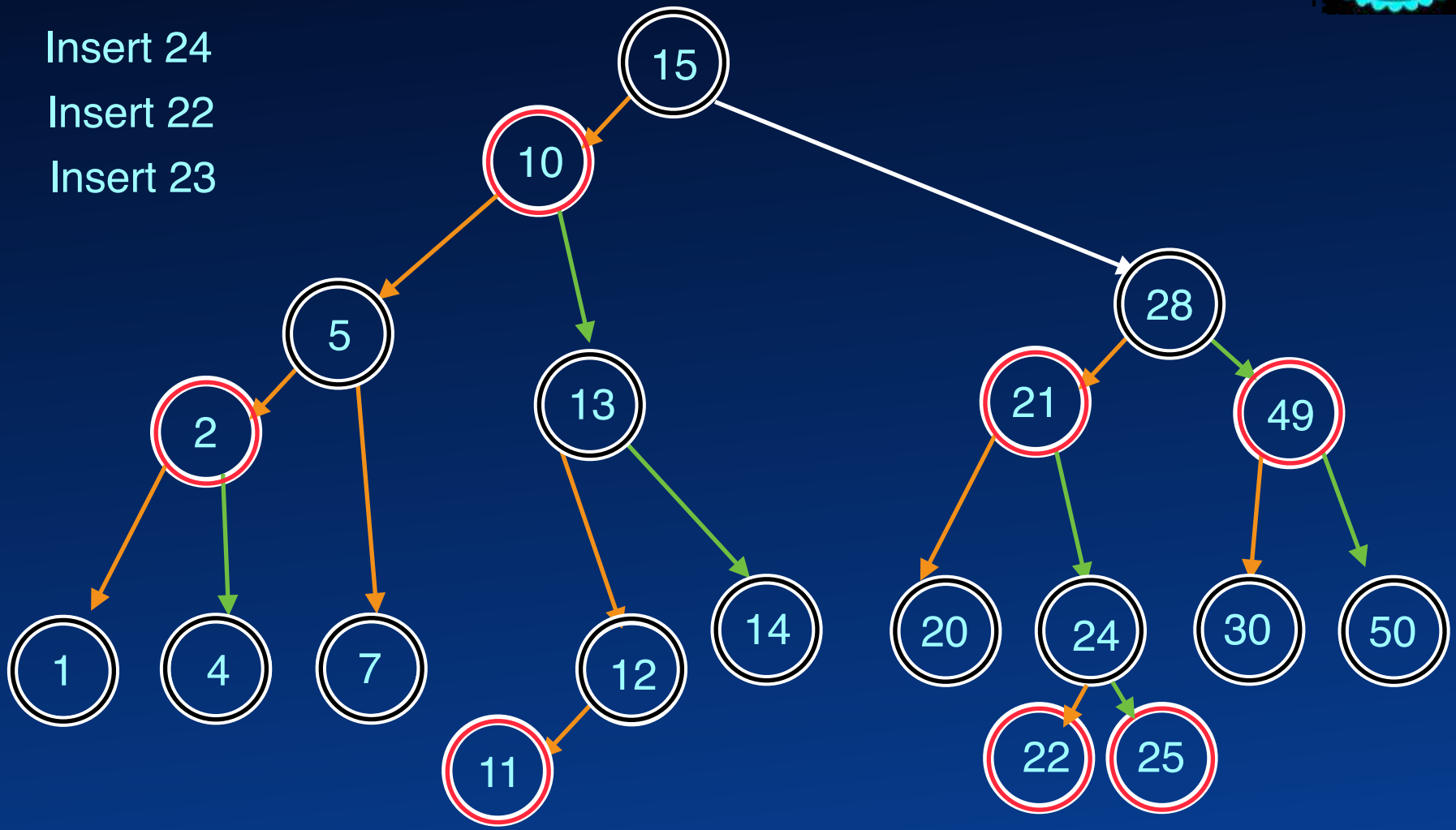

Insert @null in node ***n***

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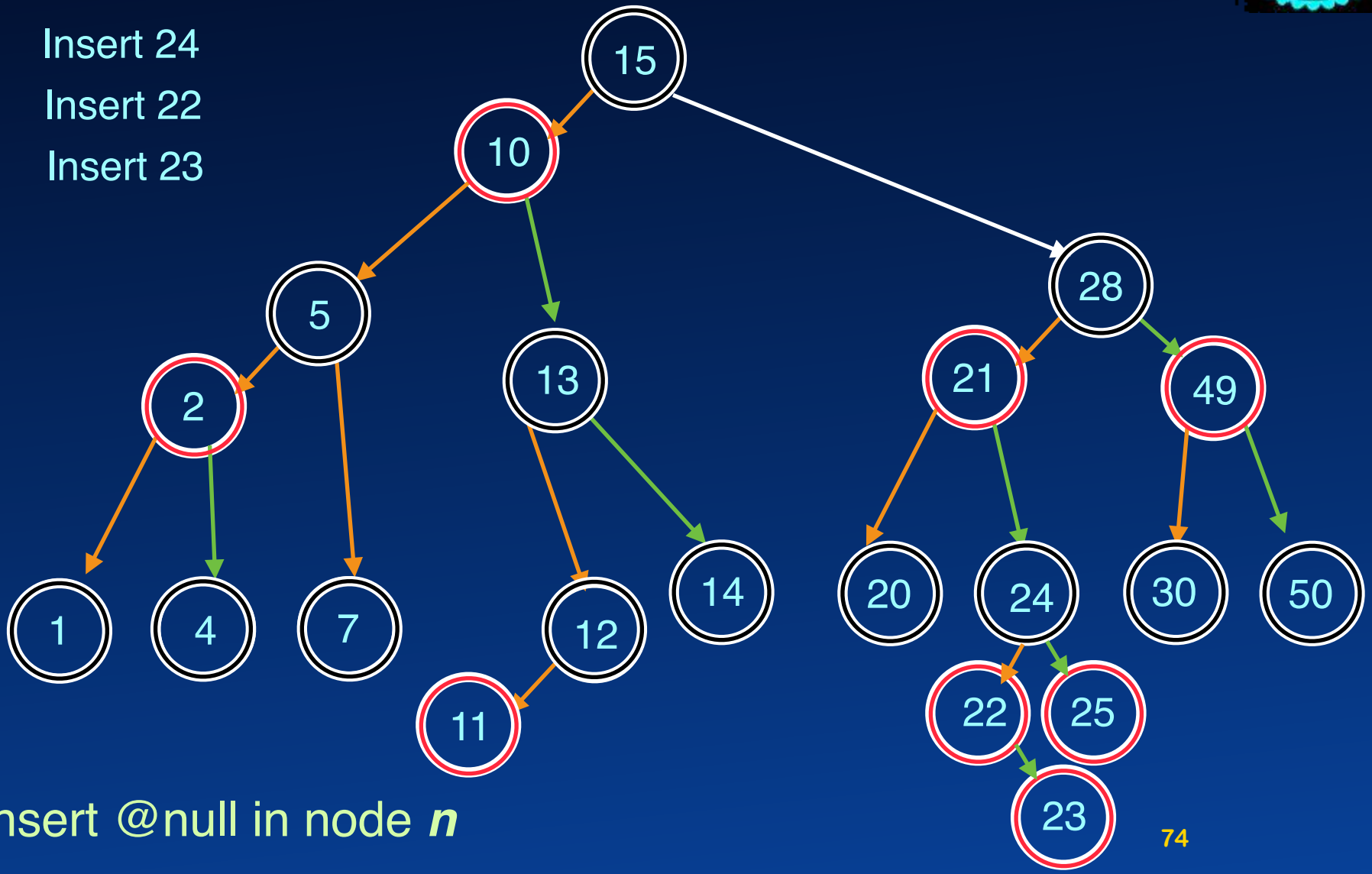

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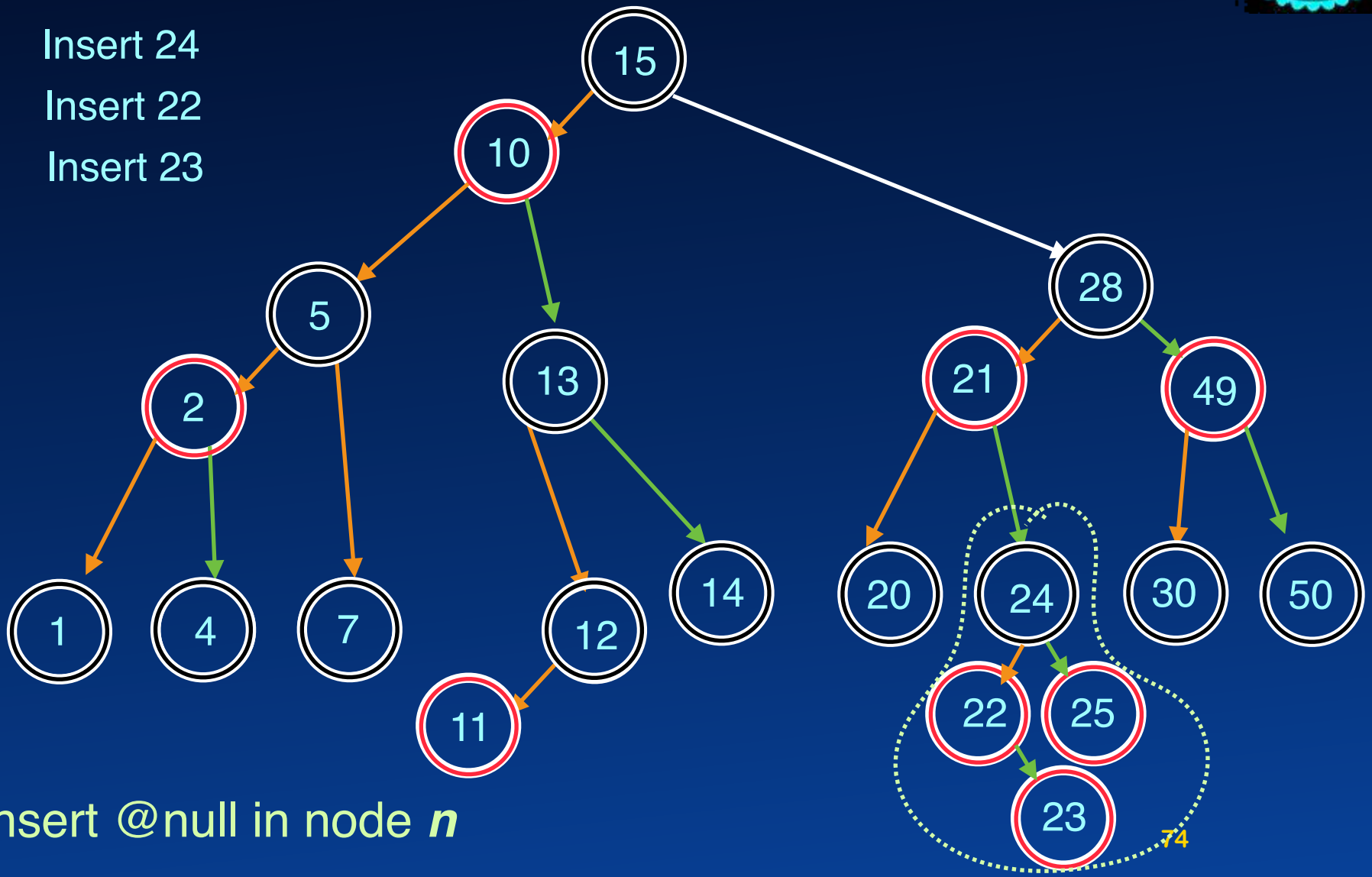

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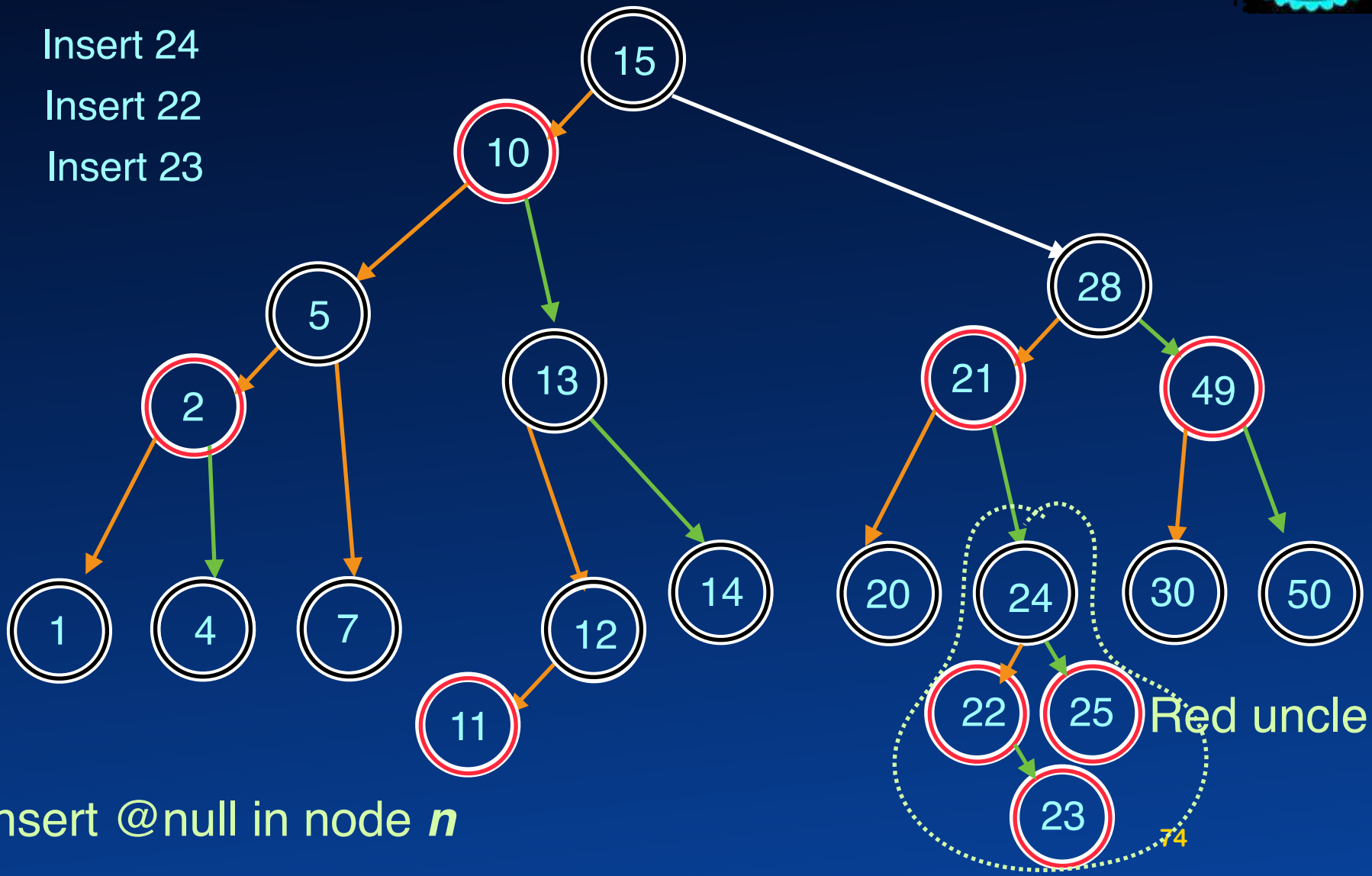

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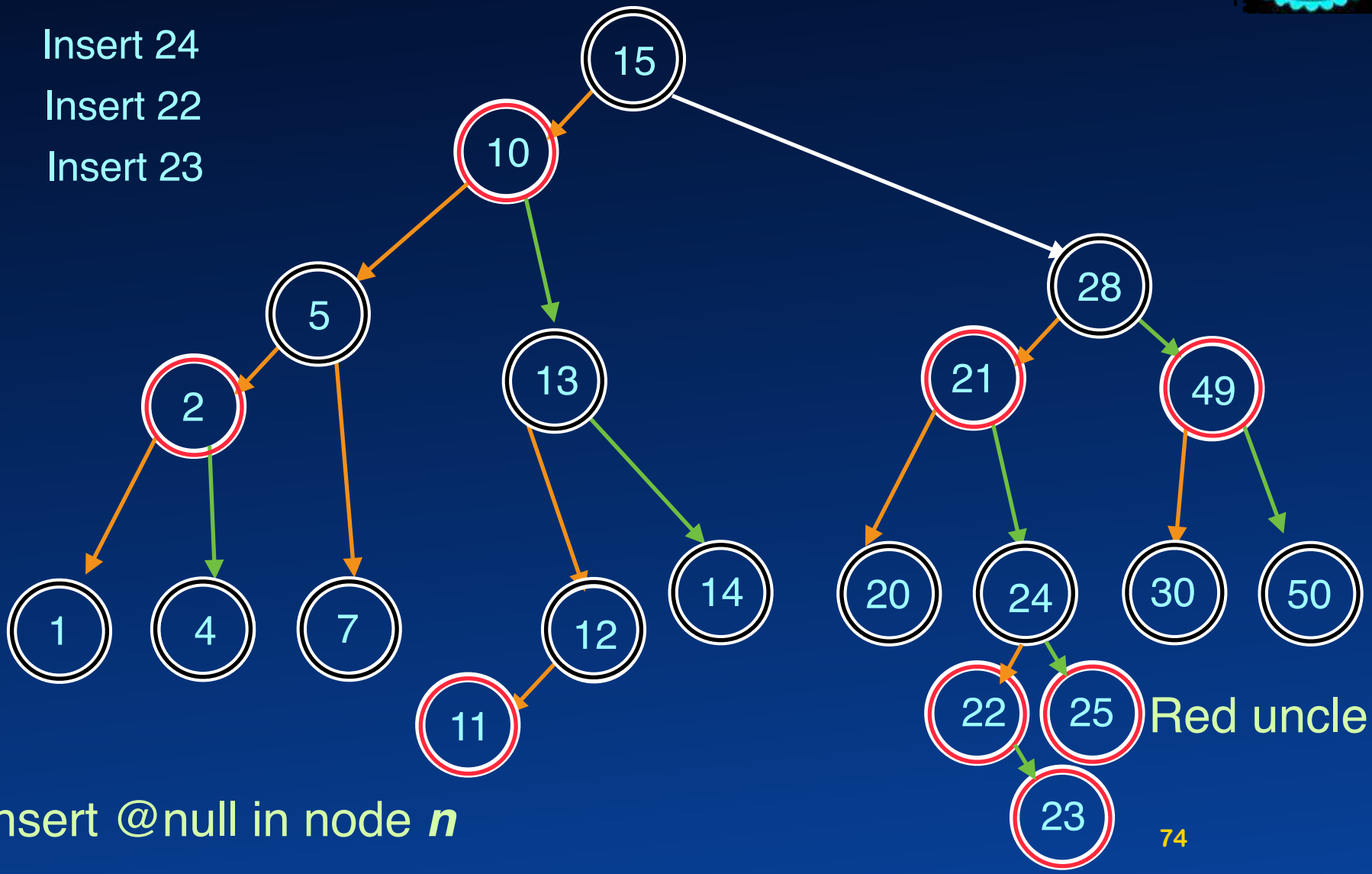

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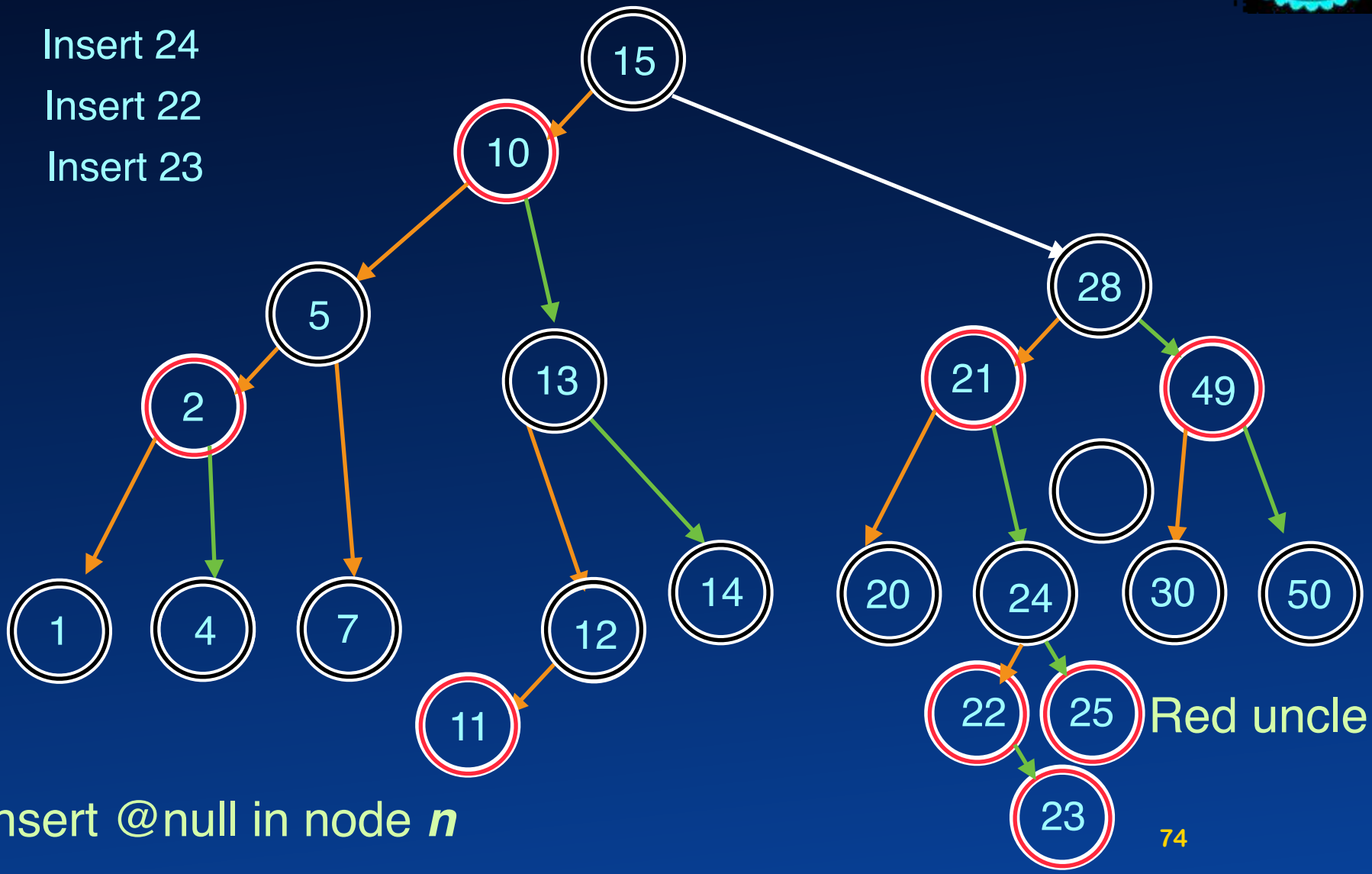

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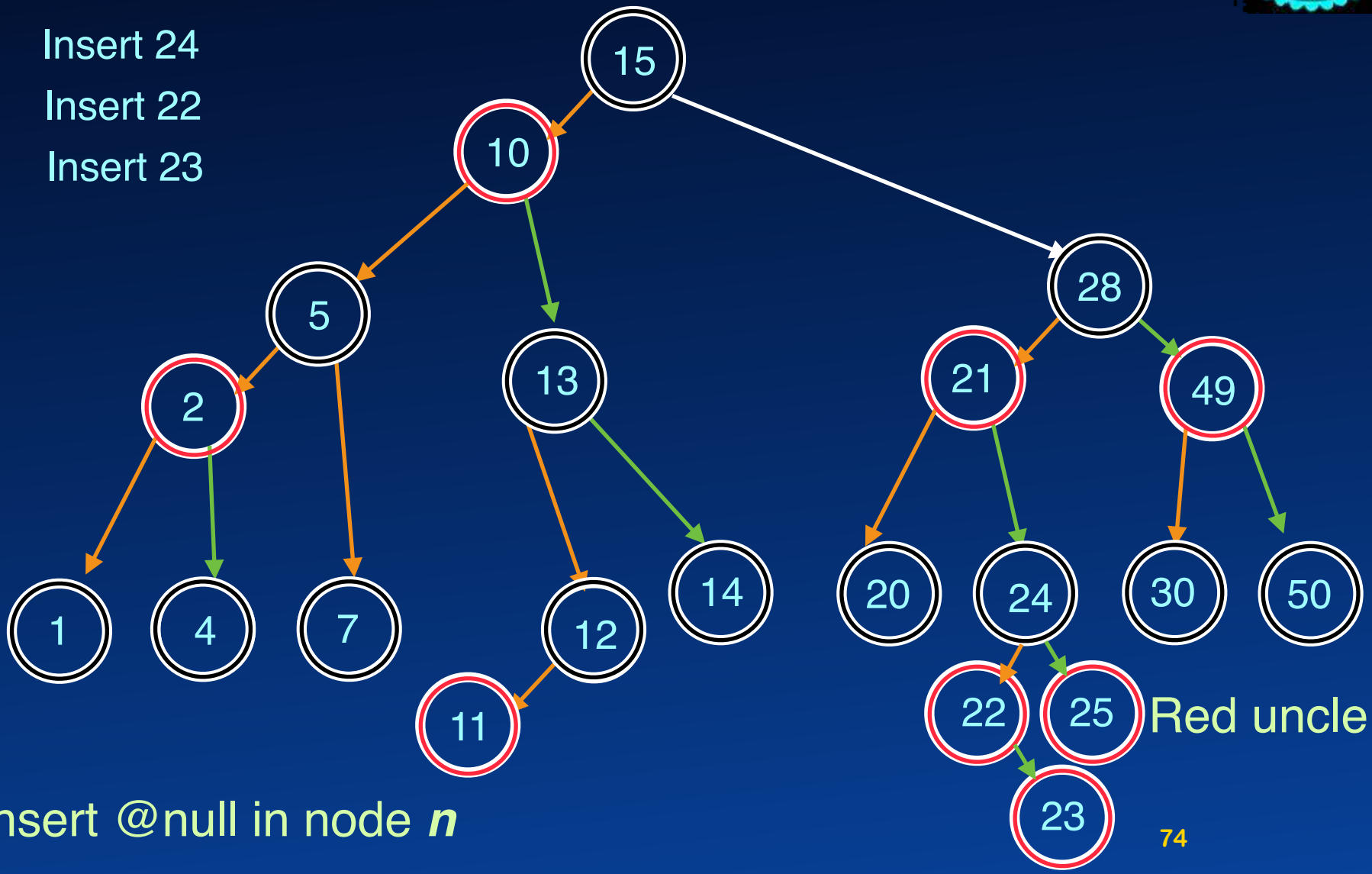
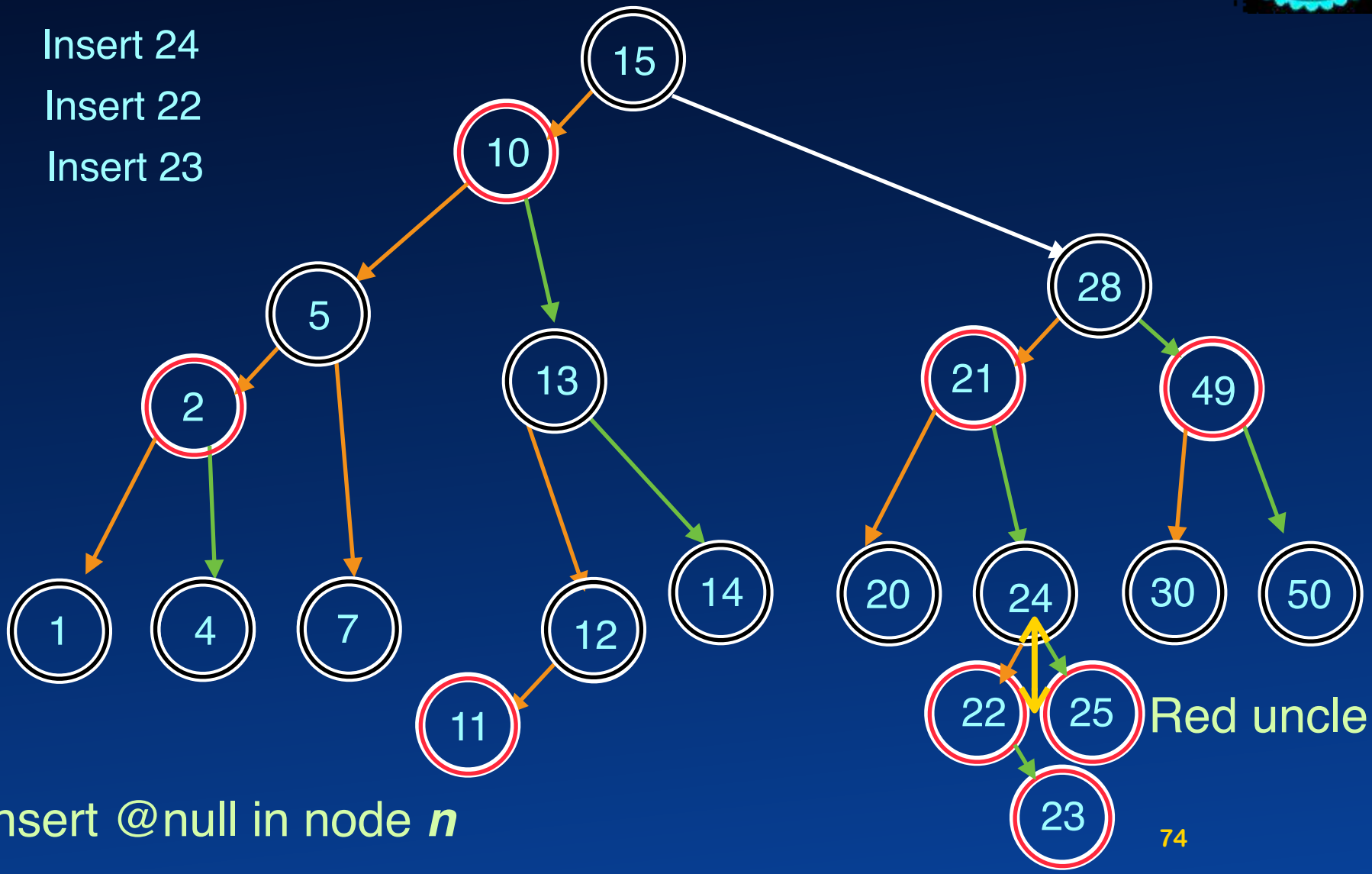


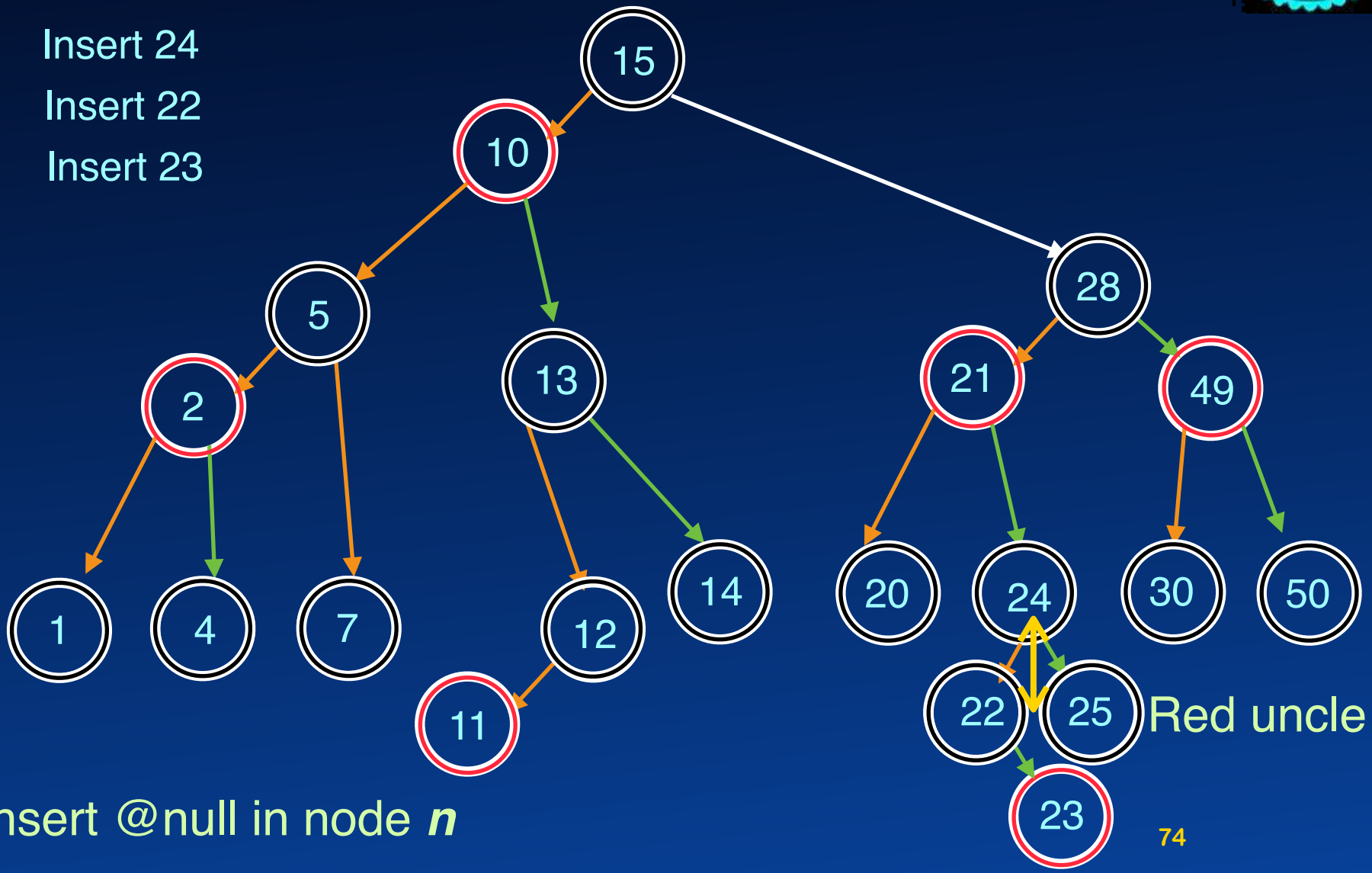
Insert @null in node ***n***

Red-Black Tree Insert



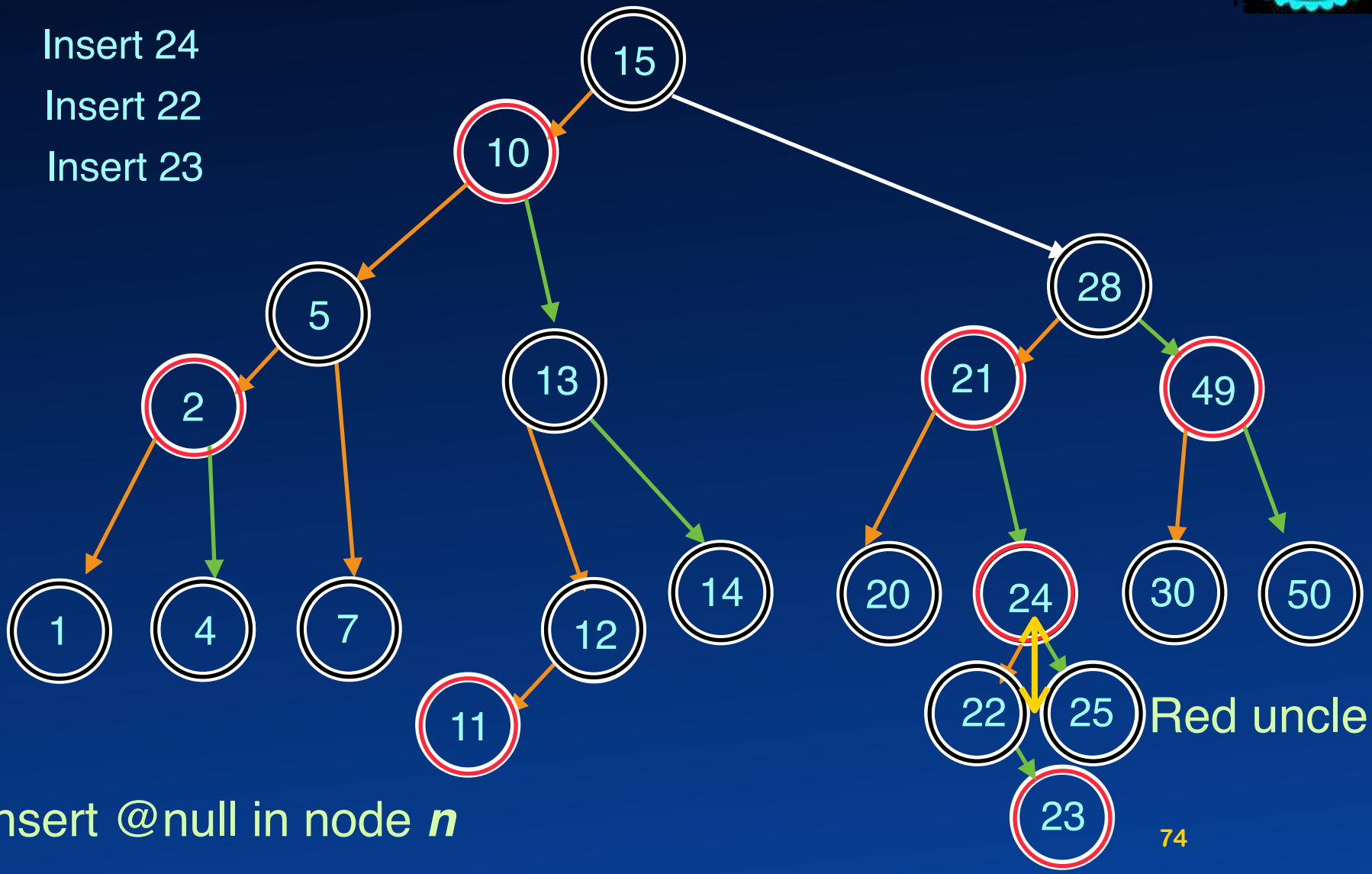
Insert @null in node ***n***

Red-Black Tree Insert



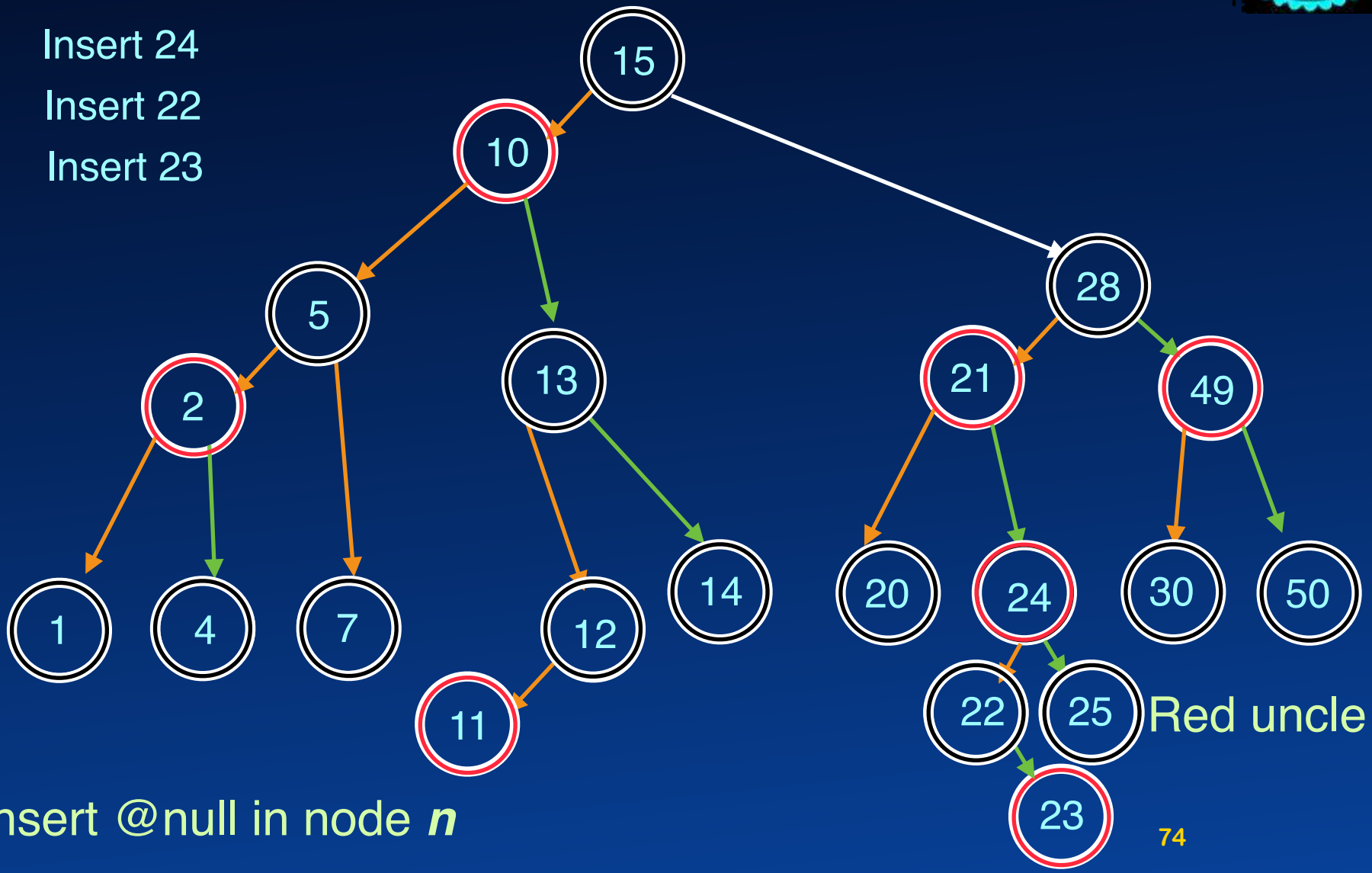
Insert @null in node ***n***

Red-Black Tree Insert



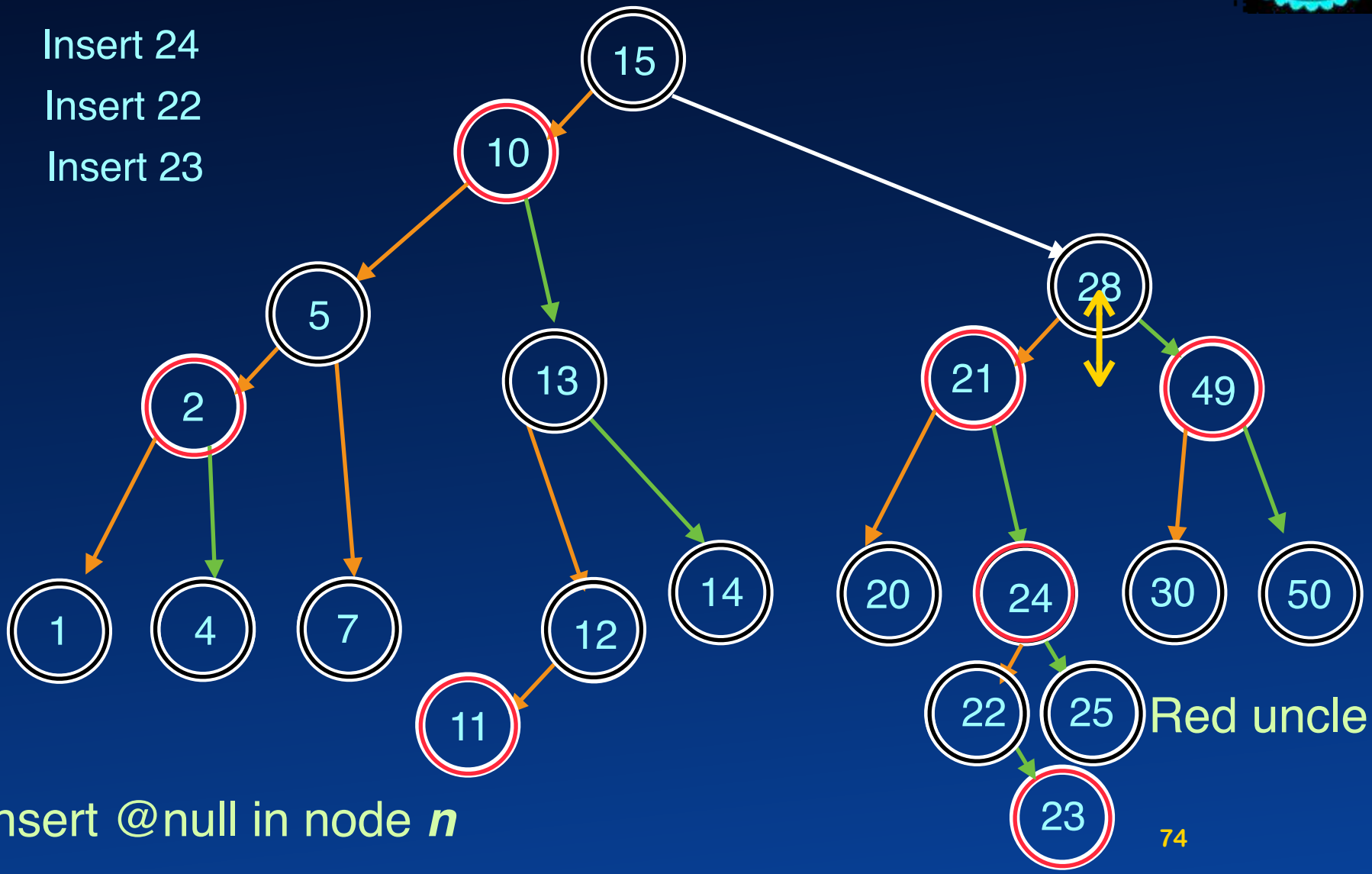
Insert @null in node ***n***

Red-Black Tree Insert



Insert @null in node ***n***

Red-Black Tree Insert



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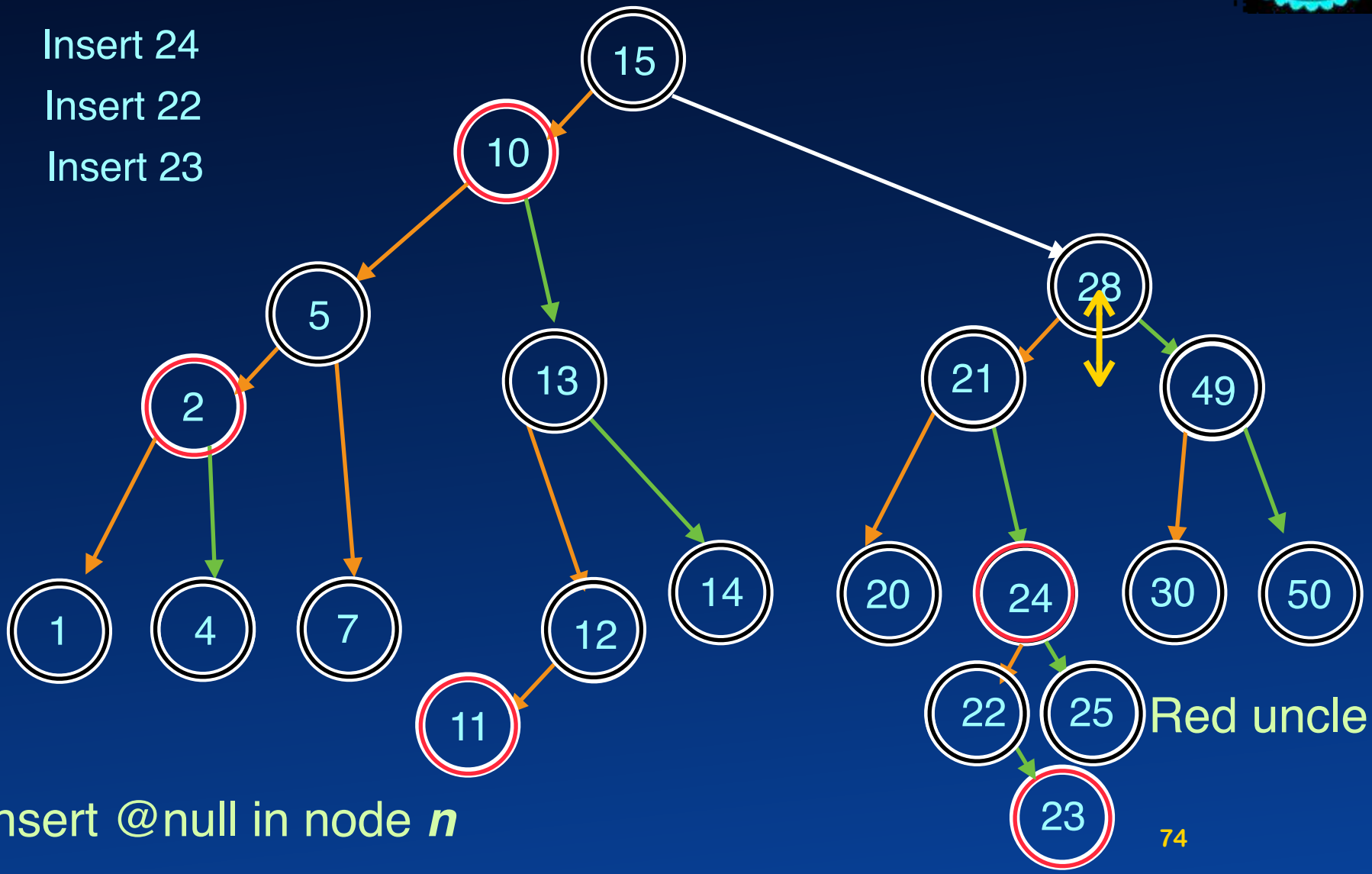
Red-Black Tree Insert



