MATH 308 - D400
Linear Optimization
Project: Surrey Central City Mall Diet Problem

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Abstract

This report addresses the development of a binary "Linear Program (LP)" aimed at minimizing the cost of a one-week diet plan, encompassing three meals per day. The diet plan sources food items exclusively from Surrey Central City Mall's Food Court. The objective is to create a solution that adheres to the five essential daily nutritional factors: Calories, Protein, Carbohydrates, Dietary Fiber, and Cholesterol, while emphasizing diversity in the food items included in the plan. The approach involves implementing a binary minimization LP, incorporating nutrition factor constraints and food item diversity constraints tailored to the unique aspects of this problem.

1. Introduction

The challenge addressed in this project involves formulating an effective one-week diet plan with three meals daily, utilizing the availability of food items at Surrey Central City Mall's Food Court. Our goal is to minimize the overall cost of the diet plan while meeting essential daily nutritional requirements. To achieve this, we employ a binary Linear Program (LP), emphasizing five key Daily Nutrition factors: Calories, Protein, Carbohydrates, Dietary Fiber, and Cholesterol. Additionally, we introduce various relevant constraints to enhance the diversity of food items in the diet plan.

2. Model Development

2.1. Parameters

Let I be the set of all food items, where each food item $i \in I = 1, ..., 115$. Let N be the set of all nutritional components, where each nutritional component $n \in N =$ Calories (K), Protein (P), Carbohydrates (C), Dietary Fiber (F), Cholesterol (T). Let D be the days of the week, where each day $d \in D = 1, ..., 7$. And let $M =$ Breakfast (B), Lunch (L), Dinner (D) be the set of meals each day, where each meal $m \in M$.

Then:

- p_i be the price for item food i
- $v_{n,i}$ be the value of nutritional component n of food item i,
- l_n be the lower bound requirement for the nutritional component n,
- u_n be the upper bound requirement for the nutritional component n,

2.2. Decision Variables

We need to decide which food to be chosen each day and for each meal. Let $x_{d,m,i}$ with $d \in D$, $m \in M$, and $i \in I$ be the binary decision variable:

$$x_{d,m,i} = 1 \text{ if food item } i \text{ is selected on day } d \text{, for meal } m,$$

$$x_{d,m,i} = 0 \text{ otherwise.}$$

2.3. Objective Function and Criterion

The total cost for a one-week meal plan consisting of three meals per day, the *objective function*, will then be:

$$\sum_{d \in D} \sum_{m \in M} \sum_{i \in I} p_i \cdot x_{d,m,i}$$

The *objective criterion* is to **minimize** the objective function.

2.4. Constraints

2.4.1. Nutritional Factors Constraint

We have three sets of constraints for balancing the daily dietary requirements for an adult male. Firstly, we considered the lower bounds of a subset of the nutritional components $N' =$ K, P, C, F. This means that the daily intake amount of each nutritional component $n \in N'$ should be greater than or equal to the lower bound requirement of that component:

$$\sum_{m \in M} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \geq l_n \ , \forall d \in D \ , \forall n \in N'$$

Secondly, we considered the upper bounds of a subset of the nutritional components $N'' =$ T. This means that the daily intake amount of nutritional component $n \in N''$ should be lower than or equal to the upper bound requirement of that component:

$$\sum_{m \in M} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \leq u_n \ , \forall d \in D \ , \forall n \in N''$$

And finally, we considered the distribution of adequate amounts of Calories (K) across the three daily meals. For Breakfast (B), we considered amounts no less than 400 and no more than 700. For Lunch (L), we considered amounts no less than 700 and no more than 1,200. And for Dinner (D), we considered amounts no less than 1,200.

$$\sum_{m = B} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \geq 400 \ , \forall d \in D \ , n = K$$

$$\sum_{m = B} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \leq 700 \ , \forall d \in D \ , n = K$$

$$\sum_{m = L} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \geq 700 \ , \forall d \in D \ , n = K$$

$$\sum_{m = L} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \leq 1200 \ , \forall d \in D \ , n = K$$

$$\sum_{m = D} \sum_{i \in I} x_{d,m,i} \cdot v_{n,i} \geq 1200 \ , \forall d \in D \ , n = K$$

2.4.2. Food Items Diversity Constraint

We employed three sets of constraints to ensure diversity in the food items selected for our diet problem. Firstly, to ensure that the same food items are not selected on two consecutive days, we introduced seven constraints that disallow the selection of the same food item on two consecutive days.

$$\sum_{d=\{1,2\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

$$\sum_{d=\{2,3\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

$$\sum_{d=\{3,4\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

$$\sum_{d=\{4,5\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

$$\sum_{d=\{5,6\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

$$\sum_{d=\{6,7\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

$$\sum_{d=\{1,7\}} \sum_{m \in M} x_{d,m,i} \leq 1 \ , \ \forall i \in I$$

Secondly, to ensure that no food item is selected more than two times within one week, we devised another constraint.

