

Trapezoidal Rule of Integration

What is Integration

Integration:

The process of measuring the area under a function plotted on a graph.

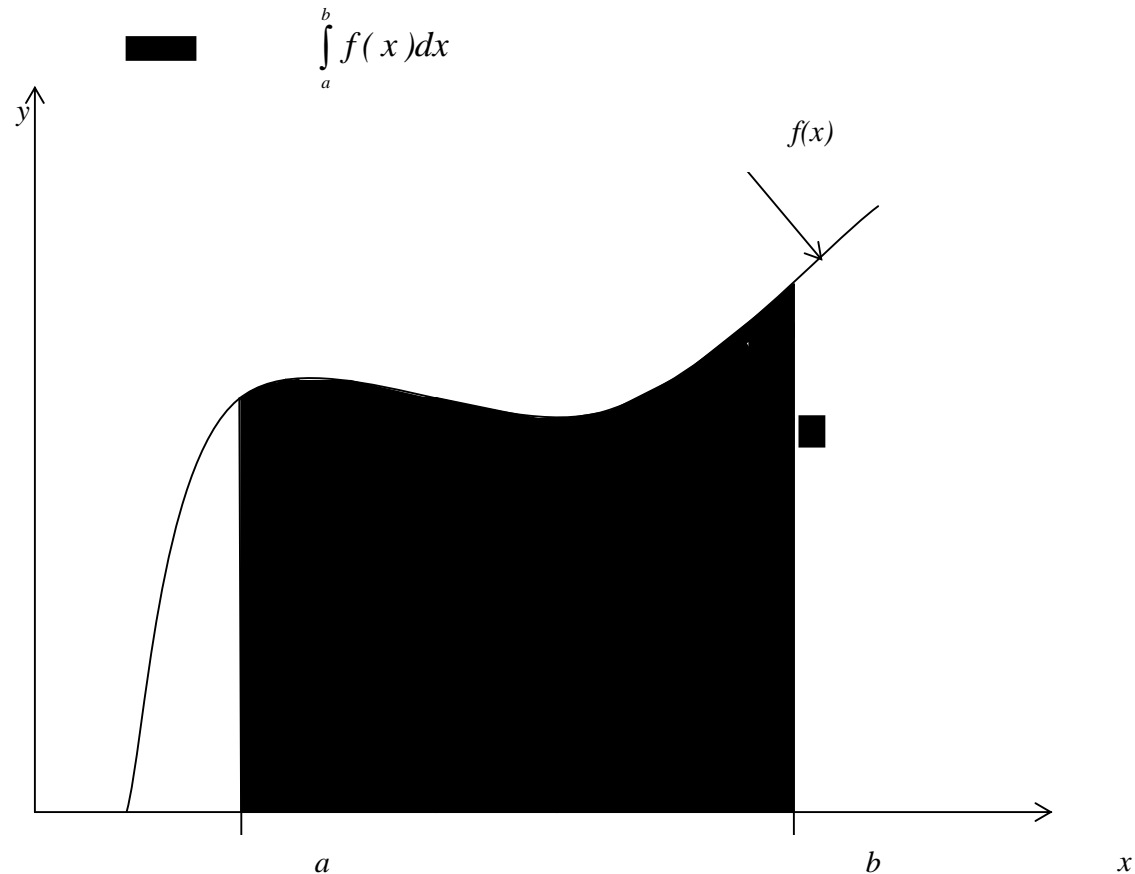
$$I = \int_a^b f(x) dx$$

Where:

$f(x)$ is the integrand

a = lower limit of integration

b = upper limit of integration



Basis of Trapezoidal Rule

Trapezoidal Rule is based on the Newton-Cotes Formula that states if one can approximate the integrand as an n^{th} order polynomial...

$$I = \int_a^b f(x) dx \quad \text{where} \quad f(x) \approx f_n(x)$$

and
$$f_n(x) = a_0 + a_1x + \dots + a_{n-1}x^{n-1} + a_nx^n$$

Basis of Trapezoidal Rule

Then the integral of that function is approximated by the integral of that n^{th} order polynomial.

$$\int_a^b f(x) \approx \int_a^b f_n(x)$$

Trapezoidal Rule assumes $n=1$, that is, the area under the linear polynomial,

$$\int_a^b f(x) dx = (b-a) \left[\frac{f(a) + f(b)}{2} \right]$$

Derivation of the Trapezoidal Rule

Method Derived From Geometry

The area under the curve is a trapezoid.
The integral

$$\begin{aligned}\int_a^b f(x)dx &\approx \text{Area of trapezoid} \\ &= \frac{1}{2}(\text{Sum of parallel sides})(\text{height}) \\ &= \frac{1}{2}(f(b) + f(a))(b - a) \\ &= (b - a) \left[\frac{f(a) + f(b)}{2} \right]\end{aligned}$$