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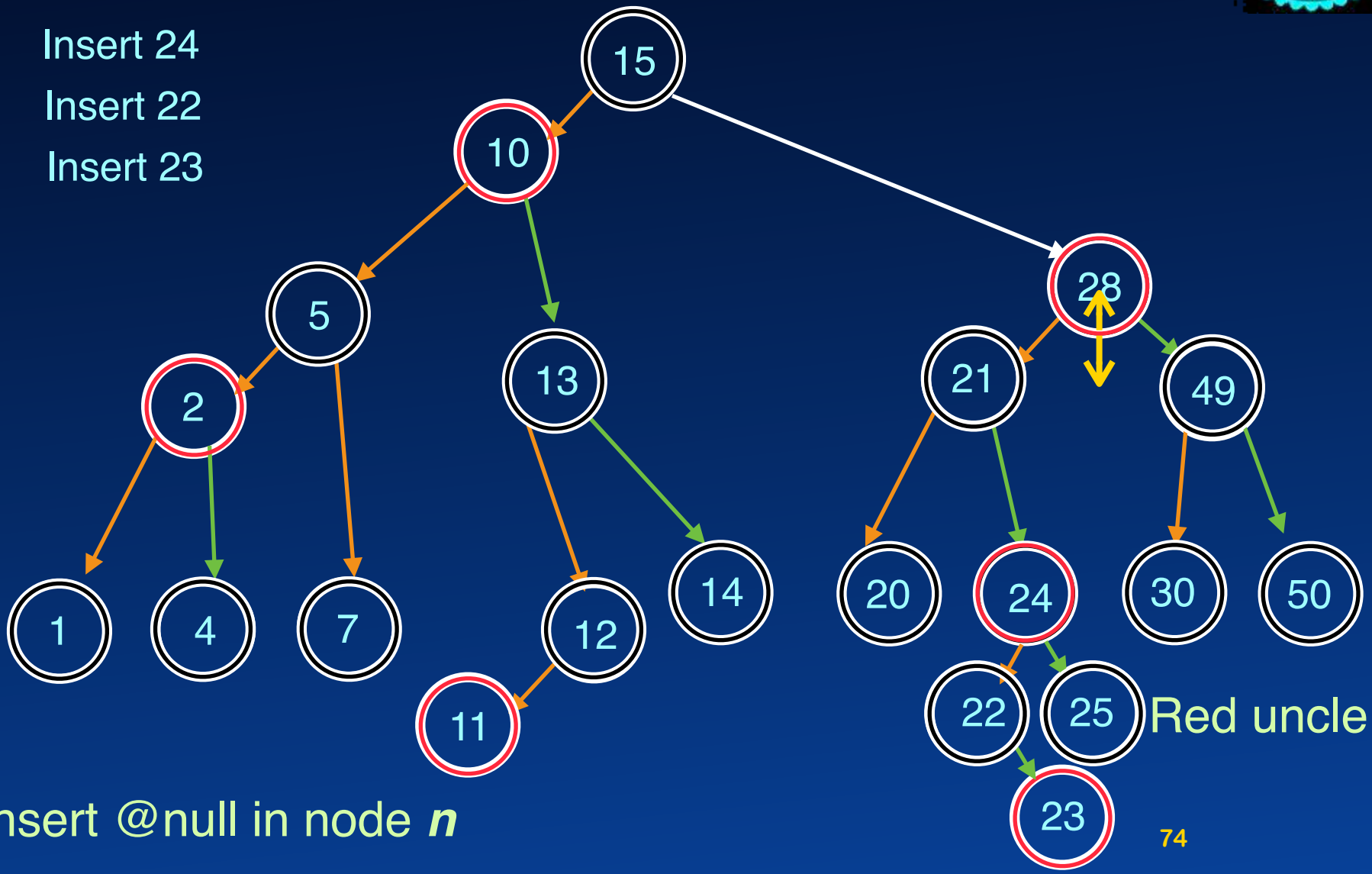
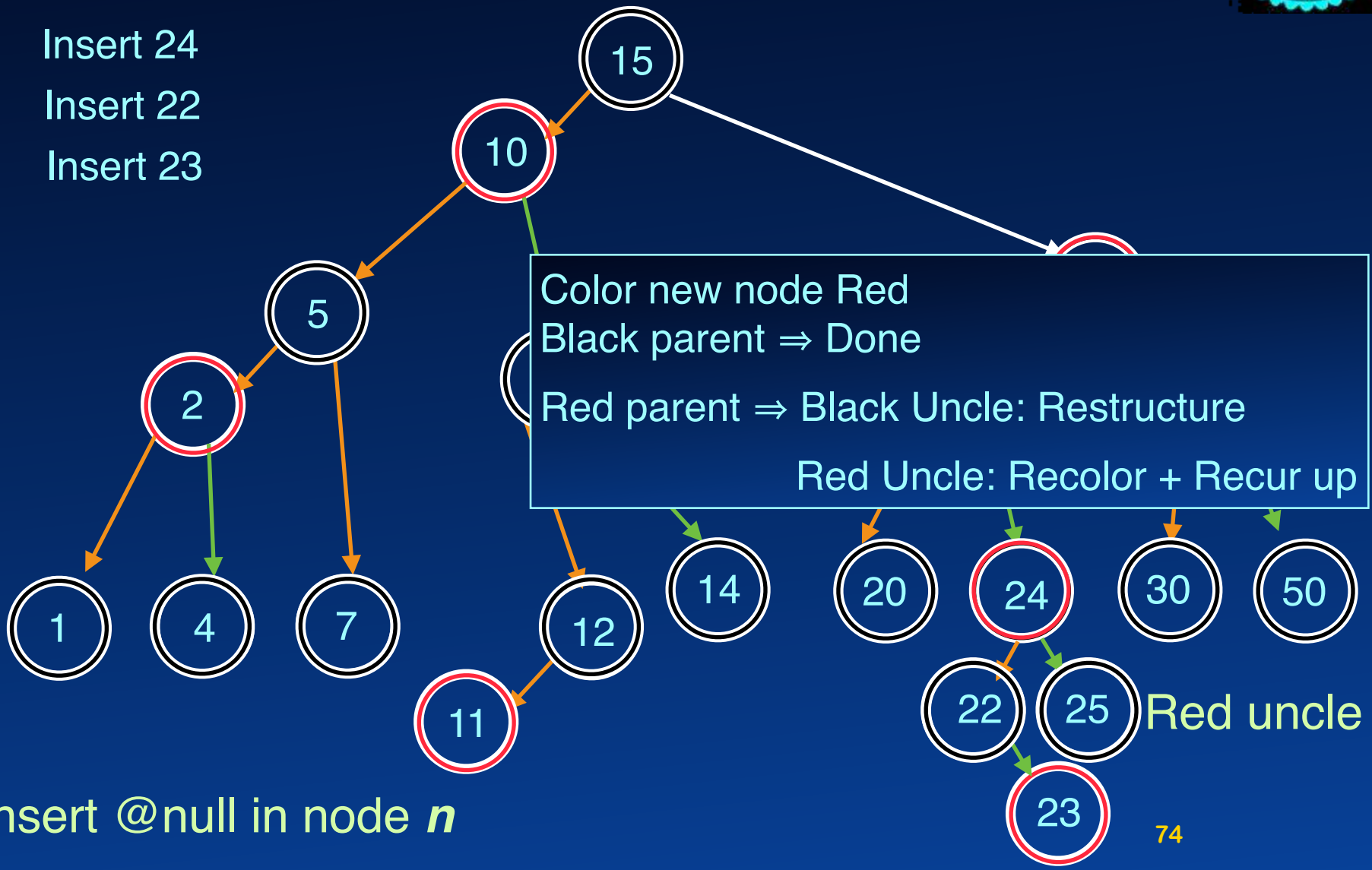
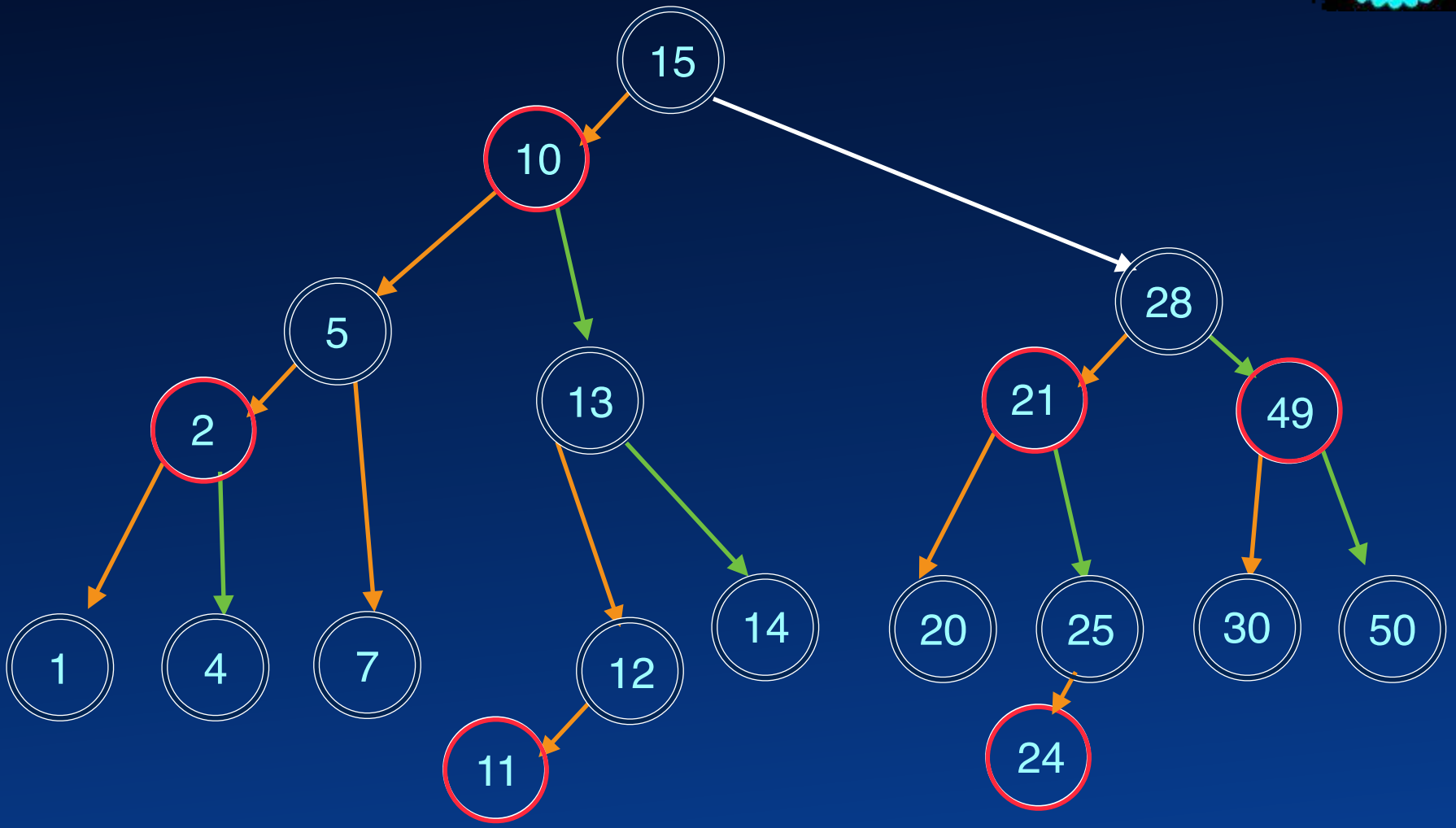


Insert @null in node ***n***

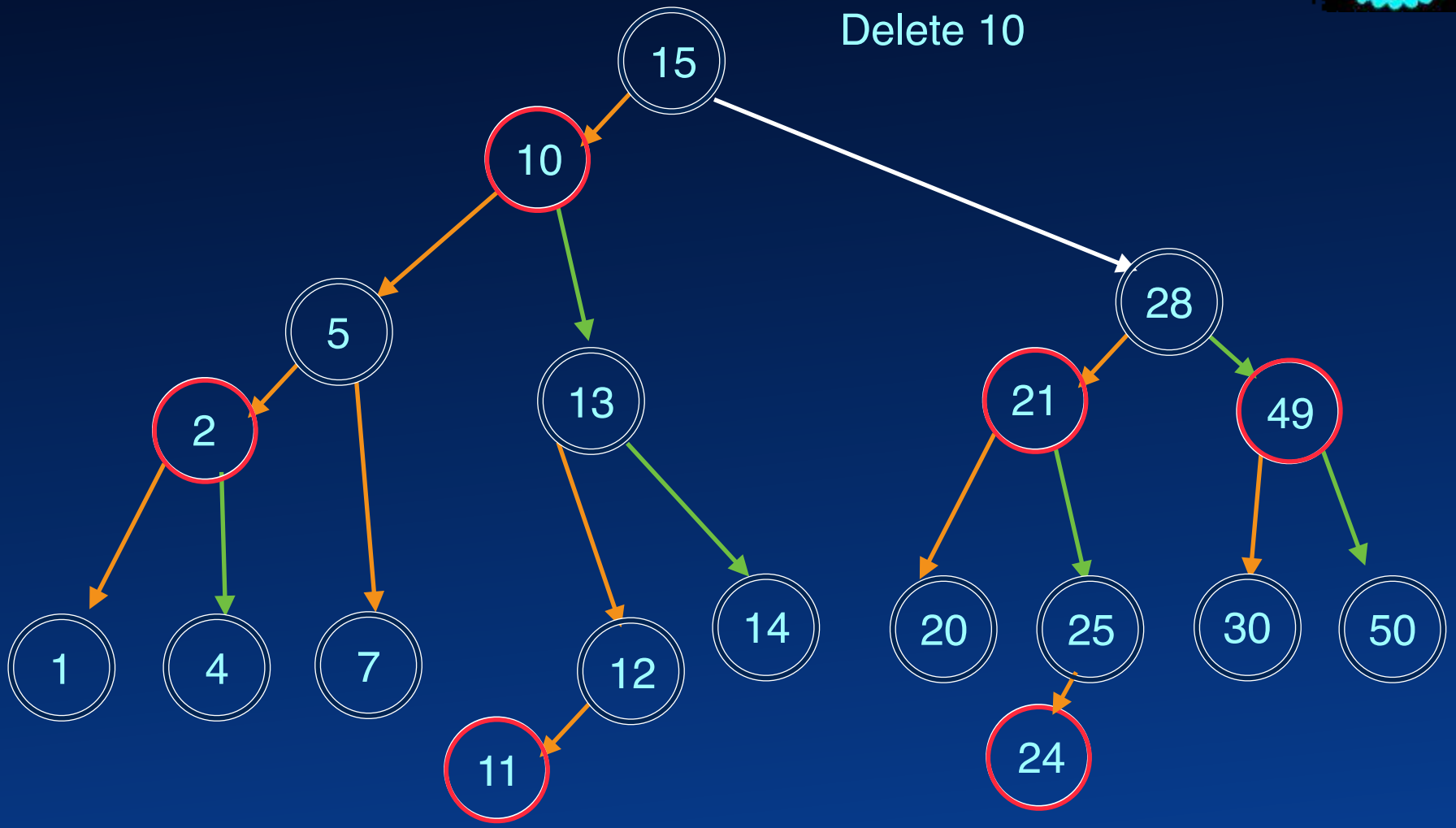
Red-Black Tree Insert



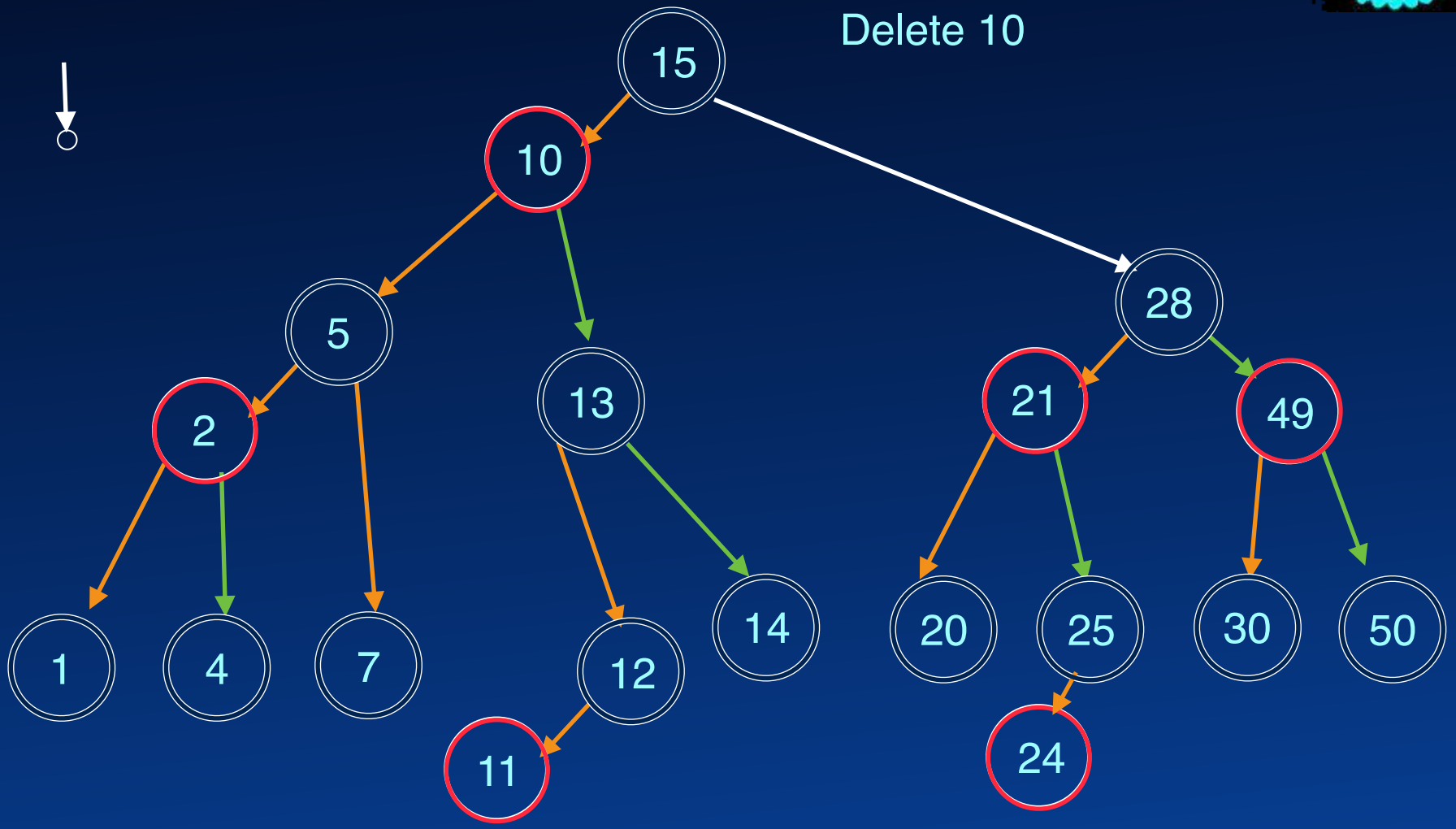
R-B Tree Deletion



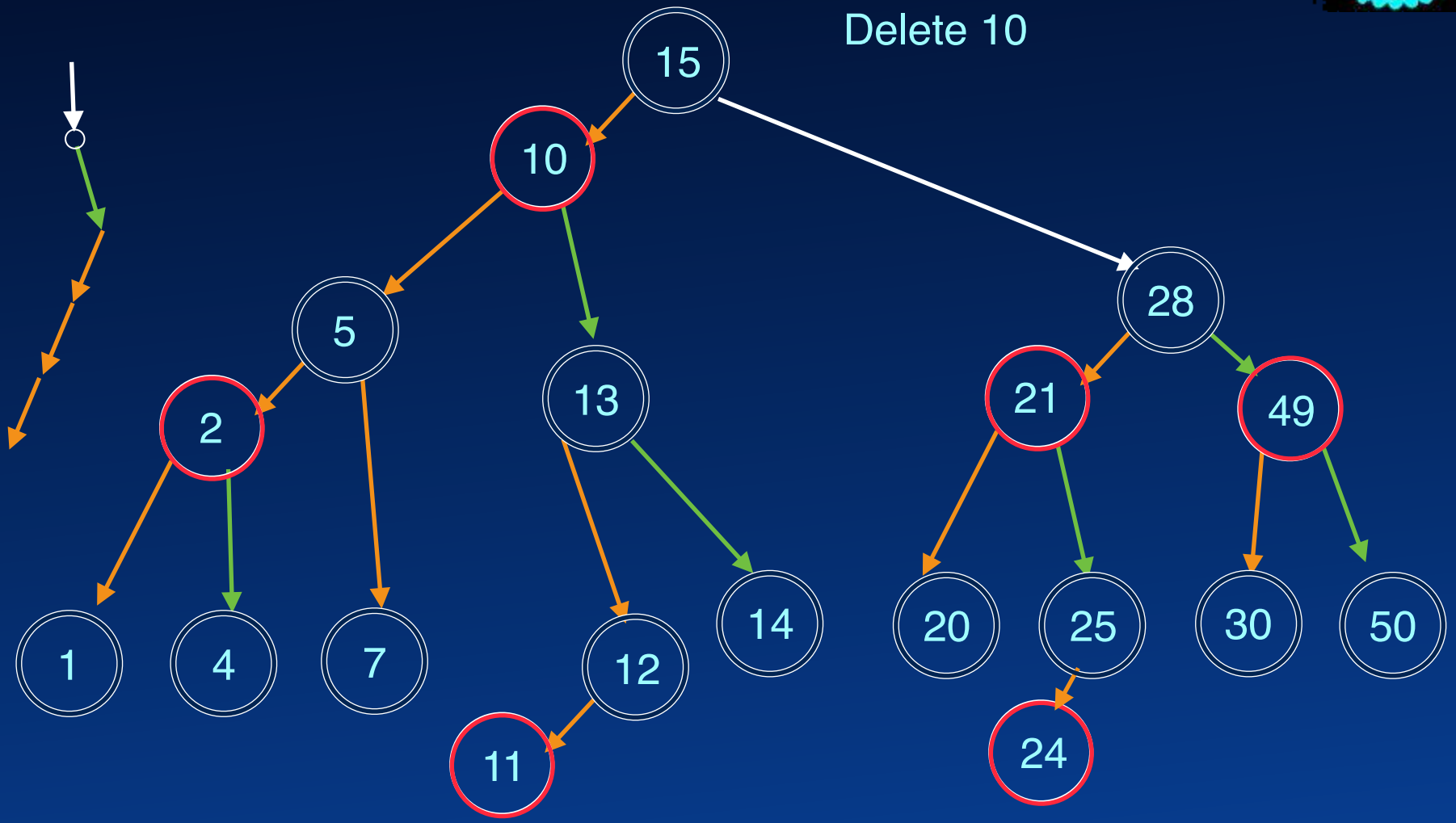
R-B Tree Deletion



R-B Tree Deletion



R-B Tree Deletion



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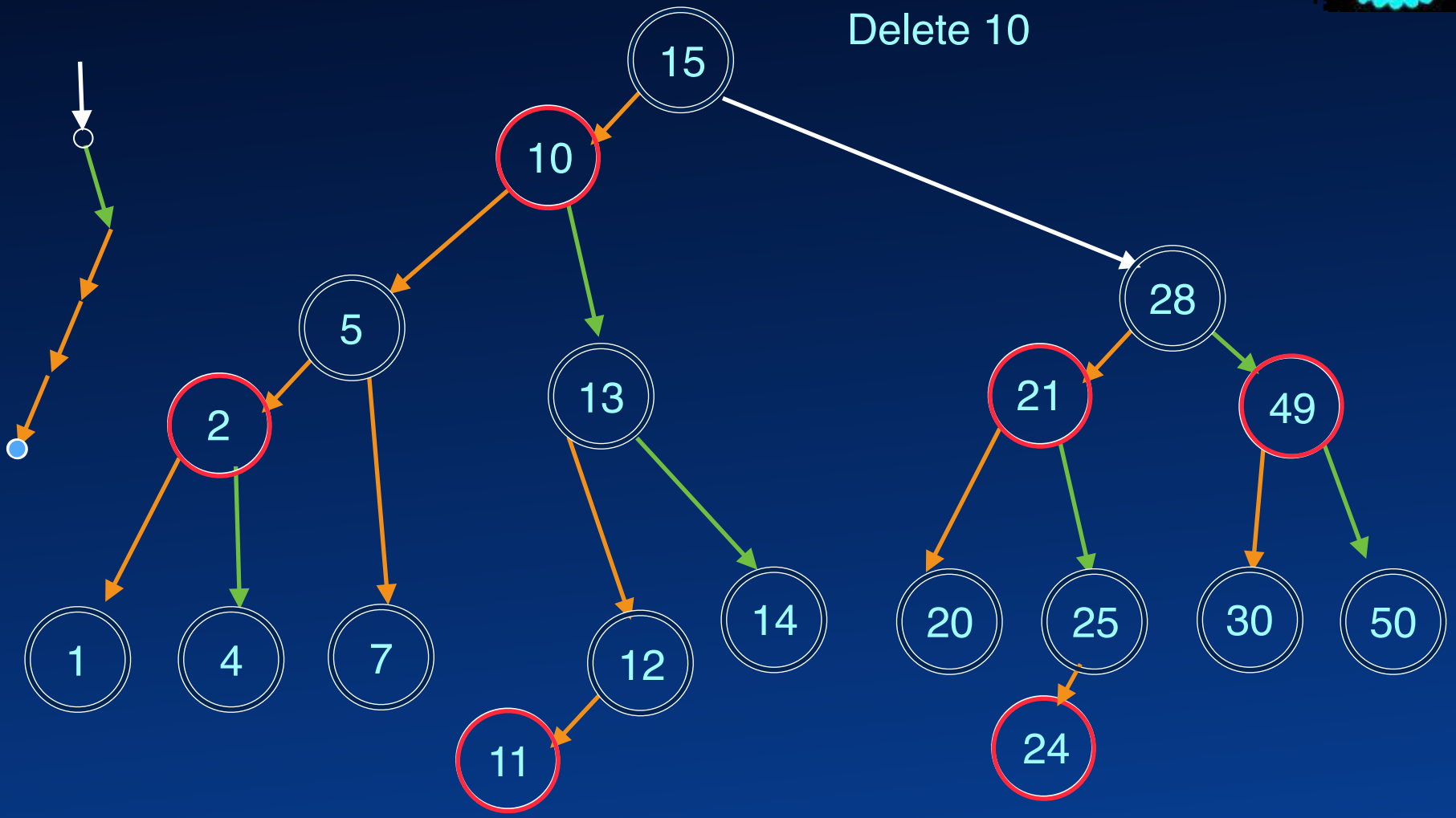
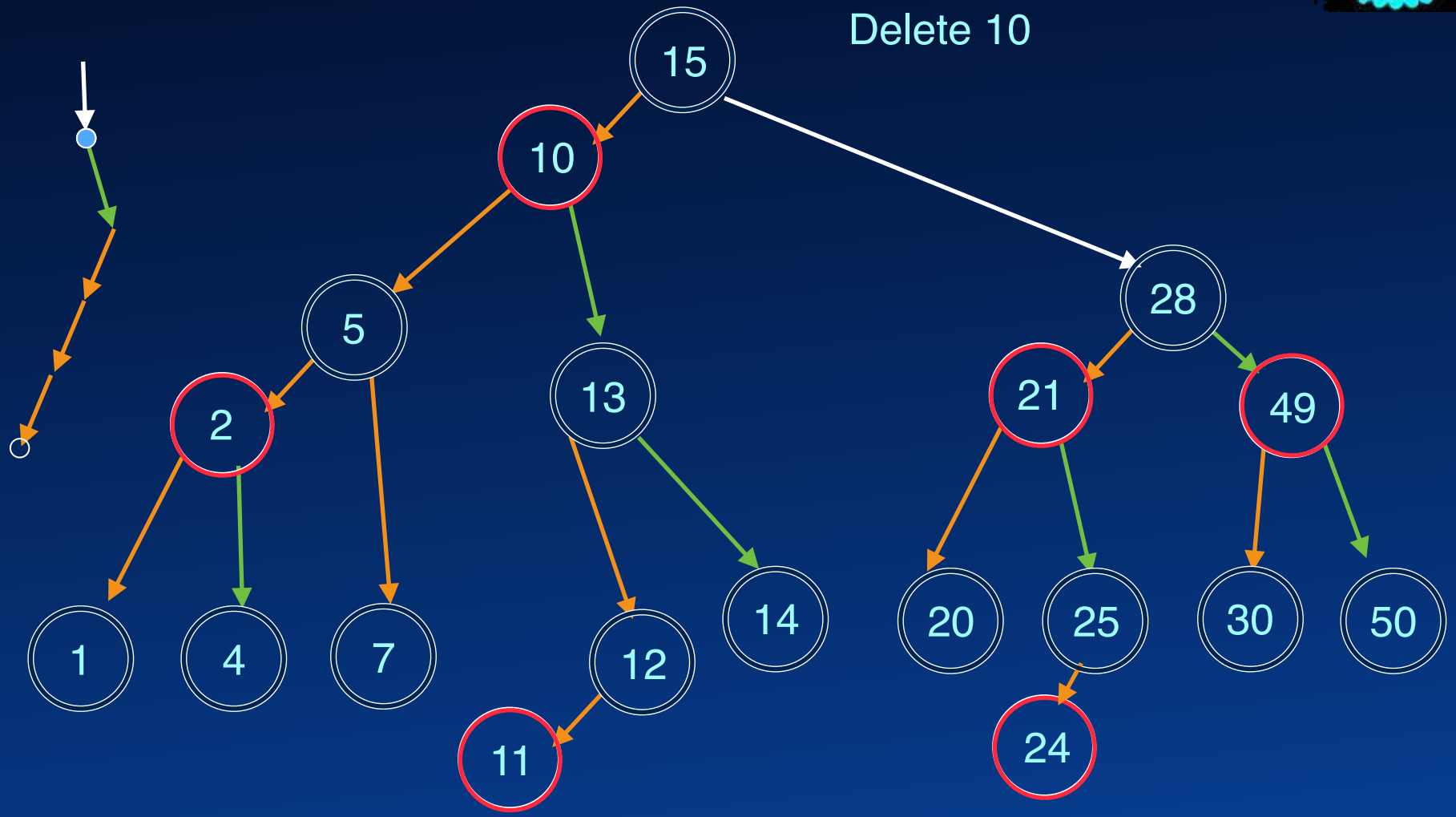
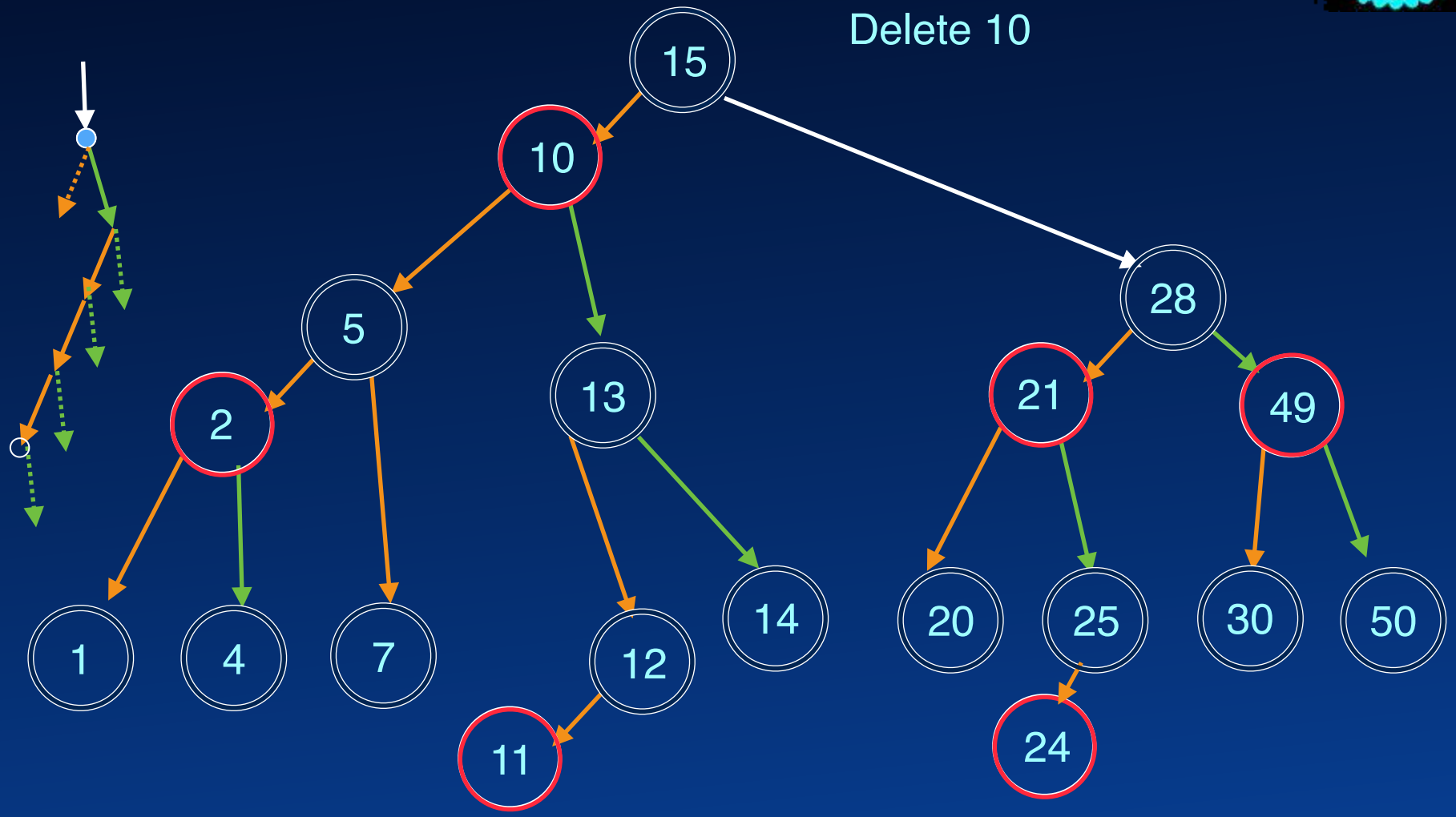


