Diet planning for humans using mixed-integer linear programming

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Human diet planning is generally carried out by selecting the food items or groups of food items to be used in the diet and then calculating the composition. If nutrient quantities do not reach the desired nutritional requirements, foods are exchanged or quantities altered and the composition recalculated. Iterations are repeated until a suitable diet is obtained. This procedure is cumbersome and slow and often leads to compromises in composition of the final diets. A computerized model, planning diets for humans at minimum cost while supplying all nutritional requirements, maintaining nutrient relationships and preserving eating practices is presented. This is based on a mixed-integer linear-programming algorithm. Linear equations were prepared for each nutritional requirement. To produce linear equations for relationships between nutrients, linear transformations were performed. Logical definitions for interactions such as the frequency of use of foods, relationships between exchange groups and the energy content of different meals were defined, and linear equations for these associations were written. Food items generally eaten in whole units were defined as integers. The use of this program is demonstrated for planning diets using a large selection of basic foods and for clinical situations where nutritional intervention is desirable. The system presented begins from a definition of the nutritional requirements and then plans the foods accordingly, and at minimum cost. This provides an accurate, efficient and versatile method of diet formulation.

Computerized diet planning: Humans

Diet formulation is generally carried out to date by first defining the food items or groups of food items to be used in the diet and then calculating the nutrient composition. If nutrient quantities do not fulfil the desired nutritional requirements, food quantities are altered or food items are exchanged and the composition recalculated. This is repeated until required levels of all nutrients are reached and a suitable diet is obtained. This iterative procedure is cumbersome and slow and, if this process is manual, often leads to compromises in the composition of the final diets.

An approach using a part of the formalism of linear programming for pediatric dietetics has been recently described by Colavita & D'Orsi (1990). Methodology of operations research was used to examine by geometrical representation the 'feasible region' and 'objective function'.

Linear programming has been used for formulating rations for farm animals for many years (Gass, 1969; Sklan & Bondi, 1987), solving a series of linear equations while minimizing cost. However, planning animal rations is simpler than using this methodology for planning diets for humans. In particular, the need to plan some foods as whole units (yoghurts, eggs etc.) and the possibility of using exchange groups and taste preferences must be available in any human diet planning. In addition, eating practices must be maintained while using automatic diet planning, long-term planning should be available and economic considerations cannot be discounted.

Algorithms for mixed-integer linear programming have been described (Gomory, 1958; Land & Doig, 1960), these maintain as whole units the variables defined as integers. These algorithms require greater computing resources than the non-integer algorithms and, to our knowledge, have not been reported to have been applied to diet formulation.

This report presents a method of diet planning based on a mixed-integer linear-programming algorithm for calculating nutrition at minimum costs for institutions or individuals.

METHODS

Linear programming solves a series of linear equations to satisfy the conditions of the problem while optimizing an objective function, where the objective function is usually the cost. Mathematically the problem can be stated as follows.

Minimize the objective function

$$c_1 x_1 + c_2 x_2 + c_3 X_3 \ldots + c_n x_n,$$

where c_n is the cost of variable x_n.

Subject to:

$$\begin{aligned} a_{11} x_1 + a_{12} x_2 + a_{13} X_3 \ldots + a_{1n} x_n &> = b_1, \\ a_{21} x_2 + a_{22} x_2 + a_{23} X_3 \ldots + a_{2n} x_n &> = b_2, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \\ a_{m1} x_2 + a_{m2} x_2 + a_{m3} X_3 \ldots + a_{mn} x_n &> = b_m, \end{aligned}$$

where a_{mn} is the value of constraint m in variable x_n.

To solve this set of equations, a version of the Simplex algorithm was used (Ferguson & Sargent, 1958; Gass, 1969). A simplex is essentially a n-sided polyhedron, where the equation for each constraint defines a face of the polyhedron (Gale, 1960). Theory shows that the objective function must reach its minimum value at one of the vertices of this polyhedron (Ferguson & Sargent, 1958). The number of vertices is finite and can be readily found from the equations. The objective function is evaluated at each vertex and the lowest value is the least cost solution to this problem.

