Strongly Connected Components

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Strong Connectivity

Strongly Connected Graph

A graph G(V,E) is called Strongly Connected Graph if every pair of nodes is **mutually reachable**

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Remark

Strong Connectivity is the property of **Directed Graph**

Example of Strong Connectivity

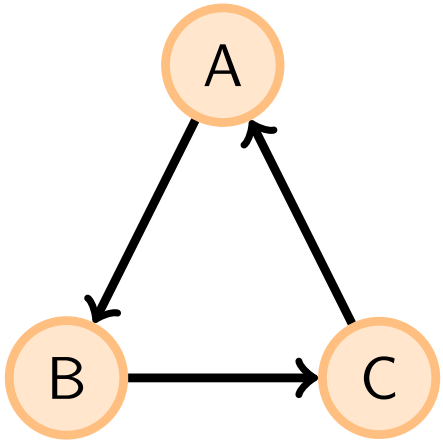


Figure: Strongly Connected Graph

Example of Strong Connectivity

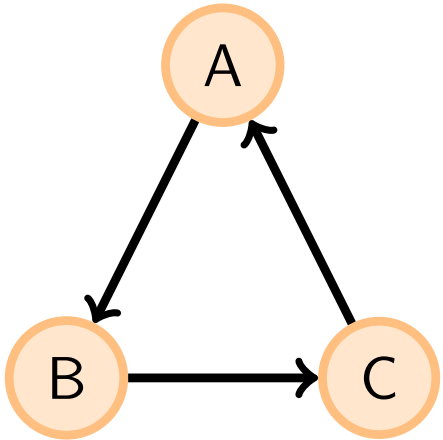


Figure: Strongly Connected Graph

In this graph every nodes are mutually reachable

Example of Strong Connectivity(Contd.)

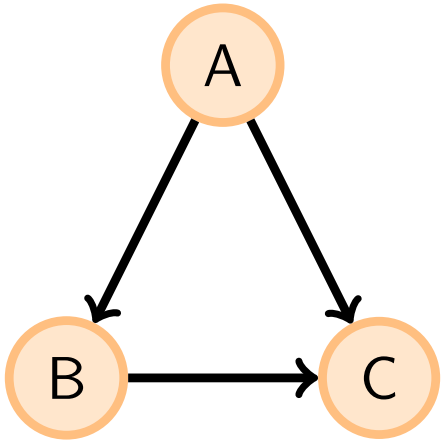


Figure: Not Strongly Connected Graph

Example of Strong Connectivity(Contd.)

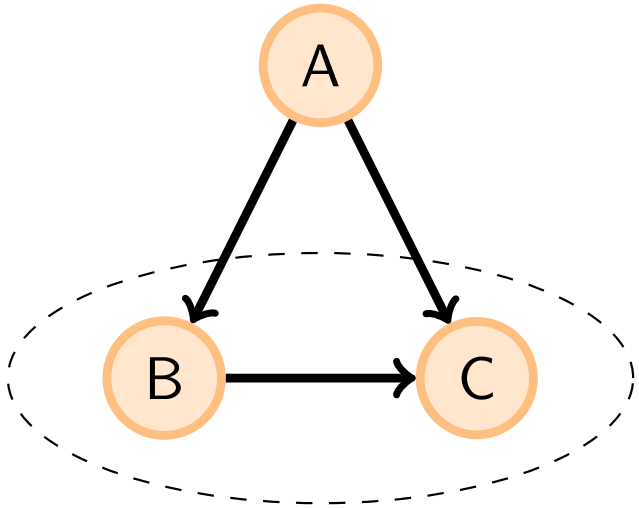


Figure: Not Strongly Connected Graph

In this graph from **B** only **C** is reachable. **A** is not reachable

Strongly Connected Component

Strongly Connected Components

Strongly Connected Components of a directed graph are the subgraphs which are **individually strongly connected**

Example of Strongly Connected Components

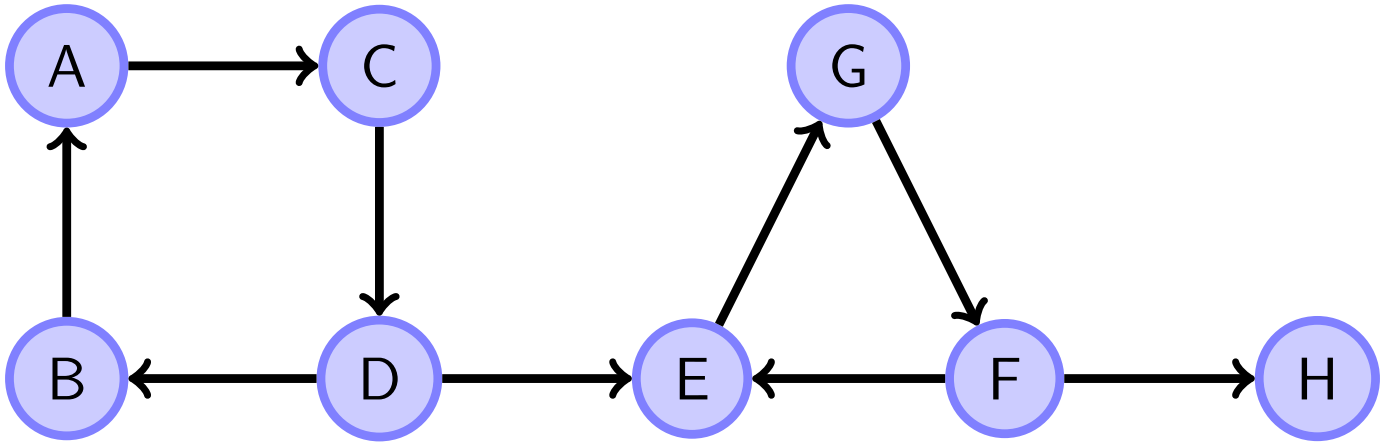
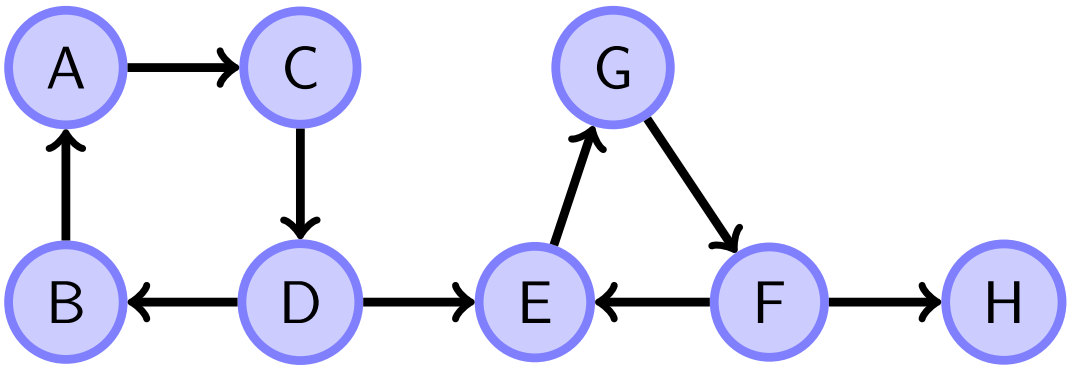


Figure: A Directed Graph

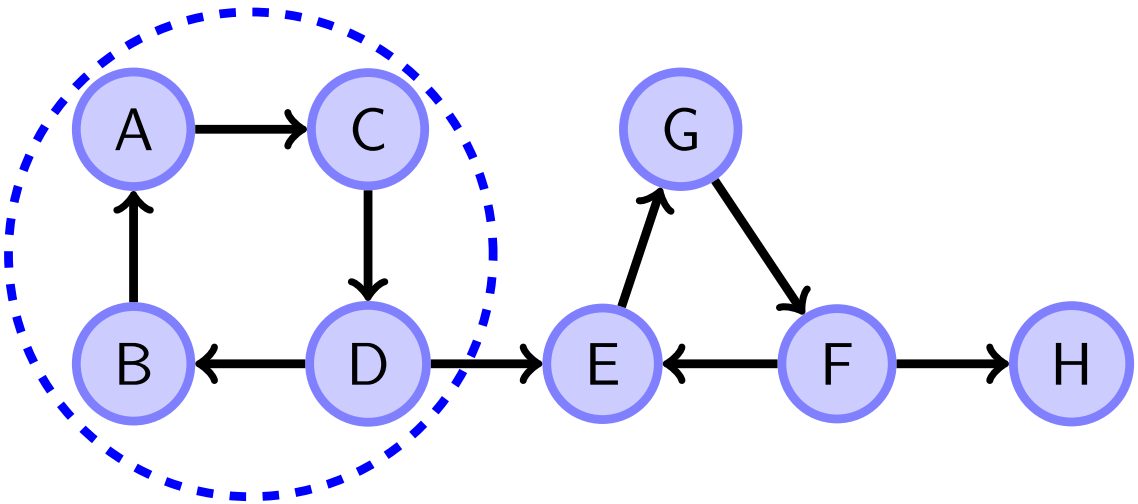
Example of Strongly Connected Components (Continued)