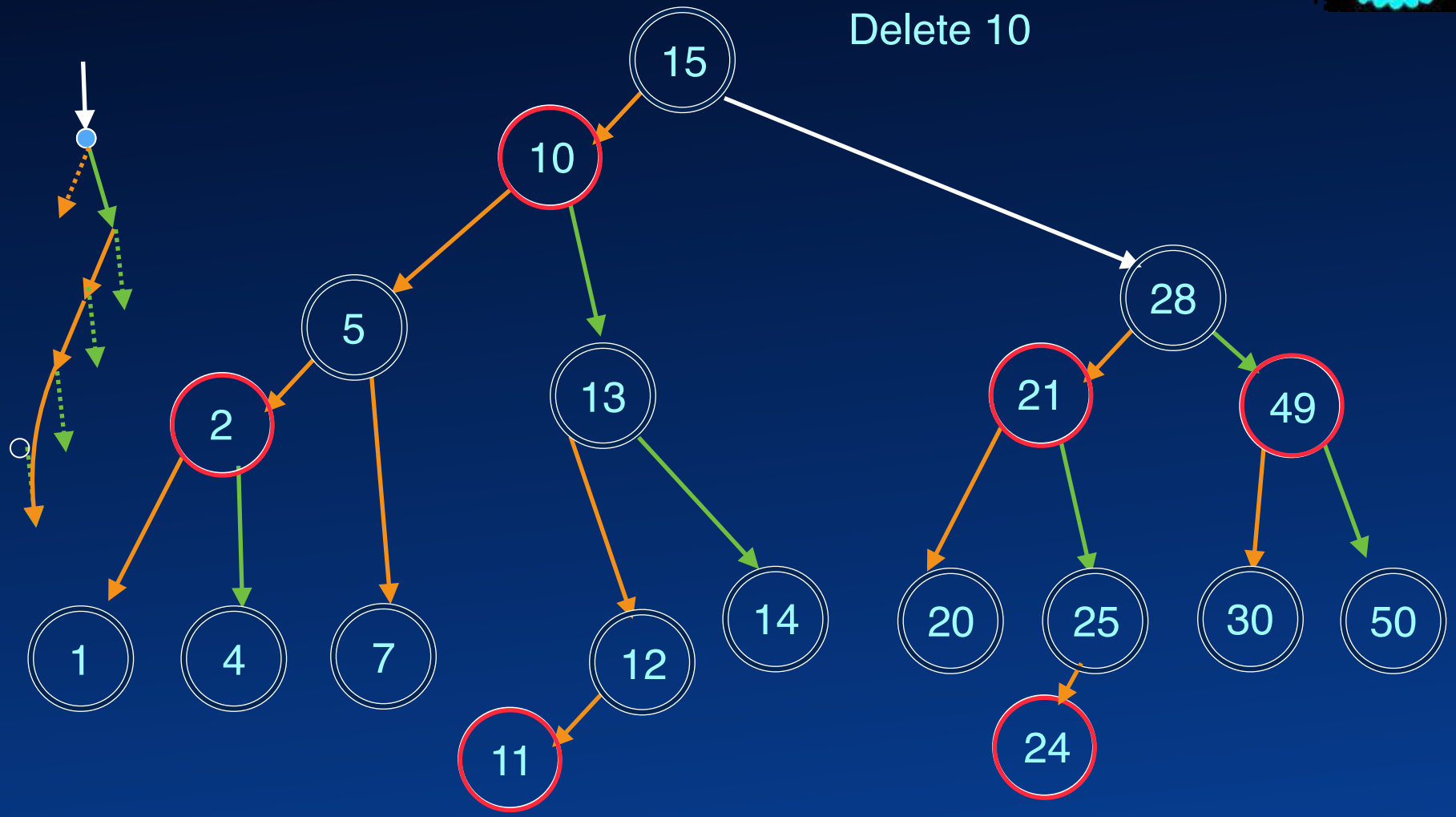
R-B Tree Deletion



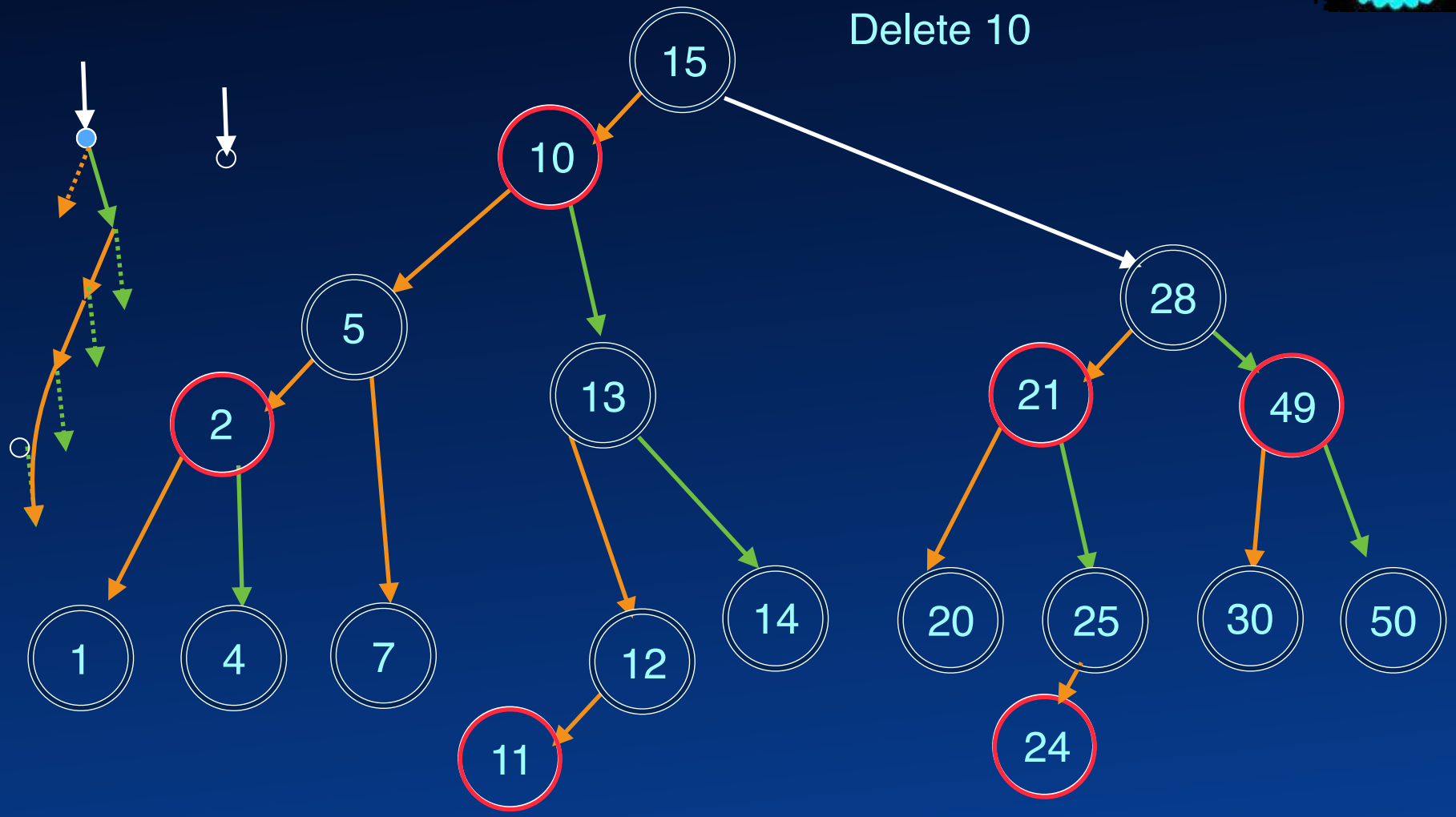
R-B Tree Deletion



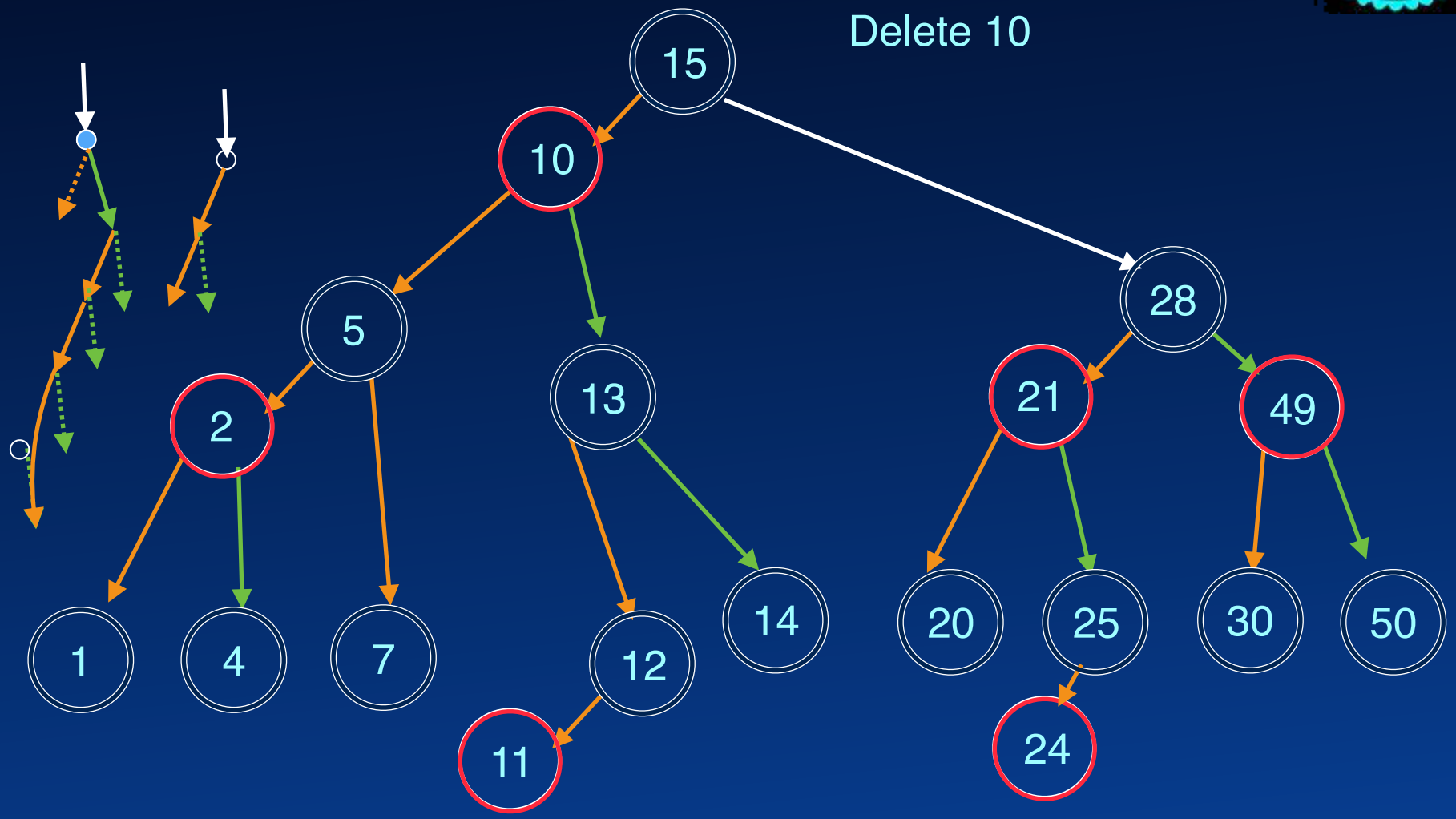
R-B Tree Deletion



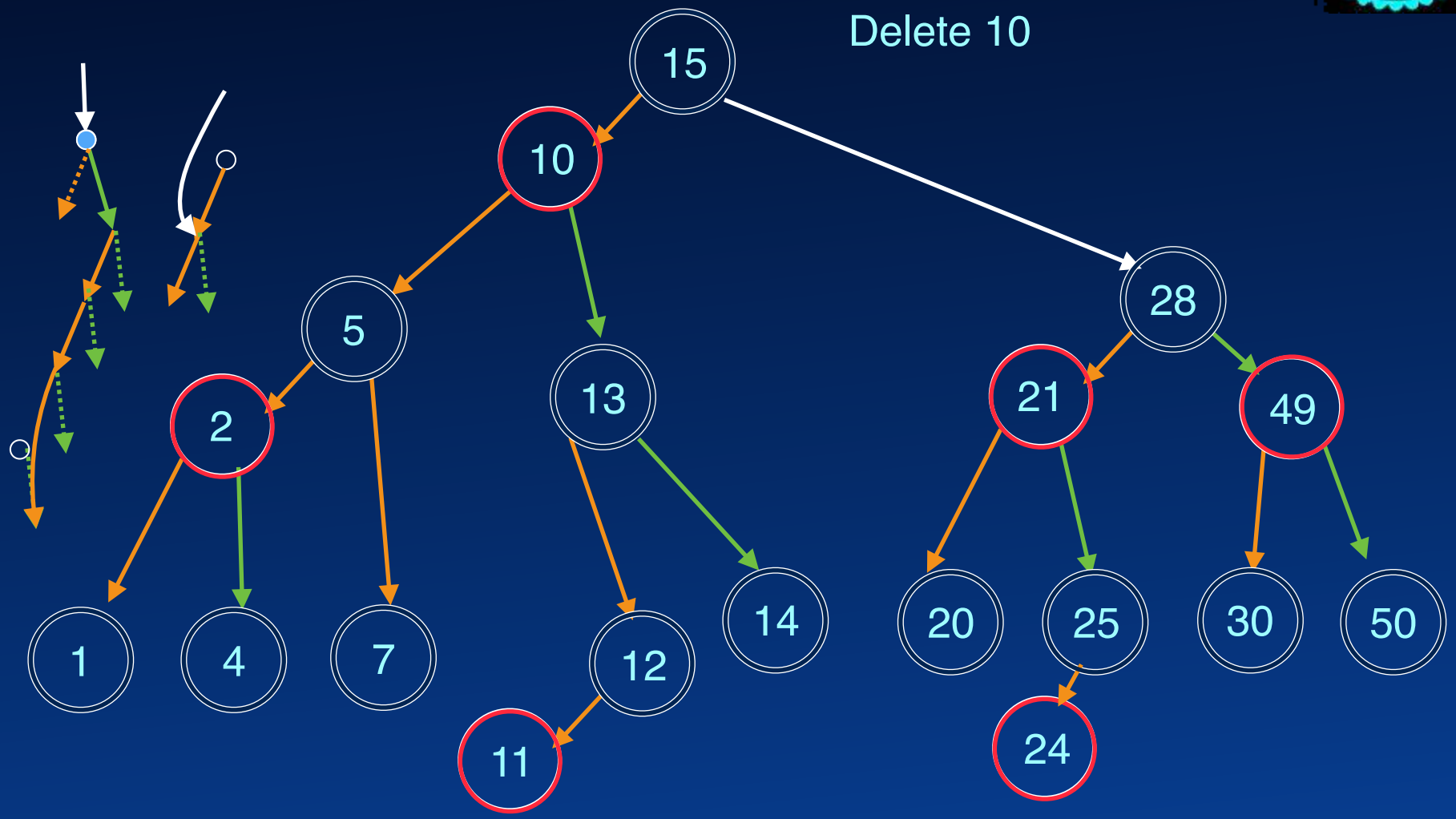
R-B Tree Deletion



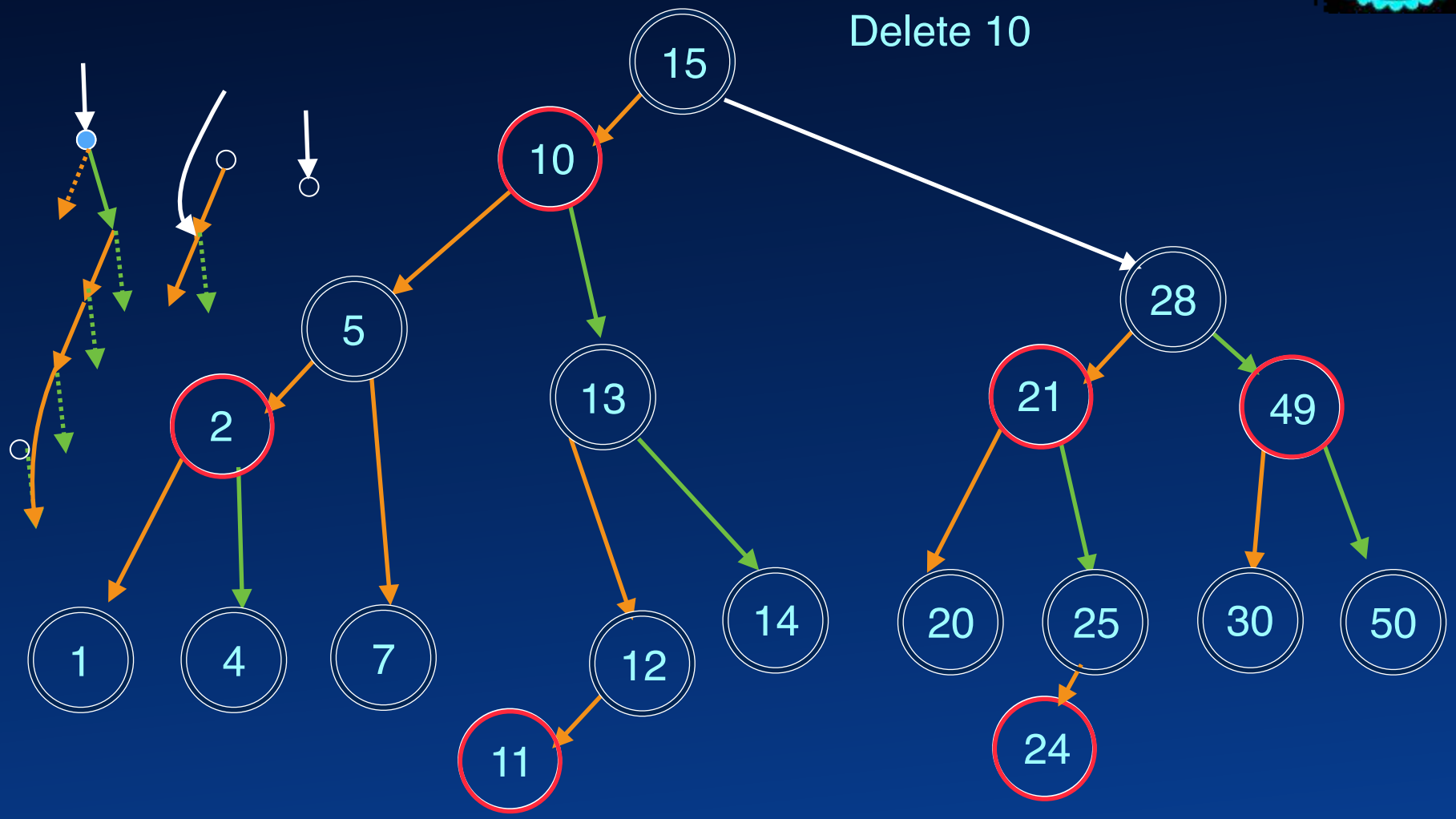
R-B Tree Deletion



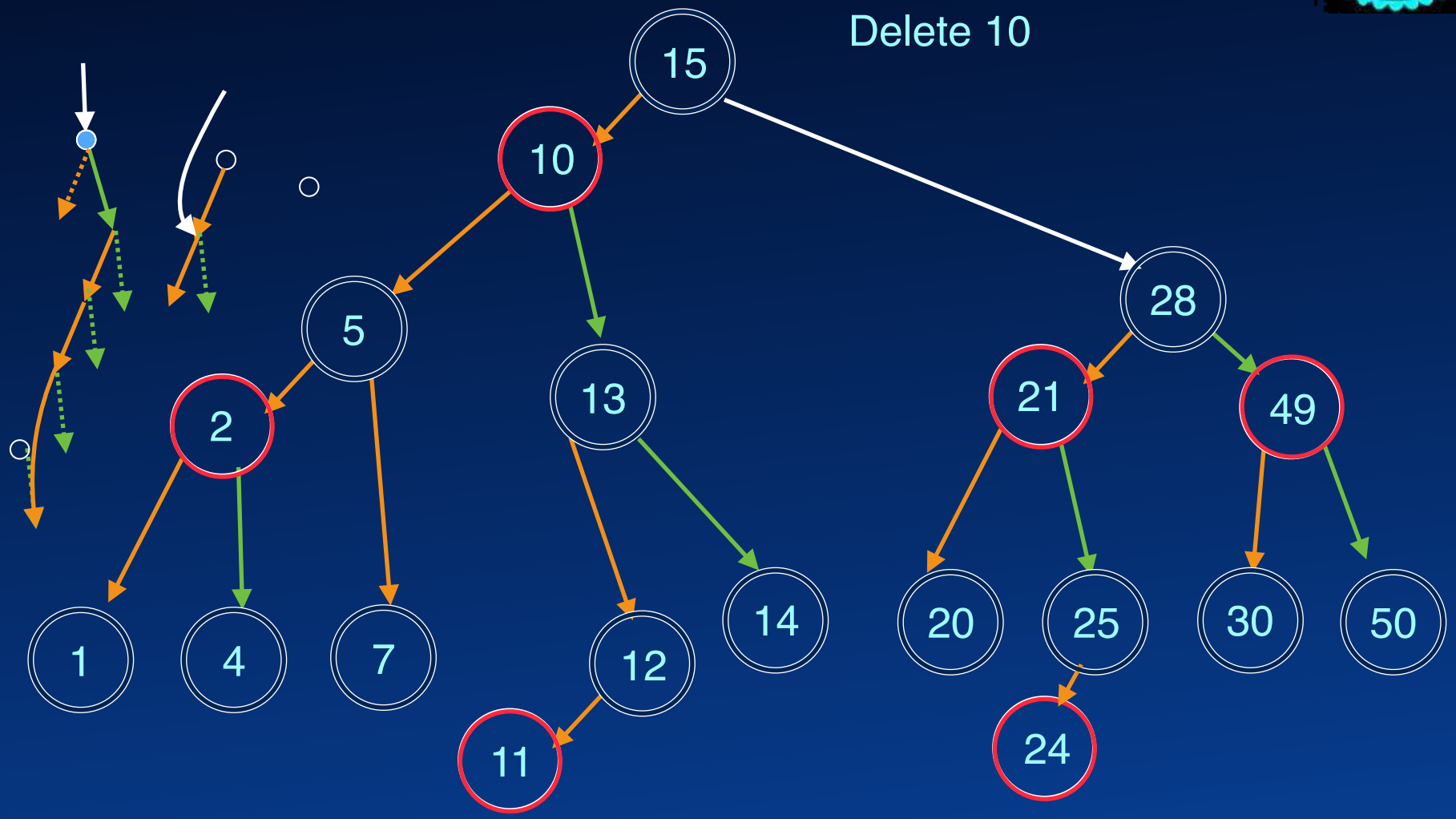
R-B Tree Deletion



R-B Tree Deletion



R-B Tree Deletion



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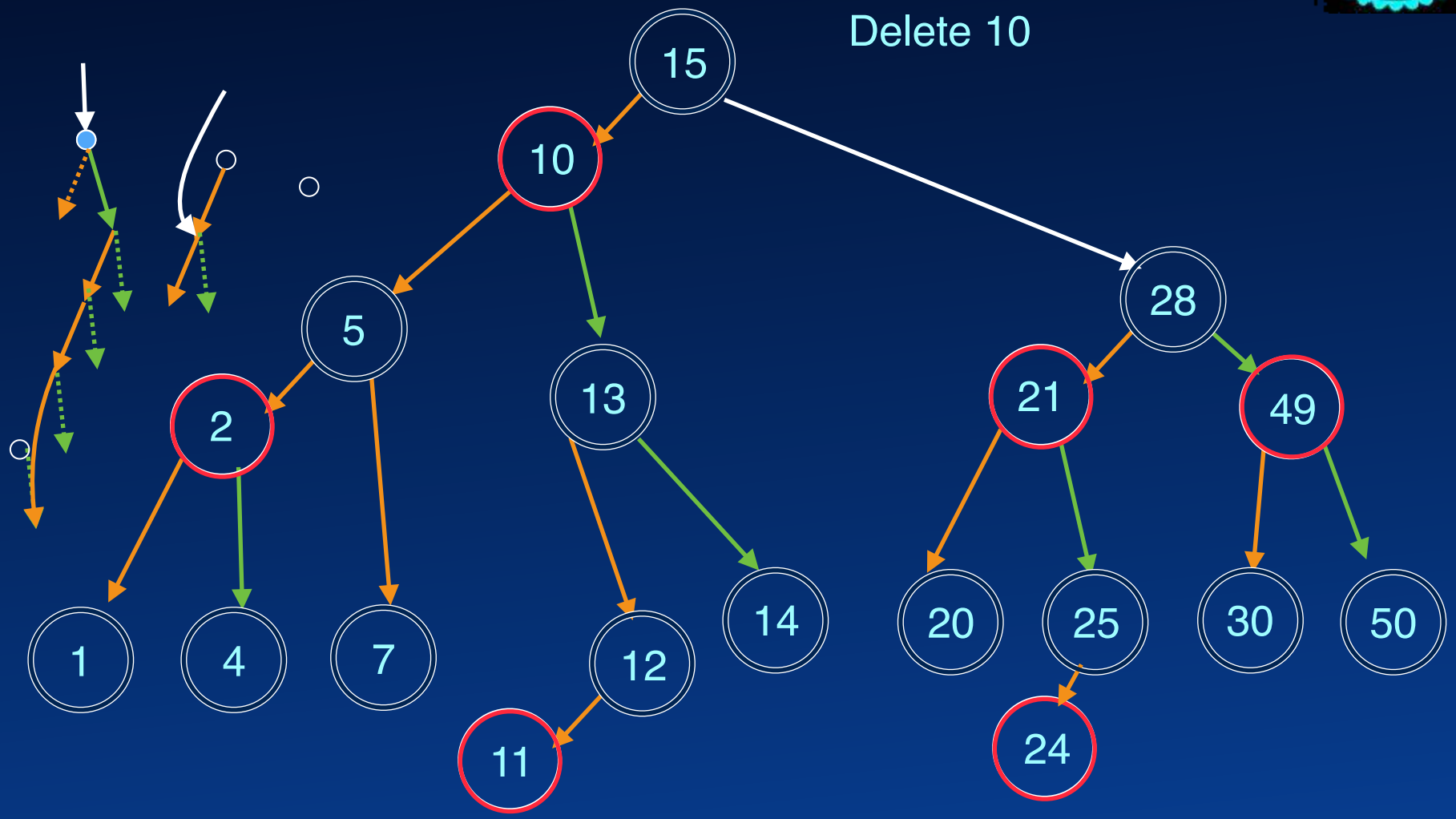
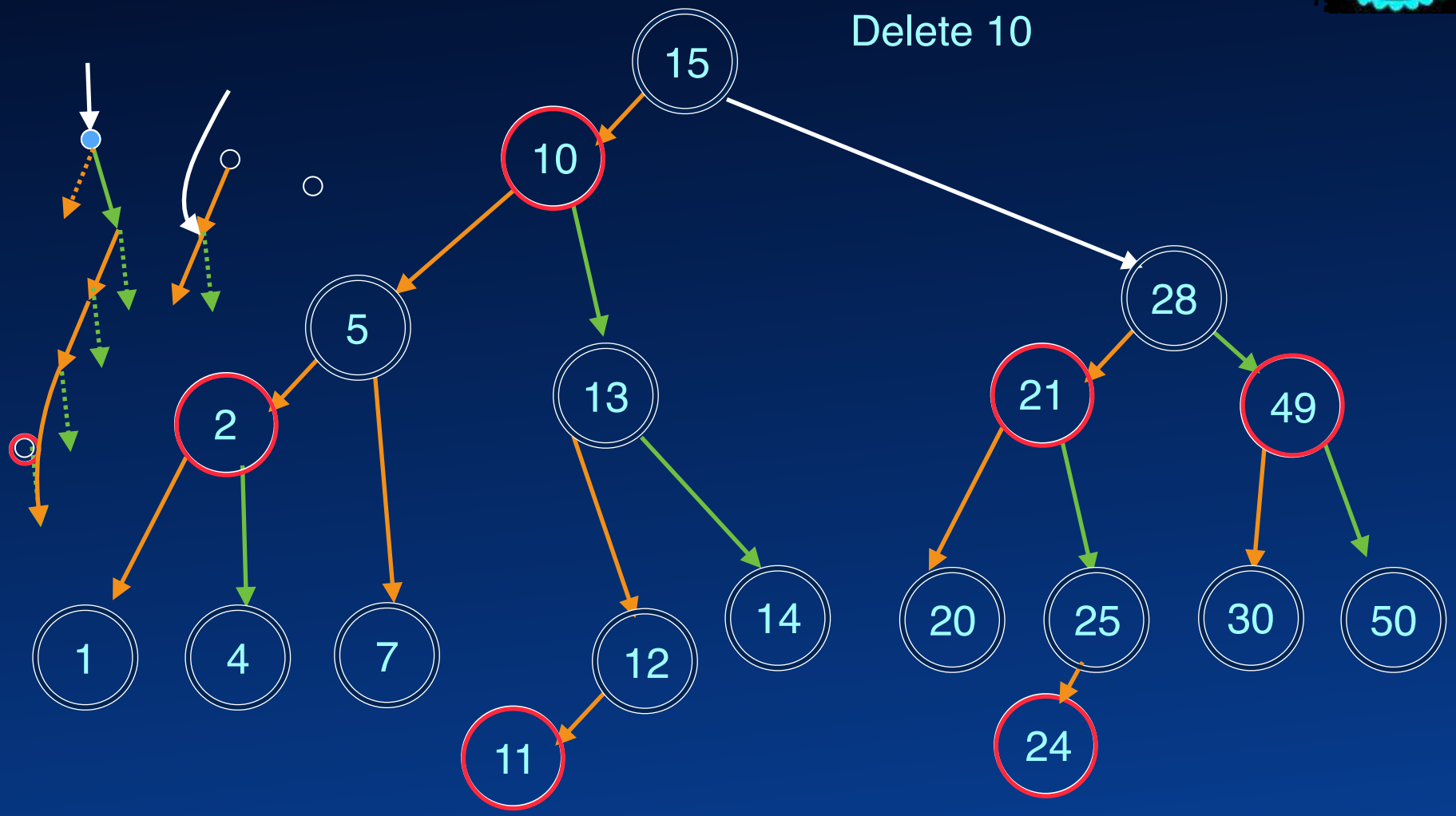


Expunge node ***n*** with @null in child.

R-B Tree Deletion



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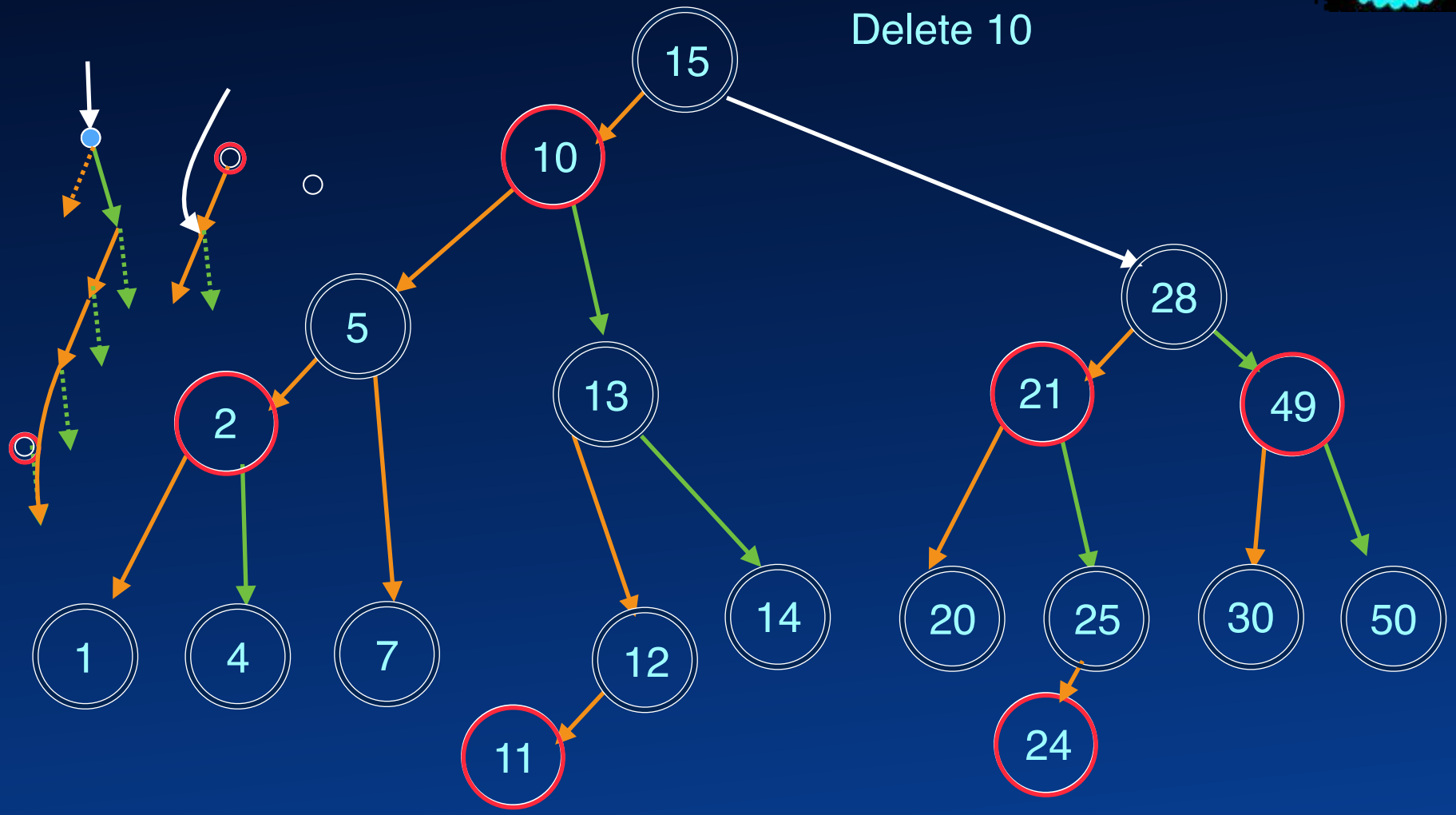

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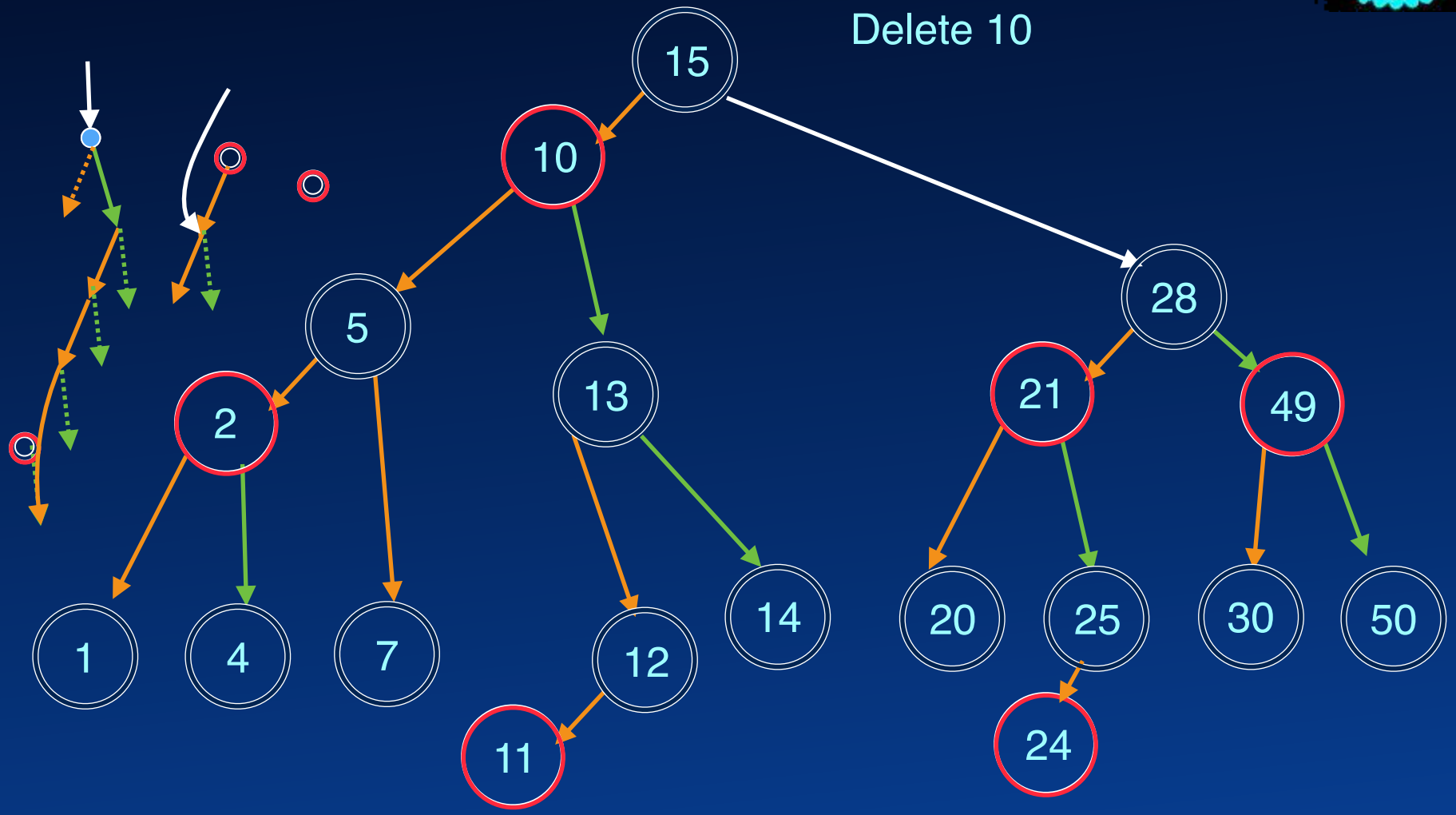

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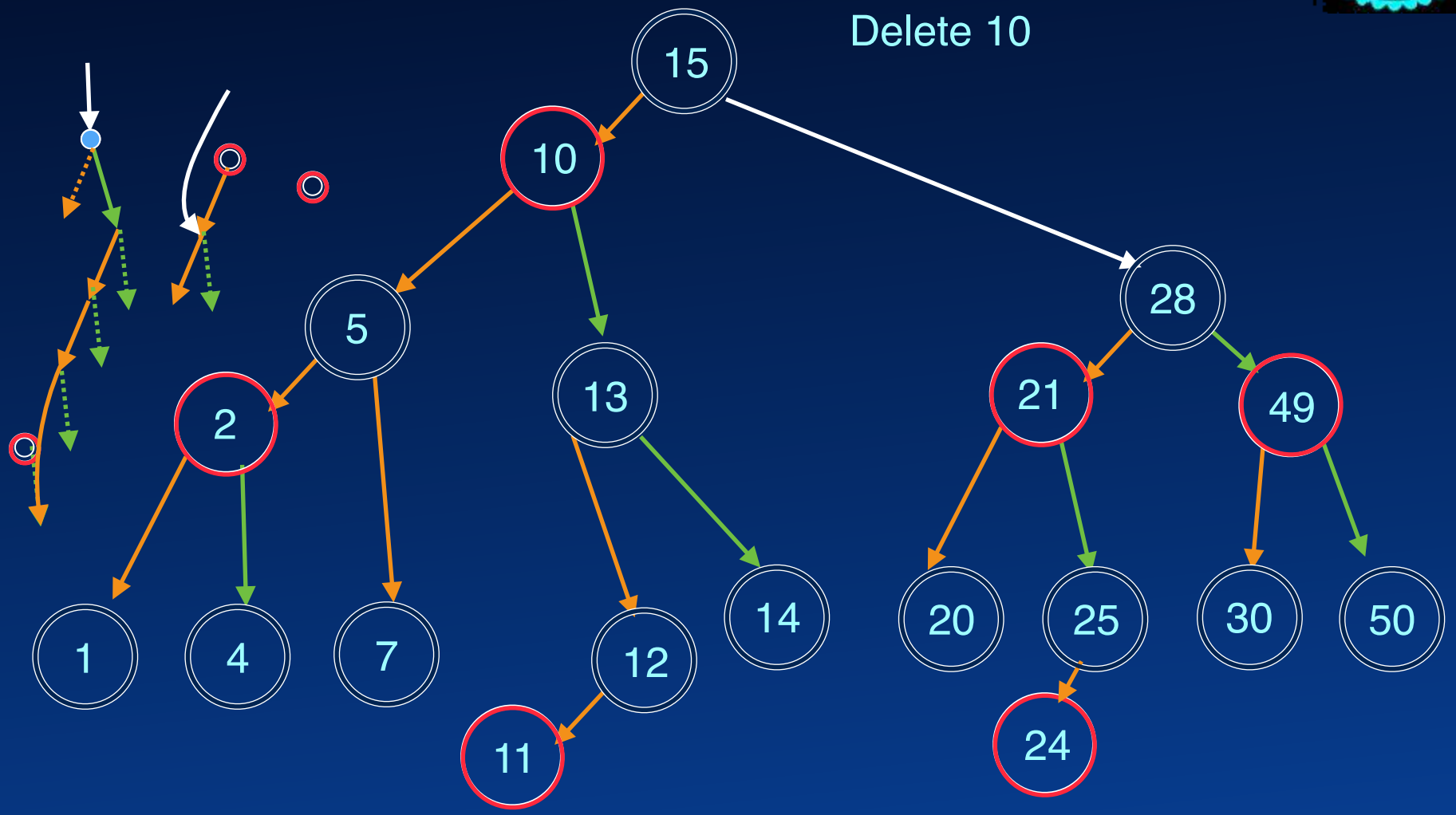

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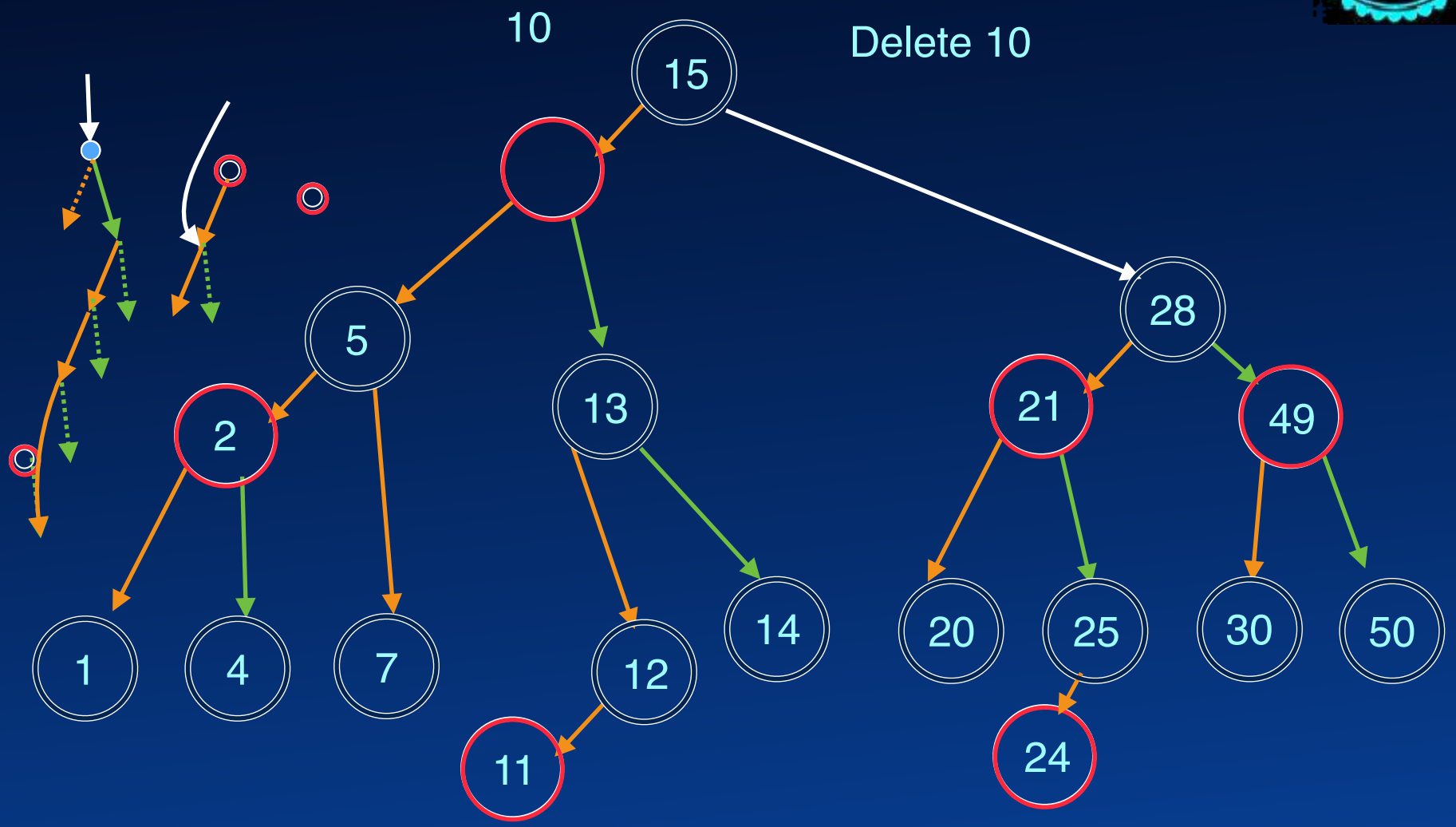
R-B Tree Deletion



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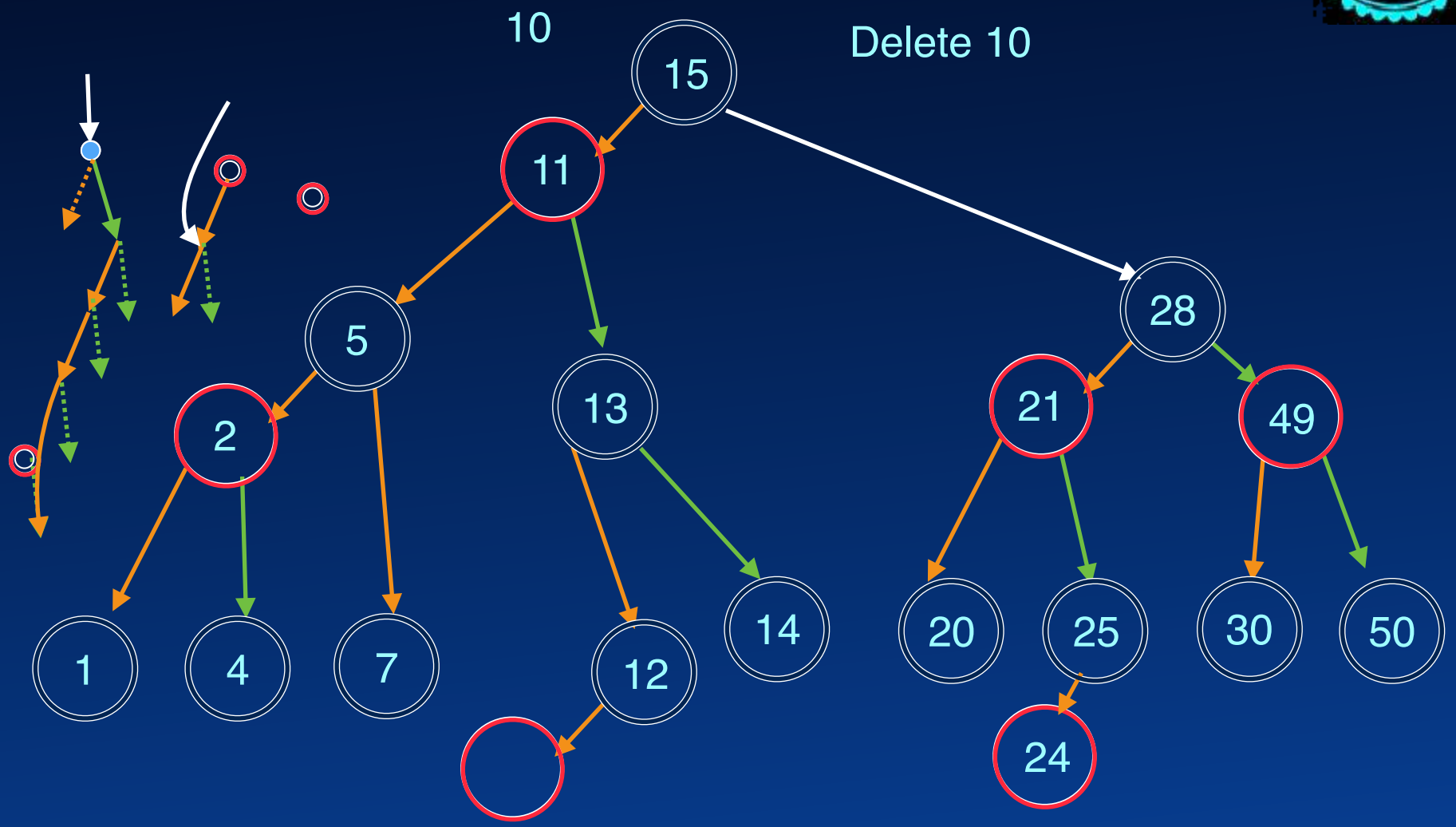

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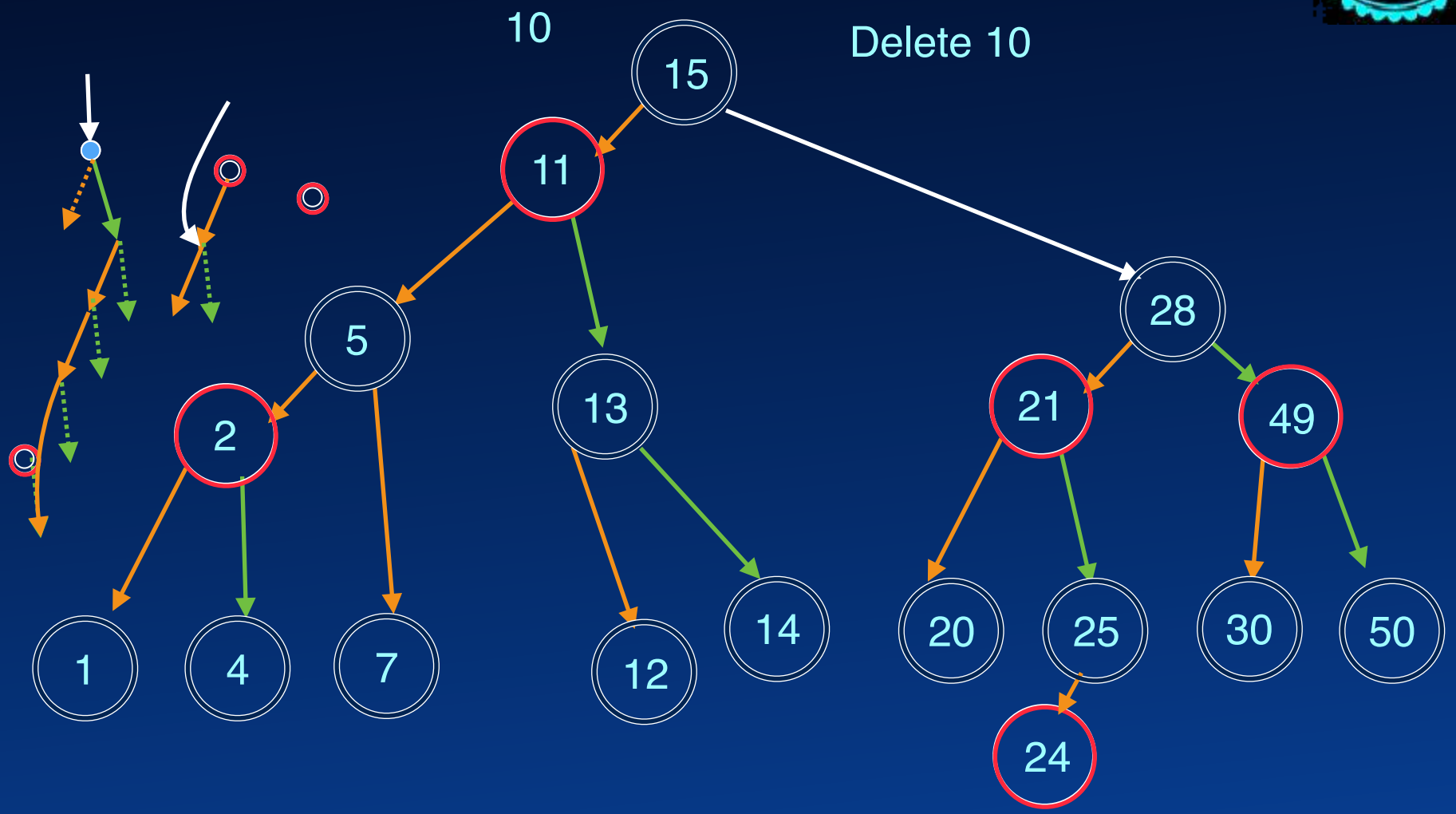

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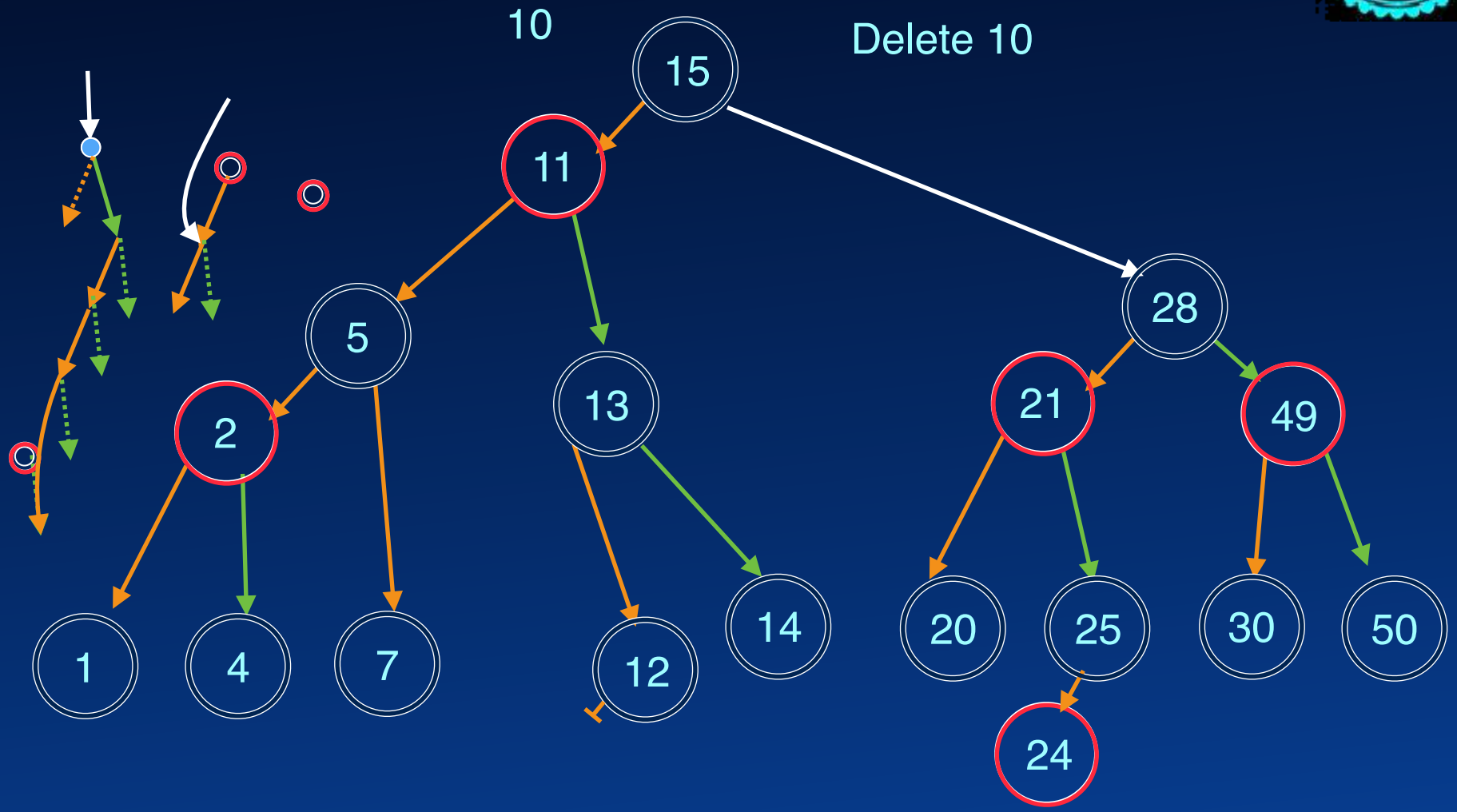

