QCD and collider physics

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Today's high energy colliders

Collider	Process	status
Tevatron	$p\bar{p}$	closes this month
LHC	pp	started Mar. '10

current and upcoming experiments collide protons

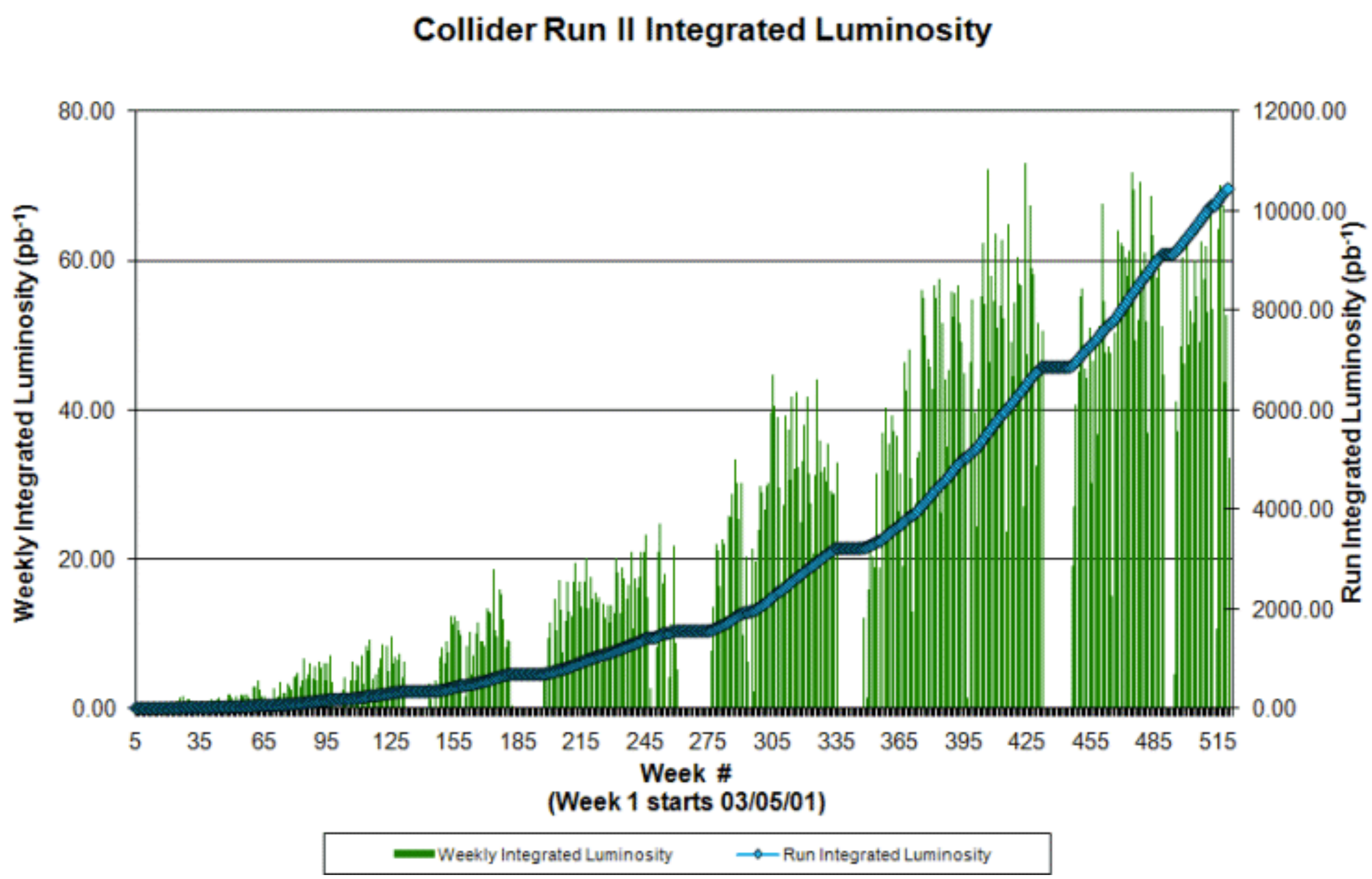
$\Rightarrow$ *all involve QCD*

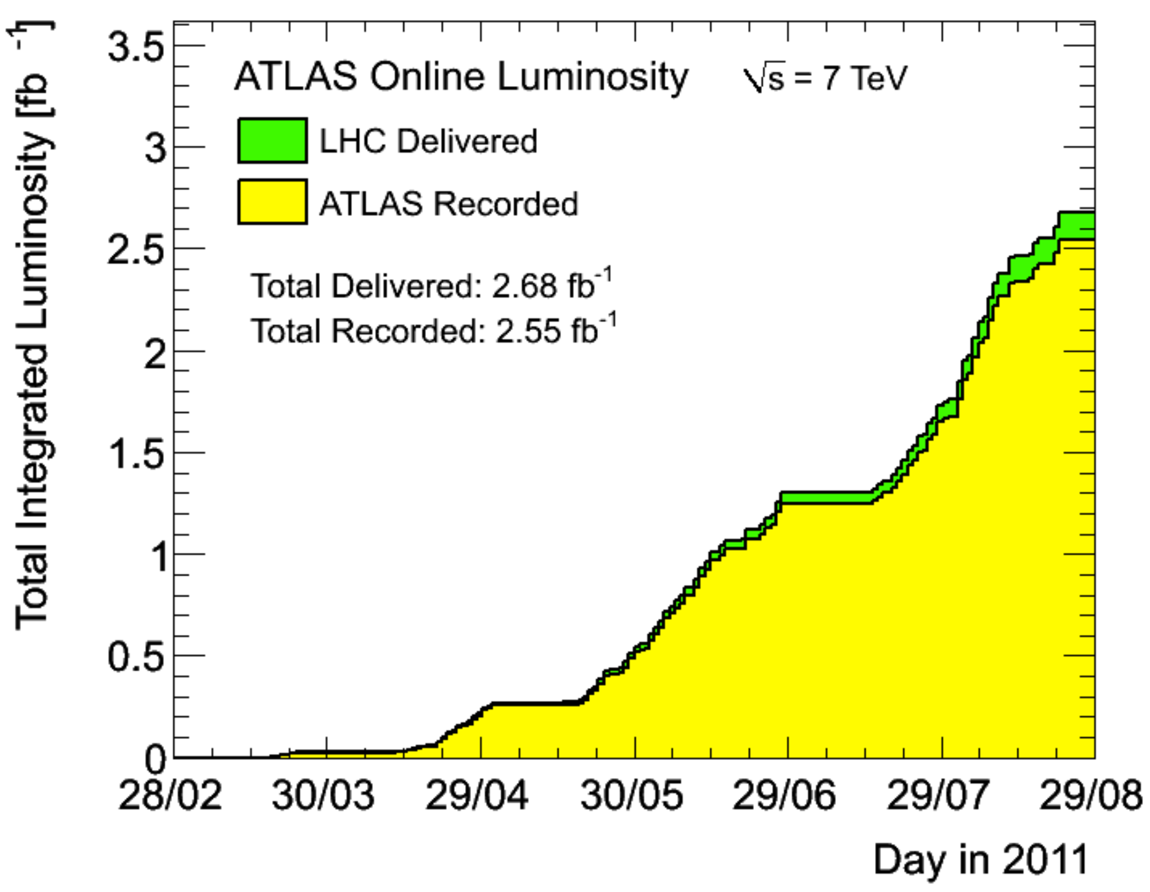
- Tevatron: discovery of top (1995) and many QCD measurements
- LHC designed to
  - understand the mechanics of electro-weak symmetry breaking (Higgs?)
  - unravel possible BSM physics

Tevatron and LHC figures

Tevatron: 1.8 TeV [run I], 1.96 TeV [run II]
LHC: 7 TeV [Mar. '10 - Dec. '12], 14TeV ? [after '14]

Tevatron: > 10 fb^{-1} [Sep. '11]
LHC: 2.5 fb^{-1} [Sep. '11], 5-8 fb^{-1} ['11-'12] ?, ? [after '14, SLHC?]





These lectures

These lectures will try to give you a theoretical basics for the analysis and interpretation of collider data

Mains aims of today's collider are to understand the EW symmetry breaking and/or the Beyond Standard Model particles that we might see. For this purpose one needs to

✓measure cross-sections
✓measure particle properties (spin, masses, couplings ...)

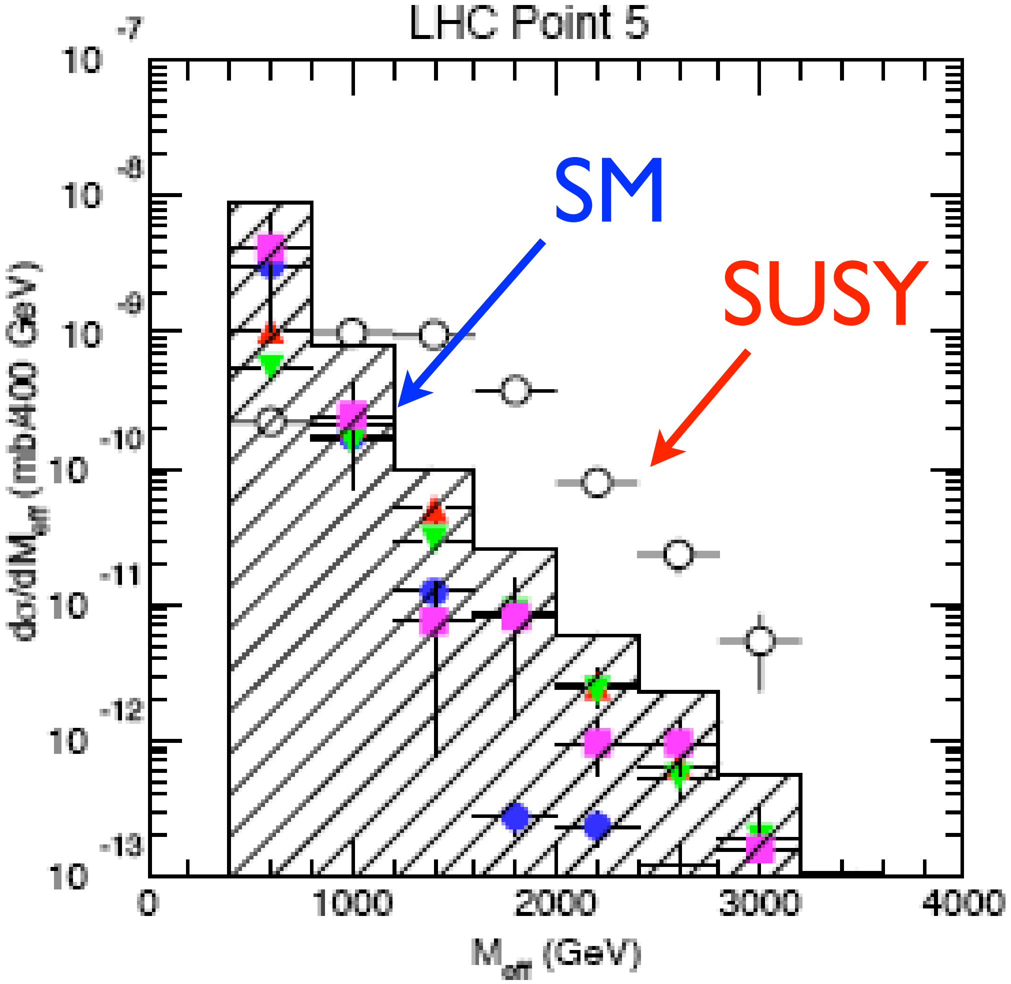
- Inclusive cross-section measurements can be done purely with data (no need for theory really)
- However, the extraction of properties requires theoretical predictions for cross-sections as a function of the "property to be measured"

These lectures will be a lot about how we can make those predictions

These lectures

For correct data interpretation it is crucial to

1. understand how much a given approximation can be trusted
2. know how to improve on it if necessary (when possible)



How reliable is the SM prediction?
If an excess is seen in the M_{eff} distribution, can one safely conclude that it is because of New Physics?

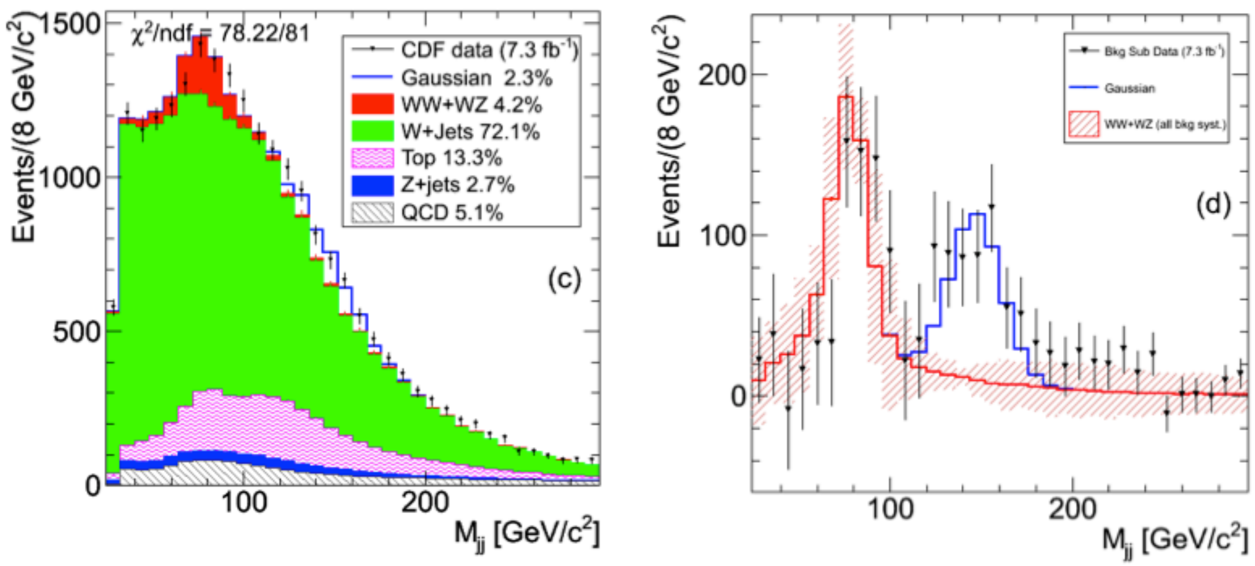
These lectures will be also a lot on understanding how reliable theoretical predictions are

Some recent excitement

CDF reported seeing a peak in M_{jj} for W + dijet events: first claim based on 4.3fb^{-1} was of 3.2 σ

CDF 1104.0699

Update to include 7.3fb^{-1} $\Rightarrow$ 4.1 σ



http://www-cdf.fnal.gov/physics/ewk/2011/wjj

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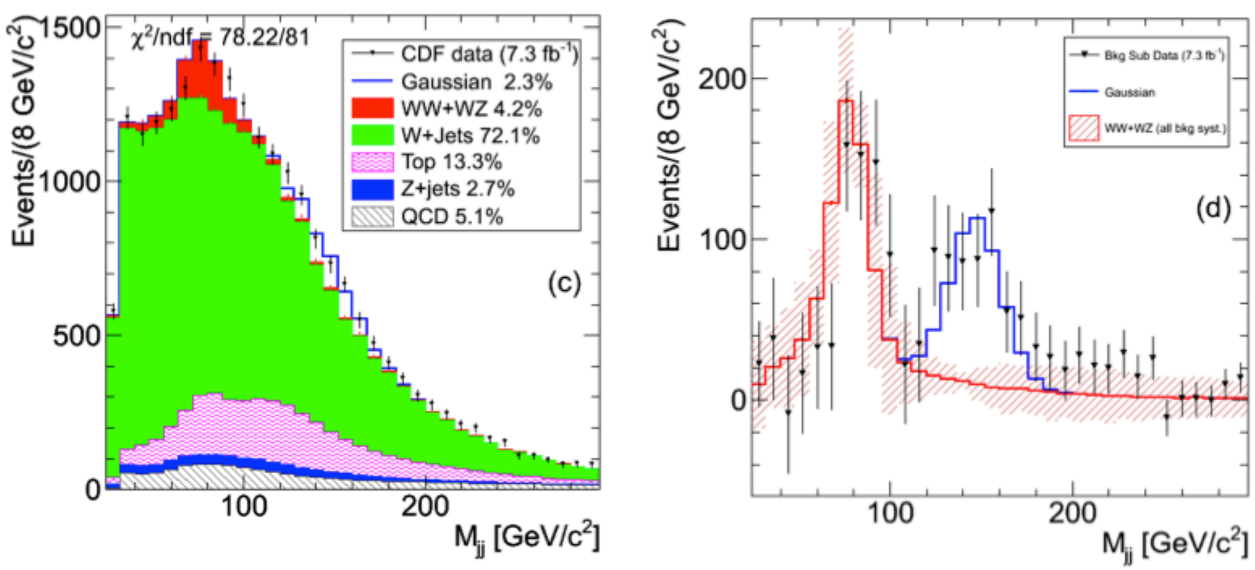

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Since then

- a large numbers of tentative BSM explanations [...]

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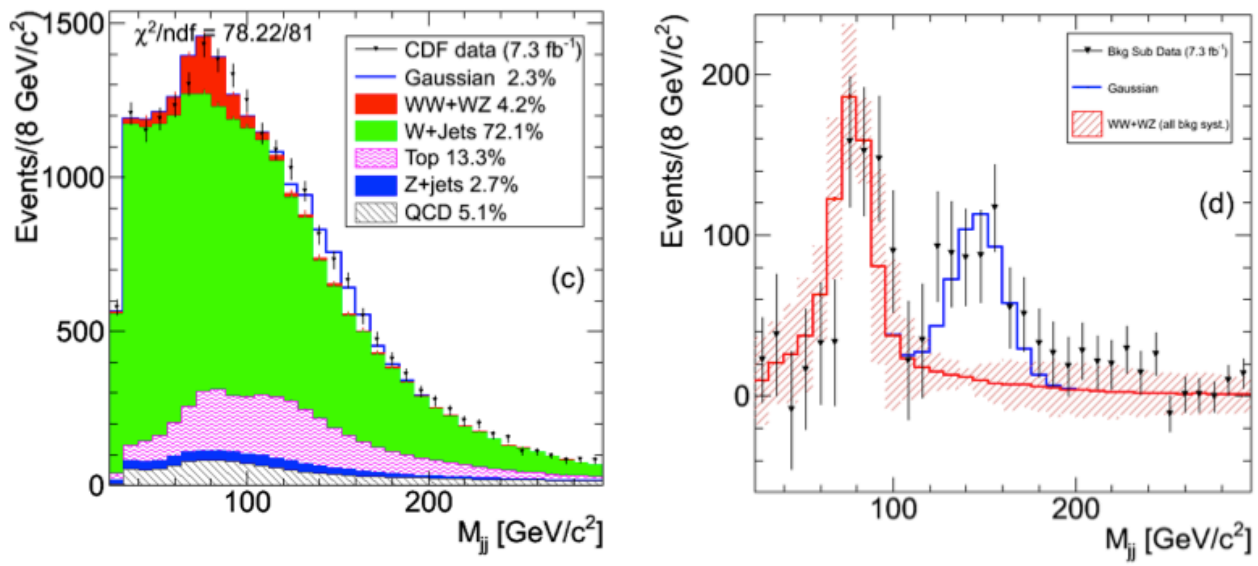

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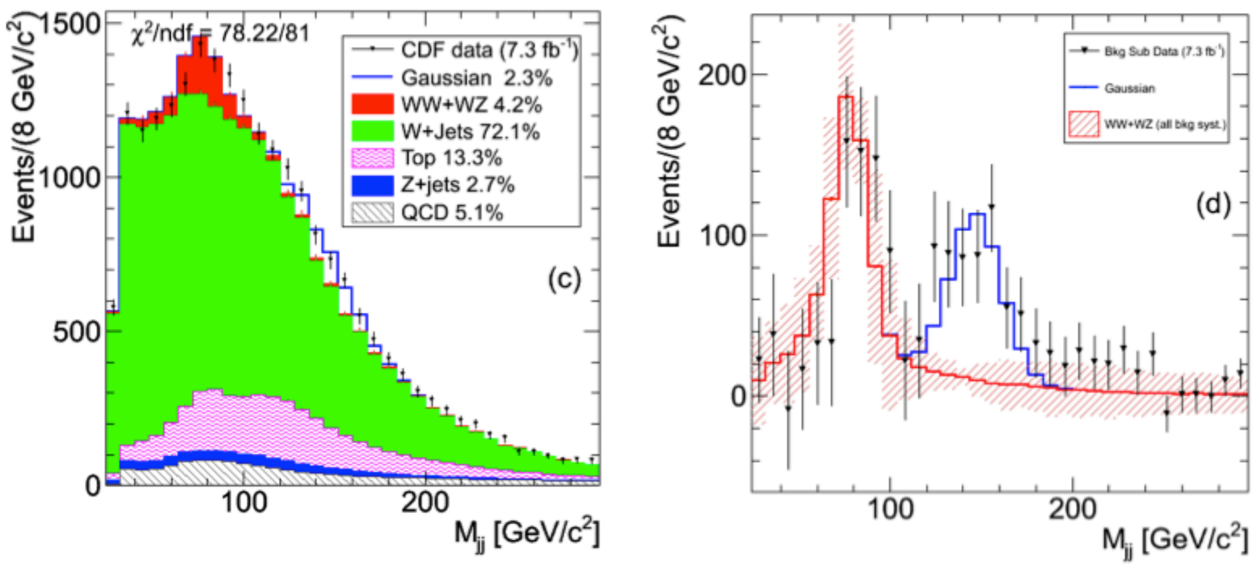

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Since then

- a large numbers of tentative BSM explanations [...]
- three SM analysis Plehn et al. 1104.4087; Sullivan & Menon 1104.3790; Campbell et al. 1105.4594
- D0 data do *not* support excess seen by CDF D0 col. 1106.1921

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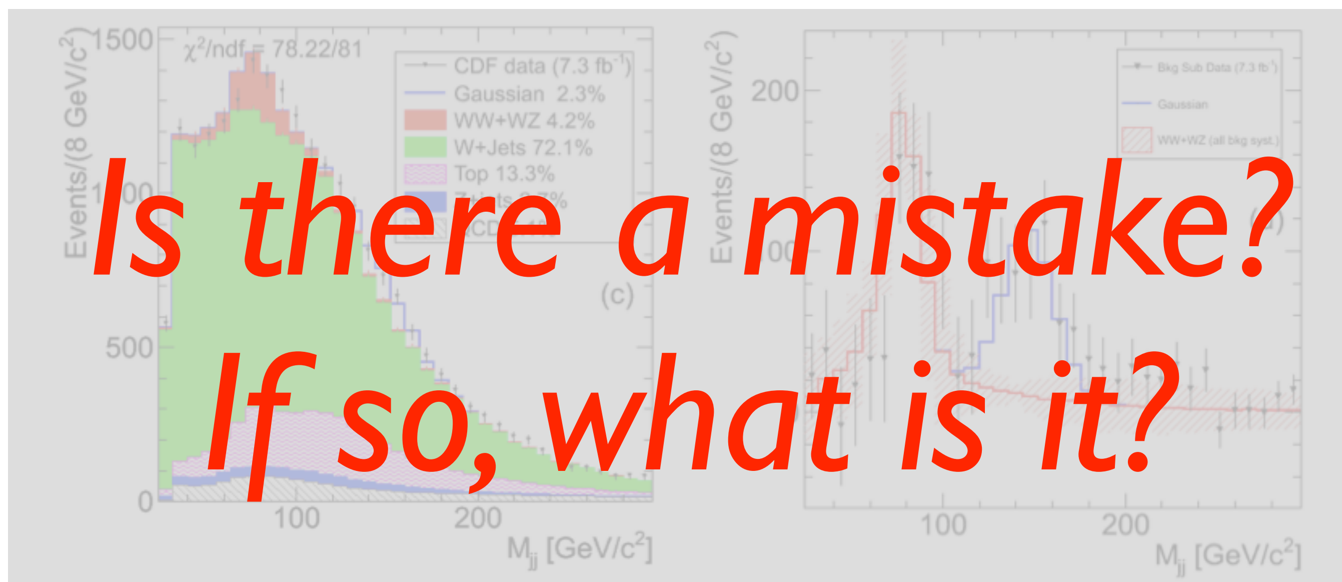

Is there a mistake?

If so, what is it?

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At the LHC expect many similar cases

- confirmation or not by a different experiment very important (re-analysis of new data not sufficiently independent)
- need robust SM predictions with reliable errors

This means that one needs to understand QCD

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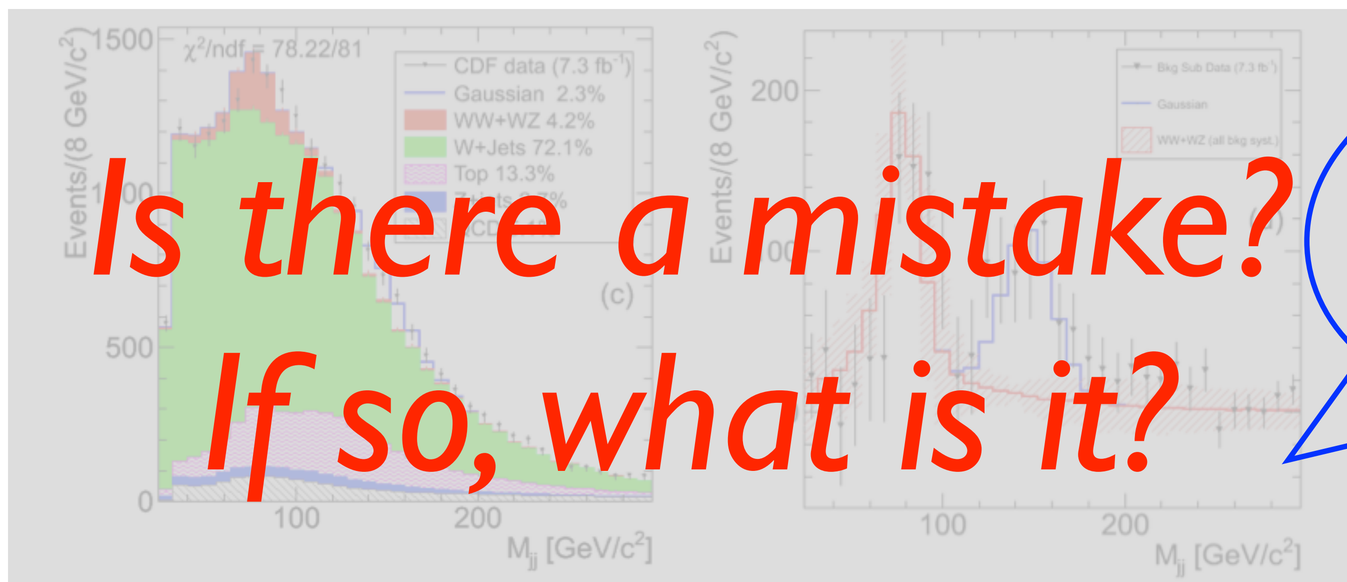

Is there a mistake? If so, what is it?

"Once we see a resonant peak on top of smooth background it's New Physics, we don't need precise SM predictions" Is not true.

http://www-cdf.fnal.gov/physics/ewk/2011/wjj

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Motivations for QCD

Satisfactory model for strong interactions: non-abelian gauge theory SU(3)

$$U^\dagger U = UU^\dagger = 1 \quad \det(U) = 1$$

Hadron spectrum fully classified with the following assumptions

- hadrons (baryons,mesons): made of spin 1/2 quarks
- each quark of a given flavour comes in N_c=3 colors
- SU(3) is an exact symmetry
- hadrons are colour neutral, i.e. colour singlet under SU(3)
- observed hadrons are colour neutral ⇒ hadrons have integer charge

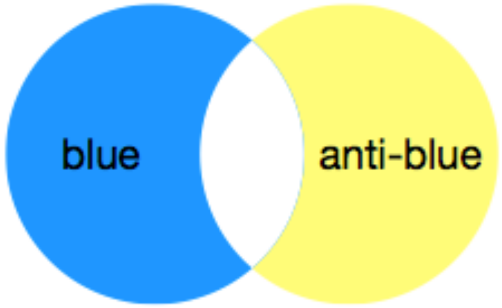
$$\begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$

Color singlet hadrons

Quarks can be combined in 2 ways into color singlets of the $SU_c(3)$ group

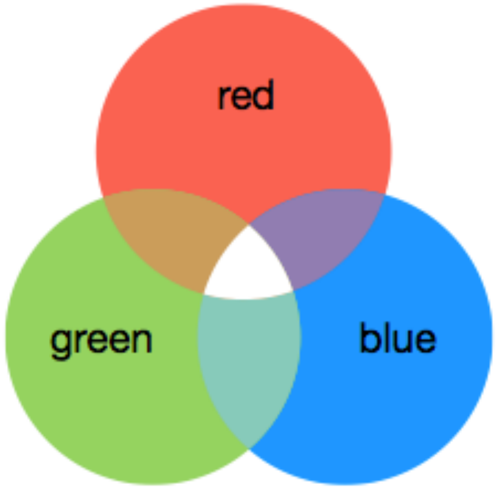
<u>Mesons</u> (bosons, e.g. pion ...)

$$\sum_i \psi_i^* \psi_i \rightarrow \sum_{ijk} U_{ij}^* U_{ik} \psi_j \psi_k = \sum_k \psi_k^* \psi_k$$



<u>Baryons</u> (fermions, e.g. proton, neutrons ...)

$$\sum_{ijk} \epsilon_{ijk} \psi_i \psi_j \psi_k \rightarrow \sum_{ii'jj'kk'} \epsilon_{ijk} U_{ii'} U_{jj'} U_{kk'} \psi_{i'} \psi_{j'} \psi_{k'} = \sum_{i'j'k'} \epsilon_{i'j'k'} \det(U) \psi_{i'} \psi_{j'} \psi_{k'}$$



First experimental evidence for colour

1. Existence of Δ^{++} particle: particle with three up quarks of the same spin and with symmetric spacial wave function. Without an additional quantum number Pauli's principle would be violated
   $\Rightarrow$ color quantum number

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I. Existence of Δ^{++} particle: particle with three up quarks of the same spin and with symmetric spacial wave function. Without an additional quantum number Pauli's principle would be violated
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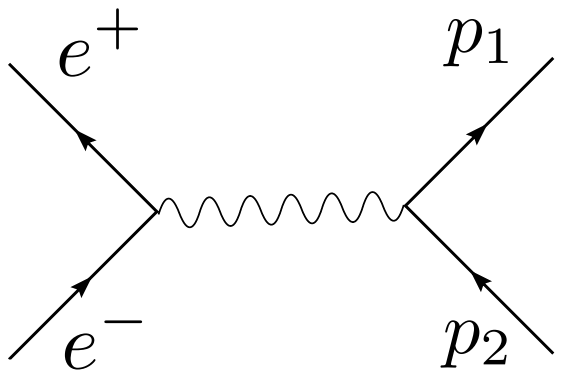
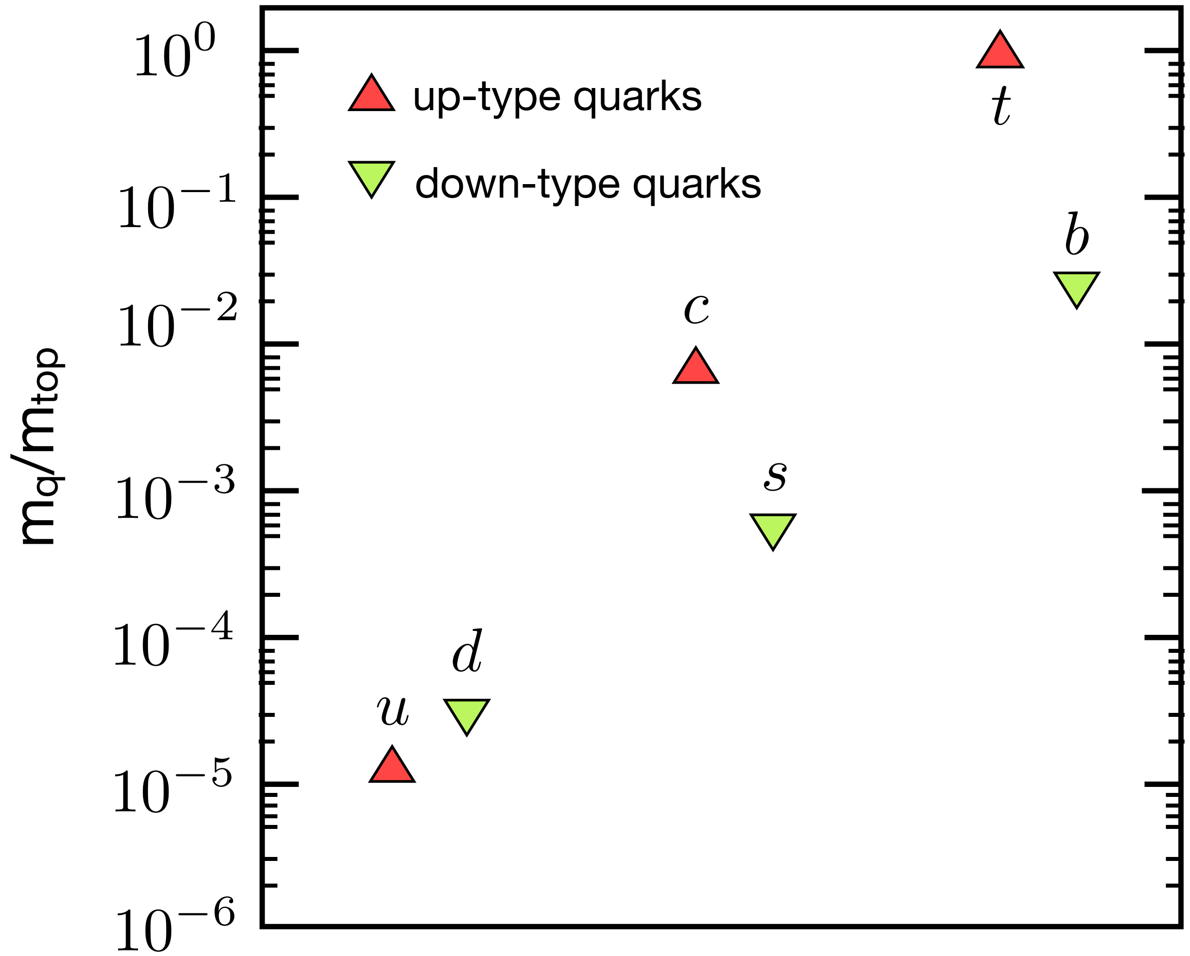
II. R-ratio: ratio of $(e^+e^- \to \text{hadrons})/(e^+e^- \to \mu^+\mu^-)$

$$R \equiv \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} \propto N_c \sum_f Q_f^2$$



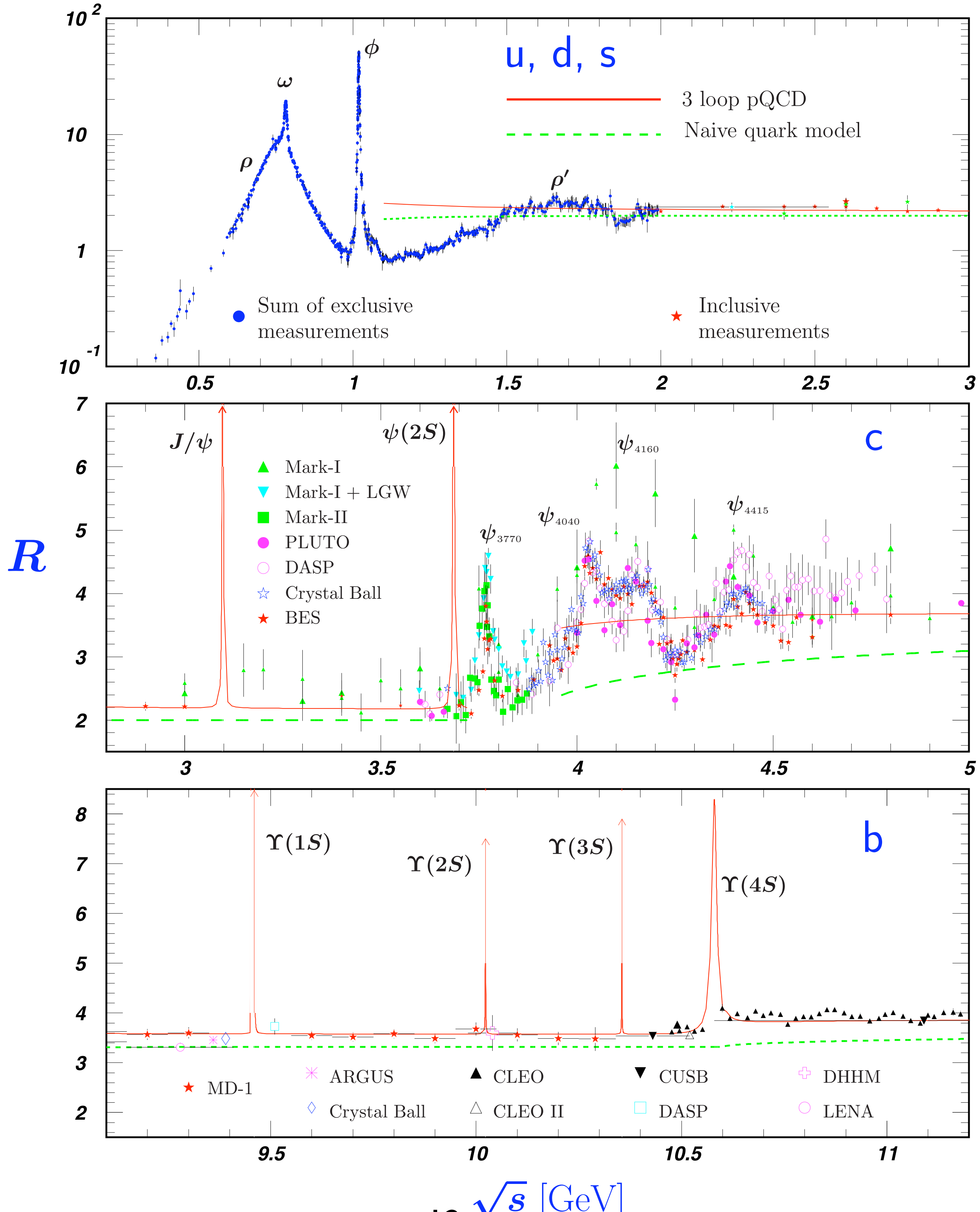
Data compatible with $N_c = 3$. Will come back to R later.

Quark mass spectrum

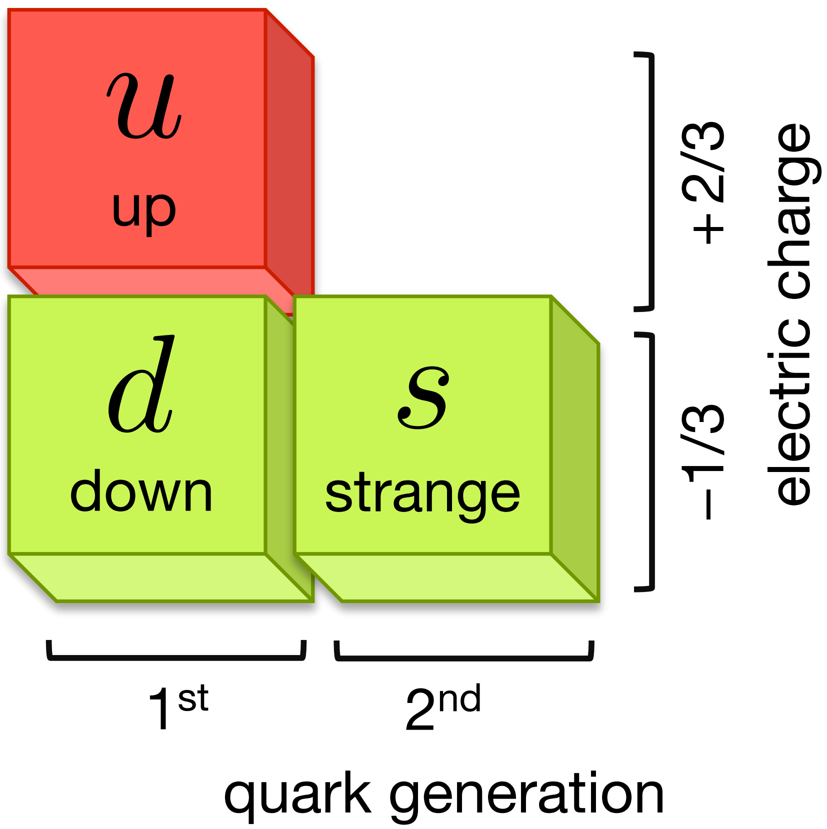


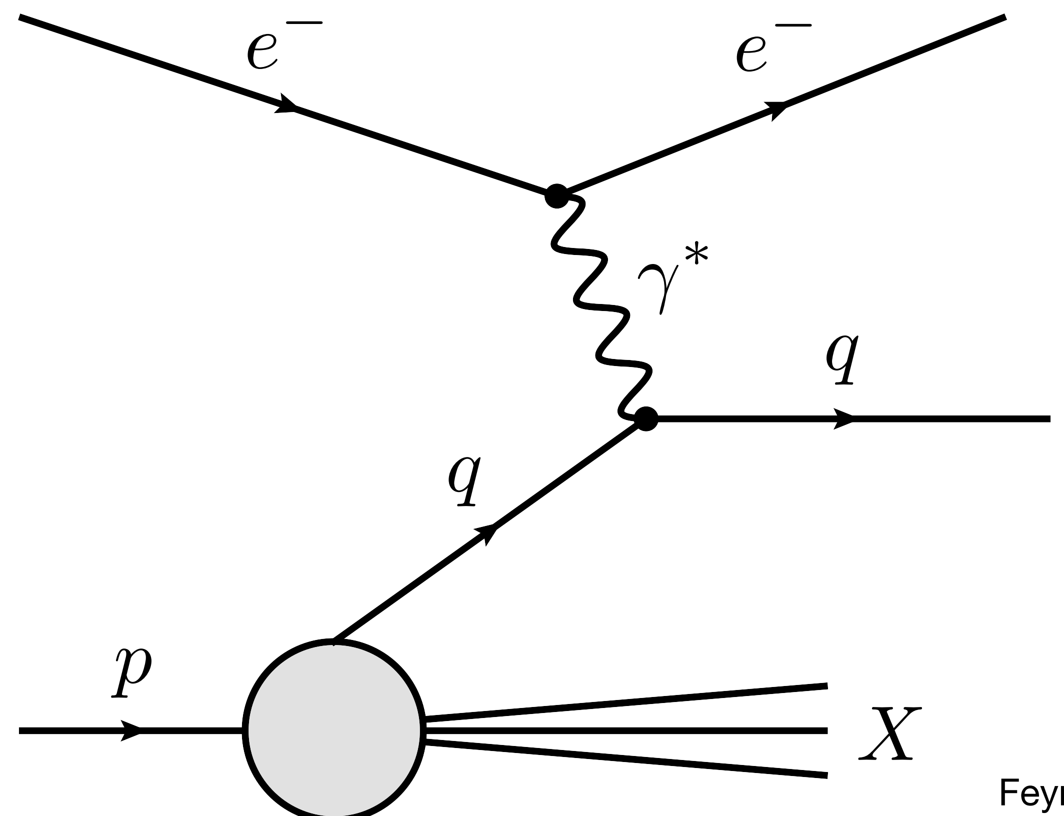
charge 2/3 mass=	up few MeV	charm ~1.6 GeV	top ~172 GeV
charge -1/3 mass =	down few MeV	strange ~100 MeV	bottom ~5 GeV

The R-ratio: comparison to data



QCD matter sector

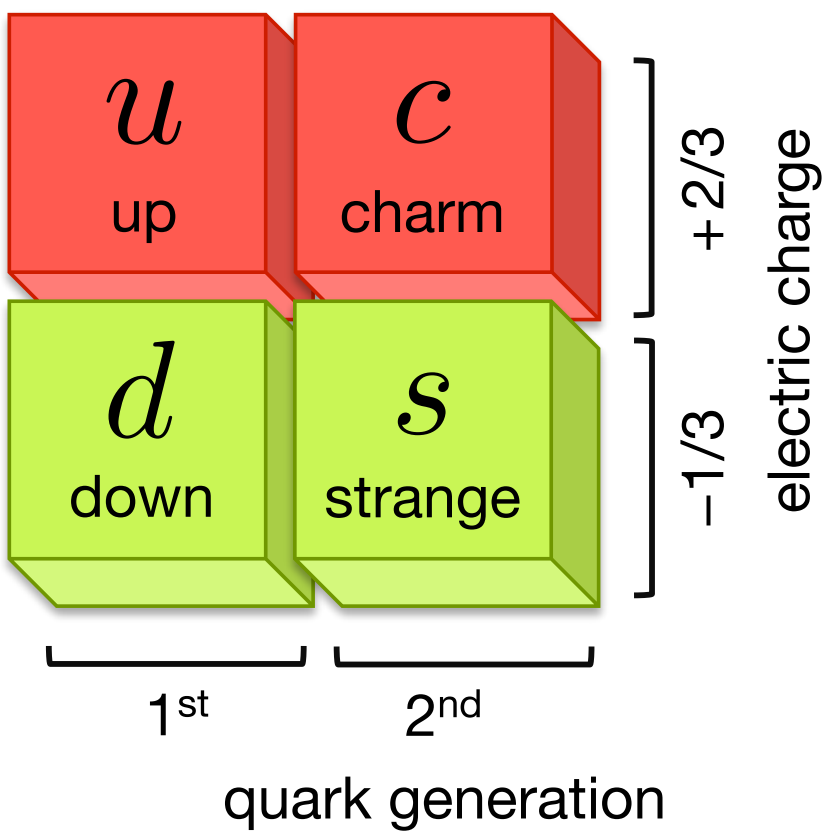




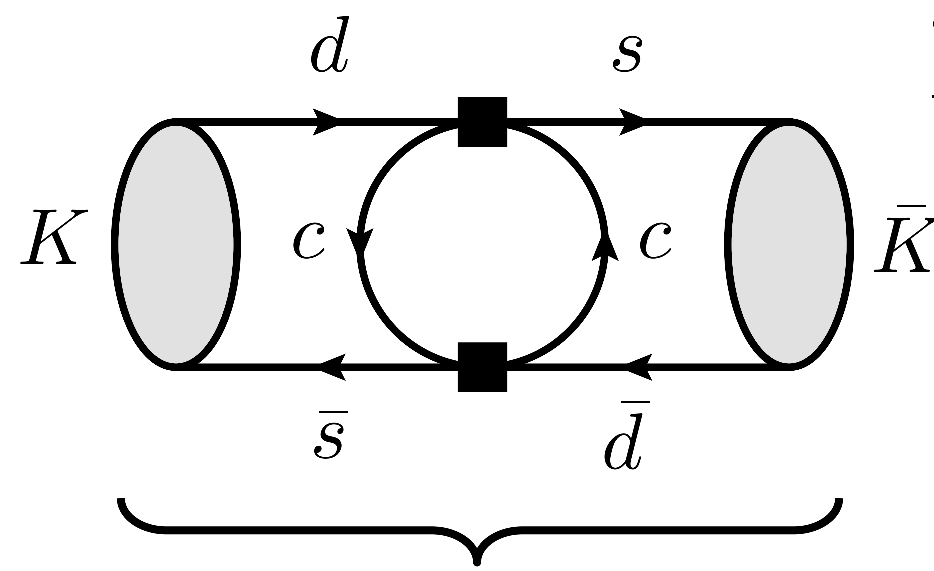
Feynman diagram describing DIS of an electron on a proton

- The light quark's existence was validated by the SLAC's deep inelastic scattering (DIS) experiments in 1968: strange was a necessary component of Gell-Mann and Zweig's three-quark model, it also provided an explanation for the kaon and pion mesons discovered in cosmic rays in 1947

QCD matter sector



Feynman diagram describing the mixing of a kaon into its anti-particle. The black boxes indicate weak effective four-fermion interactions

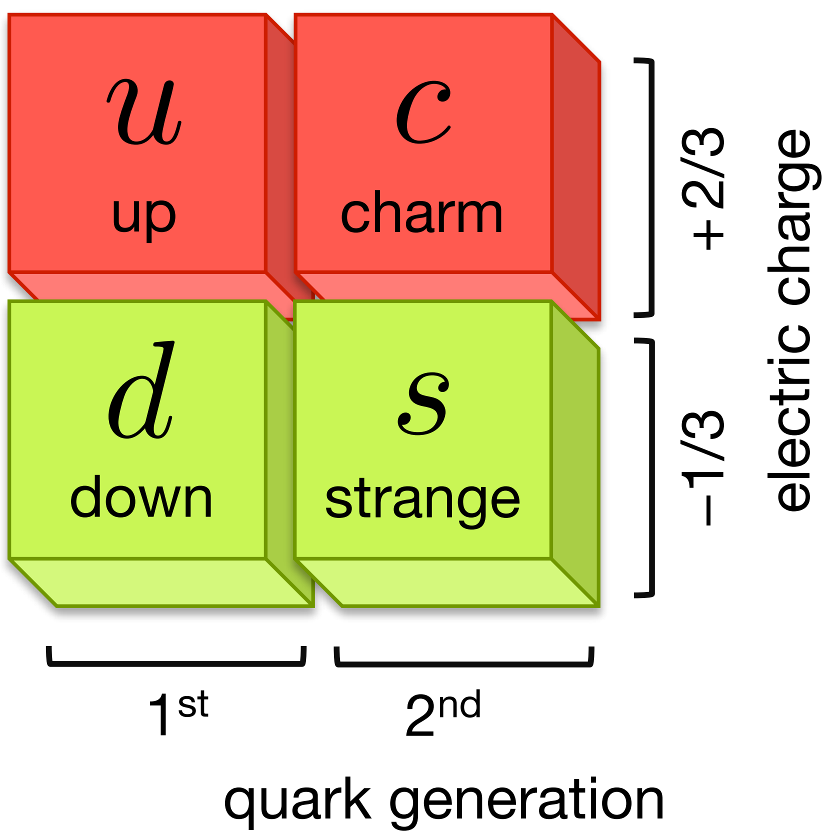


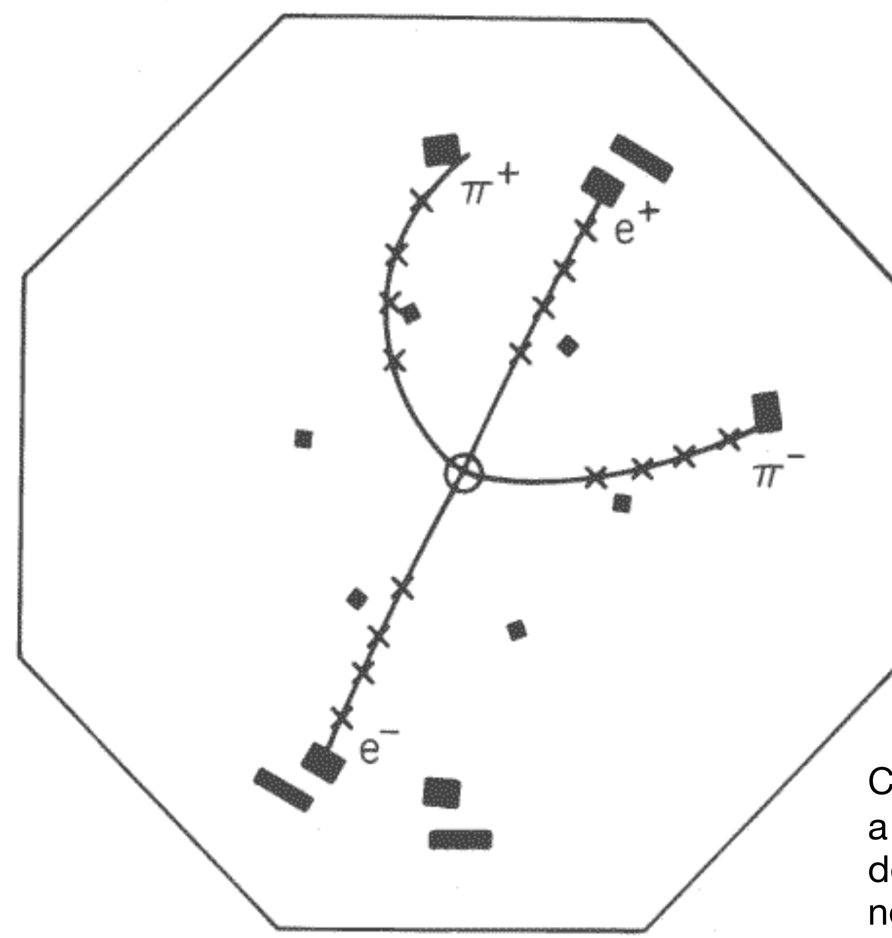
$$\propto G_F^2 \, \sin^2 \theta_c \, \frac{m_c^2}{M_W^2}$$

- In 1970 Glashow, Iliopoulos, and Maiani (GIM mechanism) presented strong theoretical arguments for the existence of the as-yet undiscovered charm quark, based on the absence of flavor-changing neutral currents

[S. L. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev.* D **2** (1970) 2]

QCD matter sector

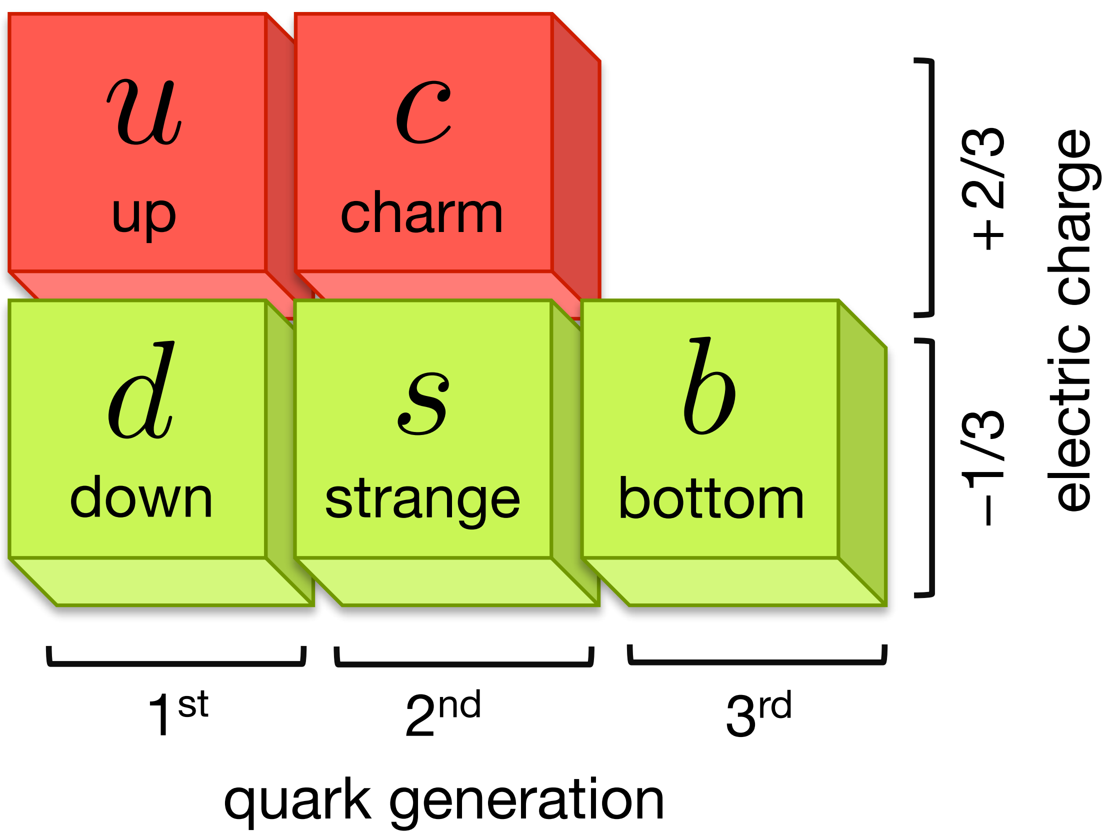


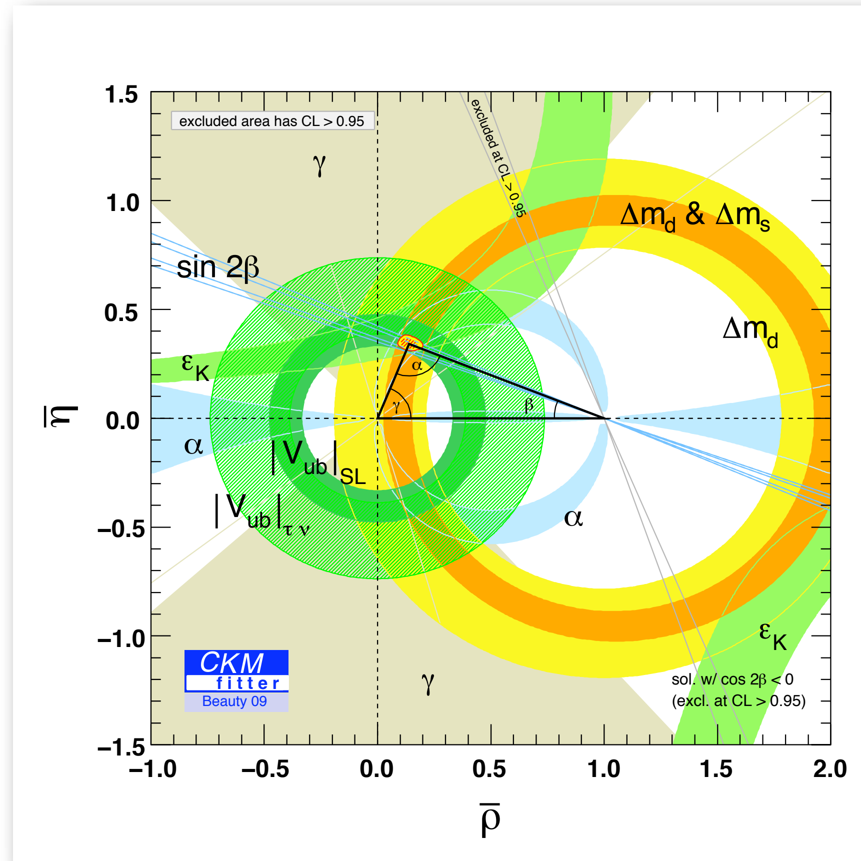


Computer reconstruction of a ψ′ decay in the Mark I detector at SLAC, making a near-perfect image of the Greek letter ψ

