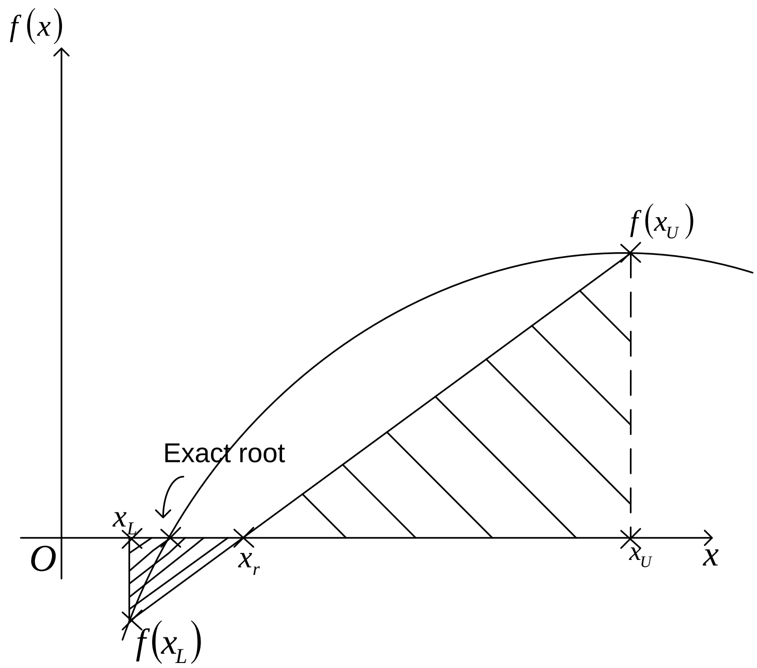
FALSE POSITION METHOD:



DEFINITION:

The false-position method takes advantage over BISECTION METHOD mathematically by drawing a secant from the function value at x_L to the function value at x_U, and estimates the root as where it crosses the *x*-axis.

Derivation:

Consider the above figure:

Based on two similar triangles, shown in Figure 1, one gets

$$\frac{0-f(x_L)}{x_r-x_L}=\frac{0-f(x_U)}{x_r-x_U} \tag{1}$$

From Equation (1), one obtains

$$(x_r-x_L)f(x_U)=(x_r-x_U)f(x_L)$$

$$x_U f(x_L)-x_L f(x_U)=x_r\{f(x_L)-f(x_U)\}$$

The above equation can be solved to obtain the next predicted root x_m as

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)} \qquad (2)$$

False-Position Algorithm

The steps to apply the false-position method to find the root of the equation $f(x)=0$ are as follows.

1. Choose x_L and x_U as two guesses for the root such that $f(x_L)f(x_U)<0$, or in other words, $f(x)$ changes sign between x_L and x_U.

2. Estimate the root, x_r of the equation $f(x)=0$ as

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

3. Now check the following

If $f(x_L)f(x_r)<0$, then the root lies between x_L and x_r; then $x_L = x_L$ and $x_U = x_r$.

If $f(x_L)f(x_r)>0$, then the root lies between x_r and x_U; then $x_L = x_r$ and $x_U = x_U$.

If $f(x_L)f(x_r)=0$, then the root is x_r. Stop the algorithm.

4. Find the new estimate of the root

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

Find the absolute relative approximate error as

$$|\in_a| = \left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| \times 100$$

where

x_r^{new} = estimated root from present iteration

x_r^{old} = estimated root from previous iteration

5. Compare the absolute relative approximate error $|\in_a|$ with the pre-specified relative error tolerance $\in_s$. If $|\in_a| > \in_s$, then go to step 3, else stop the algorithm. Note one should also check whether the number of iterations is more than the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user about it.

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