Given Graph



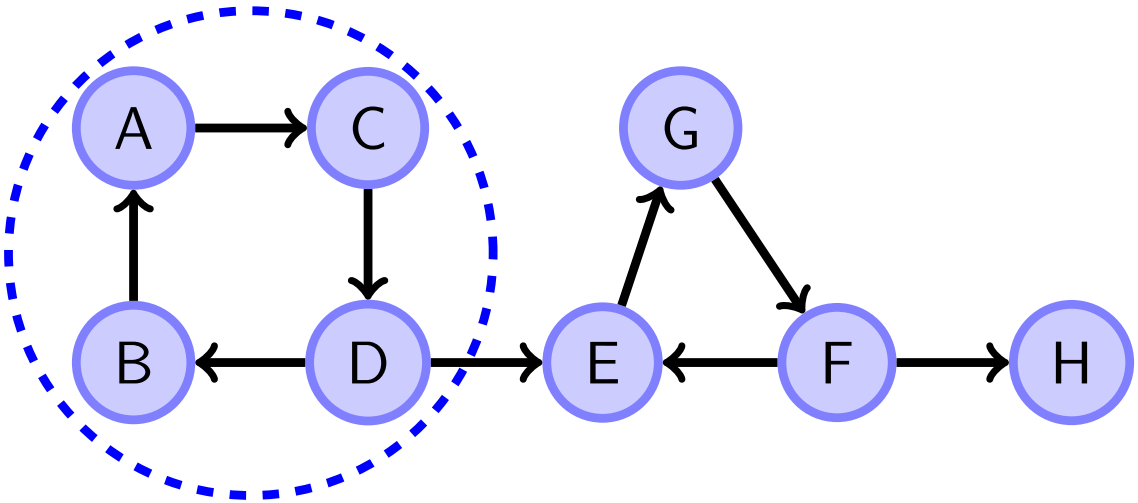
Example of Strongly Connected Components (Continued)

Given Graph

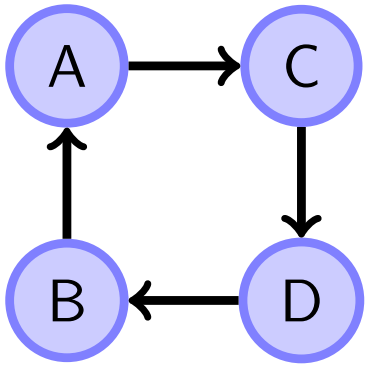


Example of Strongly Connected Components (Continued)

Given Graph

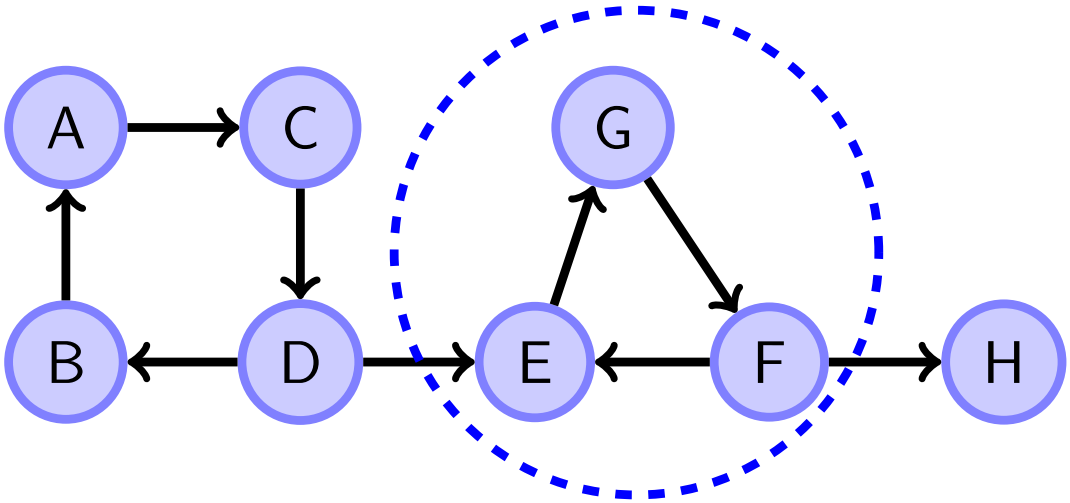


Strongly Connected Components



Example of Strongly Connected Components (Continued)

Given Graph



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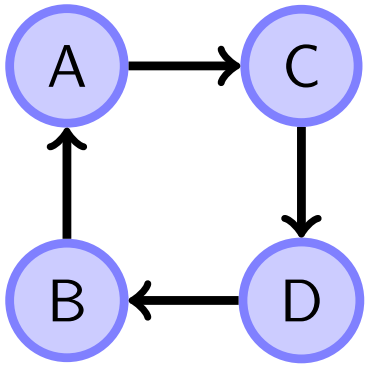
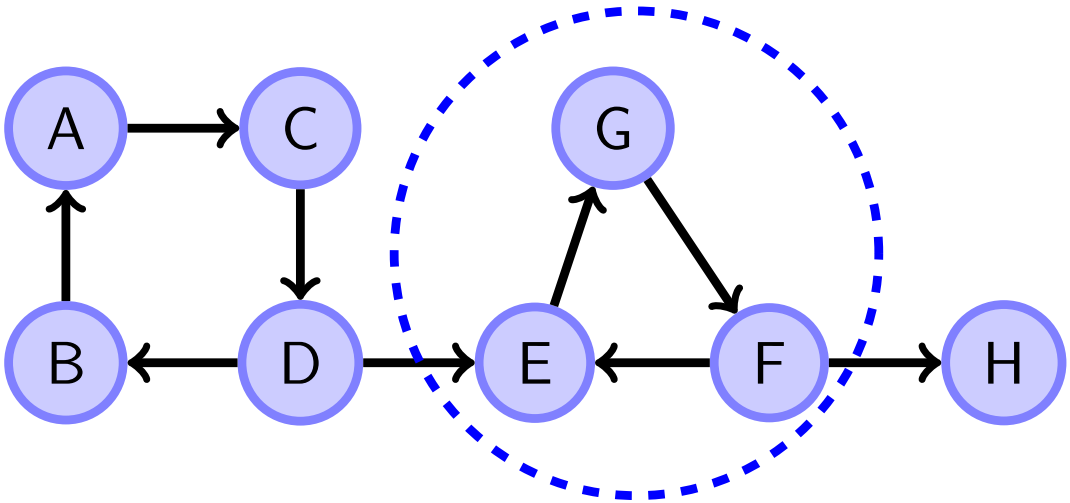


Example of Strongly Connected Components (Continued)

Given Graph



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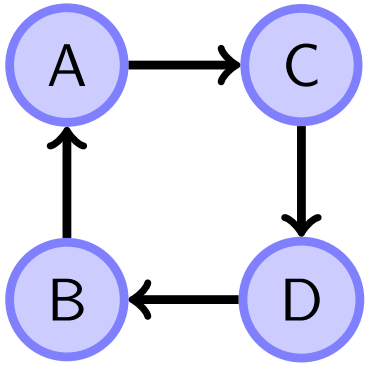
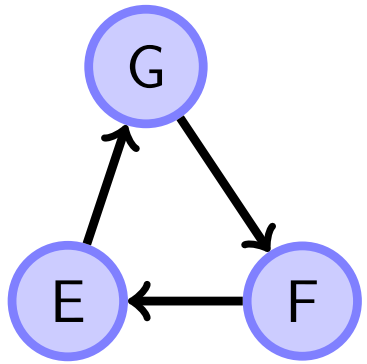
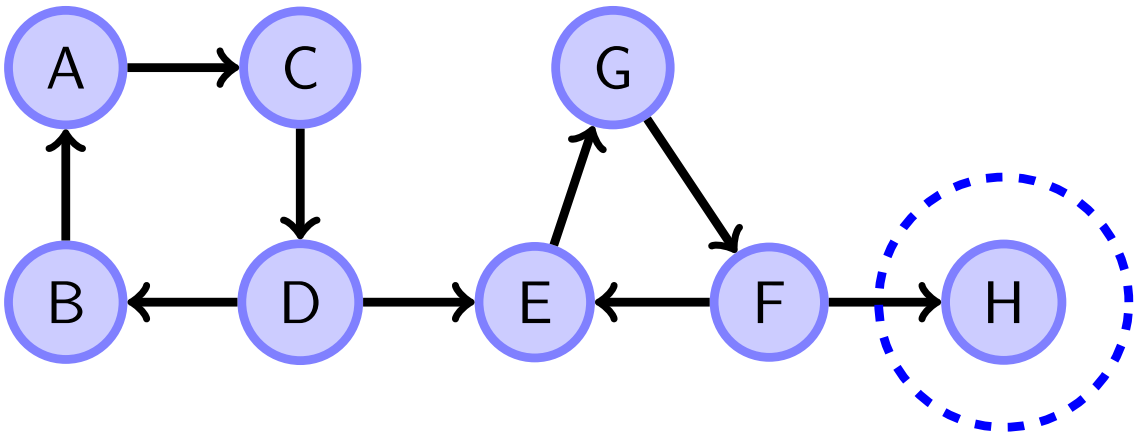