- Charm quarks were observed almost simultaneously in November 1974 at SLAC and at BNL as charm anti-charm bound states (charmonium). The two groups had assigned the discovered meson two different symbols, *J* and ψ. Thus, it became formally known as the *J*/ψ meson (Nobel Prize 1976)

QCD matter sector



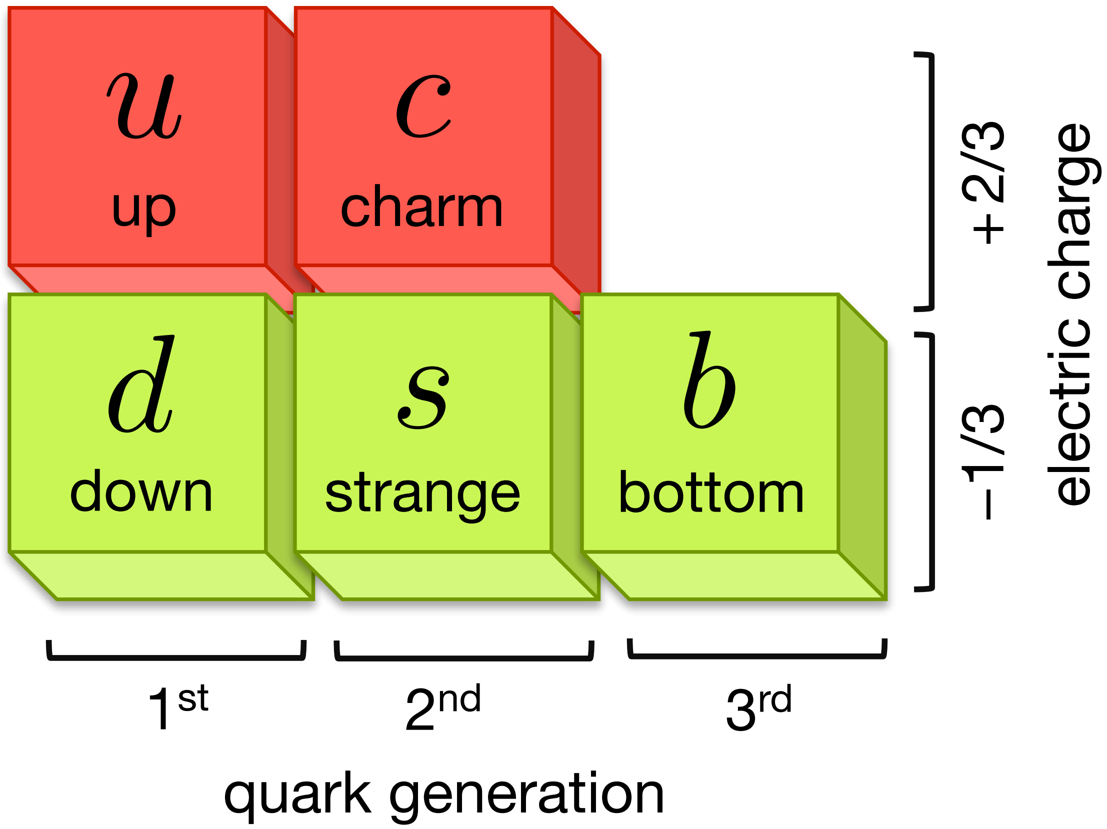


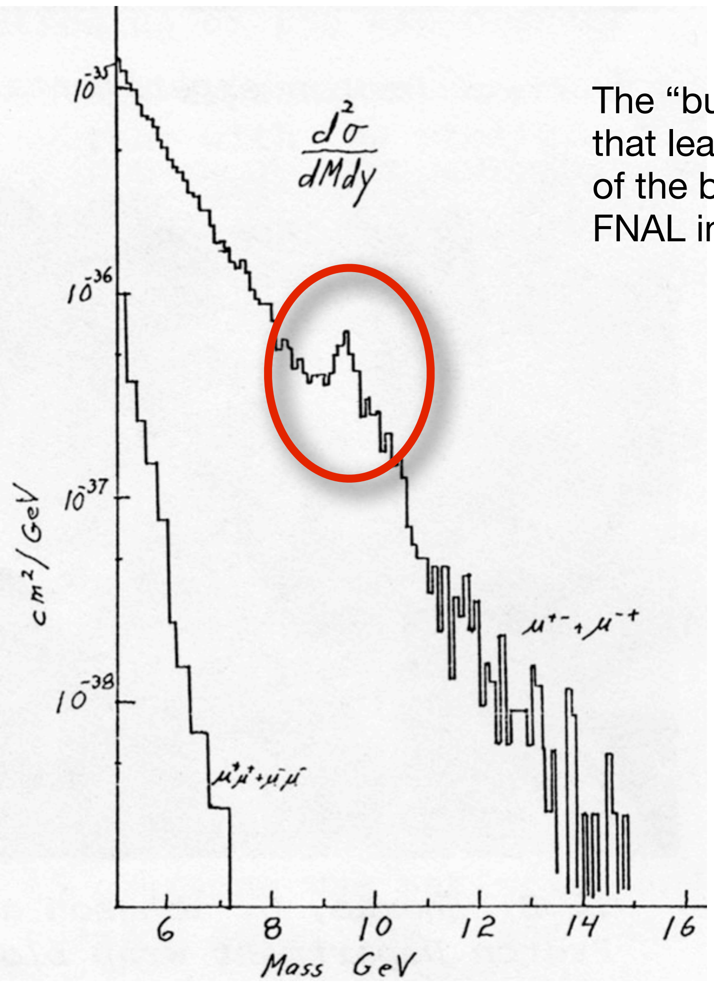
Unitarity triangle measuring the amount of CP violation in the standard model

- The bottom quark was theorized in 1973 by Kobayashi and Maskawa in order to accommodate the phenomenon of CP violation, which requires the existence of at least three generations of quarks in Nature (Nobel Prize 2008)

[M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652]

QCD matter sector





The “bump” at 9.5 GeV that lead to the discovery of the bottom quark at FNAL in 1977

- In 1977, physicists working at the fixed target experiment E288 at FNAL discovered the Υ (Upsilon) meson. This discovery was eventually understood as being the bound state of the bottom and its anti-quark (bottomonium)

QCD matter sector

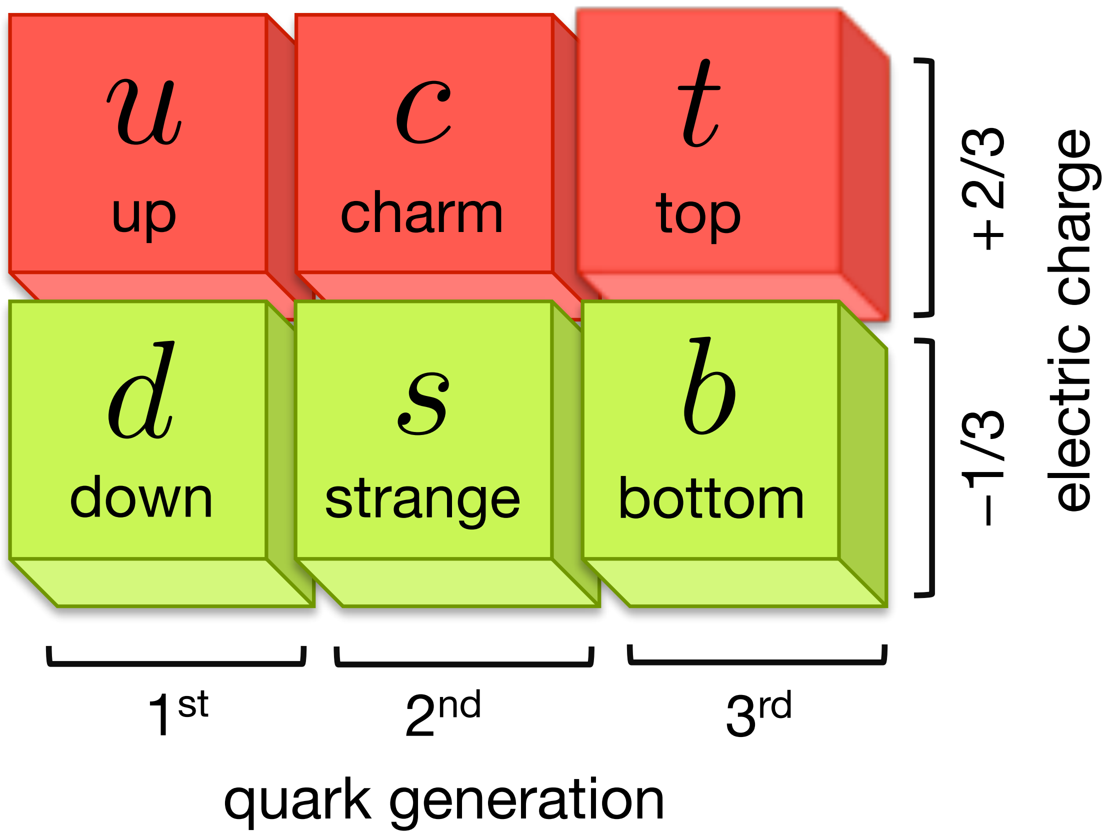
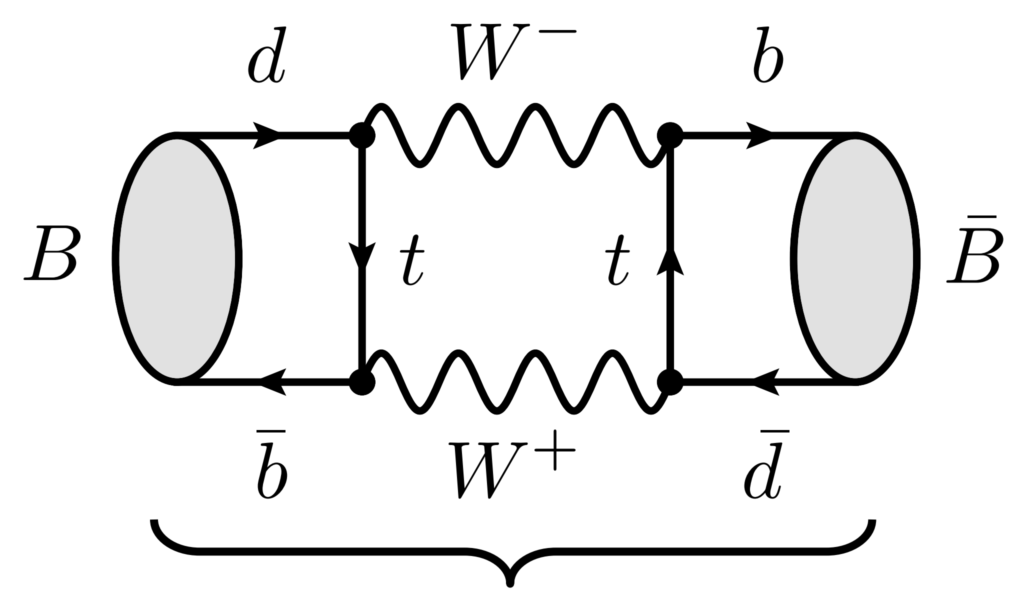


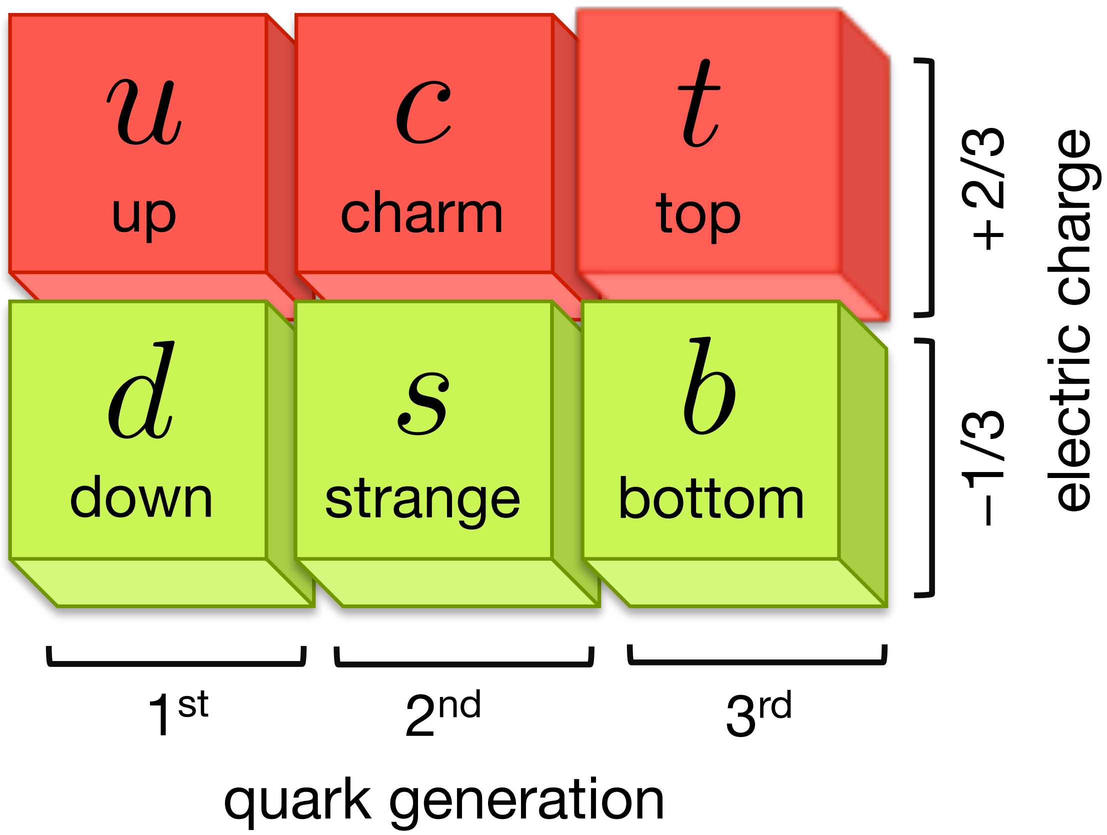
Diagram involving the virtual exchange of top quarks that induces a mass difference in the *B* meson system

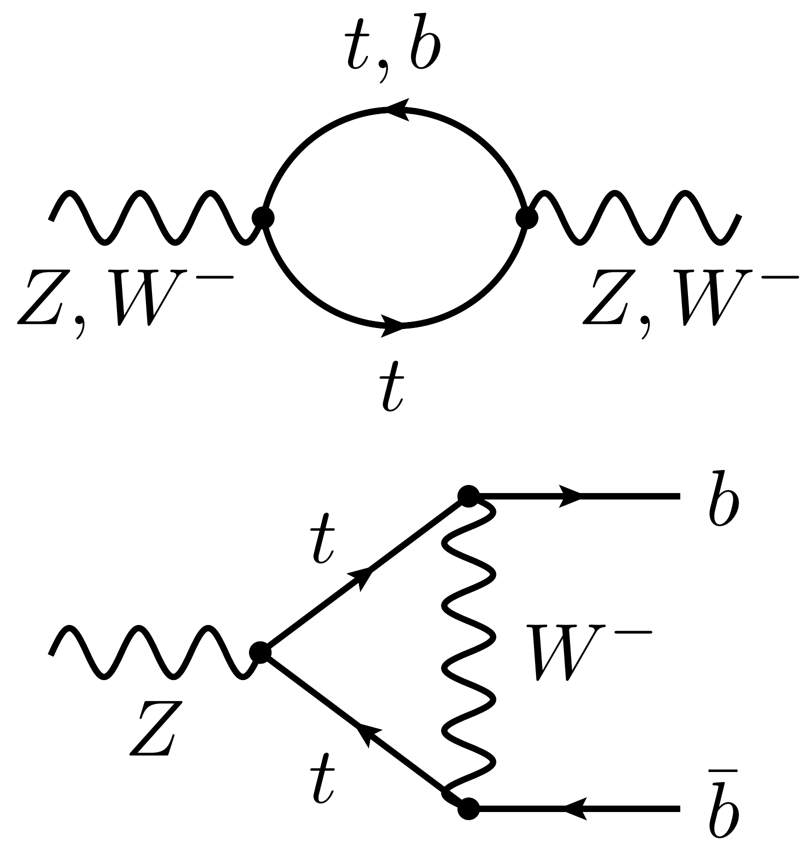


$$\Delta M_B \propto G_F^2\, m_B\, f_B^2\, |V_{td}|^2\, m_t^2$$

- The measurement of the oscillations of *B* mesons into its own anti-particles in 1987 by ARGUS led to the conclusion that the top-quark mass has to be larger than 50 GeV. This was a big surprise at that time, because in 1987 the top quark was generally believed to be much lighter

QCD matter sector

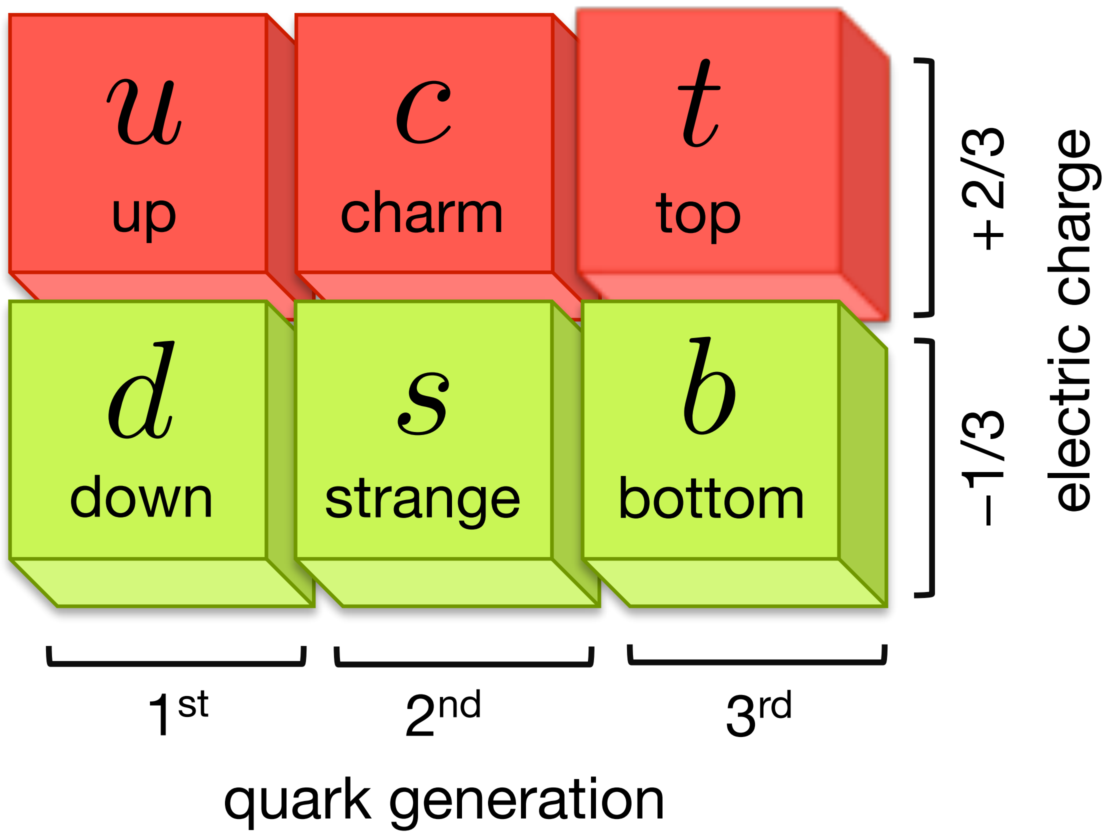


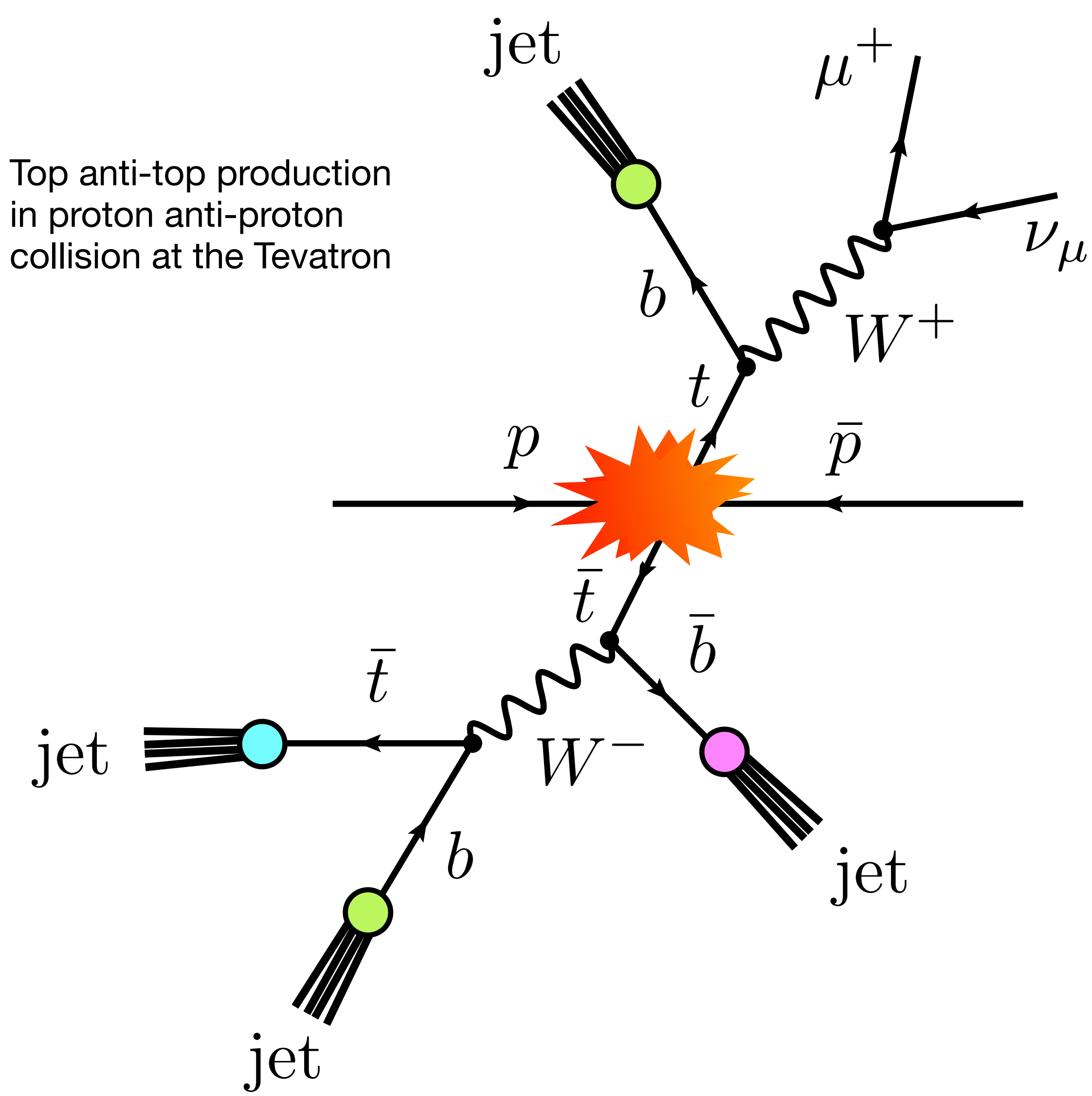


Diagrams that feature a quadratic dependence on the top-quark mass

- It was also realized that certain precision measurements of the electroweak vector-boson masses and couplings are very sensitive to the value of the top-quark mass. By 1994 the precision of these indirect measurements led to a prediction of the top-quark mass between 145 GeV and 185 GeV

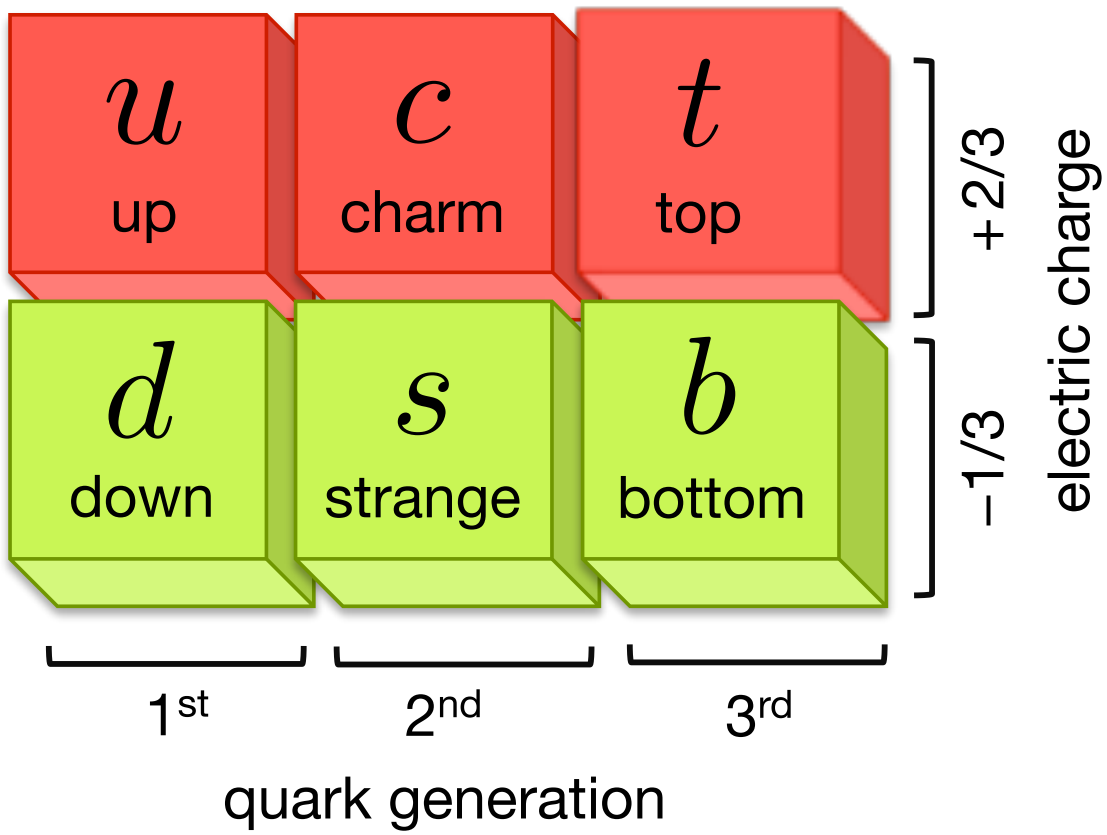
QCD matter sector

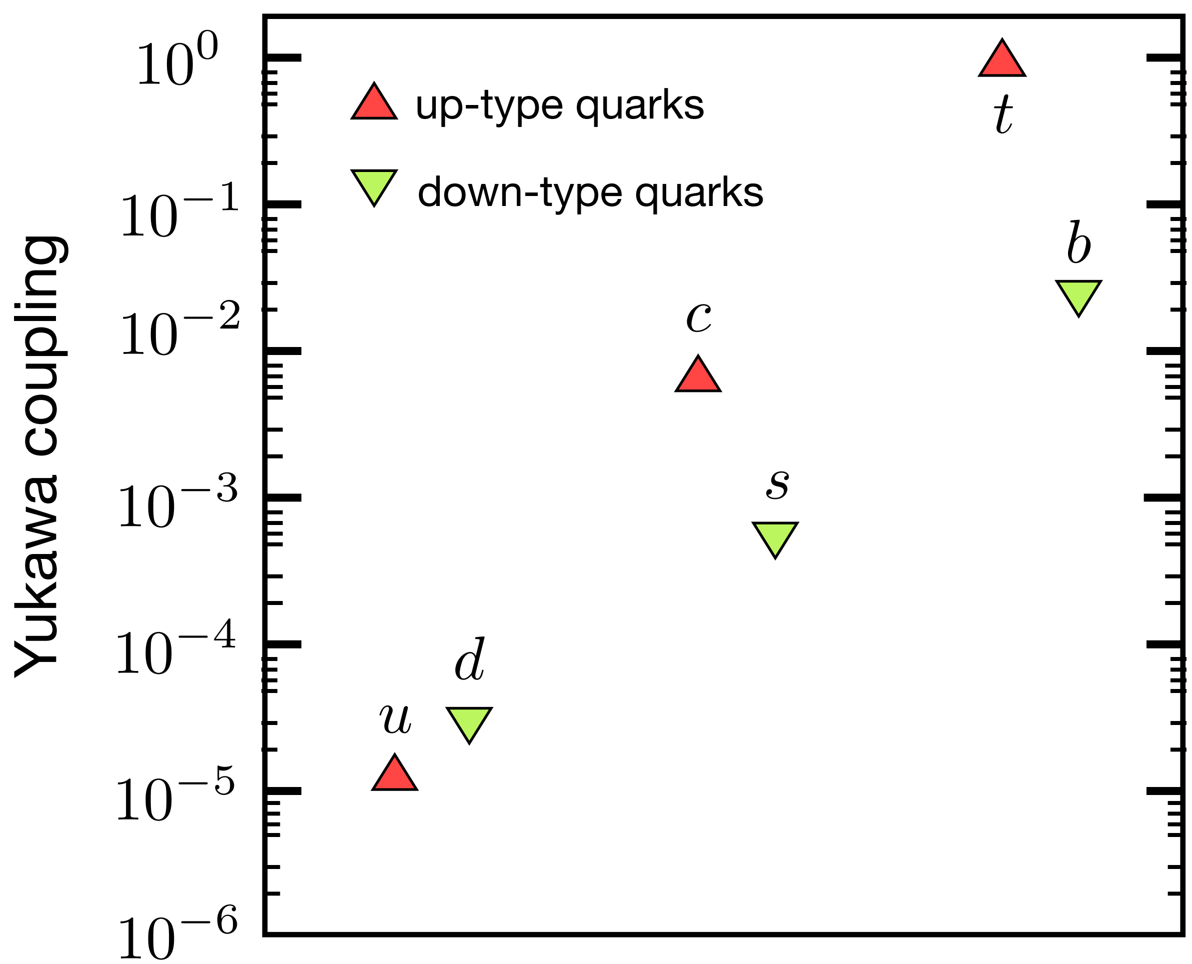




- The top quark was finally discovered in 1995 by CDF and D0 at FNAL. While the mass of the top quark is today quite well known, m_t = (173.1 ± 1.3) GeV, its charge is measured to be + 2/3 only at the 90% confidence level

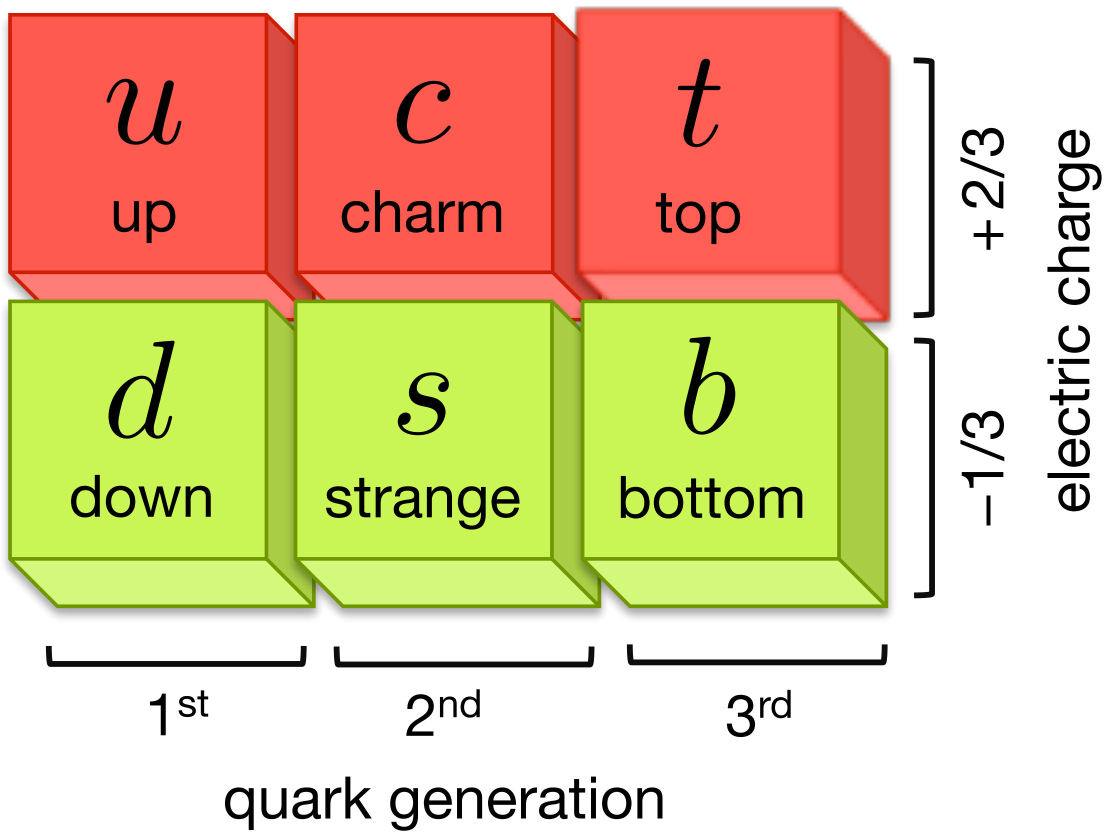
QCD matter sector

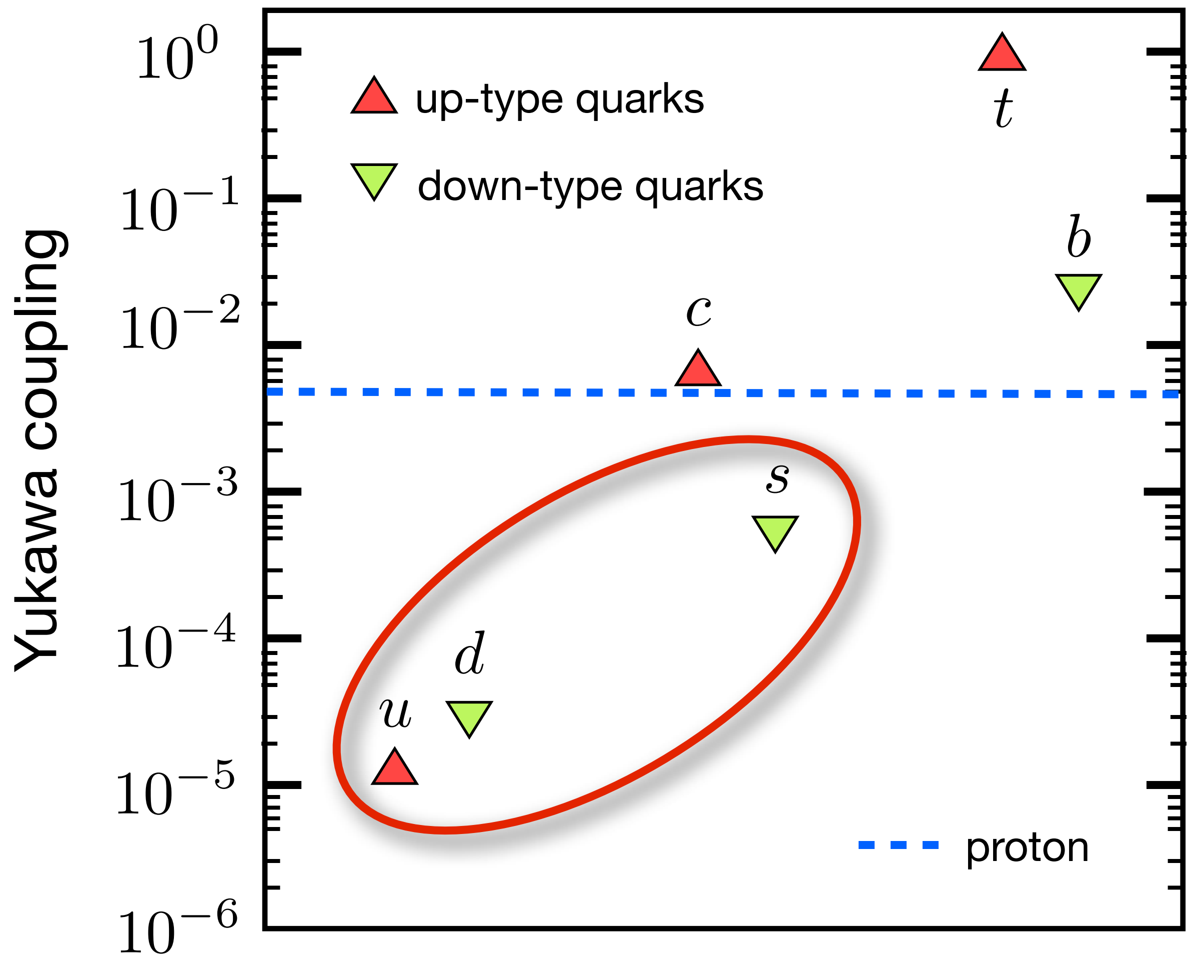




- The masses of the six different quark flavors range from around 2 MeV for the up quark to around 175 GeV for the top. Why these masses are split by almost six orders of magnitude is one of the big mysteries of particle physics

QCD matter sector





- The masses of the up, down, and strange are much lighter than the proton. If one takes these light flavors to have an identical mass, the quarks become indistinguishable under QCD, and one obtains an effective $SU(3)_f$ symmetry

QED and QCD

QED and QCD are very similar, yet very different theories

- quarks are a bit like leptons, but there are three of each
- gluons are a bit like photons, but there are eight of them
- gluons interact with themselves
- the QCD coupling is also small at collider energies, but larger then the QED one
- the similarities and differences are evident from the two Lagrangians

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So, let's start by looking at the QED Lagrangian

The QED Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{QED}} &= \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}} \\
&= \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi - \frac{1}{2}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi \boxed{A_\mu} \\
&= \bar{\psi}\,(i\boxed{D\!\!\!\!/} - m)\,\psi - \frac{1}{2}(\boxed{F_{\mu\nu}})^2
\end{aligned}
$$

electromagnetic vector potential A_μ

field strengh tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

covariant derivative $D_\mu = \partial_\mu + ieA_\mu$

QED Feynman rules

$$
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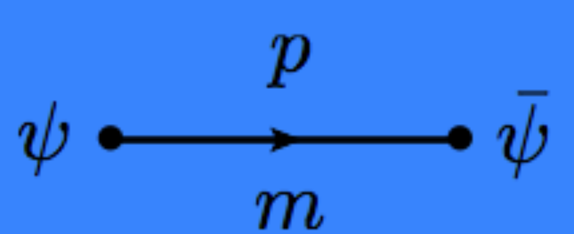
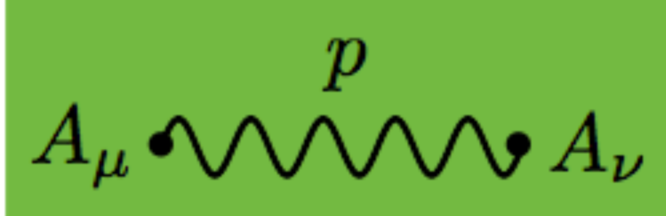
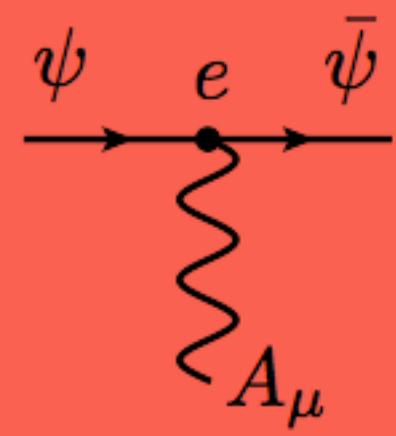


$$= \frac{i\,(p\!\!\!/ + m)}{p^2 - m^2}$$



$$= \frac{-ig^{\mu\nu}}{p^2}$$



$$= ie\gamma^{\mu}$$

QED gauge invariance

$$\mathcal{L}_{\text{QED}} = \bar{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{2}(F_{\mu\nu})^2$$

A crucial property of the QED Lagrangian is that it is invariant under

$$\boxed{\psi(x) \to e^{i\alpha(x)}\,\psi(x)\,, \qquad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)}$$

which acts on the Dirac field as a local phase transformation

<u>Exercise:</u> Check that the QED Lagrangian is invariant under the above transformations

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Yang and Mills (1954) proposed that the local phase rotation in QED could be generalized to invariance under any continues symmetry

[C. N. Yang and R. L. Mills, Phys. Rep. 96 (1954) 191]

The QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} \left(i \not{D}_{ij} - m_f \delta_{ij} \right) \psi_j^{(f)}$$

$$D^\mu_{ij} \equiv \partial^\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a \,, \qquad F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

⇒ covariant derivative ⇒ field strength

The QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} \left(i \not{D}_{ij} - m_f \delta_{ij} \right) \psi_j^{(f)}$$

$$D^\mu_{ij} \equiv \partial^\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a \,, \qquad F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

⇒ covariant derivative ⇒ field strength

- only one QCD parameter g_s regulating the strength of the interaction (quark masses have EW origin)

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- QCD flavour blind (differences only due to EW)

The generators of SU(N)

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So, the fundamental representation of SU(N) has N^2-1 generators t^a :
N×N traceless hermitian matrices ⇒ N^2-1 gluons

$$U = e^{i\theta_a(x)t^a} \qquad\qquad a = 1, \cdots N^2 - 1$$

The Gell-mann matrices

One explicit representation: $t^A = \frac{1}{2}\lambda^A$

λ^A are the Gell-mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Standard normalization: $\mathrm{Tr}(t^a t^b) = T_R\, \delta^{ab} \quad T_R = \frac{1}{2}$

Notice that the first three Gell-mann matrices contain the three Pauli matrices in the upper-left corner

The generators of SU(N)

Infinitesimal transformations (close to the identity) give complete information about the group structure. The most important characteristic of a group is the commutator of two transformations:

$$\begin{aligned}[U(\delta_1), U(\delta_2)] &\equiv U(\delta_1)U(\delta_2) - U(\delta_2)U(\delta_1) \\ &= (i\delta_1^a)\,(i\delta_2^b)\,[t^a, t^b] + \mathcal{O}(\delta^3)\end{aligned}$$

The two matrices to not commute, therefore the transformations don't. Such a group is called non-abelian

- Familiar abelian groups: translations, phase transformations U(1) ...

- Familiar non-abelian groups: 3D-rotations

The generators of SU(N)

Consider the commutator

$$Tr([t_a, t_b]) = 0 \quad \Rightarrow \quad [t_a, t_b] = i f_{abc} t^c$$

f_{abc} are the (real) structure constants of the SU(N_c) algebra, they generate a representation of the algebra called adjoint representation

Clearly, f_{abc} is anti-symmetric in (ab). It is easy to show that it is fully antisymmetric

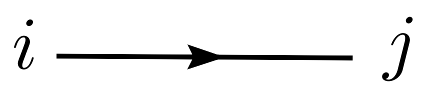
$$f_{abc} = -f_{bac} = -f_{acb}$$

and that

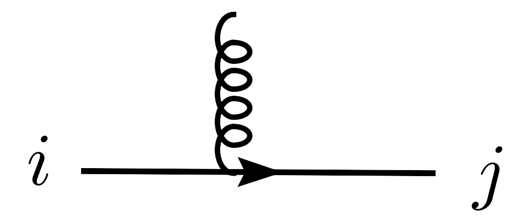
$$i f_{abc} = 2\,\mathrm{Tr}\left([t_a, t_b] t_c\right)$$

Color algebra: fundamental identities

Fundamental representation 3:

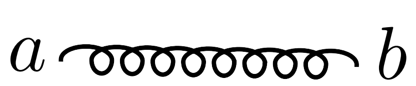


$= \delta_{ij}$

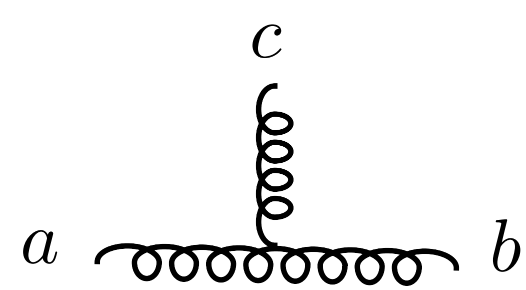


$= t^a_{ij}$

Adjoint representation 8:

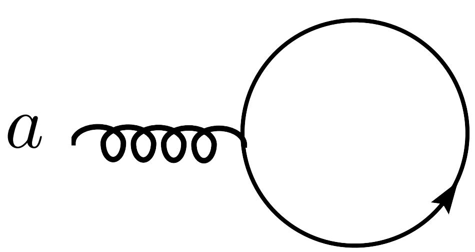


$= \delta_{ab}$



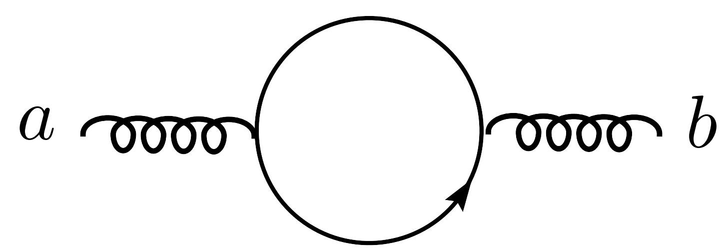
$= i f_{abc}$

Trace identities:

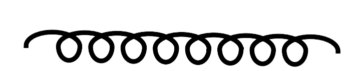


$= 0$

$\mathrm{Tr}(t^a) = 0$

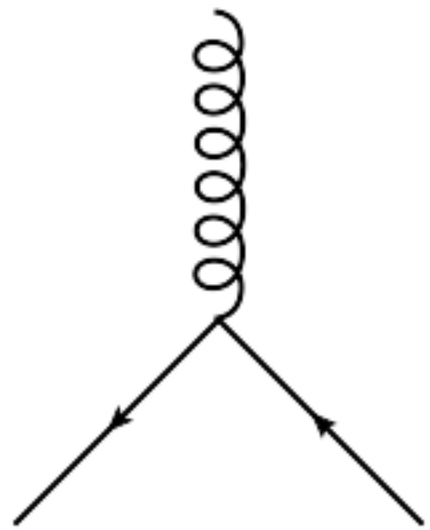


$= T_R$



$\mathrm{Tr}(t^a\, t^b) = \mathrm{T_R}\delta^{ab}$

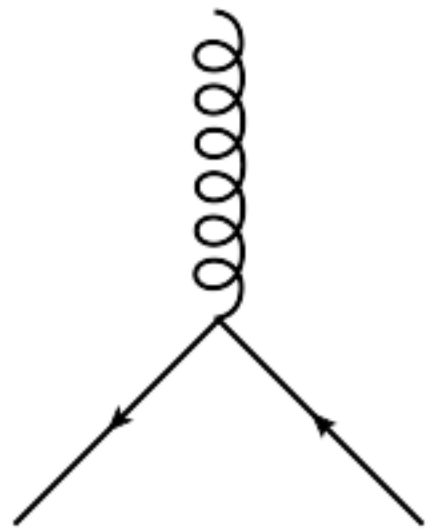
What do color identities mean physically



$$\bar{\psi}_i \, t^A_{ij} \, \psi_j$$

What does this really mean?

What do color identities mean physically



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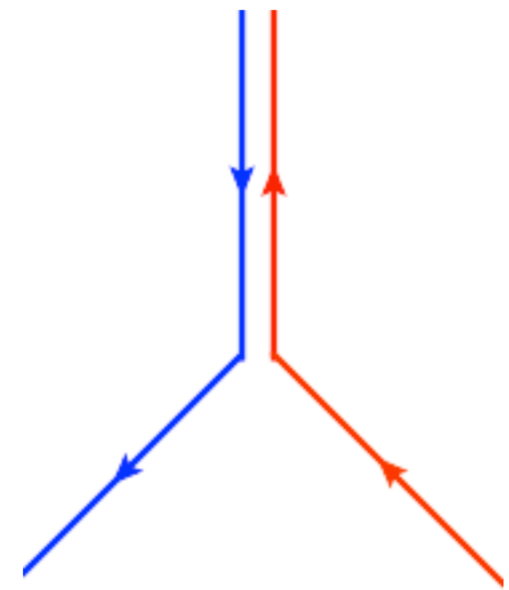


$$(1, 0, 0) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

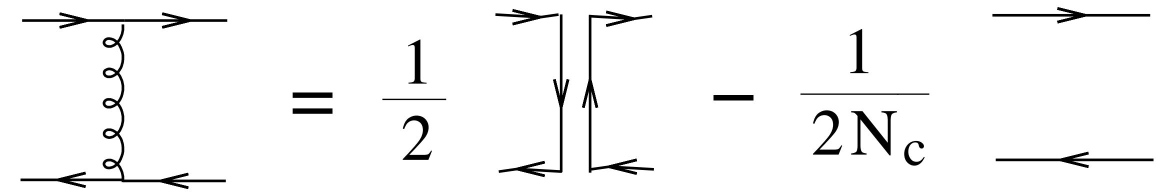
What does this really mean? $\quad \bar{\psi}_i \quad t^1_{ij} \quad \psi_j$

Gluons carry color and anti-color. They repaint quarks and other gluons.

Color algebra: Casimirs & Fierz identity

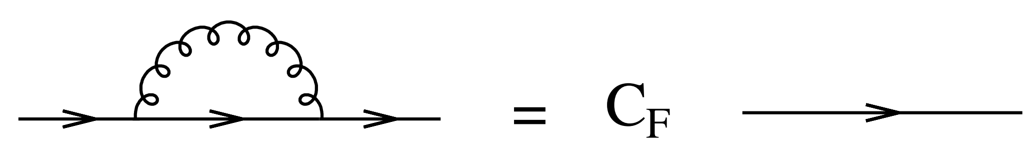
Fierz identity:

$$(t^a)^i_k (t^a)^l_j = \frac{1}{2}\delta^i_j \delta^l_k - \frac{1}{2N_c}\delta^i_k \delta^l_j$$



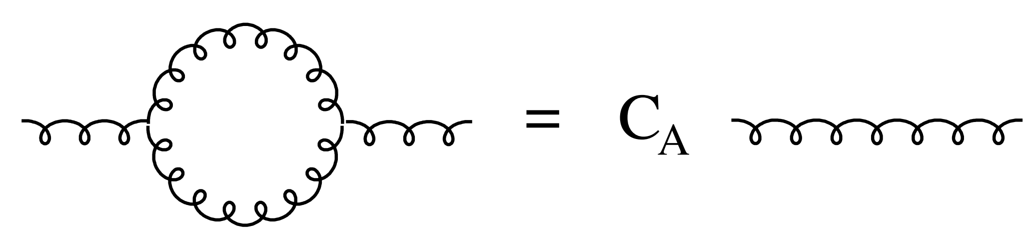
Fundamental representation 3:

$$\sum_a (t^a_{ij})(t^a_{kj}) = C_F \delta_{ij} \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$



Adjoint representation 8:

$$\sum_{cd} f^{acd} f^{bdc} = C_A \delta^{ab} \qquad C_A = N_c$$



Gauge invariance

The QCD Lagrangian is invariant under local gauge transformations, i.e. one can redefine the quark and gluon fields independently at every point in space and time without changing the physical content of the theory

- Gauge transformation for the quark field

$$\boxed{\psi \to \psi' = U(x)\psi}$$

- The covariant derivative $(D_\mu)_{ij} = \partial_\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a$ must transform as (covariant = transforms "with" the field)

$$\boxed{D_\mu \psi \to D'_\mu \psi' = U(x) D_\mu \psi}$$

- From which one derives the transformation property of the gluon field

$$\boxed{t^a A_a \to t^a A'_a = U(x) t^a A_a U^{-1}(x) + \frac{i}{g_s} \left(\partial U(x)\right) U^{-1}(x)}$$

Gauge invariance

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- It follows that

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi} U^\dagger(x)$$

$$t^a F^a_{\mu\nu} \to t^a F^{a'}_{\mu\nu} = U(x) t^a F^a_{\mu\nu} U^{-1}(x)$$

e.g. because $i\, g_s\, t^a F^a_{\mu\nu} = [D_\mu, D_\nu]$

- Therefore the QCD Lagrangian is indeed gauge invariant

$$-\frac{1}{4} F'^{\mu\nu}_a F'^a_{\mu\nu} = -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu}$$

$$\sum_f \bar{\psi}'^{(f)}_i \left(i \not{D}'_{ij} - m_f \delta_{ij} \right) \psi'^{(f)}_j = \sum_f \bar{\psi}^{(f)}_i \left(i \not{D}_{ij} - m_f \delta_{ij} \right) \psi^{(f)}_j$$

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Remarks:

- the field strength alone is not gauge invariant in QCD (unlike in QED) because of self interacting gluons (carries of the force carry colour, unlike the photon)

- a gluon mass term violate gauge invariance and is therefore forbidden (as for the photon). On the other hand quark mass terms are gauge invariant.

$$\cancel{m^2 A_\mu A^\mu}$$

Isospin symmetry

Isospin SU(2) symmetry: invariance under u ↔ d

Particles in the same isospin multiplet have very similar masses (proton and neutron, neutral and charged pions)

The QCD Lagrangian has isospin symmetry if $m_u = m_d$ or $m_u, m_d \to 0$

The fermionic Lagrangian becomes

$$\mathcal{L}_F = \sum_f \left(\bar{\psi}_L^{(f)} \not{D} \psi_L^{(f)} + \bar{\psi}_R^{(f)} \not{D} \psi_R^{(f)} \right) - \sum_f m_f \left(\bar{\psi}_R^{(f)} \psi_L^{(f)} + \bar{\psi}_L^{(f)} \psi_R^{(f)} \right)$$

$$\psi_L = P_L \psi \,, \quad \psi_R = P_R \psi \,, \quad P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$$

So neglecting fermion masses the Lagrangian has the larger symmetry

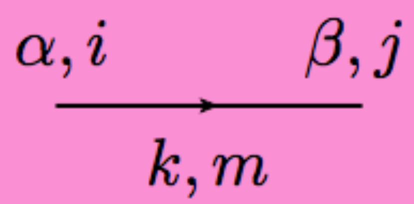
$$SU_L(N_f) \times SU_R(N_f) \times U_L(1) \times U_R(1)$$

Feynman rules: propagators

Obtain quark/gluon propagators from free piece of the Lagrangian

<u>Quark propagator:</u> replace $i\partial \to k$ and take the $i \times$ inverse

$$\mathcal{L}_{\mathrm{q,free}} = \sum_f \bar{\psi}_i^{(f)} \left(i\not{\partial} - m_f\right) \delta_{ij} \psi_j^{(f)}$$



$$= \left(\frac{i}{\not{k} - m}\right)_{\alpha\beta} \delta_{ij}$$

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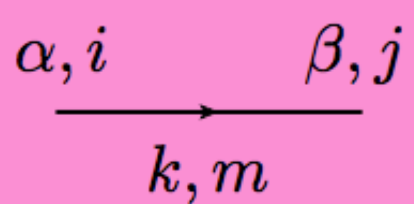


$$= \left(\frac{i}{\not{k} - m} \right)_{\alpha\beta} \delta_{ij}$$

<u>Gluon propagator:</u> replace i∂ → k and take the i × inverse ?

$$\mathcal{L}_{\rm g,free} = \frac{1}{2} A^\mu \left(\Box g_{\mu\nu} - \partial_\mu \partial_\nu \right) A^\nu$$

➡ inverse does not exist, since $\left(\Box g_{\mu\nu} - \partial_\mu \partial_\nu \right) \partial_\mu = \Box \partial_\nu - \Box \partial_\nu = 0$

How can one to define the propagator ?