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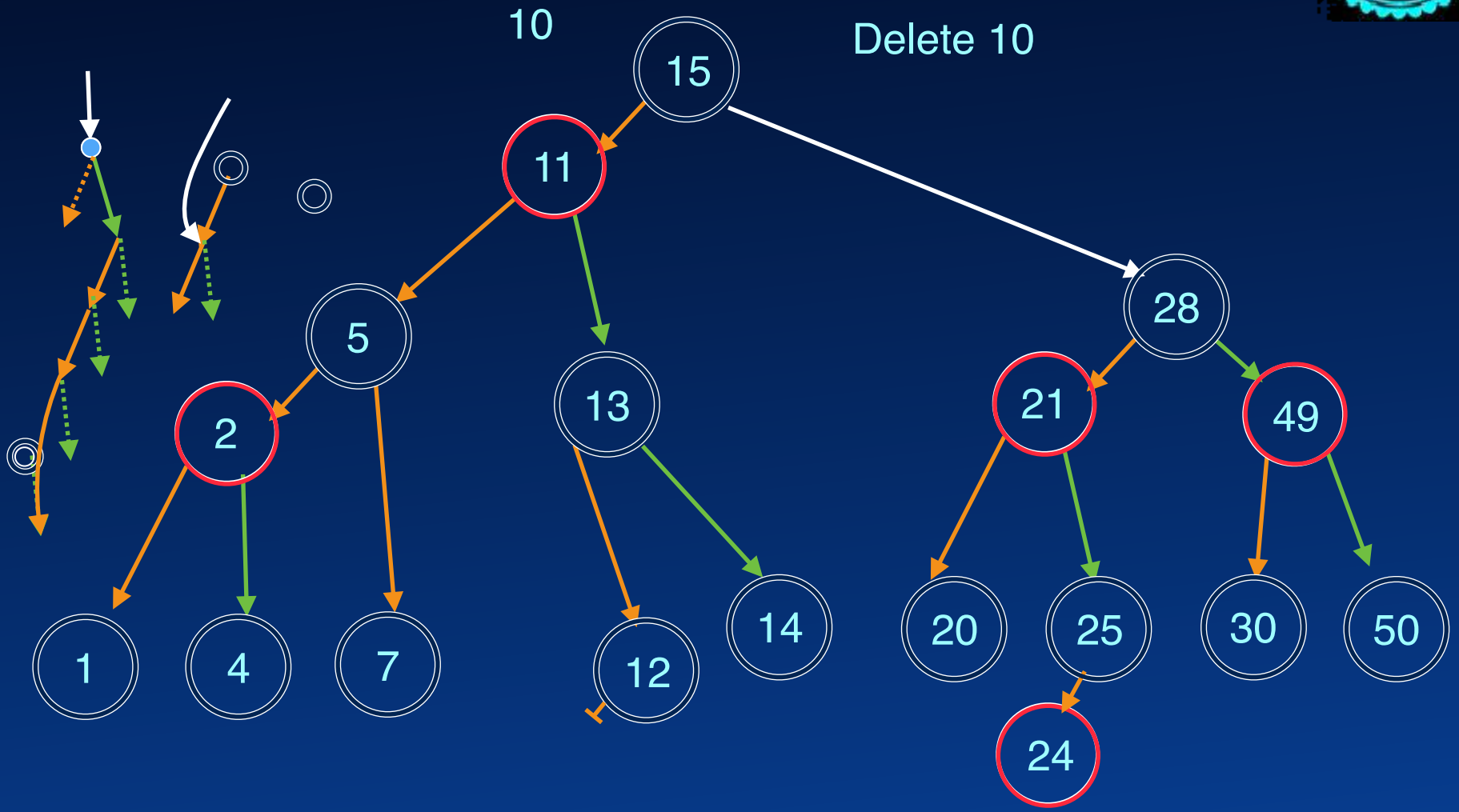

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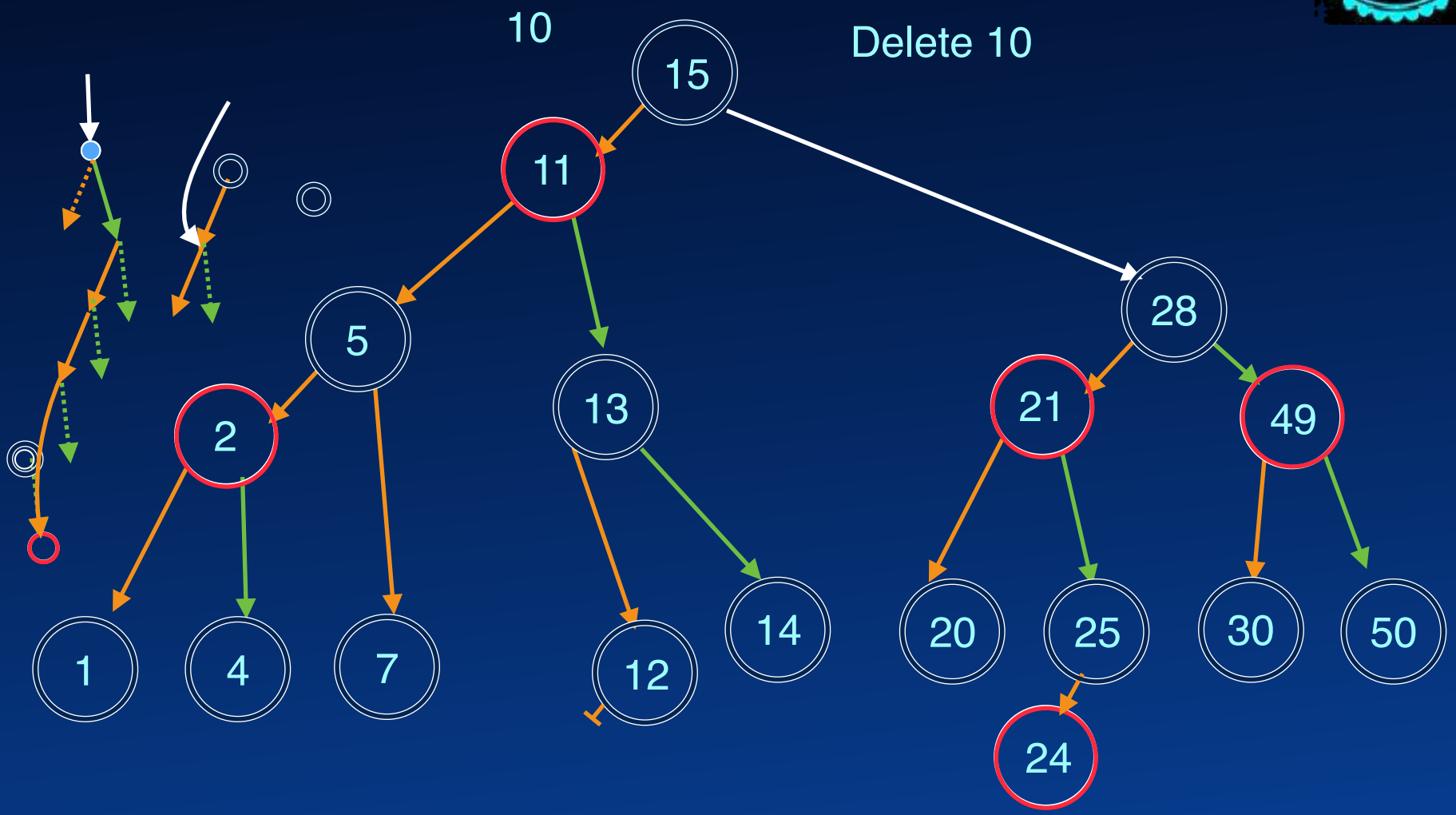

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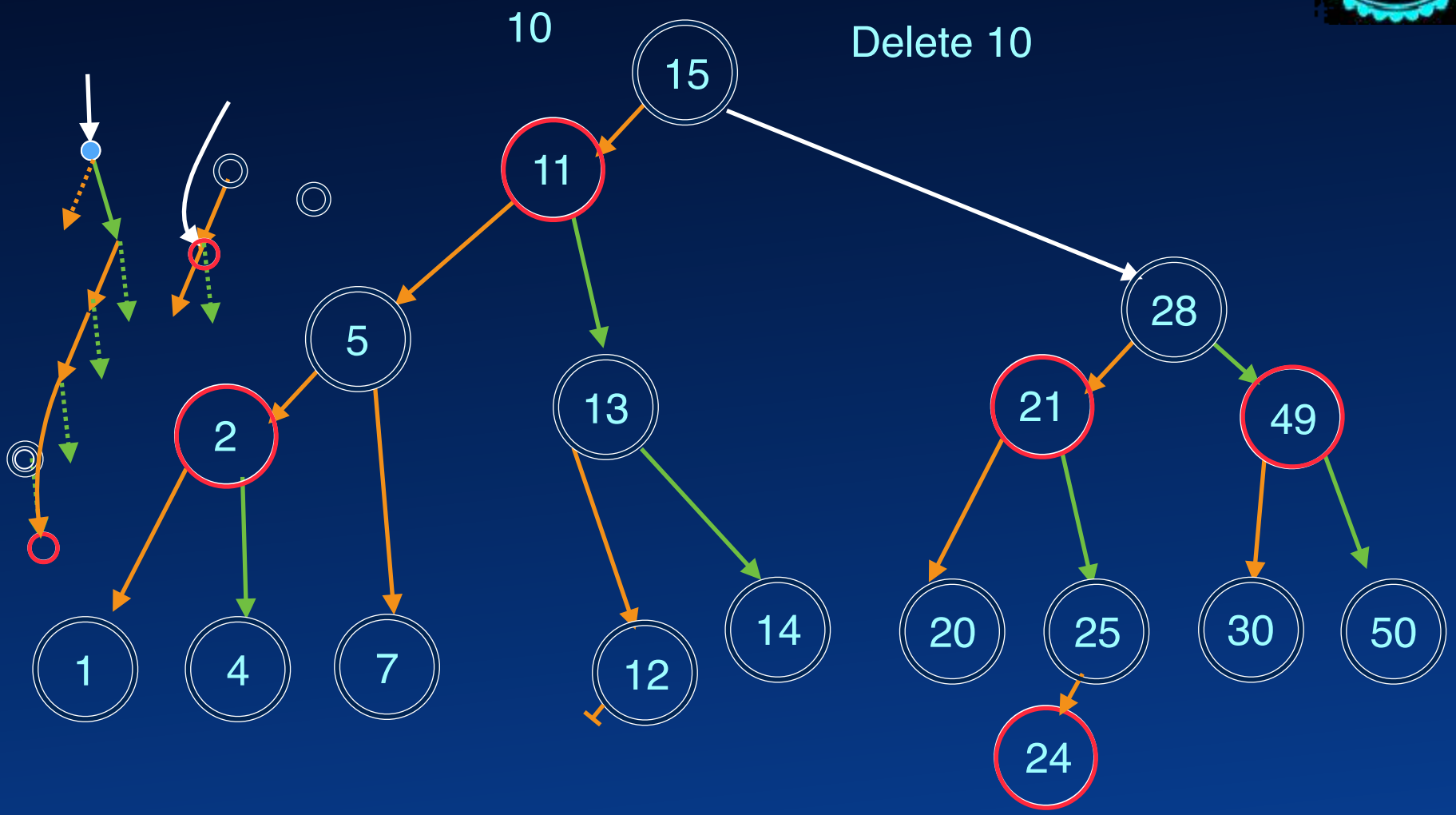

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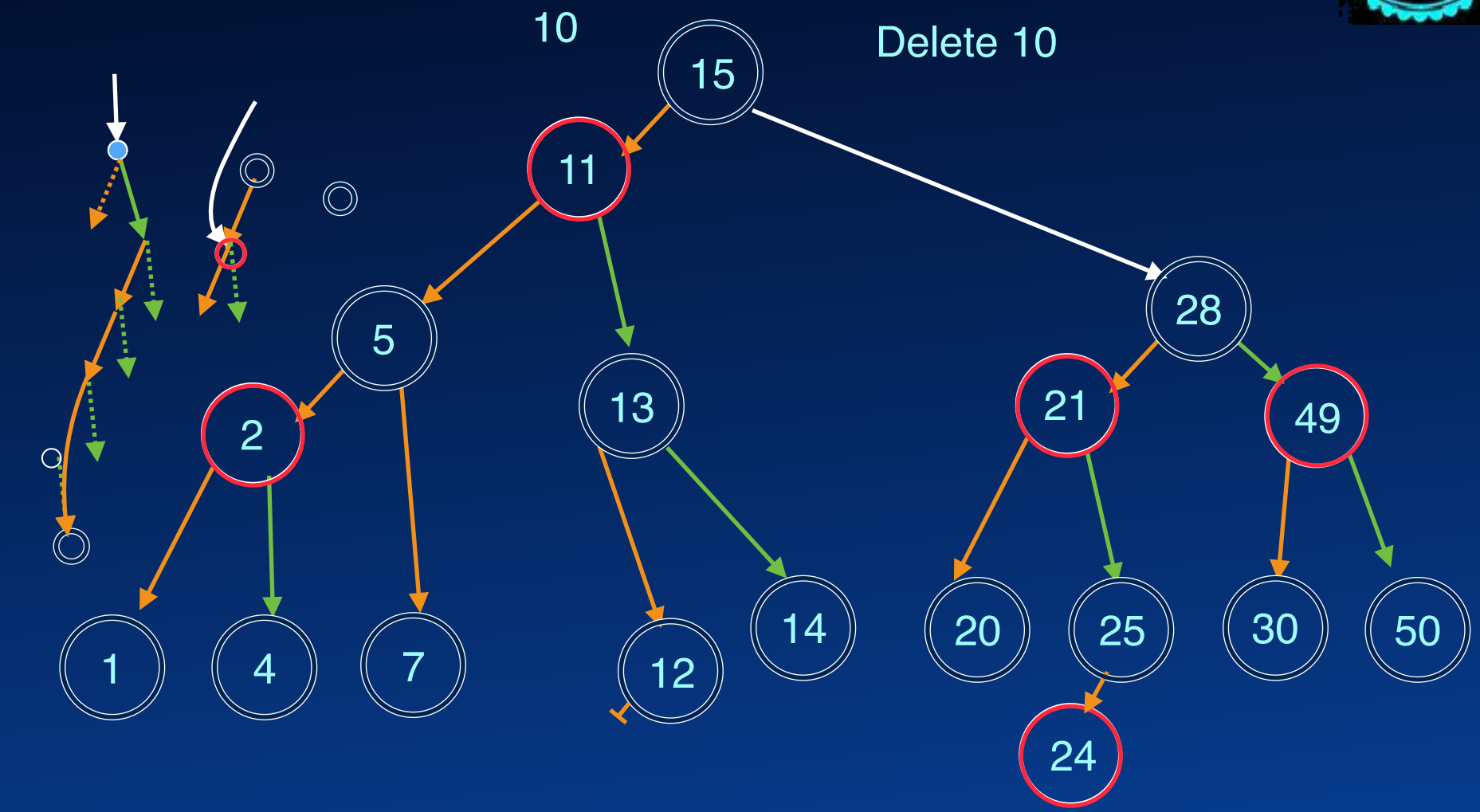

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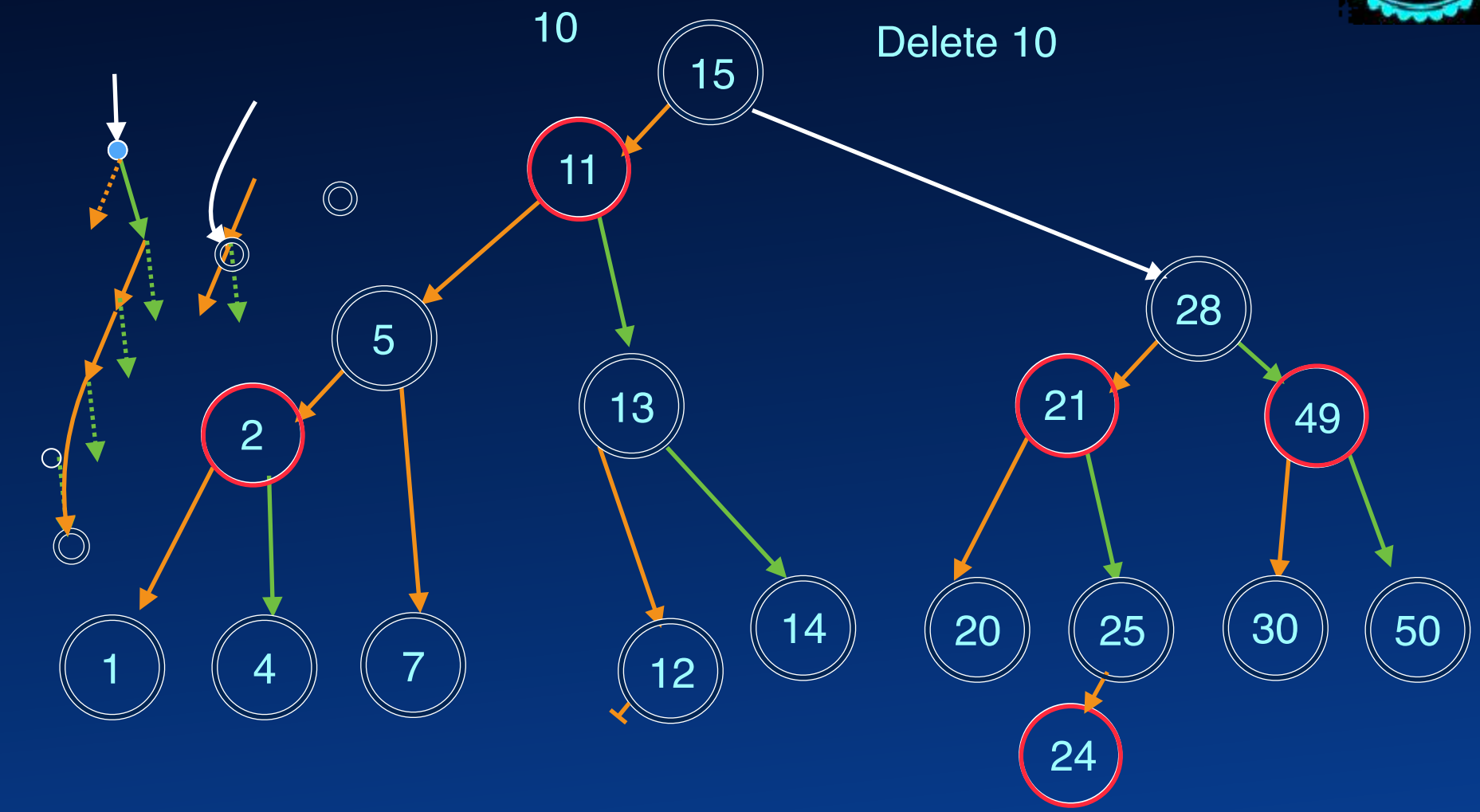

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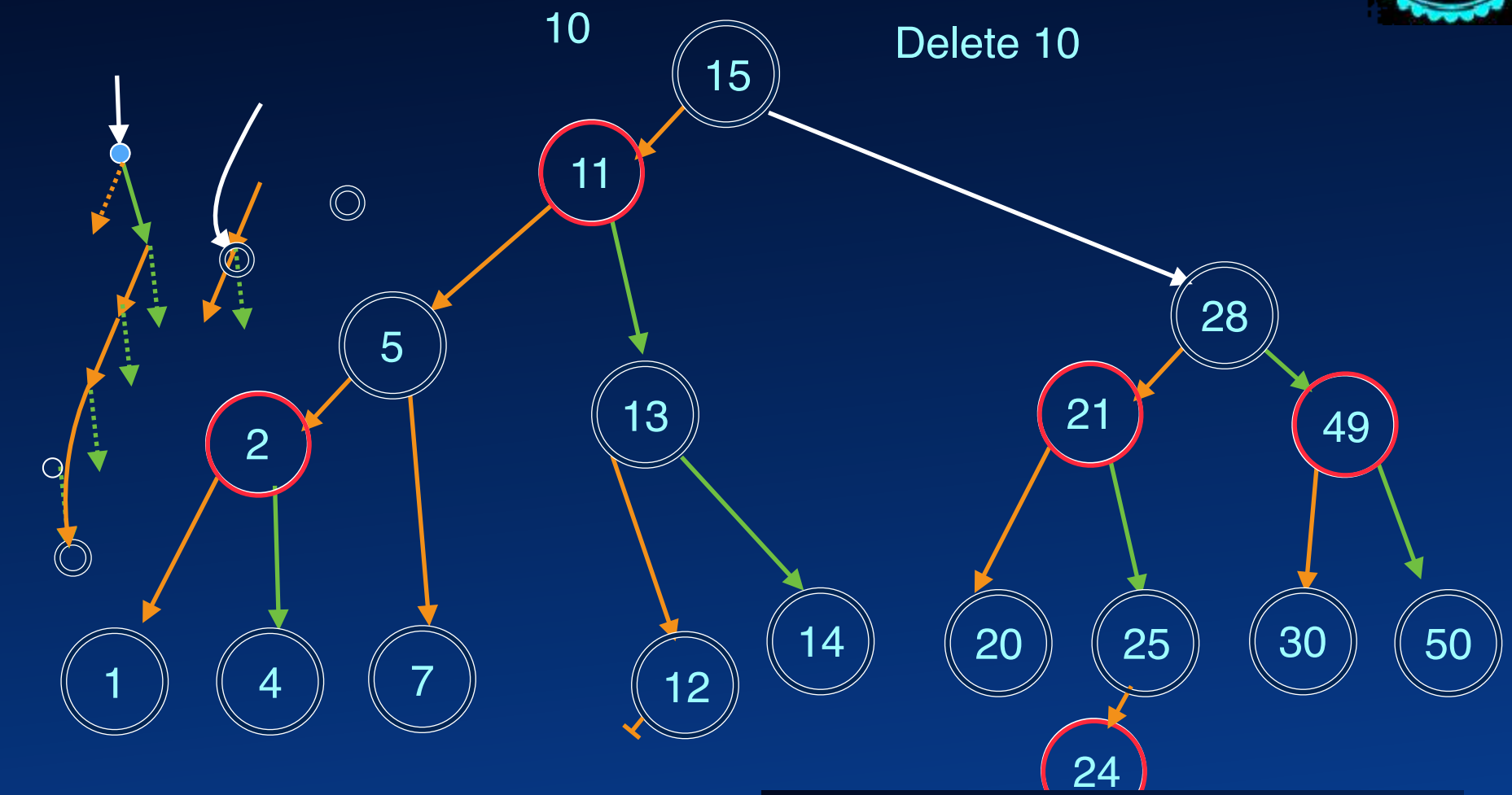

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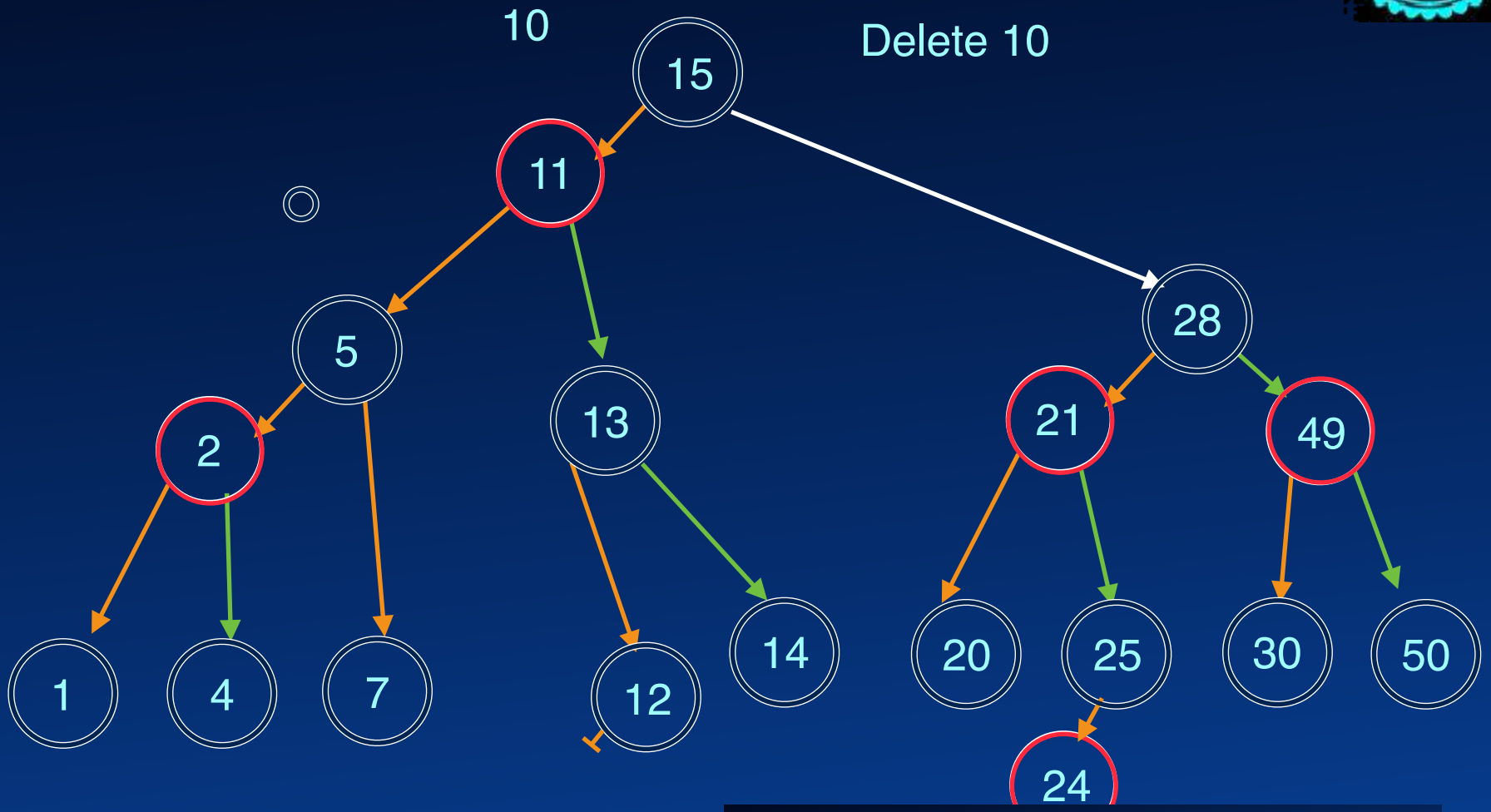
R-B Tree Deletion



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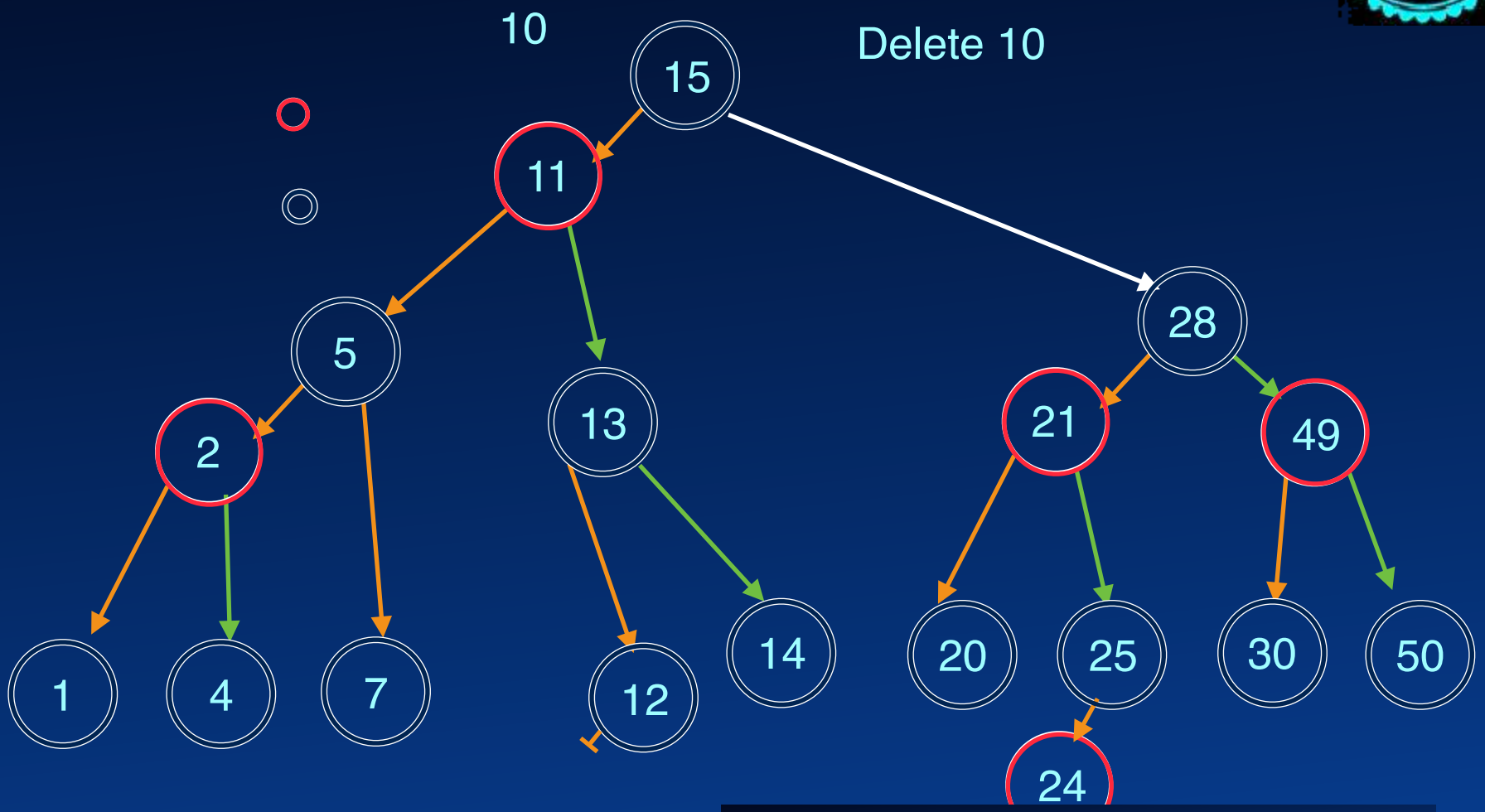

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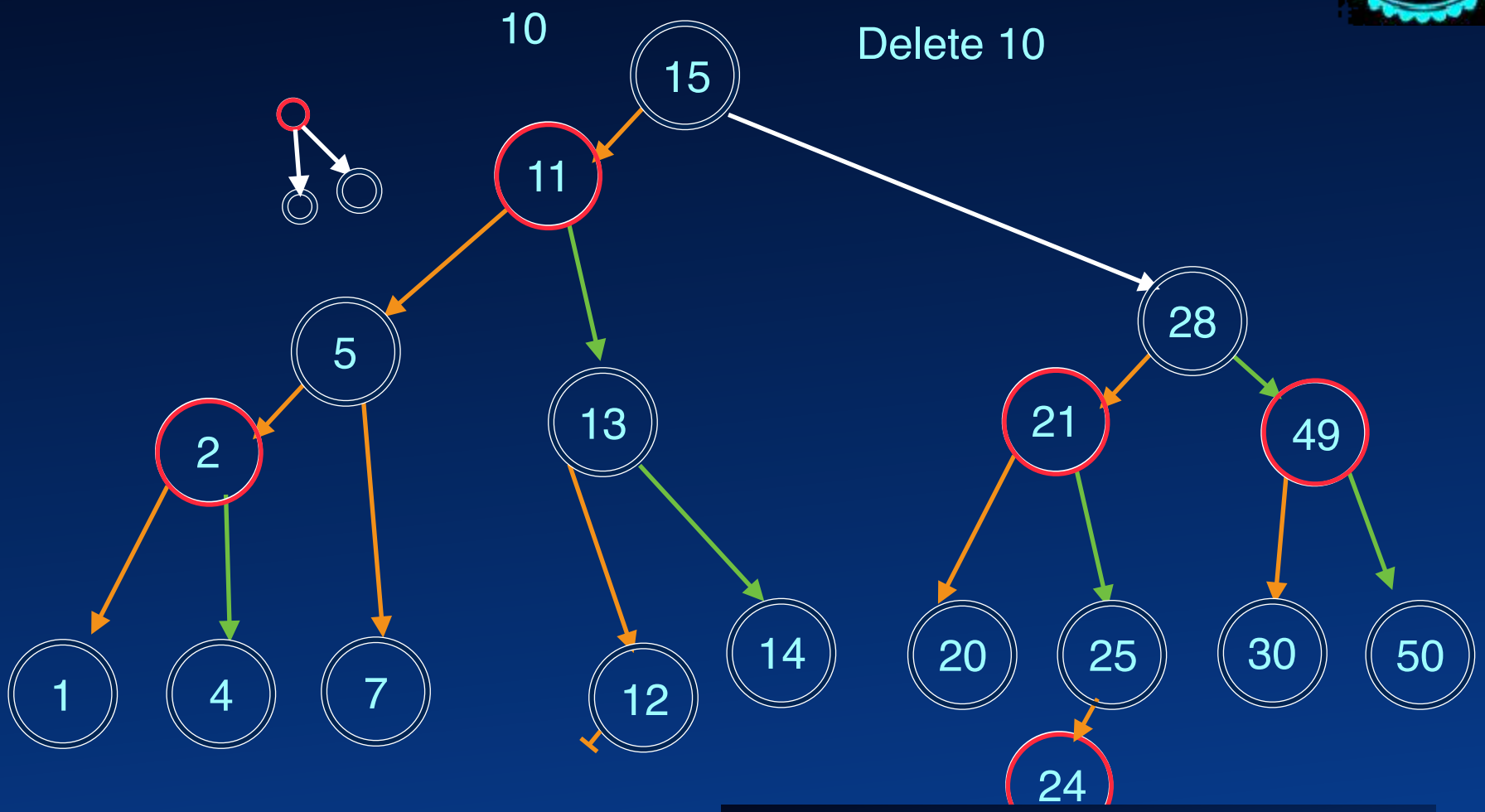

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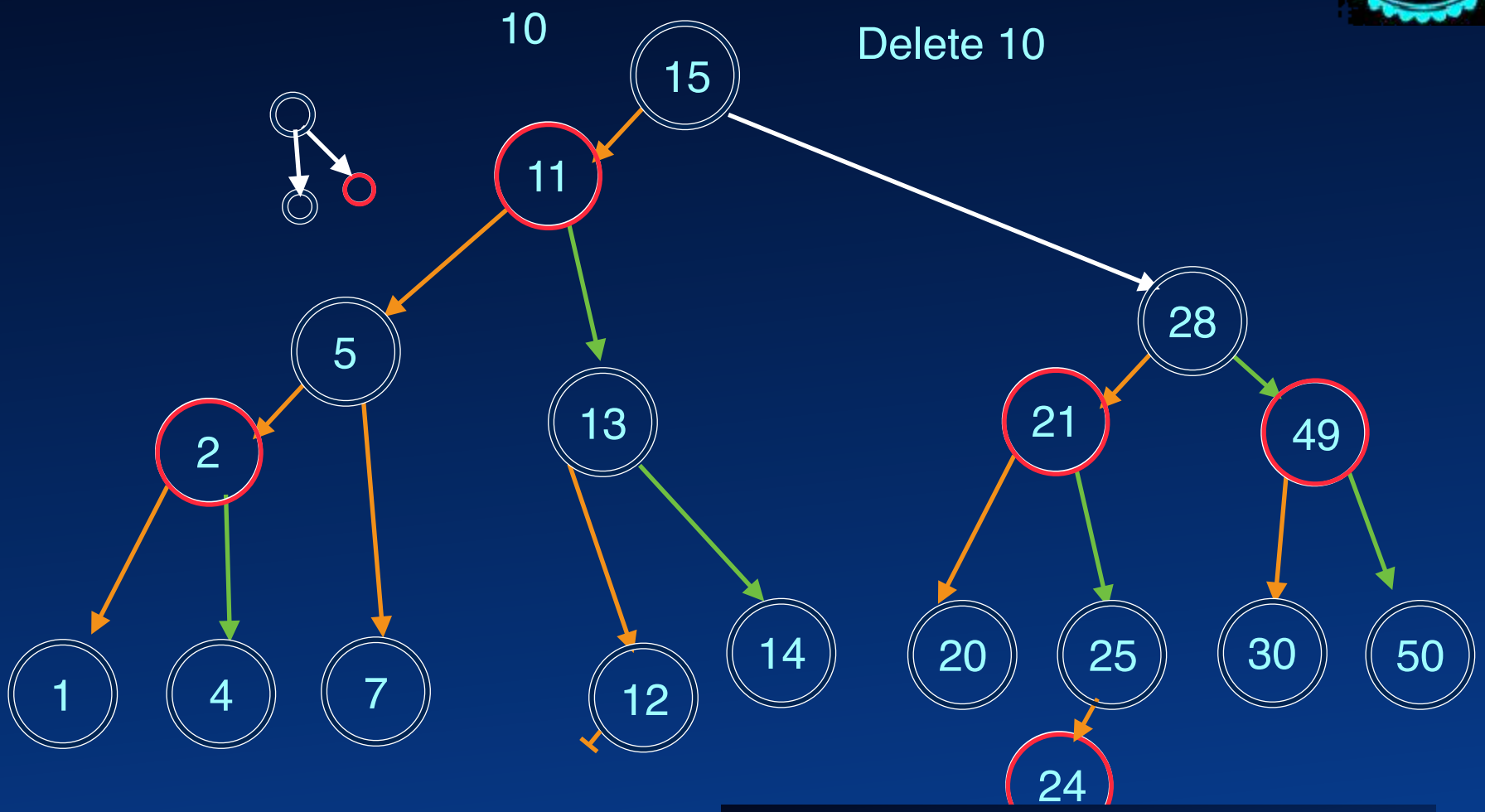

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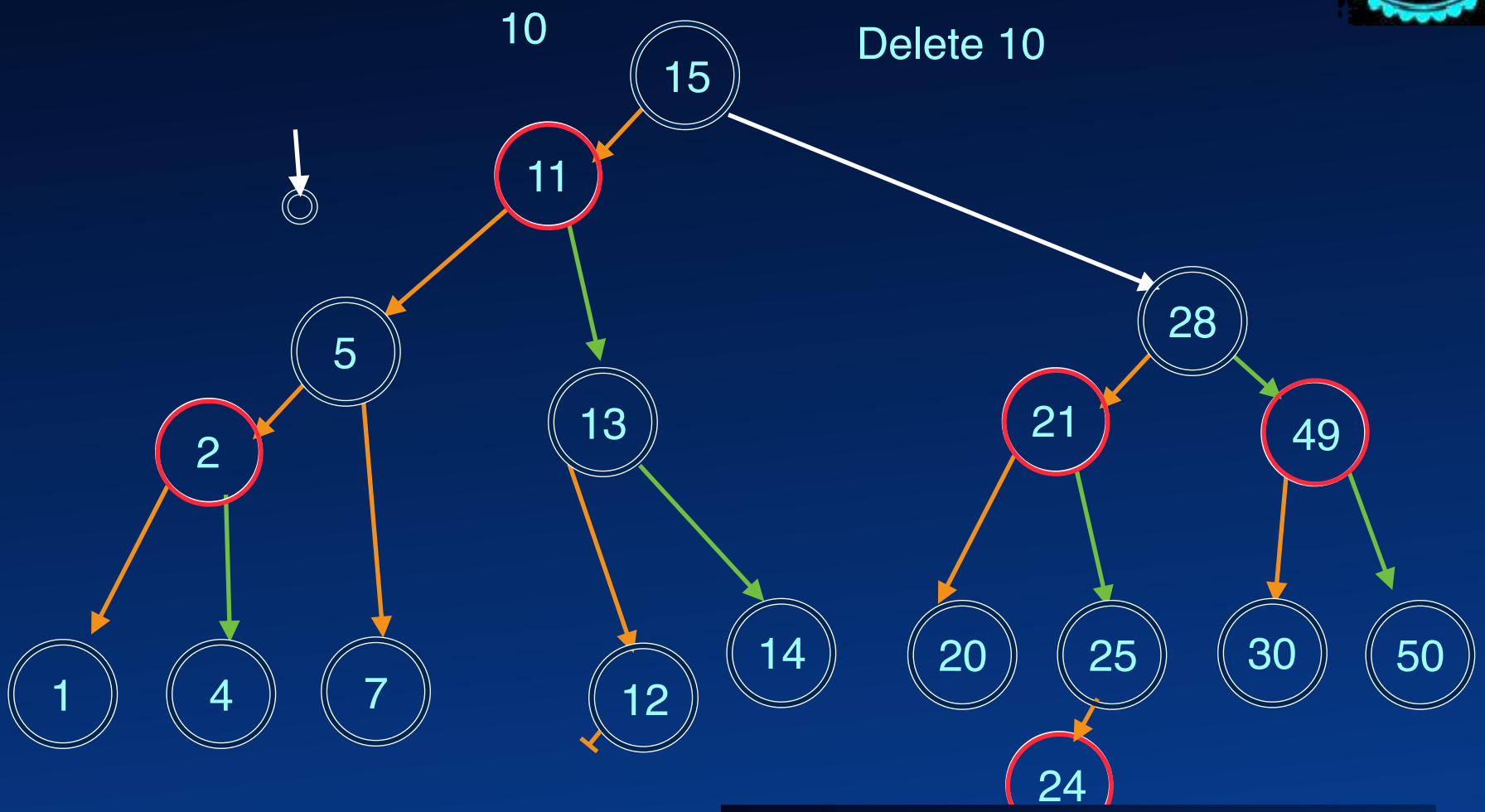

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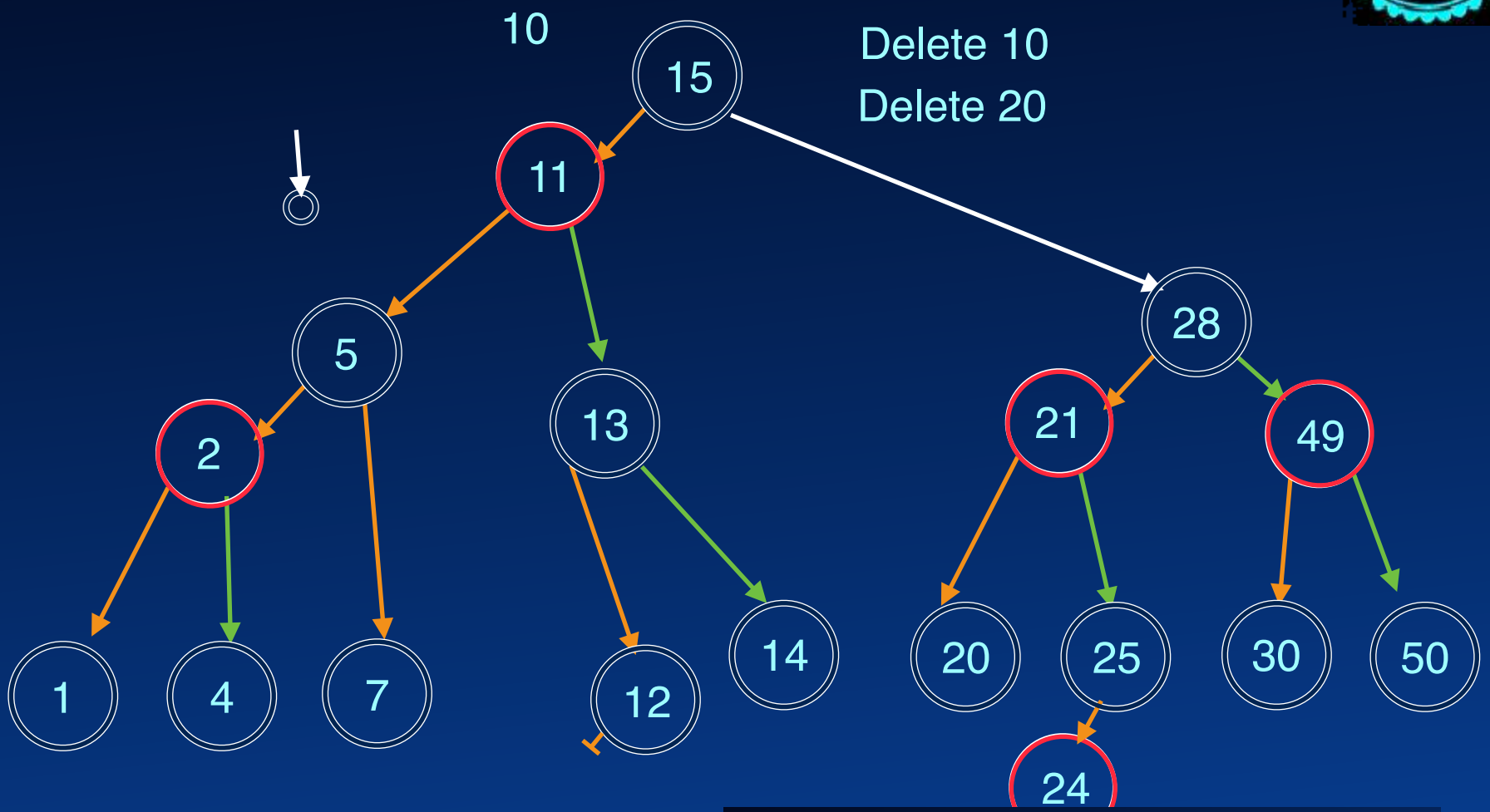

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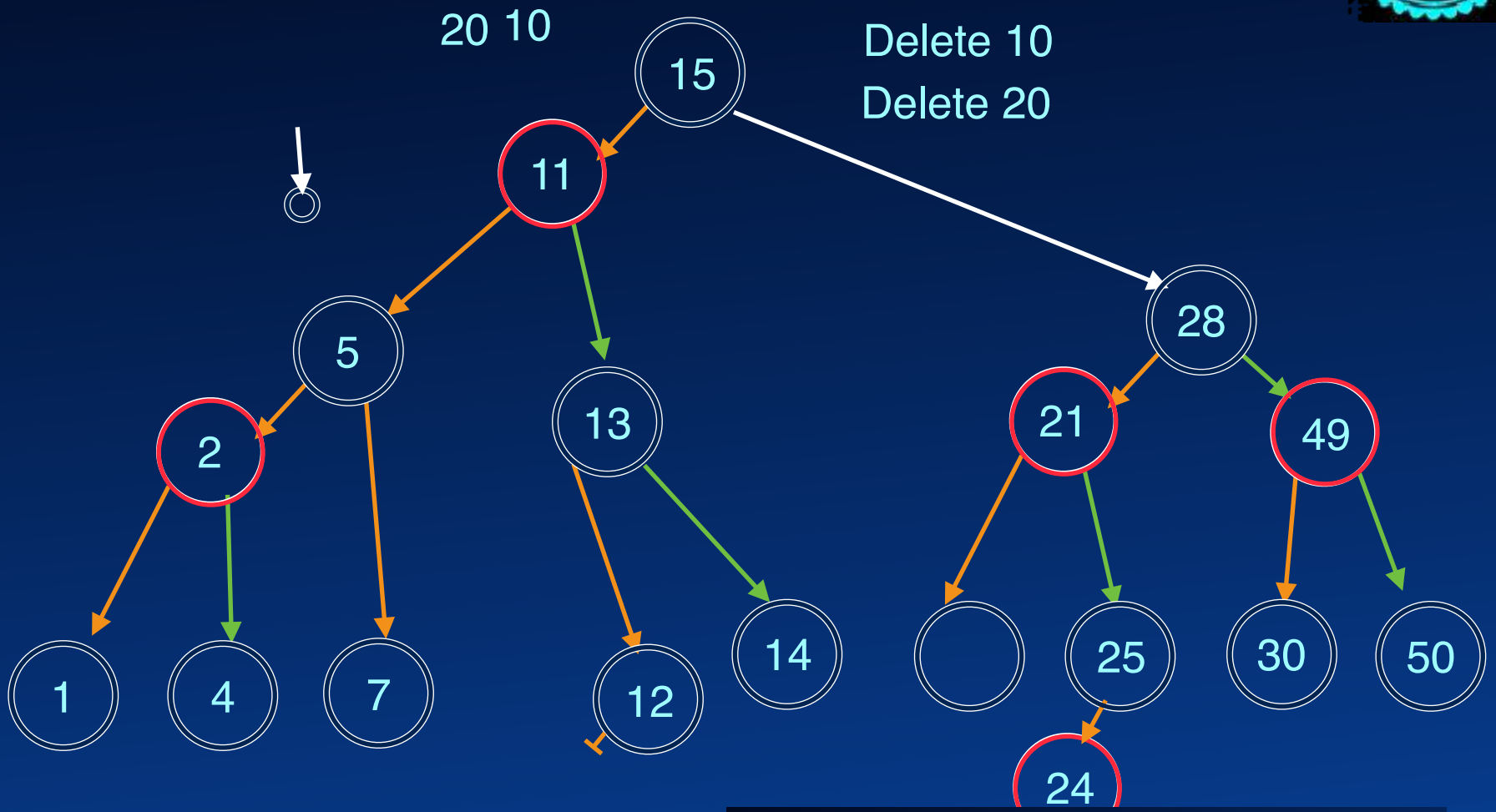