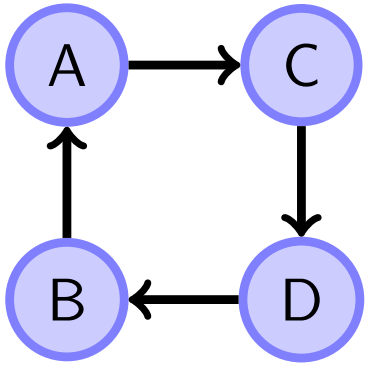


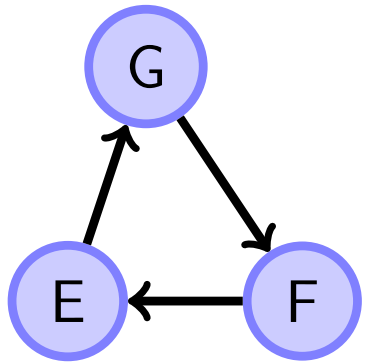
Example of Strongly Connected Components (Continued)

Given Graph



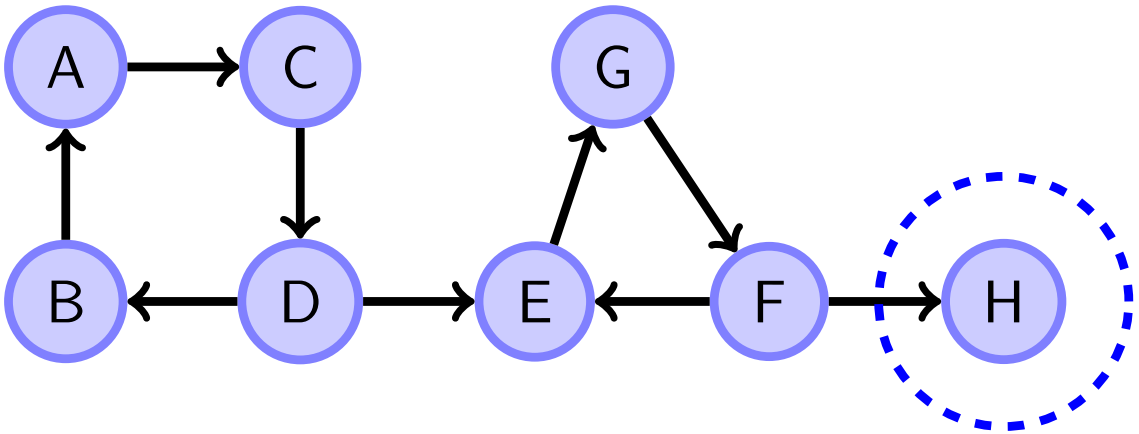
Strongly Connected Components





Example of Strongly Connected Components (Continued)

Given Graph



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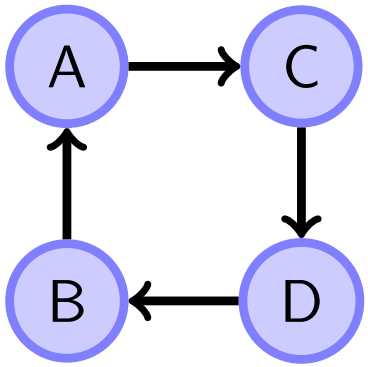
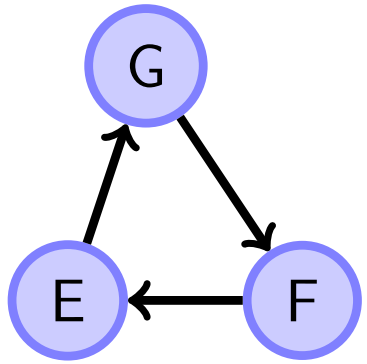
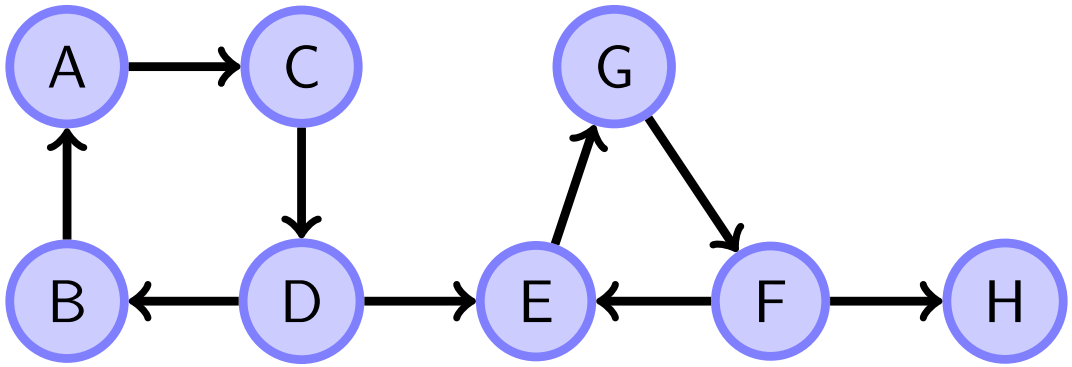




Strongly Connected Components of a Directed Graph

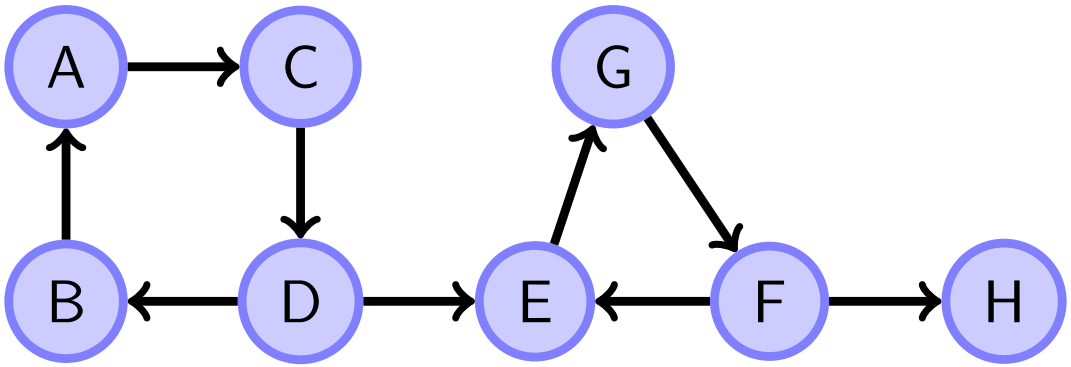
Given Graph



So finally the Strongly Connected Components are :
{A,B,C,D},{E,F,G},{H}

Finding Strongly Connected Components

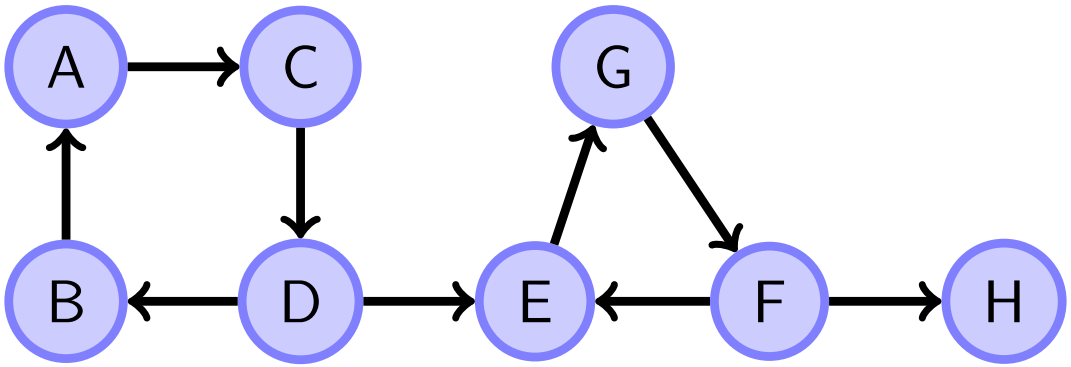
Given Graph



- **How to decompose a directed graph into strongly connected components?**

Finding Strongly Connected Components

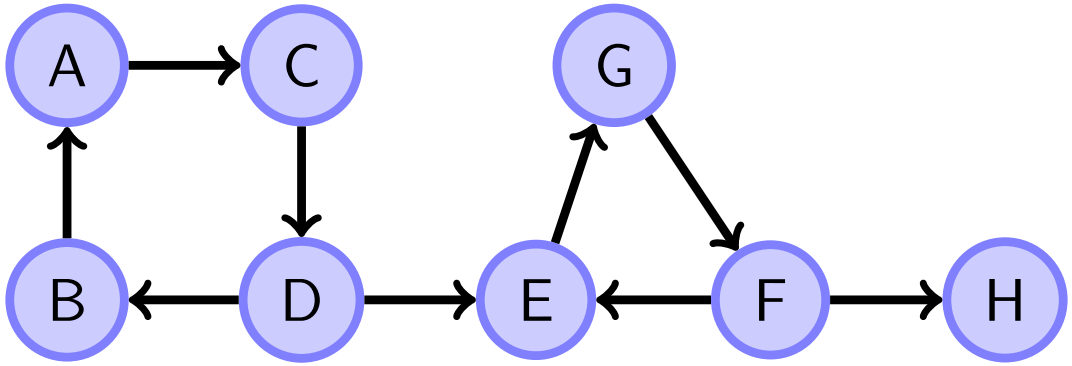
Given Graph



- **How to decompose a directed graph into strongly connected components?**
- **The idea is to use Depth First Search , but in a tricky way!**