Gauge fixing

<u>Solution:</u> add to the Lagrangian a gauge fixing term which depends on an arbitrary parameter ξ

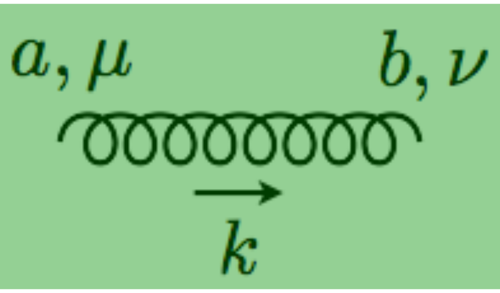
<u>In covariant gauges:</u>

$$\mathcal{L}_{\text{gauge fixing}} = -\frac{1}{\xi}\left(\partial^{\mu} A_{\mu}^{A}\right)^{2}$$

$\xi=1$ Feynman gauge
$\xi=0$ Landau gauge

<u>Gluon propagator:</u>

$$\frac{-i}{k^2}\left(g_{\mu\nu}-(1-\xi)\,\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab} =$$

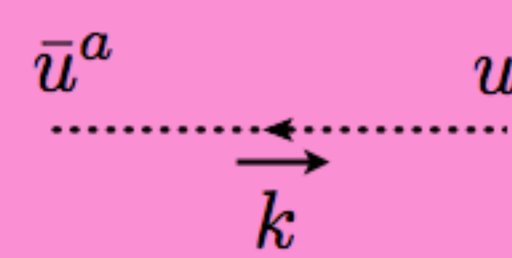


Gauge fixing explicitly breaks gauge invariance. However, in the end physical results are independent of the gauge choice. Powerful check of higher order calculations: verify that the ξ dependence fully cancels in the final result

Ghosts

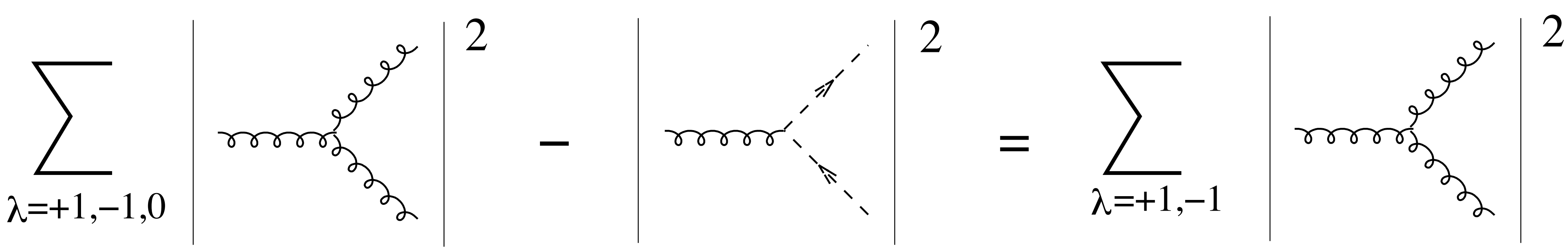
In covariant gauges gauge fixing term must be supplemented with ghost term to cancel unphysical longitudinal degrees of freedom which should not propagate

$$\mathcal{L}_{\text{ghost}} = \partial_\mu \eta^{a\dagger} D^\mu_{ab} \eta^b$$



$$= \frac{i}{k^2}\,\delta^{ab}$$

η: complex scalar field which obeys Fermi statistics



Axial gauges

Alternative: choose an axial gauge (introduce an arbitrary direction n)

$$\mathcal{L}_{\text{axial gauge}} = -\frac{1}{\xi}\left(n^\mu A_\mu^A\right)^2$$

The gluon propagator becomes

$$d_{\mu\nu} = \frac{i}{k^2}\left(-g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} + \frac{(n^2+\xi k^2)k_\mu k_\nu}{(n\cdot k)^2}\right)\delta_{ab}$$

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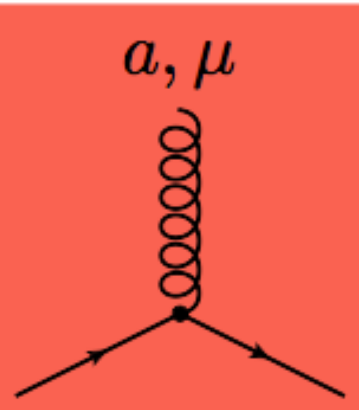
Light cone gauge: $n^2 = 0$ and $\lambda = 0$

Axial gauges for $k^2 \to 0$

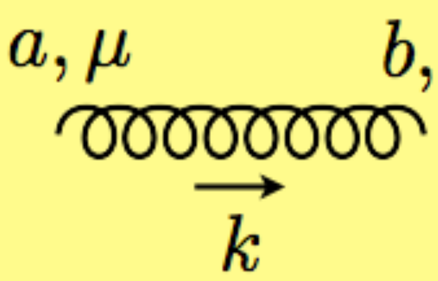
$$d_{\mu\nu}k^\mu = d_{\mu\nu}n^\mu = 0$$

i.e. only two physical polarizations propagate, that's why often the term physical gauge is used

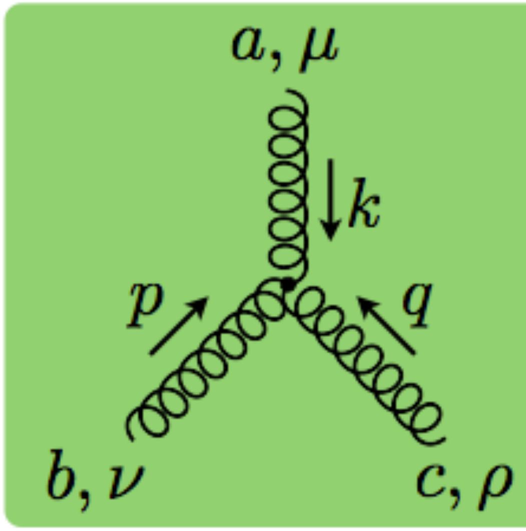
QCD Feynman rules: the vertices



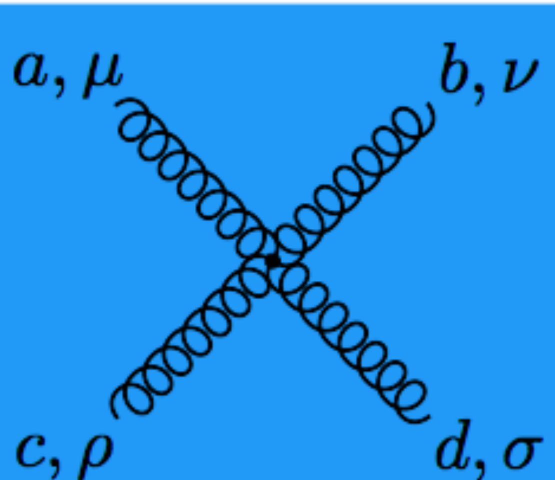
$$= \quad ig_s \gamma^\mu t^a$$



$$= \quad \left(\frac{-ig_{\mu\nu}}{k^2} \right) \delta^{ab}$$



$$= \quad g_s\, f^{abc} \Big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu \Big]$$

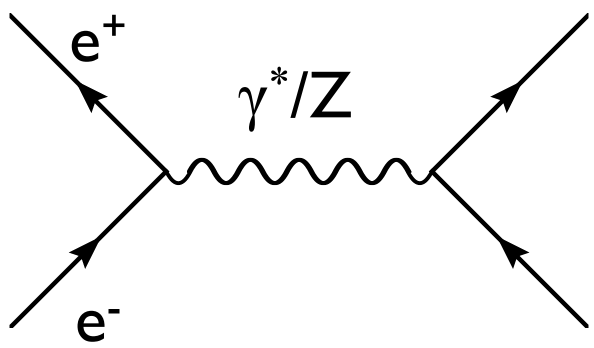


$$= \quad -ig_s^2 \Big[f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} \right) \Big]$$

Perturbative expansion of the R-ratio

The R-ratio is defined as

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



At lowest order in perturbation theory

$$\sigma(e^+e^- \to \text{hadrons}) = \sigma_0(e^+e^- \to q\bar{q})$$

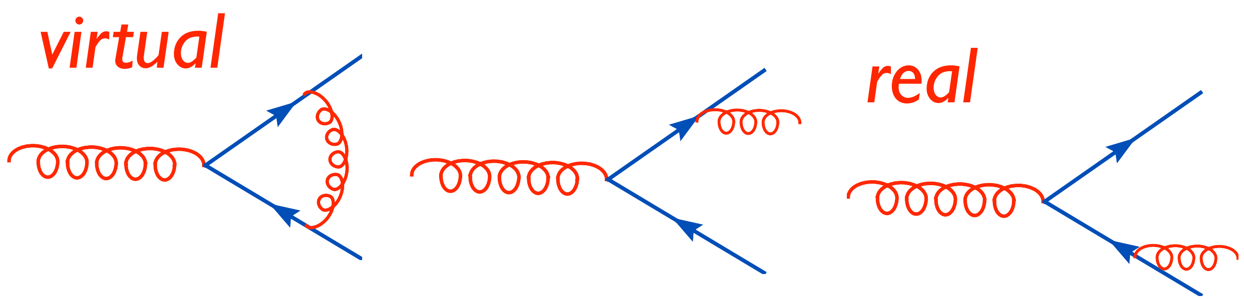
The PT treatment works since the scattering happens at large momentum transfer (short time), while hadronization happens at low momentum transfer, i.e. too late to change the original probability distribution

Since common factors cancel in numerator/denominator, to lowest order one finds

$$R_0 = \frac{\sigma_0(\gamma^* \to \text{hadrons})}{\sigma_0(\gamma^* \to \mu^+\mu^-)} = N_c \sum_f q_f^2$$

The R-ratio: perturbative expansion

First order correction



Real and virtual do not interfere since they have a different # of particles.
The amplitude squared becomes

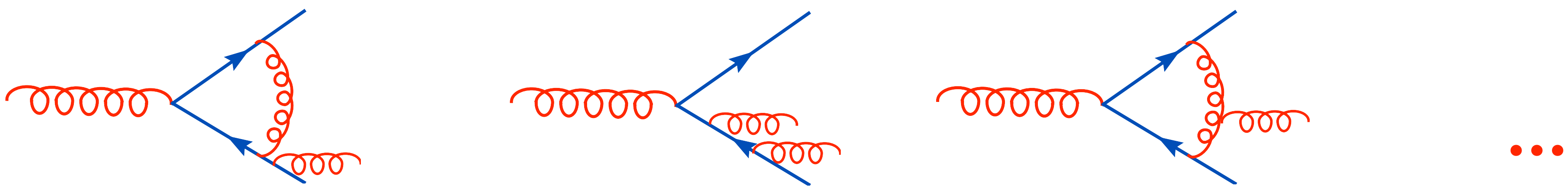
$$|A_1|^2 = |A_0|^2 + \alpha_s \left(|A_{1,r}|^2 + 2\mathrm{Re}\{A_0 A_{1,v}^*\}\right) + \mathcal{O}(\alpha_s^2) \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

Integrating over phase space, the first order result reads

$$R_1 = R_0 \left(1 + \frac{\alpha_s}{\pi}\right)$$

R-ratio and UV divergences

To compute the second order correction one has to compute diagrams like these and many more



One gets

$$R_2 = R_0 \left(1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(c + \pi b_0 \ln \frac{M_{\mathrm{UV}}^2}{Q^2}\right)\right) \qquad b_0 = \frac{11 N_c - 4 n_f T_R}{12\pi}$$

Ultra-violet divergences do not cancel. Result depends on UV cut-off.

Renormalization and running coupling

The divergence is dealt with by renormalization of the coupling constant

$$\alpha_s(\mu) = \alpha_s^{\text{bare}} + b_0 \ln \frac{M_{UV}^2}{\mu^2} \left(\alpha_s^{\text{bare}}\right)^2$$

R expressed in terms of the renormalized coupling is finite

$$R = R_0 \left(1 + \frac{\alpha_s(\mu)}{\pi} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(c + \pi b_0 \ln \frac{\mu^2}{Q^2}\right) + \mathcal{O}(\alpha_s^3(\mu))\right)$$

Renormalizability of the theory guarantees that *the same redefinition of the coupling removes all UV divergences from all physical quantities (massless case)*

Will not cover renomalization in these lectures, but it suffices to know that renormalization of S-matrix elements is achieved by replacing bare masses and bare coupling with renormalized ones

- the coupling $\Rightarrow$ β function
- the masses $\Rightarrow$ anomalous dimensions γ_m

The beta-function

$$\boxed{\beta(\alpha_s^{\text{ren}}) \equiv \mu^2 \frac{d\alpha_s(\mu^2)}{d\mu^2}}$$

The renormalized coupling is

$$\alpha_s(\mu) = \alpha_s^{\text{bare}} + b_0 \ln \frac{M_{UV}^2}{\mu^2} \left(\alpha_s^{\text{bare}}\right)^2$$

So, one immediately gets

$$\beta = -b_0 \alpha_s^2(\mu) + \ldots$$

Integrating the differential equation one finds at lowest order

$$\frac{1}{\alpha_s(\mu)} = b_0 \ln \frac{\mu^2}{\mu_0^2} + \frac{1}{\alpha_s(\mu_0)} \qquad \Rightarrow \qquad \alpha_s(\mu) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}}$$

The fundamental parameter Λ_{QCD}

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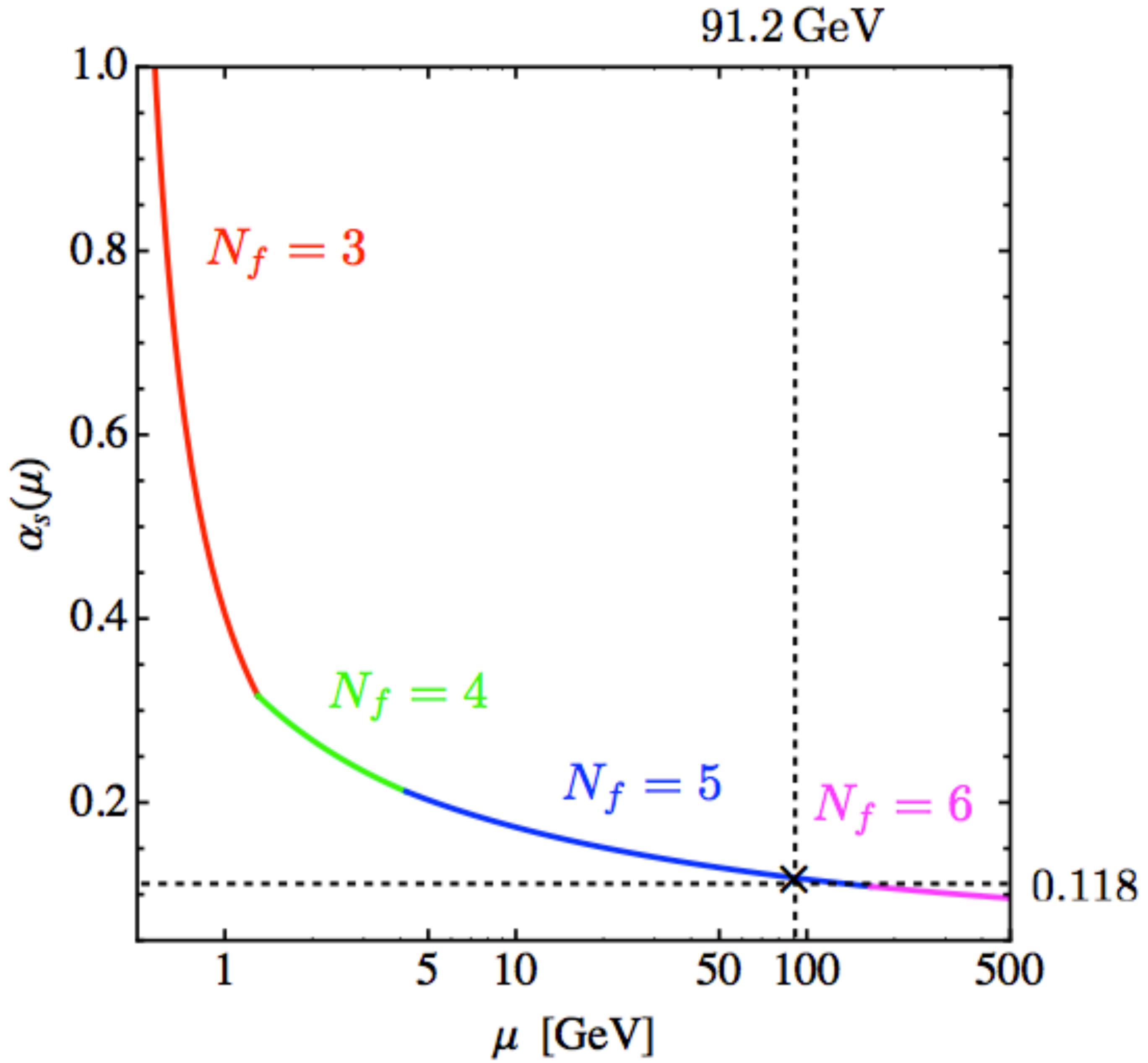
Question: why does nobody talk about Λ_{QED} ?

Active flavours & running coupling

The (active) field content of a theory modifies the running of the couplings



Renormalization Group Equation

Consider a dimensionless quantity A, function of a single scale Q. The dimensionless quantity should be independent of Q. However in quantum field theory this is not true, as renormalization introduces a second scale μ

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So, for any observable A one can write a renormalization group equation

$$\boxed{\left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right] A\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0}$$

$$\alpha_s = \alpha_s(\mu^2) \qquad \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

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The scale dependence of A enters through the running of the coupling: knowledge of $A(1, \alpha_s(Q^2))$ allows one to compute the variation of A with Q given the beta-function

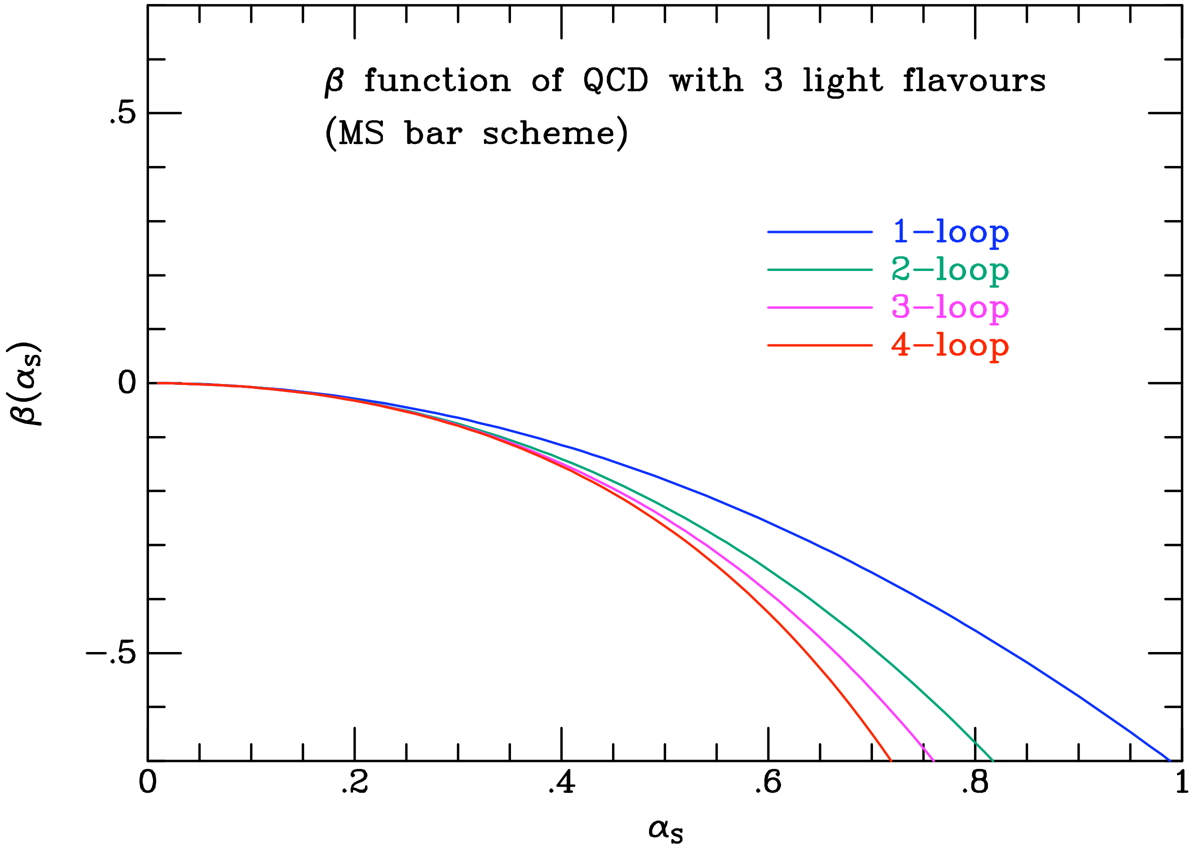
More on the beta-function

Perturbative expansion of the beta-function:

$$\beta = -\alpha_s^2(\mu) \sum_i b_i \alpha_s^i(\mu)$$

$$b_0 = \frac{11 N_c - 4 n_f T_R}{12\pi}$$

$$b_1 = \frac{17 N_c^2 - 5 N_c n_f - 3 C_F n_f}{24\pi^2}$$

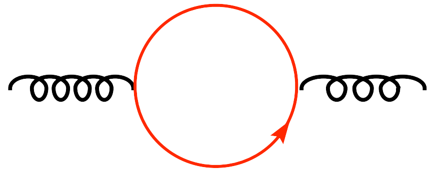


- n_f is the number of active flavours (depends on the scale)
- today, the beta-function known up to four loops, but only first two coefficients are independent of the renormalization scheme (see later)

More on the beta-function

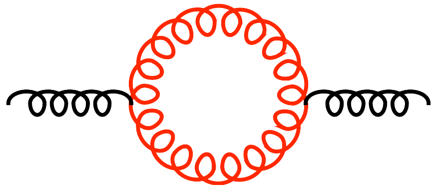
<u>Roughly speaking:</u>

(a) quark loop vacuum polarization diagram gives a negative contribution to $b_0 \sim n_f$



(a)

(b) gluon loop gives a positive contribution to $b_0 \sim N_c$



(b)

(b) > (a) $\Rightarrow$ $b_{0,QCD} > 0 \Rightarrow$ overall negative beta-function in QCD

While in QED (b) = 0 $\Rightarrow b_{0,QED} < 0$

$$\beta_{\rm QED} = \frac{1}{3\pi}\alpha^2 + \ldots$$

Asymptotic freedom

Integrating the differential equation

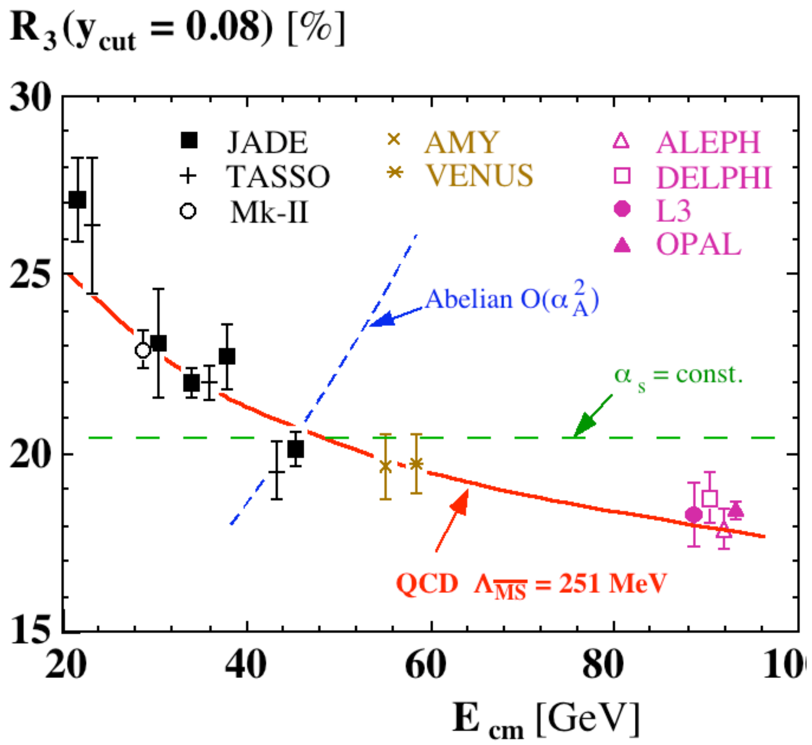
$$\frac{\partial \alpha_s(Q)}{\partial t} = -b_0 \alpha_s^2(Q) + \mathcal{O}(\alpha_s^3) \qquad t = \ln\left(\frac{Q^2}{\mu^2}\right)$$

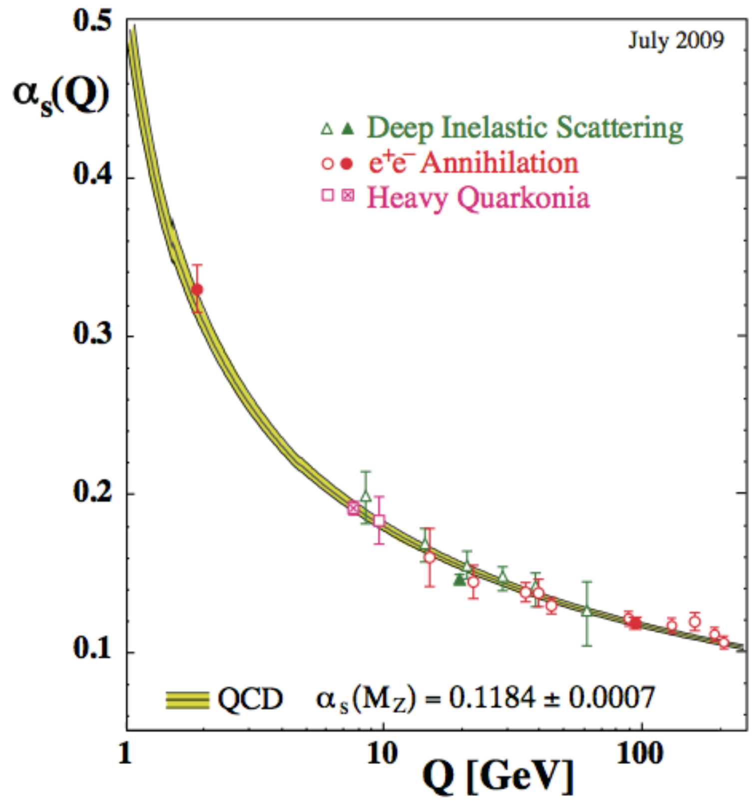
To lowest order one gets

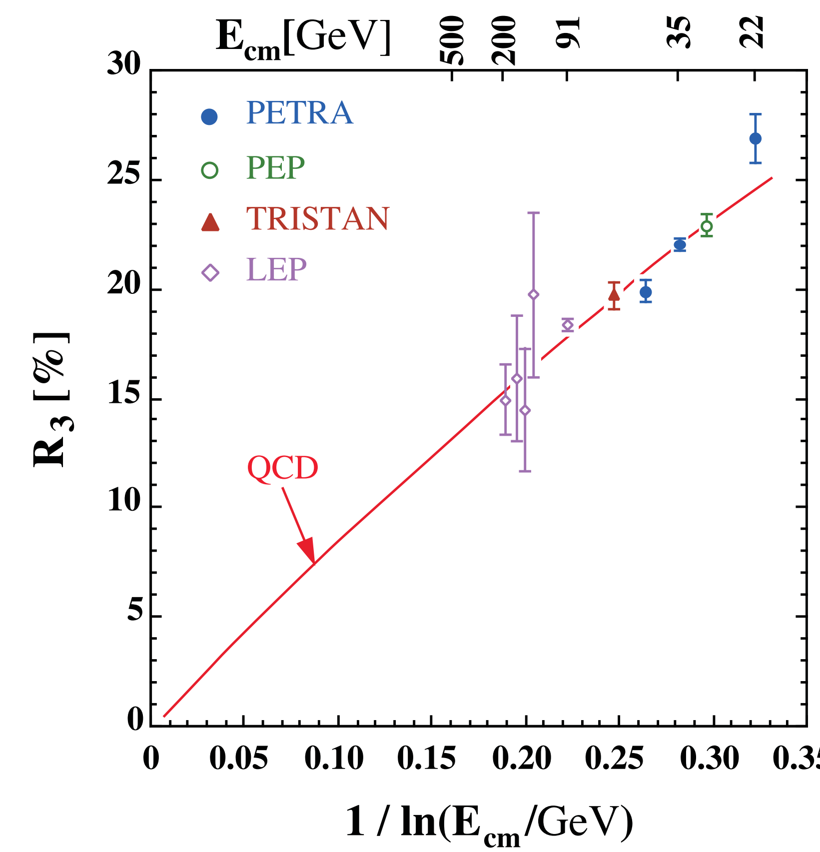
$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 + b_0 \ln \frac{Q^2}{\mu^2} \alpha_s(\mu)}$$

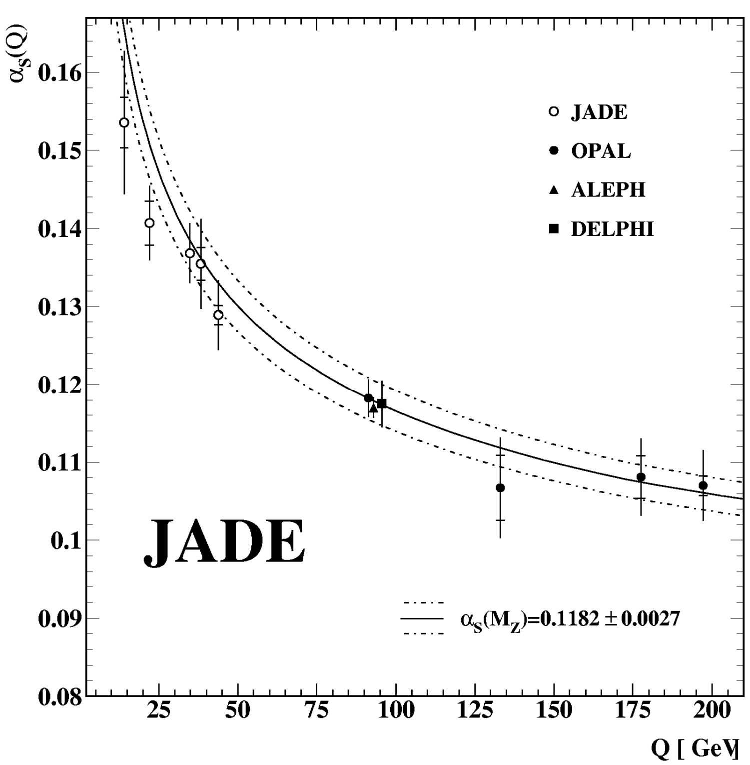
So the coupling constant decreases logarithmically with increasing energy. The statement that the theory becomes free at high energy goes under the name of asymptotic freedom [N.B. the sign of b_0 is crucial], i.e. the non-abelian vacuum polarization has an anti-screening effect

Measurements of the running coupling



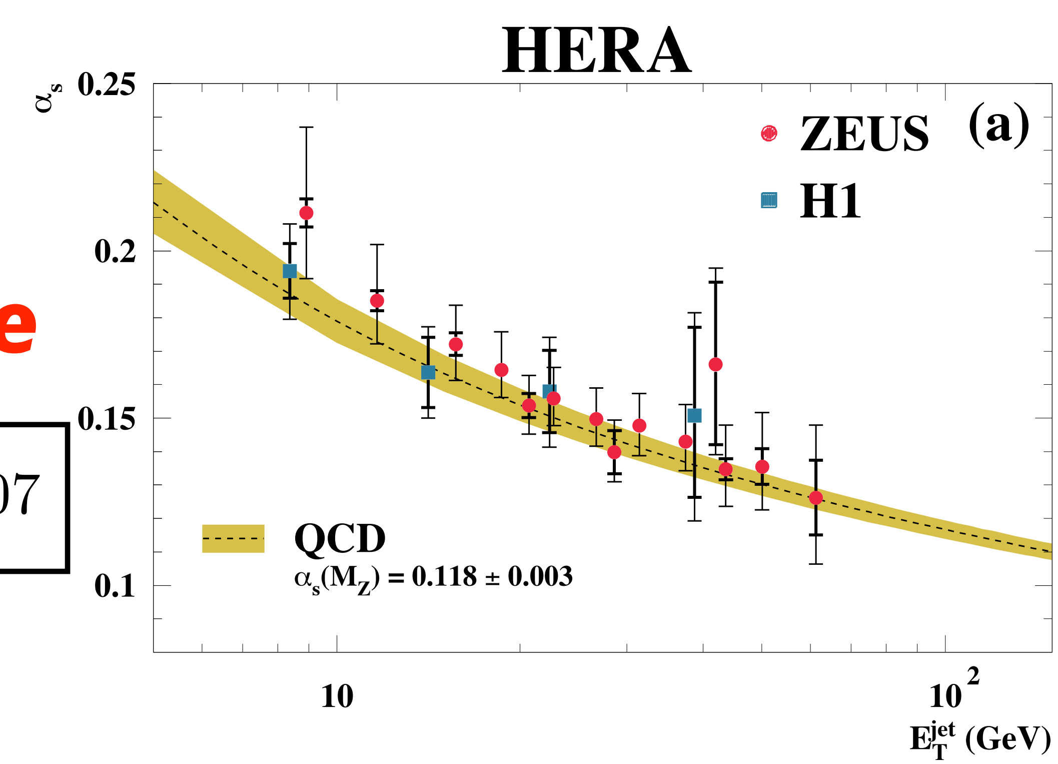






World average

$$\alpha_s(M_{Z^0}) = 0.1184 \pm 0.007$$



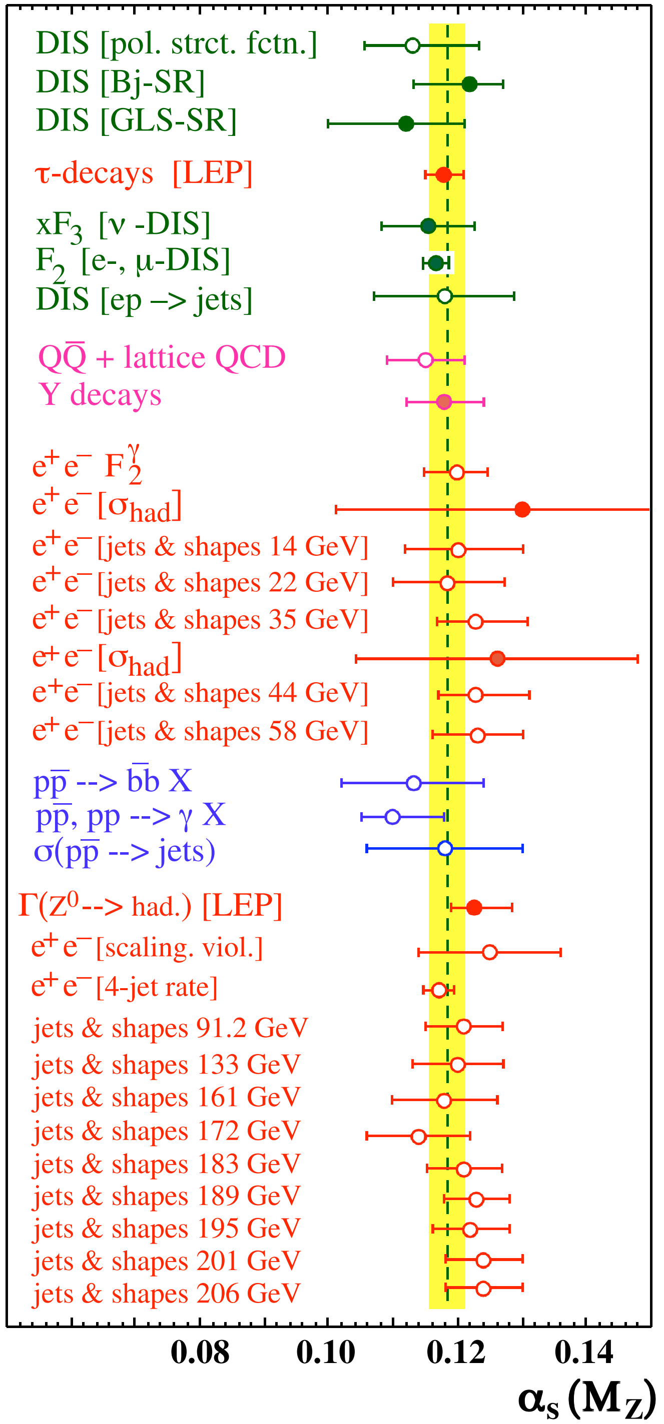
Measurements of the running coupling

<u>Summarizing:</u>

- overall consistent picture: α_s from very different observables compatible
- α_s is not so small at current scales
- α_s decreases slowly at higher energies (logarithmic only)
- higher order corrections are and will remain important

2009 World average

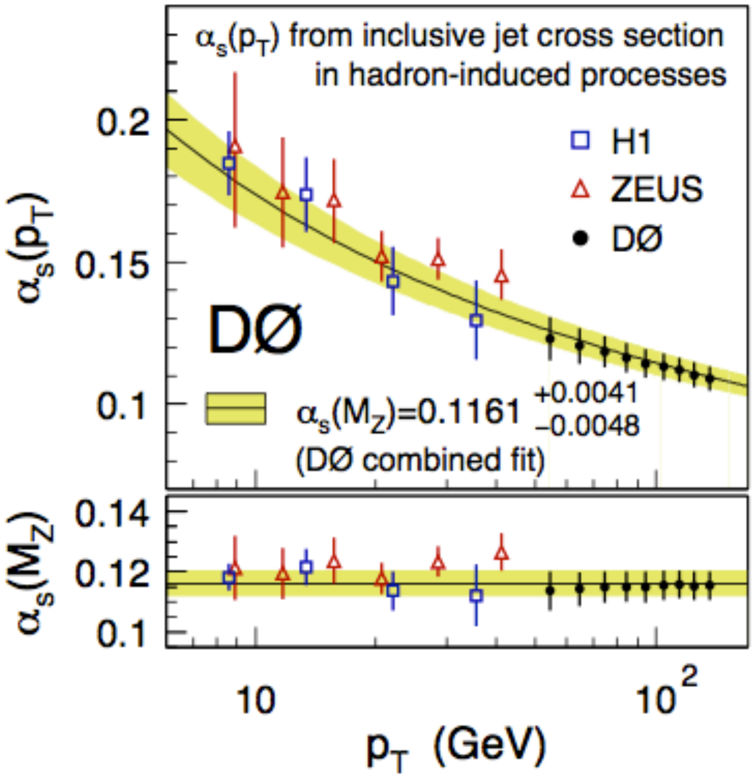
$$\boxed{\alpha_s(M_{Z^0}) = 0.1184 \pm 0.007}$$



α_s in the year 2011

Preliminary July 2011# : $\alpha_s = 0.1183 \pm 0.0010$

0911.2710



Process	Q [GeV]	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
τ-decays	1.78	0.1197 ± 0.0016	0.1180 ± 0.0011	0.9
DIS $[F_2]$	2 - 170	0.1142 ± 0.0023	0.1186 ± 0.0013	1.7
DIS [e-p $\to$ jets]	6 - 100	0.1198 ± 0.0032	0.1182 ± 0.0010	0.5
Lattice QCD	7.5	0.1183 ± 0.0008	0.1182 ± 0.0017	0.1
Υ decays	9.46	$0.119^{+0.006}_{-0.005}$	0.1183 ± 0.0010	0.1
new e^+e^- [jets & shps]	14 - 44	0.1172 ± 0.0051	0.1183 ± 0.0010	0.2
→ $p\bar{p}$ incl. jets	50-145	0.1161 ± 0.0045	0.1183 ± 0.0010	0.4
e^+e^- [ew prec. data]	91.2	0.1193 ± 0.0028	0.1182 ± 0.0010	0.4
→ e^+e^- [jets & shps]	91 - 208	0.1208 ± 0.0038	0.1182 ± 0.0011	0.7
→ e^+e^- [5-jet]	91 - 208	$0.1155^{+0.0041}_{-0.0034}$	0.1183 ± 0.0010	0.6

$$\sigma_{\rm pert} = \left(\sum_n \alpha_s^n c_n\right) \otimes f_1)(\alpha_s) \otimes f_2(\alpha_s)$$

Competitive measurements at the LHC? Combined fit with pdfs or use ratios?

Open issue: treatment of very accurate outliers e.g.

$\alpha_s = 0.1135 \pm 0.0010$ [SCET, thrust at N^3LO]

Abbate et al. 1106.3080

$\alpha_s = 0.1213 \pm 0.0014$ [τ-decays]

Pich 1001.0389

$\alpha_s = 0.1122 \pm 0.0014$ [NNLO DIS]

Alekhin et al. 1001.0389

Asymptotic freedom & confinement

Asymptotic freedom:

- coupling smaller at higher energies (smaller distances). Theory becomes effectively free
- a consequence of the sign of the beta function
- perturbation theory predicts asymptotic freedom

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Confinement:

- related to the fact that the coupling increases at small energies
- however, the behavior is theoretically unknown because perturbation theory breaks down (rely on different techniques e.g. lattice QCD)
- we do not have a rigorous explanation for confinement
- we just observe that all partons are confined into color singlet hadrons: if one tries to separate partons it becomes favorable to extract from the QCD vacuum $q\bar{q}$-pairs and create hadrons
- we assume that confinement always holds

Intermediate Recap

- QCD is in principle a simple theory based on a simple Lagrangian with gauge group is SU(3)
- Simple color algebra and Feynman rules are the necessary ingredients for perturbative calculations (see later)
- Today, we know three families of quarks, we briefly revisited the experiments which lead to their discovery
- There are UV divergences but they are dealt with by renormalization (coupling + masses)
- This is intimately related to the fact that the coupling runs $\Rightarrow$ beta-function
- The theory is asymptotically free and consistent with confinement

Next

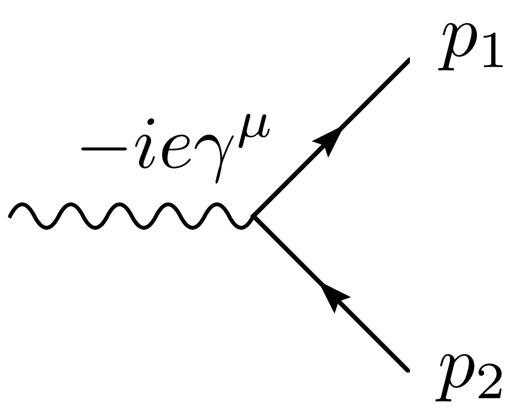
- Infrared and collinear divergences and IRsafety
- Parton model: incoherent sum of all partonic cross-sections
- Sum rules (momentum, charge, flavor conservation)
- Determination of parton densities from data (electron & neutrino scattering in DIS or Drell-Yan)
- Radiative corrections: failure of parton model
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ measure gluon PDF

The soft approximation

Let's consider again the R-ratio. This is determined by $\gamma^* \to q\bar{q}$

At leading order:

$$M_0^\mu = \bar{u}(p_1)(-ie\gamma^\mu)v(p_2)$$



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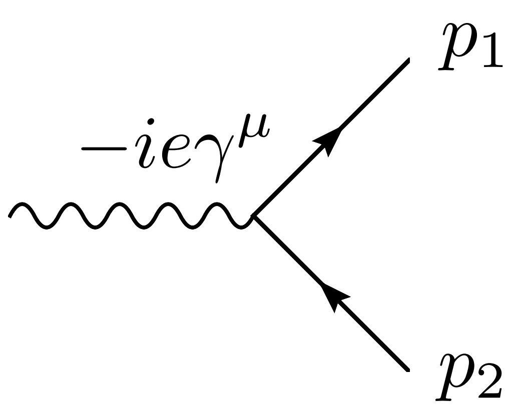
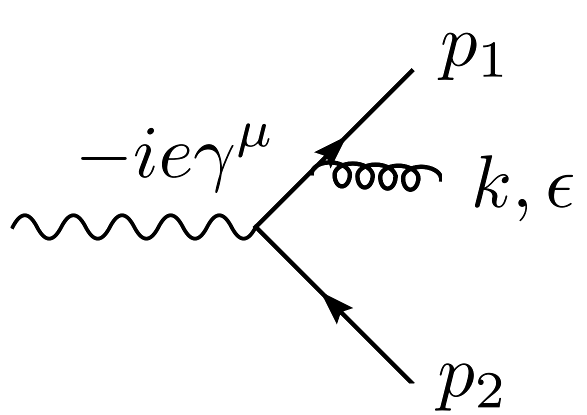


Emit one gluon:

$$\begin{aligned} M^\mu_{q\bar{q}g} &= \bar{u}(p_1)(-ig_s t^a \not{\epsilon})\frac{i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie\gamma^\mu)v(p_2) \\ &+ \bar{u}(p_1)(-ie\gamma^\mu)\frac{i(\not{p}_2-\not{k})}{(p_2-k)^2}(-ig_s t^a \not{\epsilon})v(p_2) \end{aligned}$$



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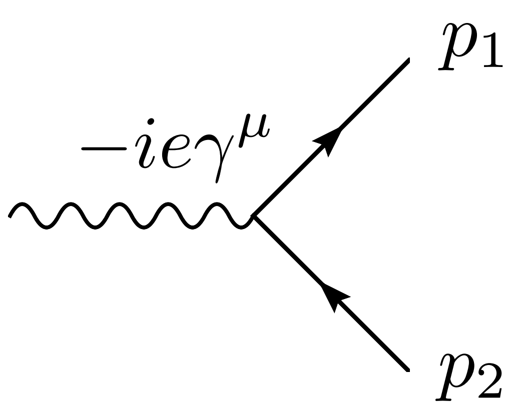

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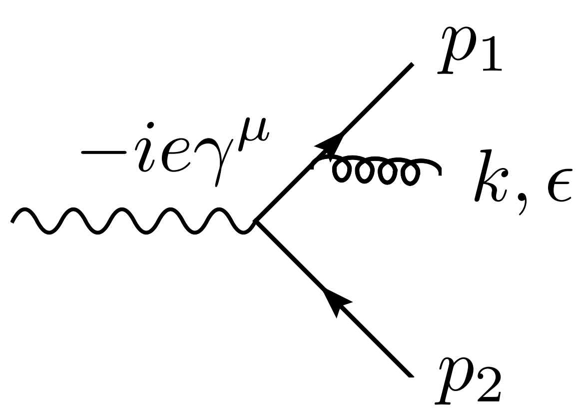

Consider the soft approximation: $k \ll p_1, p_2$

$$M^\mu_{q\bar{q}g} = \bar{u}(p_1)\left((-ie\gamma^\mu)(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)$$

$\Rightarrow$ factorization of soft part

Soft divergences

The squared amplitude becomes

$$|M^{\mu}_{q\bar{q}g}|^2 = \sum_{\text{pol}} \left| \bar{u}(p_1)\left((-ie\gamma^{\mu})(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)\right|^2$$

$$= \quad |M_{q\bar{q}}|^2 C_F g_s^2 \frac{2p_1p_2}{(p_1k)(p_2k)}$$

Soft divergences

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Including phase space

$$d\phi_{q\bar{q}g}|M_{q\bar{q}g}|^2 \quad = \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \frac{d^3k}{2\omega(2\pi)^3} C_F g_s^2 \frac{2p_1p_2}{(p_1k)(p_2k)}$$
$$= \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi}\frac{2\alpha_s C_F}{\pi}\frac{1}{\omega^2(1-\cos^2\theta)}$$

Soft divergences

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$$= \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi}\frac{2\alpha_s C_F}{\pi}\frac{1}{\omega^2(1-\cos^2\theta)}$$

The differential cross section is

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\frac{d\theta}{\sin\theta}\frac{d\phi}{2\pi}$$

Soft & collinear divergences

Cross section for producing a q$\bar{q}$-pair and a gluon is infinite (IR divergent)

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$\underline{\omega \to 0}$: soft divergence

$\underline{\theta \to 0}$: collinear divergence

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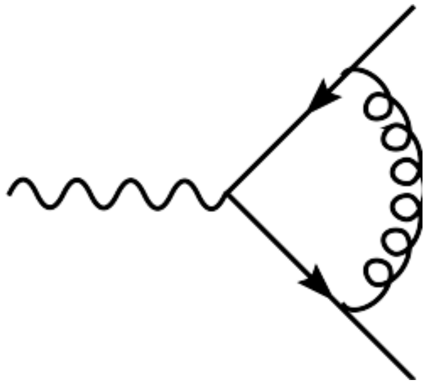
$\underline{\omega \to 0}$: soft divergence

$\underline{\theta \to 0}$: collinear divergence

But the full O(α_s) correction to R is finite, because one must include a virtual correction which cancels the divergence of the real radiation

$$d\sigma_{q\bar{q},v} \sim -d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$



NB: here we kept only soft terms, if we do the full calculation one gets a finite correction of α_s/π

Soft & collinear divergences

<u>$\omega \to 0$</u> soft divergence: the four-momentum of the emitted particle approaches zero, typical of gauge theories, even if matter (radiating particle) is massive

<u>$\theta \to 0$</u> collinear divergence: particle emitted collinear to emitter. Divergence present only if all particles involved are massless

NB: the appearance of soft and collinear divergences discussed in the specific context of $e^+e^- \to qq$ are a general property of QCD

Infrared safety (= finiteness)

So, the R-ratio is an infrared safe quantity.

In perturbation theory one can compute only IR-safe quantities, otherwise get infinities, which can not be renormalized away (why not?)

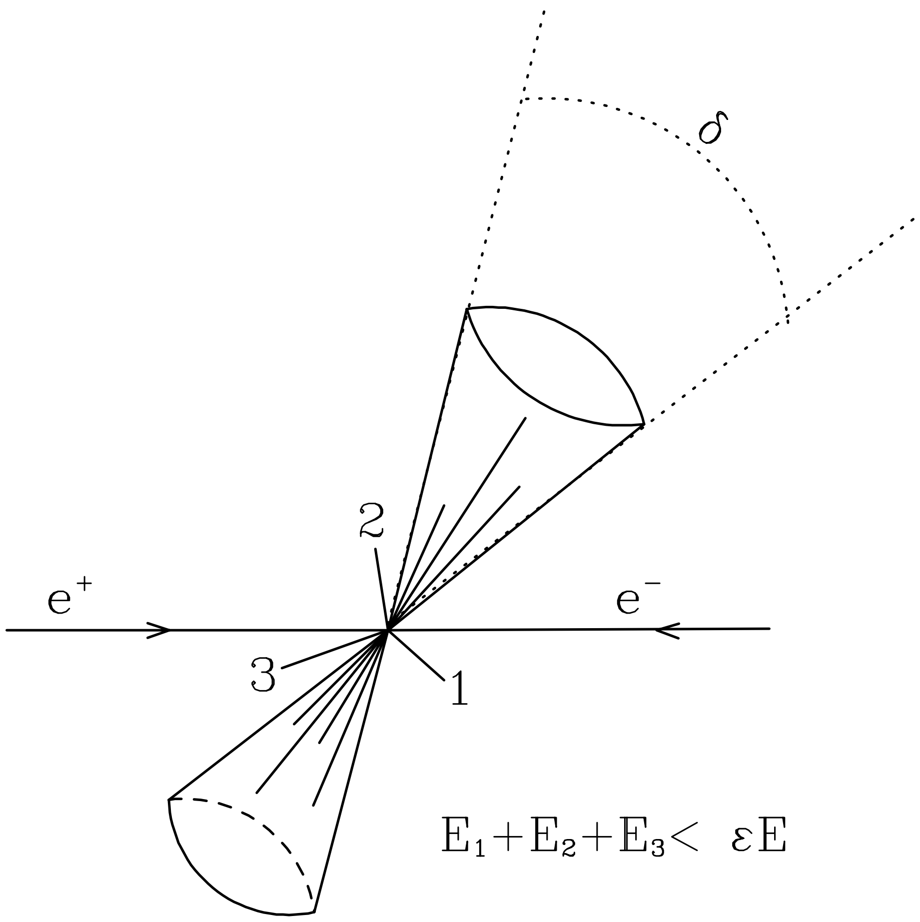
So, the natural questions are:

- are there other IR-safe quantities?
- what property of R guarantees its IR-safety?