$$\sum_{d \in D} \sum_{m \in M} x_{d,m,i} \leq 2 \ , \ \forall i \in I$$

And lastly, to ensure that food items from all restaurants are selected by our linear program, we introduced another constraint. However, this constraint is redundant due to the previous two food item diversity constraints and our data having only four distinct restaurants to choose food items from. Nonetheless, we decided to keep the constraint because with larger data, this constraint will be useful. Let R be the set of all restaurants, where $r \in R = 1, ..., 4$.

$$\sum_{d \in D} \sum_{m \in M} \sum_{i \in I} x_{d,m,i} \geq 3 \ , \ \forall r \in R$$

3. Data Collection

Our data consists of four distinct fast-food restaurants located at Surrey Central City Mall. We selected these four stores based on the availability of online information on prices and Nutrition Facts. The four stores we chose are A&W, Burger King, Charley's Cheesesteaks, and Subway. We used their mobile apps to gather information on prices and their official websites to extract information on the five nutrition facts mentioned in the introduction section, as well as the nutrition facts of the items on Total Fat. However, we chose not to use Total Fat as a constraint in the modeling approach; we will elaborate on the reasons in the next section (Experimental Results). In total, we randomly selected 115 food items from these four stores and collected the required information and facts from a reliable source.

We also gathered data on the daily intake recommendations for nutritional factors, namely Calories, Protein, Carbohydrates, Dietary Fiber, Total Fat, and Cholesterol, for an adult male. This information on nutritional factors was collected from the British Nutrition Foundation Website and the University of California San Francisco (UCSF) Health Website. The table below illustrates the daily intake recommendations for the six nutritional factors.

Nutritional Factors	**Daily Intake Amounts**
Calories (kcal)	≥ 2500
Protein (g)	≥ 55
Carbohydrates (g)	≥ 333
Dietary Fiber (g)	≥ 30
Total Fat (g)	≤ 79
Cholesterol (mg)	≤ 300

Table 1: Nutritional Factors and their Daily Intake Amounts Recommendation

4. Experimental Results

Initially, we planned to use the lower bound of the nutritional factor, Total Fat, as a less than or equal to constraint. However, using this constraint makes the linear program (LP) infeasible. This is because the food items in fast-food restaurants generally contain high-fat content, and considering the upper bounds of other nutrition factor constraints, the problem turned out to be infeasible. For instance, meeting the upper bound of Daily Dietary Fiber intake as well as the lower bound of Total Fat is not possible using the data at hand. As a result, we did not use the lower bound constraint for Total Fat.

Solving the minimization LP with the constraints outlined in section 2.4, we obtained a total cost of \$297.67 for a one-week diet plan consisting of three meals each day. We observed that, on average, the Breakfast meal costs \$54.90 each week, which is the least, followed by the lunch meal costing \$76.84, and then the most expensive meal being Dinner, which costs \$165.95. The trend we see in the optimal solution is somewhat expected due to the distribution constraint of adequate amounts of Calories (K) across the three daily meals. Figures 1 and 2 below summarize the individual cost of each meal across a week (from Day 1 to Day 7), and Figure 3 represents the total cost over the one-week diet plan consisting of three meals per day.

	Day-1			Day-2			Day-3			Day-4		
	Breakfast	*Lunch*	*Dinner*	*Breakfast*	*Lunch*	*Dinner*	*Breakfast*	*Lunch*	*Dinner*	*Breakfast*	*Lunch*	*Dinner*
Cost per Meal	7.58	12.68	16.87	3.29	10.58	26.65	6.59	12.98	22.56	7.58	9.37	29.06

Figure 1: Cost of the individual three meals for Day 1 to 4

	Day-5			Day-6			Day-7		
	Breakfast	*Lunch*	*Dinner*	*Breakfast*	*Lunch*	*Dinner*	*Breakfast*	*Lunch*	*Dinner*
Cost per Meal	11.28	15.48	15.07	8.19	5.58	25.65	10.39	10.17	30.07

Figure 2: Cost of the individual three meals for Day 5 to 7

Total Cost	297.67

Figure 3: Total cost of a one-week diet plan

Our other findings include the total number of food items selected for the one-week diet plan. The results showed a total of 53 food items selected, and among them, 12 are from A&W, 27 from Burger King, 4 from Charley's Cheesesteaks, and 10 from Subway.

5. Conclusion

The problem presented mainly mentioned the use of the upper limit on Calories; however, we extended the problem to a more comprehensive approach by considering four other important nutritional factors, namely Protein, Carbohydrates, Dietary Fiber, and Cholesterol. We also incorporated a large selection of food items across four distinct fast-food restaurants and employed various variety constraints to ensure variety in the food selection. As a result, a single food item was not selected on two consecutive days, and no more than two times in a one-week period. On average, each meal costs approximately $14.20, which can be argued to be inexpensive. Overall, the binary linear program produces cost-effective results that are well-founded on balancing the daily nutritional requirements of an adult male and ensuring diversity in the food items in the diet plan.