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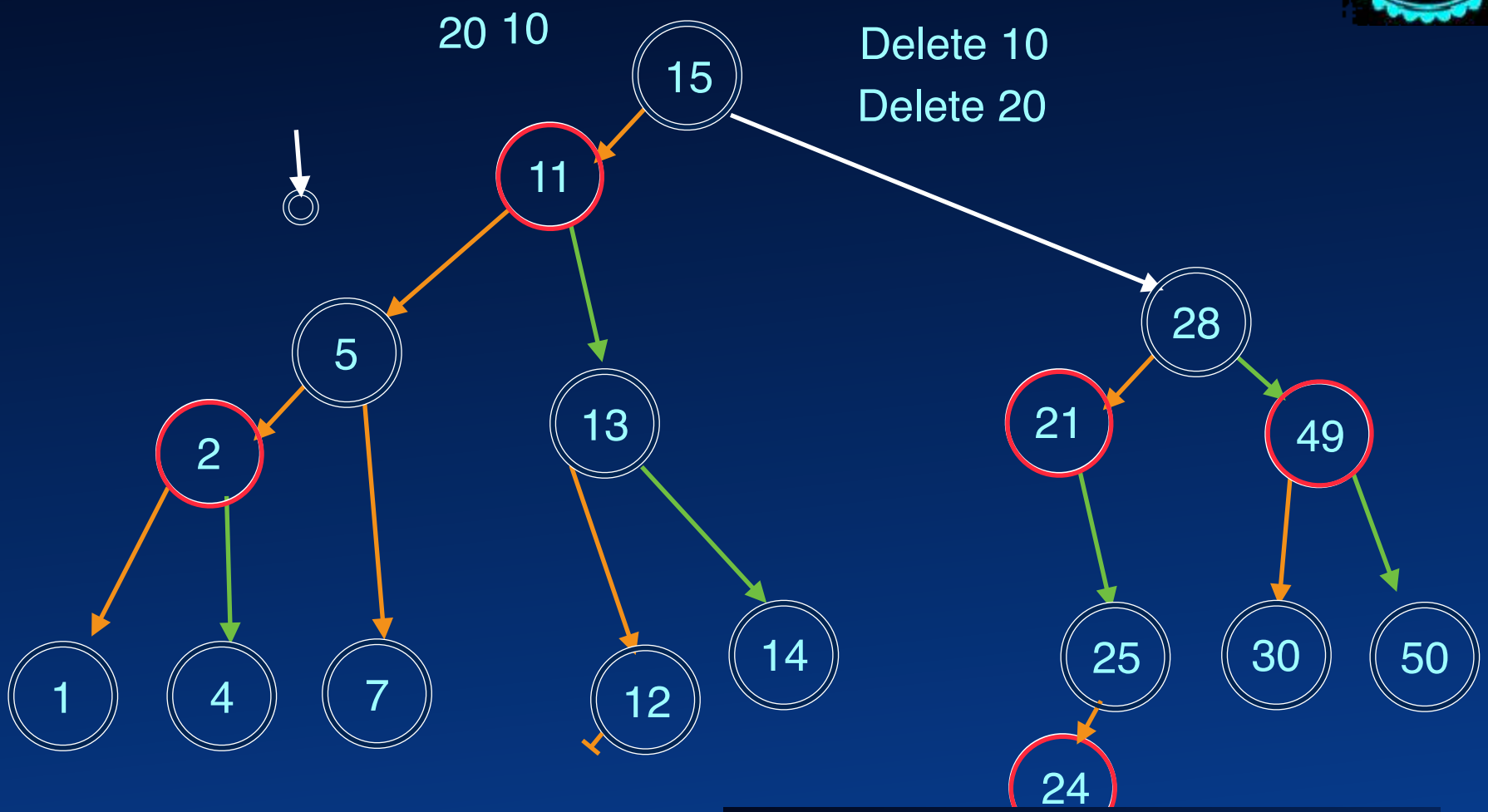

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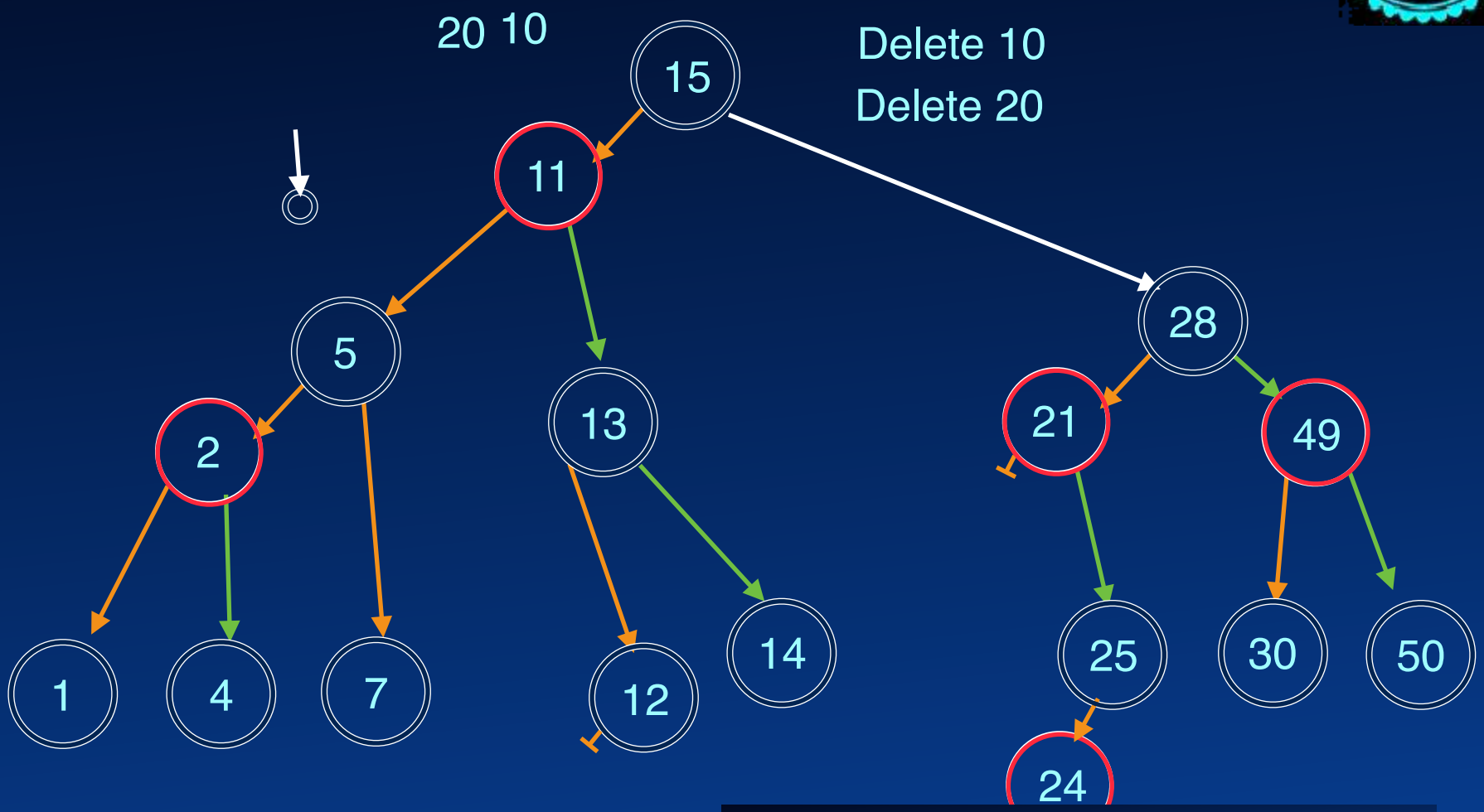

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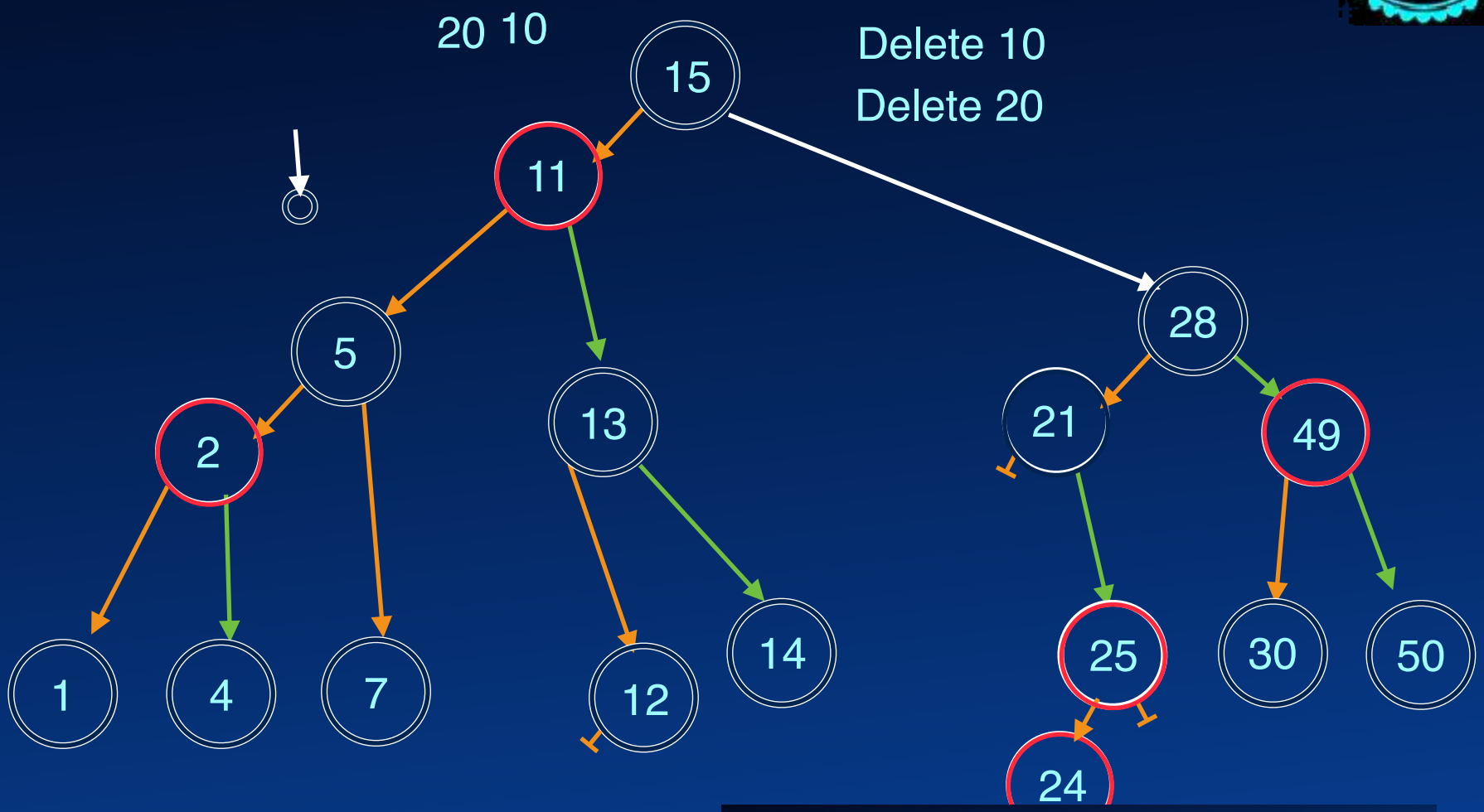

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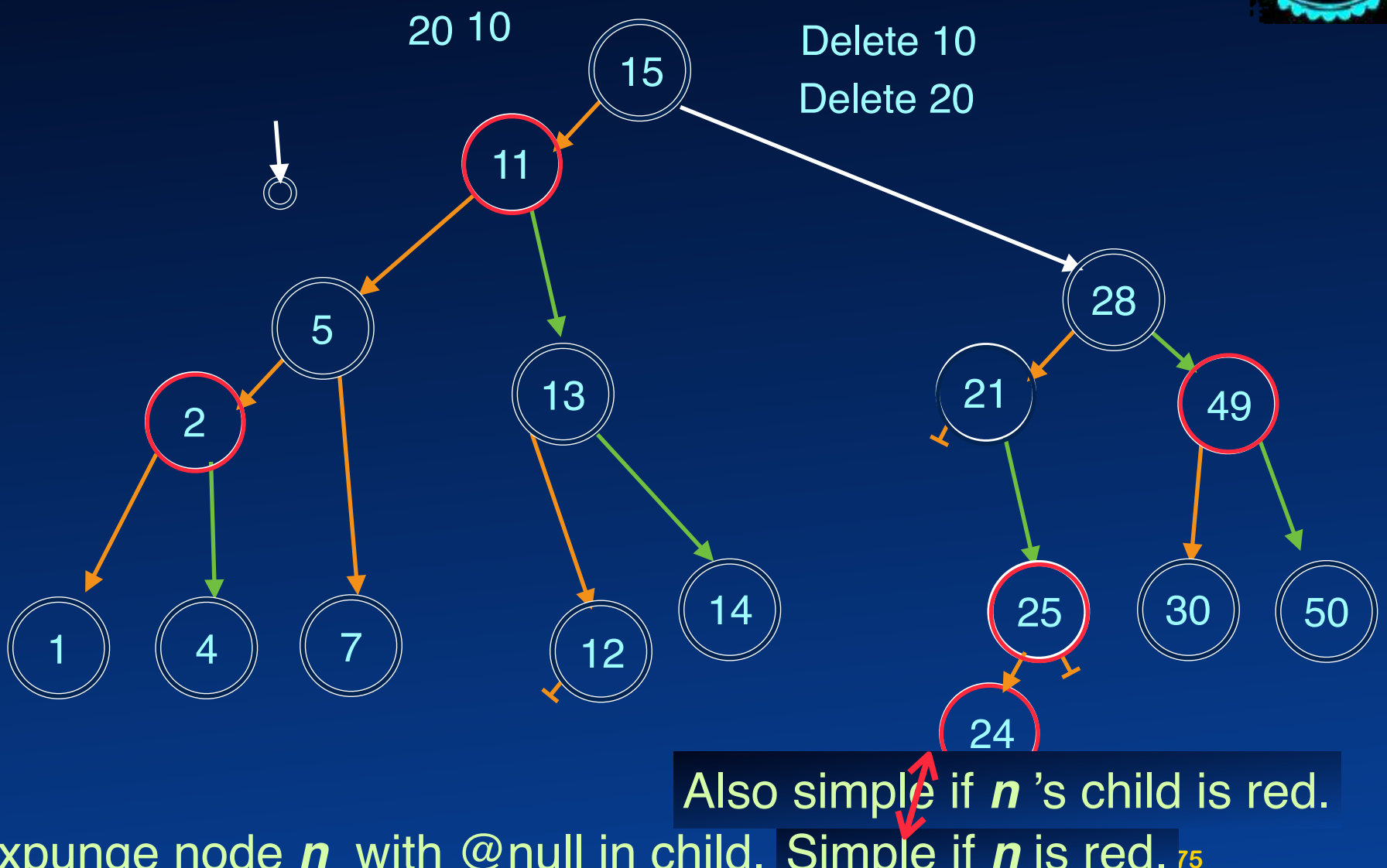
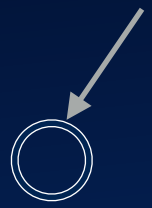


Also simple if ***n*** 's child is red.

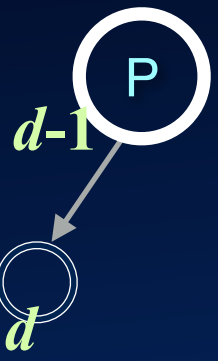
Expunge node ***n*** with @null in child. Simple if ***n*** is red.

Complex Delete Cases

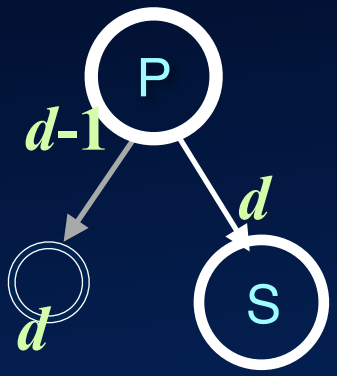
Complex Delete Cases



Complex Delete Cases



Complex Delete Cases



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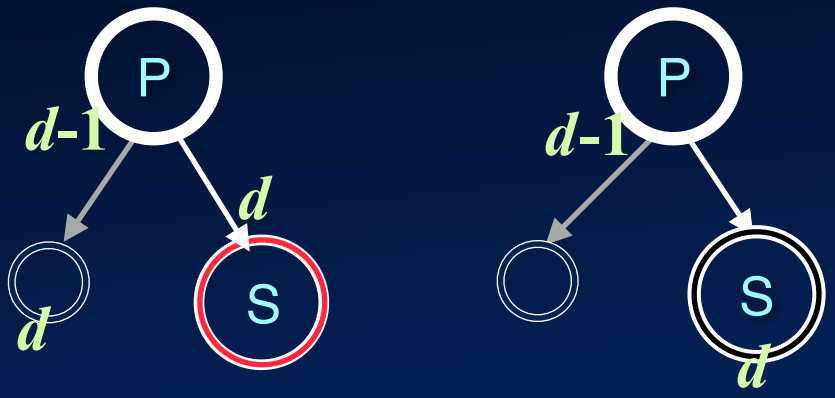

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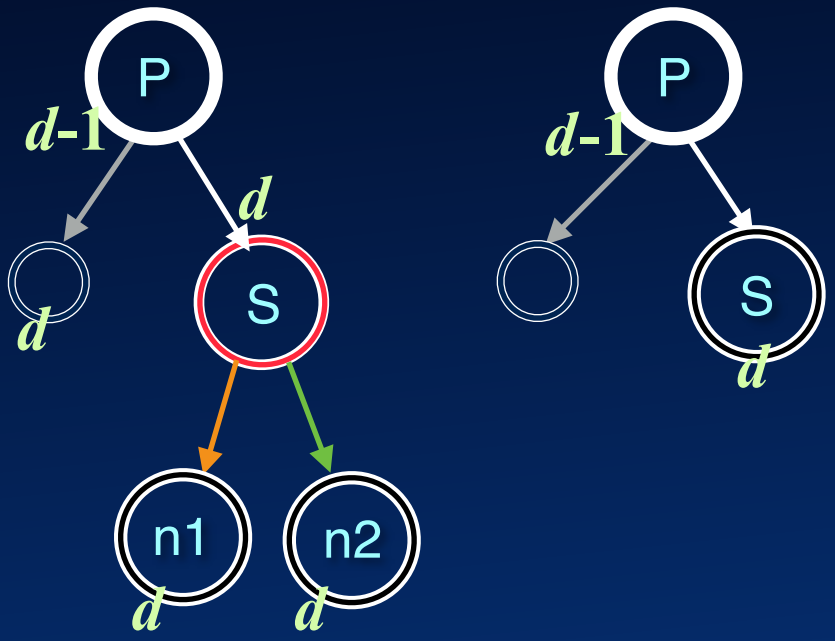

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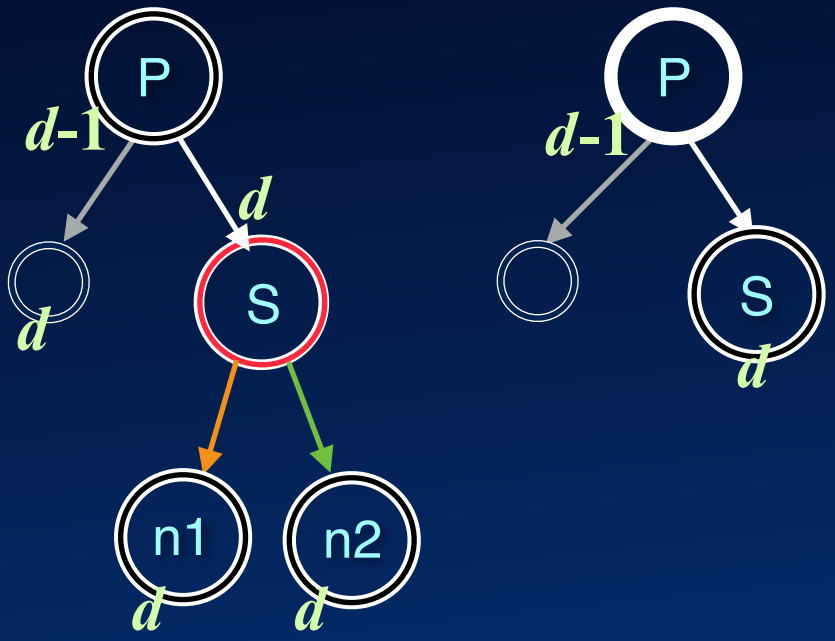

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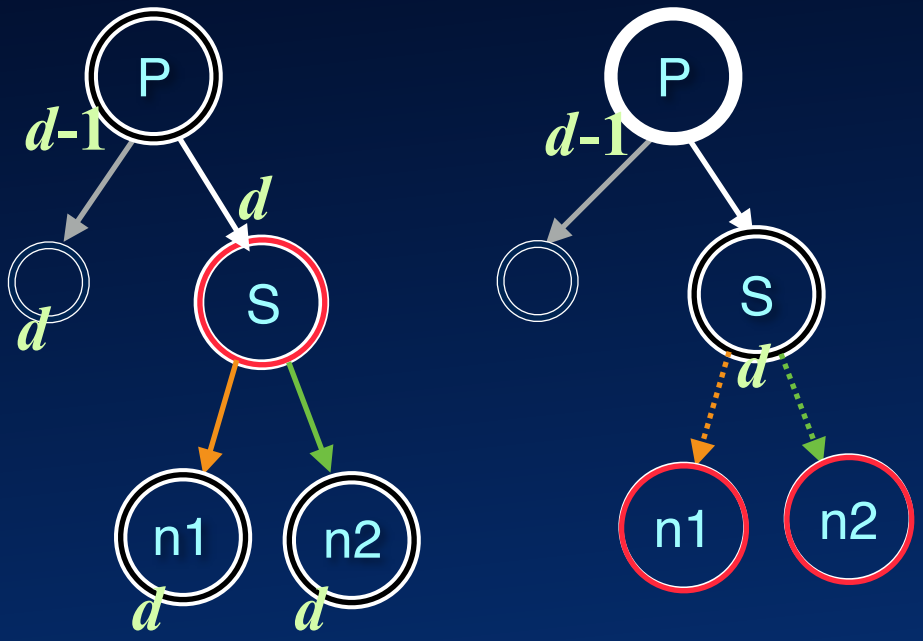

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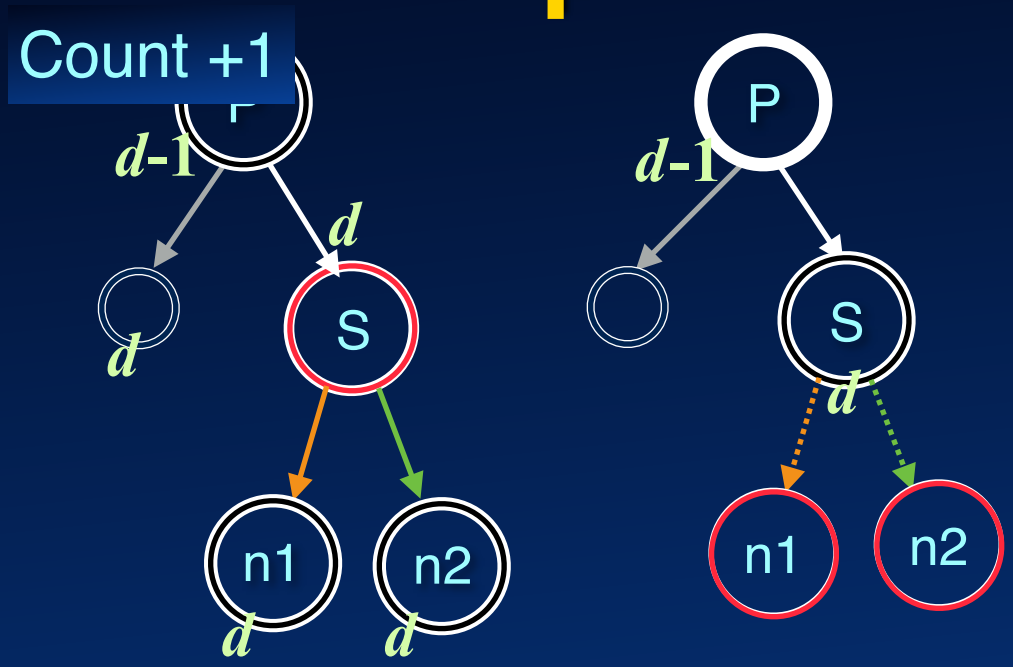

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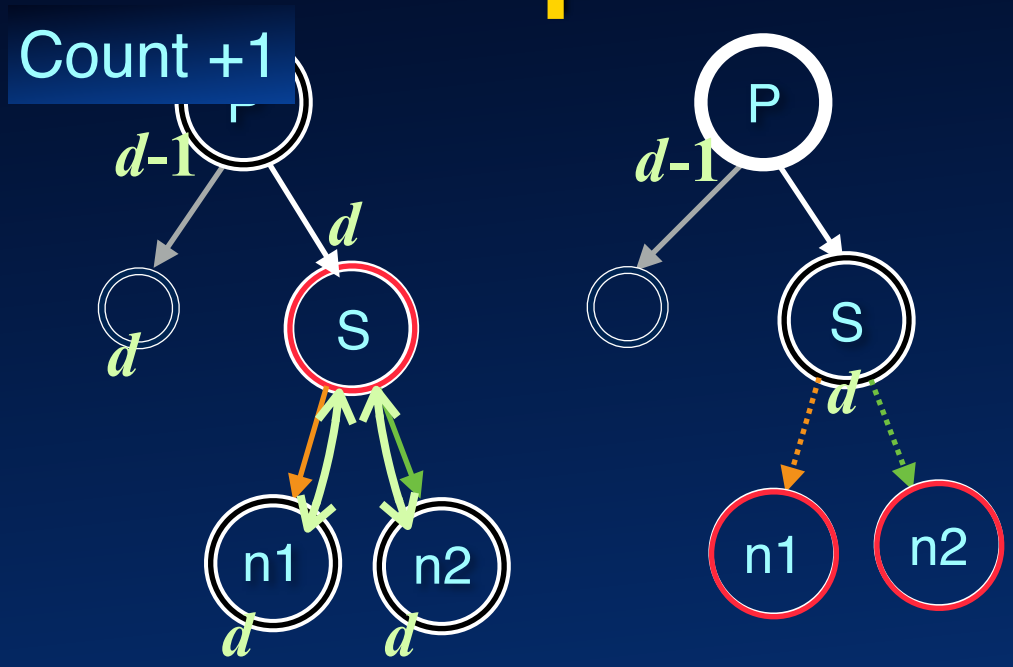

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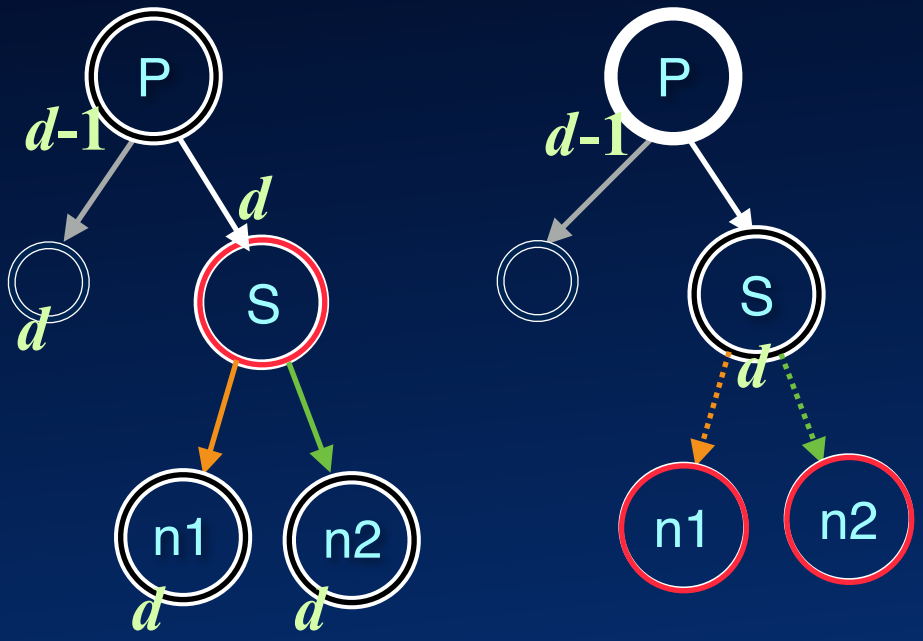

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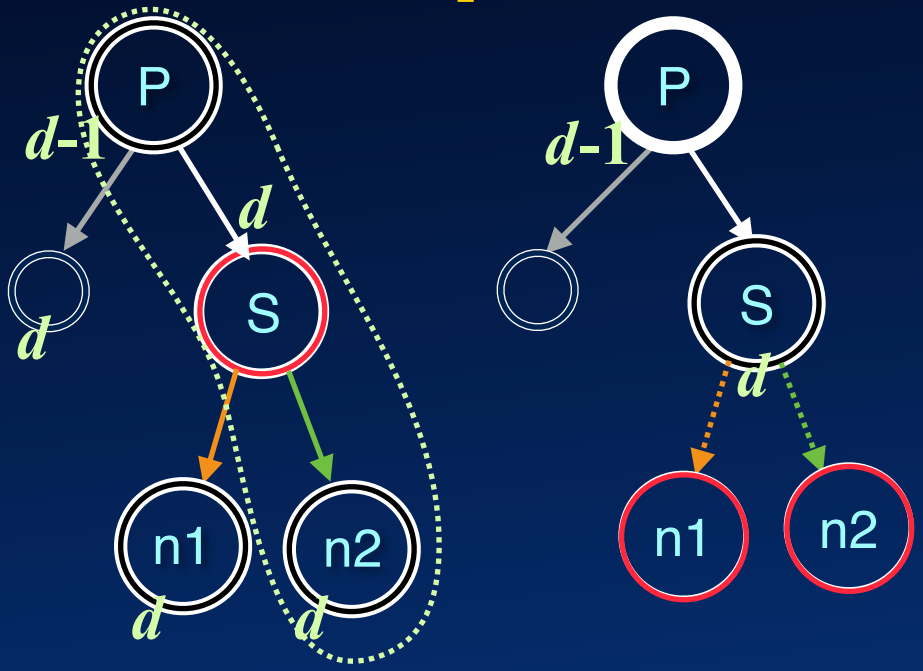

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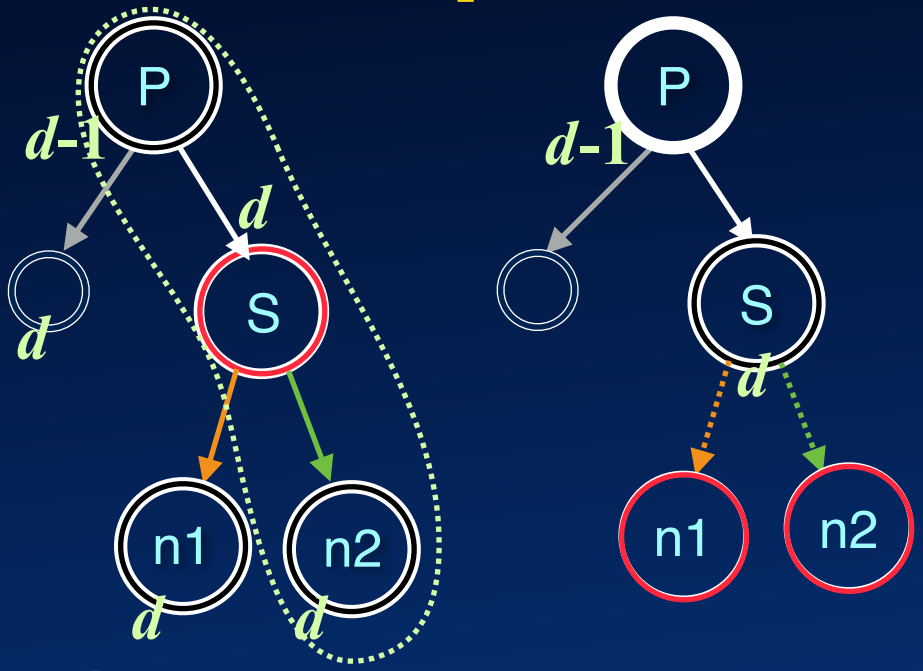

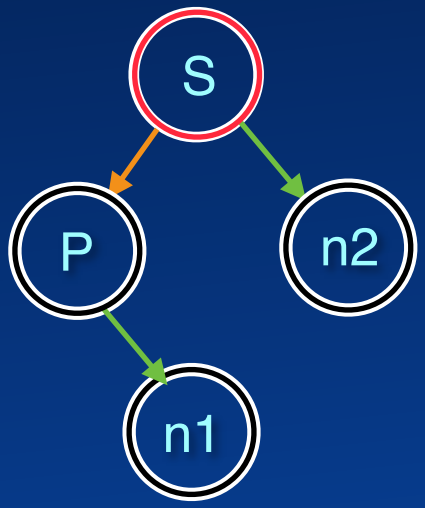

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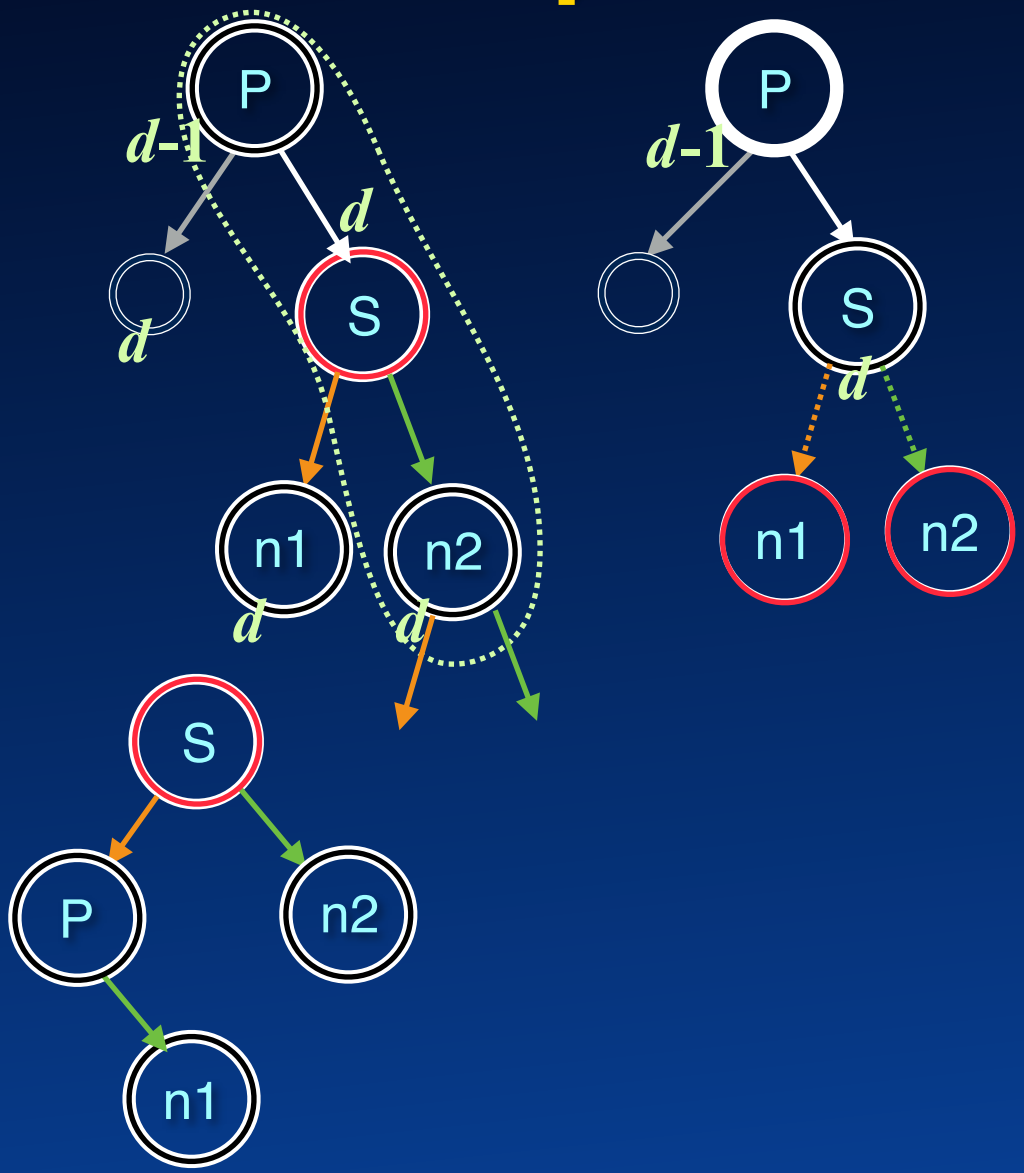

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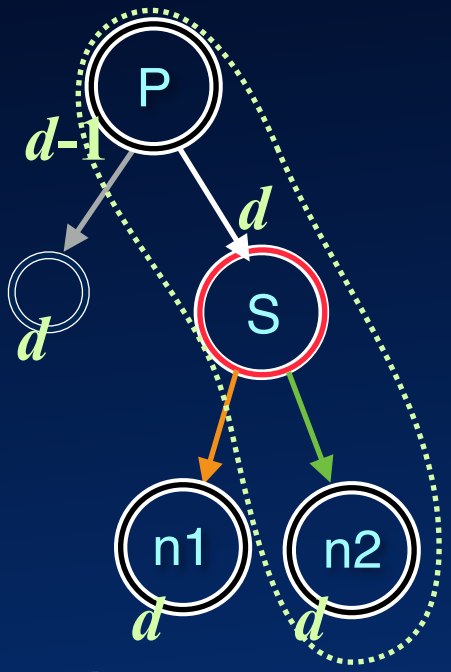

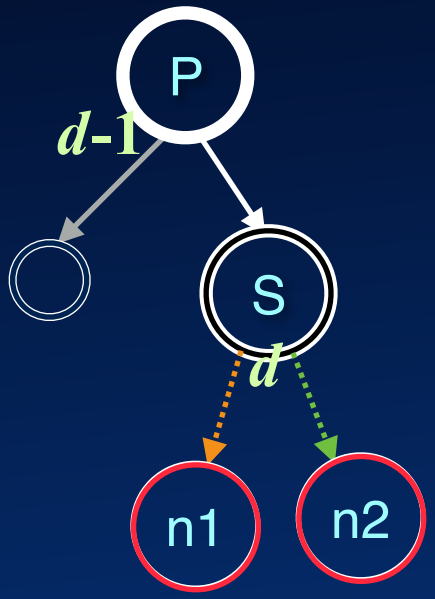

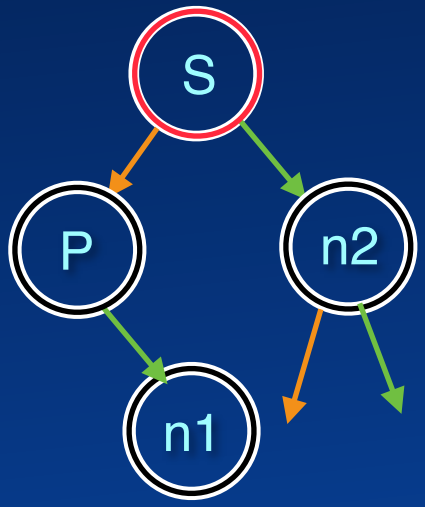

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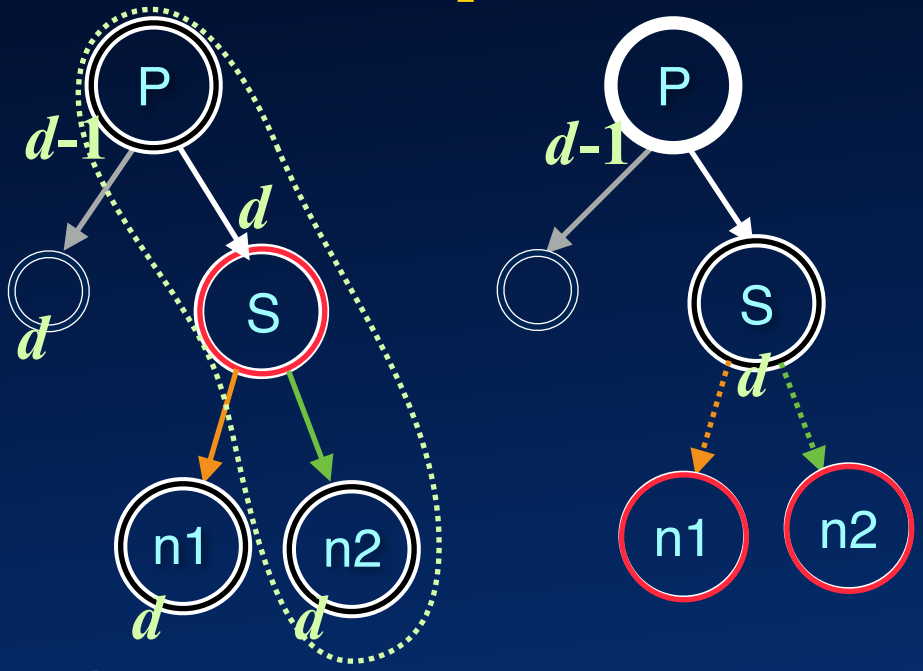

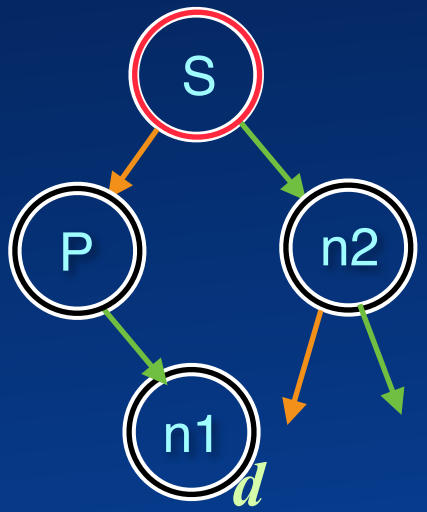

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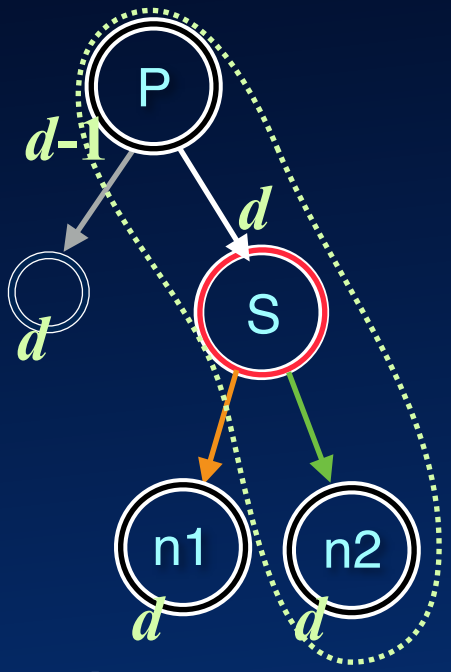

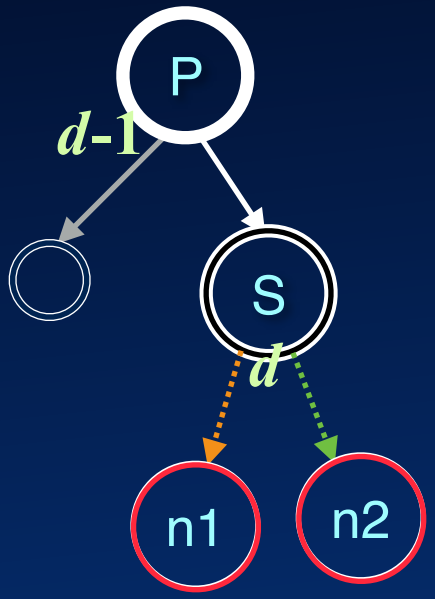

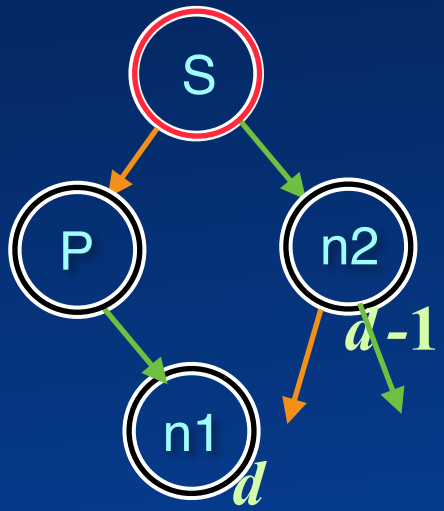

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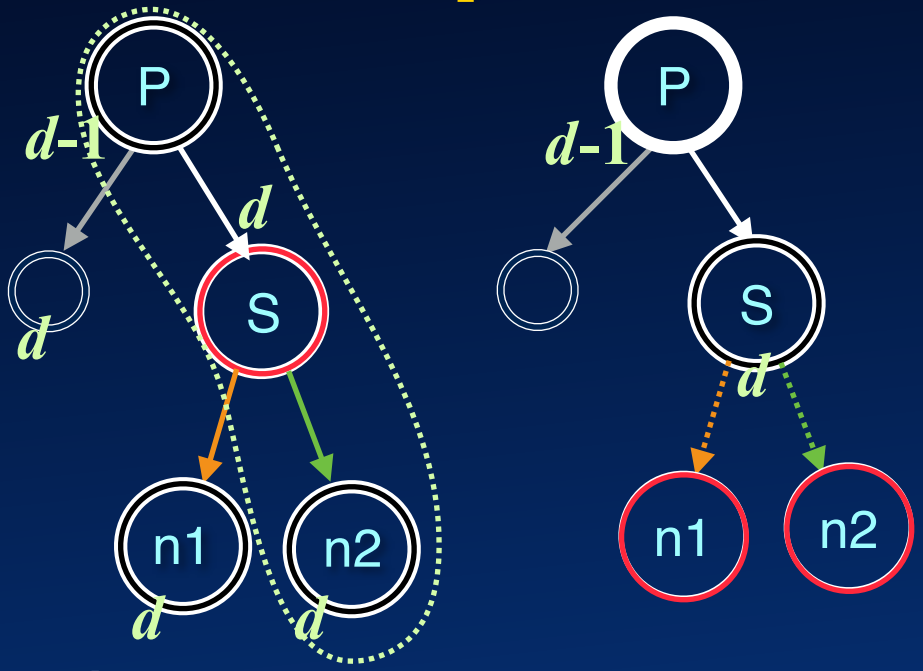