Sterman-Weinberg jets

First formulation of cross-sections which are finite in perturbation theory and describe the hadronic final state

Introduce two parameters ε and δ:
a pair of Sterman-Weinberg jets are two cones of opening angle δ that contain all the energy of the event excluding at most a fraction ε



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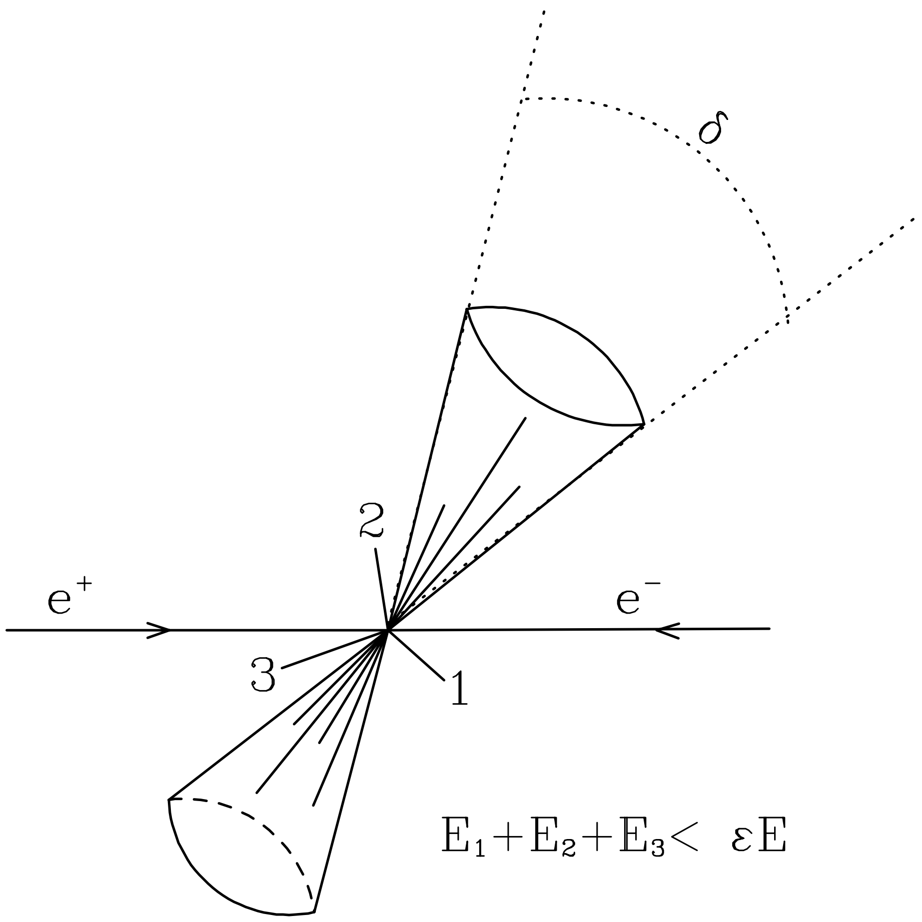

Why finite? the cancelation between real and virtual is not destroyed in the soft/collinear regions

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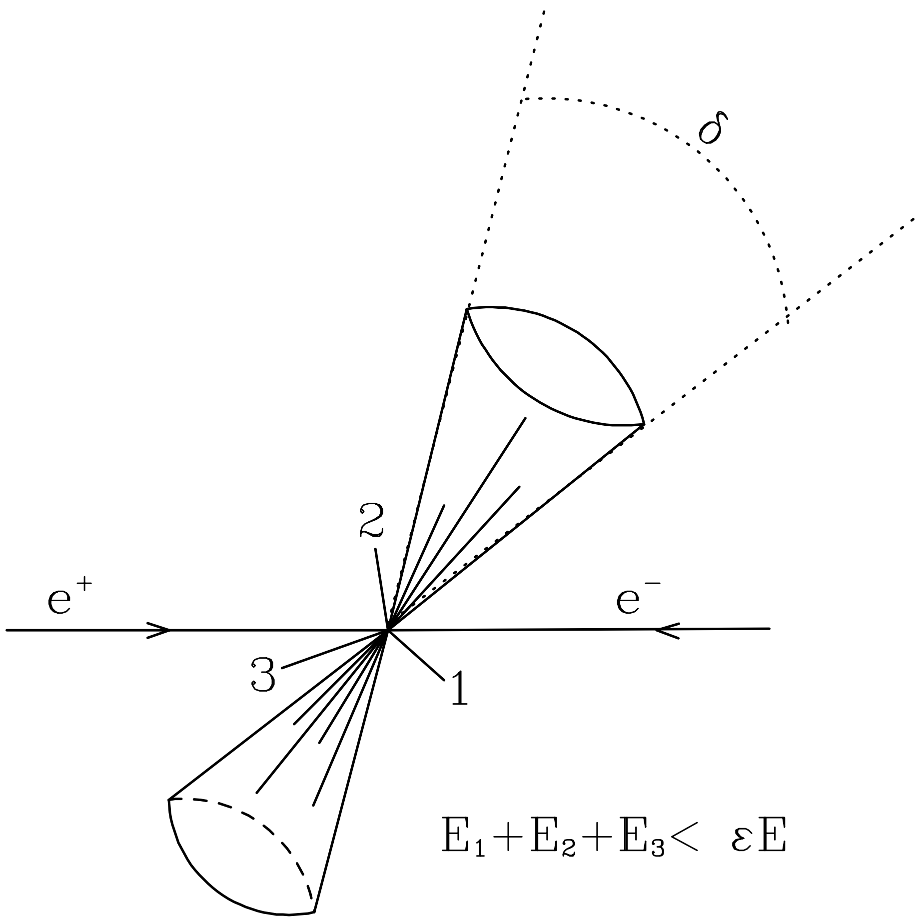
Introduce two parameters ε and δ:
a pair of Sterman-Weinberg jets are two cones of opening angle δ that contain all the energy of the event excluding at most a fraction ε



Why finite? the cancelation between real and virtual is not destroyed in the soft/collinear regions

Kinoshita-Lee-Nauenberg (KLN) theorem:
final-state infrared divergences cancel in measurable quantities (transition probabilities, cross-sections summed over indistinguishable states...)

Sterman-Weinberg jets

The Sterman-Weinberg jet cross-section up to $O(\alpha_s)$ is given by

$$\sigma_1 = \sigma_0 \left(1 + \frac{2\alpha_s C_F}{\pi} \ln\epsilon \ln\delta^2 \right)$$

Effective expansion parameter in QCD is often $\alpha_s C_F/\pi$ not α_s

α_s-expansion enhanced by a double log: left-over from real-virtual cancellation

- if more gluons are emitted, one gets for each gluon
  - a power of $\alpha_s C_F/\pi$
  - a soft logarithm $\ln\varepsilon$
  - a collinear logarithm $\ln\delta$
- if ε and/or δ become too small the above result diverges
- if the logs are large, fixed order meaningless, one needs to resum large infrared and collinear logarithms to all orders in the coupling constant

Infrared safety: definition

An observable $\mathcal{O}$ is infrared and collinear safe if

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

i.e. the observable is insensitive to emission of soft particles or to collinear splittings

Infrared safety: examples

Infrared safe ?

- energy of the hardest particle in the event
- multiplicity of gluons
- momentum flow into a cone in rapidity and angle
- cross-section for producing one gluon with $E > E_{min}$ and $\theta > \theta_{min}$
- jet cross-sections

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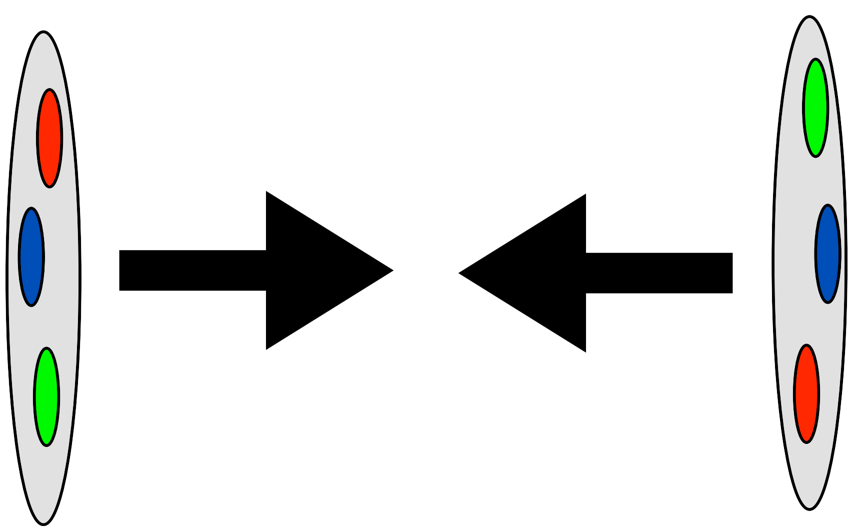
- jet cross-sections DEPENDS

Partons in the initial state

- We talked a lot about final state QCD effects
- This is the only thing to worry about at e^+e^- colliders (LEP)
- Hera/Tevatron/LHC involve protons in the initial state
- Proton are made of QCD constituents

Next we will focus mainly on aspects related to initial state effects

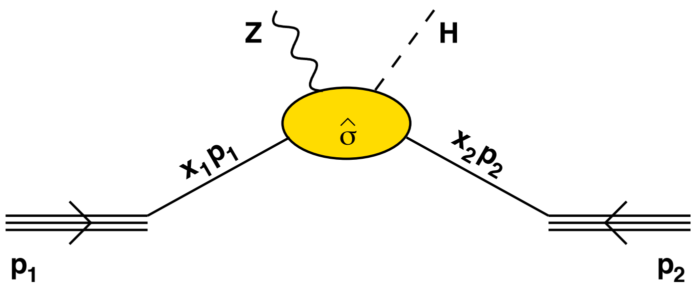


The parton model

Basic idea of the parton model: intuitive picture where in a high transverse momentum scattering partons behave as quasi free in the collision
$\Longrightarrow$ cross section is the incoherent sum of all partonic cross-sections

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \qquad \hat{s} = x_1 x_2 s$$

NB: This formula is wrong/incomplete (see later)



$f_i^{(P_j)}(x_i)$: parton distribution function (PDF) is the probability to find parton i in hadron j with a fraction x_i of the longitudinal momentum (transverse momentum neglected), extracted from data

$\hat{\sigma}(x_1 x_2 s)$: partonic cross-section for a given scattering process, computed in perturbative QCD

Sum rules

Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

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Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

$$\int_0^1 dx \left(f_s^{(p)}(x) - f_{\bar{s}}^{(p)}(x) \right) = 0$$

In the proton: u, d valence quarks, all other quarks are called sea-quarks

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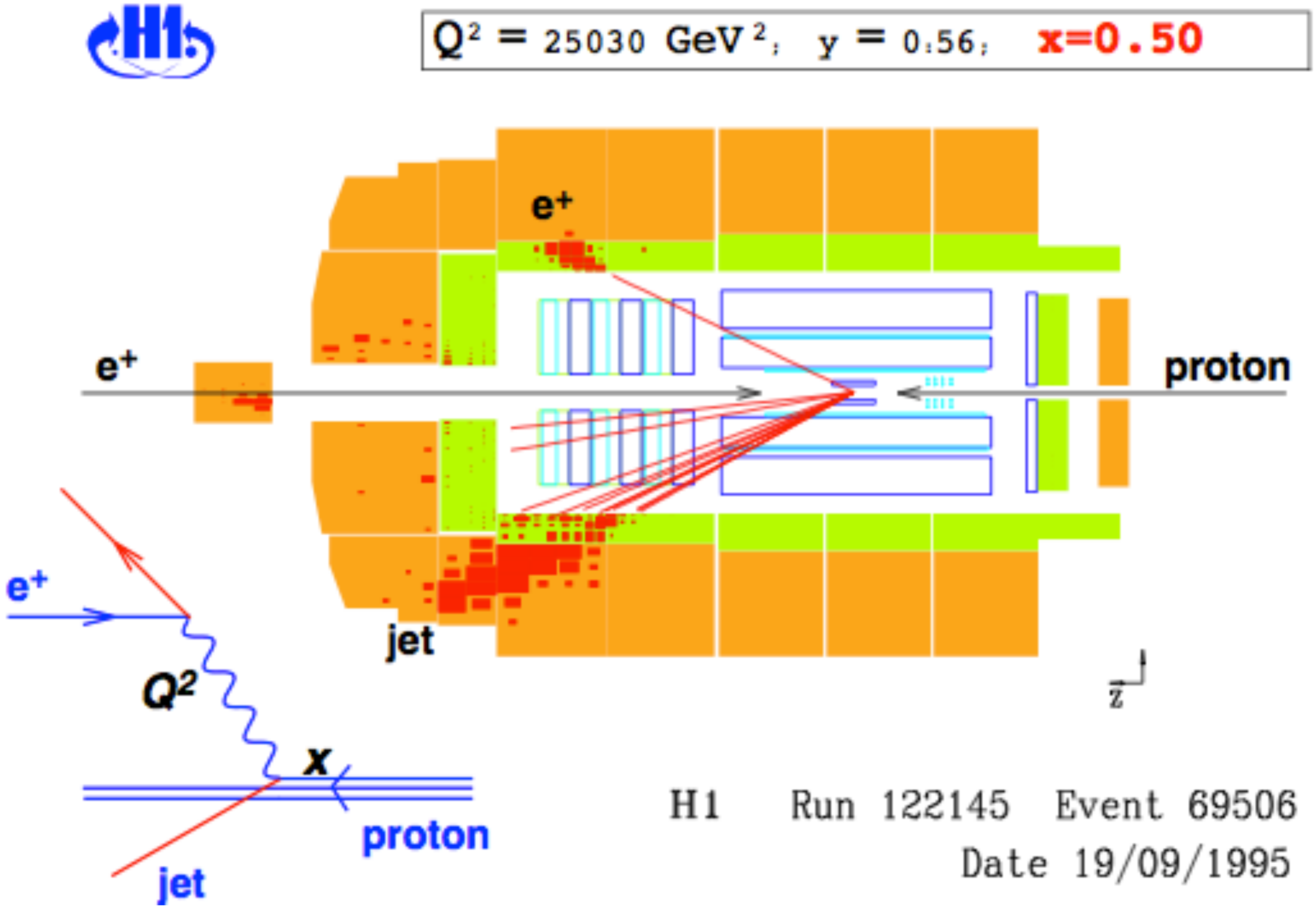
How can parton densities be extracted from data?

Deep inelastic scattering

Easier than processes with two incoming hadrons is the scattering of a lepton on a (anti)-proton



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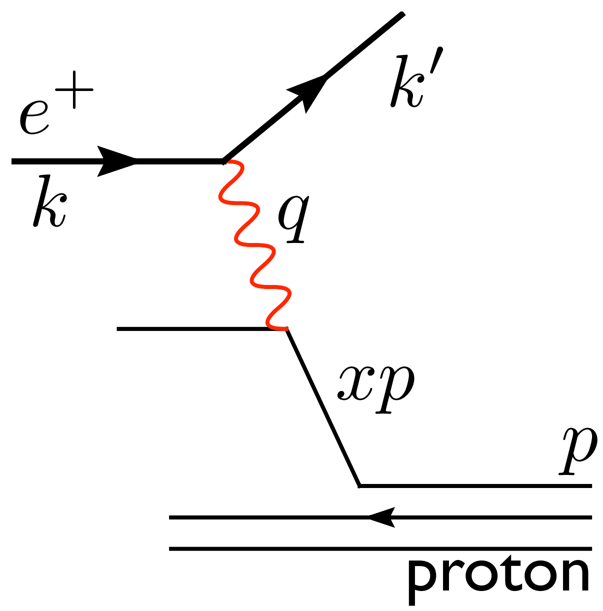

<u>Kinematics:</u>

$$Q^2 = -q^2 \quad s = (k+p)^2 \quad x_{Bj} = \frac{Q^2}{2p \cdot q} \quad y = \frac{p \cdot q}{k \cdot p}$$

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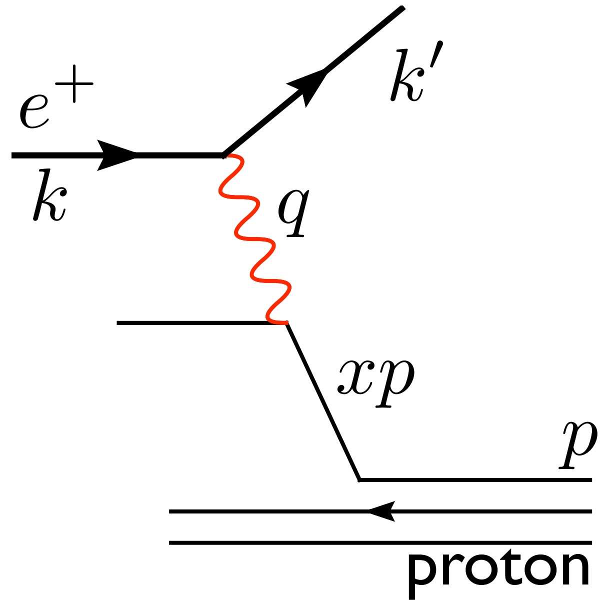

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$$\hat{p} = xp \quad \hat{s} = (k+\hat{p})^2 = 2k \cdot \hat{p} \quad \hat{y} = \frac{\hat{p} \cdot q}{k \cdot \hat{p}} = y \quad (\hat{p}+q)^2 = 2\hat{p} \cdot q - Q^2 = 0$$

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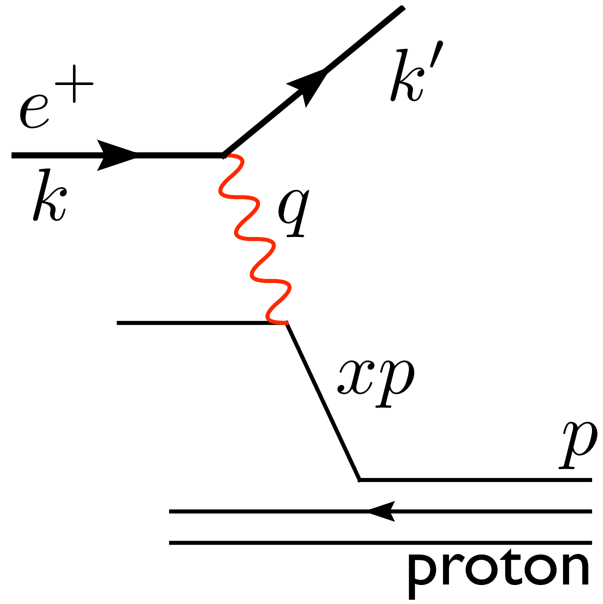

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$$\Rightarrow x = x_{Bj}$$

Partonic cross section:

(just apply QED Feynman rules and add phase space)

$$\boxed{\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\,\pi\,\alpha_{em}\left(1+(1-\hat{y})^2\right)}$$

Deep inelastic scattering

Hadronic cross section:

$$\frac{d\sigma}{dy} = \int dx \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$$

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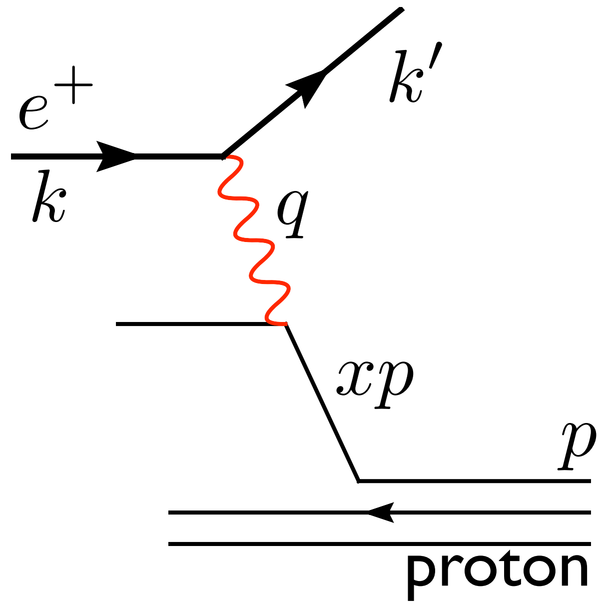
$$\frac{d\sigma}{dy\,dx_{Bj}} = \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$$

$$= \frac{2\pi\,\alpha_{em}^2 s x_{Bj}}{Q^4} \left(1 + (1-y)^2\right) \sum_l q_l^2 f_l^{(p)}(x_{Bj})$$



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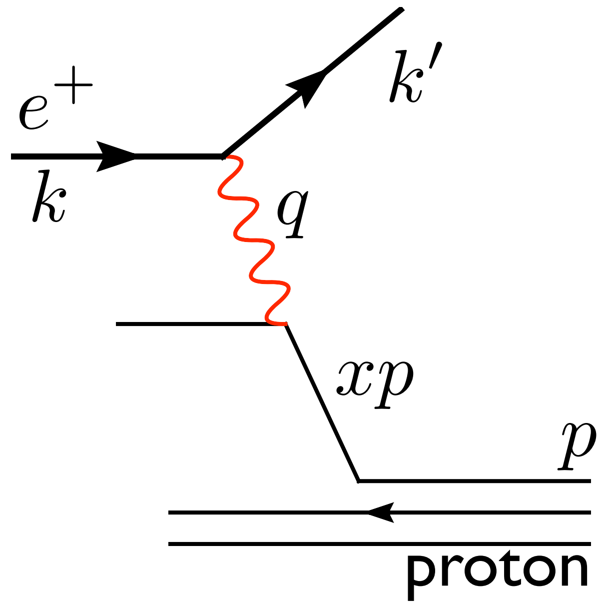

1. at fixed x_{Bj} and y the cross-section scales with s
2. the y-dependence of the cross-section is fully predicted and is typical of vector interaction with fermions ⇒Callan-Gross relation
3. can access (sums of) parton distribution functions
4. Bjorken scaling: pdfs depend on x and not on Q^2

The structure function F_2

$$\frac{d\sigma}{dydx} = \frac{2\pi\alpha_{em}^2 s}{Q^4}\left(1+\left(1-y^2\right)F_2(x)\right) \qquad F_2(x) = \sum_l x q_l^2 f_l^{(p)}(x)$$

F_2 is called **structure function** (describes structure/constituents of nucleus)

For electron scattering on proton

$$F_2(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

NB: use perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Question: F_2 gives only a linear combination of u and d. How can they be extracted separately?

Isospin

Neutron is like a proton with u & d exchanged

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For electron scattering on a proton

$$F_2^p(x) = x\left(\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right)$$

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For electron scattering on a neutron

$$F_2^n(x) = x\left(\frac{1}{9}d_n(x) + \frac{4}{9}u_n(x)\right) = x\left(\frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)\right)$$

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F_2^n and F_2^p allow determination of u_p and d_p separately

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F_2^n and F_2^p allow determination of u_p and d_p separately

NB: experimentally get F_2^n from deuteron: $F_2^d(x) = F_2^p(x) + F_2^n(x)$

Sea quark distributions

Inside the proton there are fluctuations, and pairs of $u\bar{u}$,$d\bar{d}$,$c\bar{c}$,$s\bar{s}$ ... can be created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx\,(u_p(x) - \bar{u}_p(x)) = 2 \qquad \int_0^1 dx\,\left(d_p(x) - \bar{d}_p(x)\right) = 1$$

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How can one measure the difference?

Question: What interacts differently with particle and antiparticle?

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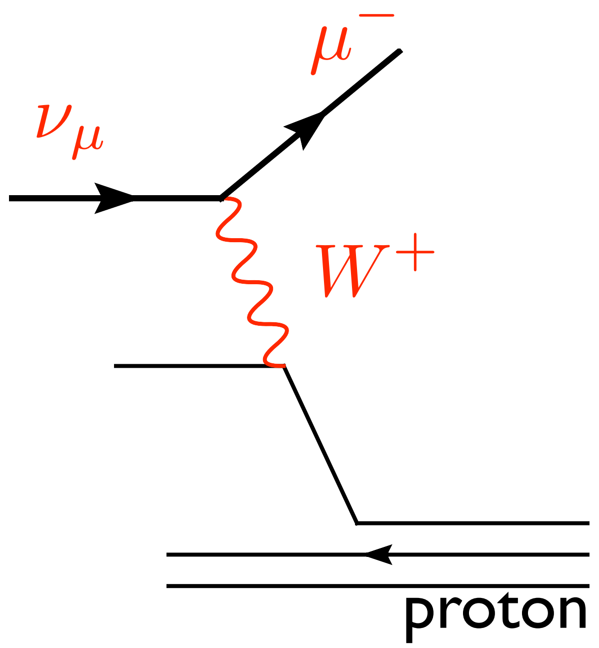
Question: What interacts differently with particle and antiparticle? W^+/W^- from neutrino scattering



Check of the momentum sum rule

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u_v	0.267
d_v	0.111
u_s	0.066
d_s	0.053
s_s	0.033
c_c	0.016
total	0.546

➟ *half of the longitudinal momentum is missing*

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$\gamma/W^{+/-}$ don't interact with gluons

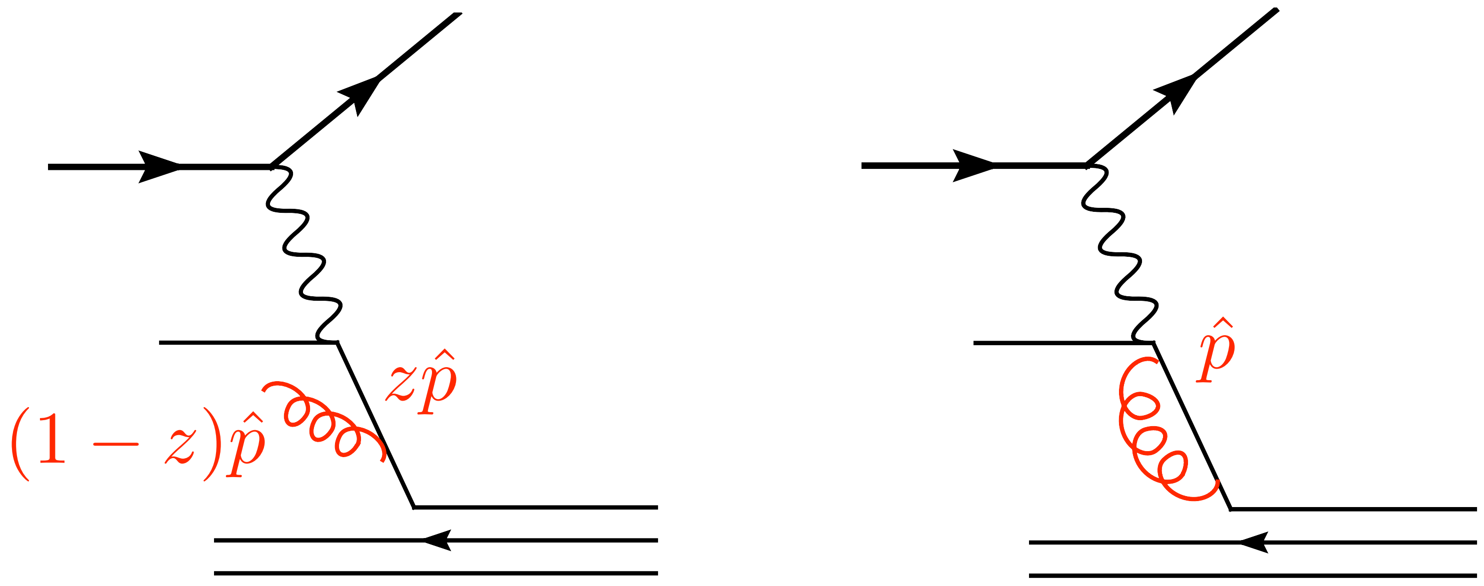
How can one measure gluon parton densities?

We need to discuss radiative effects first

Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



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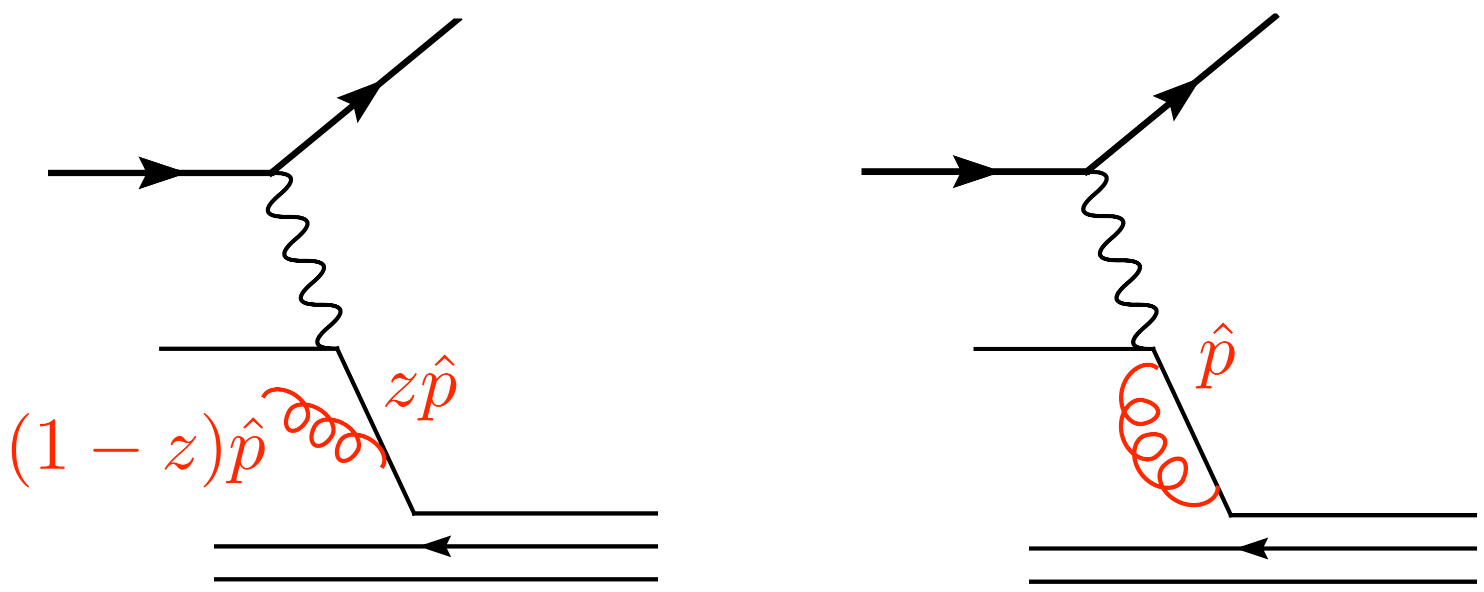

Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right), \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $k_\perp \to 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

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Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

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Plus prescription makes the universal cancelation of soft singularities explicit

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 f(z) \left(g(z) - g(1) \right)$$

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Collinear singularities still there, but they factorize.

Factorization scale

Schematically use $$\ln \frac{Q}{\lambda^2} = \ln \frac{Q}{\mu_F^2} + \ln \frac{\mu_F}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+\right) \sigma^{(0)}$$

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So we define

$$f_q(x, \mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p, \mu_F) = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_{qq}^{(0)}\right) \sigma^{(0)}(p)$$

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NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently
- the factorization scale acts as a cut-off, it allows to move the divergent contribution into non-pertubative parton distribution functions

Improved parton model

Naive parton model:

$$\cancel{\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s)} \qquad \hat{s} = x_1 x_2 s$$

After radiative corrections:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

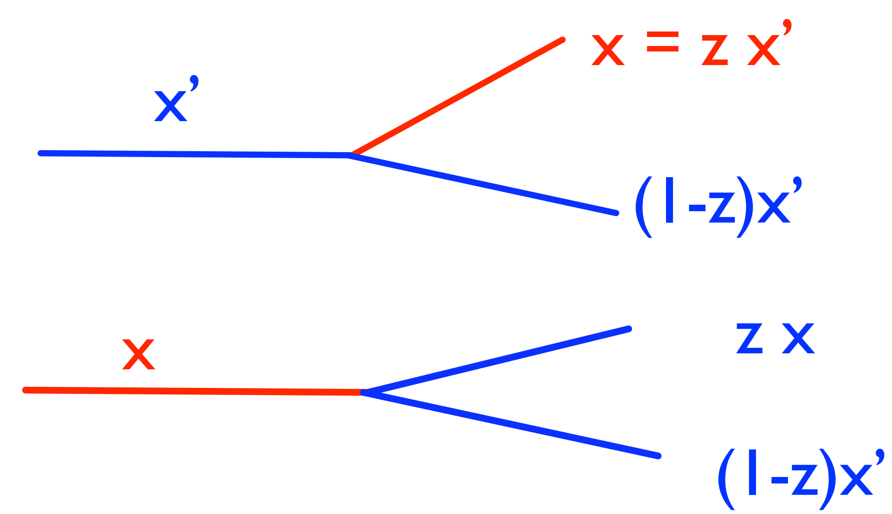
Intermediate recap

- With initial state parton collinear singularities don't cancel
- Initial state emissions with $k_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders

Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces it
- the parton itself splits



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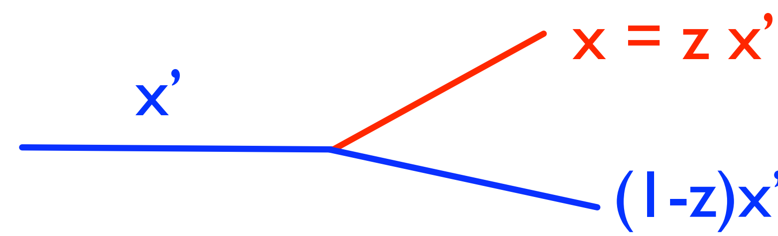

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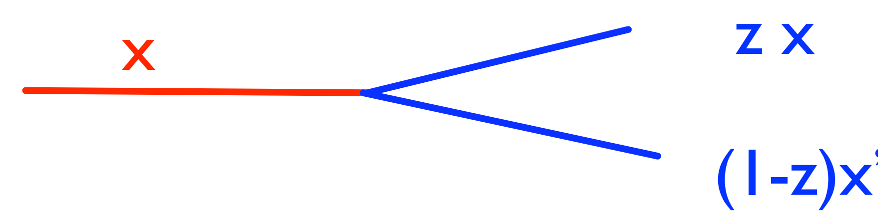


$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x',\mu^2) \delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},\mu^2\right)$$

The plus prescription

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z) \left(g(z) - g(1)\right)$$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

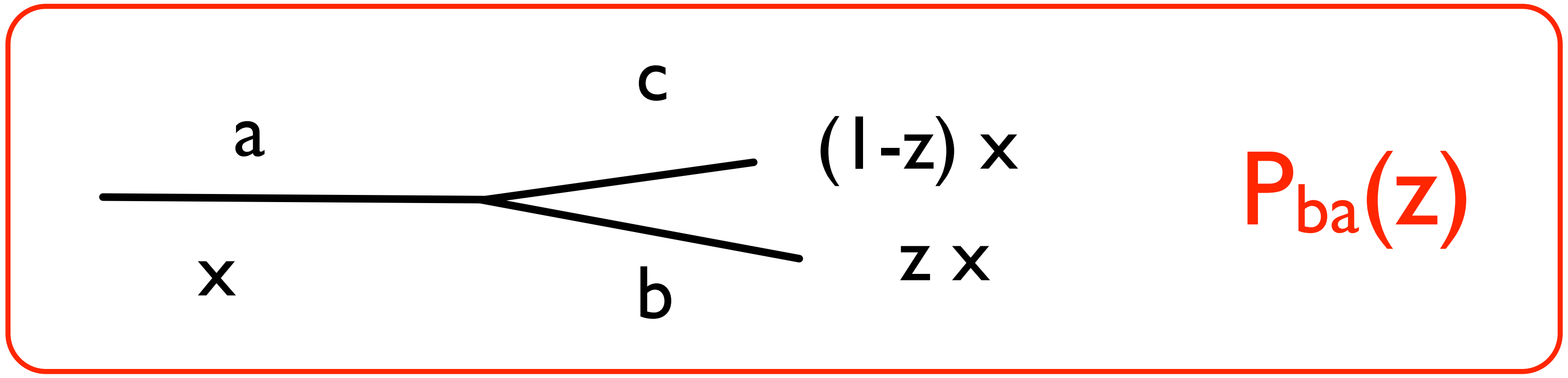
Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Plus prescription implicit from now on

Conventions for splitting functions

There are various partons flavours. Standard notation:



Accounting for the different species of partons the DGLAP equations become:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z},\mu^2\right)$$

This is a system of coupled integro/differential equations

The above convolution in compact notation:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2)$$

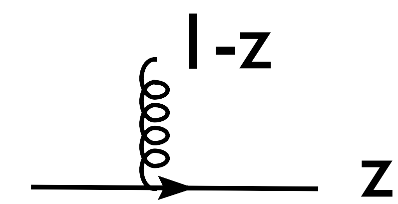
General DGLAP equation

<u>Evolution equations in the general case:</u>

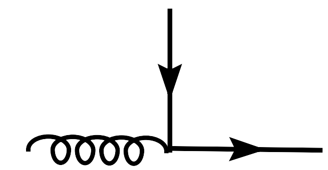
$$\mu^2 \frac{\partial f_i(z,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2) \qquad P_{ij}(x) = \frac{\alpha_s}{2\pi} P_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2)} + \dots$$

<u>Leading order splitting functions:</u>

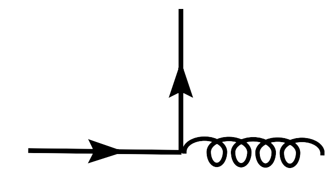
$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left(\frac{1+z^2}{1-z}\right)_+$$



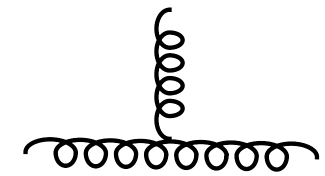
$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R\left(z^2 + (1-z)^2\right)$$



$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$



$$P_{gg}^{(0)} = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \frac{1}{6}(11C_A - 4n_f T_R)\delta(1-z)$$



<u>NB:</u> at higher orders $P_{q_iq_j}$ arise

History of splitting functions

☑ $P^{(0)}_{ab}$: Altarelly, Parisi; Gribov-Lipatov; Dokshitzer (1977)

☑ $P^{(1)}_{ab}$: Curci, Furmanski, Petronzio (1980)

☑ $P^{(2)}_{ab}$: Moch, Vermaseren, Vogt (2004)

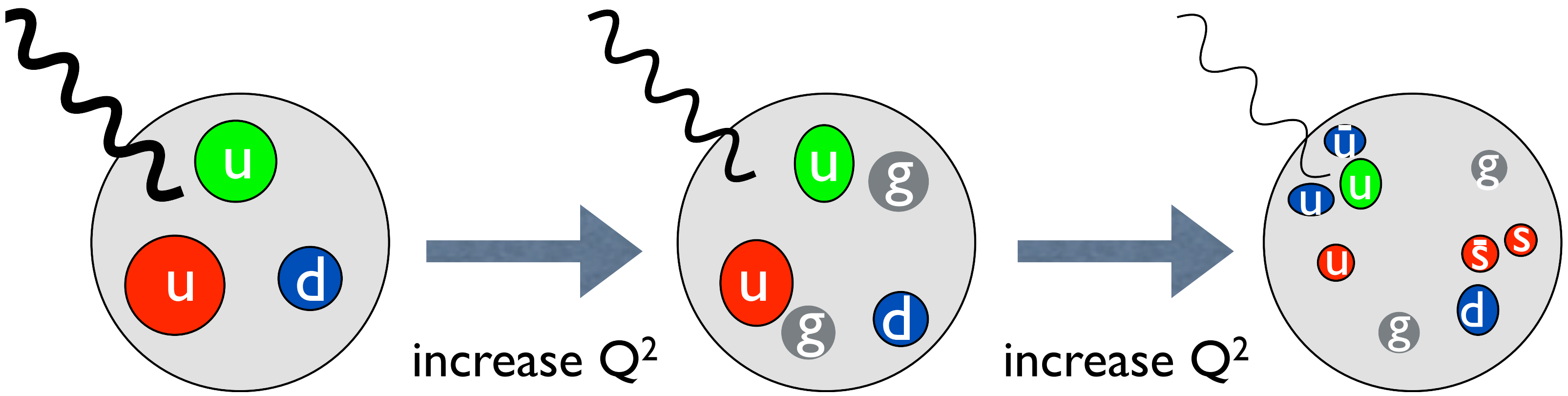
☛ $P^{(2)}_{ab}$: maybe hardest calculation ever performed in perturbative QCD

☛ Essential input for NNLO pdfs determination (state of the art today)

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



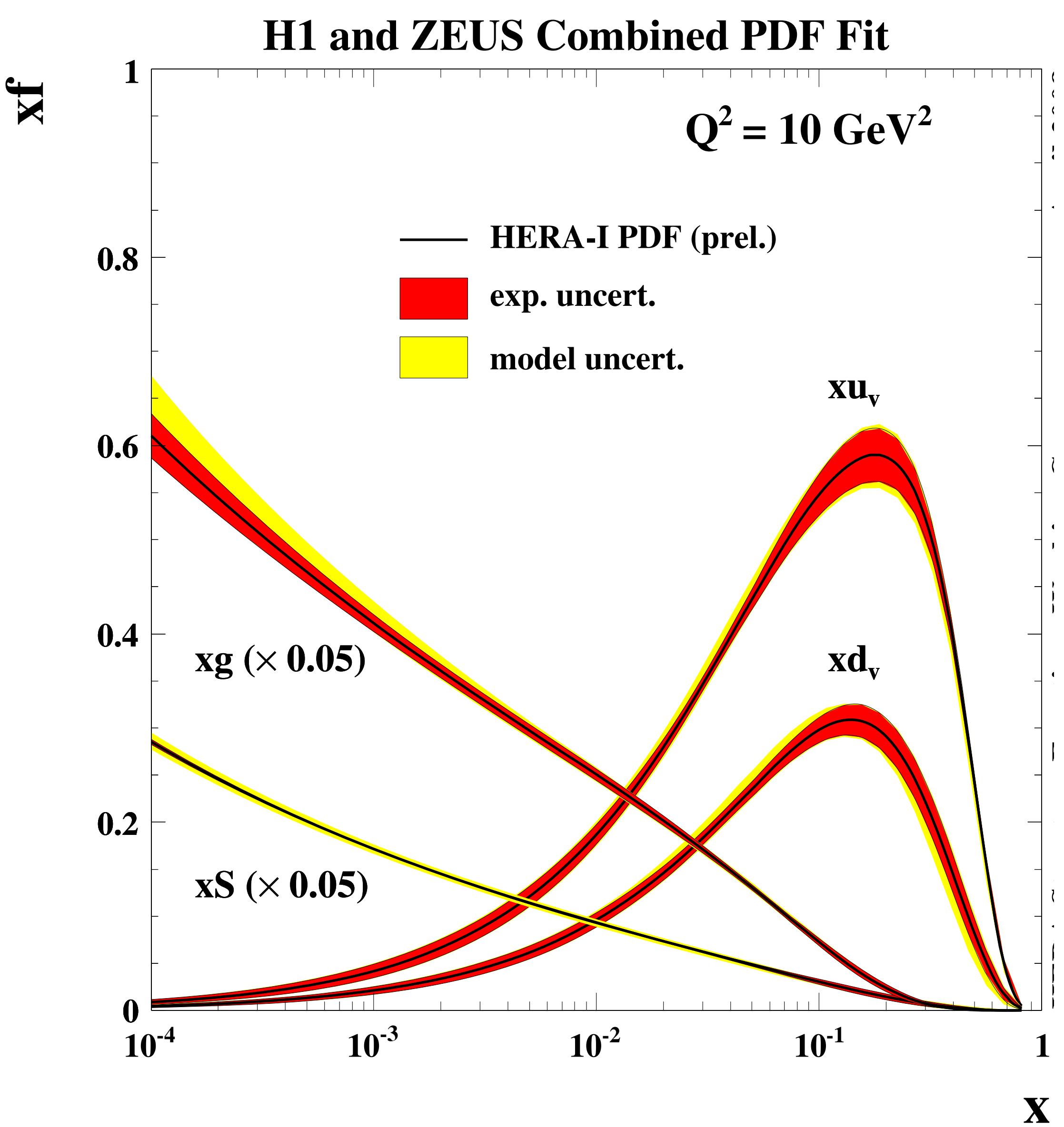
What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how one can access the gluon pdf: because of the coupled DGLAP evolution we can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs

The Hera PDF

<u>Typical features:</u>

- gluon distribution very large
- gluon and sea distributions grow at small x
- gluon dominates at small x
- valence distributions peak at x = 0.1 - 0.2
- largest uncertainties at very small or very large x

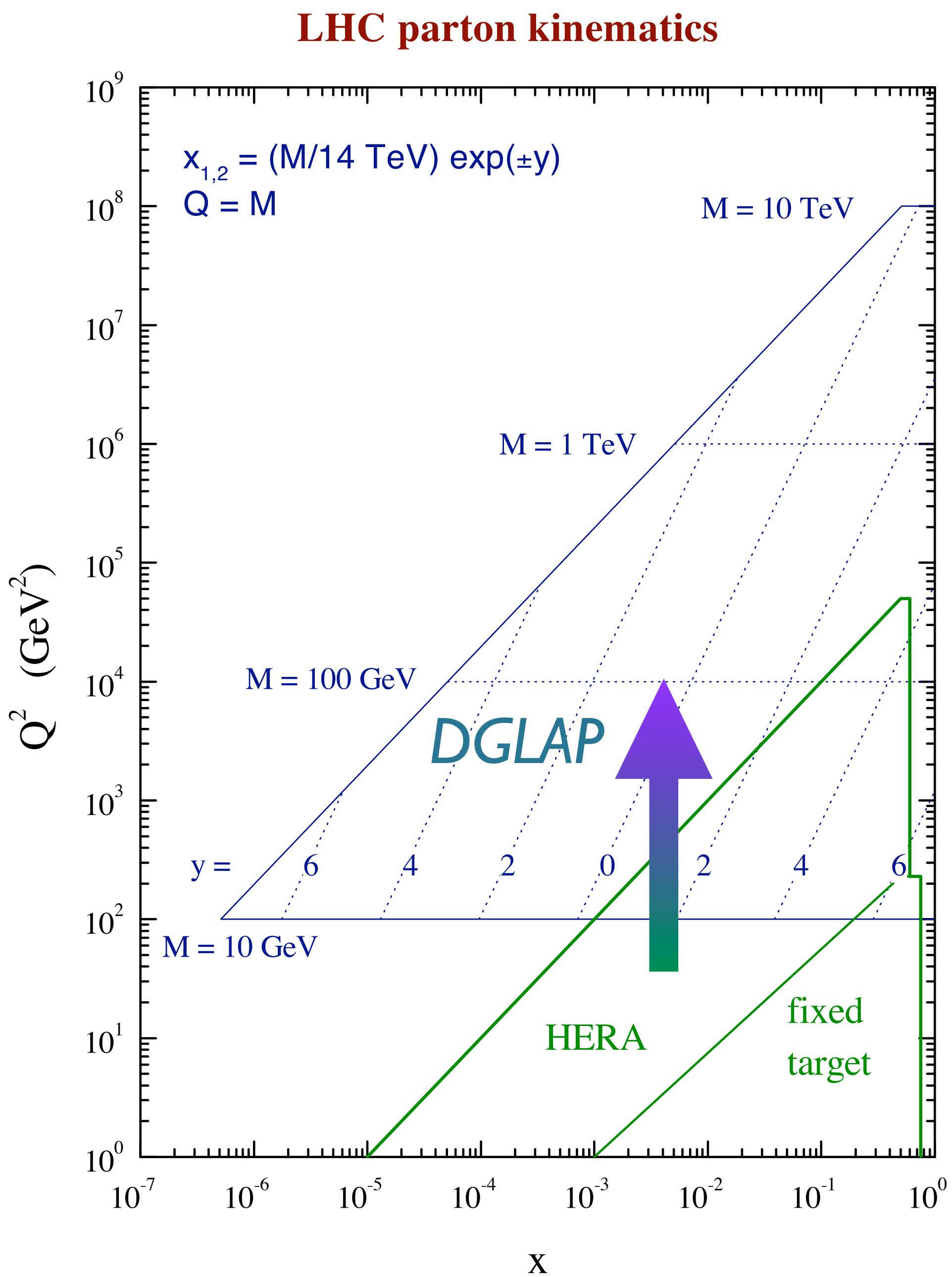


Crucial property: factorization!

Parton distributions extracted in DIS can be used at hadron colliders.
This assumption can be checked against data

Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2-evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



➠ *Hera: key and essential input to the LHC*

Parton densities: recent progress

Recent major progress:

- full NNLO evolution (previous approximate NNLO)
- more flexible parametrizations
- improved treatment of heavy flavors near the quark mass

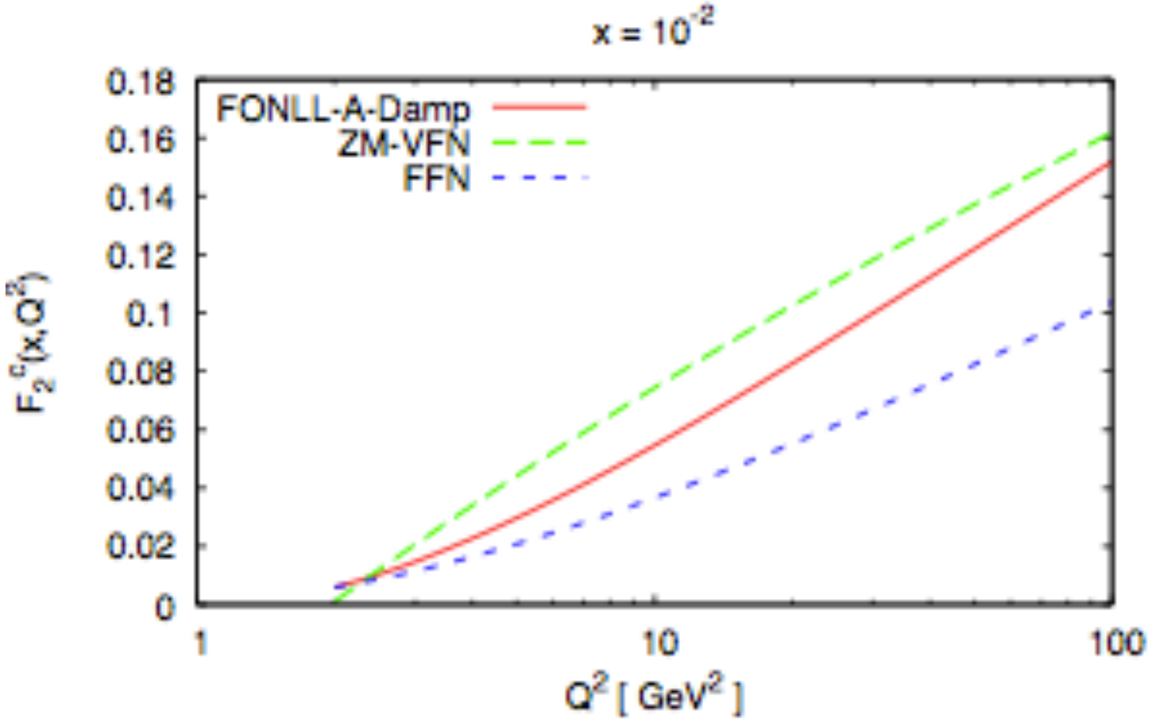
  [Numerically: e.g. (6-7)% effect on Drell-Yan at LHC]
- more systematic use of uncertainties/correlations (e.g. dynamic tolerance, combinations of PDF + α_s uncertainty)
- Neural Network (NN) PDFs

splitting functions at NNLO: Moch, Vermaseren, A. Vogt '04
+ much related theory progress '04 -'11

Parton densities: some open issues

- heavy quark treatment theoretically not 'clean' (various schemes, ad hoc procedures), but very important at the LHC

NN col '11



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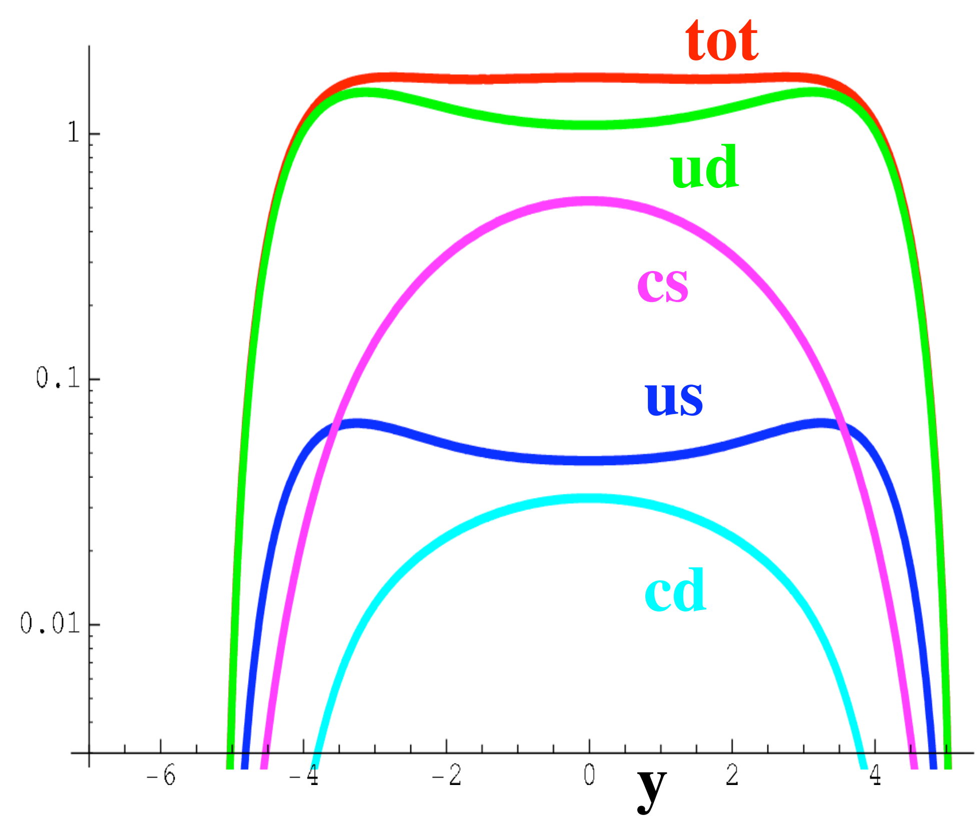
dσ/dy(W^+) at LHC



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- inconsistency between PDFs using different data sets / different heavy quark treatment

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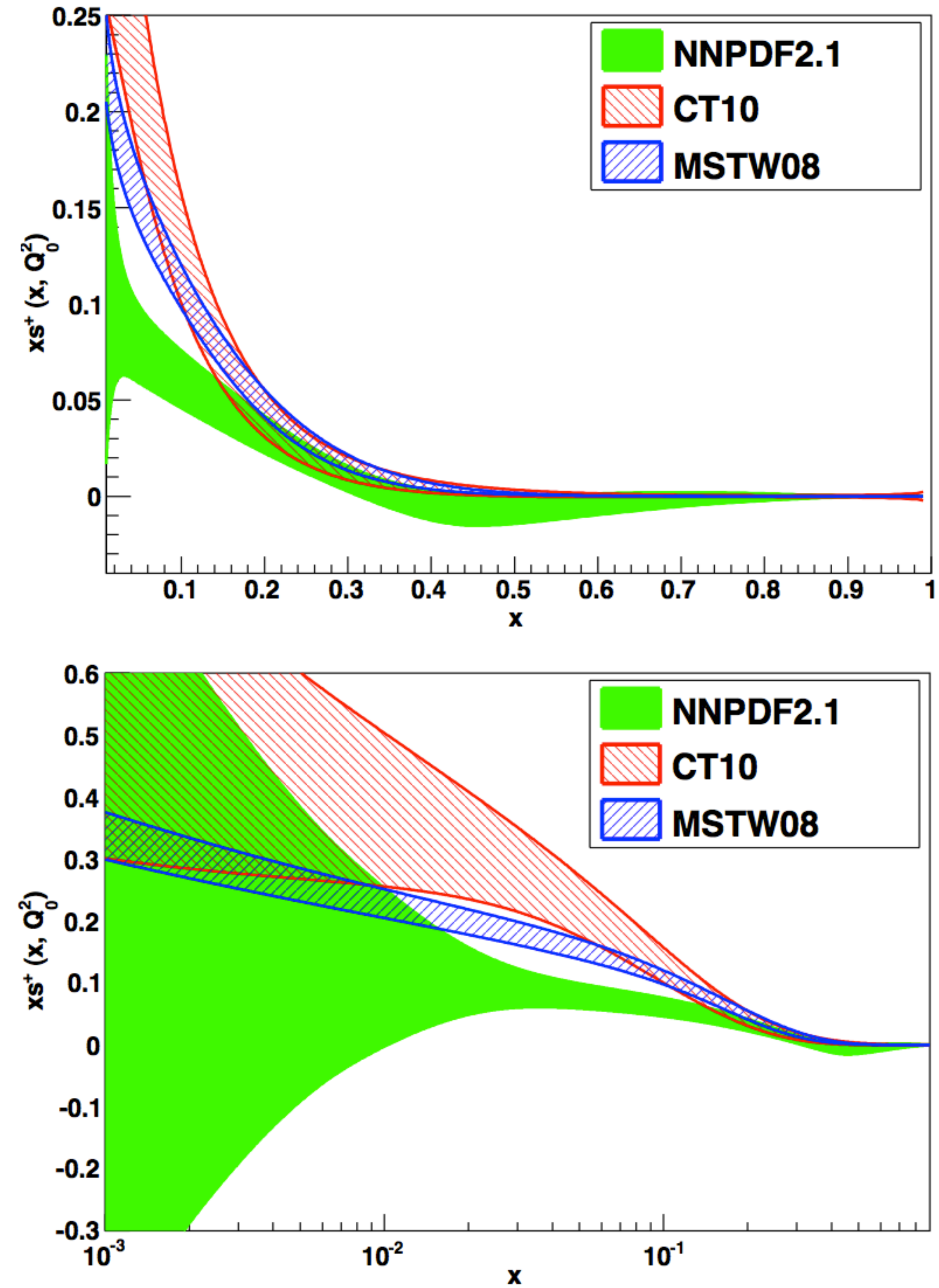
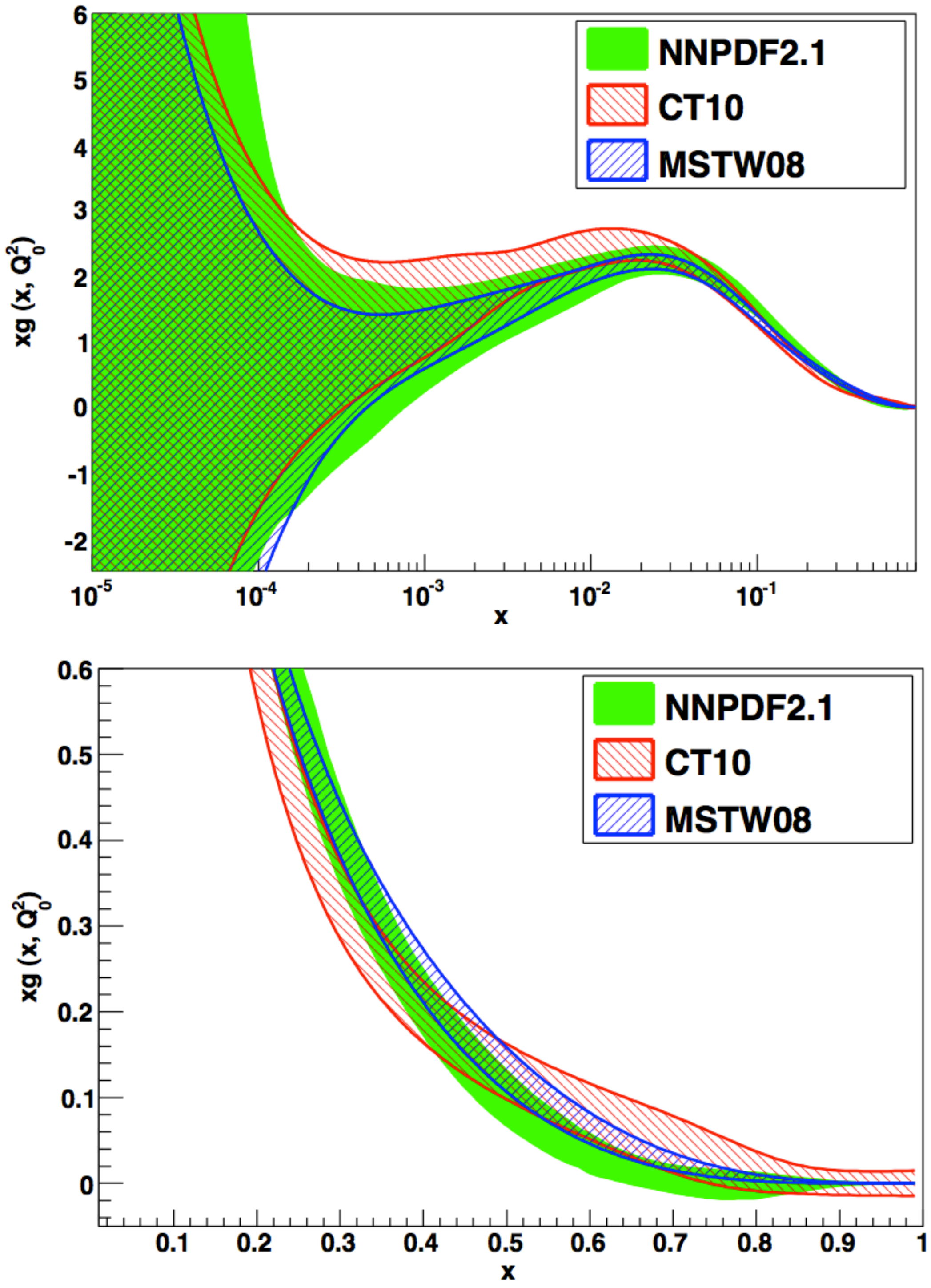


Parton densities: some open issues

- heavy quark treatment theoretically not ‘clean’ (various schemes, ad hoc procedures), but very important at the LHC
- inconsistency between PDFs using different data sets / different heavy quark treatment
- treatment of theory uncertainties (parameterizations, scheme for HQ, higher orders ...)