Simulation: Kosaraju's Algorithm

- **Step 1:** DFS and Topological Sort on the given graph



Stack

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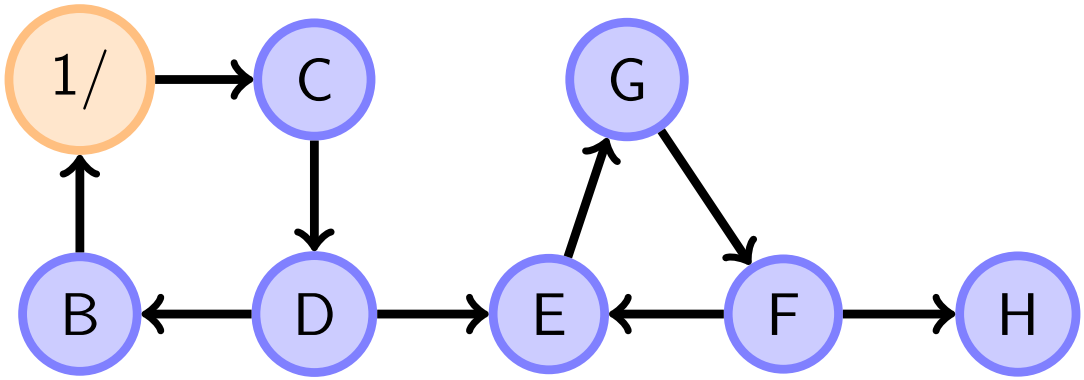

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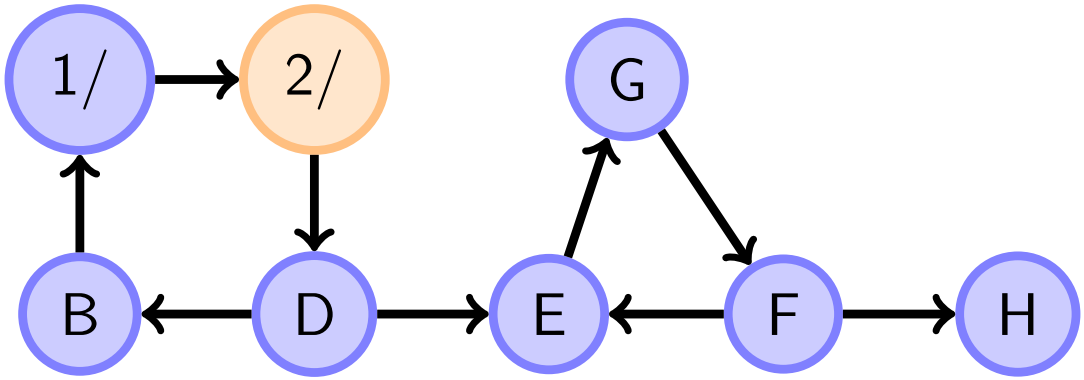

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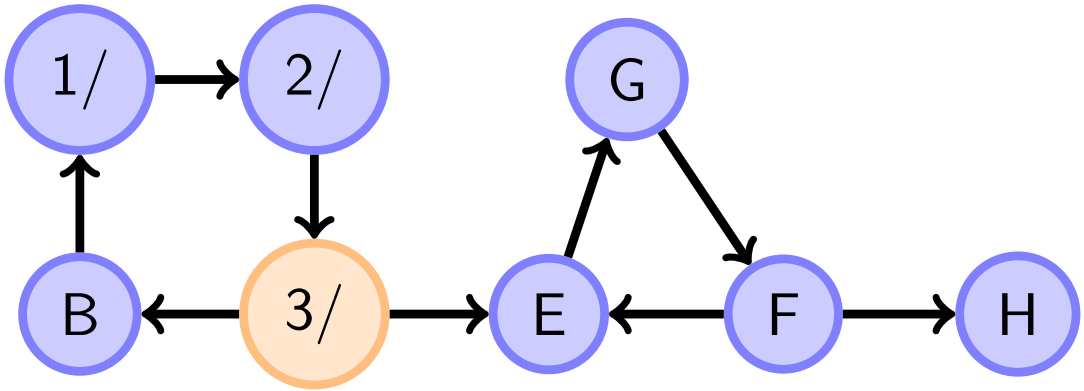

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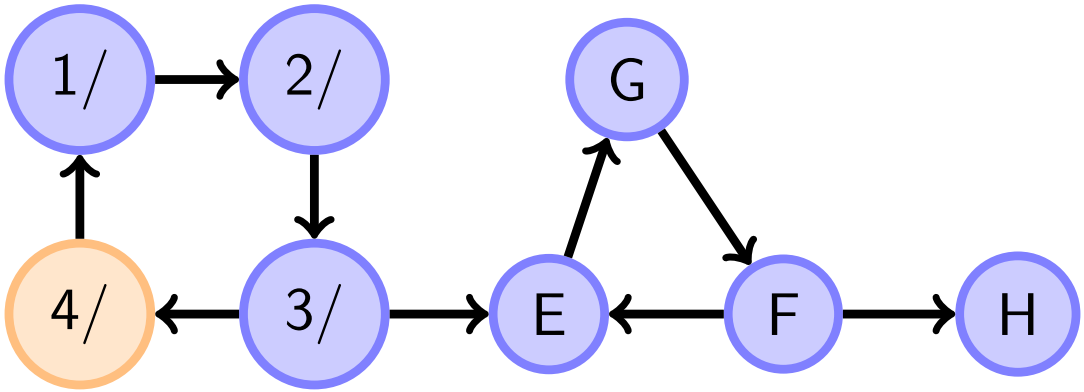

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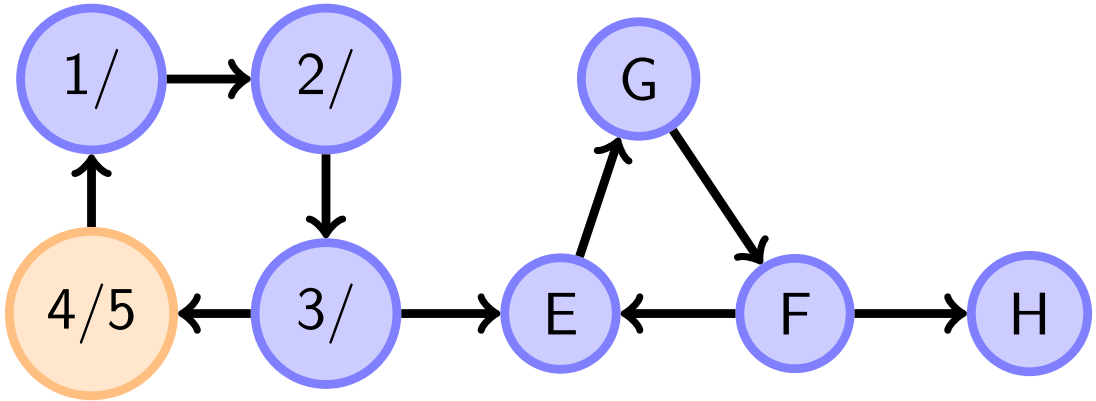

Stack | B |

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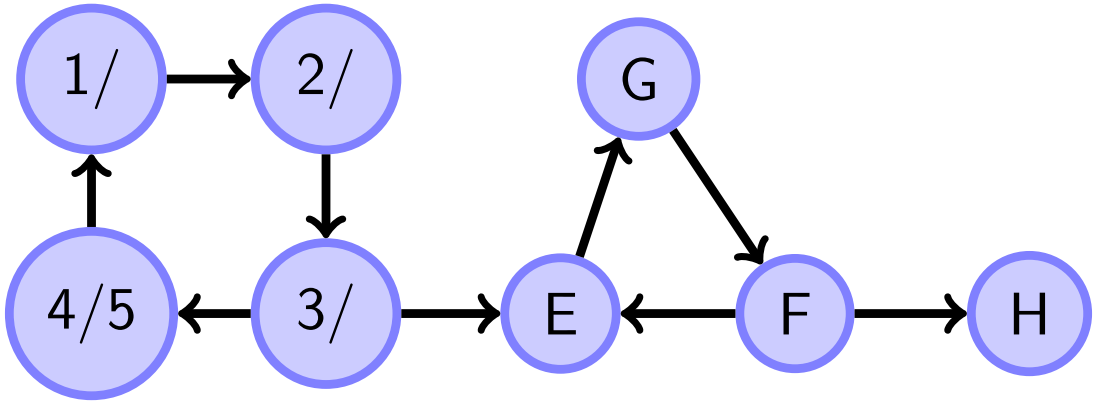

Stack | B |

Simulation: Kosaraju's Algorithm

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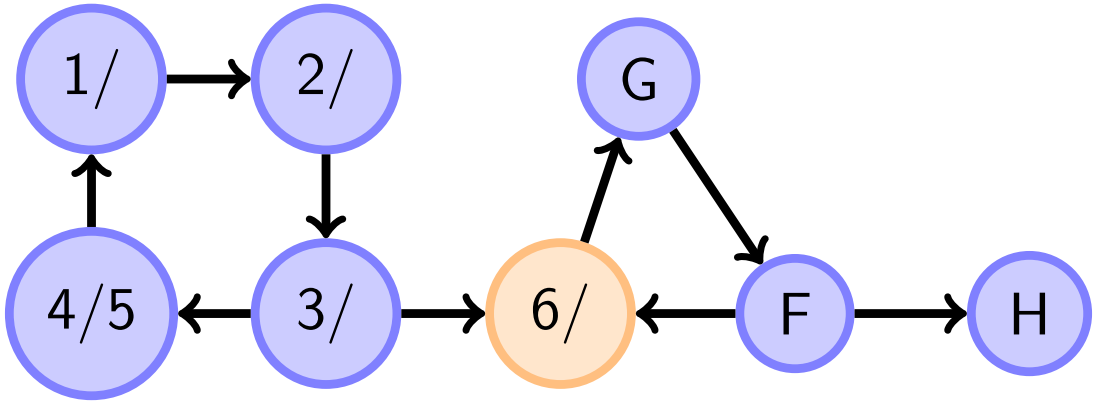

