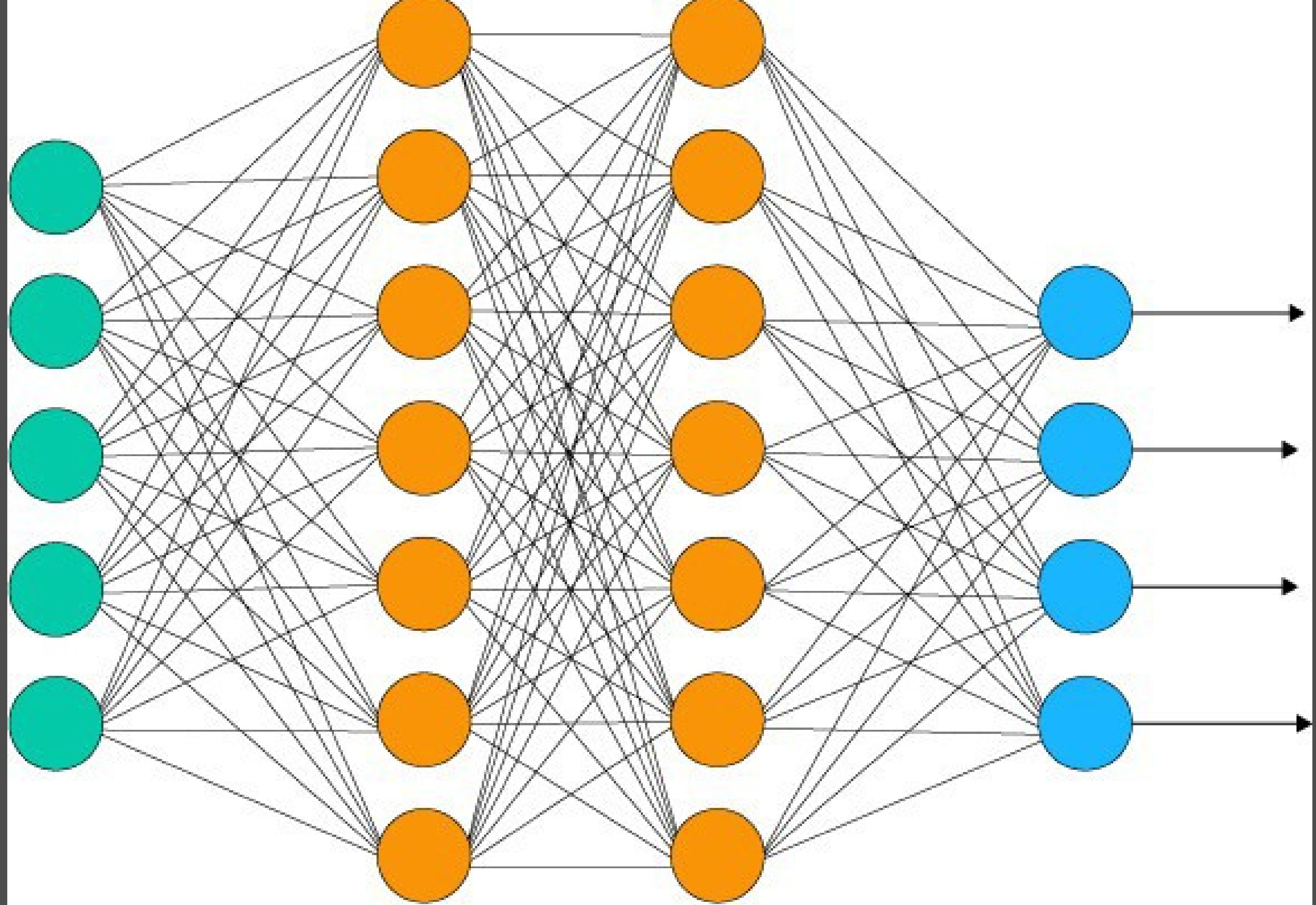


Symbol	Definition
$L$	Number of layers in the network
$l$	Layer index
$j$	Node index for layer $l$
$k$	Node index for layer $l - 1$
$y_j$	The value of node $j$ in the output layer $L$ for a single training sample
$C_0$	Loss function of the network for a single training sample
$w_j^{(l)}$	The vector of weights connecting all nodes in layer $l - 1$ to node $j$ in layer $l$
$w_{jk}^{(l)}$	The weight that connects node $k$ in layer $l - 1$ to node $j$ in layer $l$
$z_j^{(l)}$	The input for node $j$ in layer $l$
$g^{(l)}$	The activation function used for layer $l$
$a_j^{(l)}$	The activation output of node $j$ in layer $l$