An illustration of such a system applied to a dietary situation is as follows: minimize the cost of a combination of the two foods X_1 and X_2 where the price of X_1 is 3 and of X_2 is 2. The objective function to be minimized is $3X_1 + 2X_2$. The constraints of this problem are: (1) that the combination of the two feeds should be less than 100 g. This is an equation for the total amount of food and can be written as: $X_1 + X_2 < 100$. (2) that total energy should be more than 585 kJ (140 kcal) and less than 836 kJ (200 kcal). If X_1 has an energy of 6·3 kJ (1·5 kcal) and X_2 10·9 kJ (2·6 kcal) then the equations are: $2{\cdot}6X_1 + 1{\cdot}5X_2 > 140$ and $2{\cdot}6X_1 + 1{\cdot}5X_2 < 200$. (3) that X_1 be greater than 35 g, (4) that X_2 be more than 40 g. These can be stated: $X_1 > 35$ and $X_2 > 40$. The polyhedron can then be drawn as shown in Fig. 1, where the feasible area is shaded. The vertices of the polyhedron are calculated and it can be seen that the least cost solution is for 35 g of X_1 and 40 g X_2.

The model used in the present study applies additional integer and bound restrictions to some of the variables (Gomory, 1958; Land & Doig, 1960) using the revised Simplex algorithm. Upper and lower bounds are available for all variables. Lower bounds are handled by transposing the origin of the problem and untransposing the solution. Upper bounds are handled by making upper bound substitutions within the Simplex algorithm as required.

The strategy begins by relaxing the original problem and solving it using the revised Simplex algorithm as if it was a non-integer linear problem. This solution calculates non-integer values for the integer variables. When this has occurred, one of the integer variables

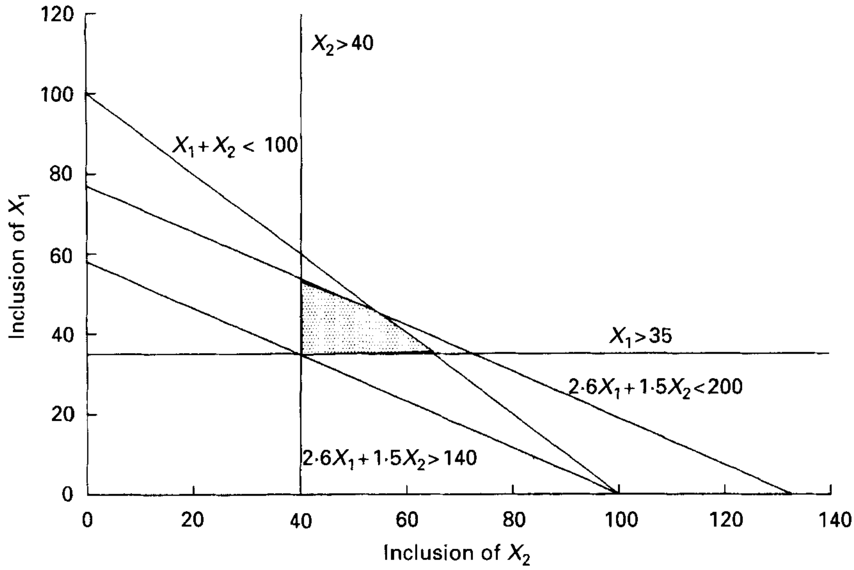

Fig. 1. Polyhedron for intake of two foods (X_1 and X_2) formed by the equations: $X_1 + X_2 < 100$, $2{\cdot}6X_1 + 1{\cdot}5X_2 > 140$, $2{\cdot}6X_1 + 1{\cdot}5X_2 < 200$, $X_1 > 35$, $X_2 > 40$. (▩), Feasible region.

is selected, and two new linear programs are created. In these two programs the integer variable is assigned the integer value either above or below the calculated value. These two programs are termed branches and, except for the new integer values, are identical to the original problem. The program then computes the lower limit of the objective function that fulfils all the integer restrictions for each of the branches. This process continues and a series of branch problems are created for all the integer variables. When a problem has integer values for all the integer variables the branch problem, solution and objective function are saved. As additional solutions are encountered they are compared with the current best solution, and if better are replaced. If a solution becomes unfeasible it is dropped from the list of solutions. If a solution to the non-integer linear program is higher than the current best solution, there is no need to develop that branch further. The procedure ends when no branches remain to examine and the best solution is then adopted. This procedure was used based on a software package (MILP88, Eastern Software Products, Alexandria, VA, USA) running on personal computers.