Stack | B |

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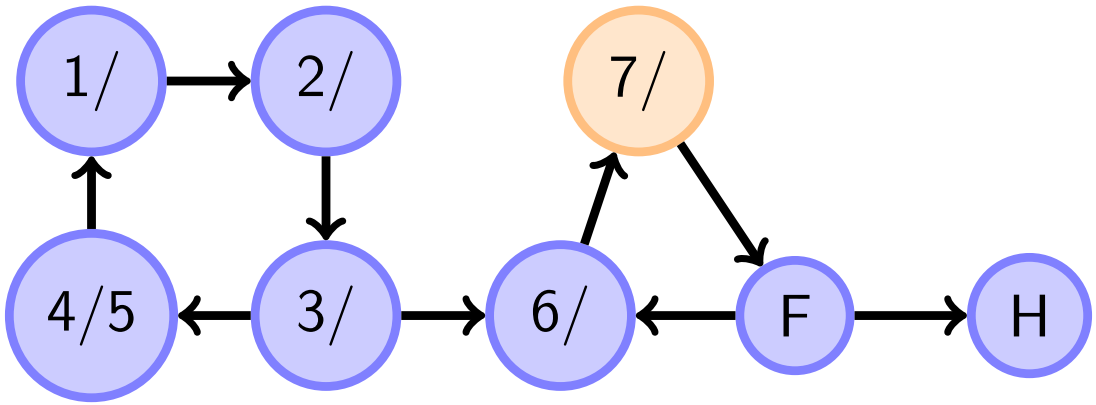

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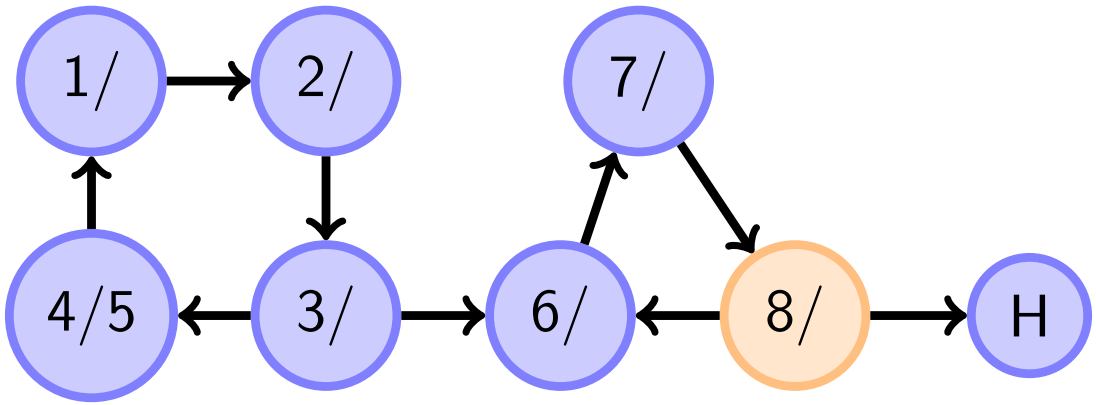

Stack | B |

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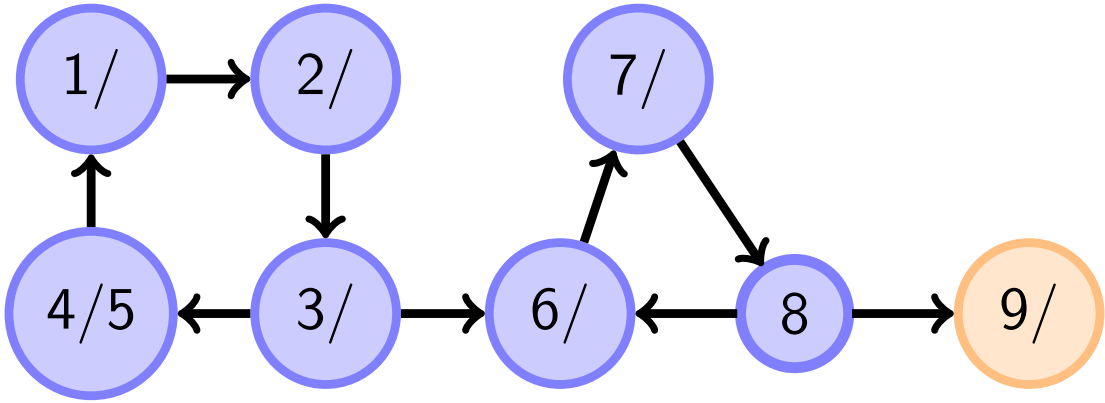

Stack | B |

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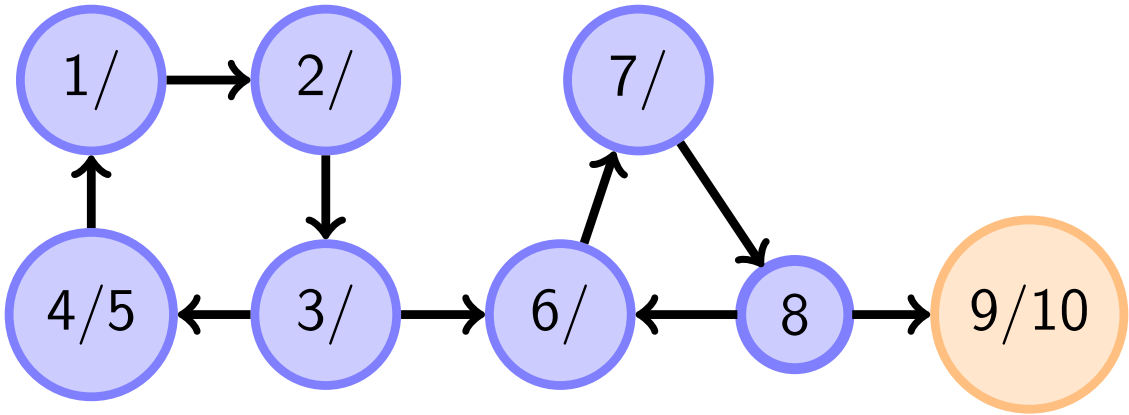
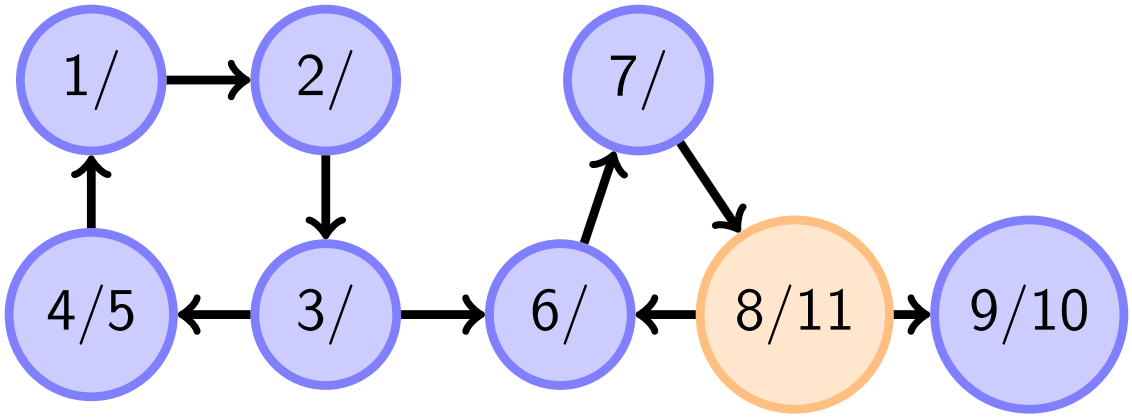


Stack | B | H |

Simulation: Kosaraju's Algorithm

- **Step 1:** DFS and Topological Sort on the given graph



Stack | B | H | F |

Simulation: Kosaraju's Algorithm

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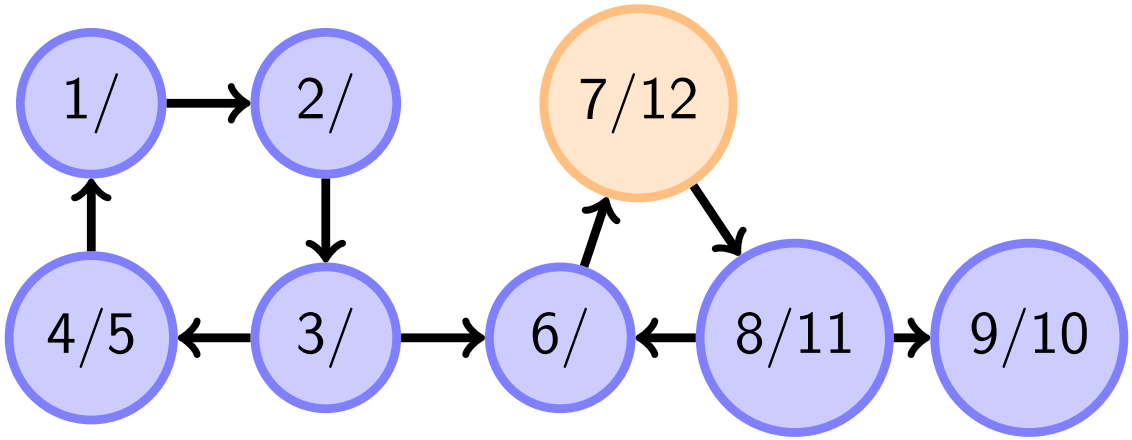