● Input Layer

● Hidden Layer

● Output Layer

# ALGORITHM

STEP 1 : Initialize all the weights and biases in the network

STEP 2 : While termination condition is not satisfied

STEP 3 :       For each training instance  $X$

STEP 4 :       **FORWARD PROPAGATE**

# FORWARD PROPAGATION

$$x = a^{(1)} \quad \text{Input layer}$$

$$z^{(2)} = W^{(1)}x + b^{(1)} \quad \text{neuron value at Hidden}_1 \text{ layer}$$

$$a^{(2)} = f(z^{(2)}) \quad \text{activation value at Hidden}_1 \text{ layer}$$

$$z^{(3)} = W^{(2)}a^{(2)} + b^{(2)} \quad \text{neuron value at Hidden}_2 \text{ layer}$$

$$a^{(3)} = f(z^{(3)}) \quad \text{activation value at Hidden}_2 \text{ layer}$$

$$s = W^{(3)}a^{(3)} \quad \text{Output layer}$$

## STEP 5 : Calculate Cost (Error) Function

### Loss $C_0$

Observe that the expression

$$\left(a_j^{(L)} - y_j\right)^2$$

is the squared difference of the activation output and the desired output for node  $j$  in the output layer  $L$ . This can be interpreted as the loss for node  $j$  in layer  $L$ .

Therefore, to calculate the total loss, we should sum this squared difference for each node  $j$  in the output layer  $L$ .

This is expressed as

$$C_0 = \sum_{j=0}^{n-1} \left(a_j^{(L)} - y_j\right)^2.$$

# Input $z_j^{(l)}$

We know that the input for node  $j$  in layer  $l$  is the weighted sum of the activation outputs from the previous layer  $l - 1$ .

An individual term from the sum looks like this:

$$w_{jk}^{(l)} a_k^{(l-1)}$$

So, the input for a given node  $j$  in layer  $l$  is expressed as

$$z_j^{(l)} = \sum_{k=0}^{n-1} w_{jk}^{(l)} a_k^{(l-1)}.$$

# Activation Output $a_j^{(l)}$

We know that the activation output of a given node  $j$  in layer  $l$  is the result of passing the input,  $z_j^{(l)}$ , to whatever activation function we choose to use  $g^{(l)}$ .

Therefore, the activation output of node  $j$  in layer  $l$  is expressed as

$$a_j^{(l)} = g^{(l)} \left( z_j^{(l)} \right).$$

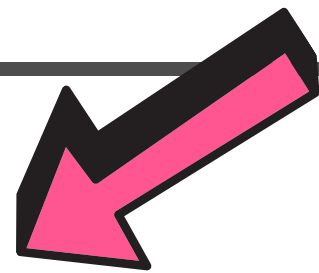


# COMPOSITION OF FUNCTIONS

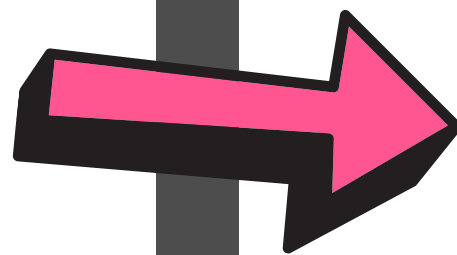
$$C_{0_j} = \left( a_j^{(L)} - y_j \right)^2.$$

We see that  $C_{0_j}$  is a function of the activation output of node  $j$  in layer  $L$ , and so we can express  $C_{0_j}$  as a function of  $a_j^{(L)}$  as

$$C_{0_j} \left( a_j^{(L)} \right).$$



$$a_j^{(L)} = g^{(L)} \left( z_j^{(L)} \right).$$



$$z_j^{(L)} = w_j^{(L)}.$$

# COMPOSITION OF FUNCTIONS

$$C_{0_j} = C_{0_j} \left( a_j^{(L)} \left( z_j^{(L)} \left( w_j^{(L)} \right) \right) \right).$$

## STEP 5 : DERIVATE OF THE LOSS FUNCTION

$$\frac{\partial C_0}{\partial w_{12}^{(L)}}.$$

Since  $C_0$  depends on  $a_1^{(L)}$ , and  $a_1^{(L)}$  depends on  $z_1^{(L)}$ , and  $z_1^{(L)}$  depends on  $w_{12}^{(L)}$ , the chain rule tells us that to differentiate  $C_0$  with respect to  $w_{12}^{(L)}$ , we take the product of the derivatives of the composed function.