Nutrient requirements were as described in *Recommended Dietary Allowances* (Food and Nutrition Board, 1989). Nutrient composition of foods was from local quality-control laboratories and, when values were not available, from Composition of Foods Handbook VIII (1987).

Application of the model

The procedure described was adapted for diet formulation, where the variables are in general the food items, the restrictions are the nutritional requirements and the objective function is the cost of the diet.

A simple linear program is shown in Table 1, where three foods are shown with equations for total amount of food, energy and protein. However, the optimal solution in Table 1 is not very suitable with respect to the amounts of bread. A method of regulating amounts of any food is shown in Table 2 where upper and lower bounds are assigned to the variables and these can be used to ensure that excess quantities of a single food were not planned.

Table 1. *A simple linear program with three foods used for diets planned for humans*

	Bread	Egg	Cheese	
Amount (g)	1·0	1·0	1·0	> = 100
Energy				
kJ	10·37	6·3	6·15	> = {1672
kcal	2·48	1·5	1·47	400
Protein (g)	0·09	0·124	0·15	> = 15
Price (NIS)	0·02	0·06	0·062	
Optimal solution (g)	113·2	78·7		

NIS, National Israeli Shekel.

Table 2. *A mixed integer linear program with bounds with three foods used for diets planned for humans*

	Bread	IEgg	Cheese	
Bounds				
Lower	0	0	0	
Upper	90	1	100	
Amount (g)	1·0	50·0	1·0	> = 100
Energy				
kJ	10·37	313·5	6·15	> = {1672
kcal	2·48	75·0	1·47	400
Protein (g)	0·09	6·2	0·15	> = 15
Price (NIS)	0·02	0·3	0·062	
Optimal solution (g)	90·0	1·0	72·0	

NIS, National Israeli Shekel.

Table 3. *A mixed integer linear program with exchange groups used for diets planned for humans*

	Bread	IEgg	Cheese	ICereal 1	ICereal 2	
Amount (g)	1·0	50·0	1·0	15	18	> = 100
Energy						
kJ	10·37	313·5	6·15	250·8	250·8	> = {1672
kcal	2·48	75·0	1·47	60·0	60·0	400
Protein (g)	0·09	6·2	0·15	1·75	2·70	> = 15
Cereal				1	1	= 1
Price (NIS)	0·02	0·3	0·062	0·4	0·45	

NIS, National Israeli Shekel.

In addition, the solution in Table 1 includes 1·65 large eggs. To realize the use of whole units, an integer variable for eggs is defined in Table 2. The optimal solution shown is more acceptable as a meal and uses one whole egg.

Exchange groups can be defined by equations limiting numbers of integer units of exchange groups available over a specified time-period. This requires that exchange groups be linked on a nutritional basis, such as energy content, rather than on a weight basis; this is shown in Table 3 for two different cereals. Ratios between nutrients can also be included in the programming following a linear transformation of the ratio. Such a transformation

Table 4. *A mixed integer program with nutrient ratios used for diets planned for humans*

	Bread	IEgg	Cheese	Protein	Energy	
Amount (g)	1·0	50·0	1·0			> = 100
Energy (E)						
kJ	10·37	313·5	6·15			> = 1672
kcal	2·48	75·0	1·47			400
Protein (P) (g)	0·09	6·2	0·15			> = 15
E balance						
kJ	10·37	313·5	6·15			
kcal	2·48	75·0	1·47		−1	= 0
P balance (g)	0·09	6·2	0·15	−1		= 0
P:E				1	−0·05	> 0
Price (NIS)	0·02	0·3	0·062	0·4	0·45	

NIS, National Israeli Shekel.