Stack | B | H | F | G |

Simulation: Kosaraju's Algorithm

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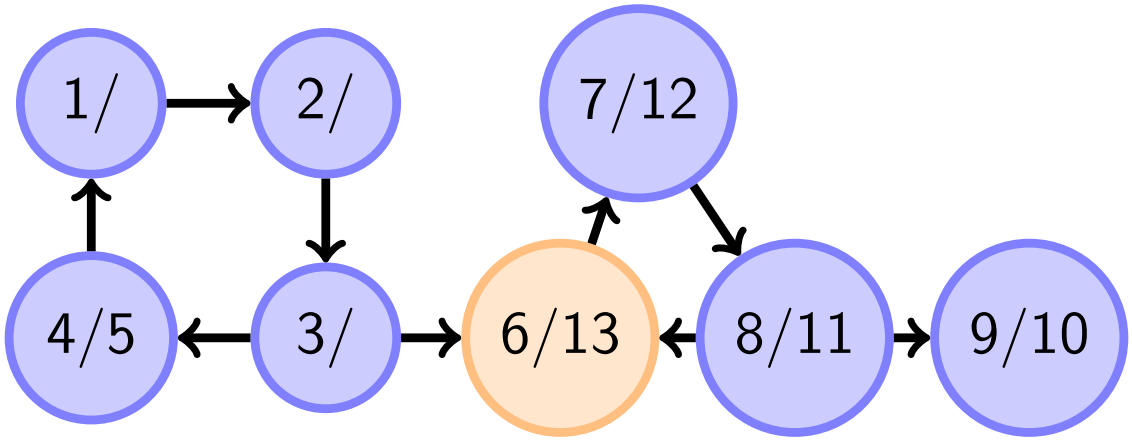

Stack | B | H | F | G | E |

Simulation: Kosaraju's Algorithm

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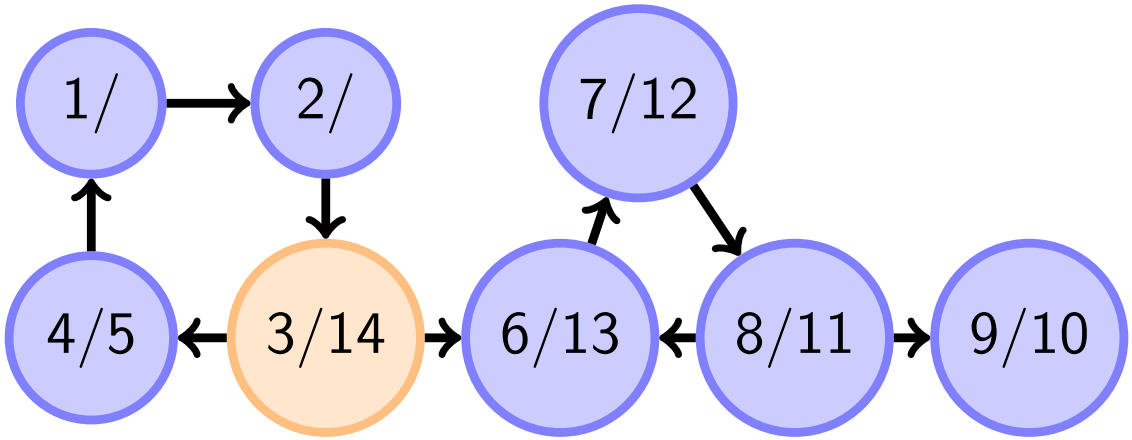
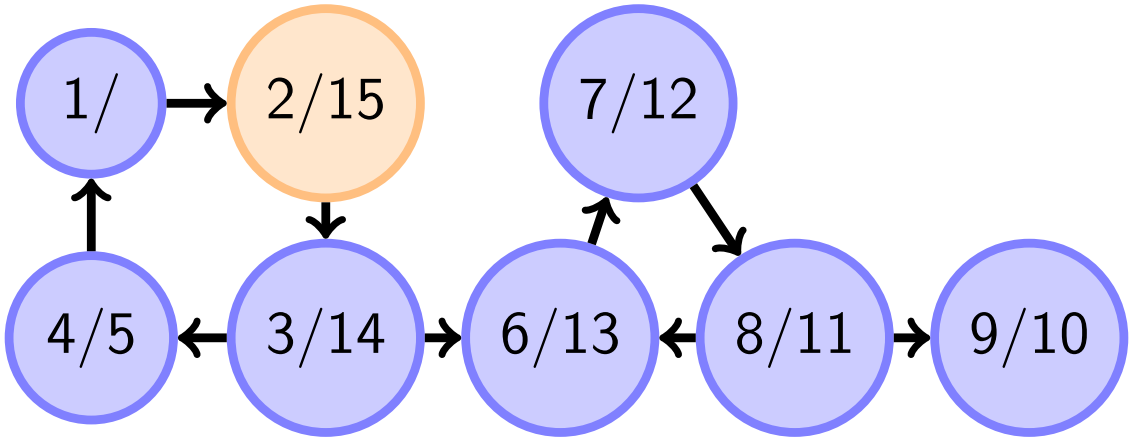


| **Stack** | B | H | F | G | E | C |

Simulation: Kosaraju's Algorithm

- **Step 1:** DFS and Topological Sort on the given graph



Stack | B | H | F | G | E | C | D |

Simulation: Kosaraju's Algorithm

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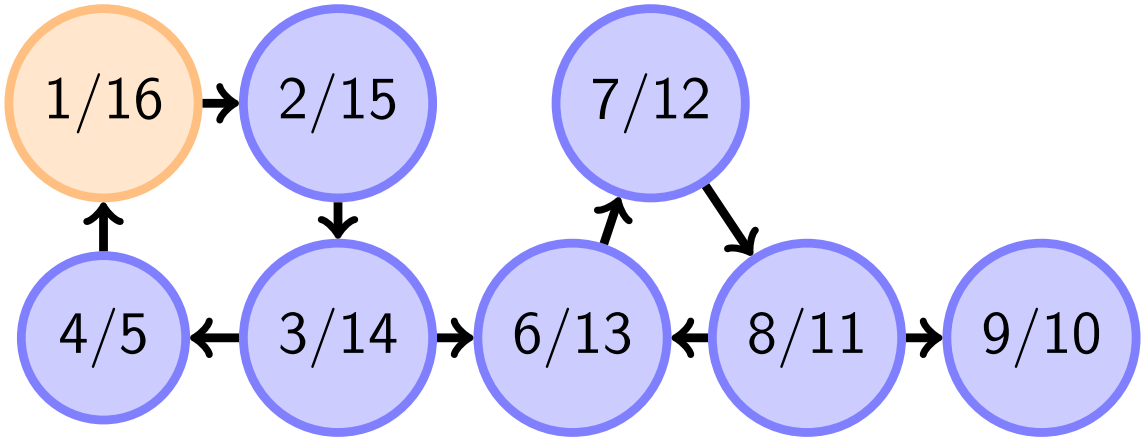

Stack	B	H	F	G	E	C	D	A

Simulation: Kosaraju's Algorithm

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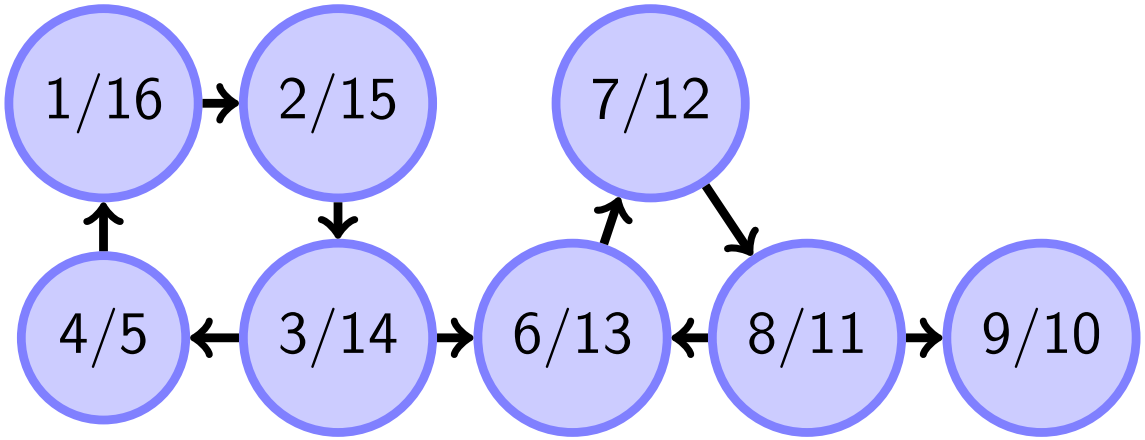