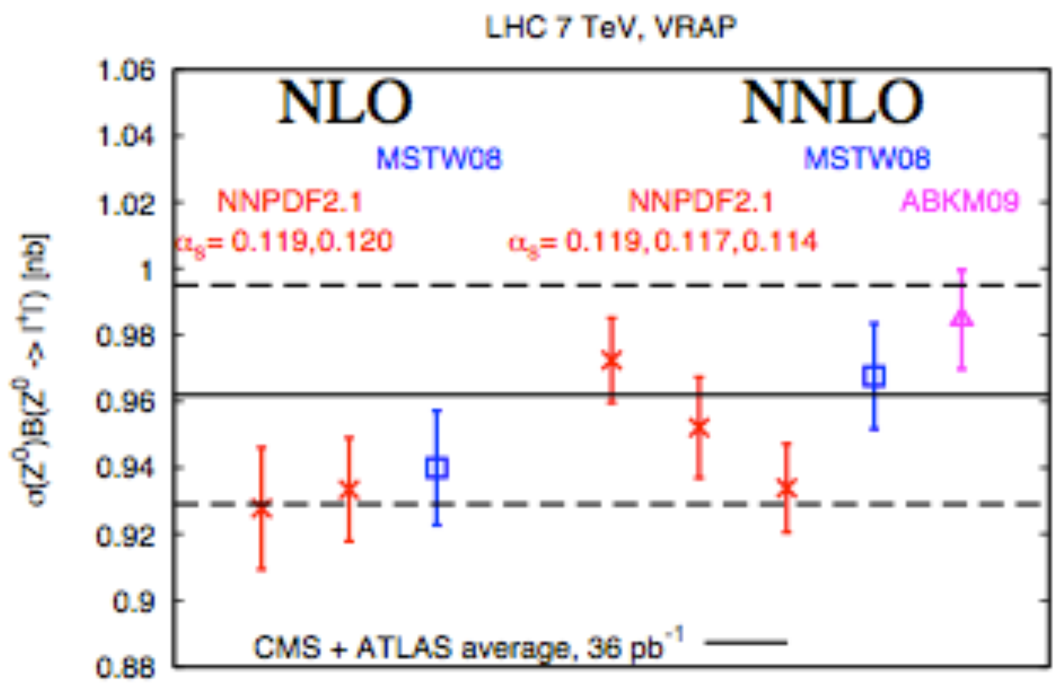
NN col ’11

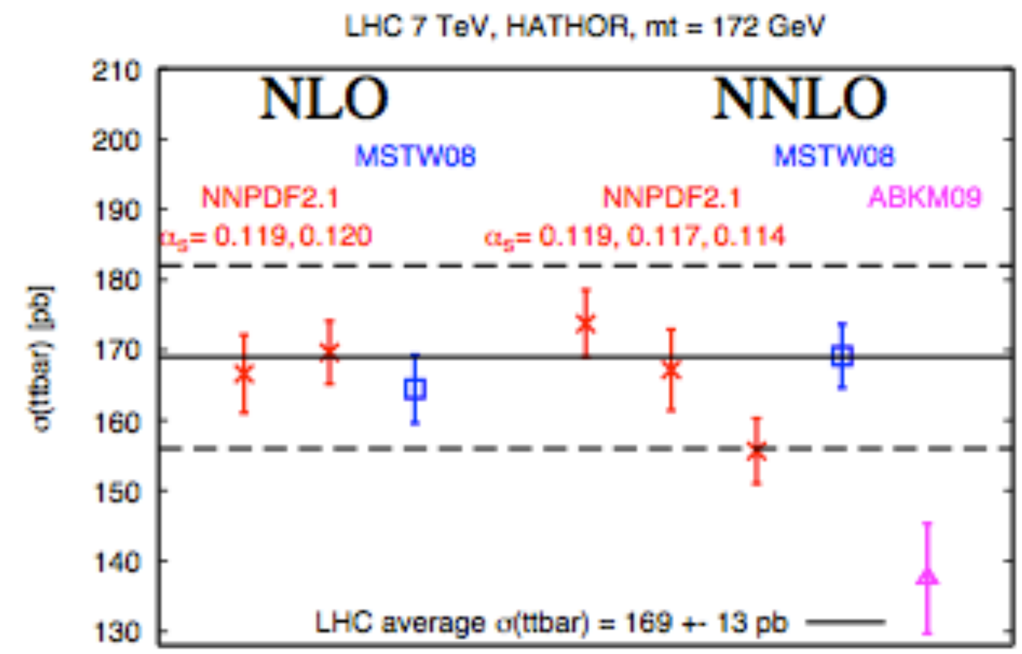


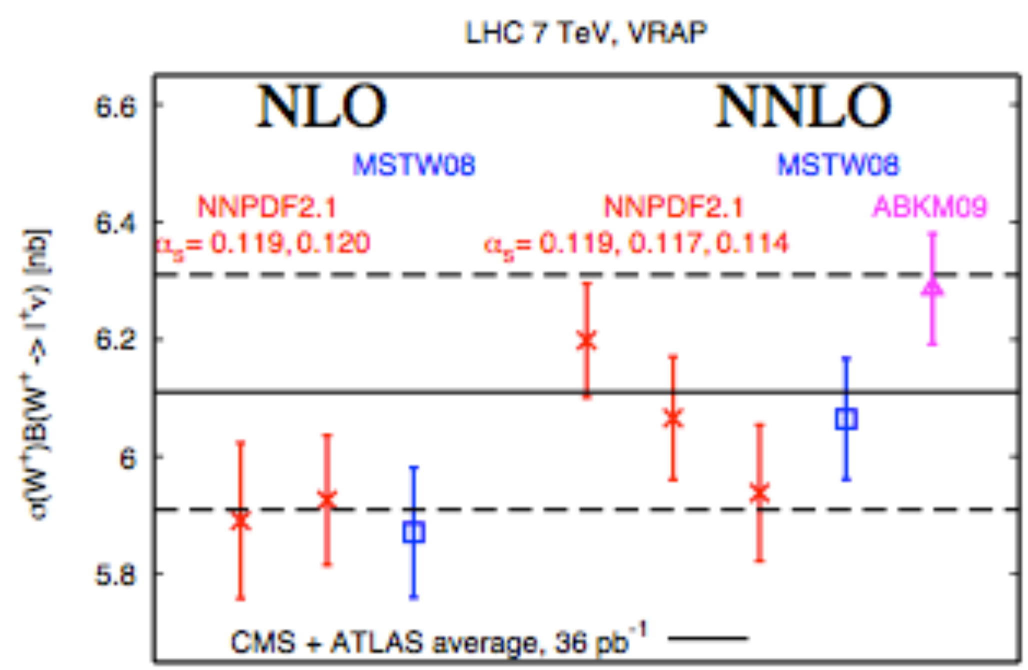
Parton densities: benchmark processes

Uncertainty from pdfs and α_s on benchmark processes

NN col. 1107.2652







Differences due to:

1) different data in fits
2) different methodology [parametrization, theory]
3) treatment of heavy quarks
4) different α_s

Intermediate recap.

- There are infrared and collinear divergences $\Rightarrow$ not all quantities can be computed in PT, only IRsafe ones
- Parton model: incoherent sum of all partonic cross-sections
- Sum rules (momentum, charge, flavor conservation)
- Determination of parton densities (electron & neutrino scattering in DIS, Drell-Yan ...)
- Radiative corrections: failure of parton model
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ measure gluon PDF
- Issues in today's determination of PDFs

Next: Perturbative calculations

Next, we will focus on perturbative calculations

- LO, NLO, NLO+MC, NNLO
- techniques, issues with divergences
- current status, sample results

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Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim \underset{\text{LO}}{A} + \underset{\text{NLO}}{B\alpha_s} + \underset{\text{NNLO}}{C\alpha_s^2} + \underset{\text{NNNLO}}{D\alpha_s^3} + \ldots$$

Perturbative calculations

- Perturbative calculations = fixed order expansion in the coupling constant, or more refined expansions that include terms to all orders
- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- Let's consider some examples

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

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$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu') = \alpha_s(\mu')^n A(p_i, \epsilon_i, \dots) = \alpha_s(\mu)^n \left(1 + n\, b_0\, \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \dots\right) A(p_i, \epsilon_i, \dots)$$

So the change of scale is a NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

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So the change of scale is a NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\rm LO}_{\rm njets}(\mu)}{\sigma^{\rm LO}_{\rm njets}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider an n-jet cross-section at NLO. At scale μ the result reads

$$\sigma^{\rm NLO}_{\rm njets}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n+1} \left(B(p_i, \epsilon_i, \dots) - n b_0 \ln \frac{\mu^2}{Q_0^2} \right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO.
- Scale dependence and normalization start being under control only at NLO, since a compensation mechanism kicks in
- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion
- Scale variation is conventionally used to estimate the theory uncertainty, but the validity of this procedure should not be overrated (see later)

Leading order: Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

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Bottlenecks

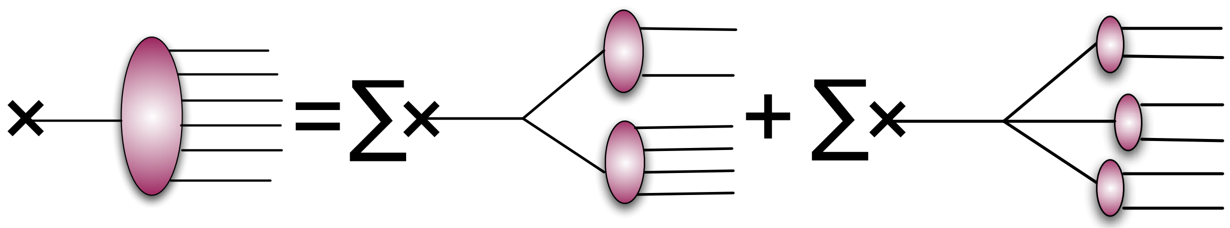
a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Techniques beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes **recursively** using off-shell currents



Berends, Giele '88

Techniques beyond Feynman diagrams

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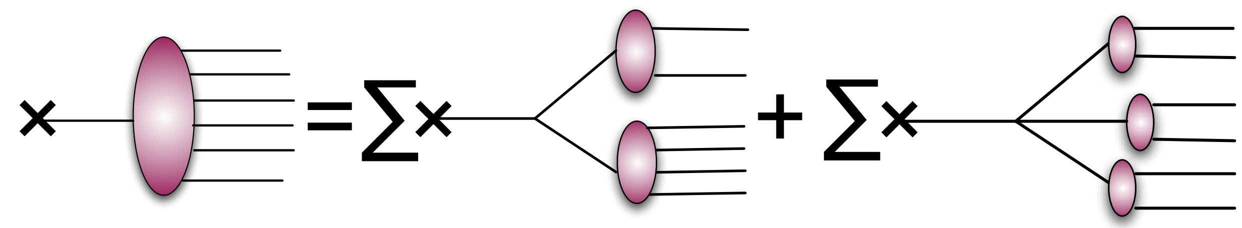
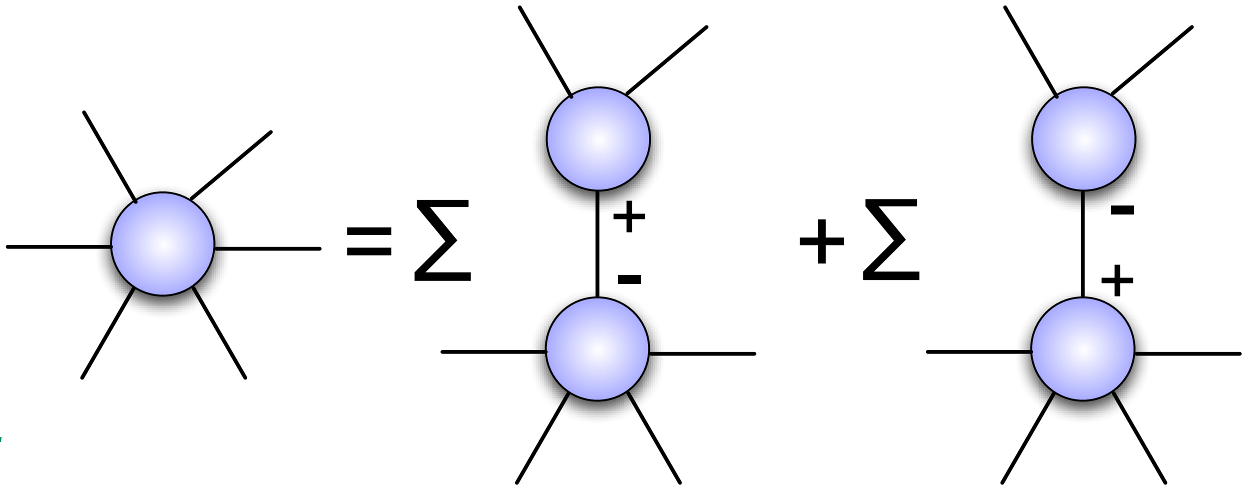


Berends, Giele '88

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)



Britto, Cachazo, Feng '04

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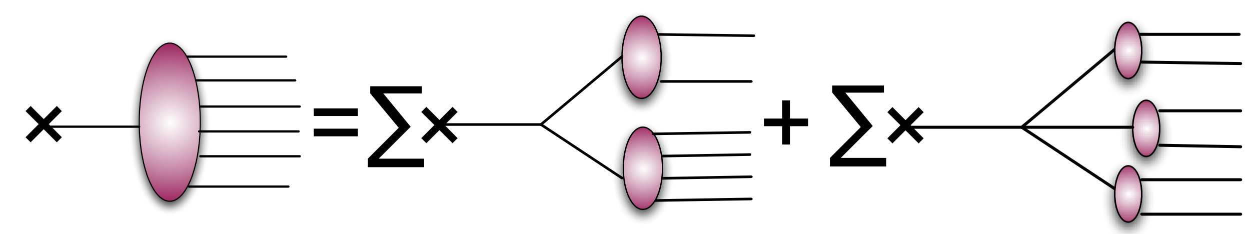

Berends, Giele '88

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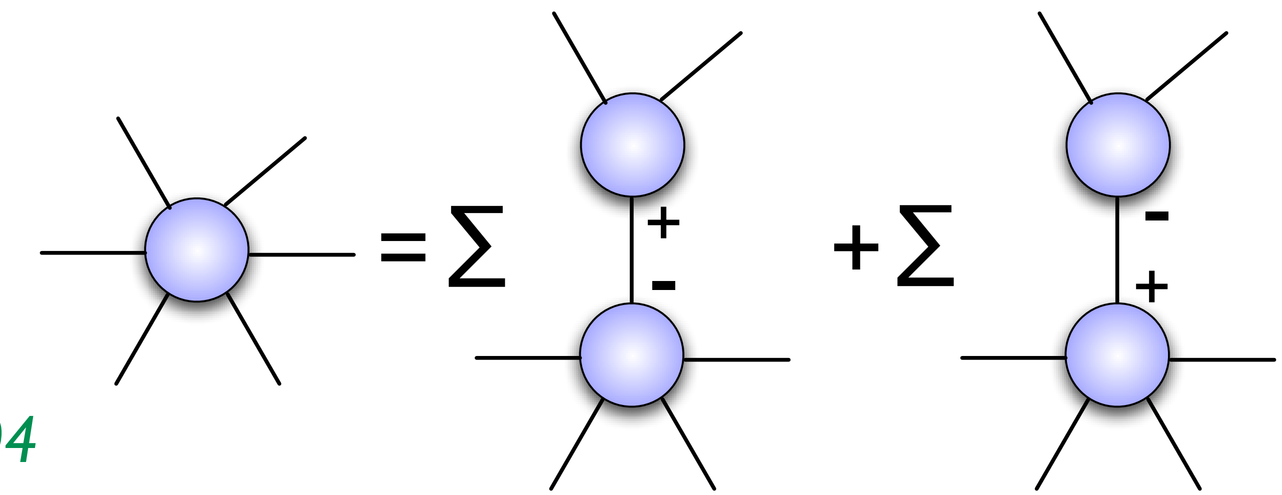
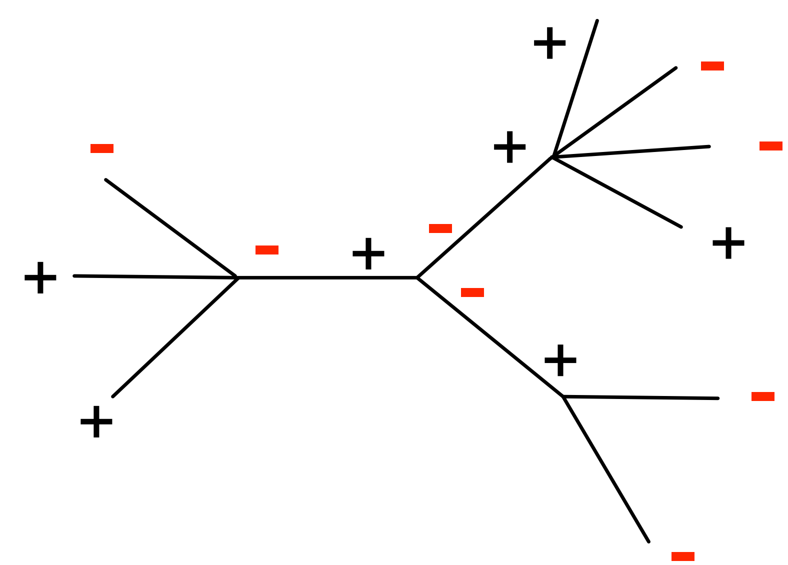


Britto, Cachazo, Feng '04

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Matrix element generators

Fully automated calculation of leading order cross-sections:

- generation of tree level matrix elements
  - Feynman diagrams [CompHEP/CalcHEP, Madgraph/Madevent, HELAS, Sherpa, ...]
  - Helicity amplitudes + off-shell Berends-Giele recursion [ALPHA/ALPGEN, Helac, Vecbos]
- phase space integration
- interface to parton showers (see later)

All these codes are currently used extensively in analysis of LHC data

Benefits and drawbacks of LO

Benefits of LO:

- fastest option; often the only one
- test quickly new ideas with fully exclusive description (new physics)
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

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Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalization
- poor control on shapes
- poor modeling of jets

Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$
Vary $\alpha_s(Q)$ by ±10% via change of Q ⇒ cross-section varies by ±40%

Next-to-leading order

<u>Benefits of next-to-leading order (NLO)</u>

- reduce dependence on unphysical scales (renormalization/ factoriaztion)
- establish normalization and shape of cross-sections
- small scale dependence at LO can be very misleading (see later), small dependence at NLO robust sign that PT is under control
- large NLO correction or large dependence at NLO robust sign that neglected other higher order are important
- through loop effects get indirect information about sectors not directly accessible

Concrete examples follow in few slides, first let's discuss briefly how one does NLO calculations

Ingredients at NLO

A full N-particle NLO calculation requires:

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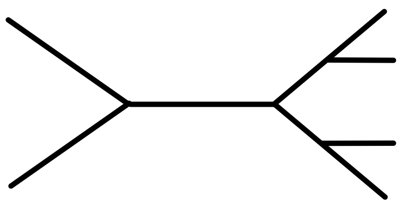

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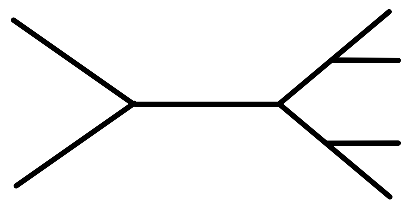
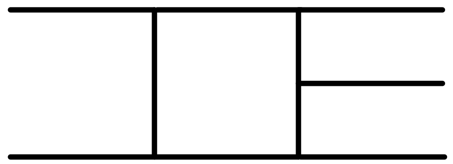


- virtual correction to N-leg process
  ➔ divergence from loop integration,
  use e.g. dimensional regularization



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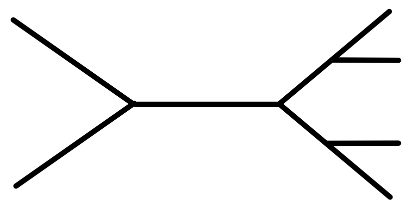

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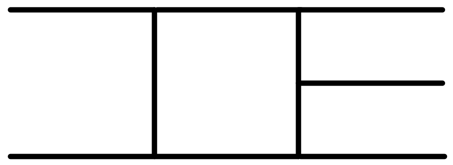

- set of subtraction terms to cancel divergences

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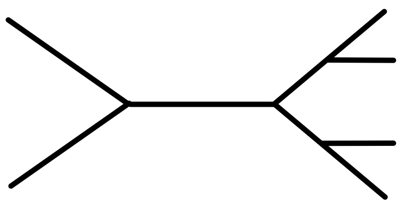

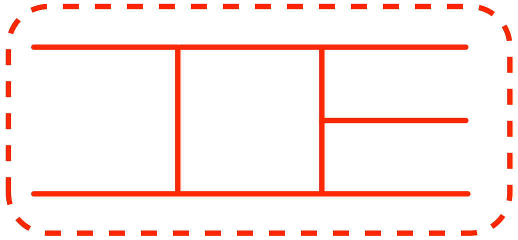

bottleneck

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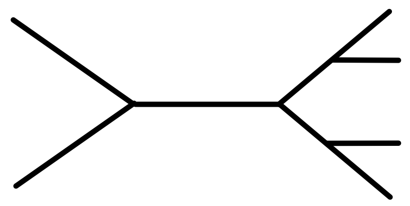

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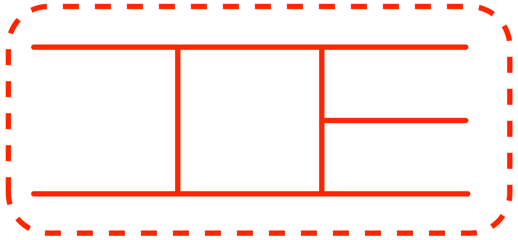

bottleneck

- ☑ set of subtraction terms to cancel divergences

We won't have time to do detailed NLO calculations, but let's look a bit more in detail at the issue of divergences/subtraction

Regularization in QCD

Regularization: a way to make intermediate divergent quantities meaningful

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Alternative regularization schemes: photon mass (EW), cut-offs, Pauli-Villard ...
Compared to those methods, dimensional regularization has the big virtue that it leaves the regularized theory Lorentz invariant, gauge invariant, unitary etc.

Renormalization schemes

Renormalization: a global redefinition of couplings and masses which absorbs all UV divergences. Several schemes are possible (MS, $\overline{\text{MS}}$, OS ...)

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- Infinite parts of renormalization constants must be the same, therefore renormalized constants must be related by a finite renormalization

$$\alpha_s^{\text{ren,B}} = \alpha_s^{\text{ren,A}} (1 + c_1 \alpha_s^{\text{ren,A}} + \dots)$$

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- Note that as a consequence of this, the first two coefficients of the β-function do not change under such a transformation, i.e. they are scheme independent. This it not true for higher order coefficients.

The $\overline{\text{MS}}$ scheme

- Today standard scheme is the modified minimal subtraction scheme, $\overline{\text{MS}}$
- After regularizing integrals via the dimensional regularization, poles appear always in the combination

$$\frac{1}{\epsilon} + \ln(4\pi) - \gamma_E$$

- Therefore in the $\overline{\text{MS}}$-scheme, instead of subtracting poles minimally (MS scheme), one always subtracts that combination, and replaces the bare coupling with the renormalized one

- It is then standard to quote the coupling and Λ_{QCD} in this scheme, the current value is

$$206\text{MeV} < \Lambda_{\overline{\text{MS}}}(5) < 231\text{MeV}$$

- Uncertainties in this quantity propagate in the QCD cross-sections

Subtraction and slicing methods

- Consider e.g. an n-jet cross-section with some arbitrary infrared safe jet definition. At NLO, two divergent integrals, but the sum is finite

$$\sigma^J_{\rm NLO} = \int_{n+1} d\sigma^J_{\rm R} + \int_n d\sigma^J_{\rm V}$$

- Since one integrates over a different number of particles in the final state, real and virtual need to be evaluated first, and combined then

- This means that one needs to find a way of removing divergences before evaluating the phase space integrals

- Two main techniques to do this
  - *phase space slicing* ⇒ obsolete because of practical/numerical issues
  - *subtraction method* ⇒ most used in recent applications

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1} |\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

where F^J is the arbitrary jet-definition

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- The matrix element has a non-integrable divergence

$$|\mathcal{M}_{n+1}|^2 = \frac{1}{x}\mathcal{M}(x)$$

  where x vanishes in the soft/collinear divergent region

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- IR divergences in the loop integration regularized by taking D=4-2ε

$$2\,\mathrm{Re}\{\mathcal{M}_V \cdot \mathcal{M}_0^*\} = \frac{1}{\epsilon}\mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma_{\mathrm{NLO}}^{J} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_{n+1}^{J}(x) + \frac{1}{\epsilon} \mathcal{V} F_n^{J}$$

Subtraction method

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$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

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- One can then add and subtract the analytically computed divergent part

$$\sigma_{\mathrm{NLO}}^{J} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_{n+1}^{J}(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_n^J + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_n^J + \frac{1}{\epsilon} \mathcal{V} F_n^J$$

Subtraction method

- This can be rewritten exactly as

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) \left(F^J_1(x) - \mathcal{V} F^J_0\right) + \mathcal{O}(1) \mathcal{V} F^J_0$$

$\Rightarrow$ Now both terms are finite and can be evaluated numerically

- Subtracted cross-section must be calculated separately for each process (but mostly automated now). It must be valid everywhere in phase space
- Systematized in the seminal papers of Catani-Seymour (dipole subtraction, '96) and Frixione-Kunszt-Signer (FKS method, '96)
- Subtraction used in all recent NLO applications and public codes (Event2, Disent, MCFM, NLOjet++, ...)

Approaches to virtual (loop) part of NLO

Two complementary approaches:

- Numerical/traditional Feynman diagram methods:
  use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

  Bottleneck:
  factorial growth, 2 → 4 doable, very difficult to go beyond

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- Numerical/traditional Feynman diagram methods:
  use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

  Bottleneck:
  factorial growth, 2 → 4 doable, very difficult to go beyond

- Analytical approaches:
  improve understanding of field theory [e.g. unitarity, onshell methods, OPP, recursion relations, twistor methods, ...]

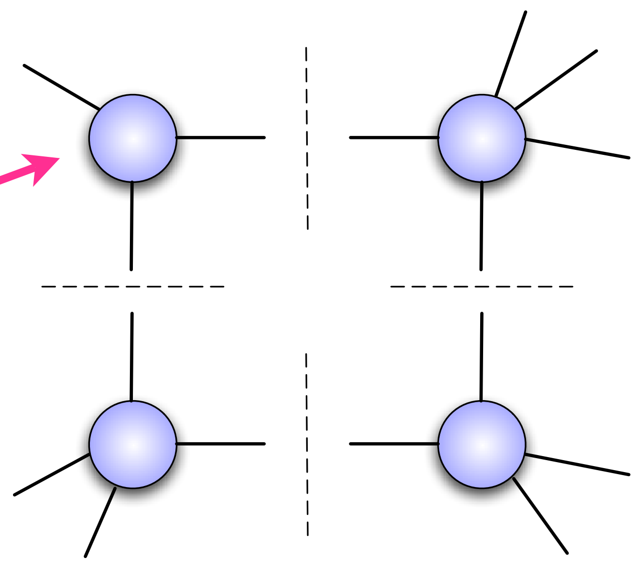
  Bottleneck:
  still lack of complete automation, fermions in general more difficult

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."

NB: non-zero because cut gives complex momenta



Britto, Cachazo, Feng '04

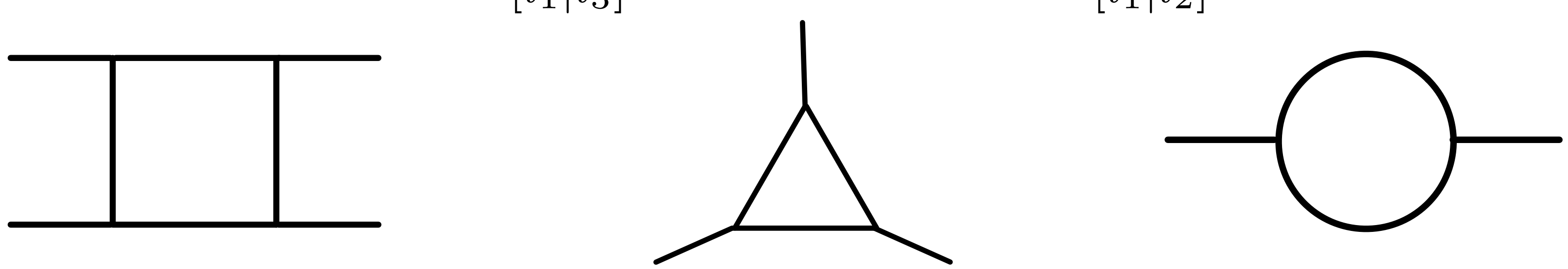
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from $D=4-2\varepsilon$ not 4, computed separately

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4}\ I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3}\ I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2}\ I^{(D)}_{i_1 i_2} \right)$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

Status in 2005

Table 42: The LHC "priority" wishlist for which a NLO computation seems now feasible.

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

The QCD, EW & Higgs Working group report hep-ph/0604120

The 2007 update

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6, 7]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to V\,V\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to V\,V$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

with Feynman diagrams

with Feynman diagrams or unitarity/onshell methods

The NLO multi-leg Working group report 0803.0494

Status of NLO today

Status of NLO:

☑ 2 → 2: all known (or easy) in SM and beyond

☑ 2 → 3: essentially all SM processes known

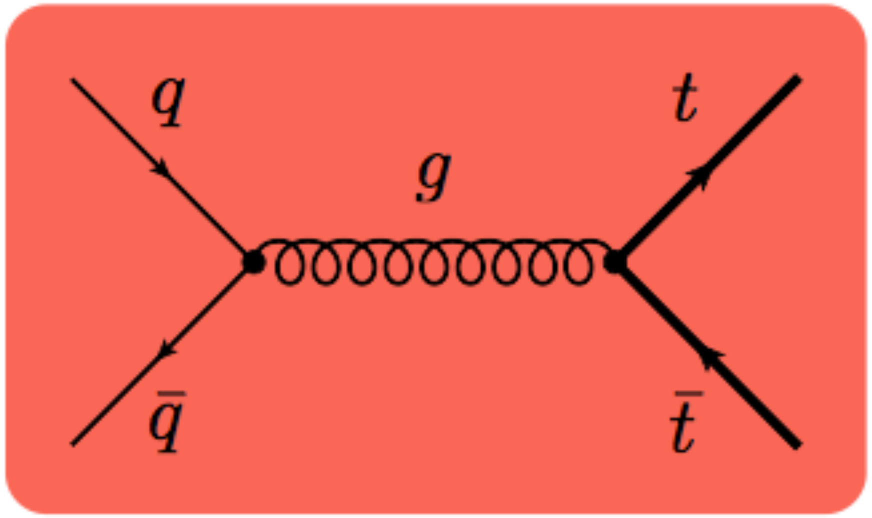
[but: often do not include decays, codes private]

☑ 2 → 4: a number of calculations performed in the last 1- or 2 years [W/Z+3jets, WW+2jets, WWbb, tt+2jets, ttbb, bbbb]. Calculations done using different techniques

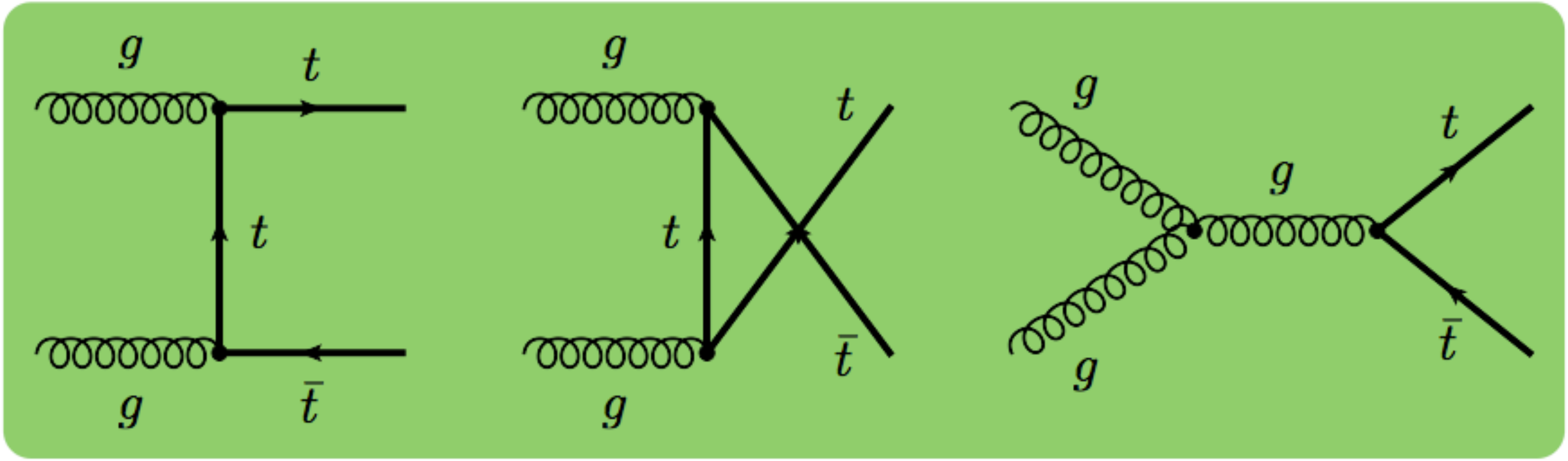
☐ 2 → 5: only dominant corrections for two processes [W/Z+4jets]

Top-pair production

Basic production mechanisms: initiated from quarks or gluons



What is the dominant production mechanism, at the Tevatron / LHC ? [And why ?]



Top-pair production: Tevatron

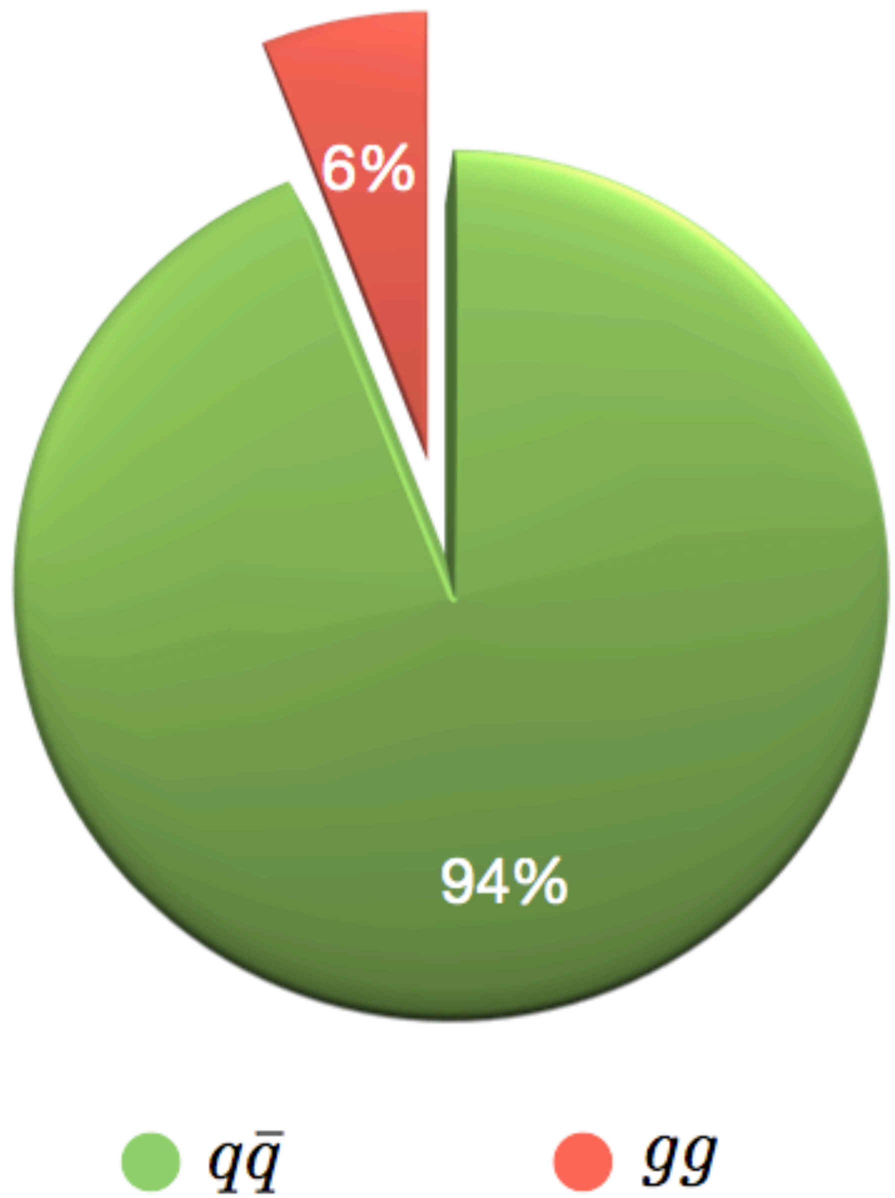
Running the program MCFM gives

```
Value of final lord integral is      9334.461 +/-      3.530 fb

 Total number of shots          :        200000
 Total no. failing cuts         :             0
 Number failing jet cuts        :             0
 Number failing process cuts :                0

   Jet efficiency : 100.00%
   Cut efficiency : 100.00%
 Total efficiency : 100.00%

 Contribution from parton sub-processes:
 ---------------------------------------
       GG     |           563.36203    6.04%
       GQ     |             0.00000    0.00%
       GQB    |             0.00000    0.00%
       QG     |             0.00000    0.00%
       QBG    |             0.00000    0.00%
       QQ     |             0.00000    0.00%
       QBQB   |             0.00000    0.00%
       QQB    |          8723.36136   93.45%
       QBQ    |            47.73759    0.51%
 ---------------------------------------
```



Top-pair production: pp @ 1.96 TeV

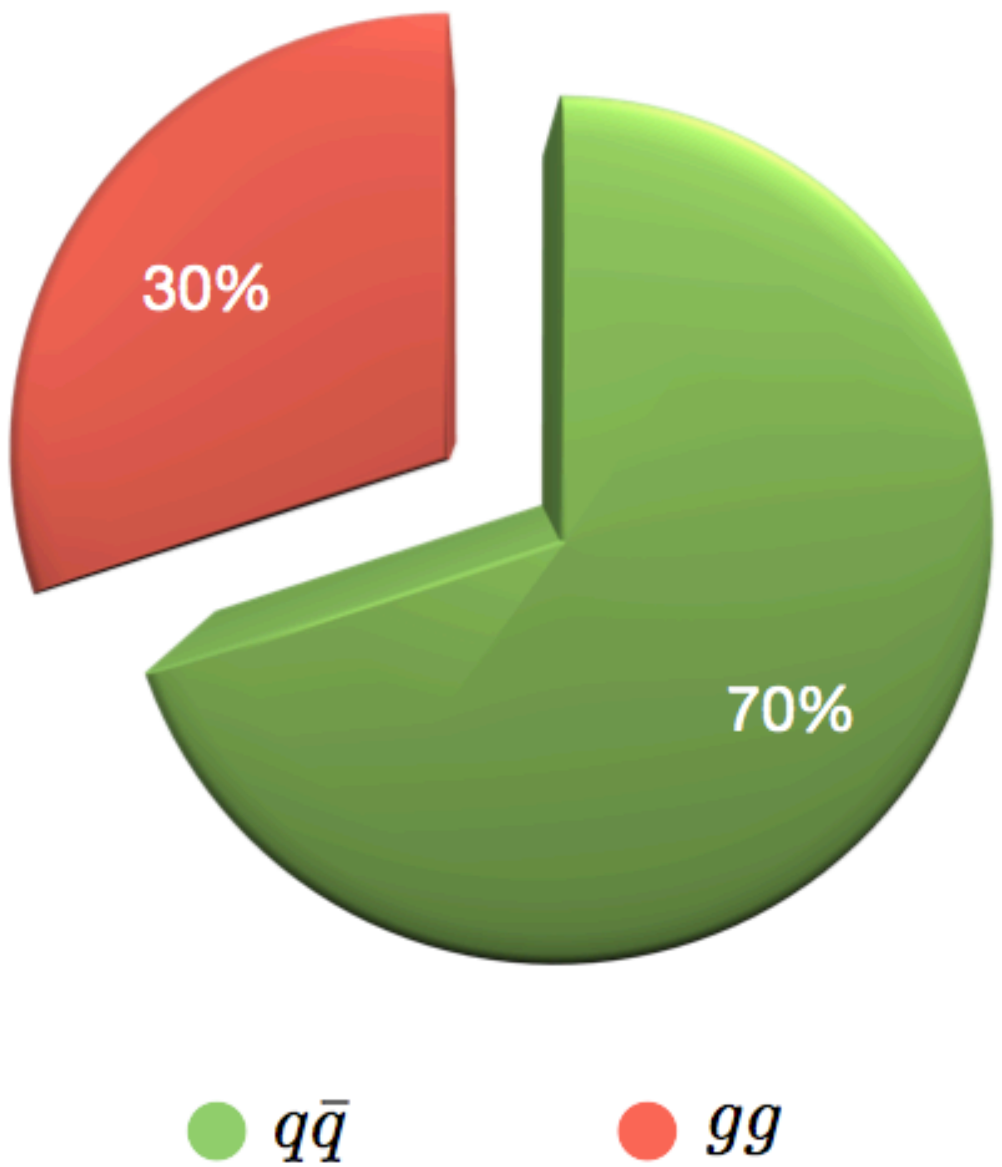
Running the program MCFM gives

```
Value of final lord integral is     1889.320 +/-     0.723 fb

 Total number of shots          :       200000
 Total no. failing cuts         :            0
 Number failing jet cuts        :            0
 Number failing process cuts :               0

   Jet efficiency : 100.00%
   Cut efficiency : 100.00%
 Total efficiency : 100.00%

 Contribution from parton sub-processes:
 ---------------------------------------
       GG     |         563.26857   29.81%
       GQ     |           0.00000    0.00%
       GQB    |           0.00000    0.00%
       QG     |           0.00000    0.00%
       QBG    |           0.00000    0.00%
       QQ     |           0.00000    0.00%
       QBQB   |           0.00000    0.00%
       QQB    |         662.81972   35.08%
       QBQ    |         663.23143   35.10%
 ---------------------------------------
```



Top-pair production: LHC

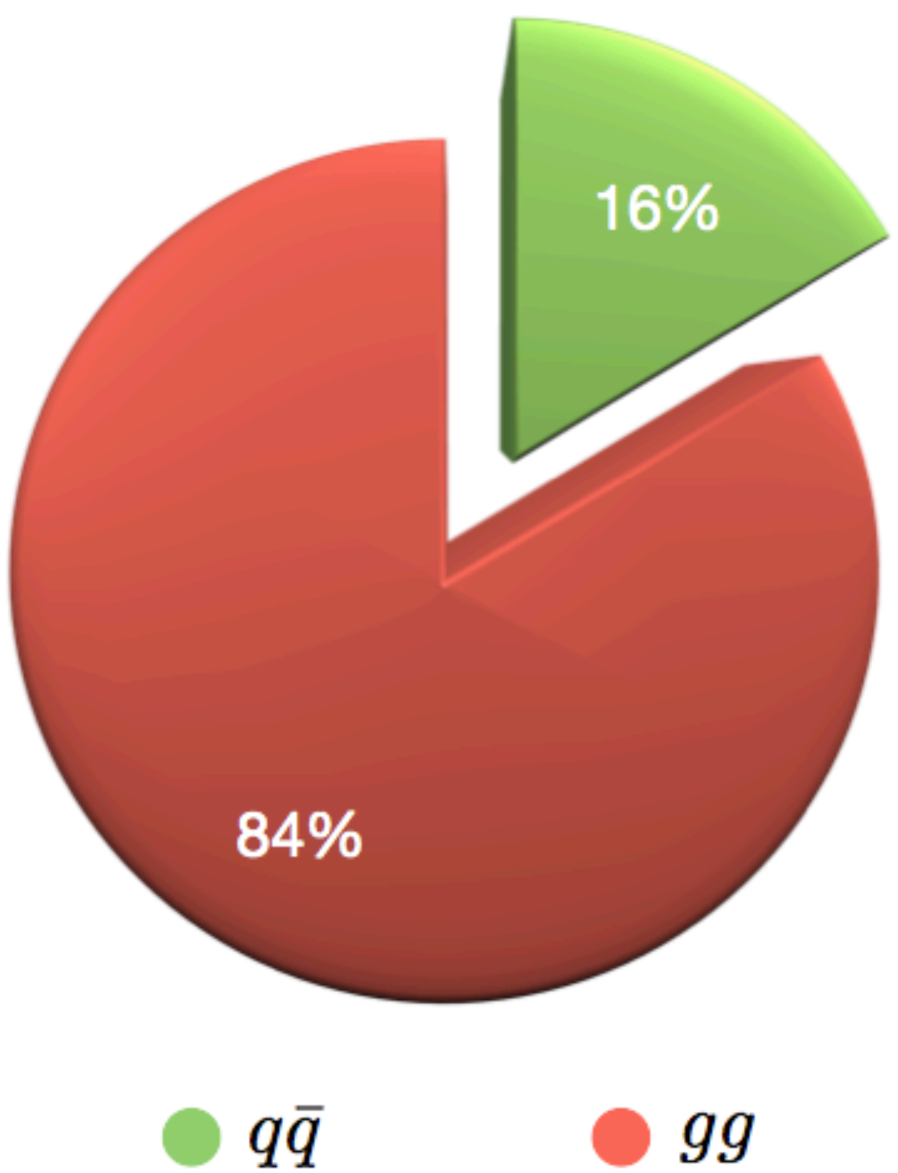
Running the program MCFM gives

```
Value of final lord integral is   373635.066 +/-   148.259 fb

 Total number of shots          :       200000
 Total no. failing cuts         :            0
 Number failing jet cuts        :            0
 Number failing process cuts    :            0

   Jet efficiency : 100.00%
   Cut efficiency : 100.00%
 Total efficiency : 100.00%

 Contribution from parton sub-processes:
 ---------------------------------------
      GG     |      312453.03253   83.63%
      GQ     |           0.00000    0.00%
      GQB    |           0.00000    0.00%
      QG     |           0.00000    0.00%
      QBG    |           0.00000    0.00%
      QQ     |           0.00000    0.00%
      QBQB   |           0.00000    0.00%
      QQB    |       30598.98764    8.19%
      QBQ    |       30583.04606    8.19%
 ---------------------------------------
```



Top-asymmetry

At the Tevatron, one interesting top measurement is its asymmetry

$$A_{fb} = \frac{N_{\rm top}(\eta > 0) - N_{\rm top}(\eta < 0)}{N_{\rm top}(\eta > 0) + N_{\rm top}(\eta < 0)}$$

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At $O(\alpha_s^3)$ the asymmetry is non-zero, an NLO calculation gives

$$A_{fb}^{\mathrm{NLO}} = 0.050 \pm 0.015$$

Kuehn et al. '99

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At $O(\alpha_s^3)$ the asymmetry is non-zero, an NLO calculation gives

$$A_{fb}^{\rm NLO} = 0.050 \pm 0.015$$

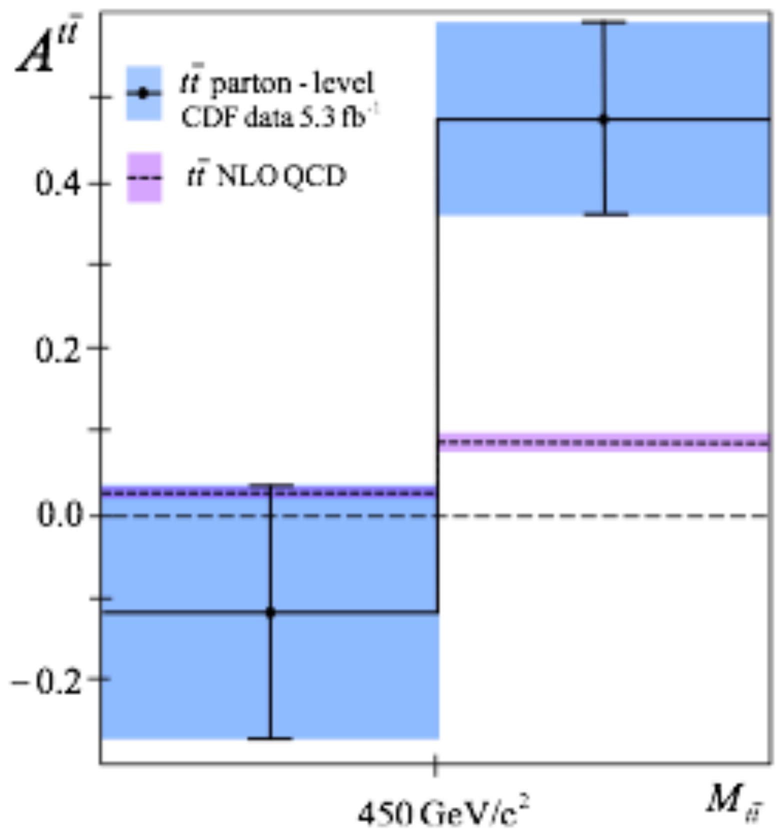
Kuehn et al. '99

But CDF & D0 measurements give

$$A_{fb}^{exp.} = 0.193 \pm 0.065\,(stat.) \pm 0.024\,(syst.)$$

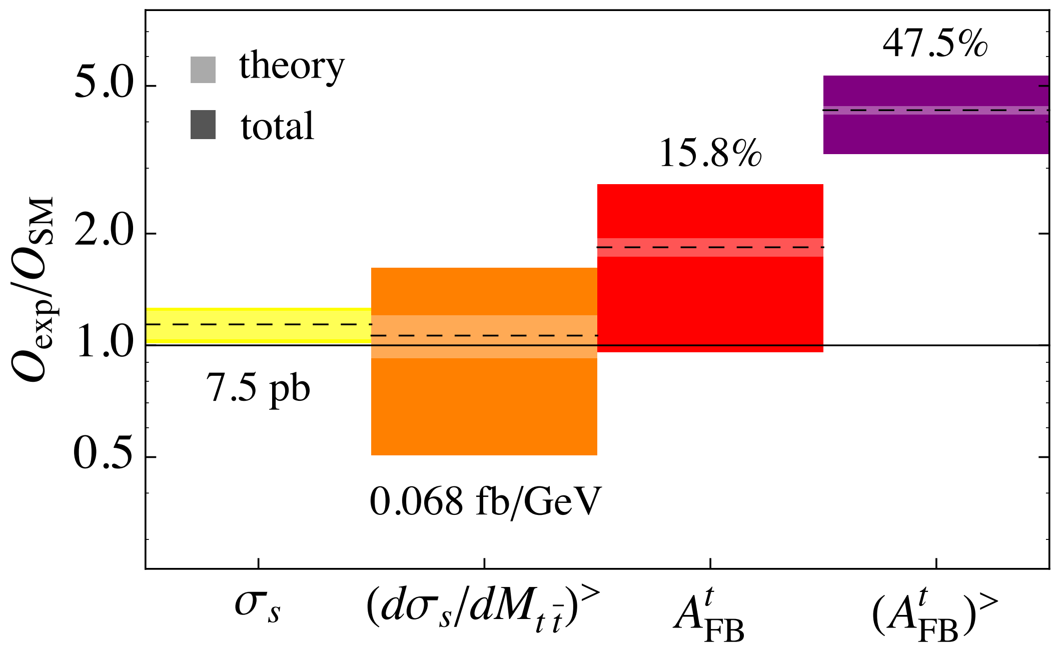
⇒ more than 2-sigma deviation from NLO

Top-asymmetry: recent update



CDF 1101.0034

Tension between symmetric and asymmetric cross-section



2.7σ / 4.2σ away from the NLO+NNLL theory. Seen both by CDF and D0, CDF effect enhanced at large M_{tt}, also in dilepton channel

Asymmetry is 0 at LO, but theoretical arguments and partial higher orders suggest that NLO is robust under higher-order corrections

Almeida et al. 0805.1885; Melnikov and Schulze 1004.3284; Ahrens et al. 1106.6051 ...

Various new models try to explain data, but difficult to preserve good agreement with symmetric cross-section, like-sign top decays, ...