Table 5. *Amounts and energy requirements for meals in the two diets planned for humans*

	Diet	
Constraint	Low-cholesterol	Low-energy
Breakfast		
Amount (g)	> 200	< 500
Energy (MJ)	> 1·67	< 1·25
(kcal)	> 400	< 300
Lunch		
Amount (g)	> 300	< 700
Energy (MJ)	> 2·93	< 2·09
(kcal)	> 700	< 500
Dinner		
Amount (g)	> 250	< 500
Energy (MJ)	> 2·51	< 1·67
(kcal)	> 600	< 400
Energy		
Daily (MJ)	> 71·1	< 45·9
(kcal)	> 1700	< 1100
Weekly (MJ)	> 497·4	< 321·9
(kcal)	> 11 900	< 7700

is demonstrated in Table 4 where the protein:energy value is restricted to a minimum of 0·05.

Nutrient requirements can be defined separately for meals, days or any desired unit of time using a structure such as that shown in Table 5. Here foods are allowed at either one or several meals, and contribute to daily and weekly total energy. We preferred to define amount of food and energy per meal, per day and per week, nutrients such as protein daily, and minerals such as Ca on a weekly basis. However, any desired time units can be used.

RESULTS

Using these considerations a practical program for determining diets was constructed. A data base of nutrient content of local foods and their costs was prepared. Foods were then

Table 6. *Daily nutritional requirements and contents for two diets planned for humans*

	Low cholesterol		Low energy	
Constraint	Constraint	Content	Constraint	Content
Amount (g)	> 2000	2074	> 1500	1594
Energy				
(MJ)	> 71·1	71·4	< 45·9	45·1
(kcal)	> 1700	1708	< 1100	1079
Fat (%)	< 30	23·7	< 30	24·3
CHO (%)	> 50	50·1	> 50	56·0
Protein (g)	> 70	75·4	> 50	54·6
Na (mg)	< 3000	2905	< 3000	1854
K (mg)	< 4000	3796	< 4000	3174
Ca (mg)	> 1000	1042	> 700	706
Fe (mg)	> 18	18·7	> 15	15·8
Cholesterol (mg)	< 300	123	< 300	85
Vitamin A (μg)	> 1200	1203	> 1200	1302
Thiamin (mg)	> 1·1	1·43	> 1·1	1·1
Vitamin C (mg)	> 45	455	> 45	505
Vitamin E (mg)	> 12	12393	> 12	11322
Fibre (g)	> 15	15·1	> 14	14·0

CHO, carbohydrate.

selected for use in the diets planned, this allowed flexibility and wide applicability for both individual and institutional use.

We demonstrate formulation of two diets using the method described with fifty foods per day with the nutritional constraints shown in Tables 5 and 6. The first diet is a low-cholesterol diet. The original target was a maximum of 300 mg/d for cholesterol and a maximum of 961 kJ (230 kcal)/4·2 MJ (1000 kcal) energy from fat with all other nutrient restrictions as recommended dietary allowances. A minimum polyunsaturated fat:saturated fat of 1·0 was set. Details of nutrient content in the optimal ration are given in Table 6. An optimal solution was found with as low as 105 mg cholesterol/d. Minimum energy was the limiting factor and fat comprised 685 kJ (140 kcal)/4·2 MJ (1000 kcal). Protein, vitamin E, thiamin, vitamin C, Ca and Fe were above minimal requirements. Daily menus were planned for 1 week with exchanges for milk products, main courses, seasonal vegetables and fruits such that the diet was sufficiently varied. A menu for a typical day is shown in Table 7.

The second diet demonstrated is a low-energy diet. The protein energy relative to total energy was set to be at least 627 kJ (150 kcal)/4·2 MJ (1000 kcal), and the upper limit on energy was planned at 5 MJ (1200 kcal) (Table 6). It was possible to achieve an optimal solution with 4·7 MJ (1125 kcal)/d. At this daily energy level protein content was higher than the minimum, whereas minimal Fe content was limiting. Minerals and vitamins (not shown) were above the planned minimum and did not restrict the solution. The diet was planned for 1 week and Sunday's menu is shown (Table 8) which presents food with sufficient volume and variety. The rest of the week was planned at the same energy level with different exchange groups for all food items such that meals were varied from day to day.