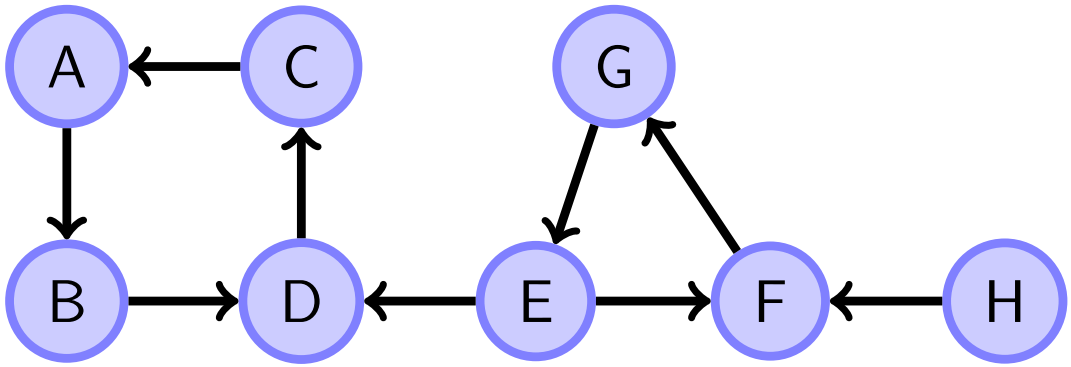
Stack | B | H | F | G | E | C | D | A |

Topological Ordering

Simulation: Kosaraju's Algorithm (Continued)

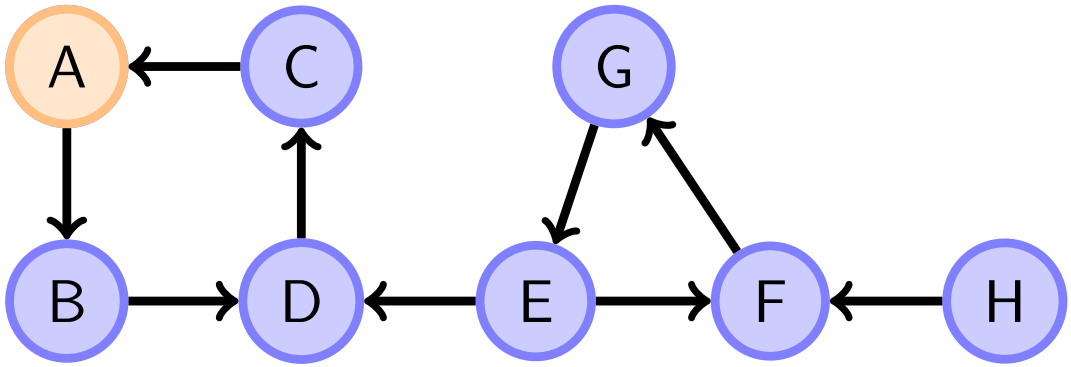
- **Step 2:** Reverse the edges and repeat DFS from topologically sorted nodes

Strongly Connected Components



Stack | B | H | F | G | E | C | D | A |

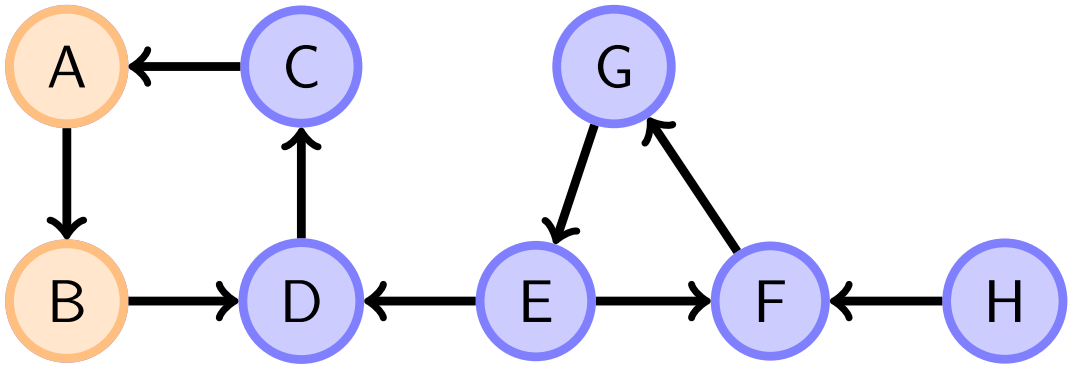
Simulation: Kosaraju's Algorithm (Continued)



Strongly Connected Components

Stack | B | H | F | G | E | C | D |

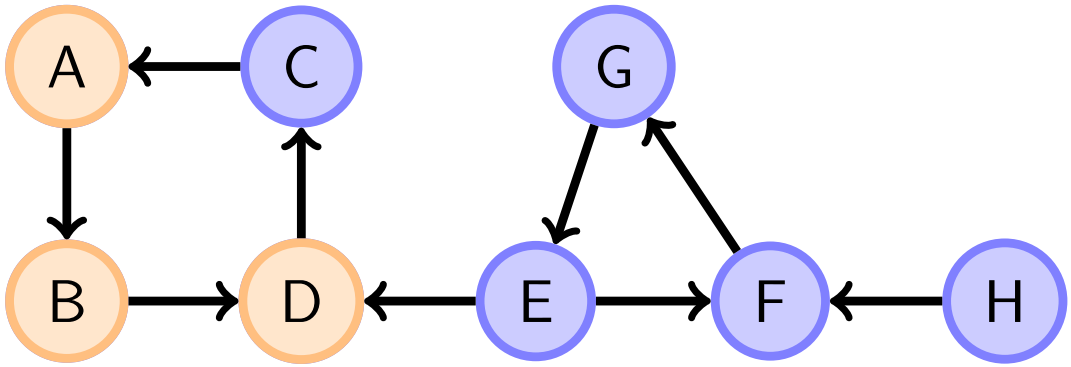
Simulation: Kosaraju's Algorithm (Continued)



Strongly Connected Components

Stack | B | H | F | G | E | C | D |

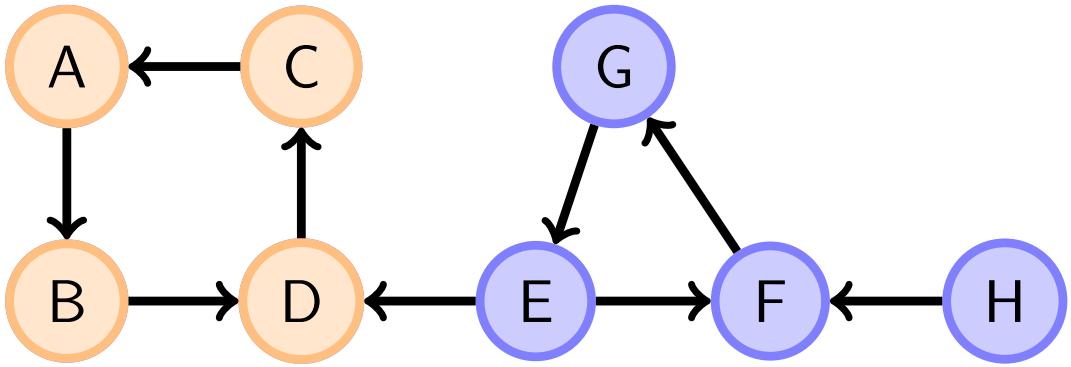
Simulation: Kosaraju's Algorithm (Continued)



Strongly Connected Components

Stack | B | H | F | G | E | C | D |

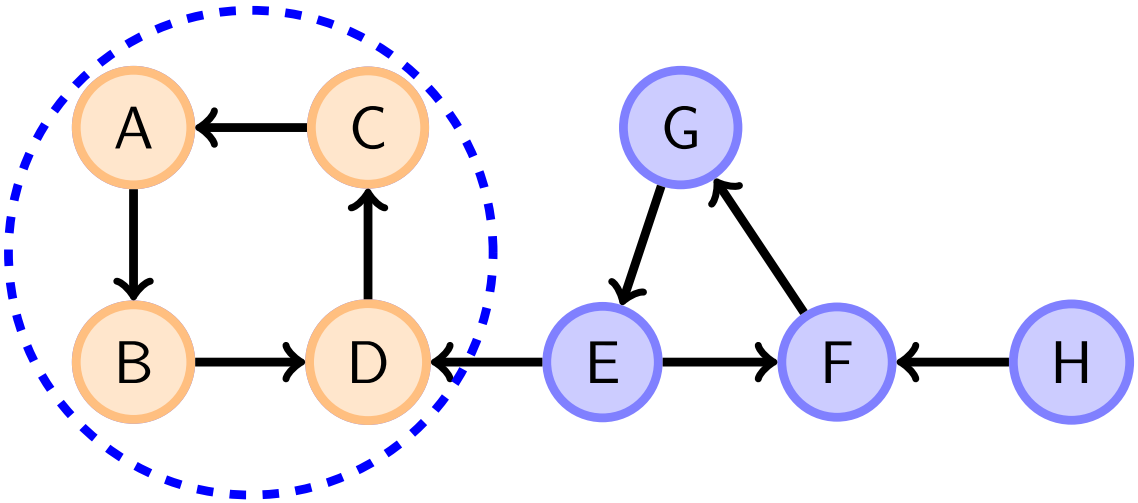
Simulation: Kosaraju's Algorithm (Continued)