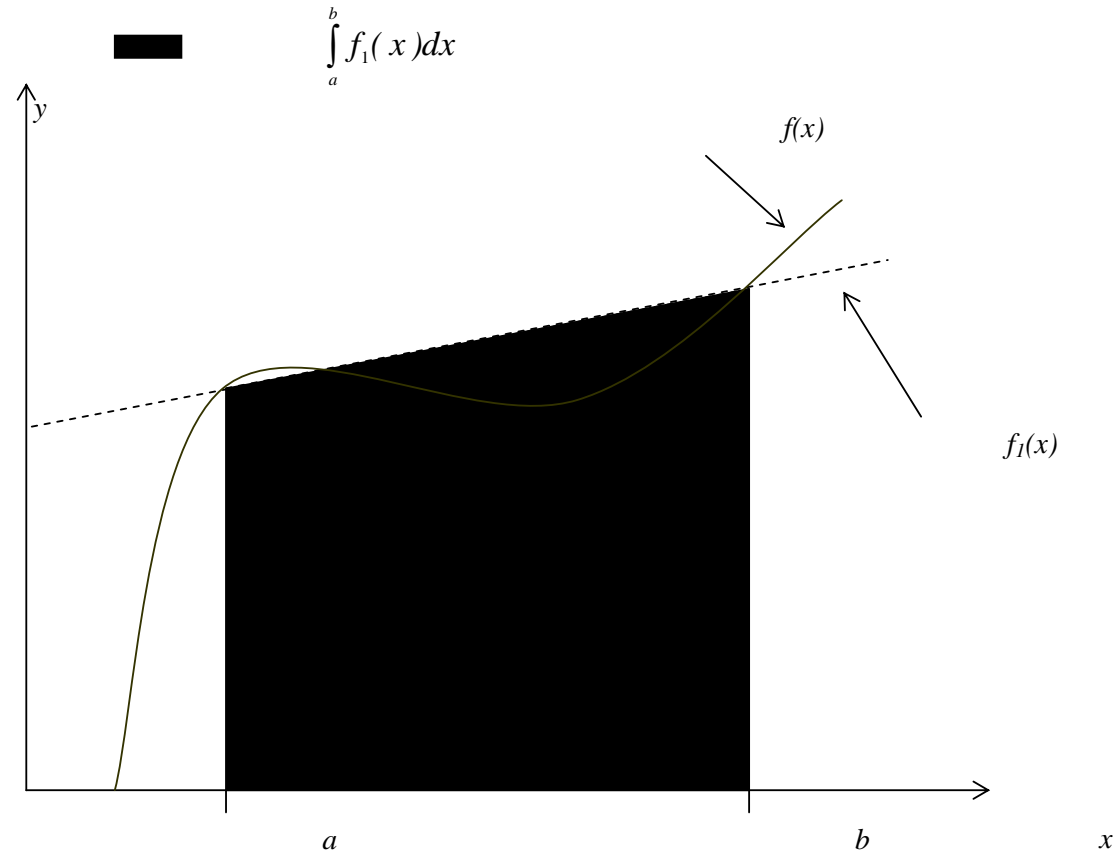


Figure 2: Geometric Representation

Example 1

The vertical distance covered by a rocket from $t=8$ to $t=30$ seconds is given by:

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

- a) Use single segment Trapezoidal rule to find the distance covered.
- b) Find the true error, E_t for part (a).
- c) Find the absolute relative true error, $|\epsilon_a|$ for part (a).

Solution

$$\text{a)} \quad I \approx (b - a) \left[\frac{f(a) + f(b)}{2} \right]$$

$$a = 8 \quad b = 30$$

$$f(t) = 2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t$$

$$f(8) = 2000 \ln \left[\frac{140000}{140000 - 2100(8)} \right] - 9.8(8) = 177.27 \text{ m/s}$$

$$f(30) = 2000 \ln \left[\frac{140000}{140000 - 2100(30)} \right] - 9.8(30) = 901.67 \text{ m/s}$$

Solution (cont)

$$\begin{aligned} \text{a)} \quad I &= (30 - 8) \left[\frac{177.27 + 901.67}{2} \right] \\ &= 11868 \, m \end{aligned}$$

b) The exact value of the above integral is

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt = 11061 \, m$$

Solution (cont)

b)

$$\begin{aligned} E_t &= \text{True Value} - \text{Approximate Value} \\ &= 11061 - 11868 \\ &= -807 \text{ m} \end{aligned}$$

c) The absolute relative true error, $|\epsilon_t|$, would be

$$|\epsilon_t| = \left| \frac{11061 - 11868}{11061} \right| \times 100 = 7.2959\%$$