This is expressed as

$$\frac{\partial C_0}{\partial w_{12}^{(L)}} = \left( \frac{\partial C_0}{\partial a_1^{(L)}} \right) \left( \frac{\partial a_1^{(L)}}{\partial z_1^{(L)}} \right) \left( \frac{\partial z_1^{(L)}}{\partial w_{12}^{(L)}} \right).$$

## STEP X : Update weights and bias

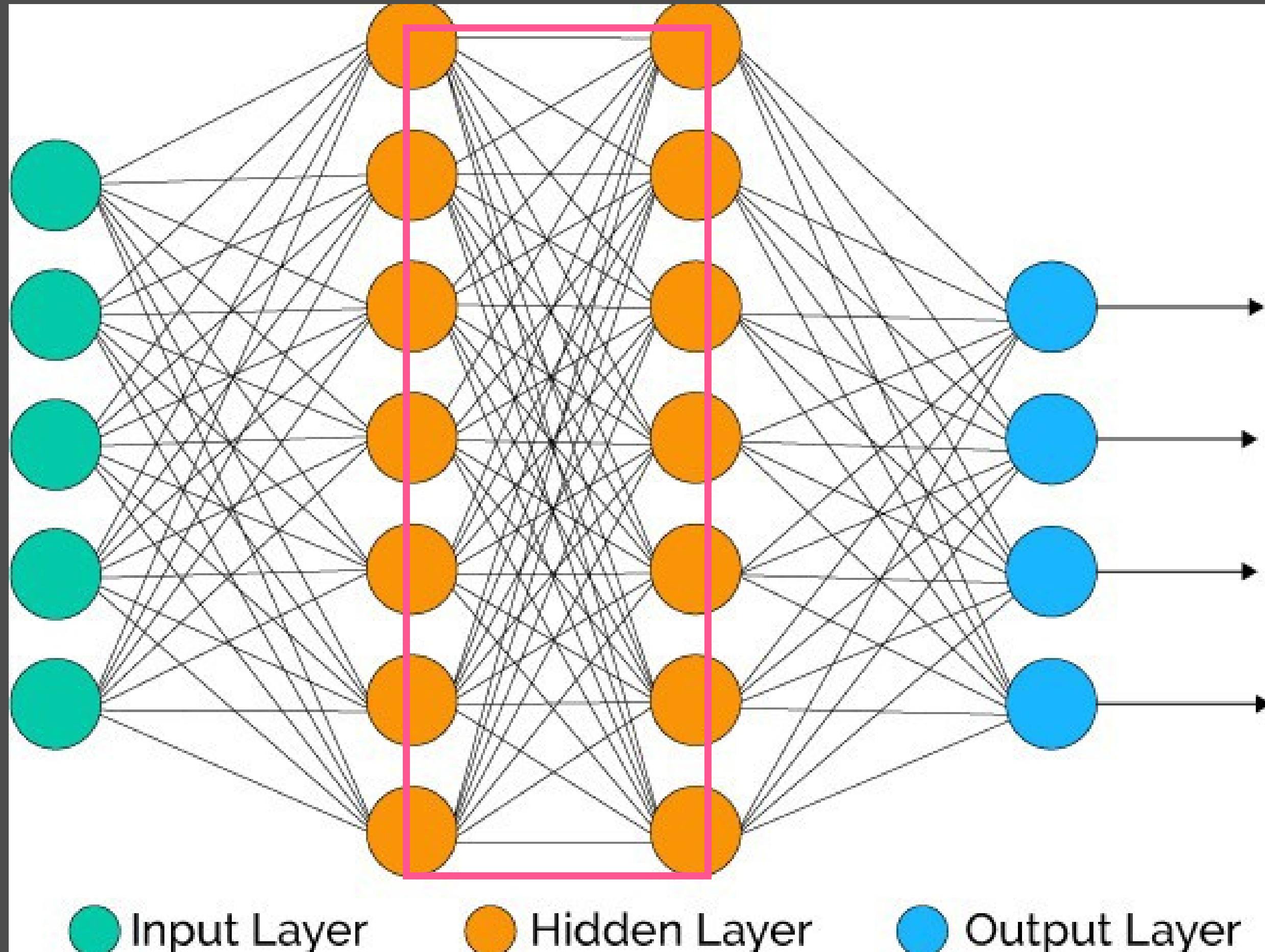
*while (termination condition not met)*

$$w := w - \epsilon \frac{\partial C}{\partial w}$$

$$b := b - \epsilon \frac{\partial C}{\partial b}$$

*end*

## STEP 6 : SIMILARLY FIND LOSS AND DERIVATE FOR HIDDEN LAYERS



## STEP 7 : Update weights and bias

*while (termination condition not met)*

$$w := w - \epsilon \frac{\partial C}{\partial w}$$

$$b := b - \epsilon \frac{\partial C}{\partial b}$$

*end*