These two examples of diets were prepared using integers for items eaten by units and exchanges for most food items. Using similar methods, diets have been planned for institutions for two weekly or monthly periods.

Table 7. *Menu for 1 d for the low-cholesterol diet planned for humans*

Meal/food item	Amount (g)
Breakfast	
Bread	70
Cottage cheese	50
Yoghurt	1 unit
Tomato	50
Cucumber	100
Coffee	5
Milk, low fat	25
Lunch	
Turkey light meat	100
Potato	200
Cabbage	300
Onion	50
Tomato sauce	40
Banana	1 unit
Apple	1 unit
Dinner	
Bread	50
Cheese, low fat	100
Tomato	100
Pepper	80
Avocado	50
Almonds	25
Orange	1 unit
Pear	1 unit

Table 8. *Menu for 1 d for the low-energy diet-planned for humans*

Meal/food item	Amount (g)
Breakfast	
Crackers, diet	50
Cottage cheese	50
Yoghurt, low fat	1 unit
Tomato	50
Cucumber	50
Coffee	5
Milk, low fat	25
Lunch	
Chicken, breast	100
Carrot, steamed	100
Cabbage	200
Onion	30
Tomato sauce	35
Grapes	100
Dinner	
Bread, light	50
Soft cheese, low fat	100
Tomato	140
Lettuce	80
Fresh mushrooms	90
Figs	2 units

DISCUSSION

An automatic iterative process is presented, which allows diet planning according to nutritional criteria and at least cost. Using this procedure varied diets can be planned, meal by meal, for any convenient period while maintaining nutrient relationships. If special eating practices are required, types of foods can be grouped together in particular meals or, using specific equations, can be mutually restricted. Essentially, we describe a completely automated procedure for diet planning based on the formalism outlined by Colavita & D'Orsi (1990) while overcoming the technical problems involved using the mixed-integer algorithm.

Using the present software the problem size is restricted to 800 equations and 4000 variables. In our hands this is easily sufficient for 60 d diet planning. Weekly diets utilize about one-tenth of these resources and such a diet is solved in seconds on a computer with a 80486 microprocessor or slightly longer on computers with earlier microprocessors.

Since least cost is the objective function, it is usually necessary to utilize the upper bound option to keep the quantities of the cheaper foods within eatable limits. On the other hand, it is also possible to force a desired food item into the solution by determining a lower bound.

The use of exchange groups linked on an energy basis involves defining a fixed relationship between these items as integers are used. In general this detracted little from the flexibility of the solution where sufficient foods were available.

Additional information is available from the solution output, and this includes the minimum and maximum values of the constraints which will cause a change in the basis of the solution, and the minimum and maximum cost of each food which will cause a change in the solution basis. In addition, owing to the ease of altering any parameter and rerunning the program, the effects of parametric changes, such as increasing Ca or the protein:energy value, can be readily examined.

We have used this program both to plan diets for many different clinical situations, of which some examples have been shown here, and for institutional feeding. The latter requires long-term dietary planning and includes a large number of food items and the economic implications are especially important. The use of this program allows examination of many nutritionally sufficient long-term diets where previously, by manual methods, the effort required to plan a single diet was such that few changes were made after achieving the first balanced solution. In planning institutional diets we have been able not only to improve the nutritional adequacy of diets but, in general, the economic benefits have been greater than 5%.

The system presented here will only provide solutions where all the nutritional requirements as determined by the dietitian are fulfilled. Thus, this is basic to the system and is not discussed here in detail. However, any specific nutritional requirements can be simply incorporated into the planning and a suitable solution will be computed.

In the recently described technique of goal programming the coefficients of a set of linear equations are adjusted to reach a predefined goal. Such an approach can be used for diet formulation but may generate several sets of diets reaching the goal. This technique, however, does not determine an optimum for the objective function.

We have utilized integer variables in the mixed-integer algorithm to allow the use of whole units of feeds as well as for defining the use of the exchange groups. This allowed us to design a flexible and practical diet formulation program which we have used in both clinical and institutional situations.

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