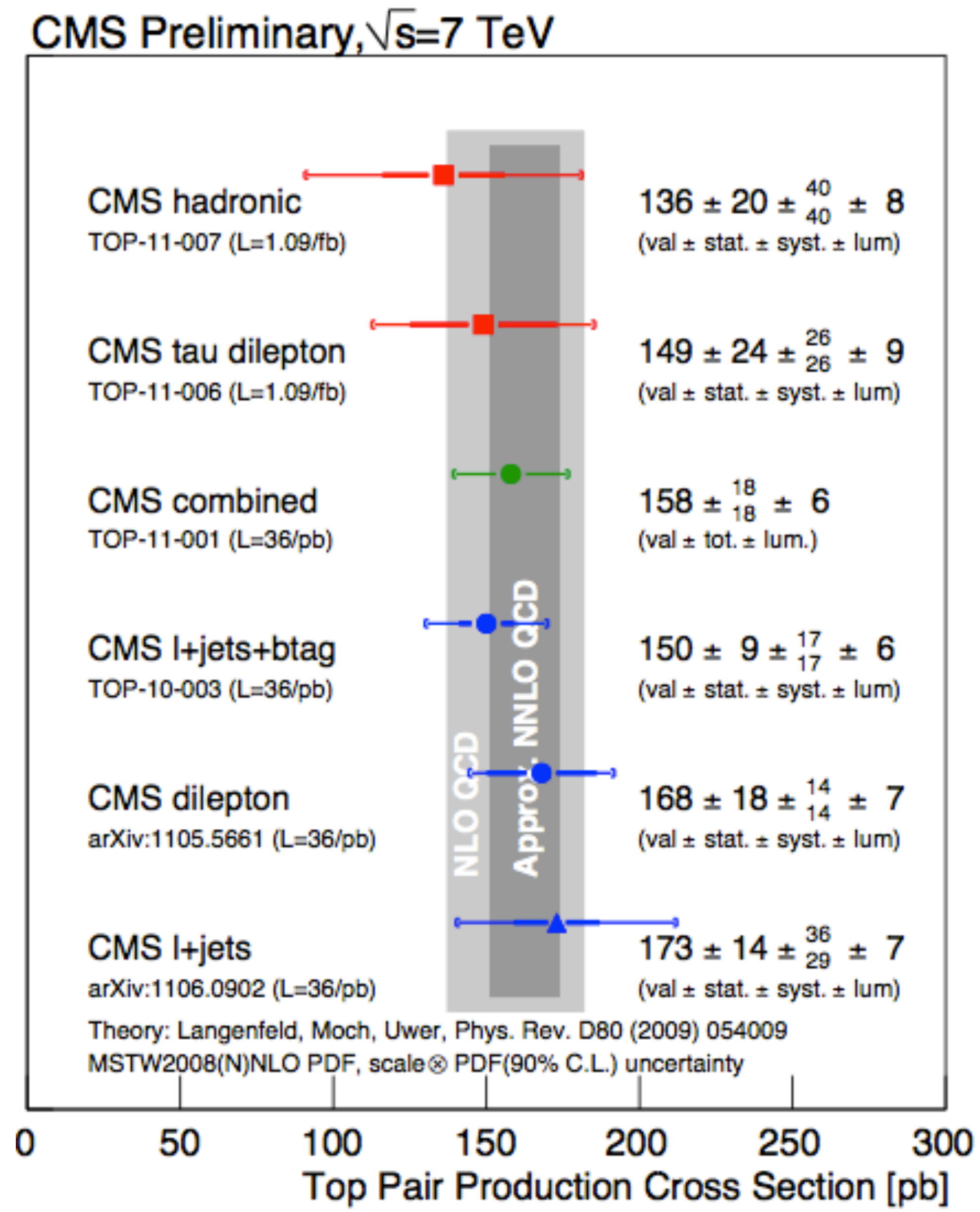
Top at the LHC

Large Yukawa coupling and prominent decay product in many new-physics models. The place where new physics will show up?

Good agreement between LHC data and NLO (and approx. NNLO) QCD

The frontier of NNLO

[...]



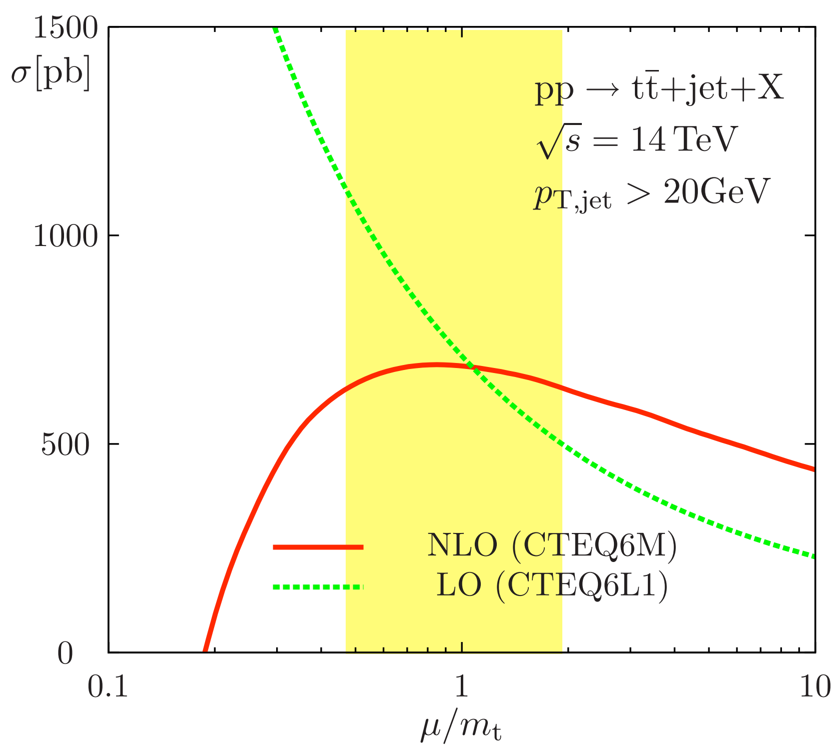
<u>Motivation for NNLO</u>

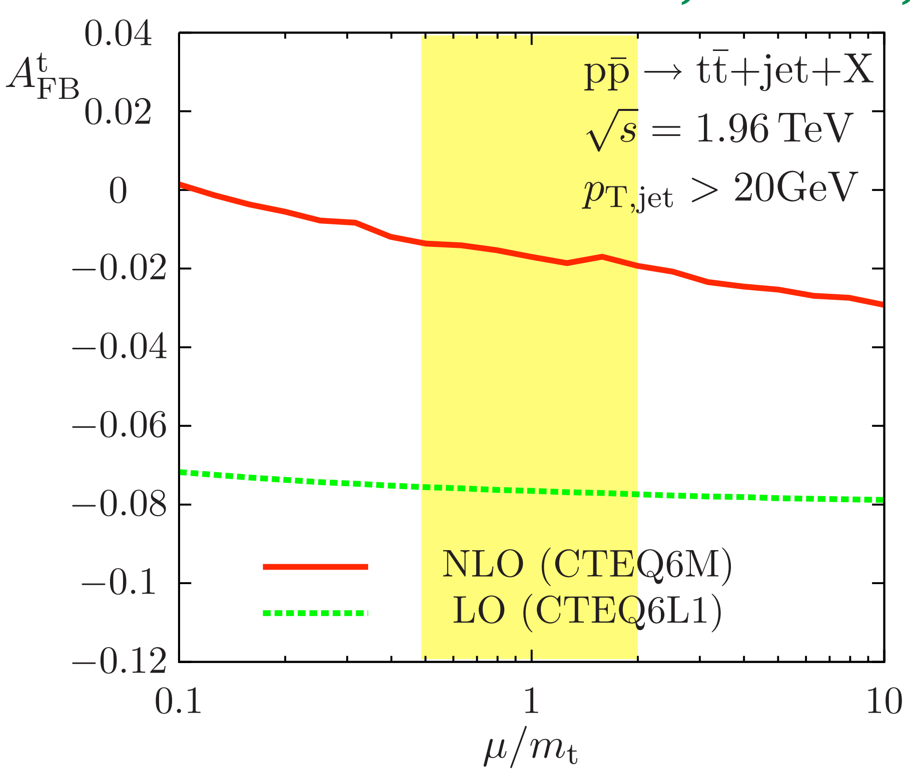
- constrain gluon pdf
- top mass from cross-section
- top FB asymmetry

tt+1jet

Calculation done with Feynman diagrams

Dittmaier, Kallweit, Uwer '07-'08

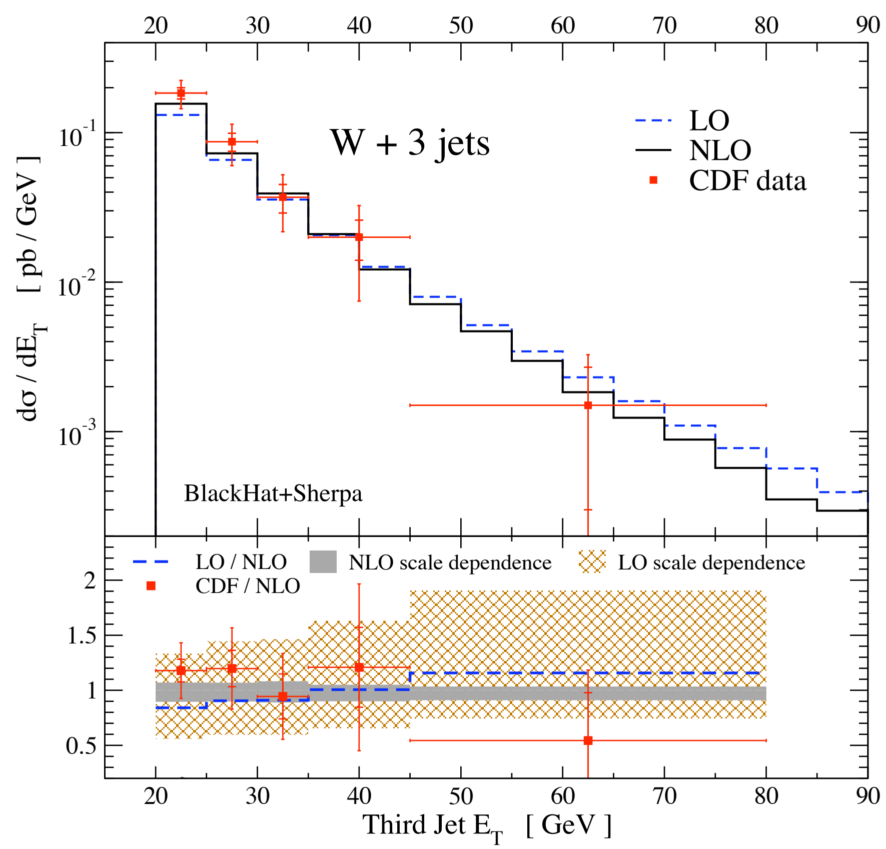




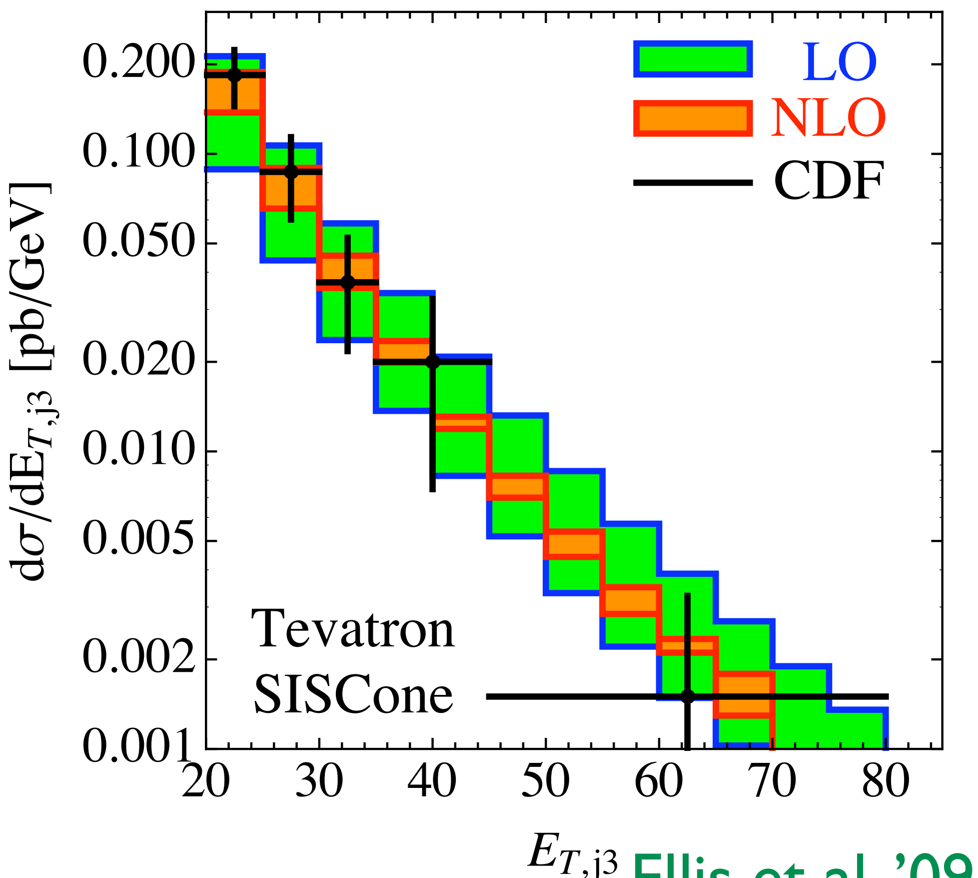
- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- essential ingredient of NNLO tt production (hot topic)

W + 3jets

Measured at the Tevatron + of primary importance at the LHC: background to model- independent new physics searches using jets + MET



Berger et al. '09



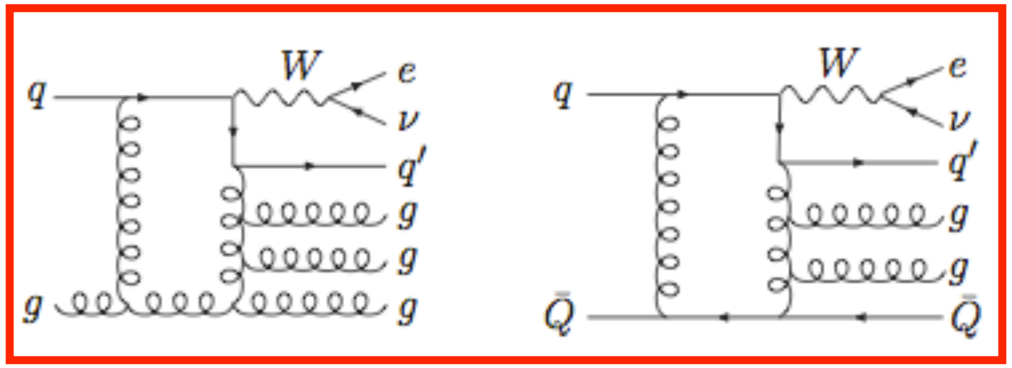
Ellis et al. '09

☺ Small K=1.0-1.1, reduced uncertainty: 50% (LO) → 10% (NLO)

☺ First applications of new techniques to 2 → 4 LHC processes

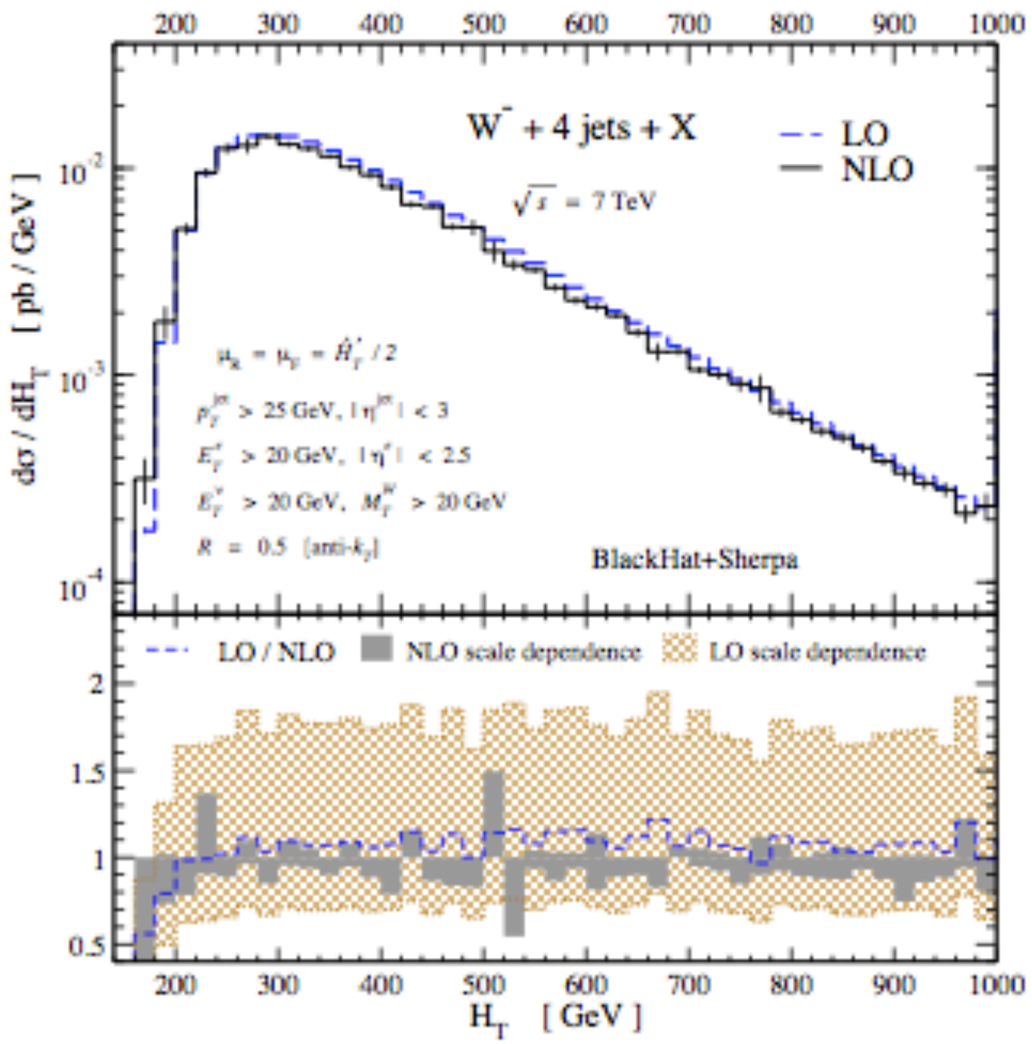
W + 4 jets at NLO

Sample diagrams*



- first pp → 5
- expected reduction of theoretical uncertainties
- key to top physics analyses: main background to tt in semi-leptonic channel

Berger et al.'10



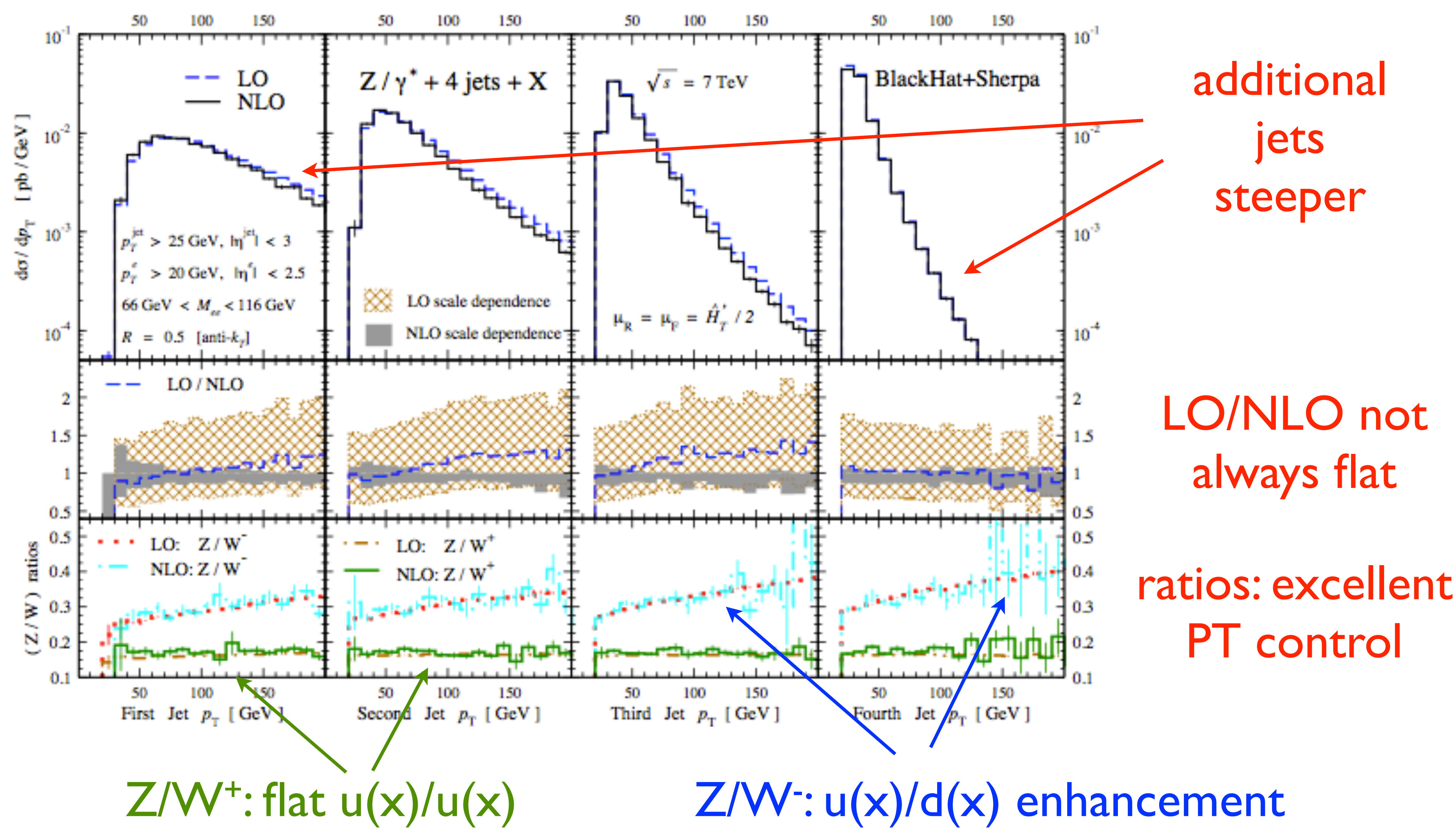
$$H_T = \sum_j p_{T,j} + p_{T,e} + p_{T,miss}$$

*Leading color calculation (OK to within 3% for lower multiplicities); missing W + 6q channels (also very small)

Z + 4 jets at NLO

4 jets + MET: important background to SUSY searches

Ita et al. '11



General NLO features?

Process	Typical scales μ_0	μ_1	Tevatron K-factor $\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	LHC K-factor $\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$
W	m_W	$2m_W$	1.33	1.31	1.21	1.15	1.05	1.15
W+1jet	m_W	p_T^{jet}	1.42	1.20	1.43	1.21	1.32	1.42
W+2jets	m_W	p_T^{jet}	1.16	0.91	1.29	0.89	0.88	1.10
WW+jet	m_W	$2m_W$	1.19	1.37	1.26	1.33	1.40	1.42
$t\bar{t}$	m_t	$2m_t$	1.08	1.31	1.24	1.40	1.59	1.48
$t\bar{t}$+1jet	m_t	$2m_t$	1.13	1.43	1.37	0.97	1.29	1.10
$b\bar{b}$	m_b	$2m_b$	1.20	1.21	2.10	0.98	0.84	2.51
Higgs	m_H	p_T^{jet}	2.33	–	2.33	1.72	–	2.32
Higgs via VBF	m_H	p_T^{jet}	1.07	0.97	1.07	1.23	1.34	1.09
Higgs+1jet	m_H	p_T^{jet}	2.02	–	2.13	1.47	–	1.90
Higgs+2jets	m_H	p_T^{jet}	–	–	–	1.15	–	–

$$\mathcal{K} = \frac{NLO}{LO}$$

[NLO report 0803.0494]

General features:

- color annihilation, gluon dominated ⇒ large K factors ?
- extra legs in the final state ⇒ smaller K-factors ?

But be careful, only full calculations can really tell!

NNLO: when is NLO not good enough?

- when NLO corrections are large (NLO correction ~ LO)

  This may happens when

  - process involve very different scales → large logarithms of ratio of scales appear
  - new channels open up at NLO (at NLO they are effectively LO)
  - master example: Higgs production

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  - W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

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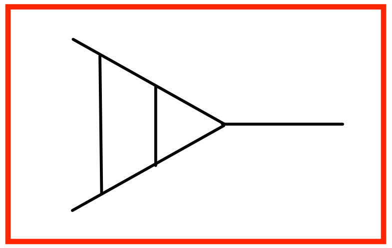
- when a reliable error estimate is needed



Collider processes known at NNLO

Collider processes known at NNLO today:

(a) Drell-Yan (Z,W)

(b) Higgs, also associated HV

(c) 3-jets in e+e-

Drell-Yan processes

Drell-Yan processes: Z/W production (W → lν , Z → l⁺l⁻)

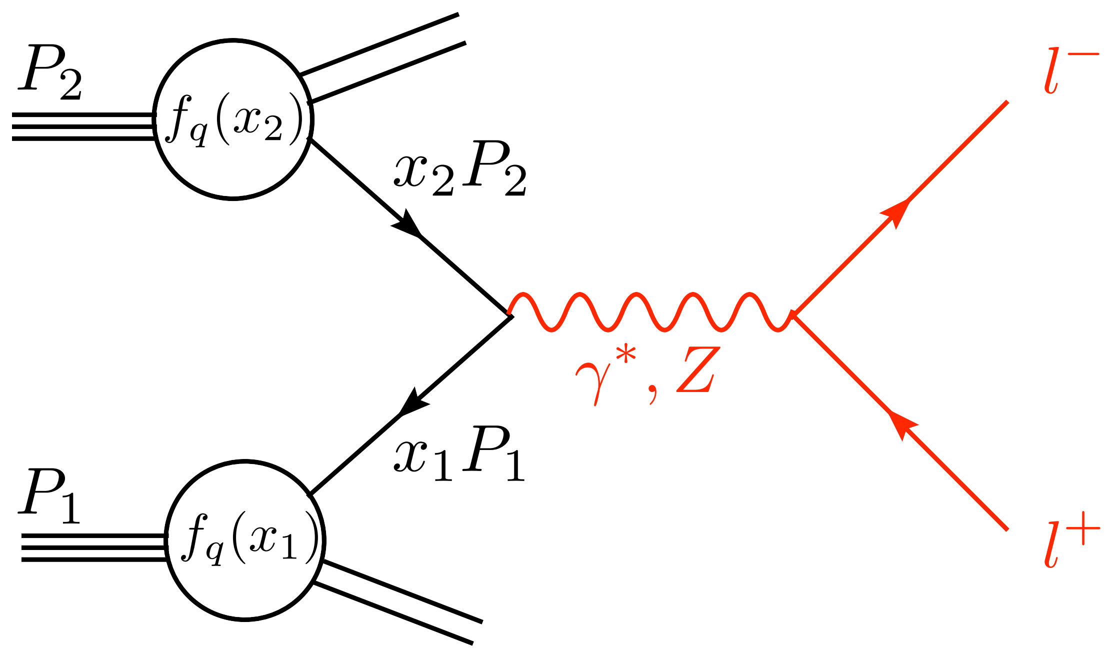
Very clean, golden-processes in QCD because

✓ dominated by quarks in the initial state

✓ no gluons or quarks in the final state (QCD corrections small)

✓ leptons easier experimentally (clear signature)

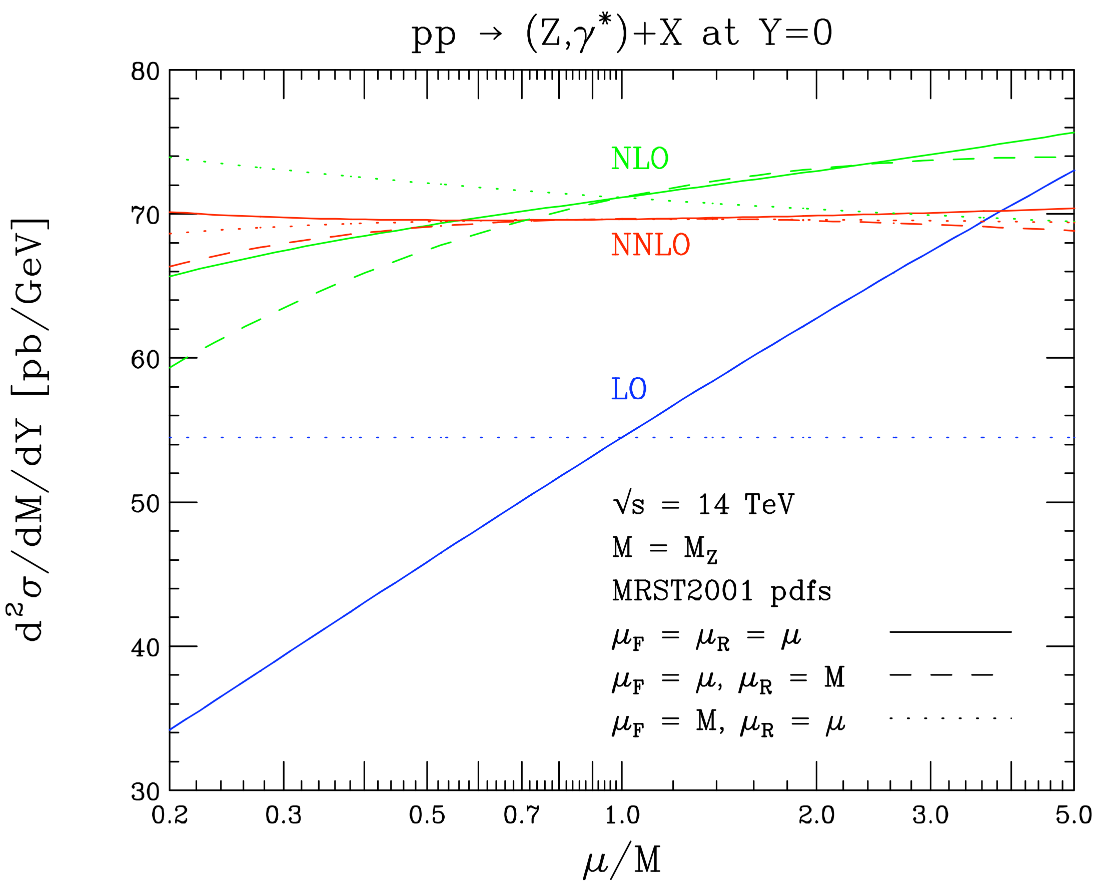
⇒ as clean as it gets at a hadron collider

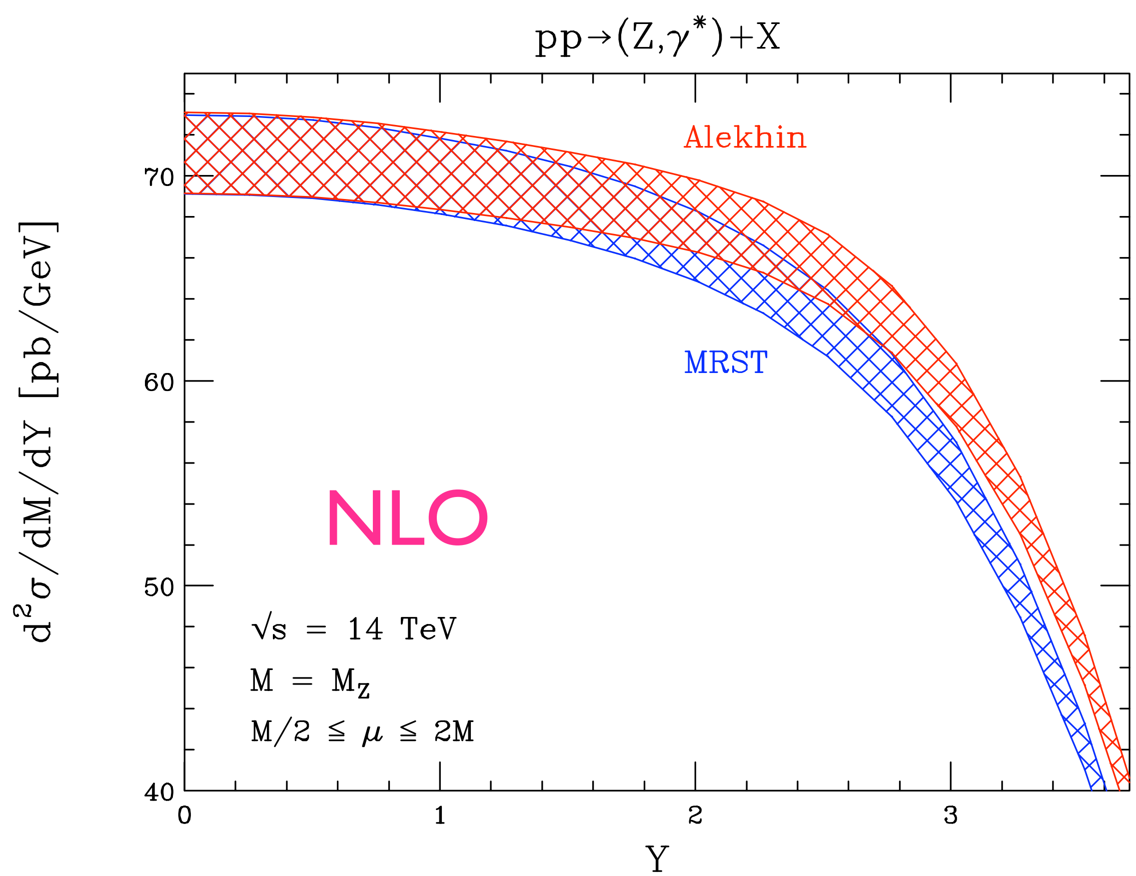


Drell-Yan processes

- most important and precise test of the SM at the LHC
- best known process at the LHC: spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta

Scale stability and sensitivity to PDFs





Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

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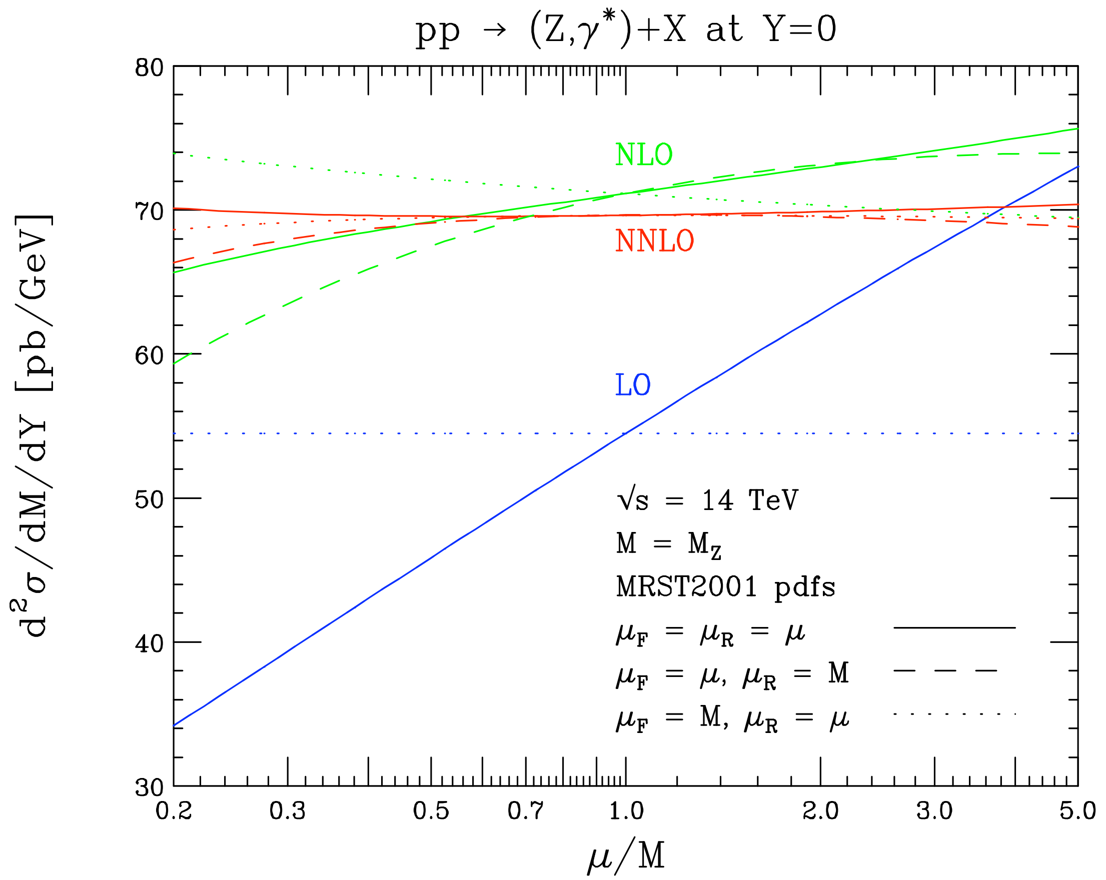
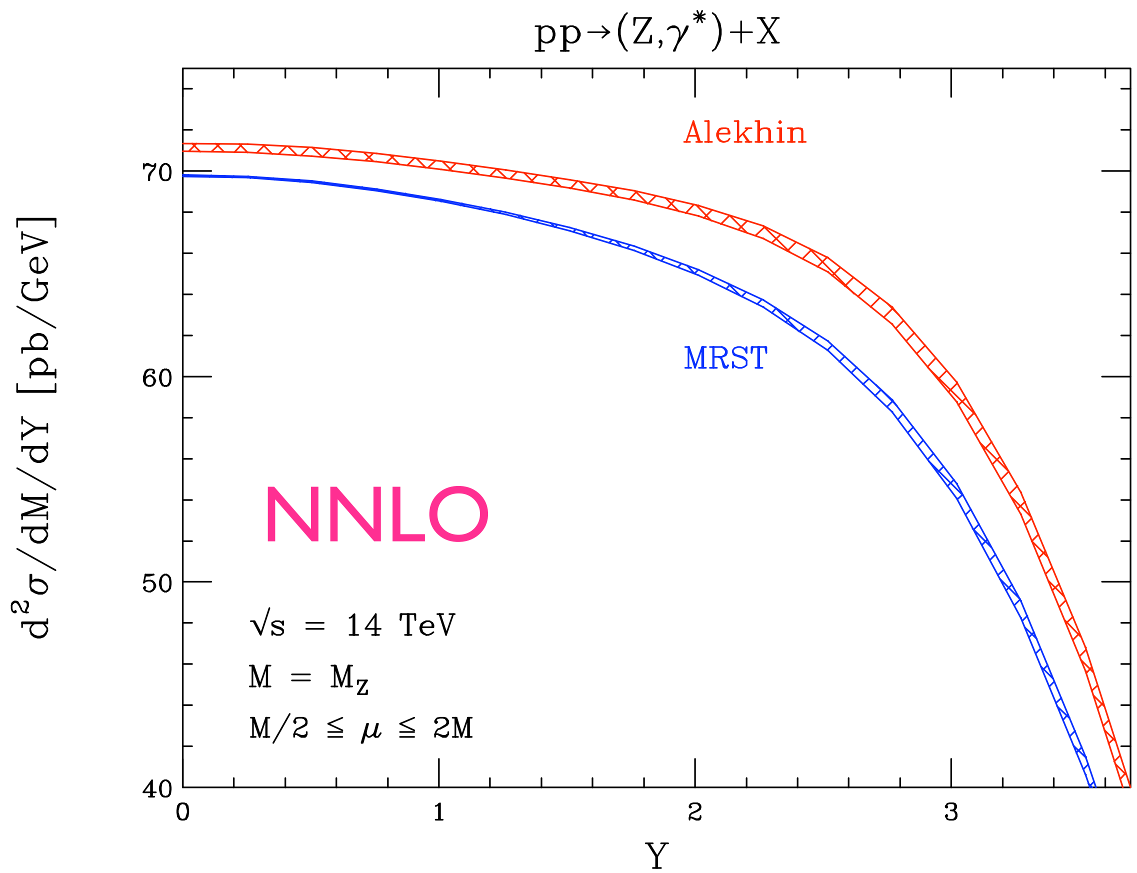
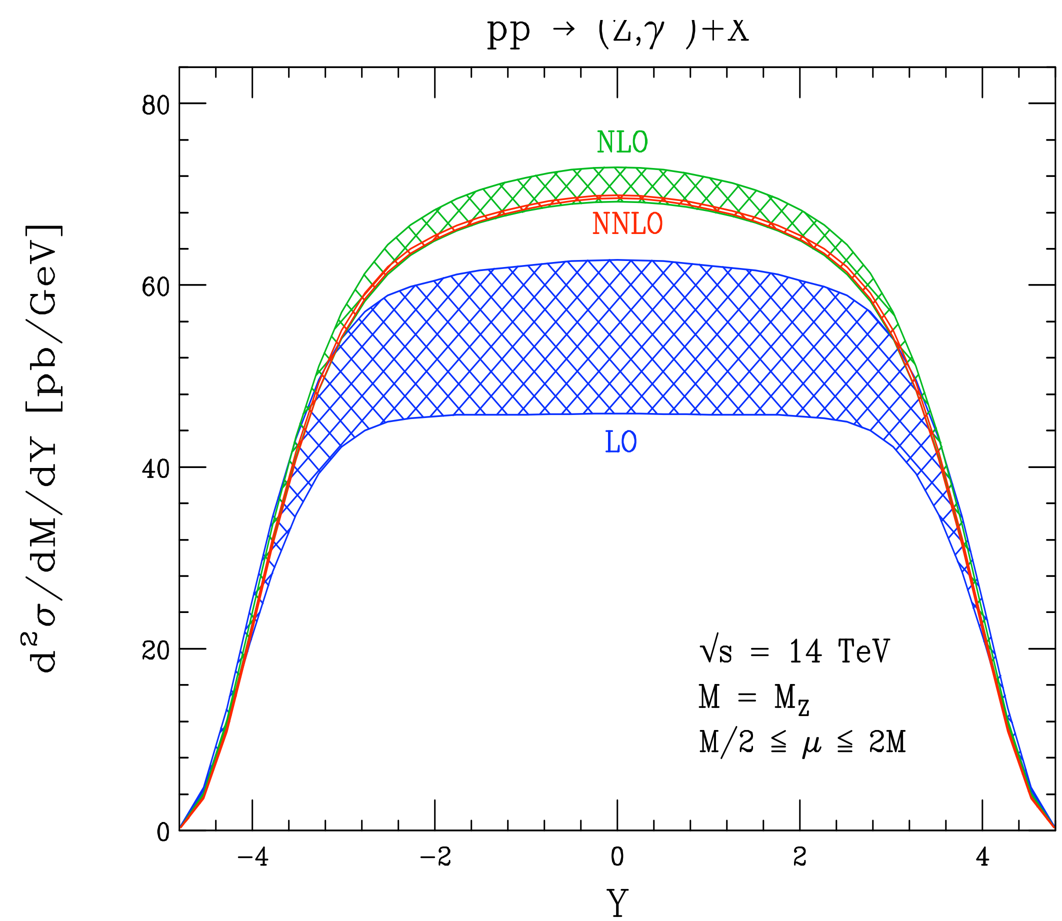
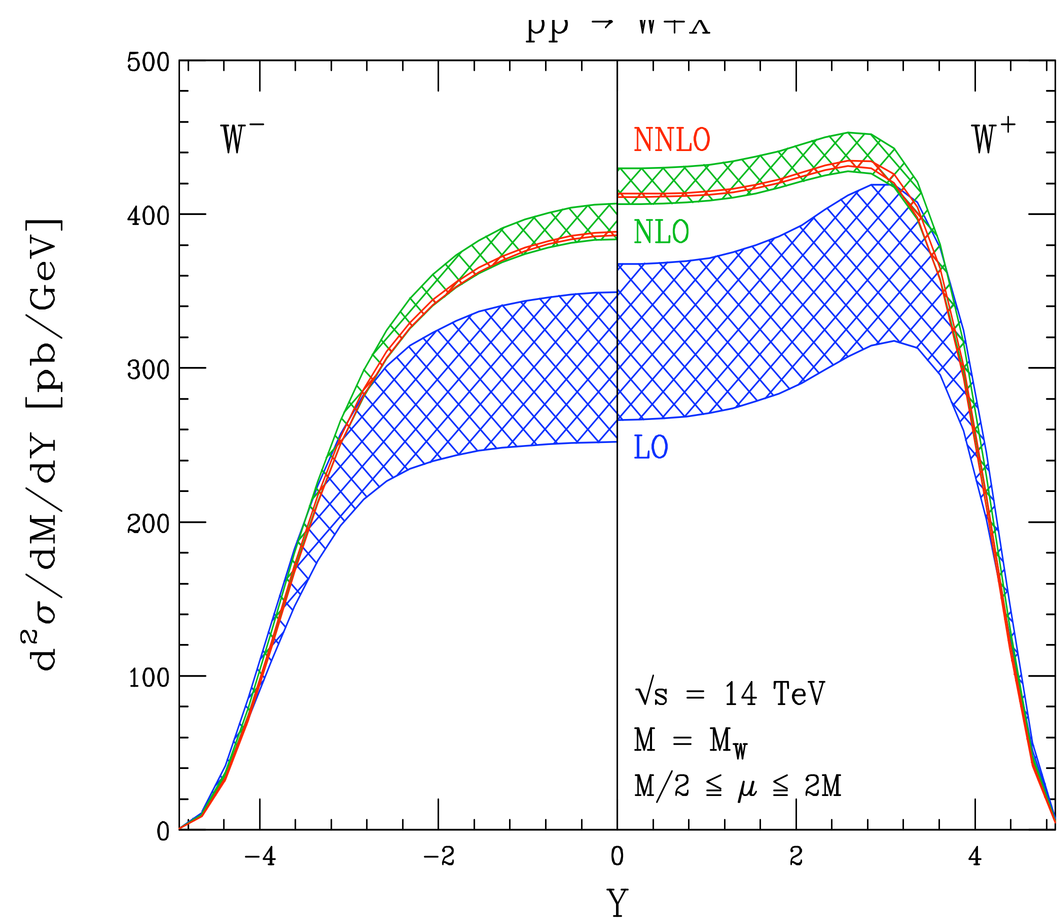




Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

Drell-Yan: rapidity distributions

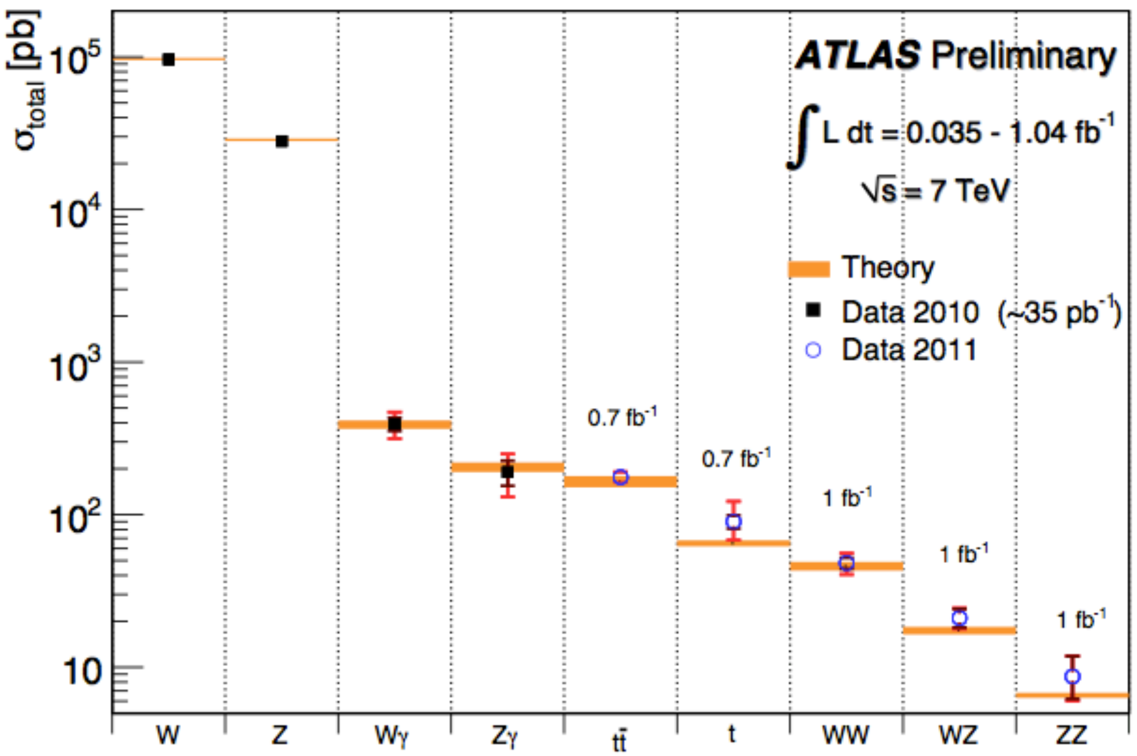


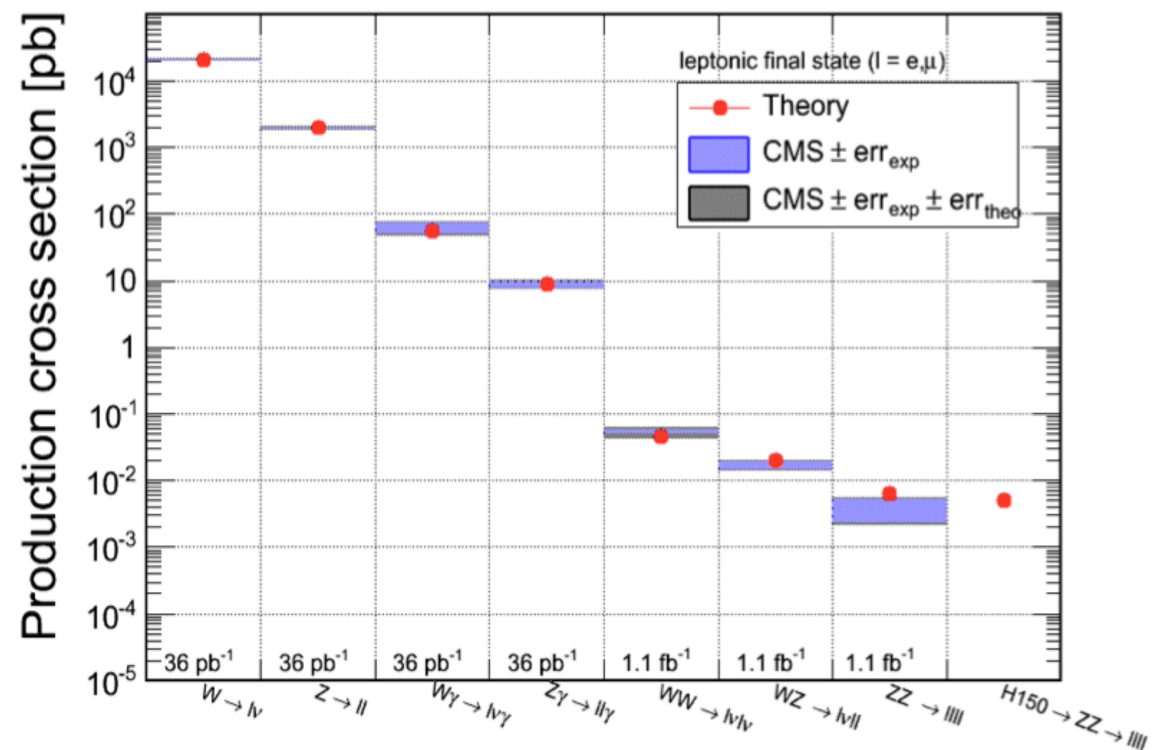


Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

☛ *LHC: perturbative accuracy of the order of 1%. This is absolutely unique!*

NNLO vs LHC data





$$\sigma(W)\cdot B(W\to e\nu)\sim 10\,\mathrm{nb} \qquad \sigma(Z)\cdot B(Z\to e^+e^-)\sim 1\,\mathrm{nb}$$

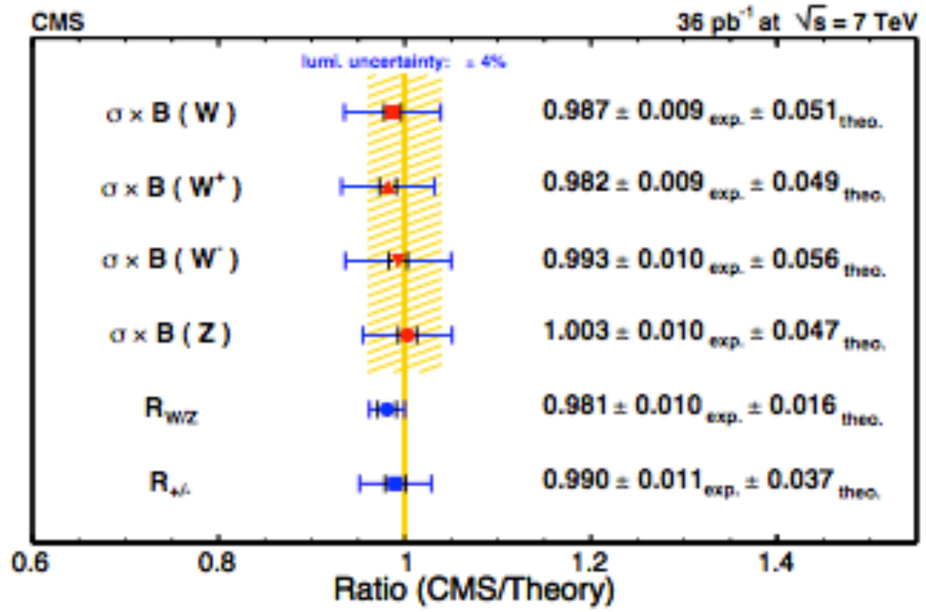
$$\sigma(WW)\cdot B(W\to l\nu)^2\sim 100\,\mathrm{fb} \qquad \sigma(ZZ)\cdot B(W\to l^+l^-)^2\sim 10\,\mathrm{fb}$$

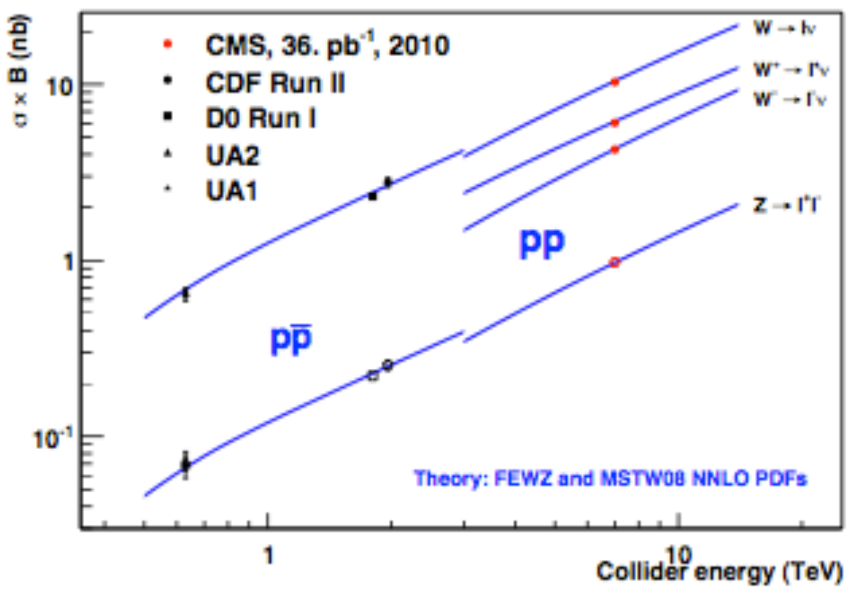
E. g. with $1\mathrm{fb}^{-1}$:

- $O(10^6)$ W and $O(10^5)$ Z events per experiment and lepton channel
- O(100) WW and O(10) ZZ per experiment including all lepton channels