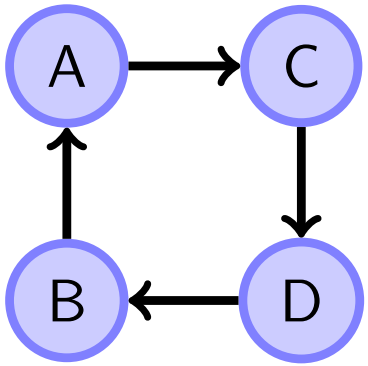
Strongly Connected Components

Stack | B | H | F | G | E | C | D |

Simulation: Kosaraju's Algorithm (Continued)

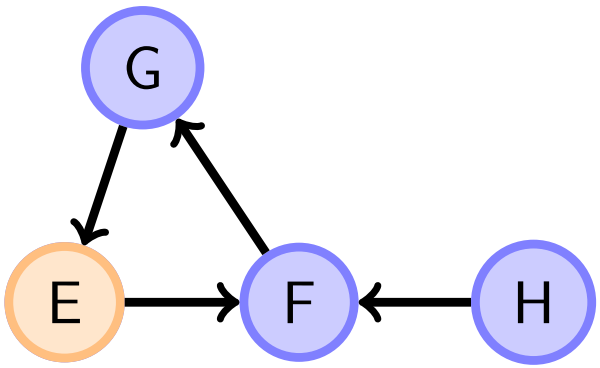


Strongly Connected Components



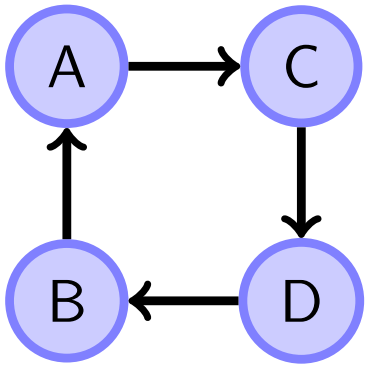
Stack | B | H | F | G | E |

Simulation: Kosaraju's Algorithm (Continued)

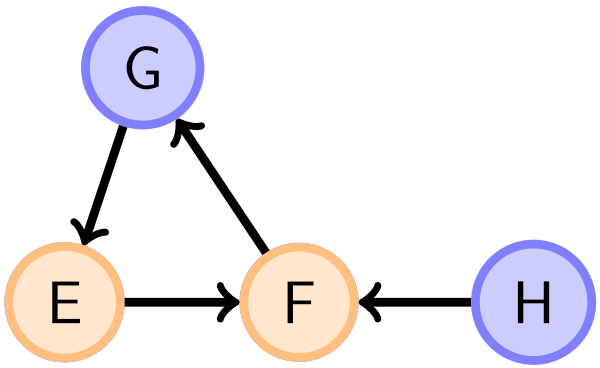


Stack | B | H | F | G |

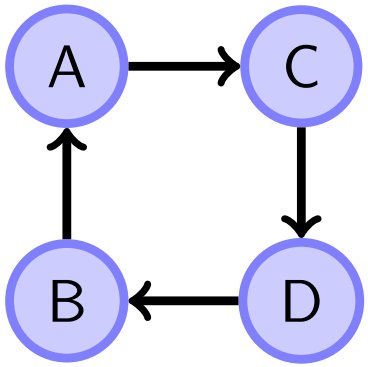
Strongly Connected Components



Simulation: Kosaraju's Algorithm (Continued)

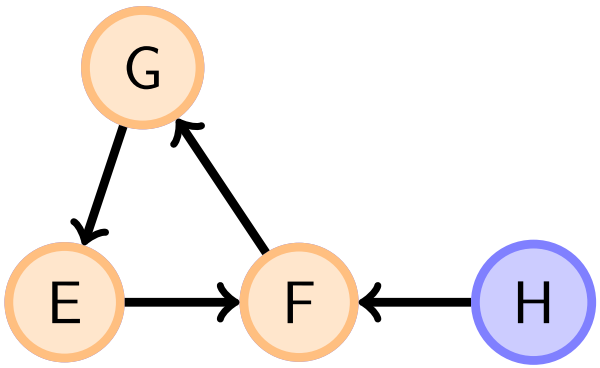


Strongly Connected Components



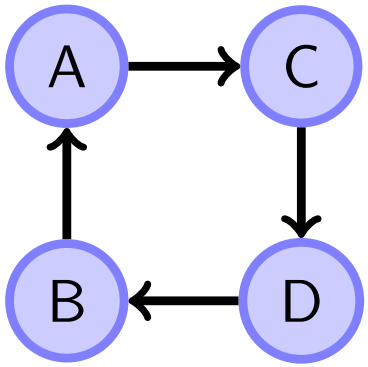
Stack | B | H | F | G |

Simulation: Kosaraju's Algorithm (Continued)

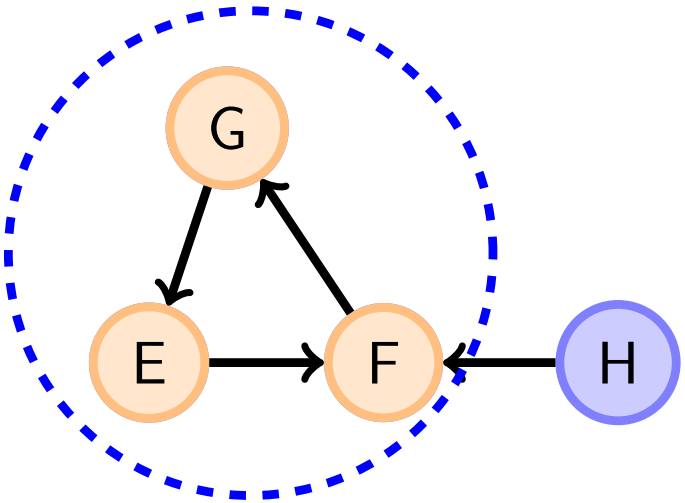


Stack | B | H |

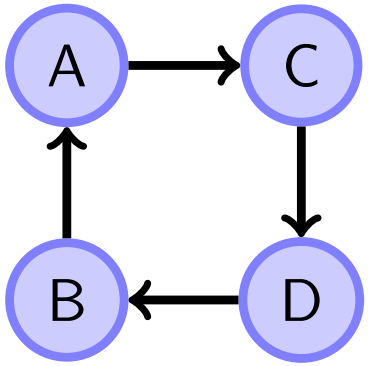
Strongly Connected Components

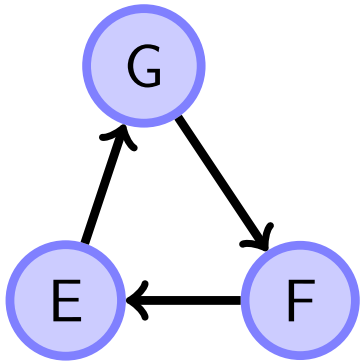


Simulation: Kosaraju's Algorithm (Continued)



Strongly Connected Components

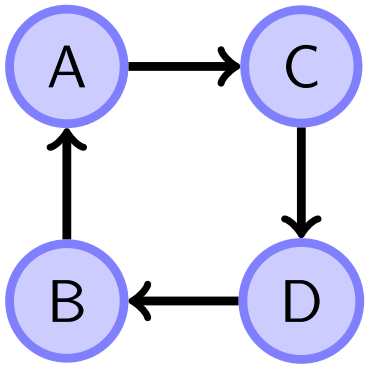


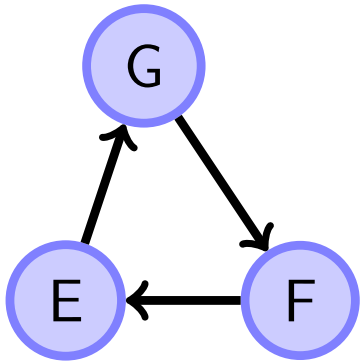


Stack | B | H |

Simulation: Kosaraju's Algorithm (Continued)

Strongly Connected Components

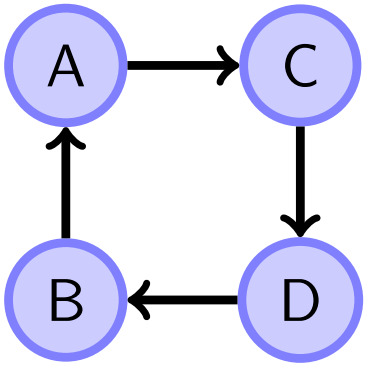


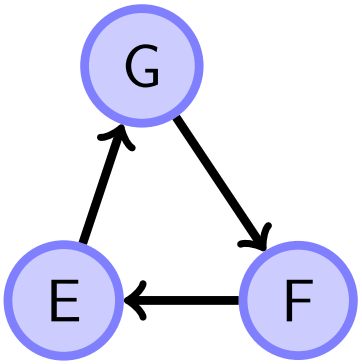


Stack | B |

Simulation: Kosaraju's Algorithm (Continued)

Strongly Connected Components





Stack |

Applications

- Directed Acyclic Subgraph Formation

Applications

- Directed Acyclic Subgraph Formation
- Social Connectivity Network Analysis

Applications

- Directed Acyclic Subgraph Formation
- Social Connectivity Network Analysis
- Map Processing and Vehicle Routing

Thank You

Any Questions?