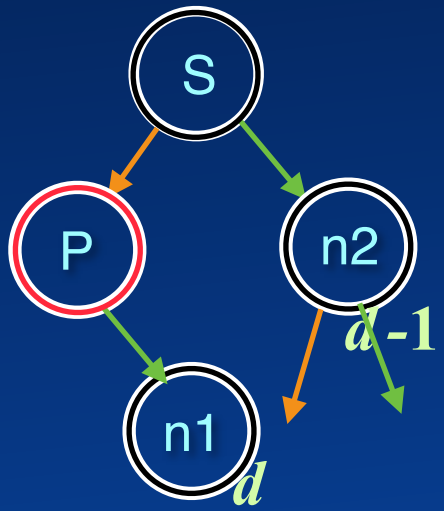

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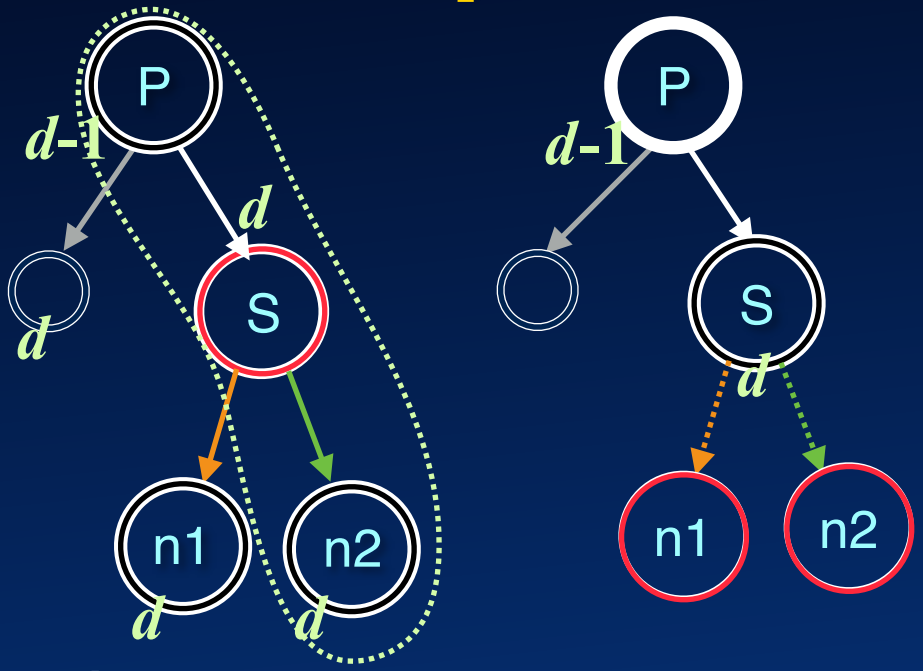

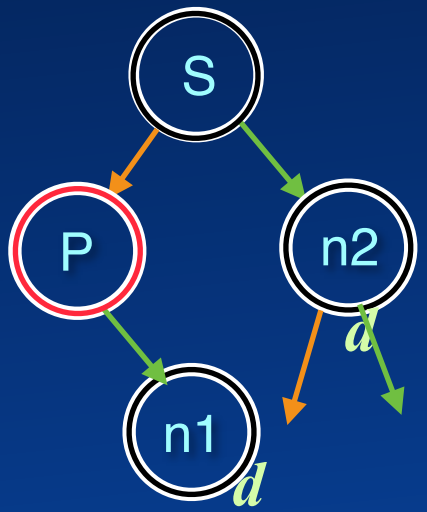

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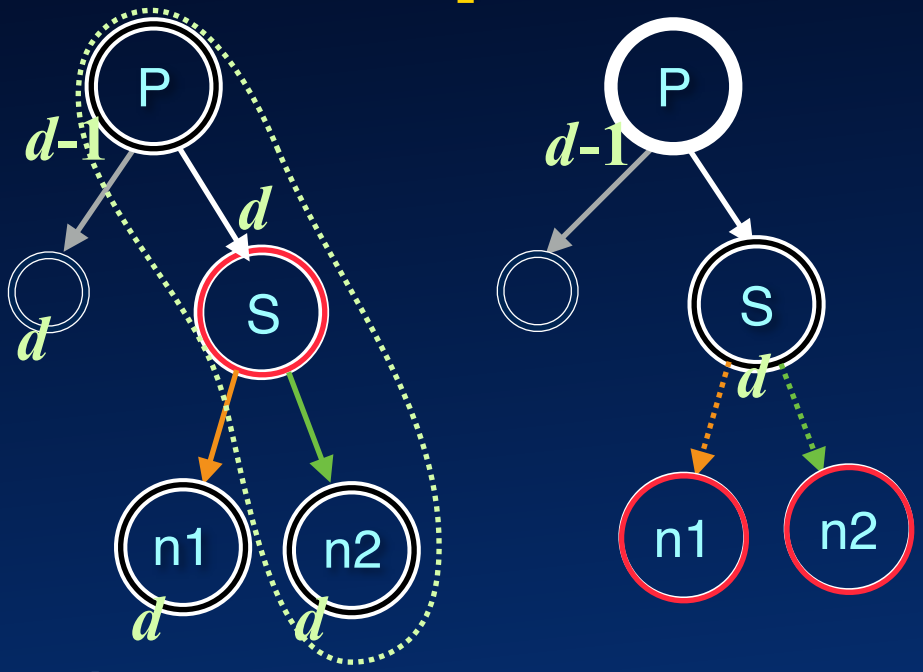

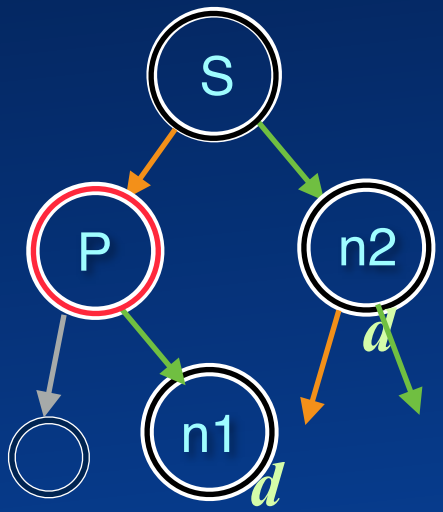

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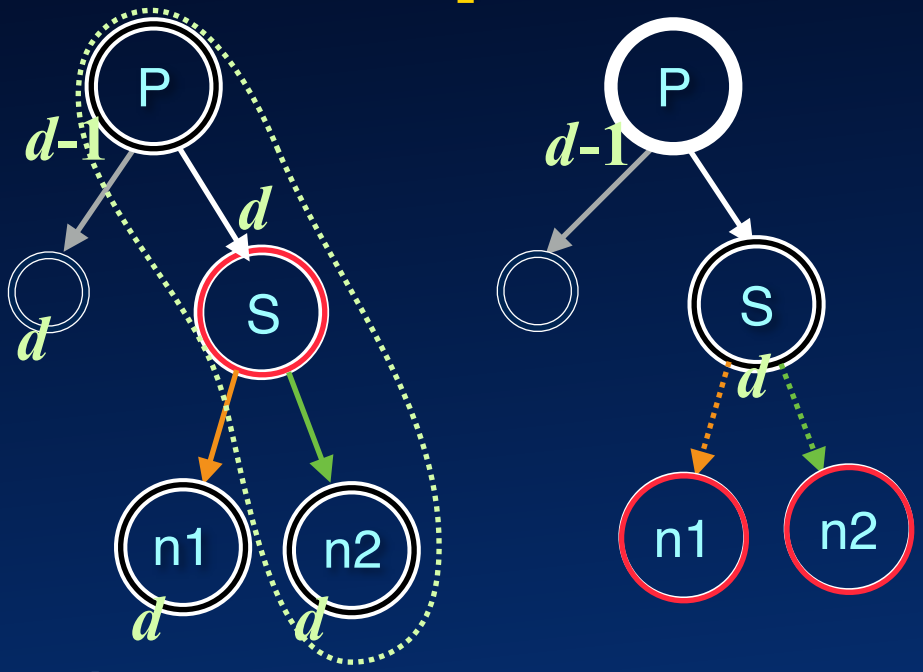

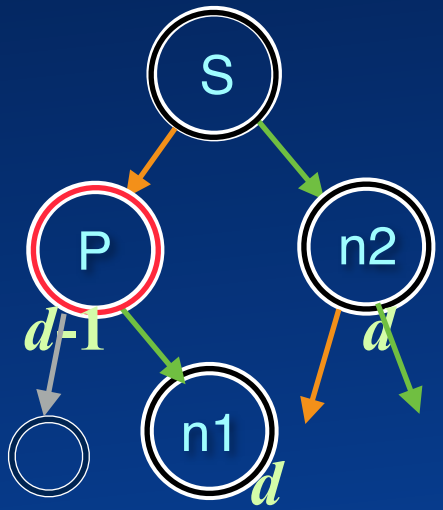

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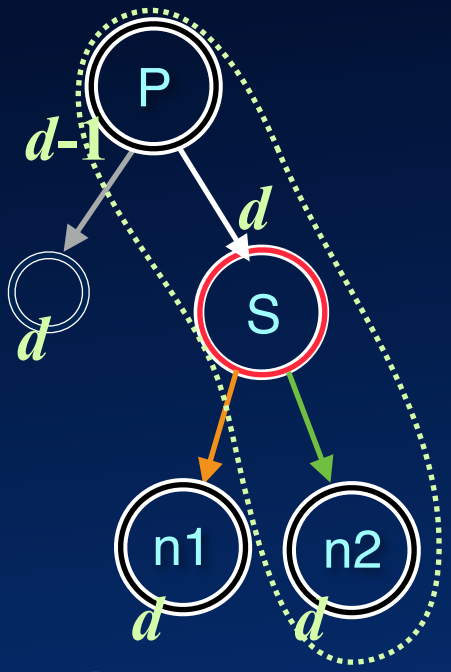

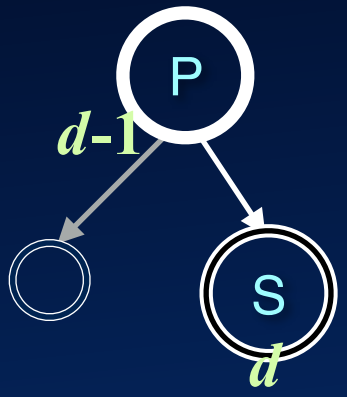

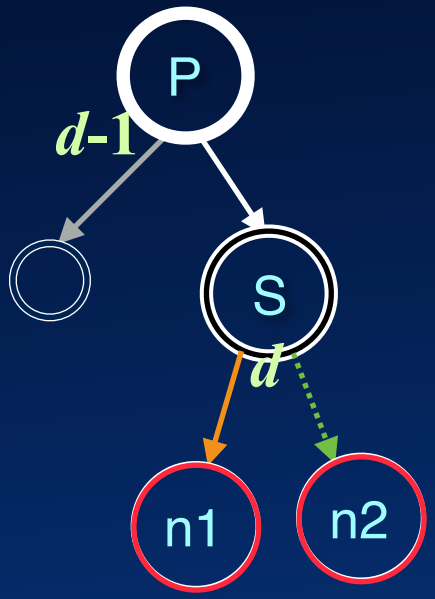

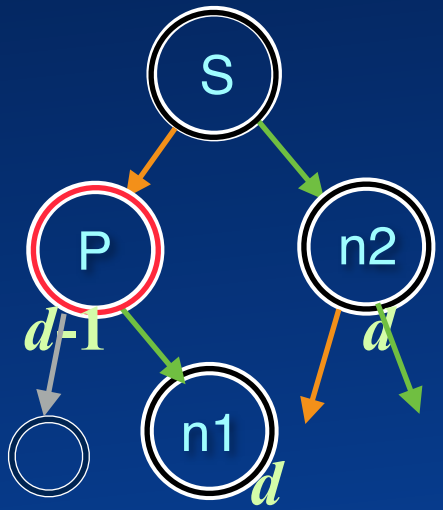

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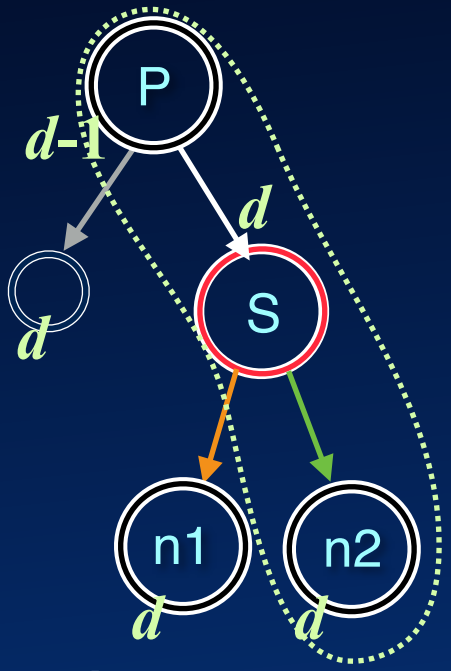
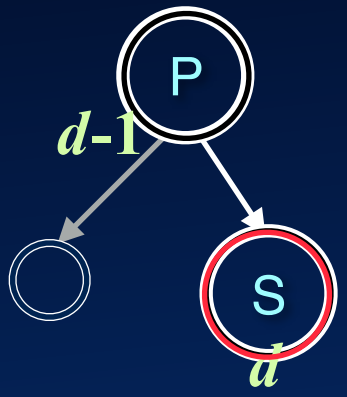
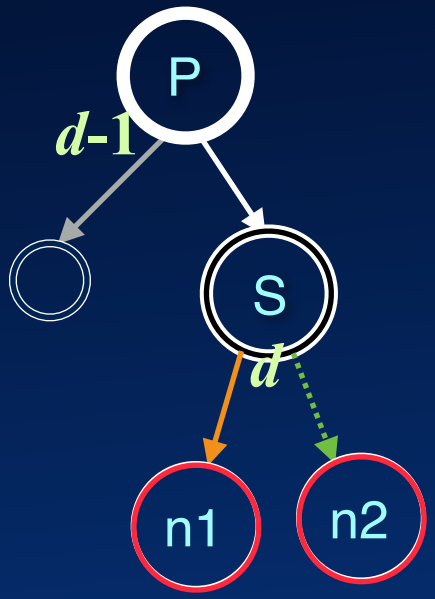
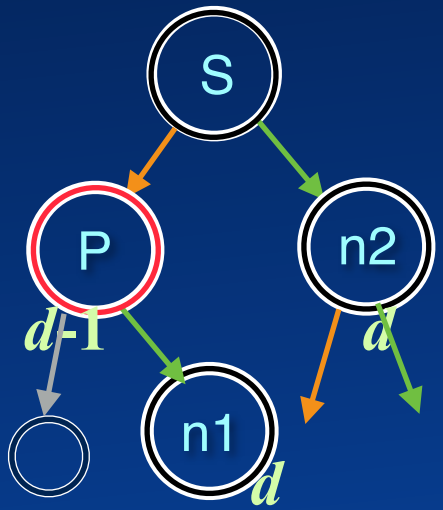
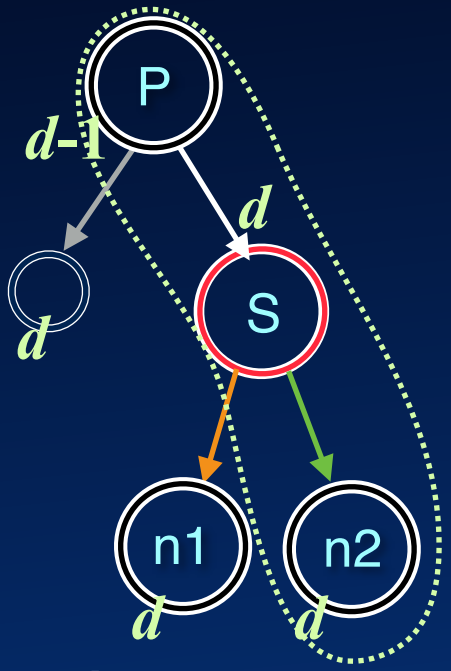


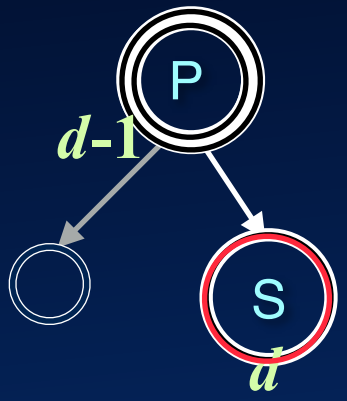


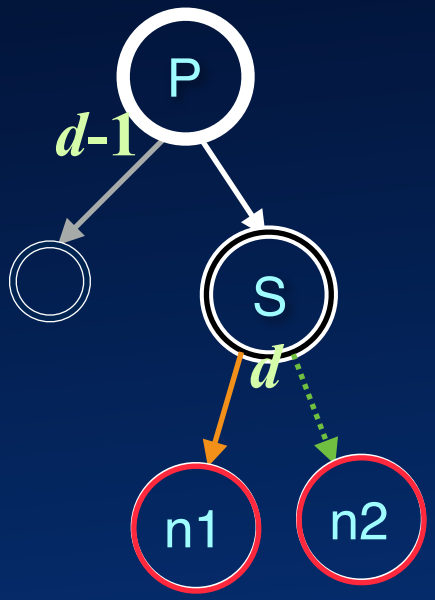


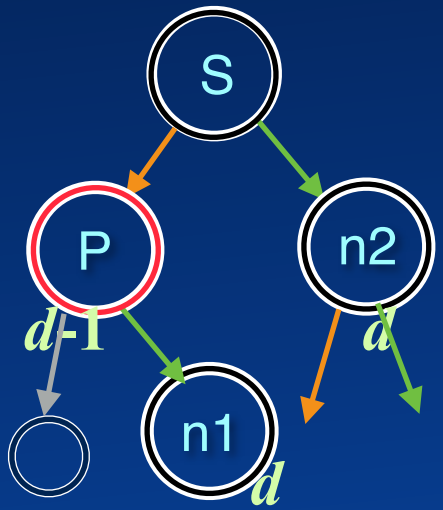


Complex Delete Cases

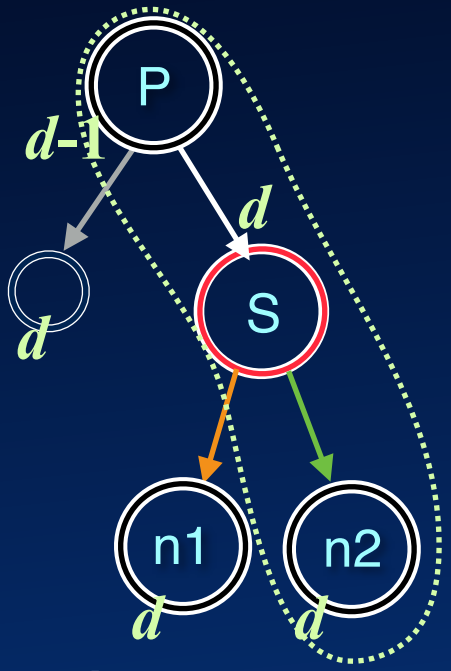


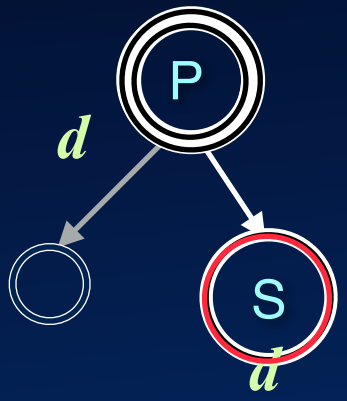


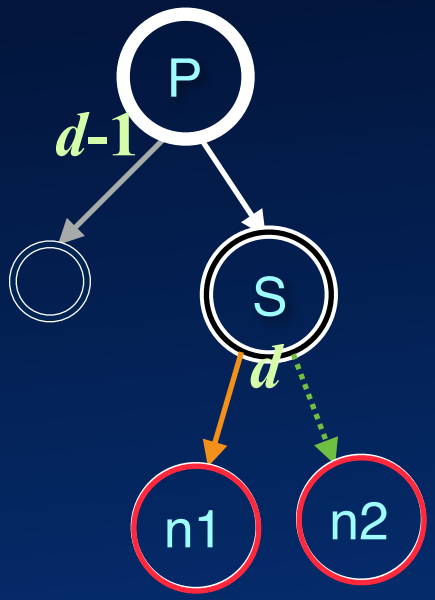


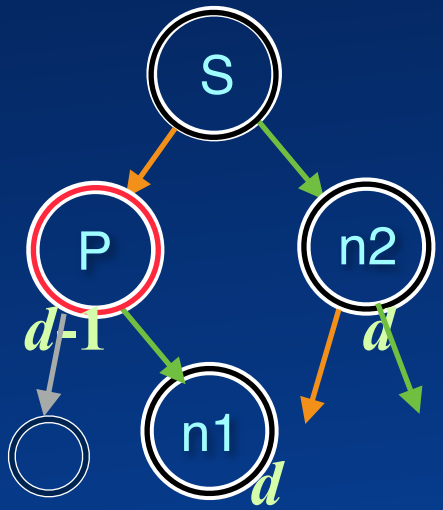


Complex Delete Cases

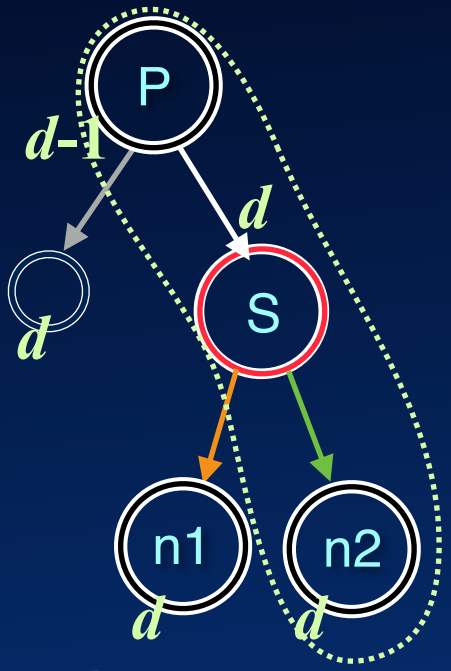


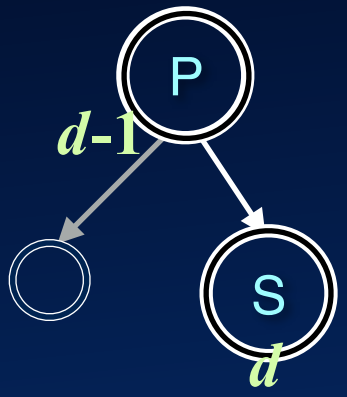


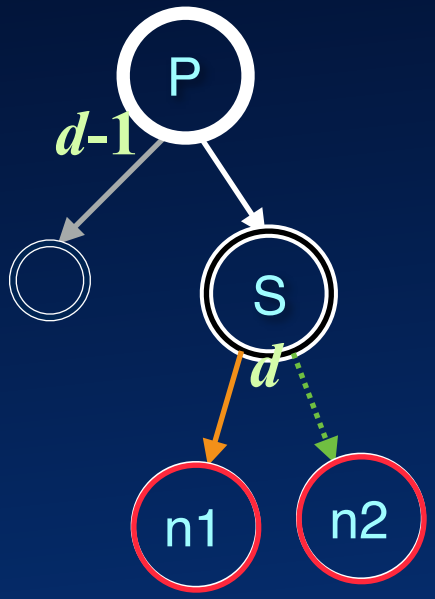


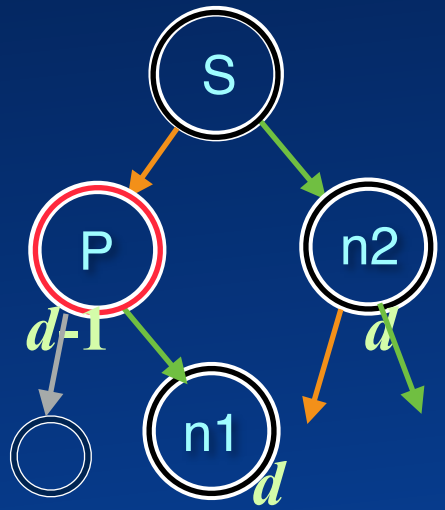


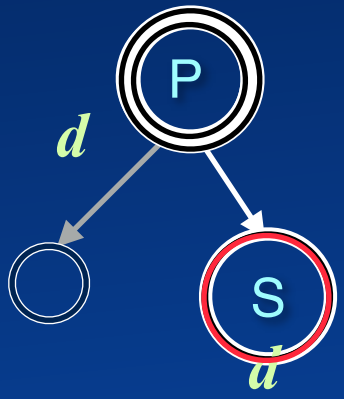
Complex Delete Cases











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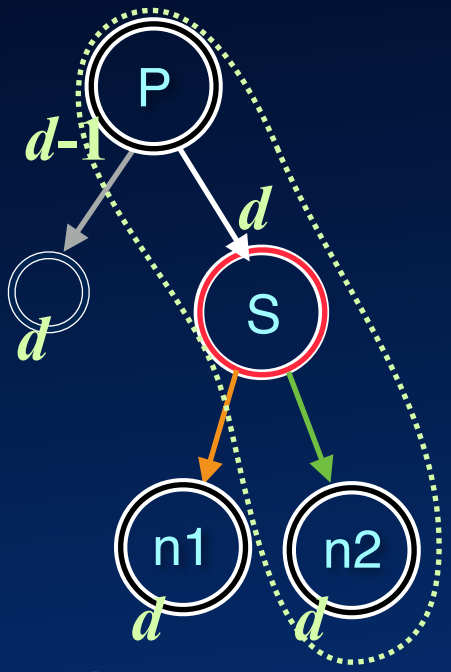

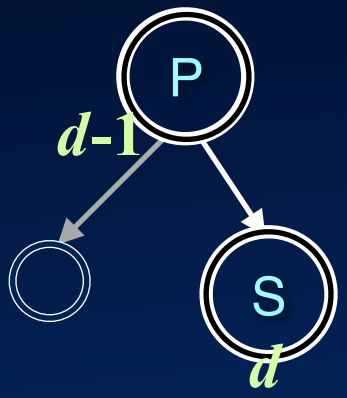

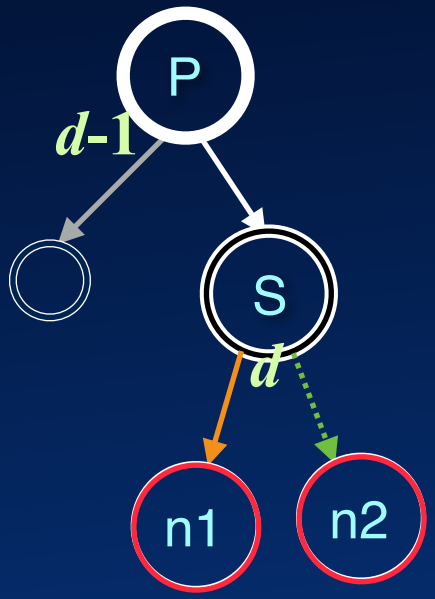

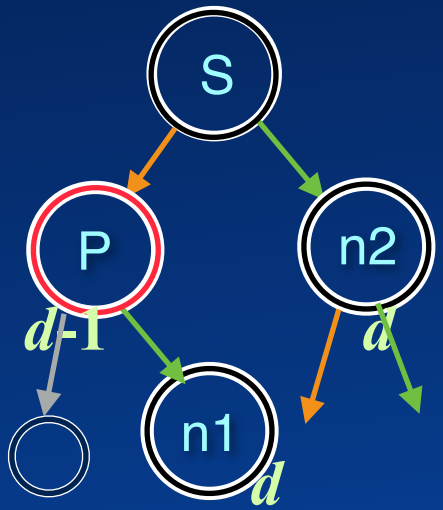

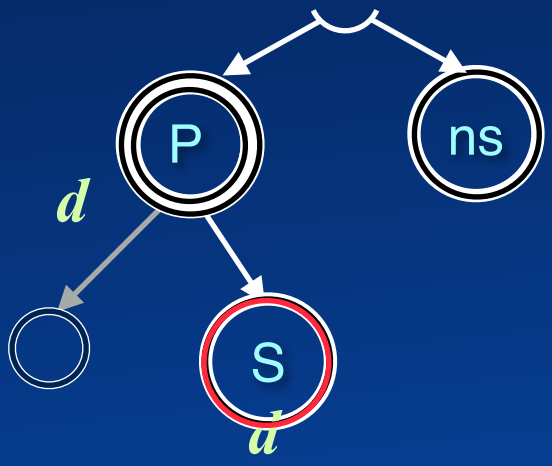

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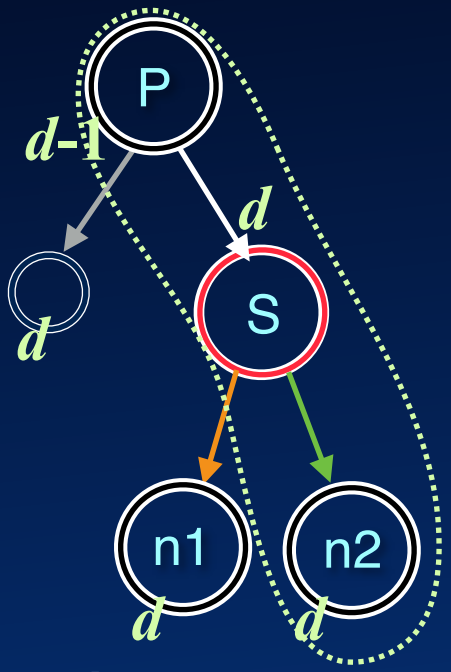

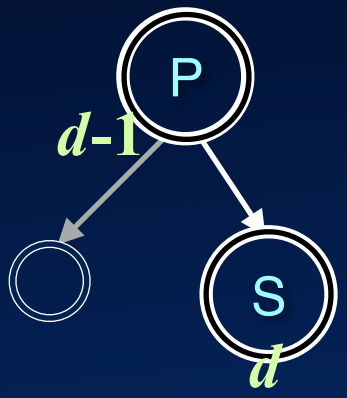

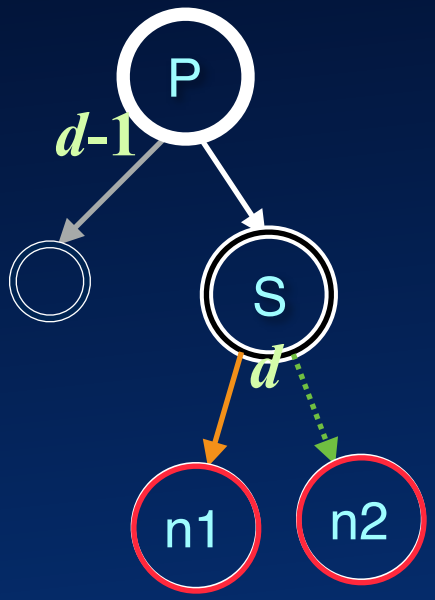

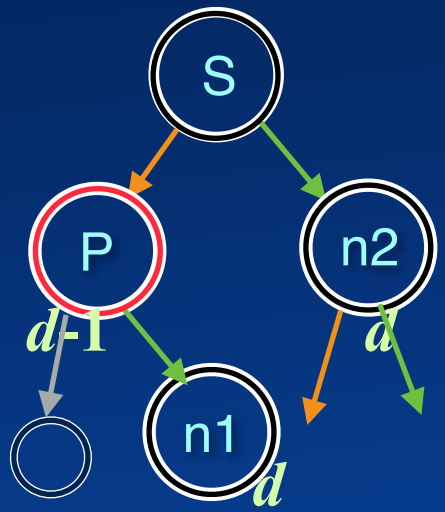

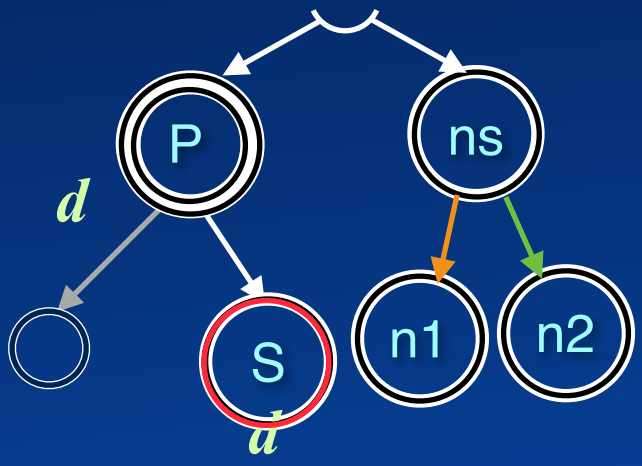

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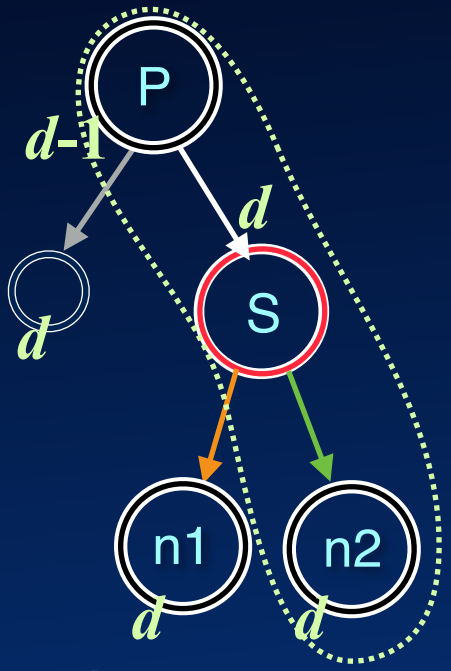

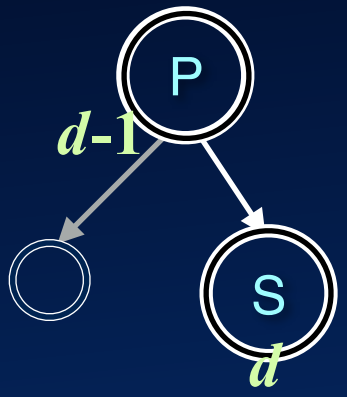

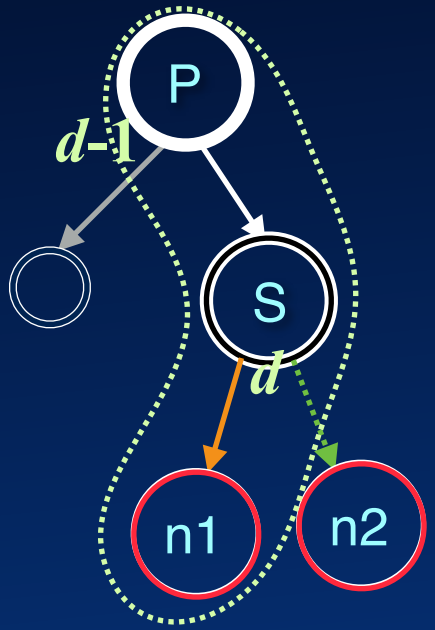

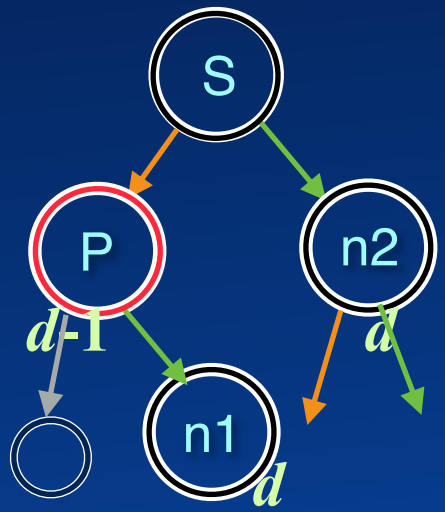

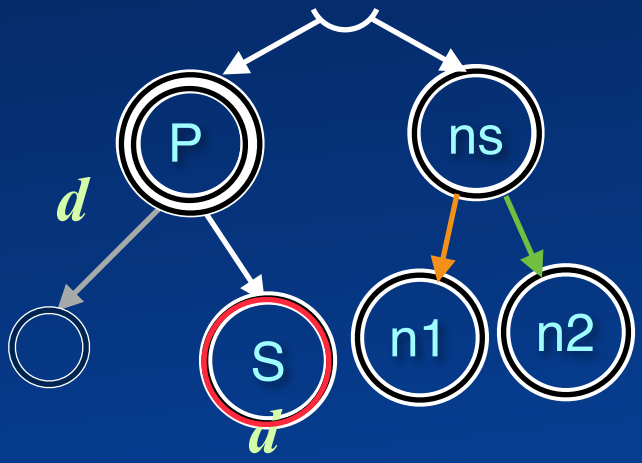

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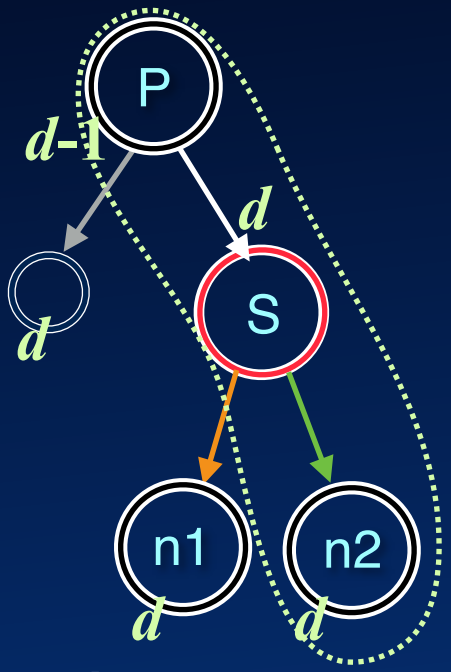

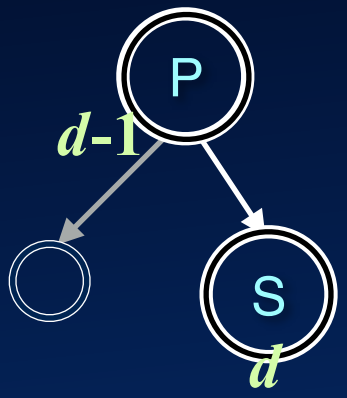

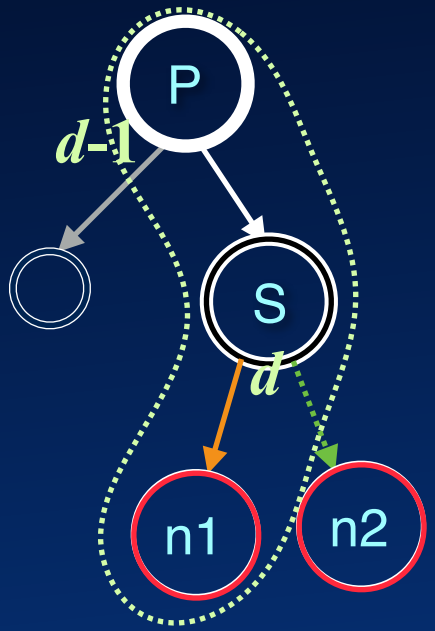

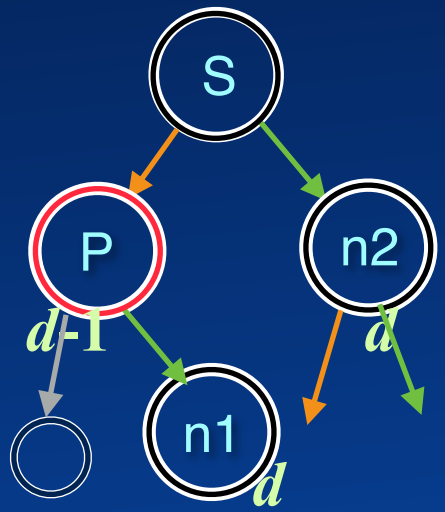

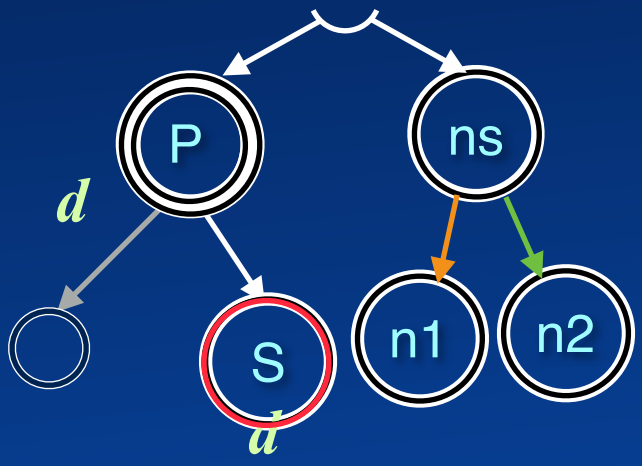

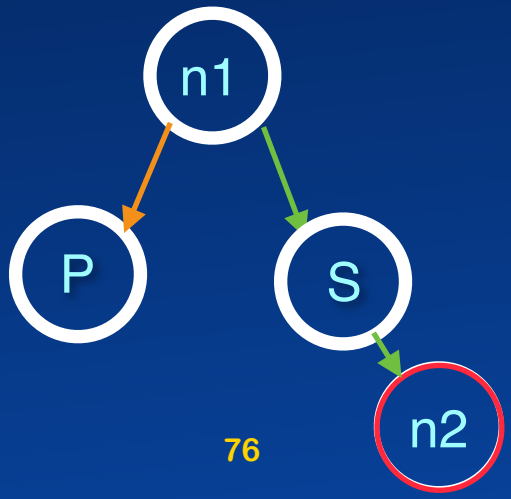

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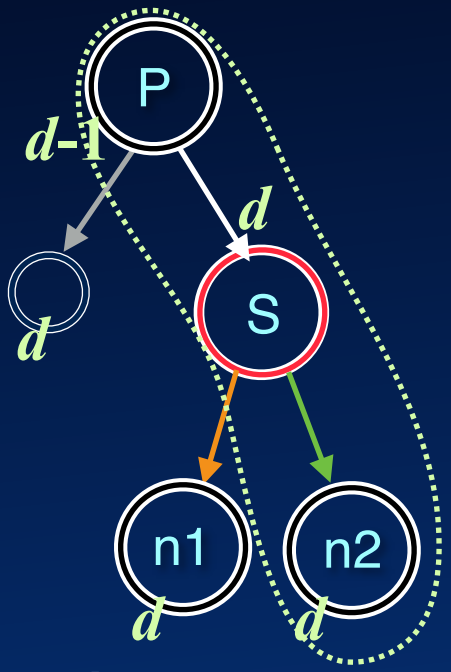

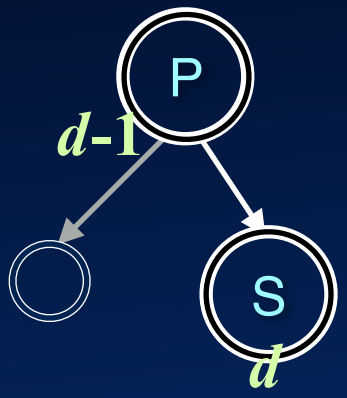

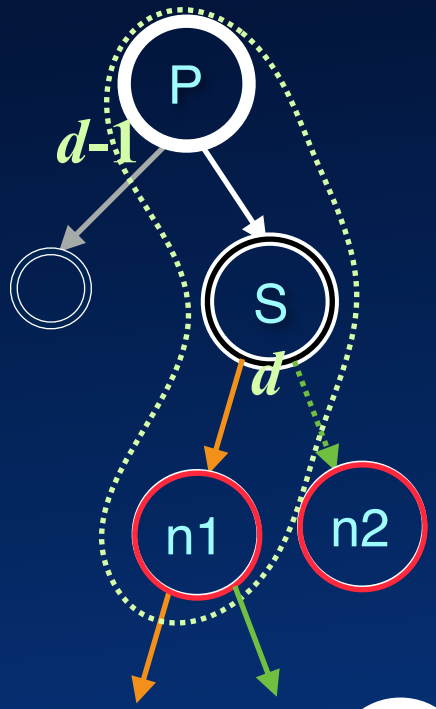

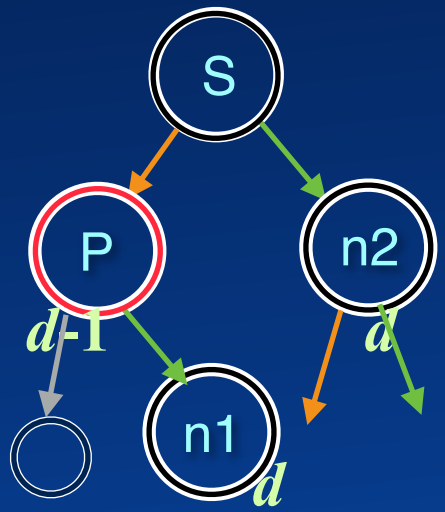

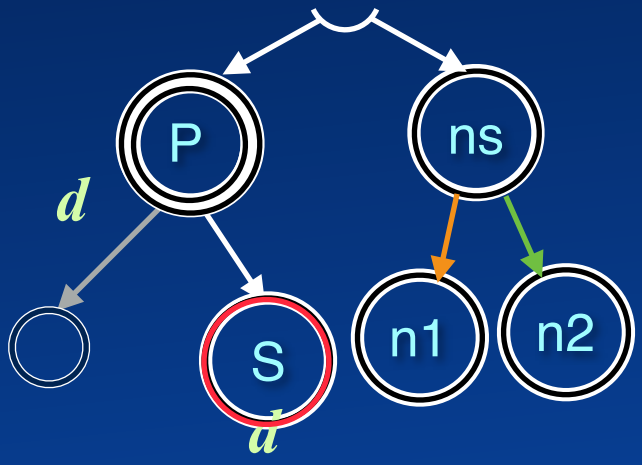

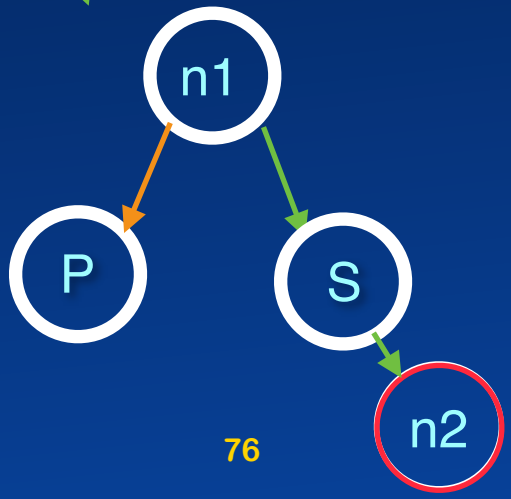

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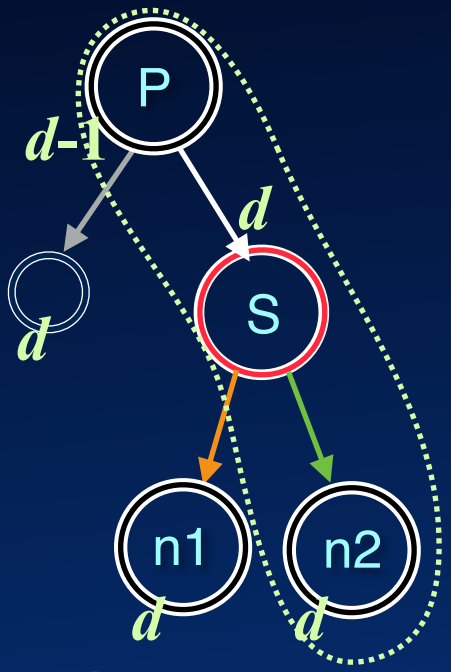