NNLO vs LHC data

Impressive agreement between experiment and NNLO theory





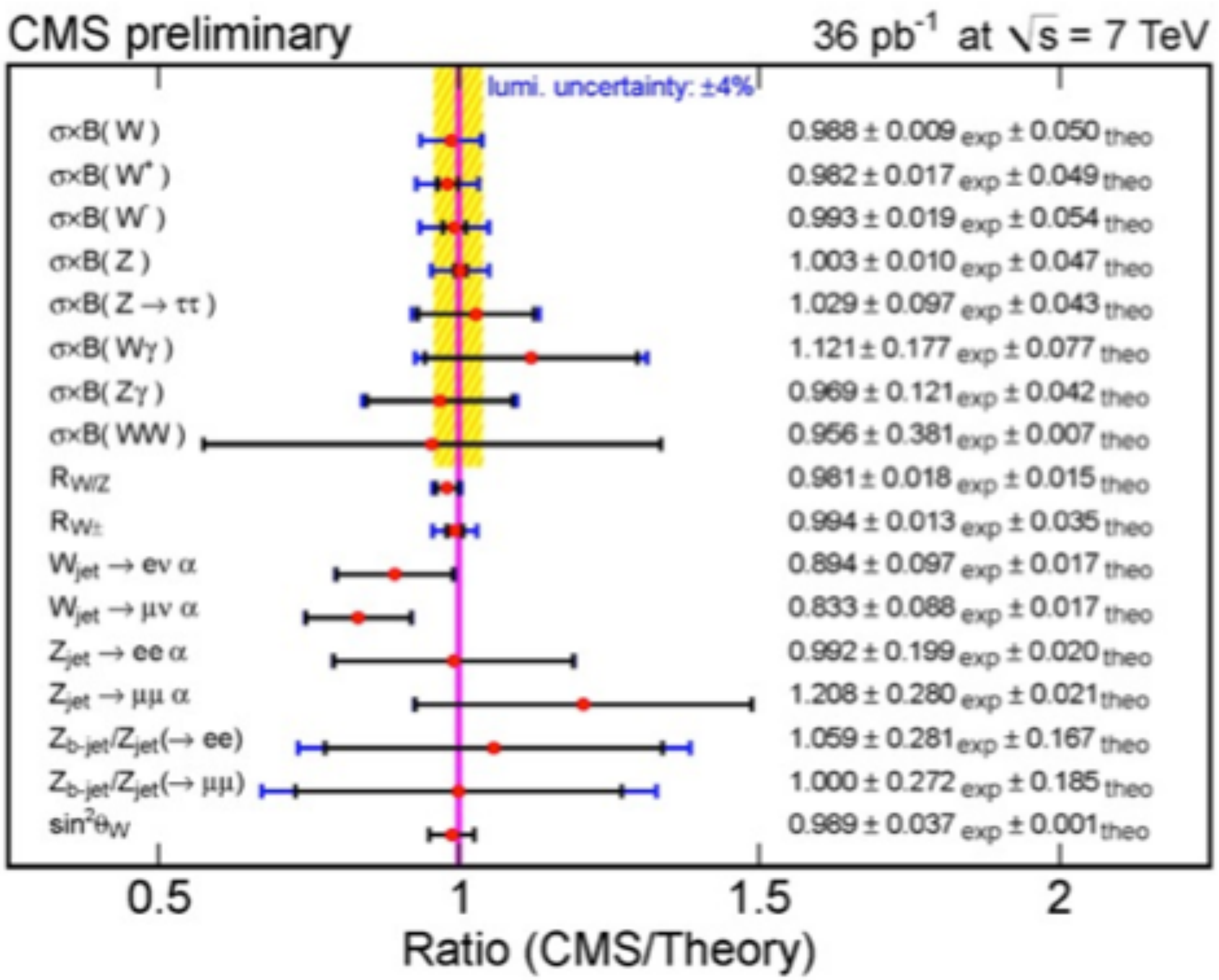
Quantity	Ratio (CMS/Theory)	Lumi. uncert. (4%)
$\sigma \times BF(W^{\pm})$	$0.987 \pm 0.009\,(ex) \pm 0.051\,(th)\ [\pm 0.051 (tot)]$	0.039
$\sigma \times BF(W^{+})$	$0.982 \pm 0.009\,(ex) \pm 0.049\,(th)\ [\pm 0.050 (tot)]$	0.039
$\sigma \times BF(W^{-})$	$0.993 \pm 0.010\,(ex) \pm 0.056\,(th)\ [\pm 0.057 (tot)]$	0.040
$\sigma \times BF(Z)$	$1.003 \pm 0.010\,(ex) \pm 0.047\,(th)\ [\pm 0.048 (tot)]$	0.040
$\sigma \times BF(W)/\sigma \times BF(Z)$	$0.981 \pm 0.010\,(ex) \pm 0.016\,(th)\ [\pm 0.019 (tot)]$	–
$\sigma \times BF(W^{+})/\sigma \times BF(W^{-})$	$0.990 \pm 0.011\,(ex) \pm 0.037\,(th)\ [\pm 0.039 (tot)]$	–

Theory error completely dominated by PDFs

CMS PAS EWK-10-005, similar results from ATLAS not shown here

NNLO vs LHC data

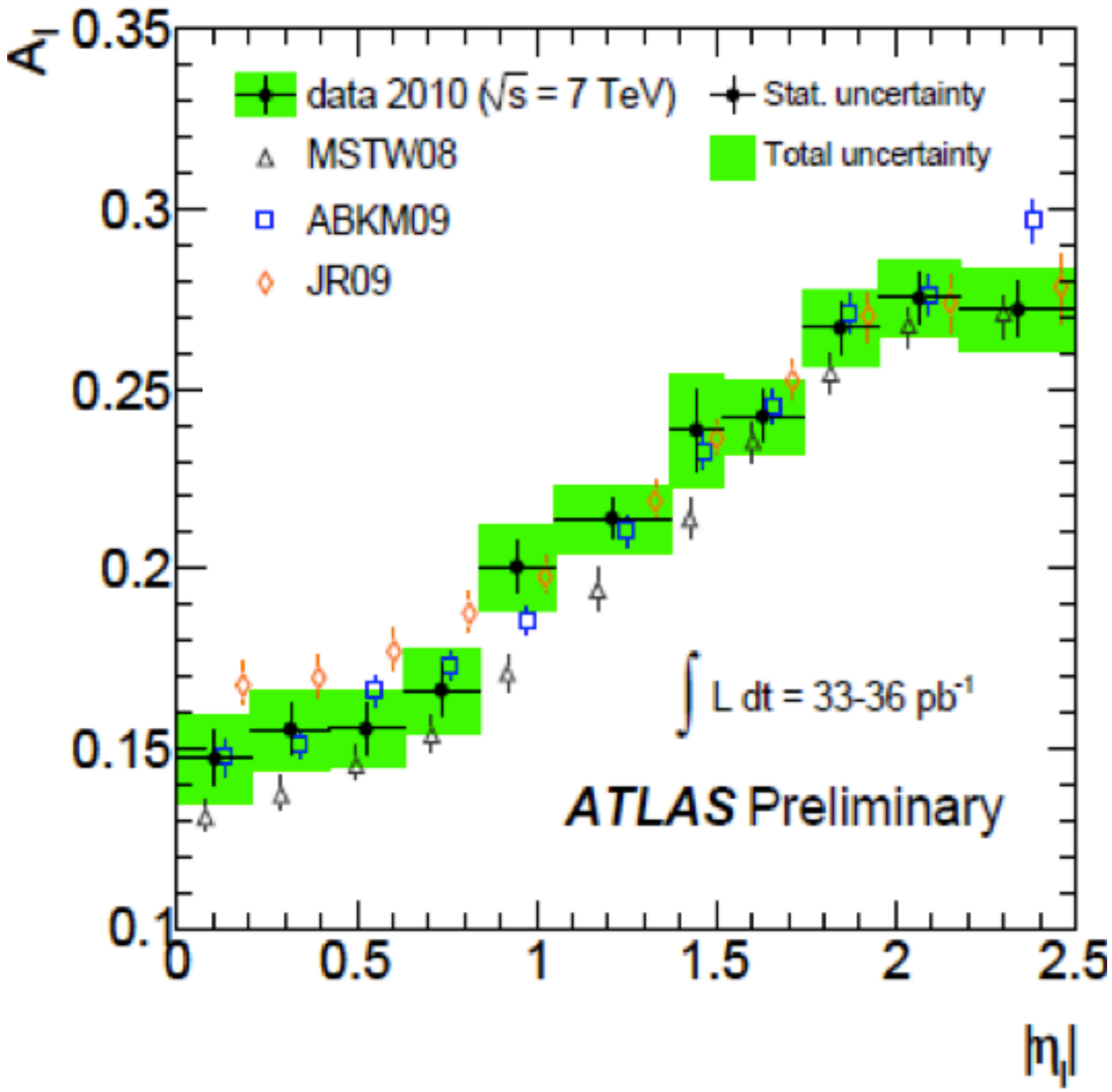
Spectacular experimental achievements in very little time !



- remarkable agreement with theory
- precise measurement of W/Z properties (also notice measurement of $\sin^2\theta_W$)
- achieved control and precision already allows improvements on PDFs

Charge asymmetry

Natural extension of the inclusive cross-section is the R_W = W+/W- ratio. Study R_W as a function of kinematics variables, e.g. charge asymmetry as a function of lepton rapidity

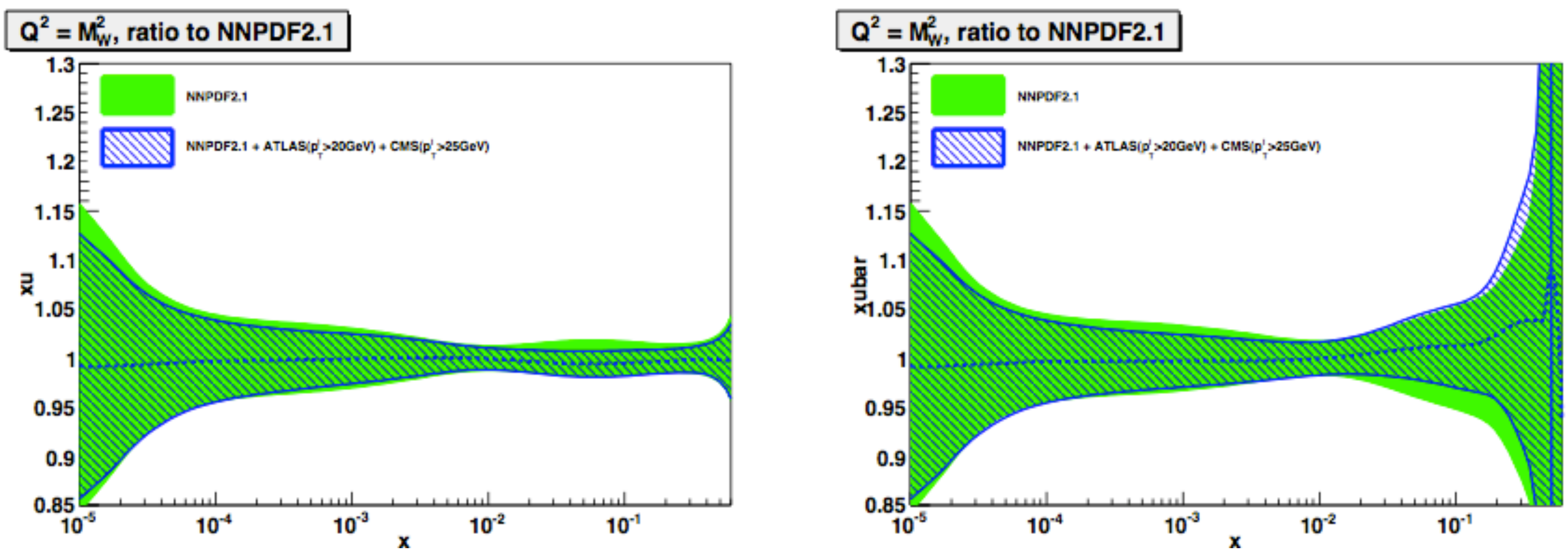


$$A(\eta) = \frac{R_W(\eta) - 1}{R_W(\eta) + 1}$$

- measurement very sensitive to PDFs since many uncertainties cancel in ratios
- good agreement with various PDFs but very sensitive to shape details
- similar results by CMS

Charge asymmetry

Effect of ATLAS and CMS lepton charge asymmetry on NNPDF global fit



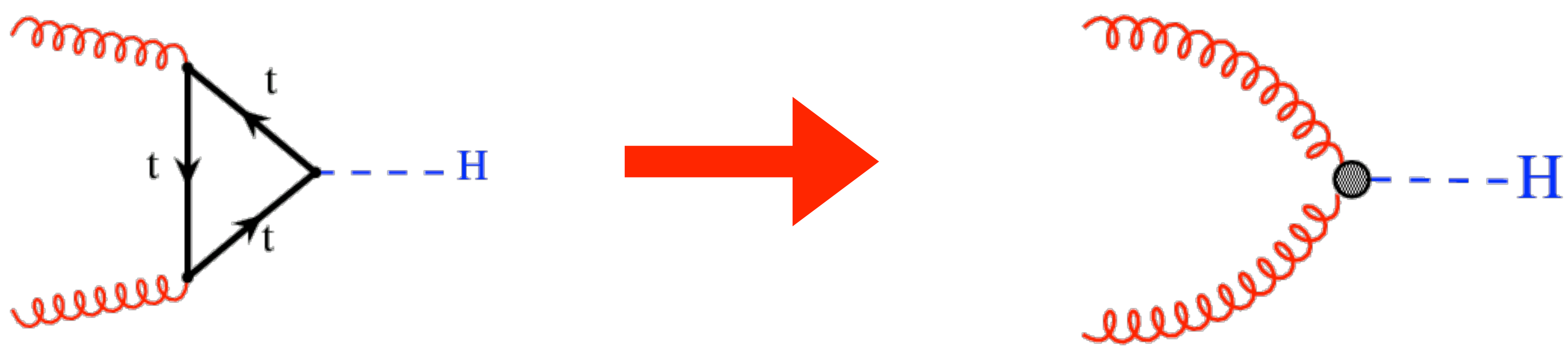
Reduction of uncertainty of the order of 10-30% in the range $x=10^{-3}-10^{-1}$
Similar results for d-quark and other sea distributions

NNPDF 1108.1758

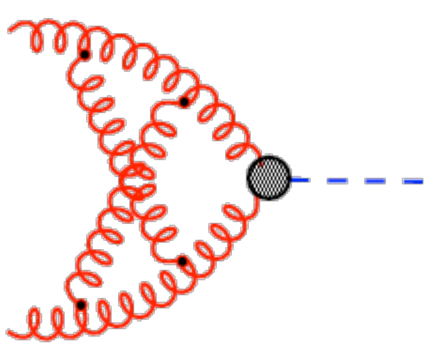
NB:
LHCb data at larger rapidities probe larger and smaller values of x that are less constraint, they will have a larger impact than ATLAS/CMS soon

Inclusive NNLO Higgs production

Inclusive Higgs production via gluon-gluon fusion in the large m_t-limit:

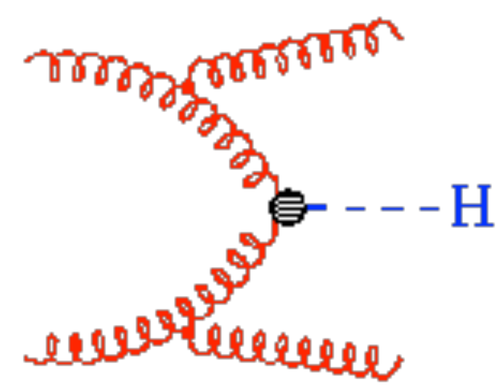


NNLO corrections known since few years now:



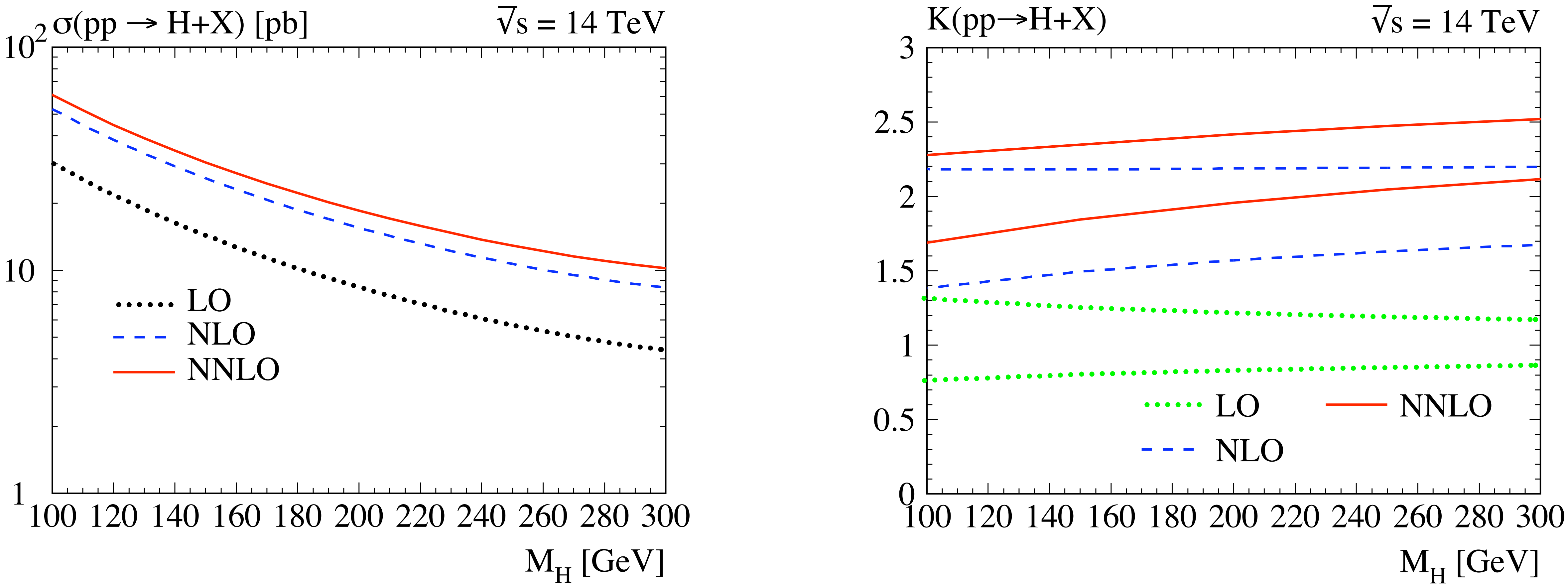
virtual-virtual

real-virtual



real-real

Inclusive NNLO Higgs production



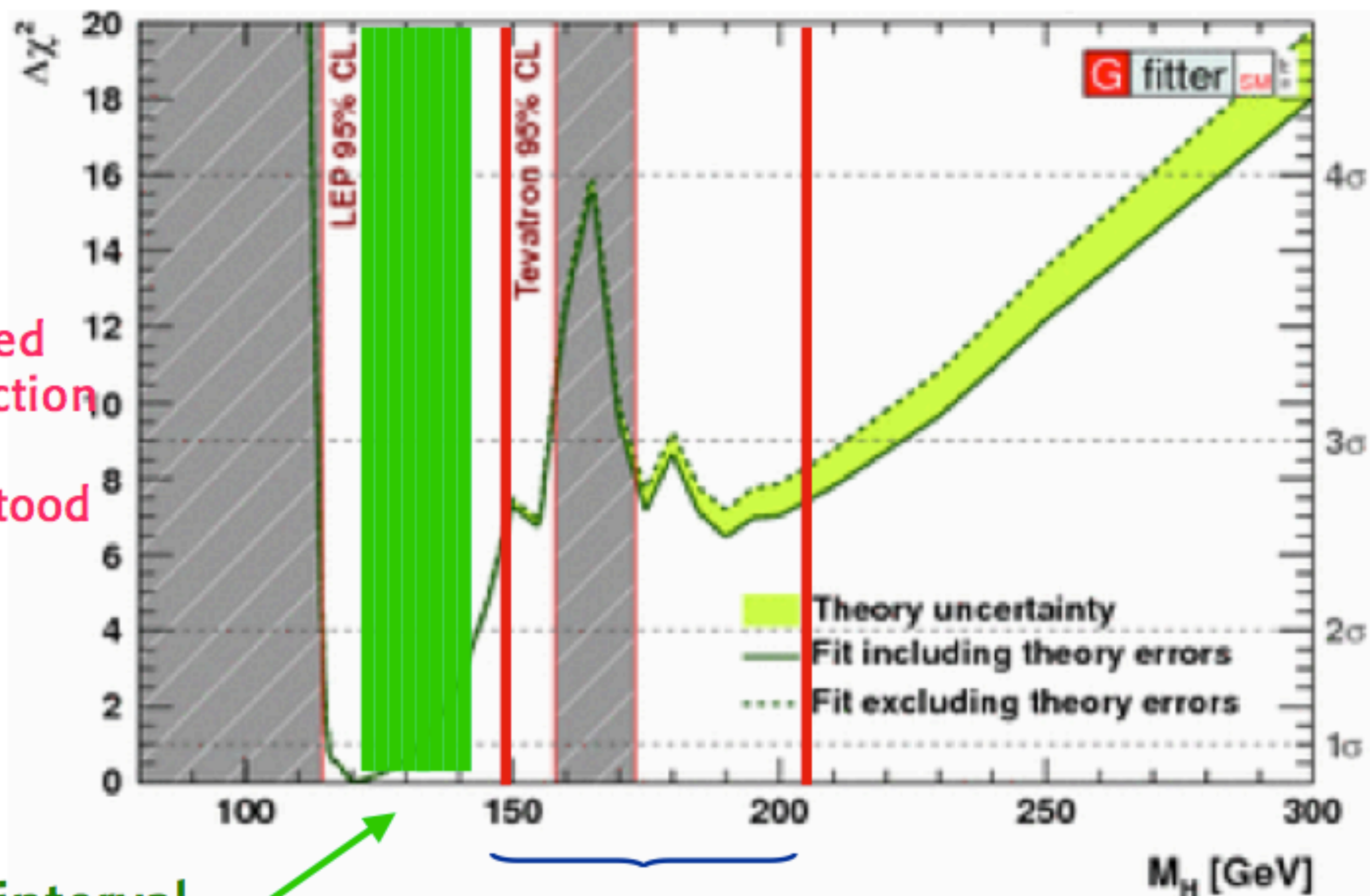
Kilgore, Harlander '02
Anastasiou , Melnikov '02

Higgs searches: status

slide taken from G. Altarelli, EPS 2011

The SM Higgs is close to be observed or excluded

This fig.
is a non
authorized
reproduction
of what
I understood



interval
with excess
~2.5 σ

Excl. by ATLAS/CMS

also $300 < m_H < 450$ GeV
is excluded

NNLO 3-jets in e^+e^-

Motivation: error on α_s from jet-observables

$$\alpha_s(M_Z) = 0.121 \pm 0.001\,(\text{exp.}) \pm 0.005\,(\text{th.})$$

Bethke '06

➥ dominated by theoretical uncertainty

NNLO 3-jet calculation in e^+e^- completed in 2007

Method: developed antenna subtraction at NNLO

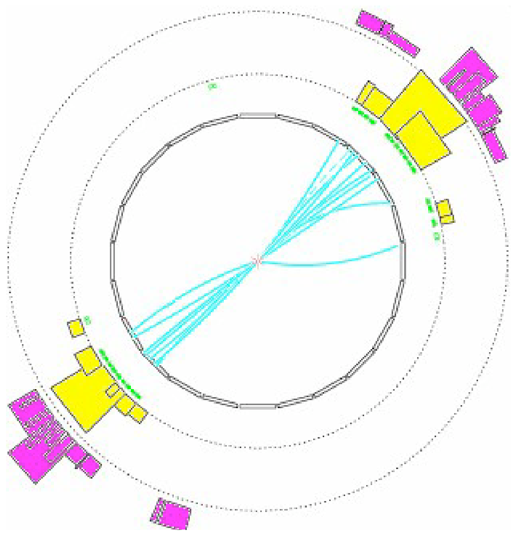
First application: NNLO fit of α_s from event-shapes

Event shapes

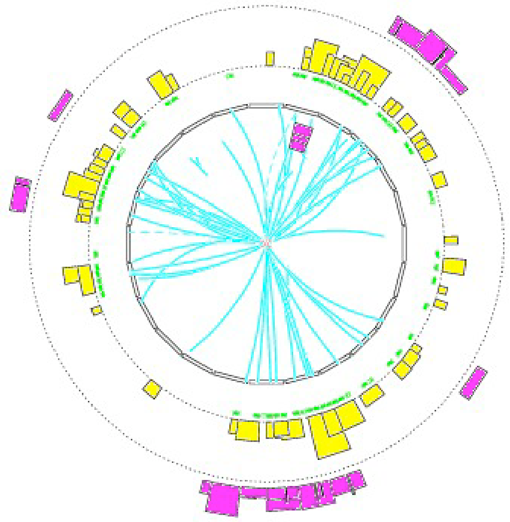
Event-shapes and jet-rates: infrared safe observables describing the energy and momentum flow of the final state.

Candle example in e^+e^-: The thrust $$T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i |\vec{p}_i|}$$

Pencil-like event: $1 - T \ll 1$

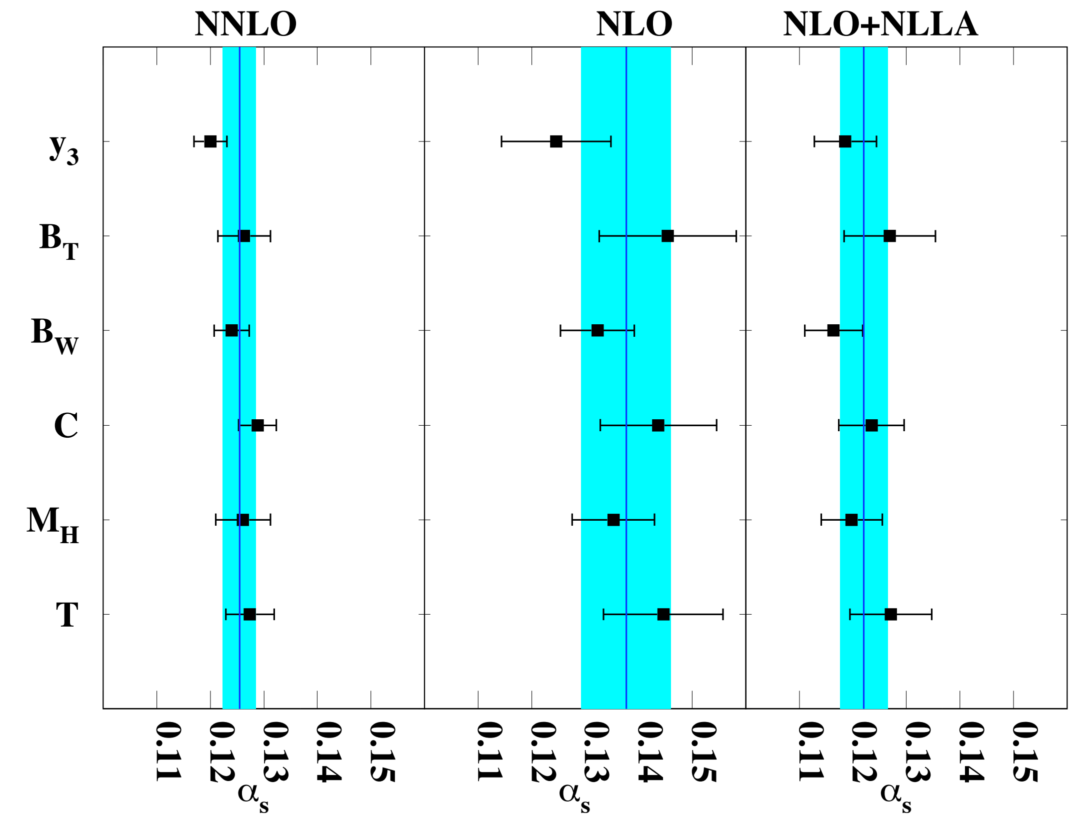


Planar event: $1 - T \sim 1$



α_s from event shapes at NNLO

- scale variation reduced by a factor 2
- scatter between α_s from different event-shapes reduced
- better χ^2, central value closer to world average



$$\alpha_s(M_Z^2) = 0.1240 \pm 0.0008\,(\mathrm{stat}) \pm 0.0010\,(\mathrm{exp}) \pm 0.0011\,(\mathrm{had}) \pm 0.0029\,(\mathrm{theo})$$

Dissertori, Gehrmann-DeRidder, Gehrmann, Glover, Heinrich, Stenzel '07
Gehrmann, Luisoni, Stenzel '08

NNLO on the horizon

- Single-jet production
  - constrain gluon PDF
  - matrix elements known for some time
  - subtraction in progress
- Top pair production
  - needed for more precise m_t determination
  - possibly for further constraining PDFs
  - top asymmetry
- Vector boson pair production
  - NLO corrections are large
  - study gauge structure of SM (triple gauge couplings)
  - most important and irreducible background for Higgs production in intermediate mass region

Recap of higher orders

Leading order

- everything can be computed in principle today (practical edge: 8 particles in the final state), many public codes
- techniques: standard Feynman diagrams or recursive methods (Berends-Giele, BCF, CSW ...)

Next-to-leading order

- current frontier $2 \to 5$ in the final state
- many new, promising techniques

Next-to-next-to-leading order

- few $2 \to 1$ processes available (Higgs, Drell-Yan)
- 3-jets in e^+e^-
- expect $2 \to 2$ calculations soon

Next

Next will focus on

- parton showers and Monte Carlo methods
- matching of parton showers and fixed order calculations
- jets

Parton shower & Monte Carlo methods

- today at the frontier of NLO calculations are processes with 4 or 5 particles in the final state. Difficult to expect much more in the coming years. However, typical LHC processes have much larger multiplicity

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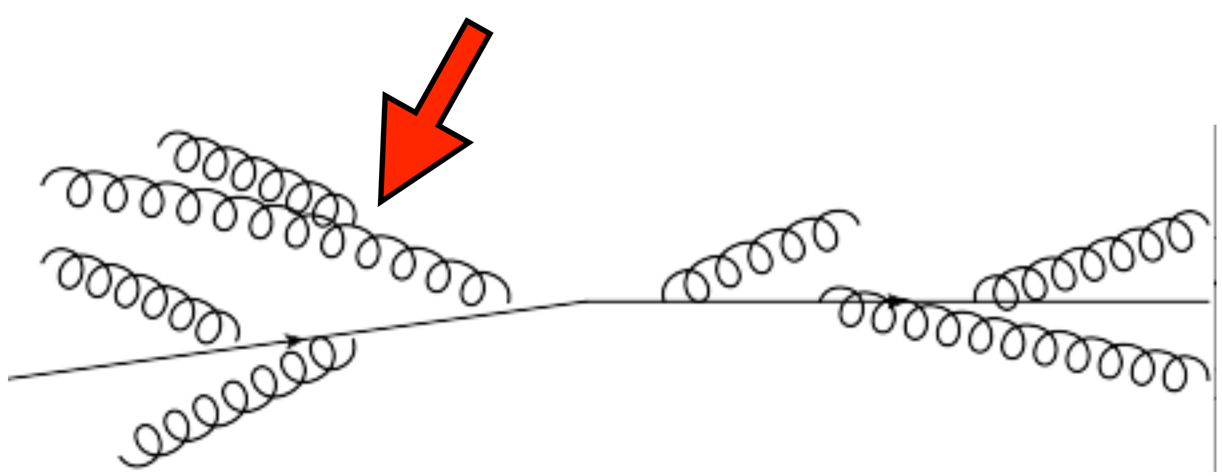
- we have also seen that large logarithms can spoil the convergence of PT, NLO results become unreliable
- instead, one can seek for an approximate result such that soft and collinear enhanced terms are taken into account to all orders
- this leads to a 'parton shower' picture, which is implemented in computer simulations, usually called Monte Carlo programs or event generators



Angular ordering

When a soft gluon is radiated from a ($p_i p_j$) dipole one gets a universal eikonal factor

$$\omega_{ij} = \frac{p_i p_j}{p_i k \, p_j k} = \frac{1 - v_i v_j \cos\theta_{ij}}{\omega_k^2 (1 - v_i \cos\theta_{ik})(1 - v_j \cos\theta_{jk})}$$

Massless emitting lines $v_i = v_j = 1$, then

$$\omega_{ij} = \omega_{ij}^{[i]} + \omega_{ij}^{[j]} \qquad \omega_{ij}^{[i]} = \frac{1}{2}\left(\omega_{ij} + \frac{1}{1 - \cos\theta_{ik}} - \frac{1}{1 - \cos\theta_{jk}}\right)$$

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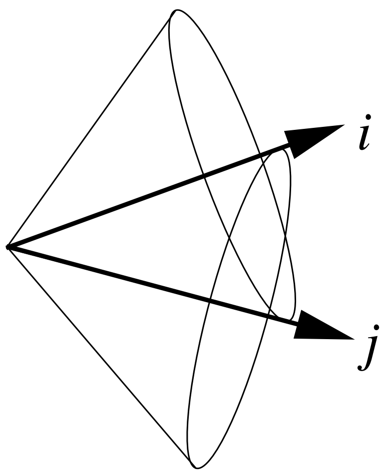
Angular ordering

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \omega_{ij}^{[i]} = \begin{cases} \frac{1}{\omega_k^2 (1-\cos\theta_{ik})} & \theta_{ik} < \theta_{ij} \\ 0 & \theta_{ik} > \theta_{ij} \end{cases}$$



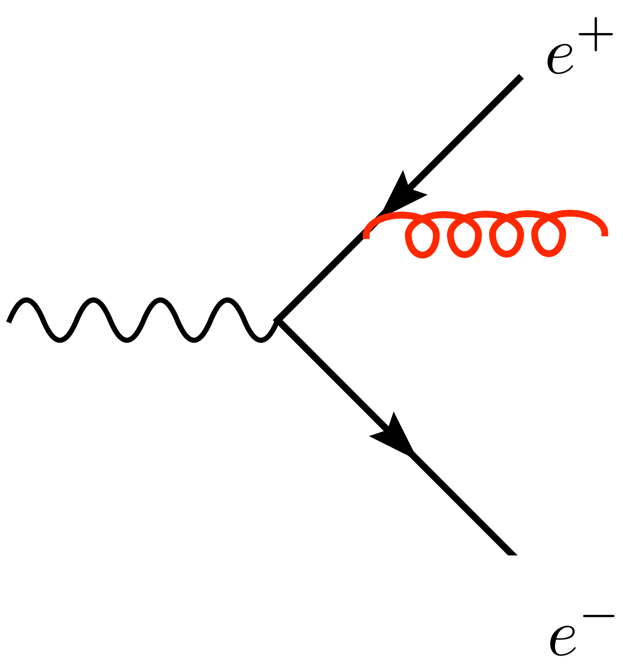
Proof: see e.g. QCD and collider physics, Ellis, Stirling, Webber

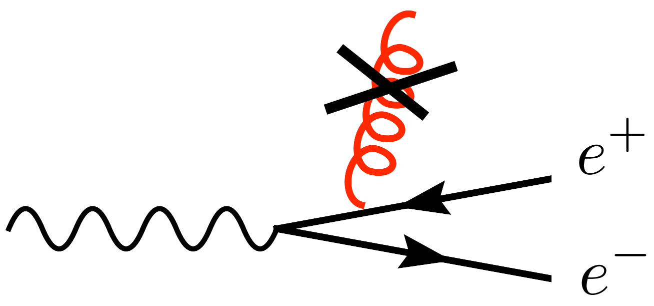
Angular ordering & coherence

A. O. is a manifestation of coherence of radiation in gauge theories

<u>In QED</u>

suppression of soft bremsstrahlung from an e+e- pair (Chudakov effect)
At large angles the e^+e^- pair is seen coherently as a system without total charge $\Rightarrow$ radiation is suppressed





Herwig use the angle as an evolution variable, therefore has coherence built in. Other Parton showers force angular ordering in the evolution.

Parton showers at the LHC

[Ariadne, Pythia, Herwig, Isajet ...]

Standard parton shower programs

- hard ($2 \to 2$) scattering
- parton shower (in the soft-collinear approximation)
- hadronization model + underlying event model

PS differ in the ordering variable of the shower, e.g. angle Herwig, transverse momentum Ariadne and Pythia (new), virtuality Pythia (old), in U.E. model, in the hadronization model

Every LHC analysis will make use of one or more PS simulation for

- the signal and/or the background
- underlying event / non-perturbative corrections
- pile-up
- efficiency studies / detector response

An example with Herwig

Select the initial state, e.g. pp collisiosn at 14 TeV

---INITIAL STATE---

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
1	P	2212	101	0	0	0	0	0.00	0.00	7000.0	7000.0	0.94
2	P	2212	102	0	0	0	0	0.00	0.00	-7000.0	7000.0	0.94
3	CMF	0	103	1	2	0	0	0.00	0.00	0.0	14000.0	14000.0

An example with Herwig

Select the hard process of interest, e.g. Z+ jet production

---HARD SUBPROCESS---

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
4	UQRK	2	121	6	8	9	5	0.00	0.00	590.8	590.8	0.32
5	GLUON	21	122	6	4	17	8	0.00	0.00	-232.1	232.1	0.75
6	HARD	0	120	4	5	7	8	0.40	-9.40	358.7	823.0	740.63
7	Z0/GAMA*	23	123	6	7	22	7	-261.59	-217.31	329.3	481.6	88.56
8	UQRK	2	124	6	5	23	4	261.59	217.31	29.4	341.3	0.32

An example with Herwig

Then Herwig dresses the process for you, both with initial state and final state shower

---PARTON SHOWERS---

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
9	UQRK	94	141	4	6	11	16	2.64	-9.83	592.2	590.2	-49.07
10	CONE	0	100	4	5	0	0	-0.27	0.96	0.1	1.0	0.00
11	GLUON	21	2	9	12	32	33	-1.02	3.59	5.6	6.7	0.75-
12	GLUON	21	2	9	13	34	35	0.25	1.46	3.6	4.0	0.75-
13	GLUON	21	2	9	14	36	37	-0.87	1.62	4.7	5.1	0.75-
14	GLUON	21	2	9	15	38	39	-0.81	4.17	3611.7	3611.7	0.75-
15	GLUON	21	2	9	16	40	41	-0.19	-1.01	1727.7	1727.7	0.75-
16	UD	2101	2	9	25	42	41	0.00	0.00	1054.6	1054.6	0.32-
17	GLUON	94	142	5	6	19	21	-2.23	0.44	-233.5	232.8	-18.36
18	CONE	0	100	5	8	0	0	0.77	0.64	0.2	1.0	0.00
19	GLUON	21	2	17	20	43	44	1.60	0.58	-2.1	2.8	0.75
20	UD	2101	2	17	21	45	44	0.00	0.00	-2687.6	2687.6	0.32
21	UQRK	2	2	17	32	46	45	0.63	-1.02	-4076.9	4076.9	0.32
22	Z0/GAMA*	23	195	7	22	251	252	-257.66	-219.68	324.8	477.5	88.56
23	UQRK	94	144	8	6	25	31	258.06	210.29	33.9	345.5	86.10
24	CONE	0	100	8	5	0	0	0.21	0.17	-1.0	1.0	0.00
25	UQRK	2	2	23	26	47	42	26.82	24.33	23.7	43.3	0.32
26	GLUON	21	2	23	27	48	49	8.50	8.18	6.0	13.3	0.75
27	GLUON	21	2	23	28	50	51	73.27	61.24	12.0	96.2	0.75
28	GLUON	21	2	23	29	52	53	73.66	58.54	-6.3	94.3	0.75
29	GLUON	21	2	23	30	54	55	67.58	52.13	-7.3	85.7	0.75
30	GLUON	21	2	23	31	56	57	6.98	4.60	2.3	8.7	0.75
31	GLUON	21	2	23	43	58	59	1.24	1.26	3.6	4.1	0.75

Add hadronization + U.E. then perform your desired physics study

Accuracy of Monte Carlos

Formally, Monte Carlos are Leading Logarithmic (LL) showers

- because they don't include any higher order corrections to the $1 \to 2$ splitting
- because they don't have any $1 \to 3$ splittings
- ....

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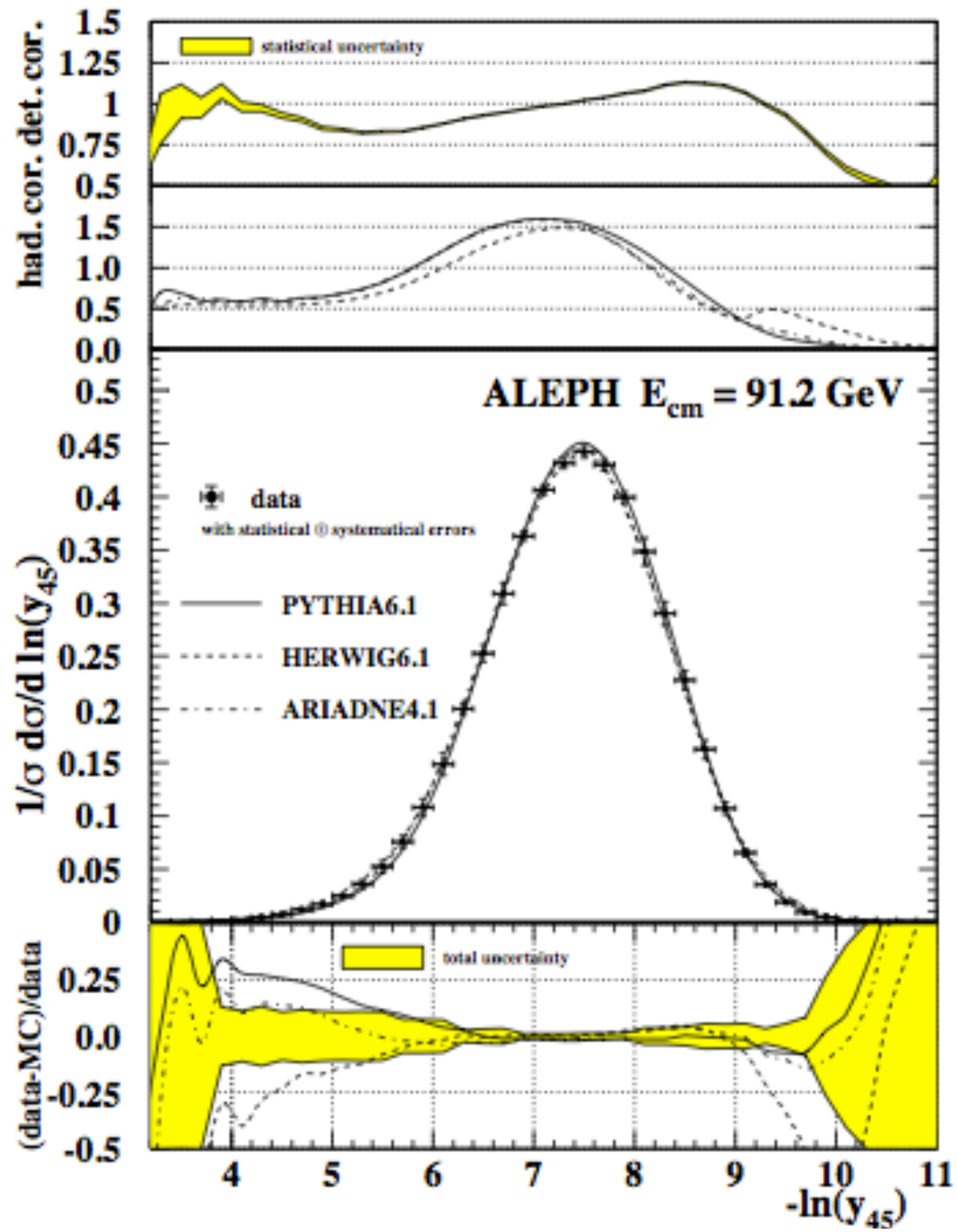
So, despite not guaranteeing any formal accuracy, they fare better than LL calculations. *The problem is that we don't know the uncertainty. Often comparison between different PS is the only way to estimate the uncertainty*

Parton shower vs data

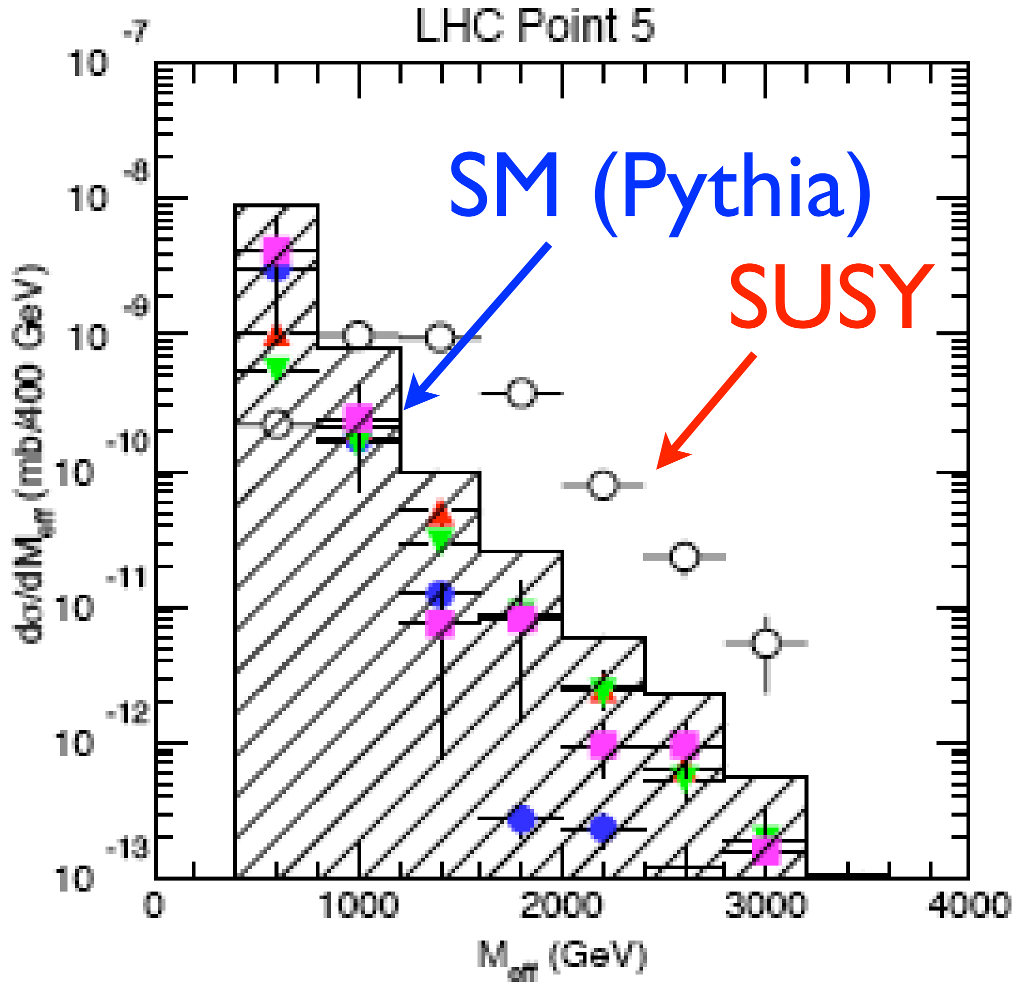
<u>Example</u>:
five-jet resolution parameter y_{45}

- Agreement over 3 orders of magnitudes for a variable that describes a multi-jet final state
- Surprising since MCs rely on the soft-collinear approximation + a model for hadronization
- Note however that MCs have been tuned to LEP data



Accuracy of parton showers

M_{eff} = total transverse energy in the event



- SUSY: position of the peak determined by the mass spectrum
- Pure PS predict steeply falling SM background
- With matrix element calculation: SM and SUSY comparable size and shape
- In this example: SUSY search much more difficult than originally thought

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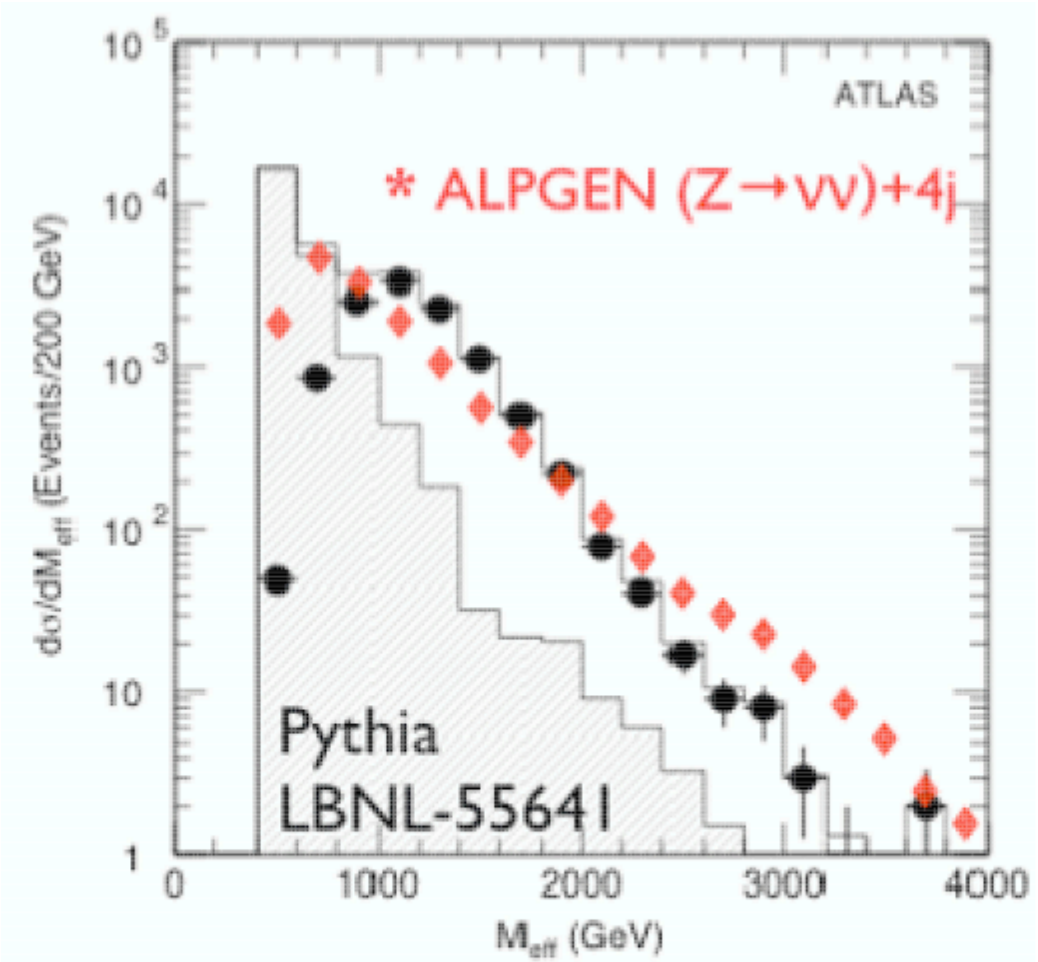

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Lesson to take away

- PS fail to describe hard radiation and it is difficult to understand the uncertainty of their predictions
- techniques and public code (Alpgen, Sherpa, Madgraph ...) exist to match matrix element calculations with Monte Carlos

NLO + parton shower

Even better than LO matrix element + shower is NLO + shower.

This combines the best features: correct rates (NLO) and hadron-level description of events (PS)

Difficult because need to avoid double counting

Two working examples:

- MC@NLO

  Frixione&Webber '02 and later refs.

- POWHEG (POWHEG-BOX)

  Nason '04 and later refs.

Processes implemented:

- W/Z boson production
- WW, WZ, ZZ production
- inclusive Higgs production
- heavy quark production
- V + 1 jet

- single-top
- dijets
- Wbb
- W^+W^+ + dijets ...
- ....

