Course Supervisor: Ms.Shaista Rais
University Of Karachi

DERIVATION
OF
LAGRANGE INTERPOLATING POLYNOMIAL

Course Supervisor: Ms.Shaista Rais University of Karachi

Lagrange Interpolation

Derivation:

we have

$$f(x) = a_0 + a_1 x + a_2 x^2 \dots a_n x^n \rightarrow (1)$$

$$\frac{f_1(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

can be rearranged as:

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) \rightarrow (2)$$

The lagrange interpolating polynomial can be derived directly from Newton's formulation. Here we are doing only for first order case.

For example, the first divided difference

$$f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \longrightarrow (A)$$

can be reformulated as

$$f[x_1, x_0] = \frac{f(x_1)}{x_1 - x_0} + \frac{f(x_0)}{x_0 - x_1} \longrightarrow (B)$$

Which is referred to as the Symmetric form.

Substituting eq (B) into eq (2), we get:

$$f_1(x) = f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1) + \frac{x - x_0}{x_0 - x_1} f(x_0)$$

finally grouping similar terms and simplifying yeids the Lagrange form.

$$\boxed{f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)}$$

The Lagrangian interpolating polynomial is given by

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

where n in $f_n(x)$ stands for the n^{th} order polynomial that approximates the function $y = f(x)$ given at $n+1$ data points as $(x_0, y_0),(x_1, y_1),......,(x_{n-1}, y_{n-1}),(x_n, y_n)$, and

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$L_i(x)$ is a weighting function that includes a product of $n-1$ terms with terms of $j = i$ omitted. The application of Lagrangian interpolation will be clarified using an example.

Course Supervisor: Ms.Shaista Rais University of Karachi