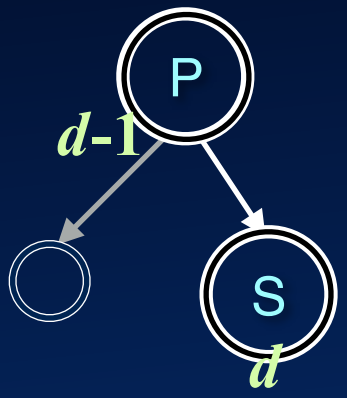

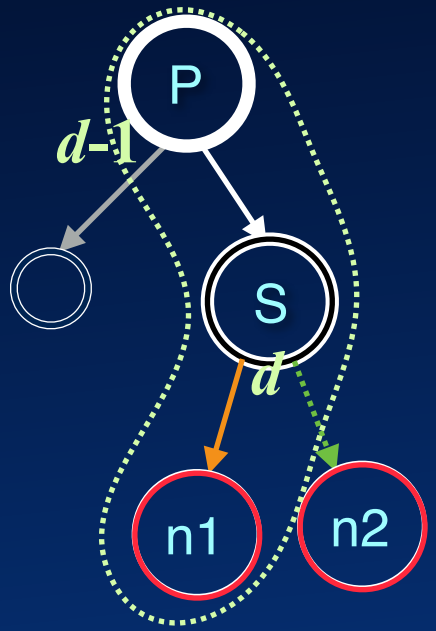

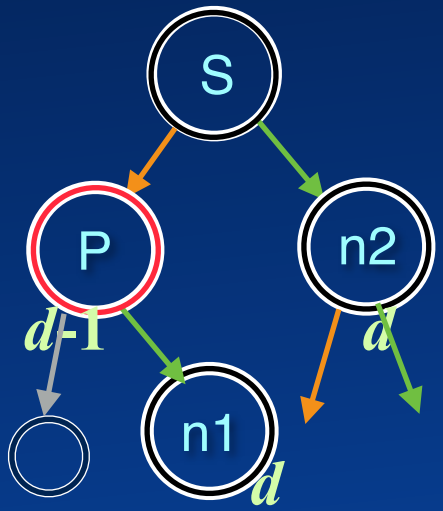

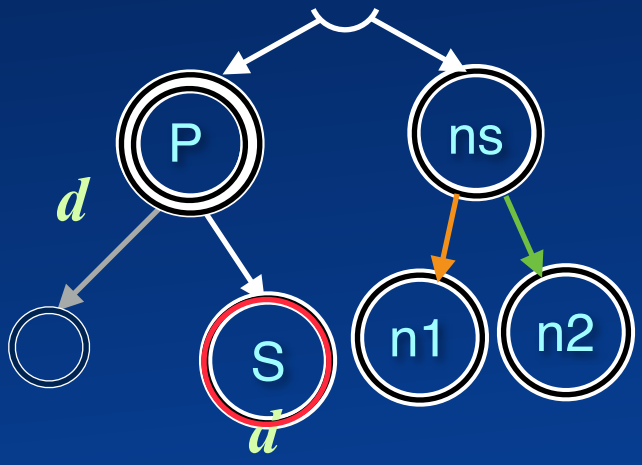

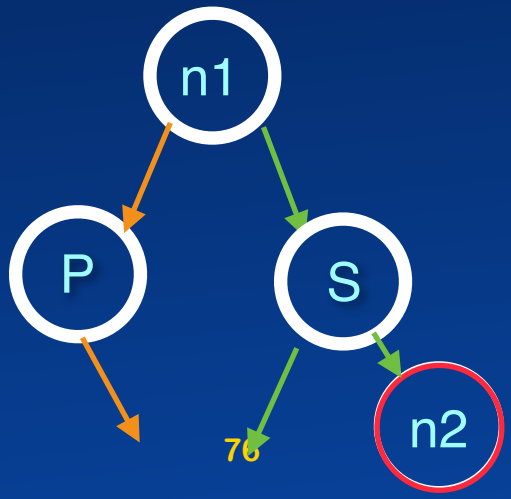

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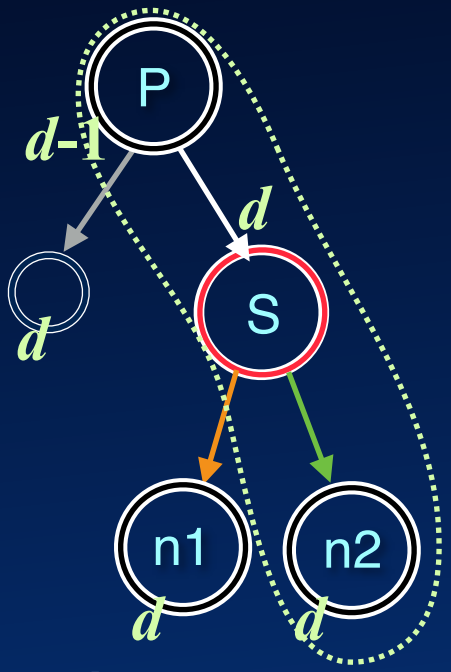

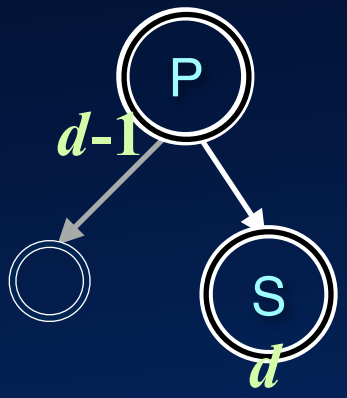

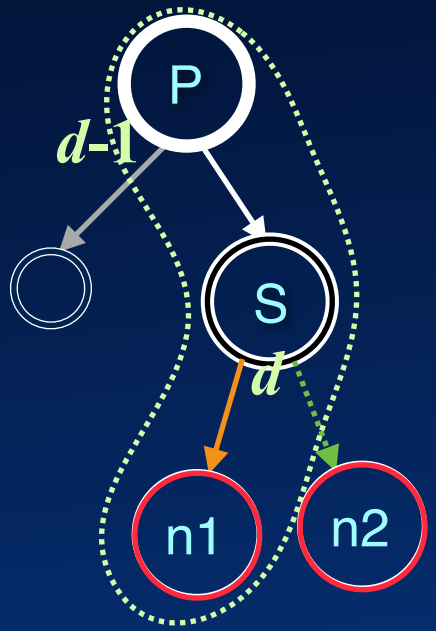

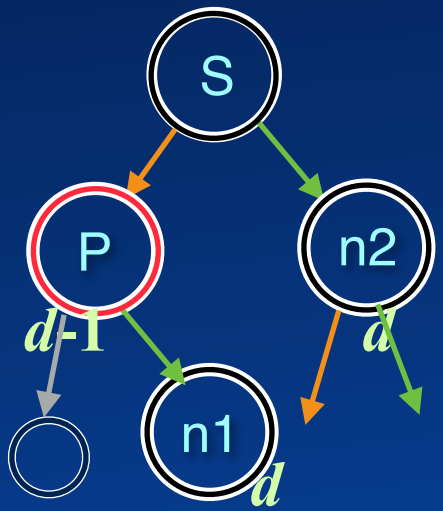

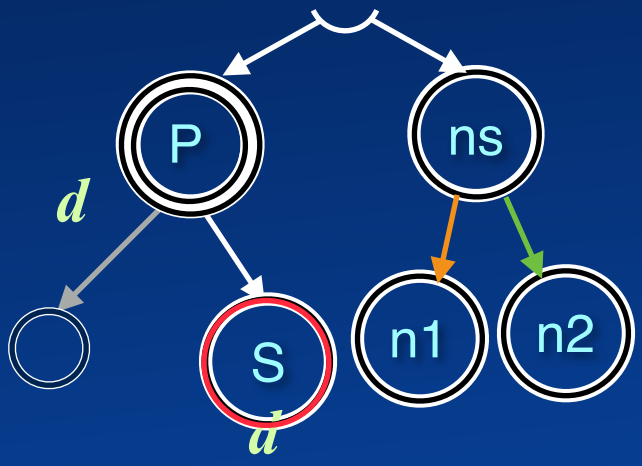

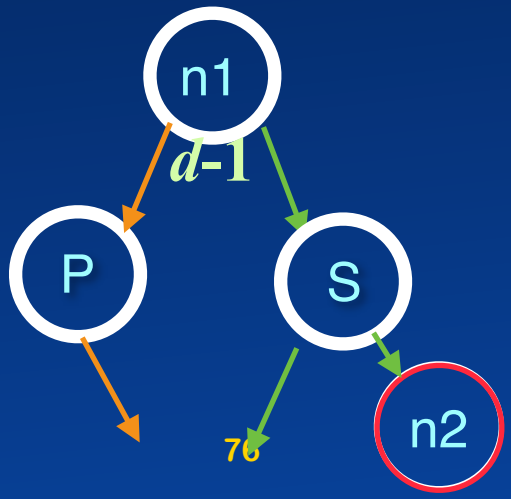

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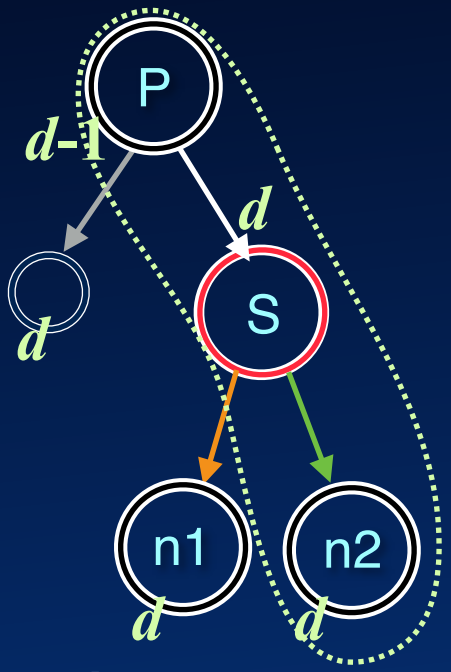

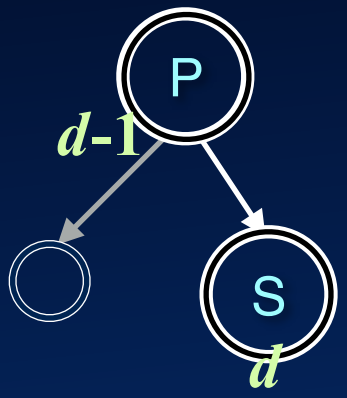

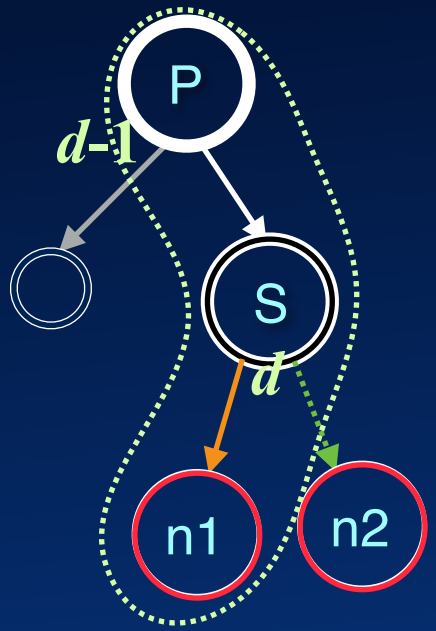

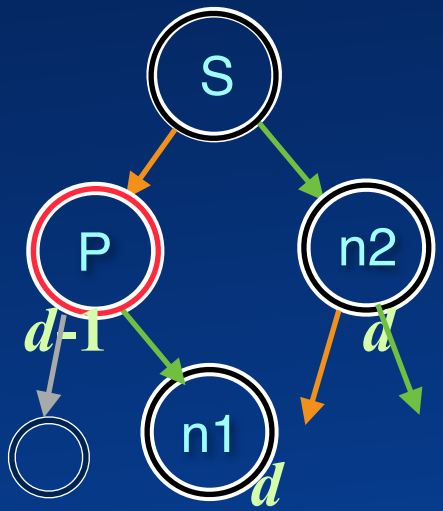

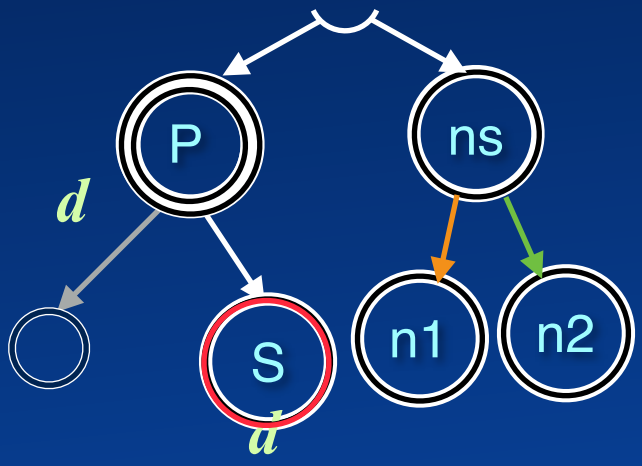

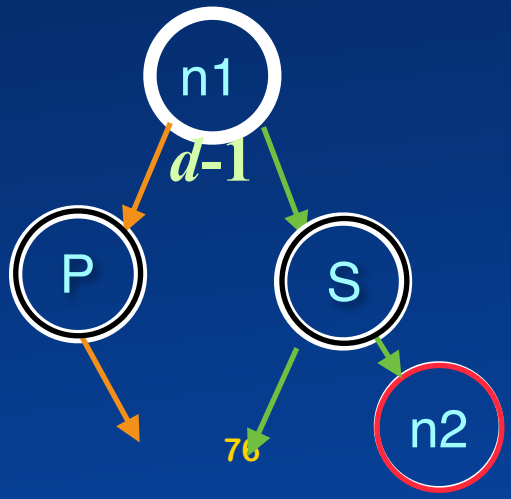

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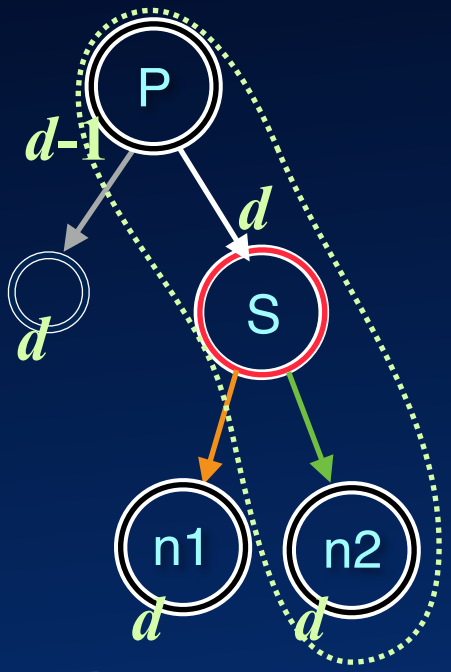

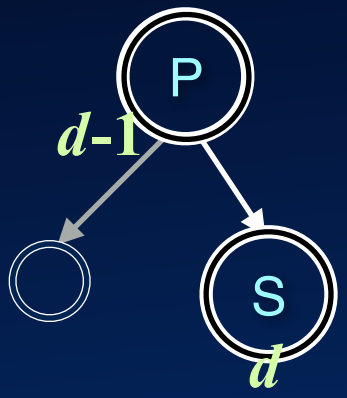

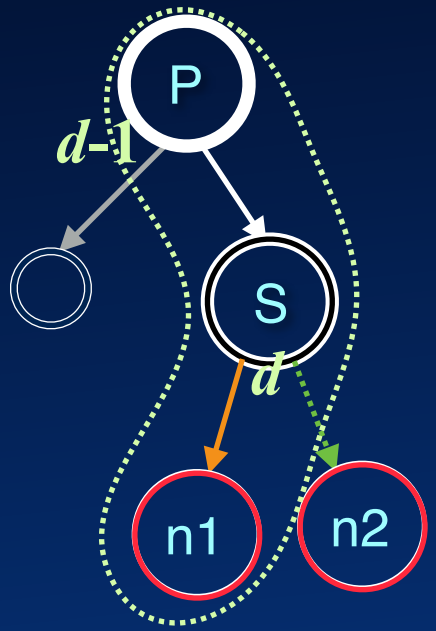

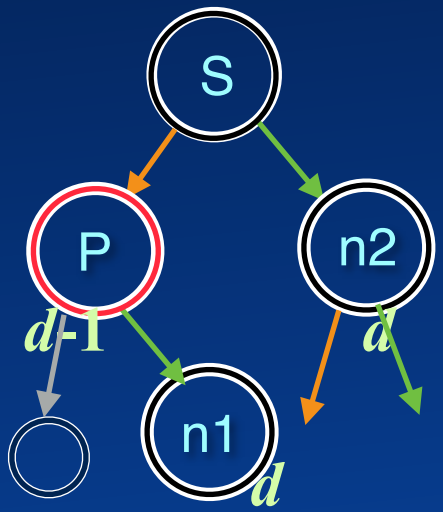

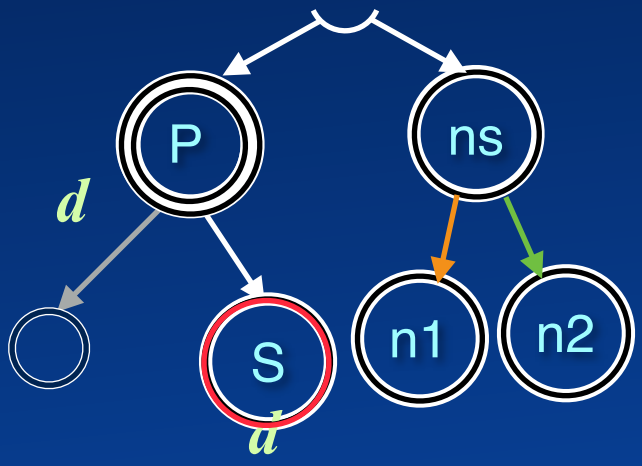

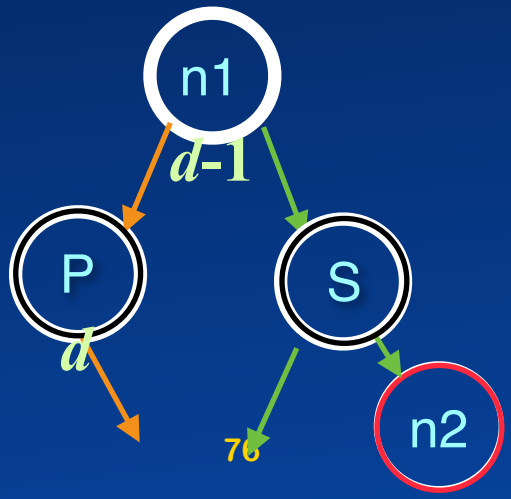

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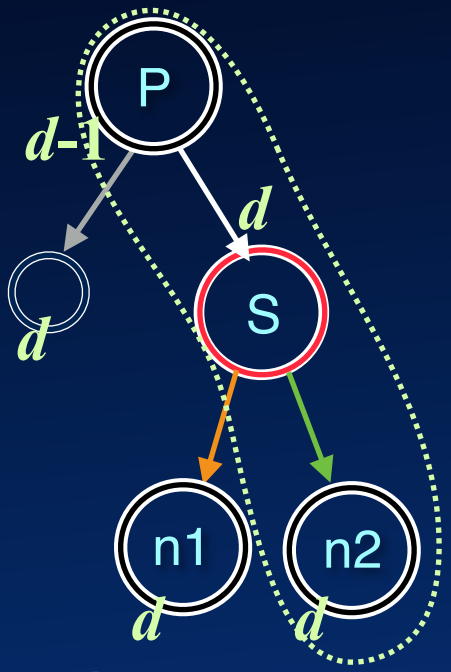
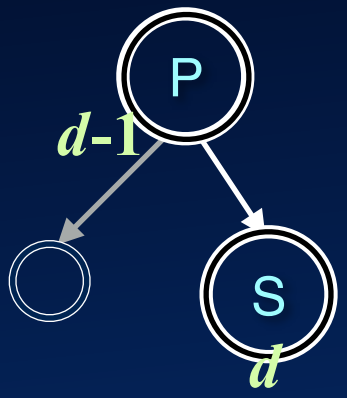
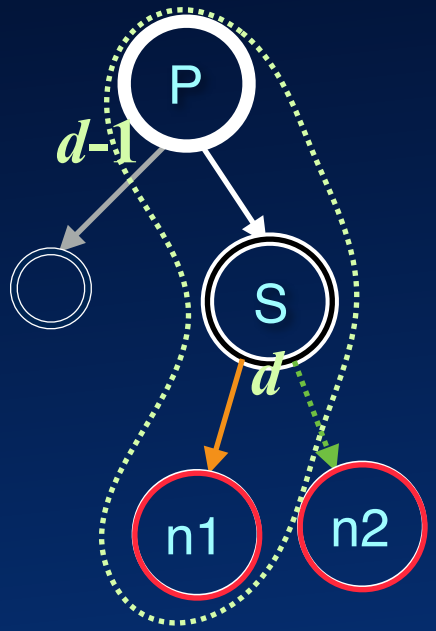
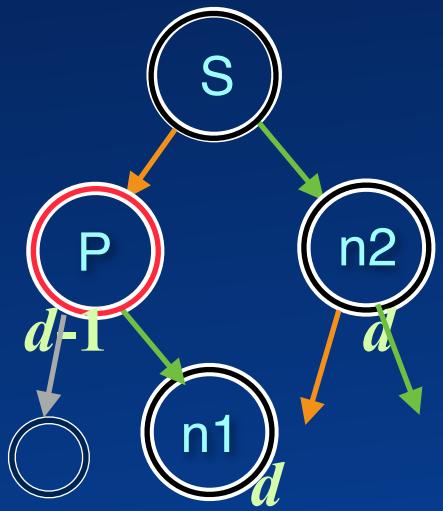
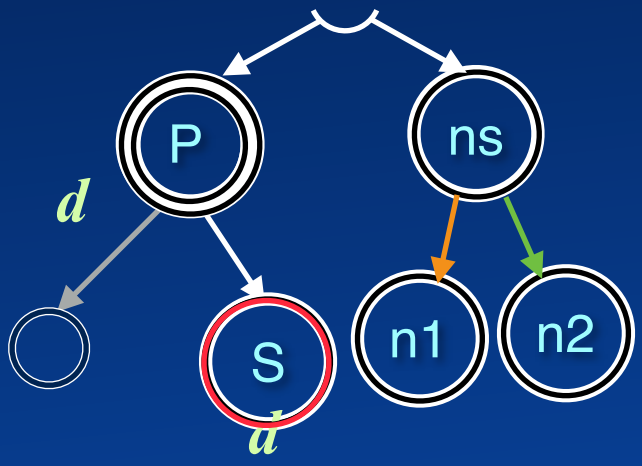
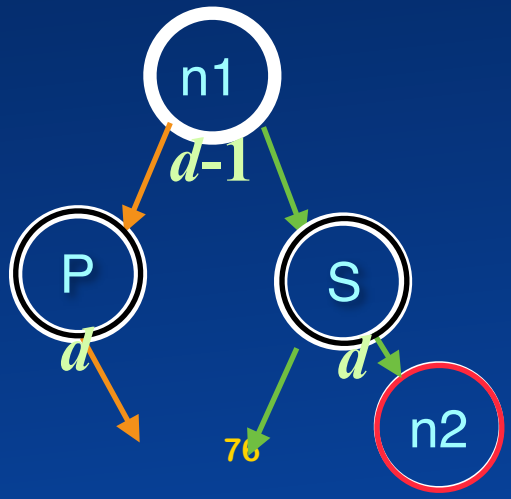




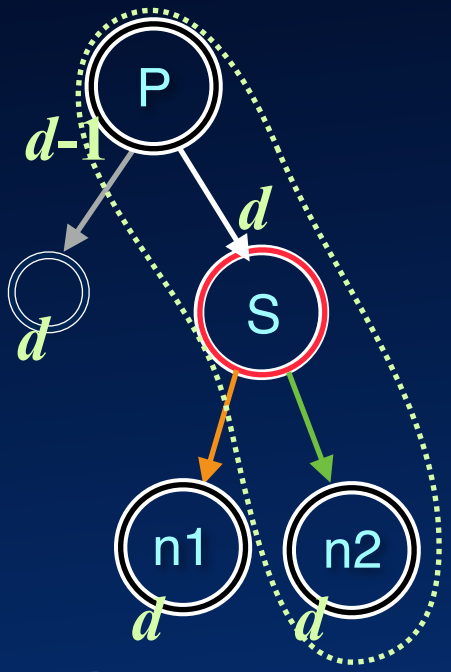


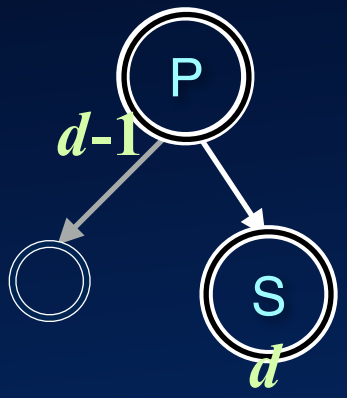


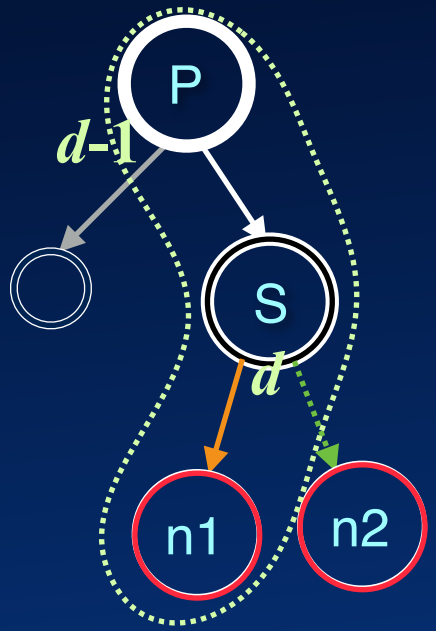


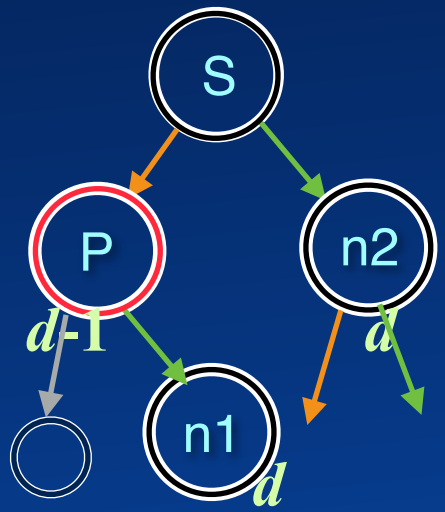


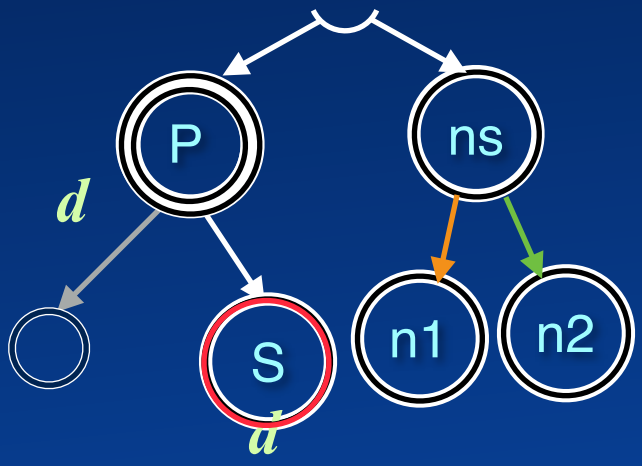
Complex Delete Cases

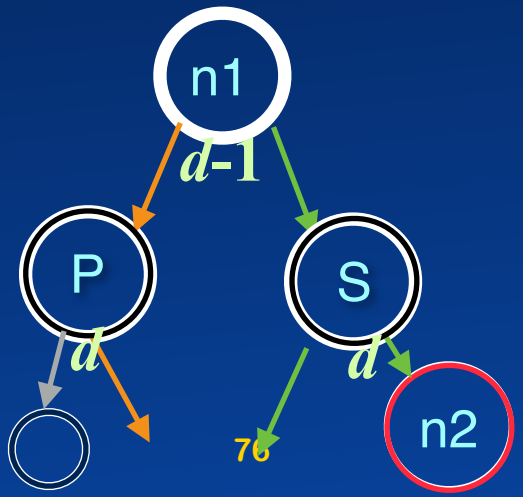








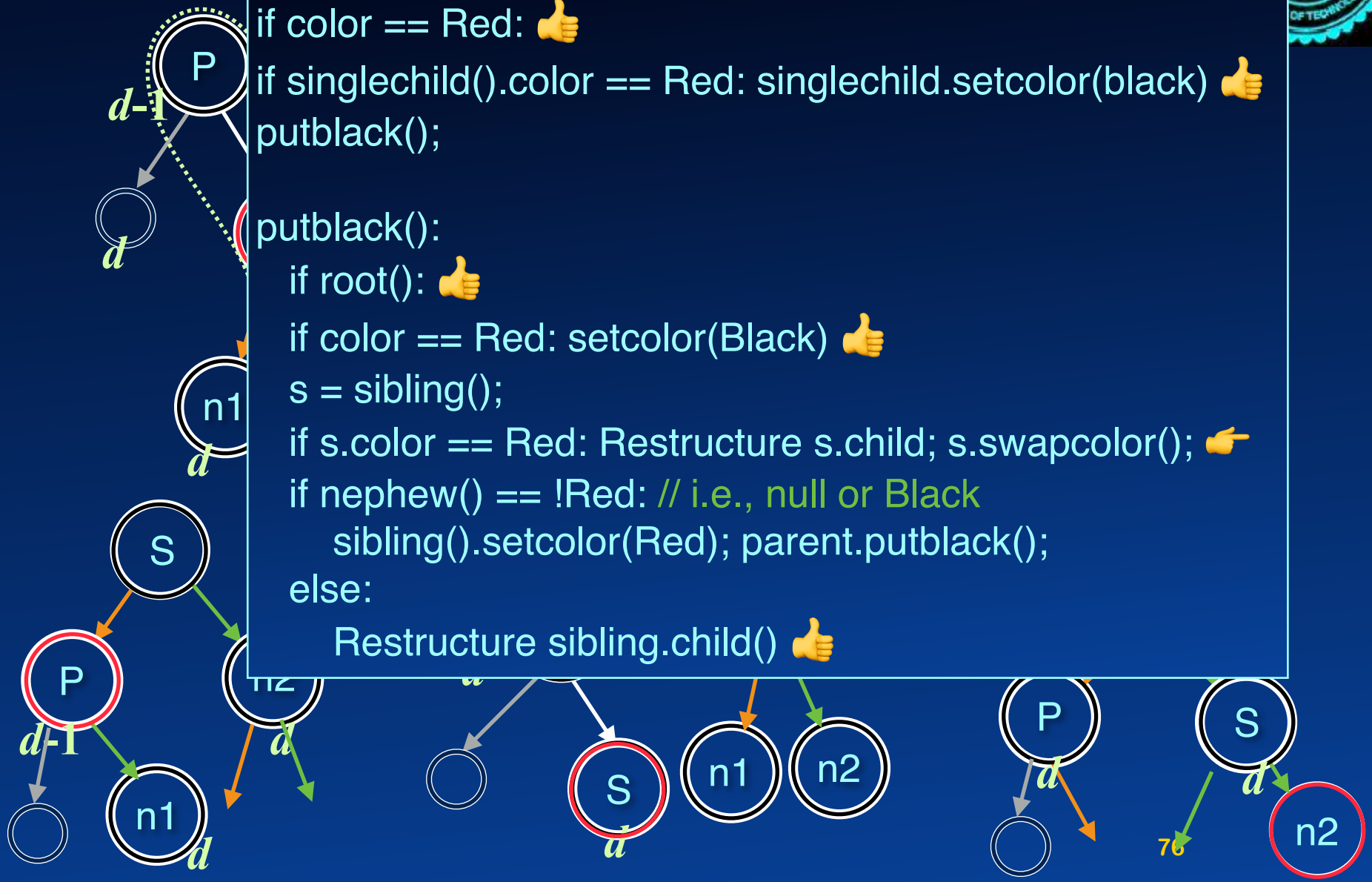




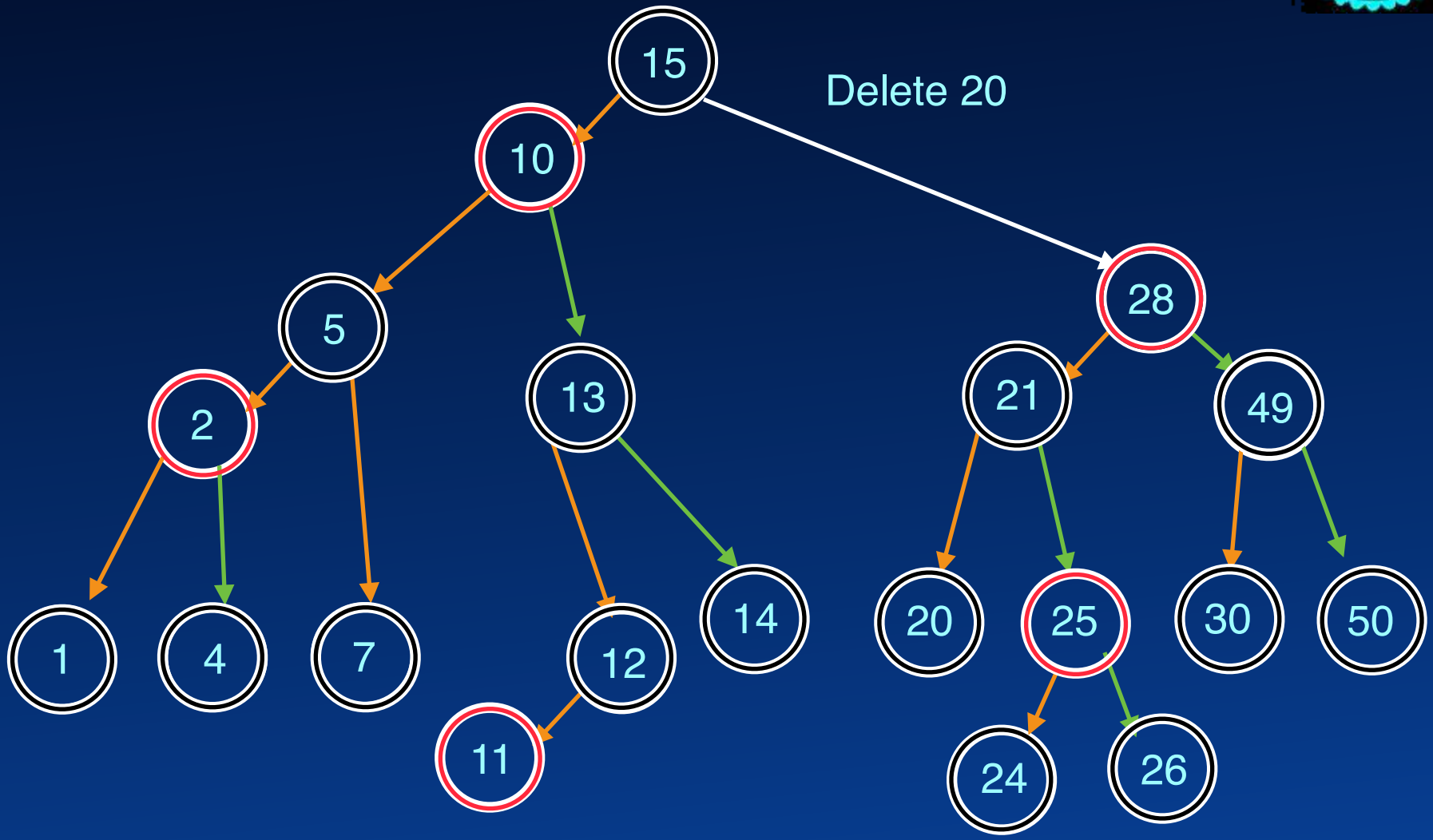
Complex Delete Cases

```
if color == Red: 👍
if singlechild().color == Red: singlechild.setcolor(black) 👍
putblack();

putblack():
   if root(): 👍
   if color == Red: setcolor(Black) 👍
   s = sibling();
   if s.color == Red: Restructure s.child; s.swapcolor(); 👉
   if nephew() == !Red: // i.e., null or Black
       sibling().setcolor(Red); parent.putblack();
   else:
       Restructure sibling.child() 👍
```

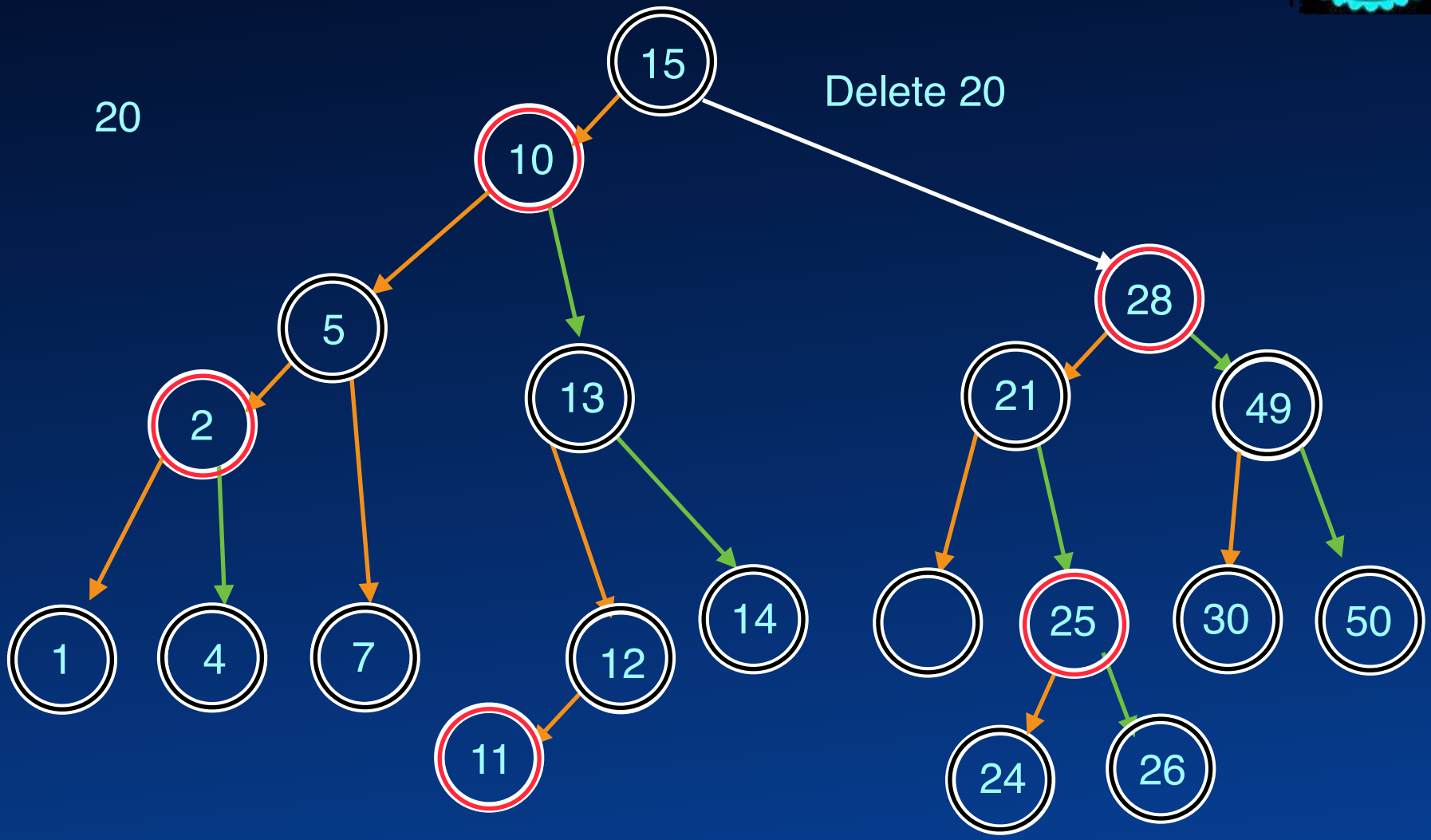


Red-Black Tree



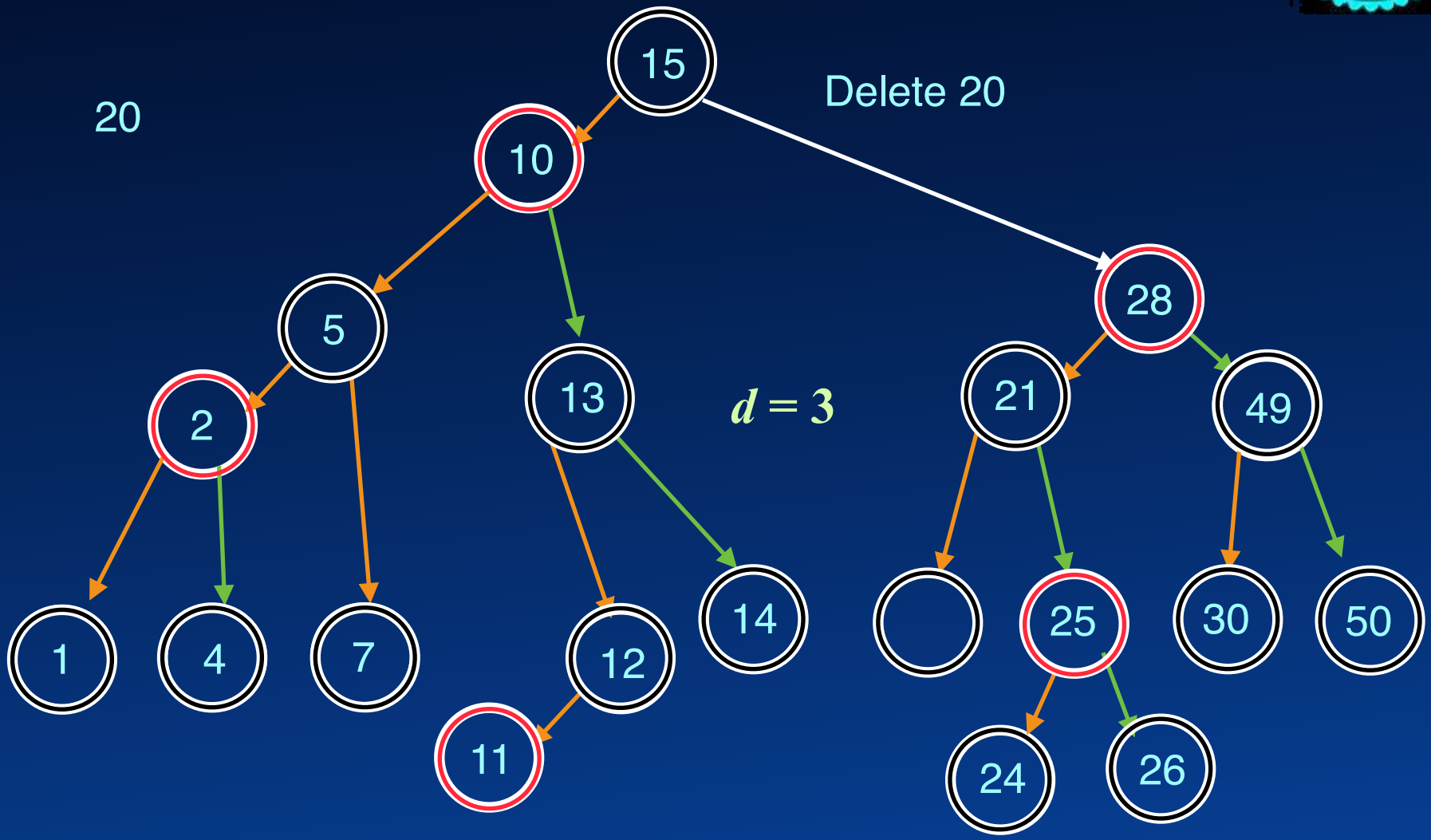
Expunge node with @null in node ***n***

Red-Black Tree



Expunge node with @null in node ***n***

Red-Black Tree



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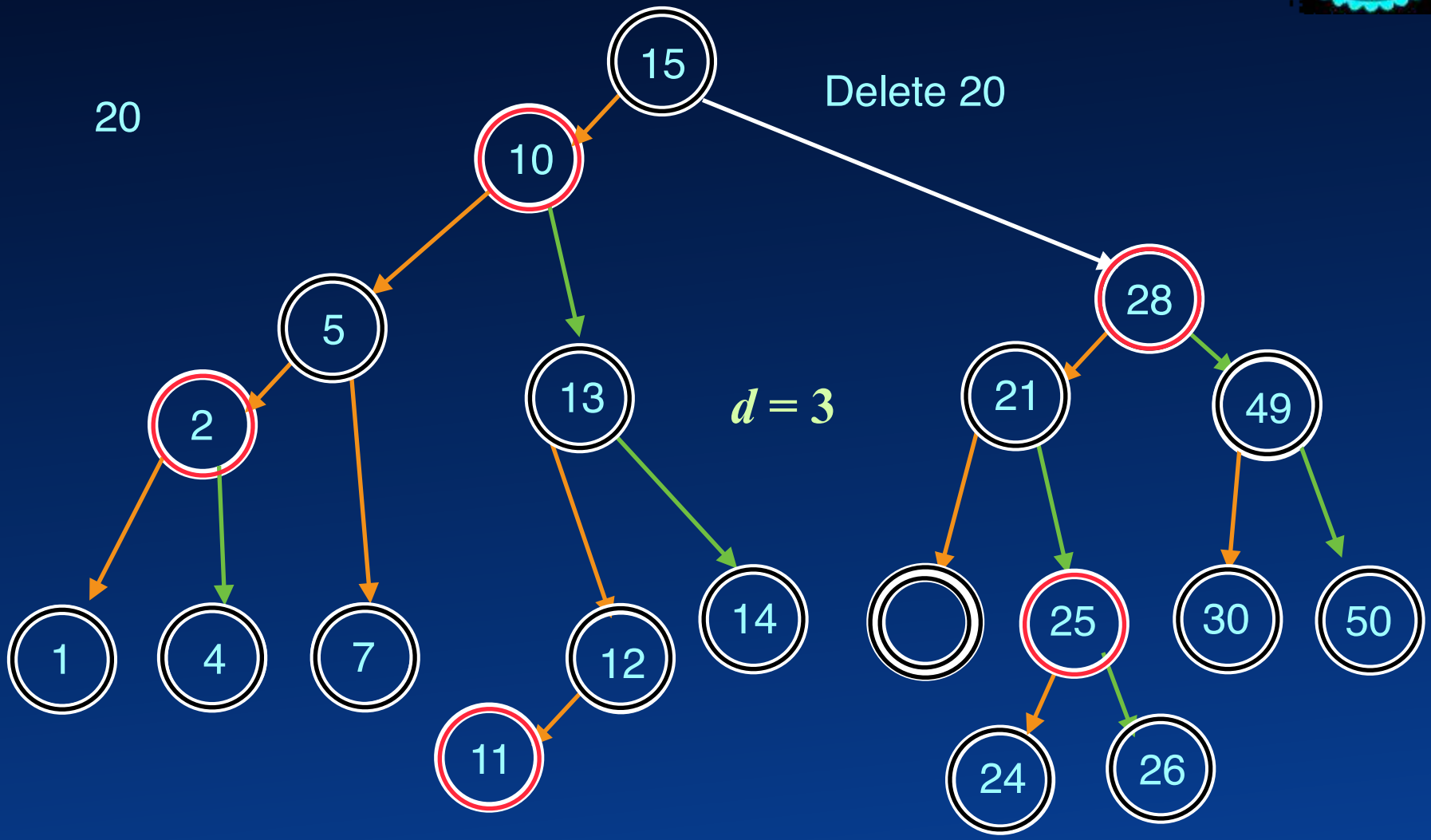
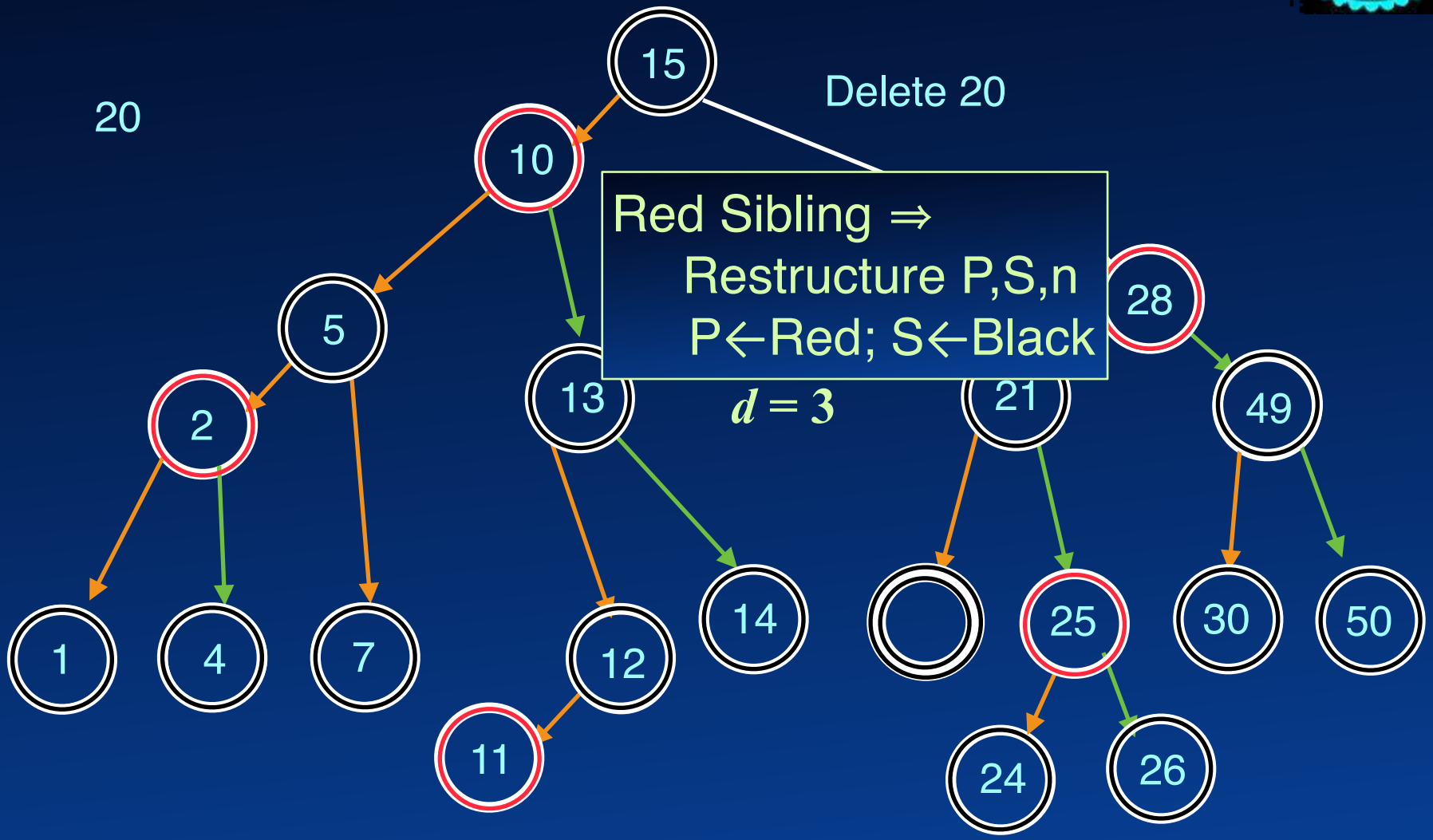


Expunge node with @null in node ***n***

Red-Black Tree



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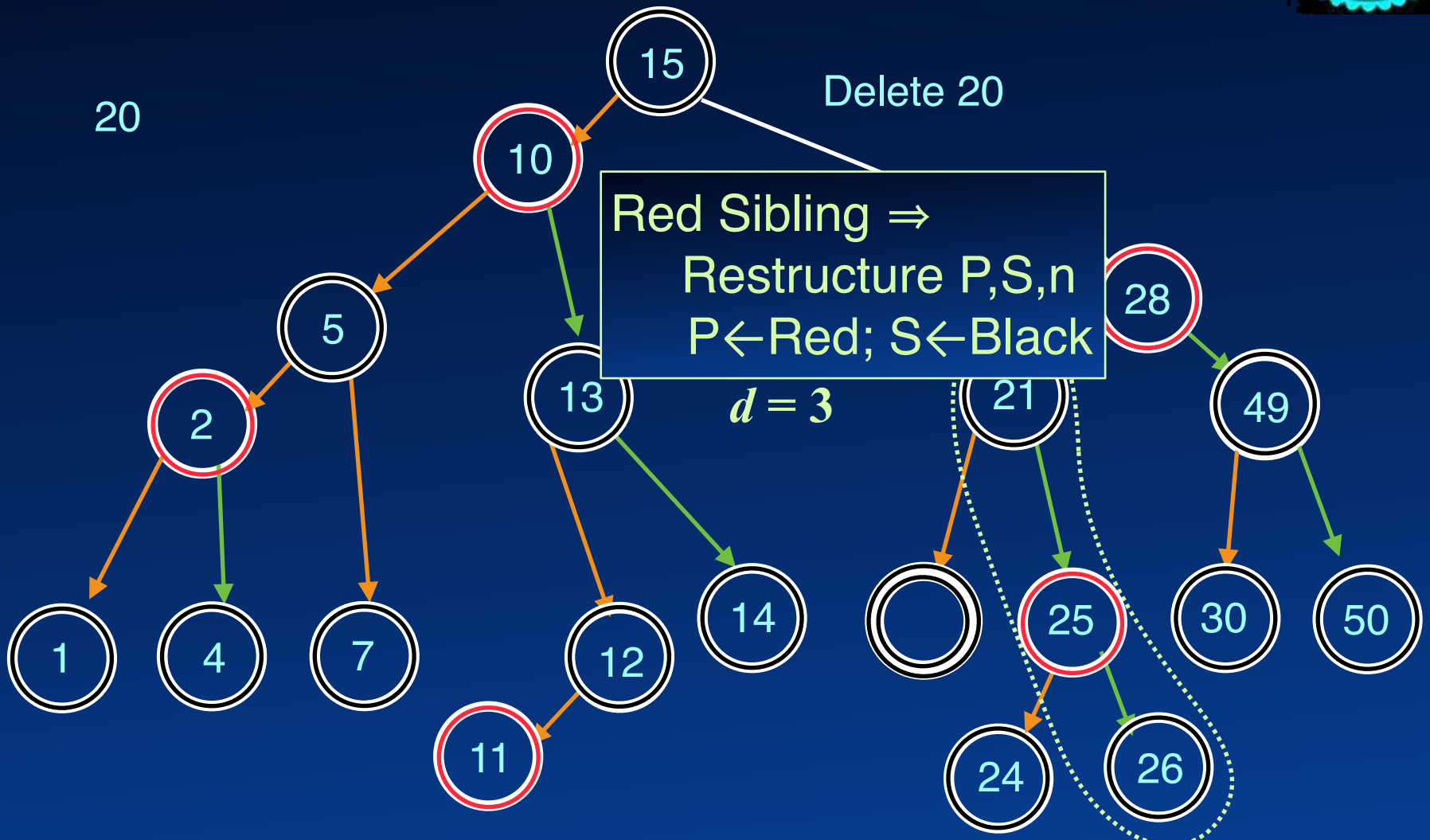

Expunge node with @null in node ***n***

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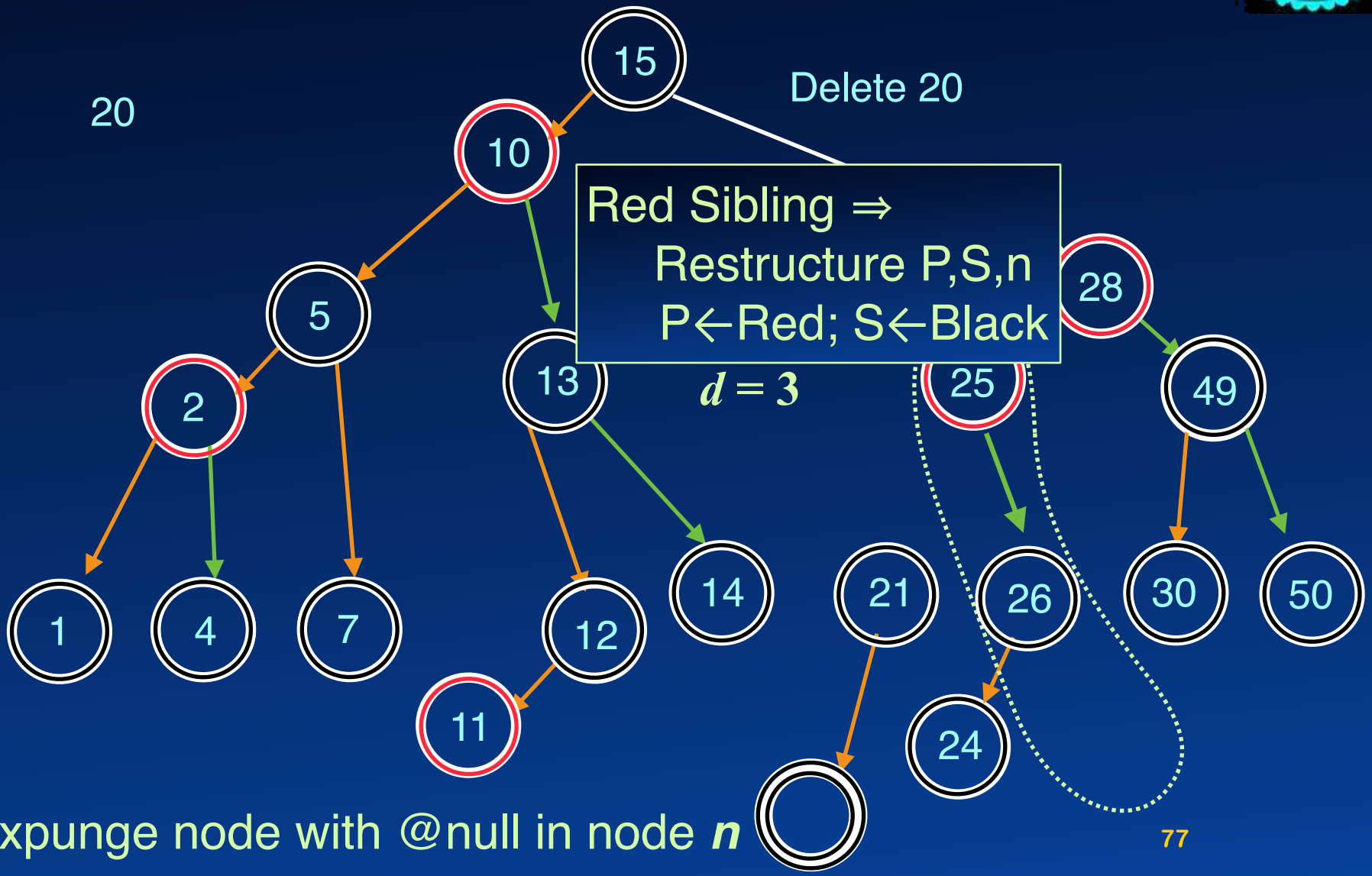
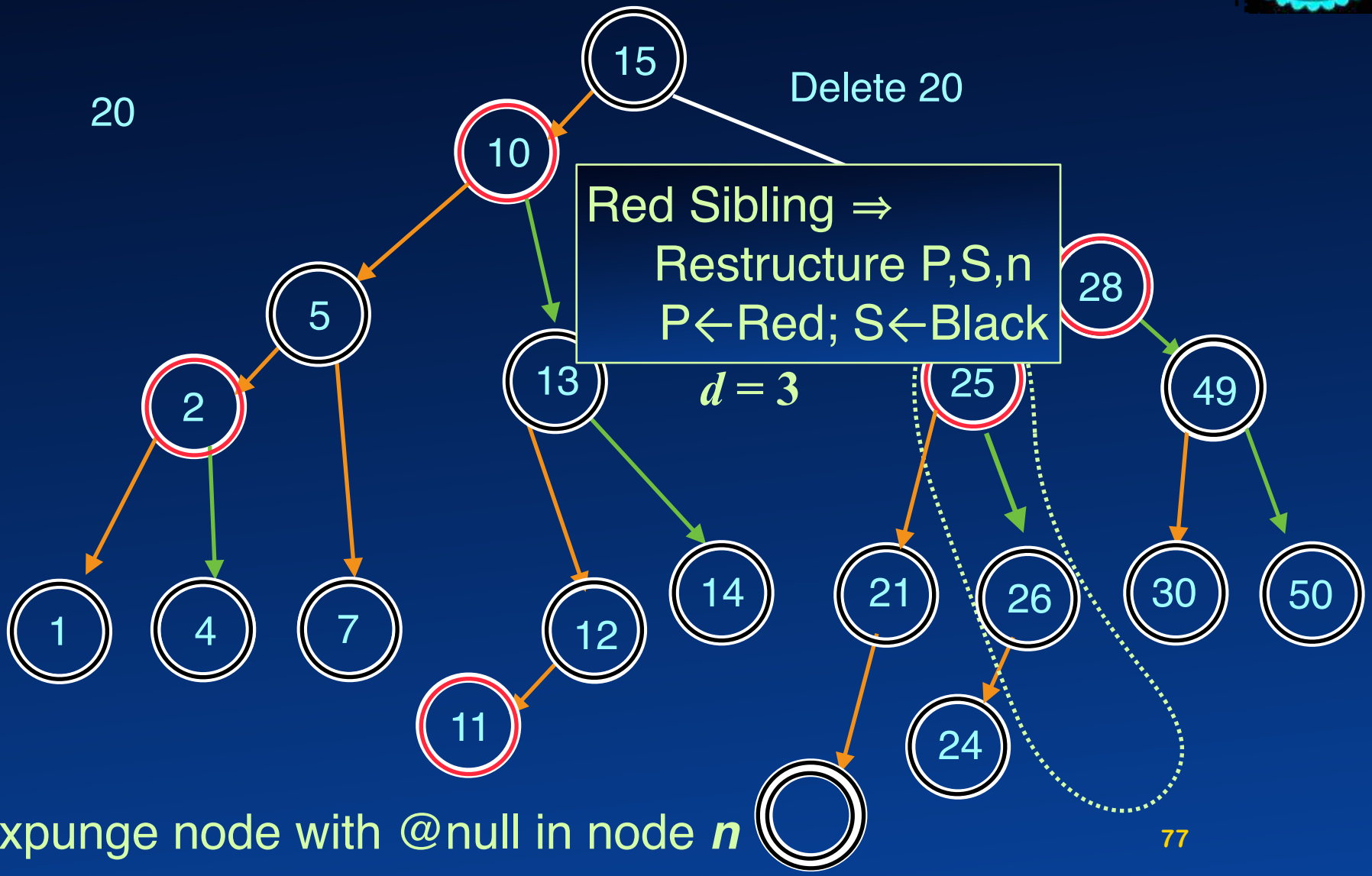


Expunge node with @null in node ***n***

Red-Black Tree



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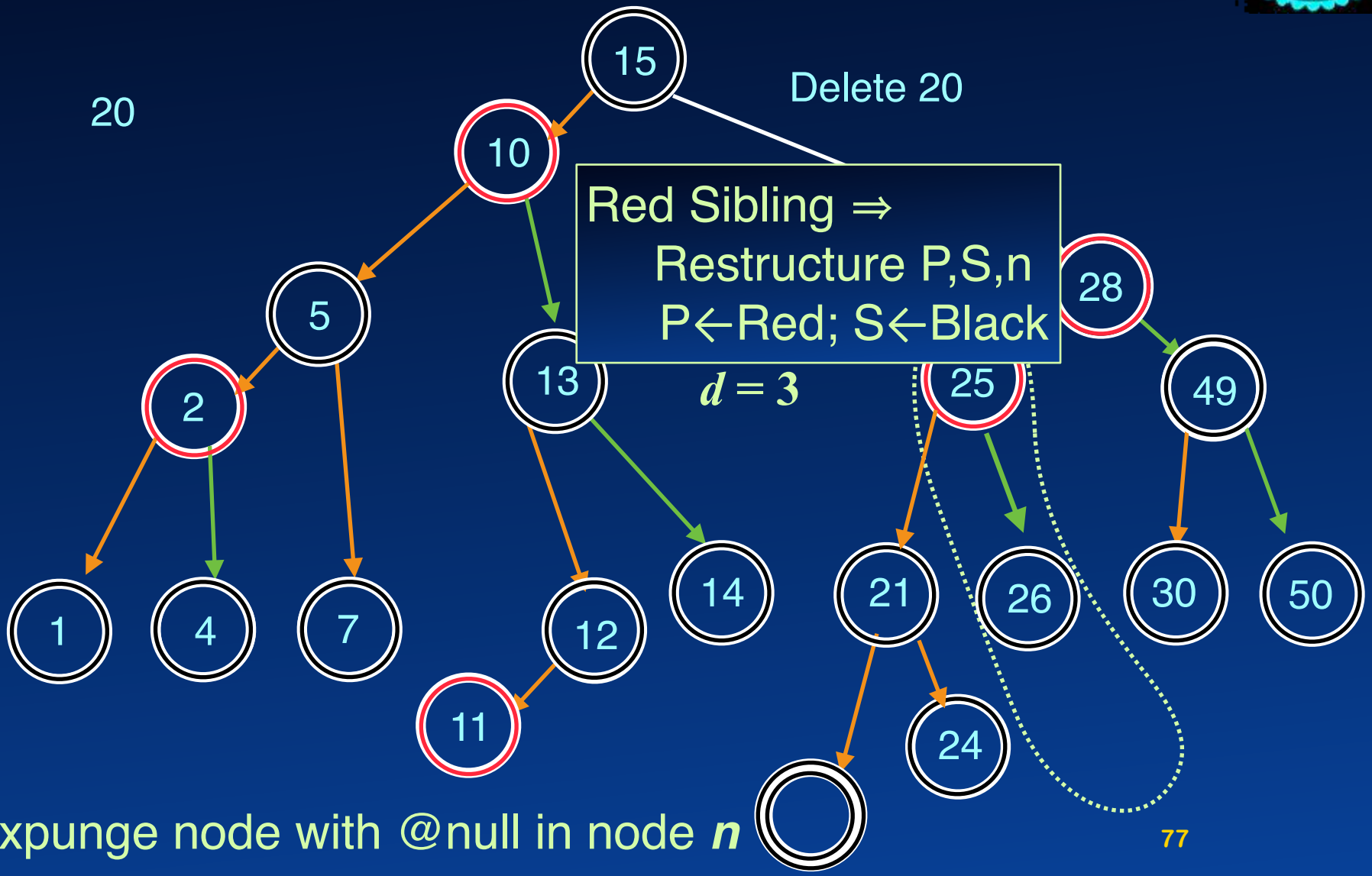

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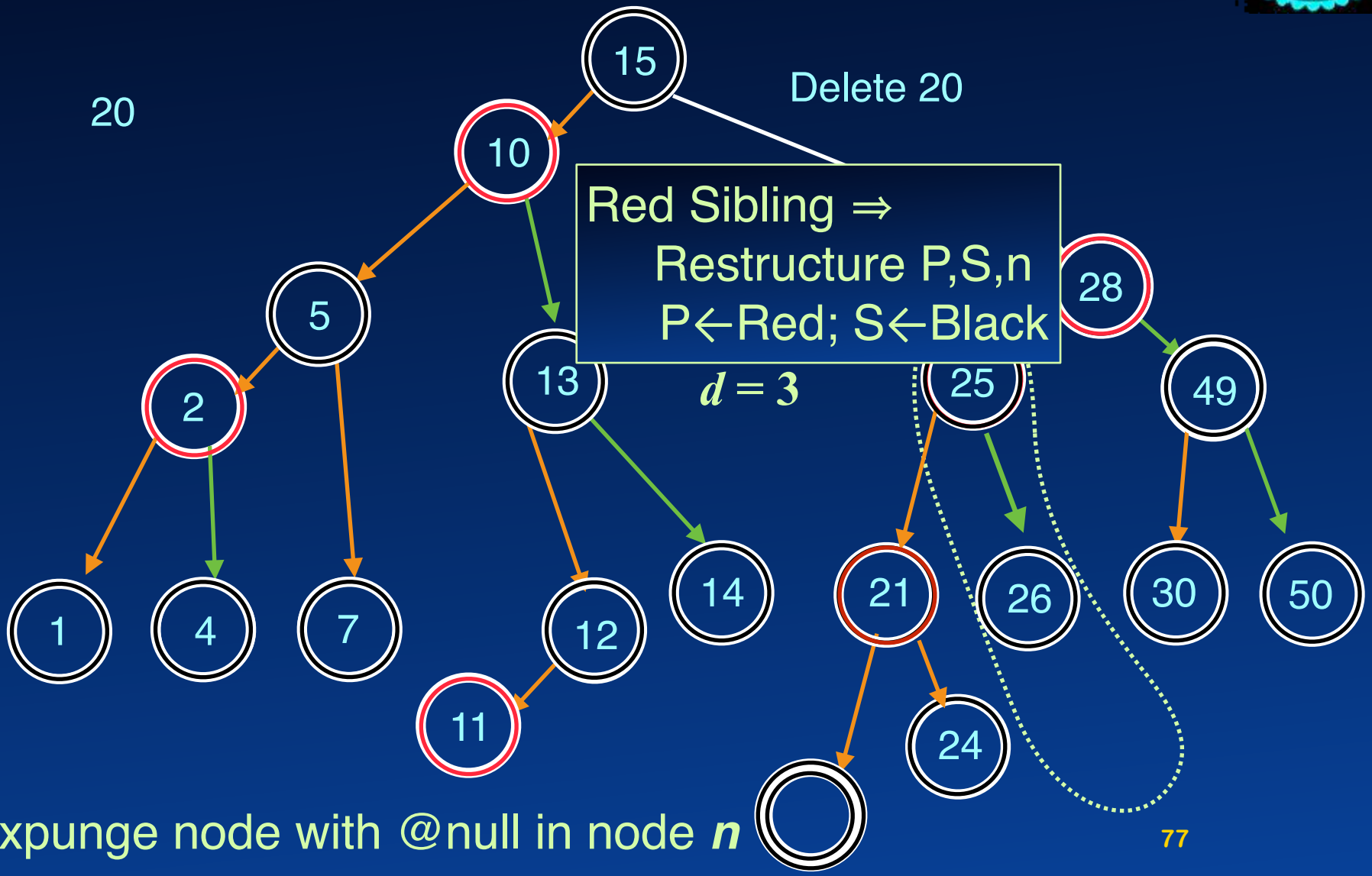

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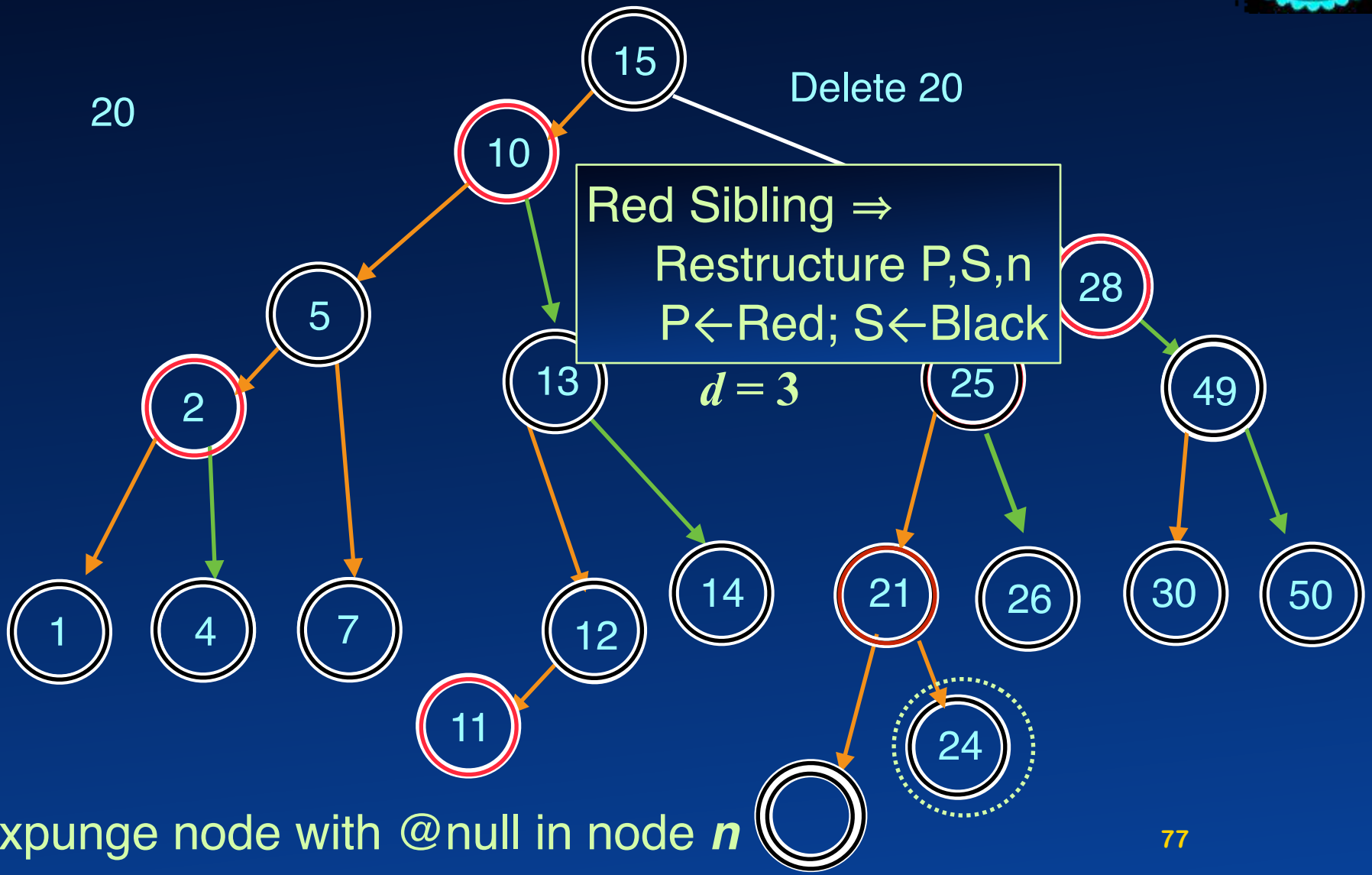


Expunge node with @null in node ***n***

Red-Black Tree

Black Sibling ⇒

No red nephew: Recolor S,P; Fix P.

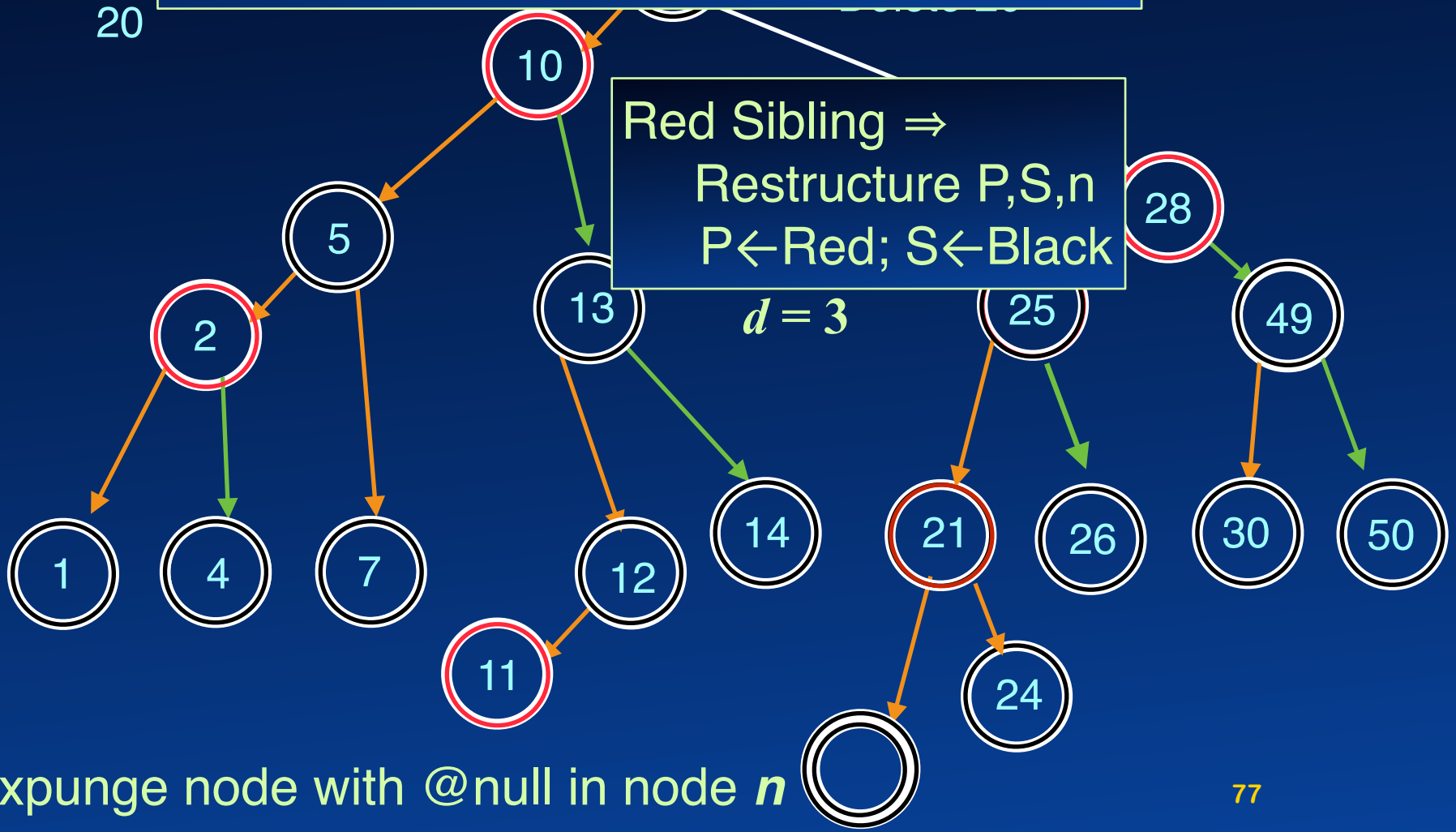
Red nephew: Restructure P,S,n; Done.

Red Sibling ⇒

Restructure P,S,n

P←Red; S←Black

$d = 3$



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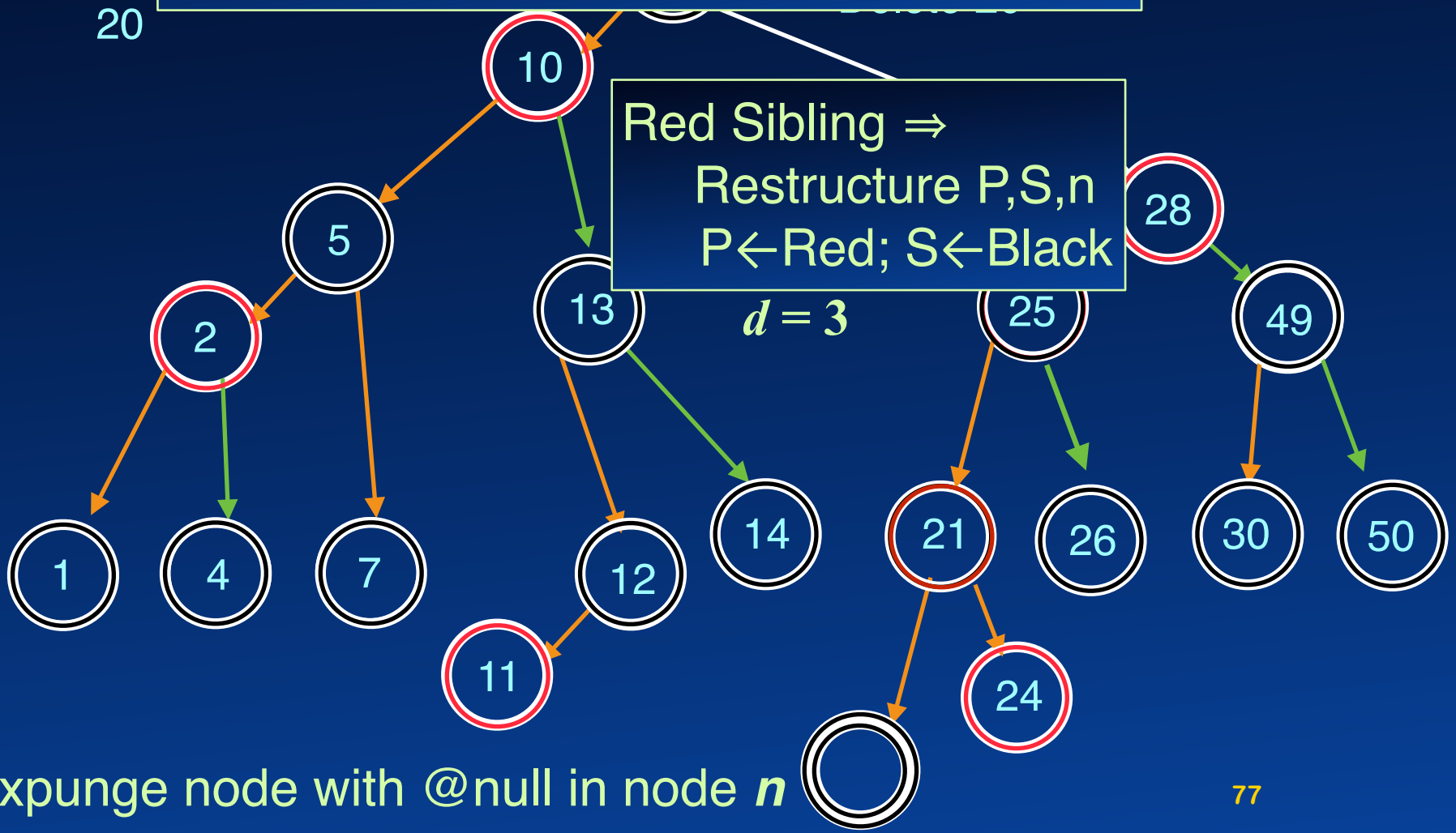

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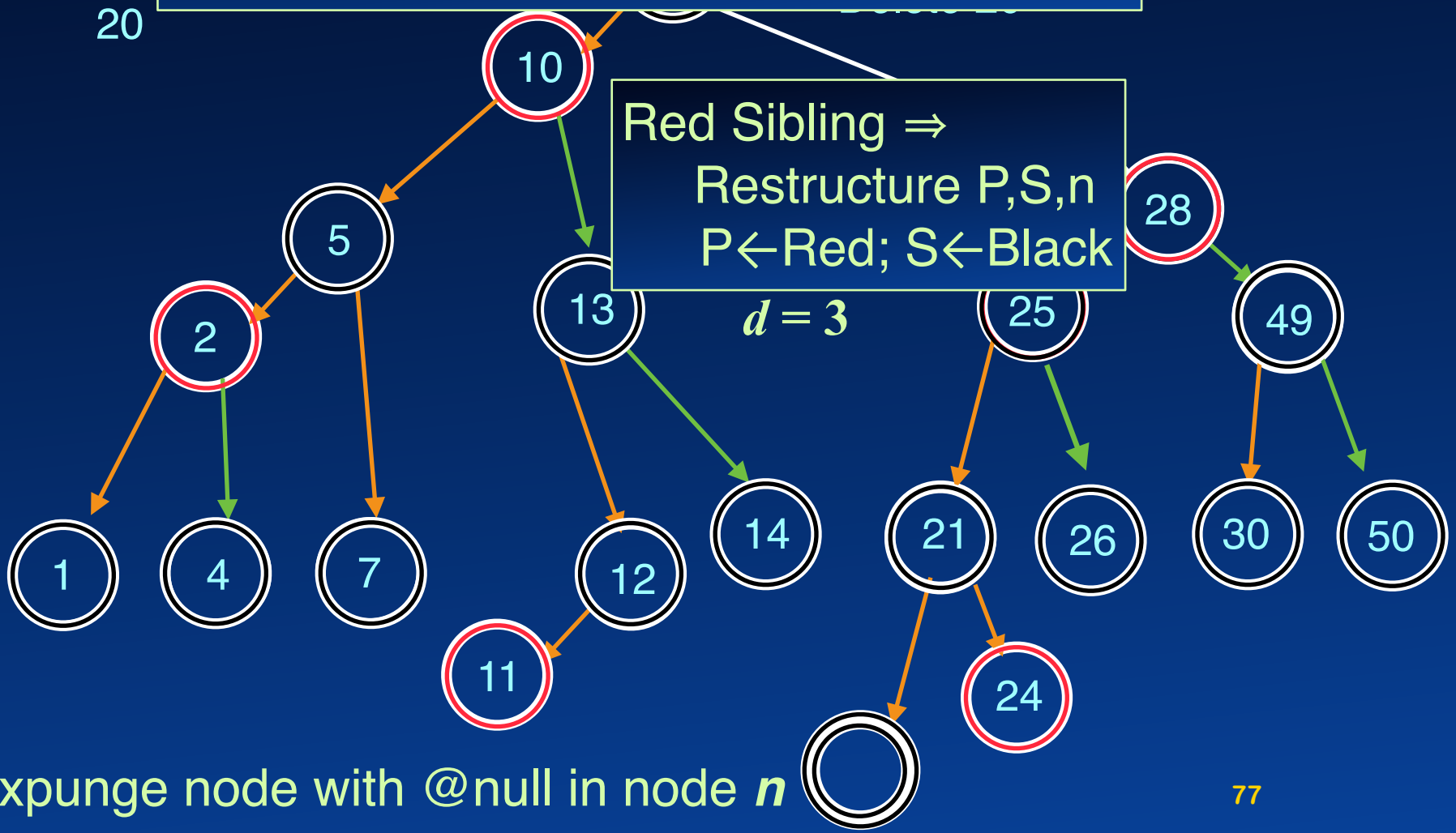

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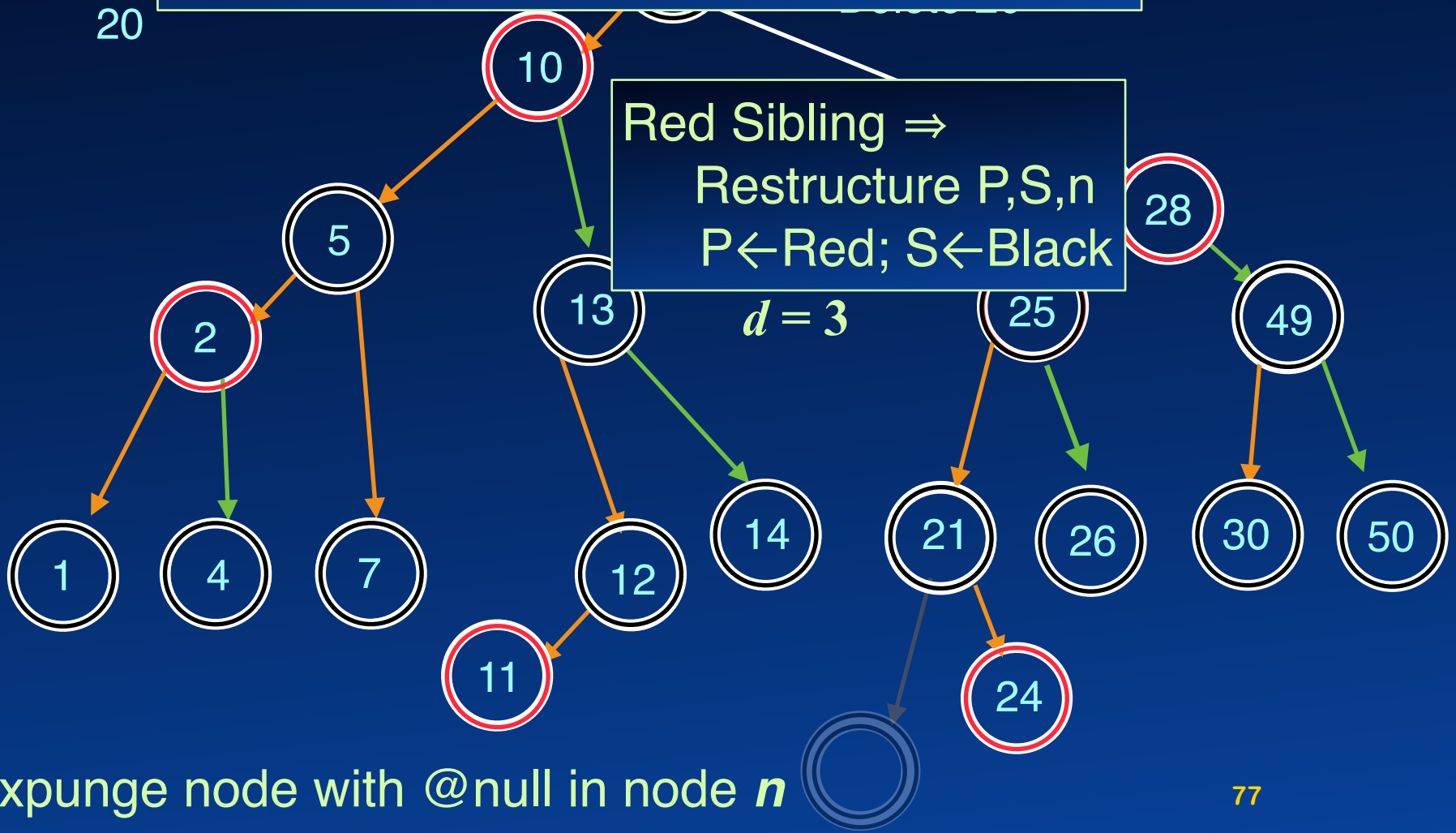

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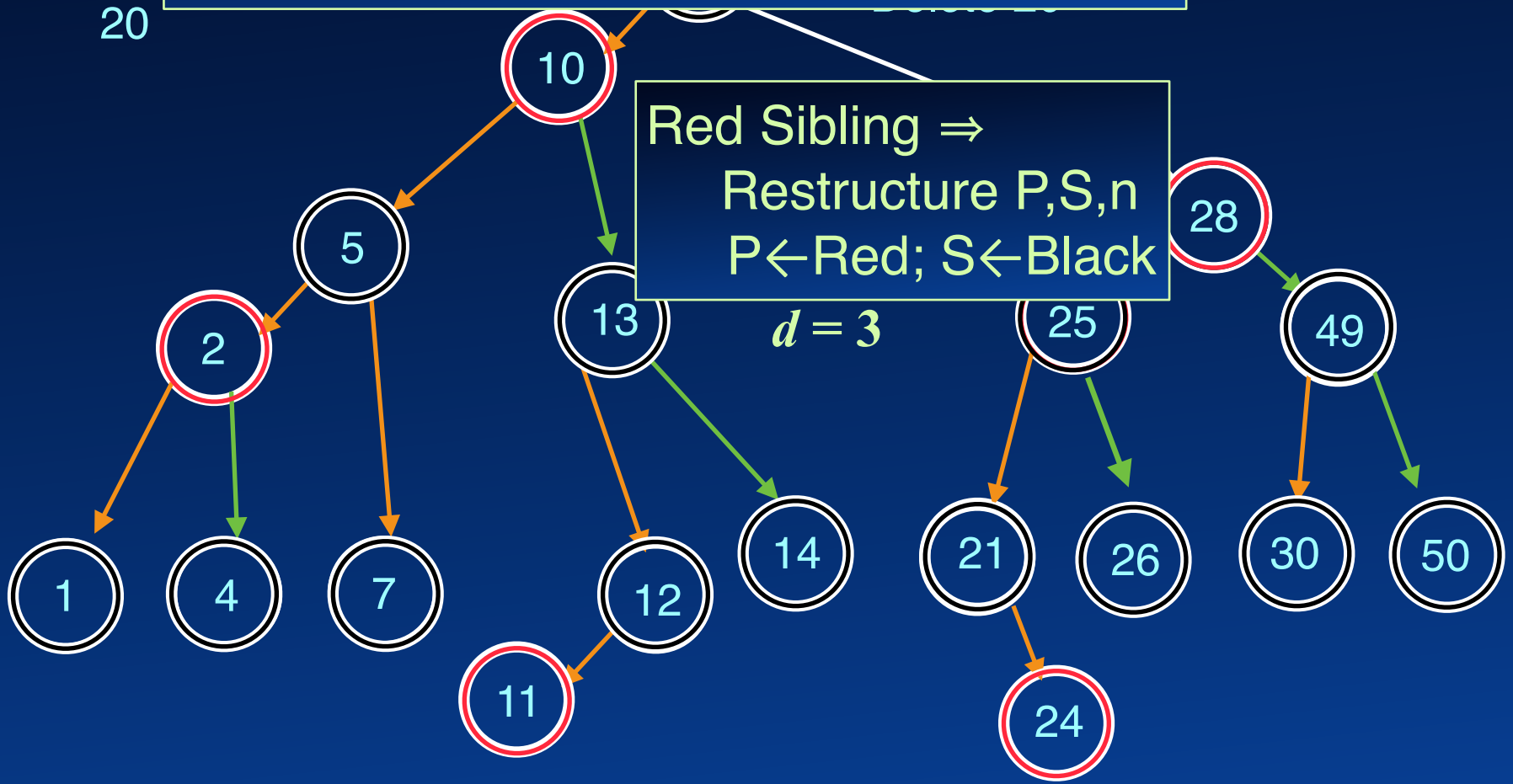

Expunge node with @null in node ***n***