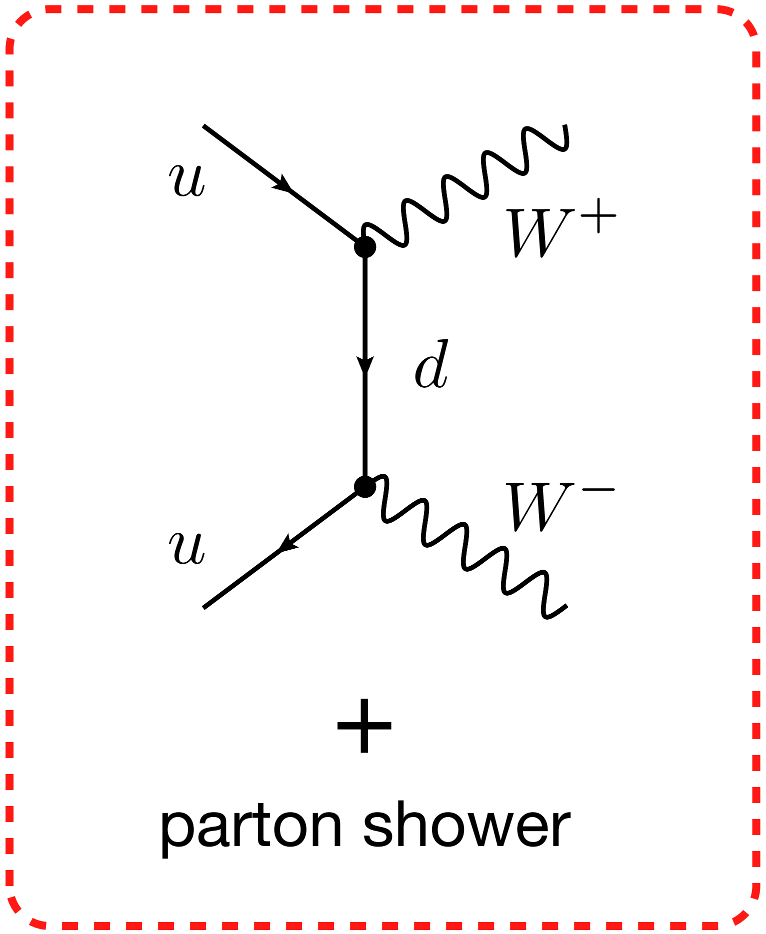
MC@NLO

IPROC	IV	IL_1	IL_2	Spin	Process
–1350–IL				✓	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1360–IL				✓	$H_1H_2 \to (Z \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1370–IL				✓	$H_1H_2 \to (\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1460–IL				✓	$H_1H_2 \to (W^+ \to) l^+_{\rm IL}\nu_{\rm IL} + X$
–1470–IL				✓	$H_1H_2 \to (W^- \to) l^-_{\rm IL}\bar{\nu}_{\rm IL} + X$
–1396				×	$H_1H_2 \to \gamma^*(\to \sum_i f_i\bar{f}_i) + X$
–1397				×	$H_1H_2 \to Z^0 + X$
–1497				×	$H_1H_2 \to W^+ + X$
–1498				×	$H_1H_2 \to W^- + X$
–1600–ID					$H_1H_2 \to H^0 + X$
–1705					$H_1H_2 \to b\bar{b} + X$
–1706		7	7	×	$H_1H_2 \to t\bar{t} + X$
–2000–IC		7		×	$H_1H_2 \to t/\bar{t} + X$
–2001–IC		7		×	$H_1H_2 \to \bar{t} + X$
–2004–IC		7		×	$H_1H_2 \to t + X$
–2030		7	7	×	$H_1H_2 \to tW^-/\bar{t}W^+ + X$
–2031		7	7	×	$H_1H_2 \to \bar{t}W^+ + X$
–2034		7	7	×	$H_1H_2 \to tW^- + X$
–2600–ID	1	7		×	$H_1H_2 \to H^0W^+ + X$
–2600–ID	1	i		✓	$H_1H_2 \to H^0(W^+ \to) l^+_i\nu_i + X$
–2600–ID	-1	7		×	$H_1H_2 \to H^0W^- + X$
–2600–ID	-1	i		✓	$H_1H_2 \to H^0(W^- \to) l^-_i\bar{\nu}_i + X$
–2700–ID	0	7		×	$H_1H_2 \to H^0Z + X$
–2700–ID	0	i		✓	$H_1H_2 \to H^0(Z \to) l_i\bar{l}_i + X$
–2850		7	7	×	$H_1H_2 \to W^+W^- + X$
–2860		7	7	×	$H_1H_2 \to Z^0Z^0 + X$
–2870		7	7	×	$H_1H_2 \to W^+Z^0 + X$
–2880		7	7	×	$H_1H_2 \to W^-Z^0 + X$

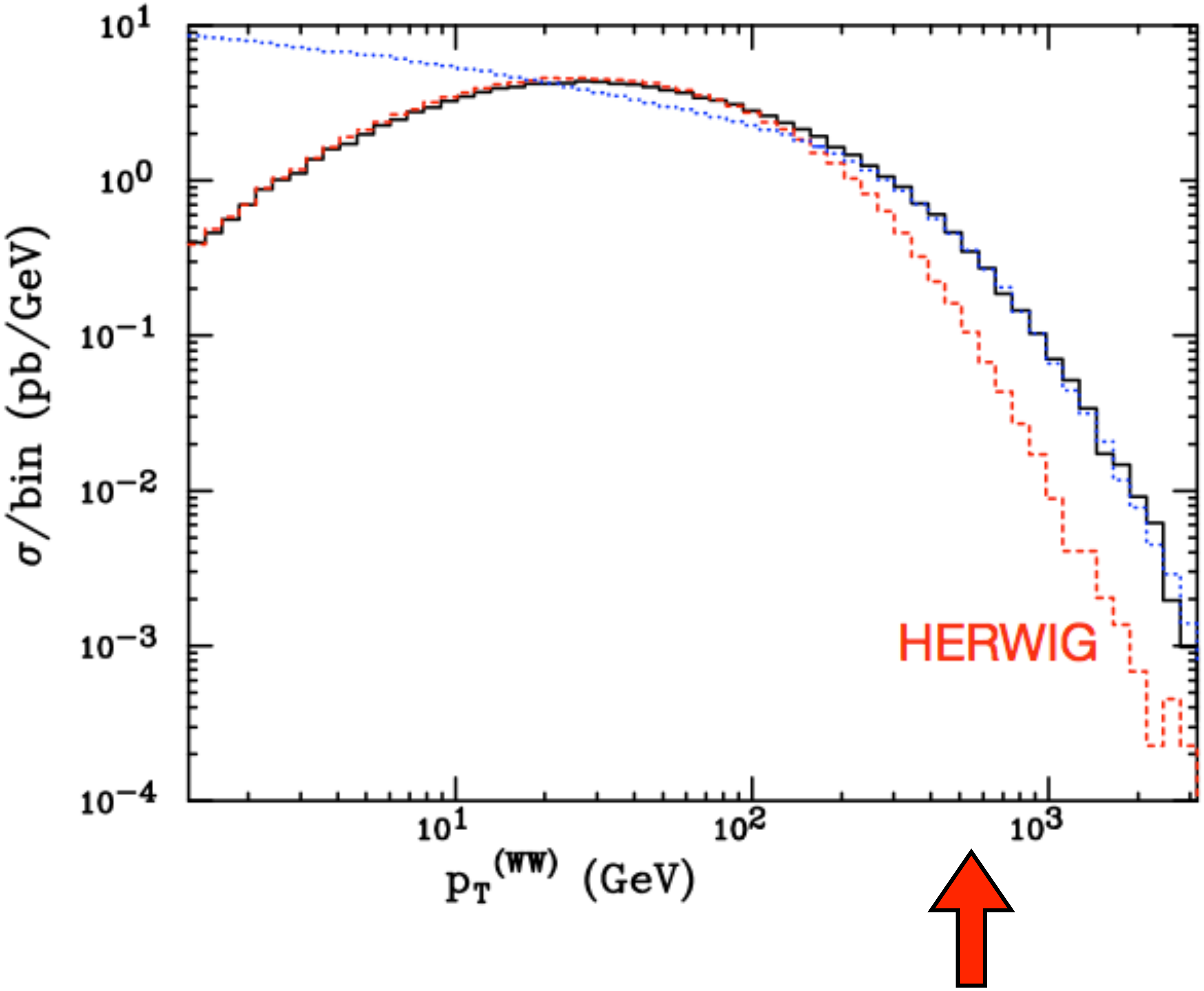
- $H_{1,2}$ denote nucleon and antinucleon
- The "Spin" indicates whether spin correlations in vector boson fusion or top decays are included (✓), neglected (×) or absent (void entry)
- The values of IV, IL, IL_1, and IL_2 control the identities of vector bosons and leptons

IPROC	IV	IL_1	IL_2	Spin	Process
–1706		i	j	✓	$H_1H_2 \to (t \to) b_kf_if'_i(\bar{t} \to)\bar{b}_lf_jf'_j + X$
–2000–IC		i		✓	$H_1H_2 \to (t \to) b_kf_if'_i/(\bar{t} \to)\bar{b}_kf_if'_i + X$
–2001–IC		i		✓	$H_1H_2 \to (\bar{t} \to)\bar{b}_kf_if'_i + X$
–2004–IC		i		✓	$H_1H_2 \to (t \to) b_kf_if'_i + X$
–2030		i	j	✓	$H_1H_2 \to (t \to) b_kf_if'_i(W^- \to) f_jf'_j/$ $(\bar{t} \to)\bar{b}_kf_if'_i(W^+ \to) f_jf'_j + X$
–2031		i	j	✓	$H_1H_2 \to (\bar{t} \to)\bar{b}_kf_if'_i(W^+ \to) f_jf'_j + X$
–2034		i	j	✓	$H_1H_2 \to (t \to) b_kf_if'_i(W^- \to) f_jf'_j + X$
–2850		i	j	✓	$H_1H_2 \to (W^+ \to) l^+_i\nu_i(W^- \to) l^-_j\bar{\nu}_j + X$

MC@NLO: W^+W^- production (LHC)

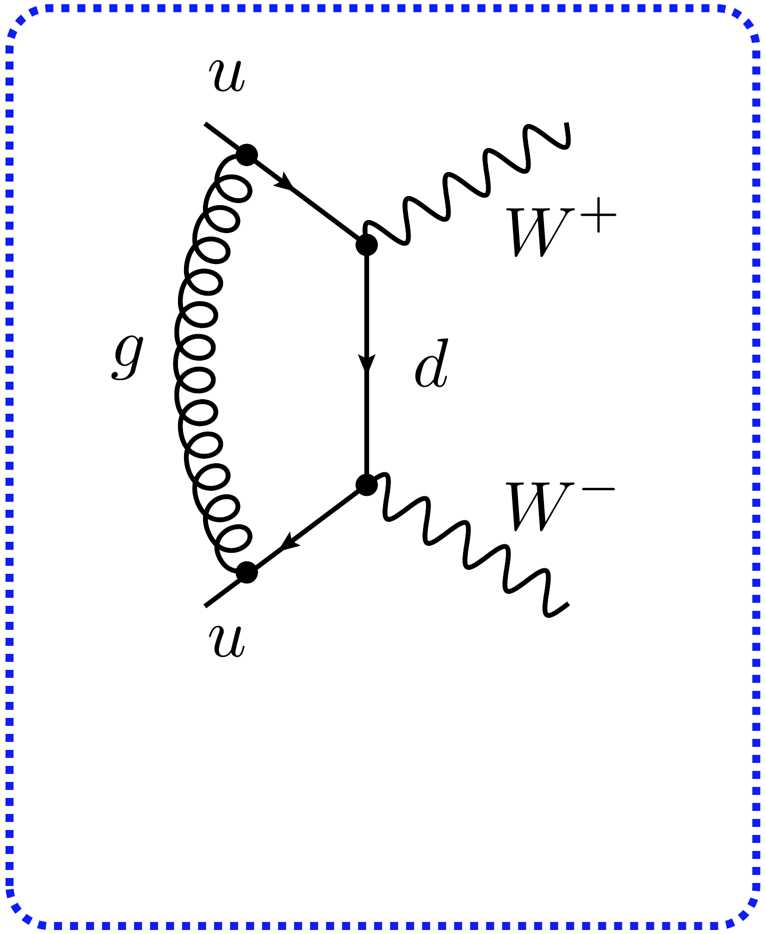


HERWIG

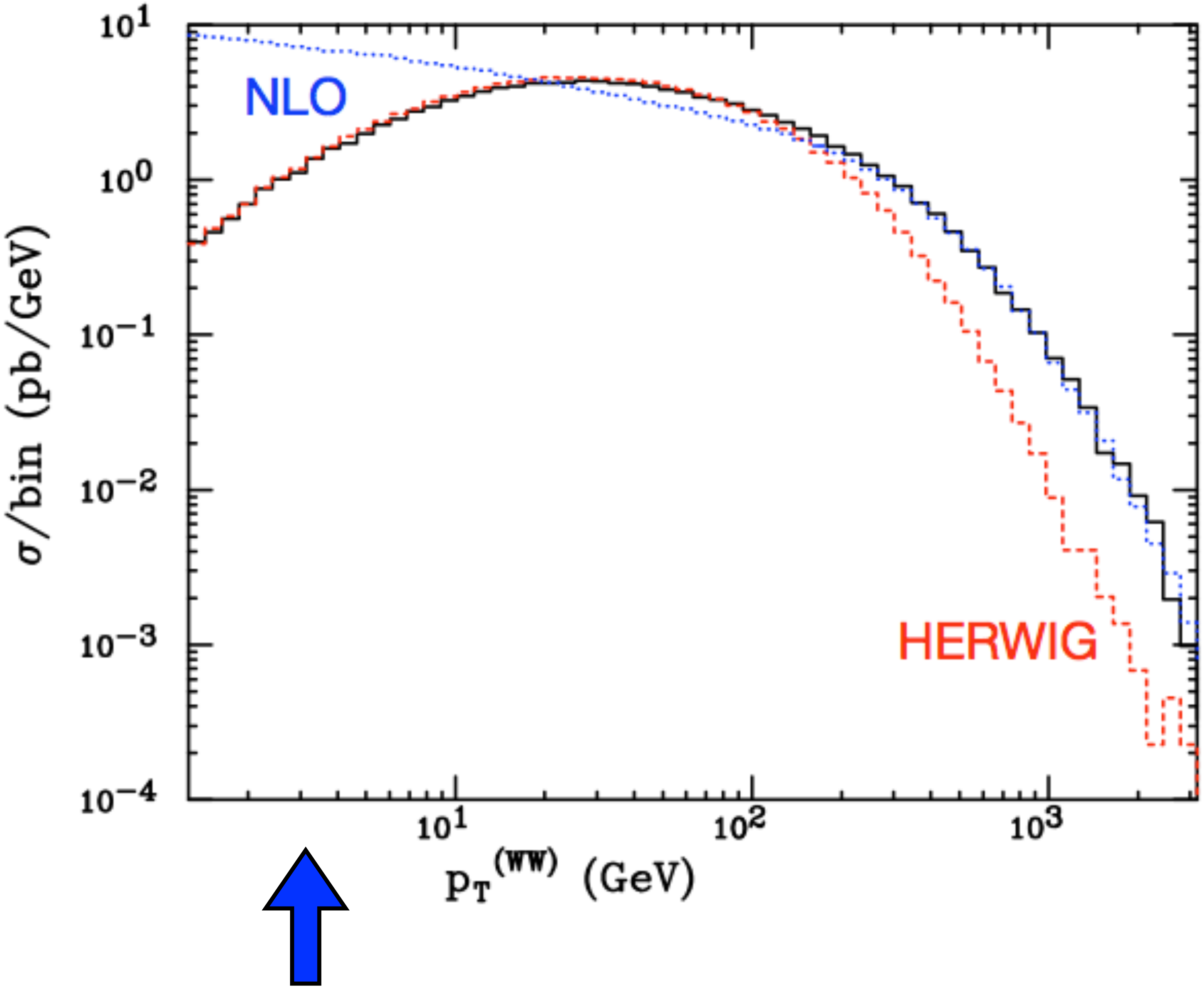


Herwig too soft in the high-p_t region

MC@NLO: W^+W^- production (LHC)

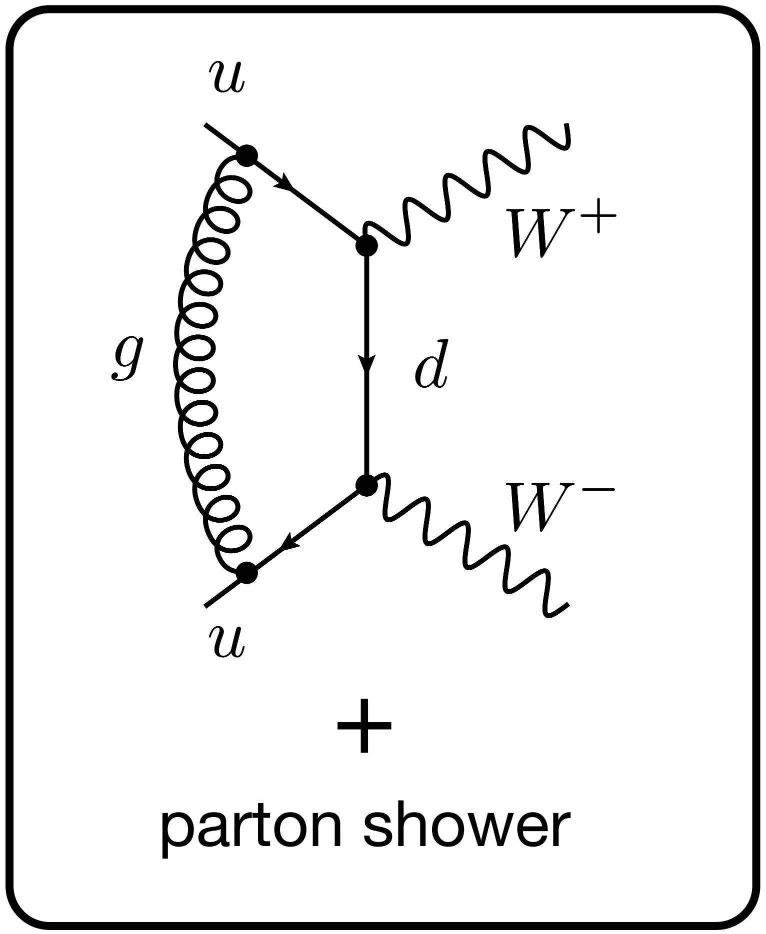


NLO

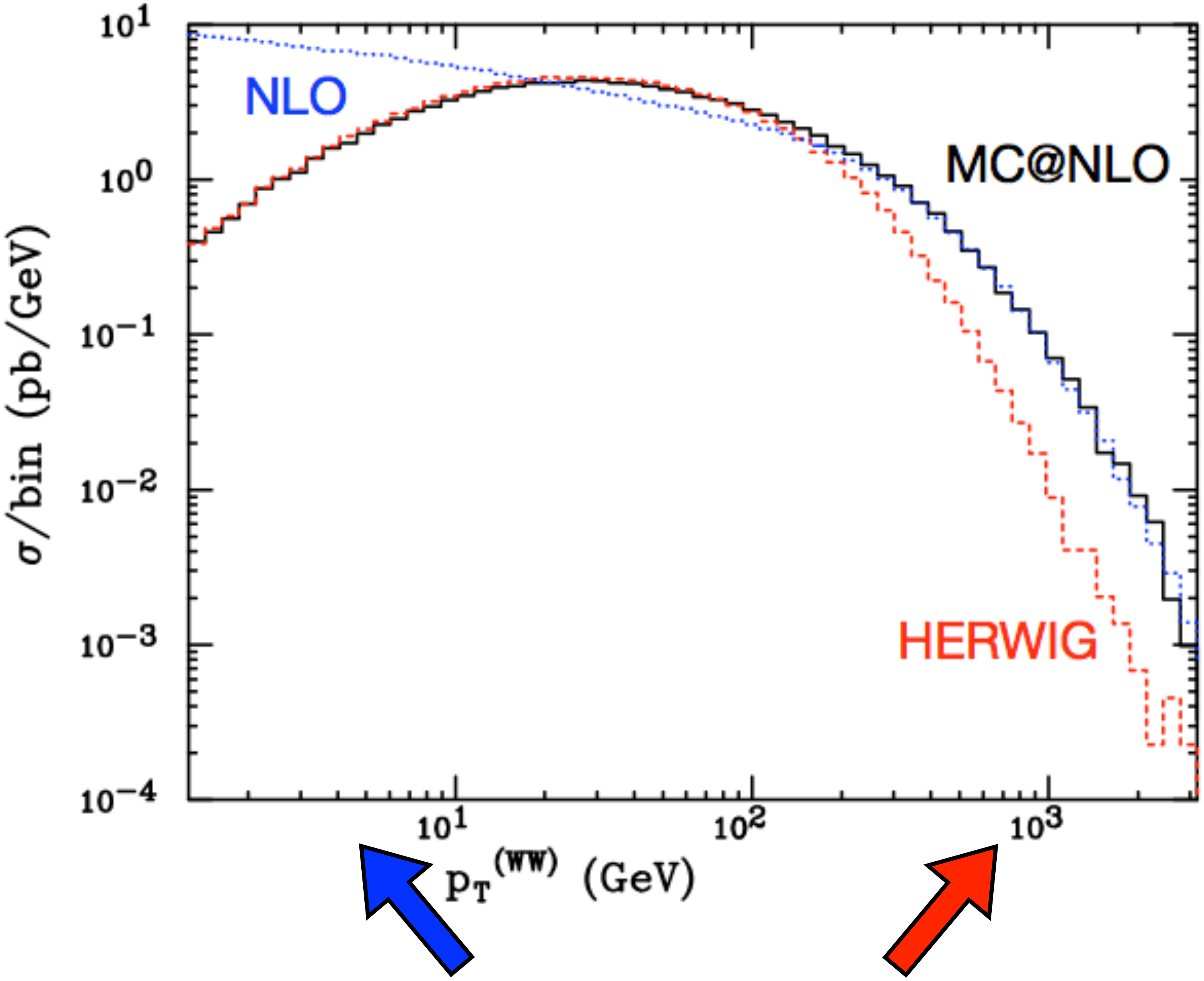


NLO divergent
in the soft region

MC@NLO: W^+W^- production (LHC)



MC@NLO



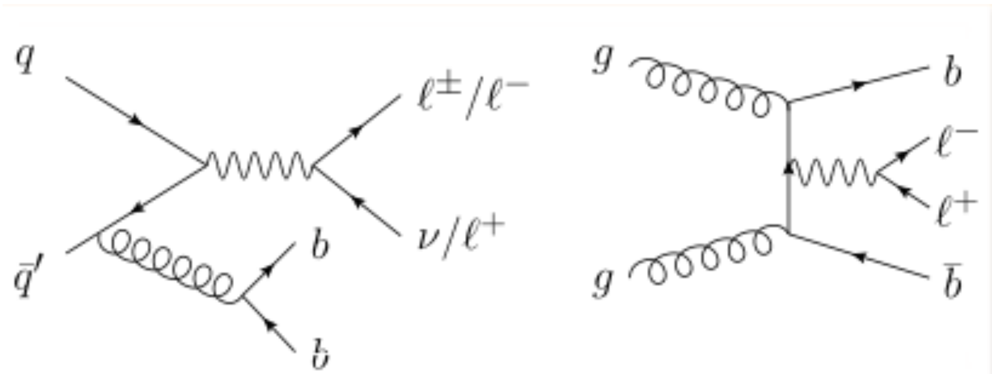
MC@NLO correctly interpolates between the two regimes

Wbb/Zbb in MC@NLO

Irreducible background to pp→ H W and pp→ HZ, with H→ bb

Accuracy: NLO+PS, with spin correlations, heavy-quark mass effects

Frederix et al. 1106.6019



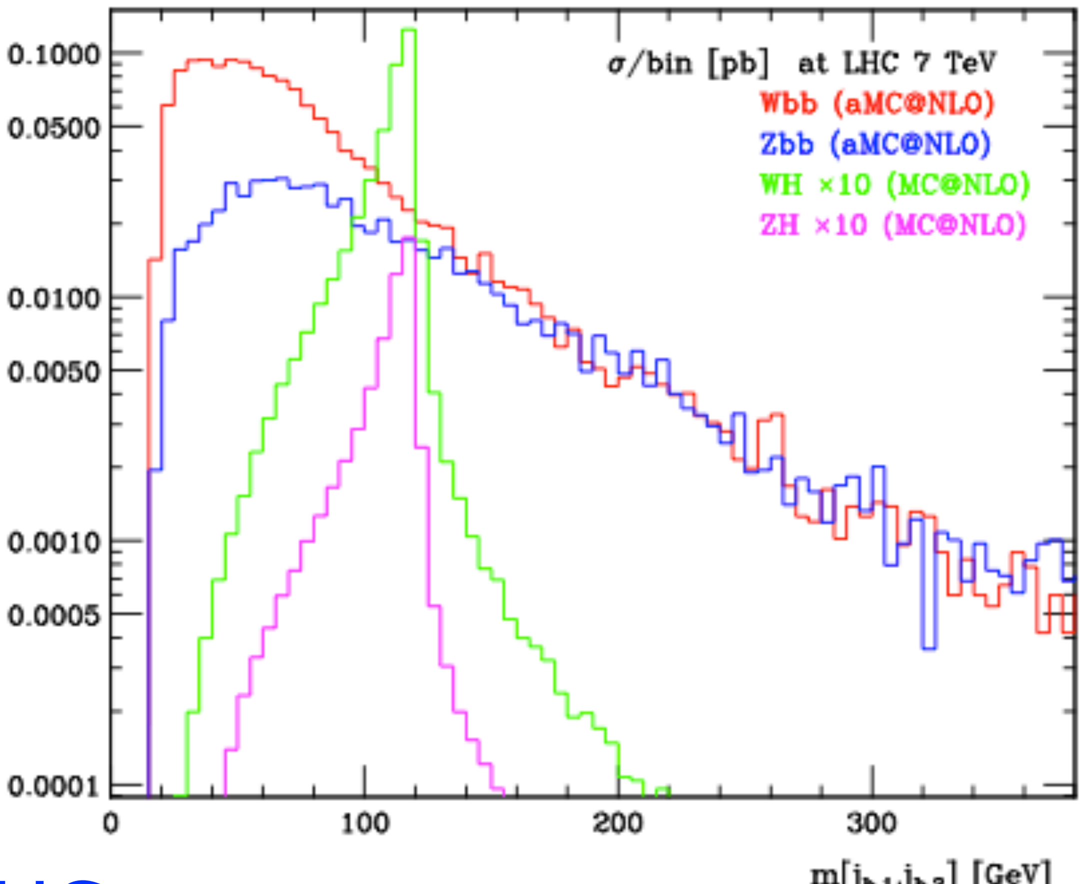
LO: gg channel present only for Zbb. Most differences Wbb vs Zbb due to this

	Cross section (pb)					
	Tevatron $\sqrt{s}$ =1.96 TeV			LHC $\sqrt{s}$ =7 TeV		
	LO	NLO	K factor	LO	NLO	K factor
$\ell\nu b\bar{b}$	4.63	8.04	1.74	19.4	38.9	2.01
$\ell^+\ell^- b\bar{b}$	0.860	1.509	1.75	9.66	16.1	1.67

Wbb/Zbb: ≈ 5 ≈ 2

Reason: gg enhancement in Zbb at the LHC

Example: signal & background with the same accuracy



Also in POWHEG: Oleari, Reina 1105.4488

Jets: five years ago

Cones are IR unsafe!

The Cone is too rigid!

IR unsafety affects jet cross-sections by less than 1%, so don't need to care!

kt collects too much soft radiation!

Cones have a well-defined circular area!

Jet area not well defined in kt: U.E. and pile-up subtraction too difficult!

What about dark towers??

After all, if D=1.35 R Cone and kt are practically the same thing....

Where do jets enter ?

Essentially everywhere at colliders!

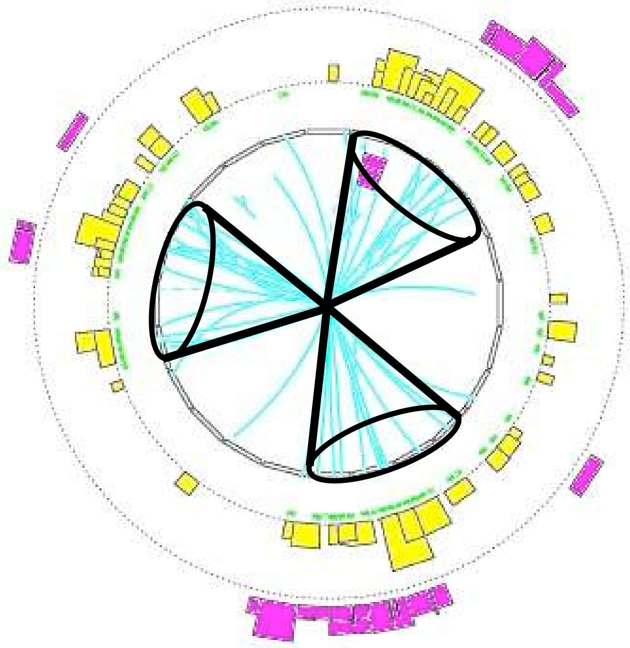
Jets are an essential tool for a variety of studies:

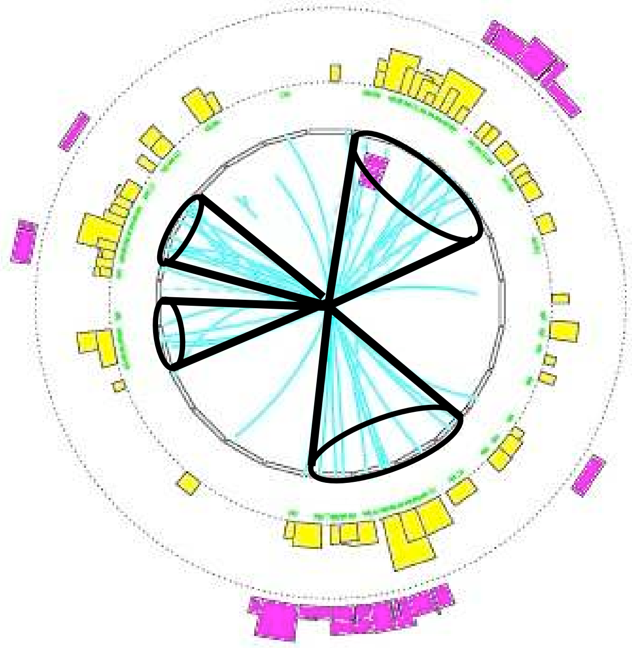
- top reconstruction
- mass measurements
- most Higgs and NP searches
- general tool to attribute structure to an event
- instrumental for QCD studies, e.g. inclusive-jet measurements
  ⇒ important input for PDF determinations

Jets

Jets provide a way of projecting away the multiparticle dynamics of an event ⇒ leave a simple quasi-partonic picture of the hard scattering

The projection is fundamentally ambiguous ⇒ jet physics is a rich subject



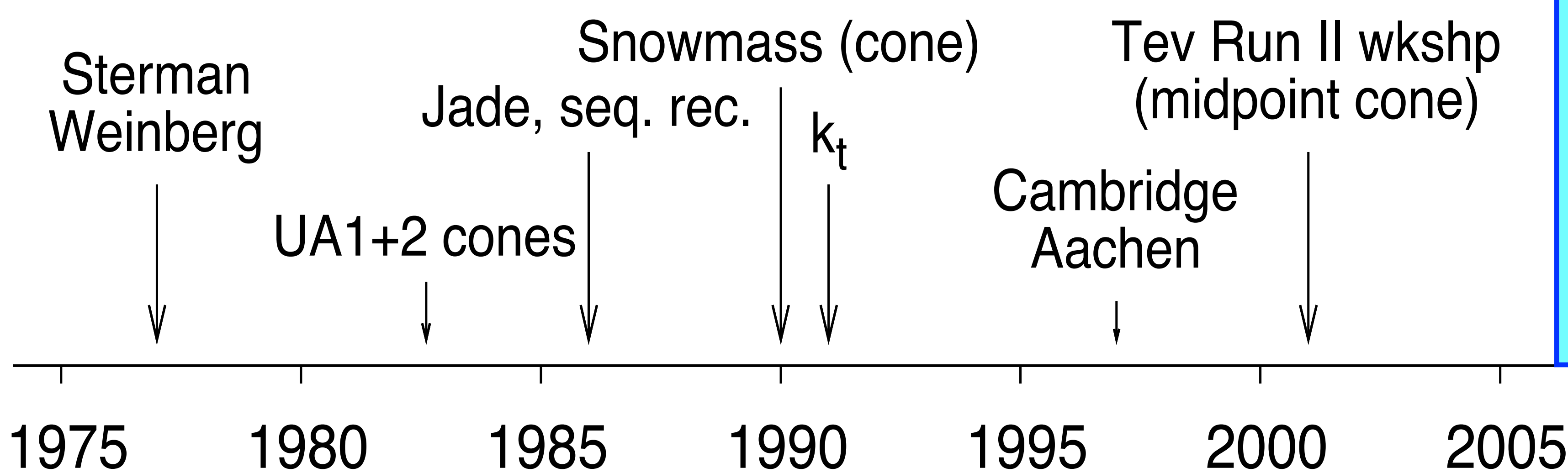


<u>Ambiguities:</u>

1) Which particles should belong to a same jet ?

2) How does recombine the particle momenta to give the jet-momentum?

Jet developments



fast-kt, SISCone, anti-kt, jet-areas, jet-flavour, non-perturbative effects, quality measures, jet-substructure, boosted jets ...

Two broad classes of jet algorithms

Today many extensions of the original Sterman-Weinberg jets.
Modern jet-algorithms divided into two broad classes

Jet algorithms

Cone type	Sequential
(UA1,JetCLU, Midpoint, SISCone..)	(kt-type, Jade, Cambridge/ Aachen...)
top down approach: cluster particles according to distance in coordinate-space	bottom up approach: cluster particles according to distance in momentum-space
Idea: put cones along dominant direction of energy flow	Idea: undo branchings occurred in the PT evolution

Jet requirements

Snowmass accord

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

Inclusive k_t/Durham-algorithm

Catani et. al '92-'93; Ellis&Soper '93

<u>Inclusive algorithm:</u>

1. For any pair of final state particles i,j define the distance

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \min\{k_{ti}^2, k_{tj}^2\}$$

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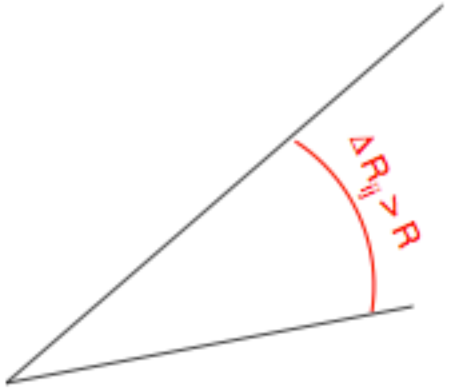
3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle ($\Longrightarrow$ recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

   NB: if $\Delta R_{ij}^2 \equiv \Delta y_{ij}^2 + \Delta \phi_{ij}^2 < R^2$ then partons (ij) are always recombined, so R sets the minimal interjet angle



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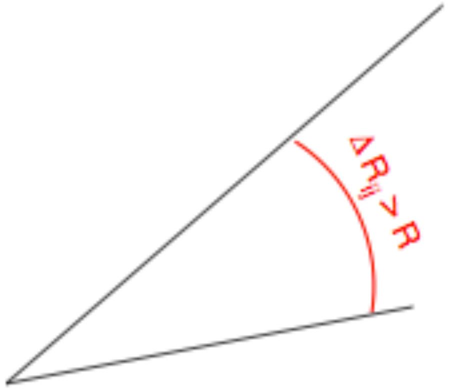


4. repeat the procedure until no particles are left

Exclusive k_t/Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive k_t/Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive version: run the inclusive algorithm but stop when either

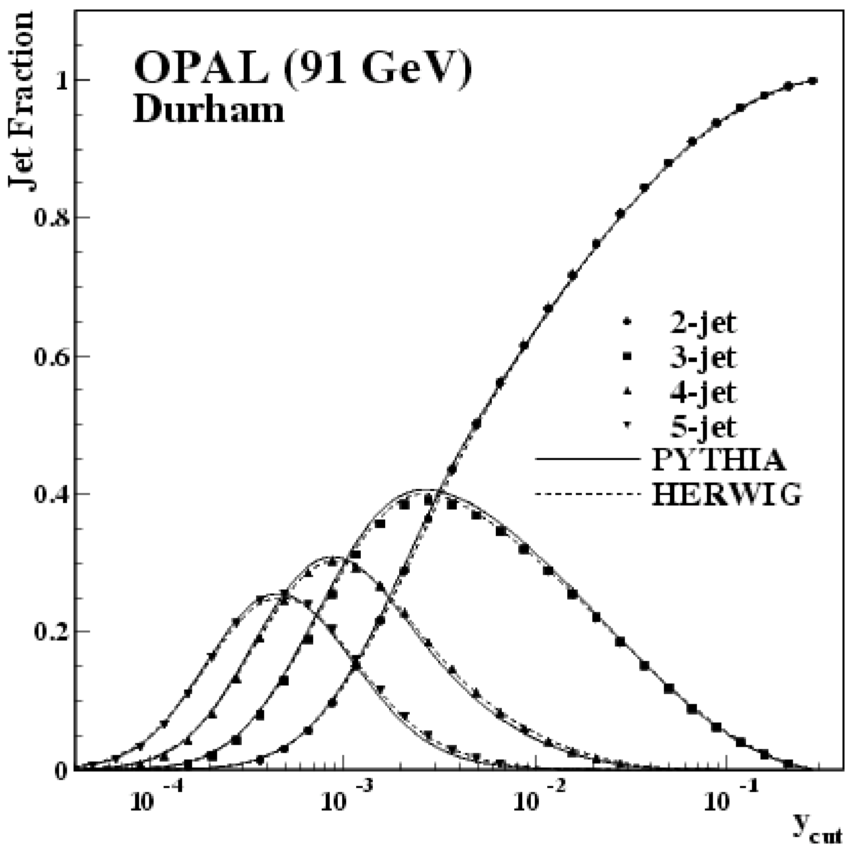
- all d_{ij}, $d_{iB} > d_{cut}$ or
- when reaching the desired number of jets n

k_t/Durham-algorithm in e^+e^-

k_t originally designed in e^+e^-, most widely used algorithm in e^+e^- (LEP)

$$y_{ij} = 2\min\{E_i^2, E_j^2\}\left(1-\cos\theta_{ij}^2\right)$$

- can classify events using y_{23}, y_{34}, y_{45}, y_{56} ...
- resolution parameter related to minimum transverse momentum between jets



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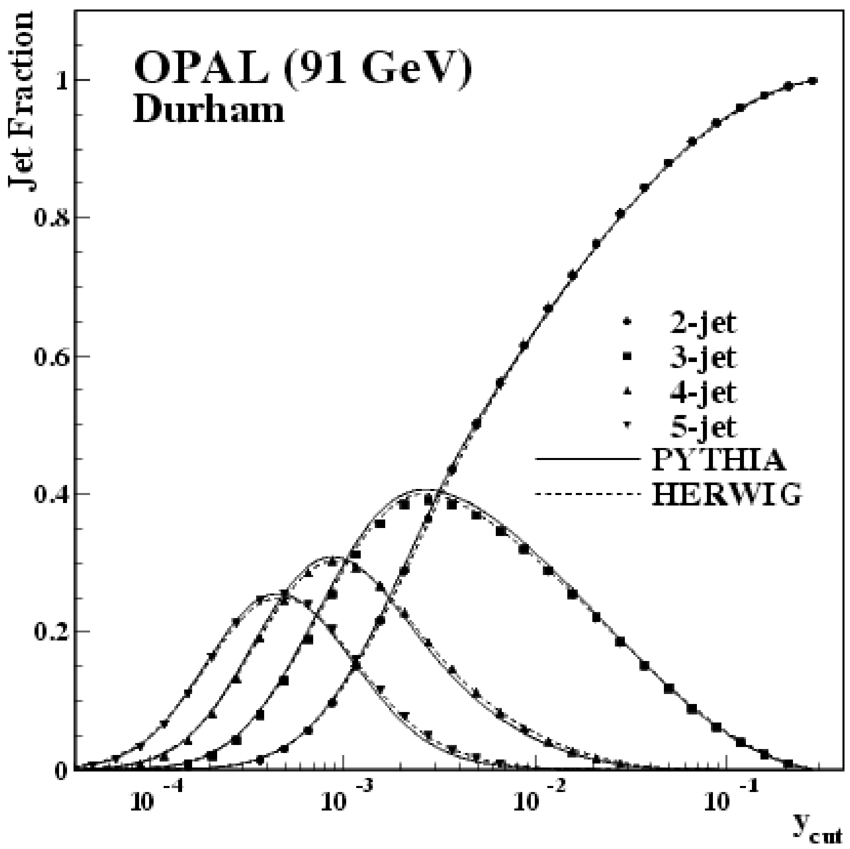

Satisfies fundamental requirements:

1. Collinear safe: collinear particles recombine early on
2. Infrared safe: soft particles do not influence the clustering sequence

$\Rightarrow$ collinear + infrared safety important: it means that cross-sections can be computed at higher order in pQCD (no divergences)!

The CA and the anti-k_t algorithm

<u>The Cambridge/Aachen</u>: sequential algorithm like k_t, but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = 1 \qquad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97; Wobisch & Wengler '99

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The anti-kt algorithm: designed not to recombine soft particles together

$$d_{ij} = \min\{1/k_{ti}^2, 1/k_{tj}^2\}\Delta R_{ij}^2/R^2 \qquad d_{iB} = 1/k_{ti}^2$$

Cacciari, Salam, Soyez '08

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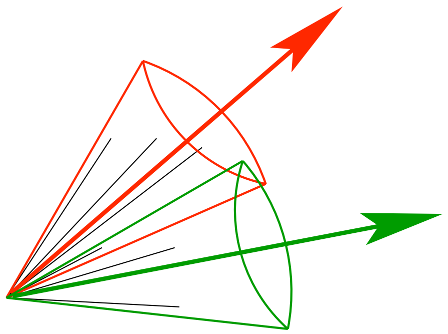
Cacciari, Salam, Soyez '08

anti-kt is the default algorithm for ATLAS and CMS

Cone algorithms

1. A particle i at rapidity and azimuthal angle (y_i, Φ_i) ⊂ cone C iff

$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \leq R_{\mathrm{cone}}$$



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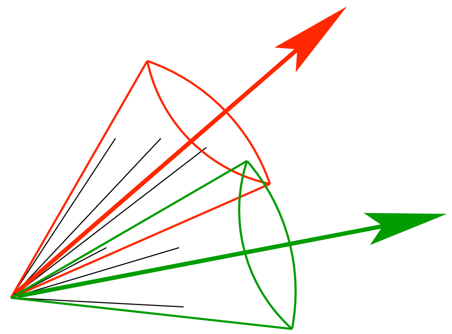
2. Define

$$\bar{y}_C \equiv \frac{\sum_{i \in C} y_i \cdot p_{T,i}}{\sum_{i \in C} p_{T,i}} \qquad \bar{\phi}_C \equiv \frac{\sum_{i \in C} \phi_i \cdot p_{T,i}}{\sum_{i \in C} p_{T,i}}$$



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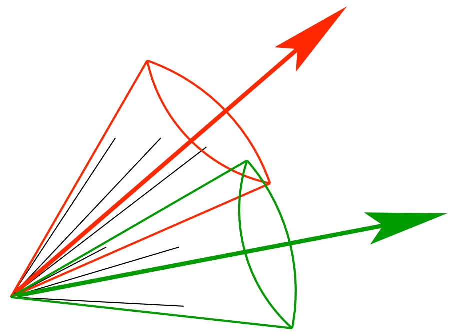

3. If weighted and geometrical averages coincide $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ a stable cone (⇒ jet) is found, otherwise set $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ & iterate

Cone algorithms

1. A particle i at rapidity and azimuthal angle (y_i, Φ_i) ⊂ cone C iff

$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \leq R_{\text{cone}}$$

2. Define

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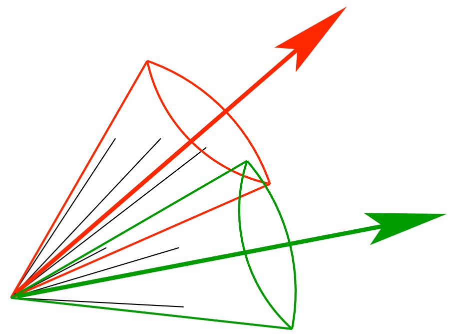

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4. Stable cones can overlap. Run a split-merge on overlapping jets: merge jets if they share more than an energy fraction f, else split them and assign the shared particles to the cone whose axis they are closer to.
   Remark: too small f (<0.5) creates hugh jets, not recommended

Cone algorithms

- The question is where does one start looking for stable cone ?
- The direction of these trial cones are called seeds
- Ideally, place seeds everywhere, so as not to miss any stable cone
- Practically, this is unfeasible. Speed of recombination grows fast with the number of seeds. So place only some seeds, e.g. at the (y, Φ)-location of particles.

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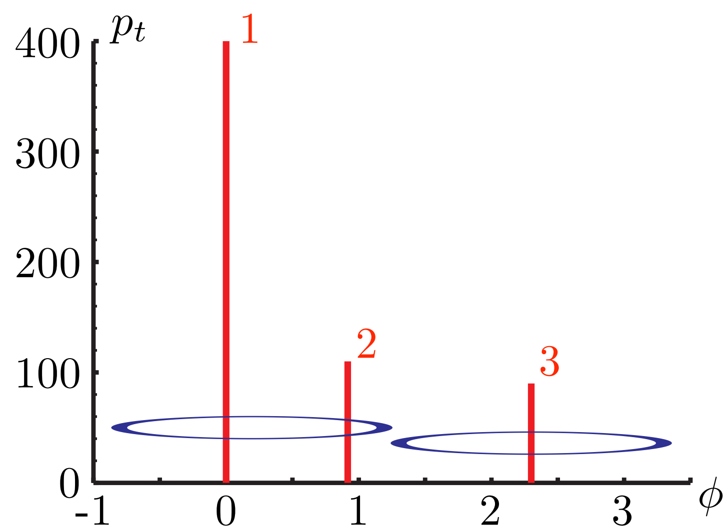
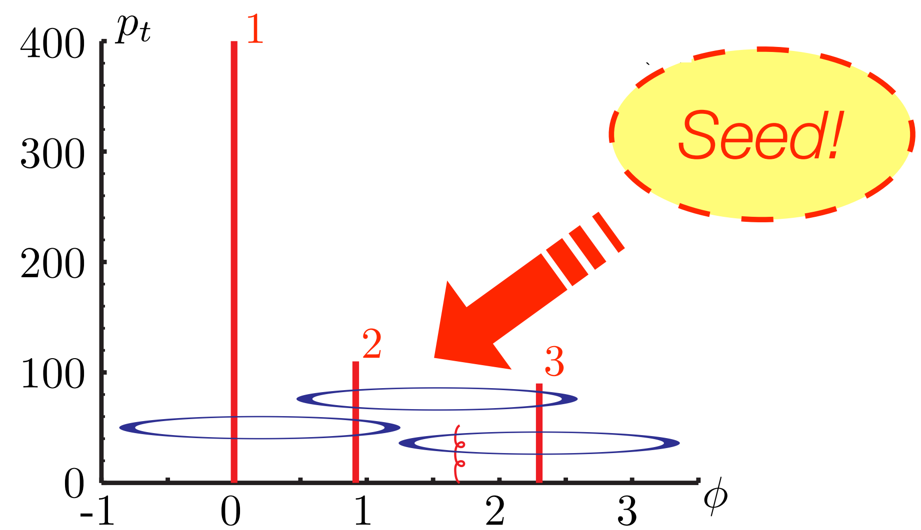
Seeds make cone algorithms infrared unsafe

Jets: infrared unsafety of cones



3 hard ⇒ 2 stable cones



3 hard + 1 soft ⇒ 3 stable cones

Soft emission changes the hard jets ⇒ algorithm is IR unsafe

Midpoint algorithm: take as seed position of emissions and midpoint between two emissions (postpones the infrared satefy problem)

Seedless cones

<u>Solution</u>:

use a seedless algorithm, i.e. consider all possible combinations of particles as candidate cones, so find all stable cones [⇒ jets]

Blazey '00

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The problem:
clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} ys to cluster the event ⇒ prohibitive beyond PT (N=4,5)

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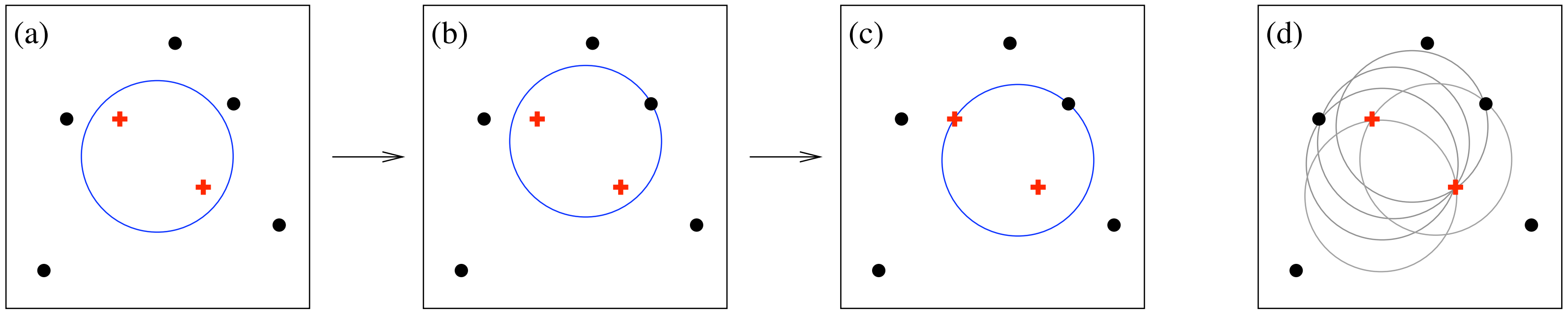
clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} ys to cluster the event ⇒ prohibitive beyond PT (N=4,5)

Better solution:

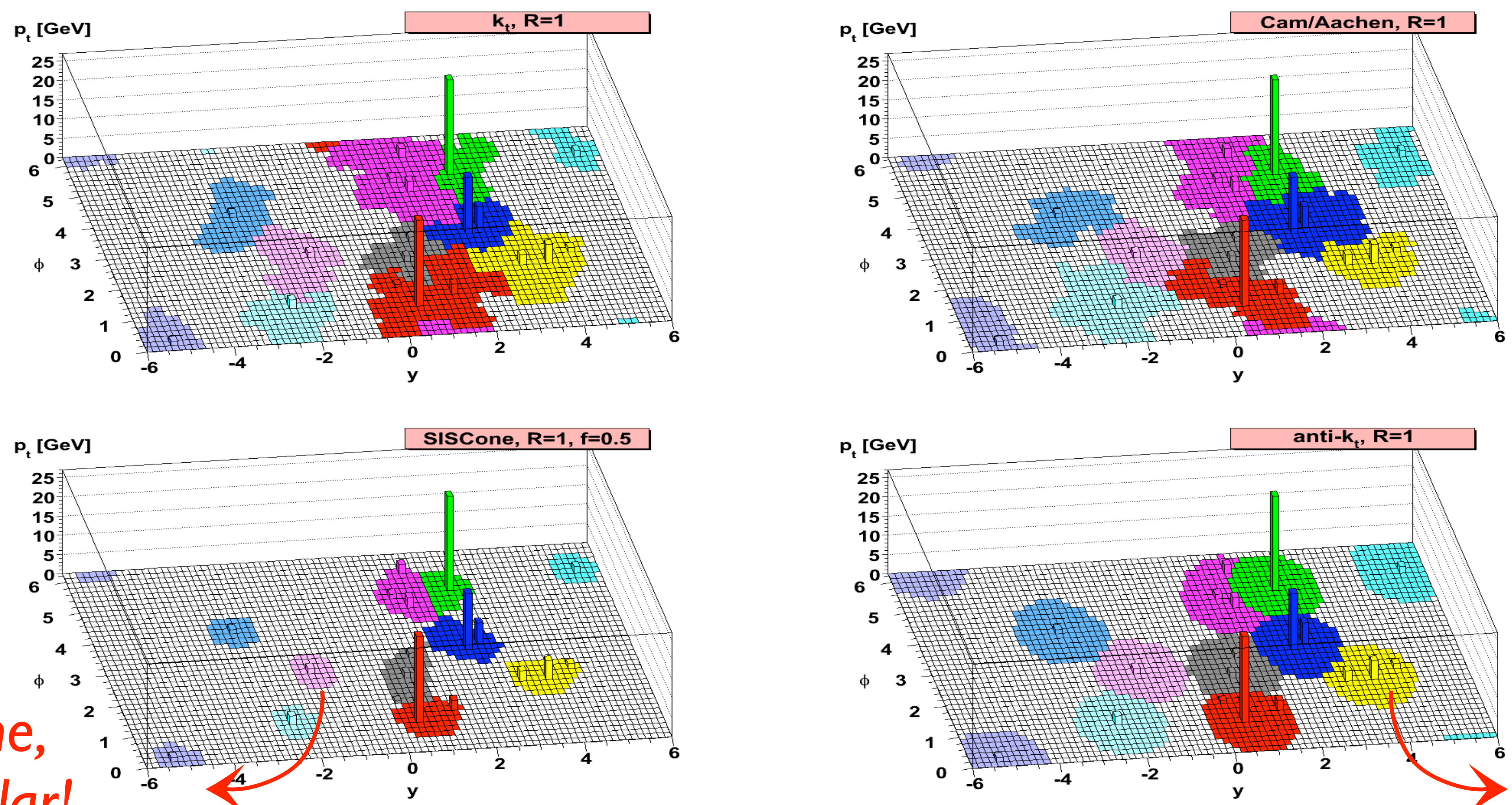
SISCone recasts the problem as a computational geometry problem, the identification of all distinct circular enclosures for points in 2D and finds a solution to that ⇒ $N^2 \ln N$ time IR safe algorithm



Salam, Soyez '07

Jet area

Given an infrared safe, fast jet-algorithm, can define the jet area A as follows: fill the event with an infinite number of infinitely soft emissions uniformly distributed in η-φ and make A proportional to the # of emissions clustered in the jet



NB: cone, not circular!

NB: new anti-kt

What jet areas are good for

jet-area ≡ catching area of the jet when adding soft emissions

⇒ use the jet area to formulate a simple area based subtraction of pile-up events

1. cluster particle with an IR safe jet algorithm
2. from *all jets* (most are pile-up ones) in the event define the median

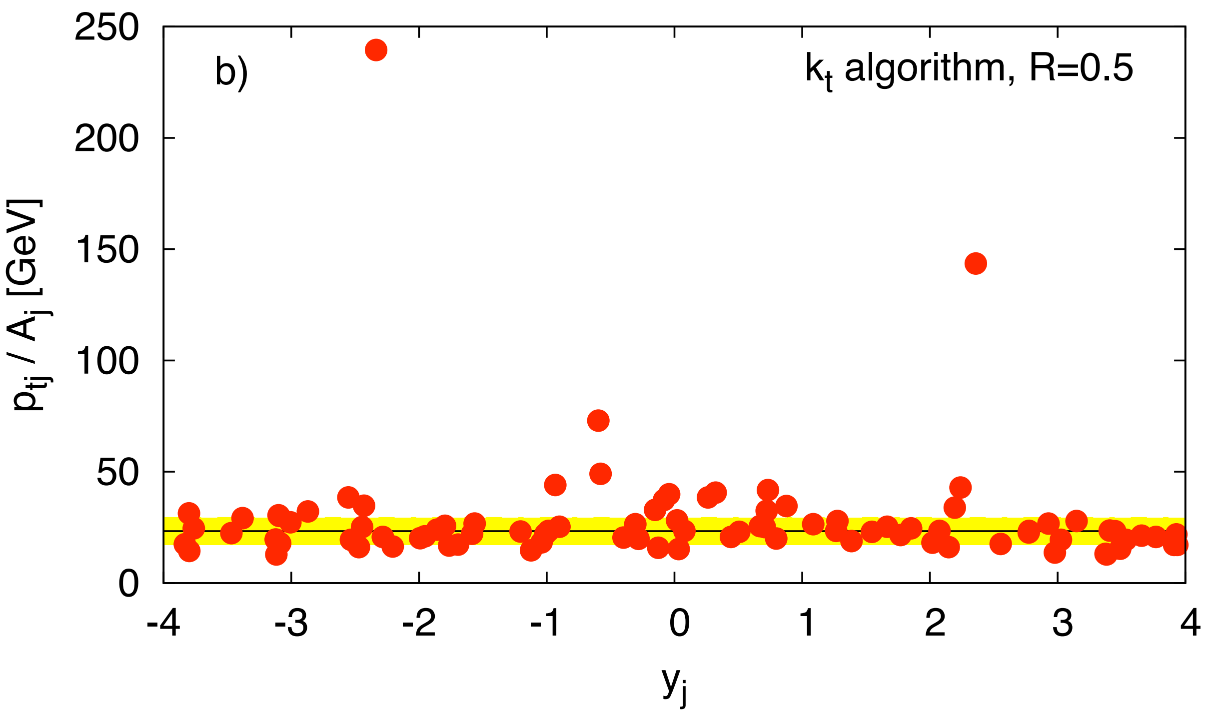
$$\rho = \frac{p_{t,j}}{A_j}$$

3. the median gives the typical p_t/A_j for a given event
4. use the median to subtract off dynamically the soft part of the soft events

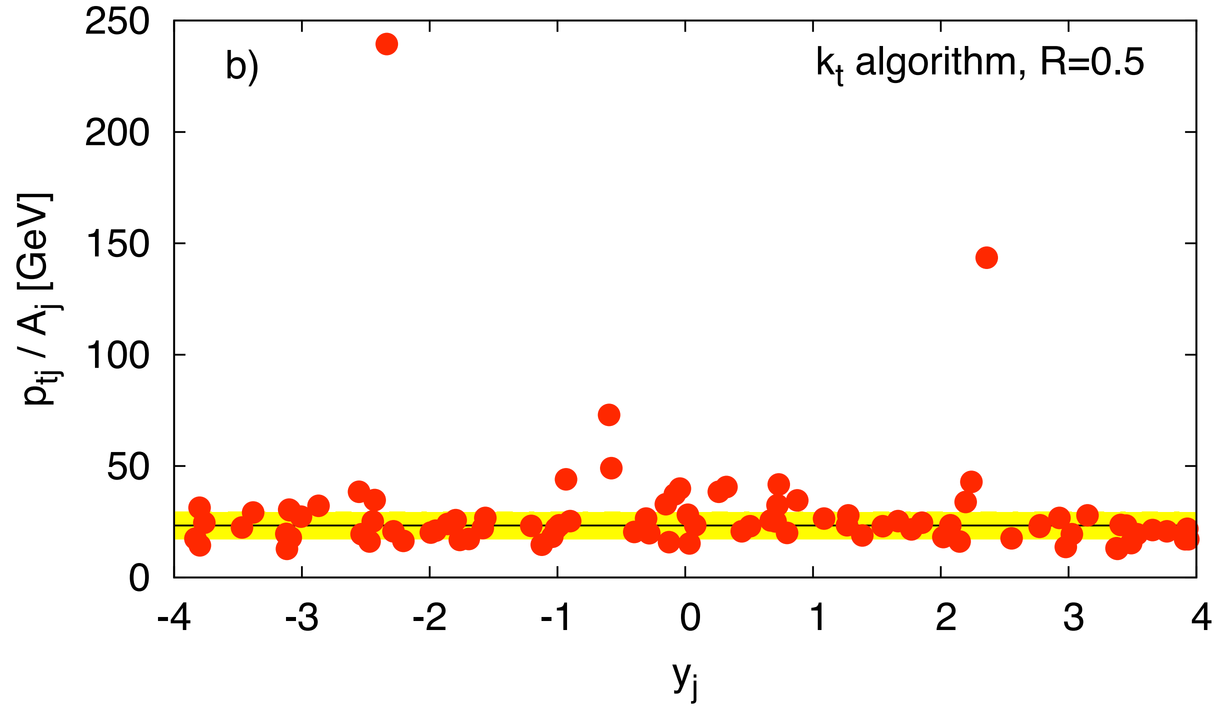
$$p_j^{\mathrm{sub}} = p_j - A_j \rho$$

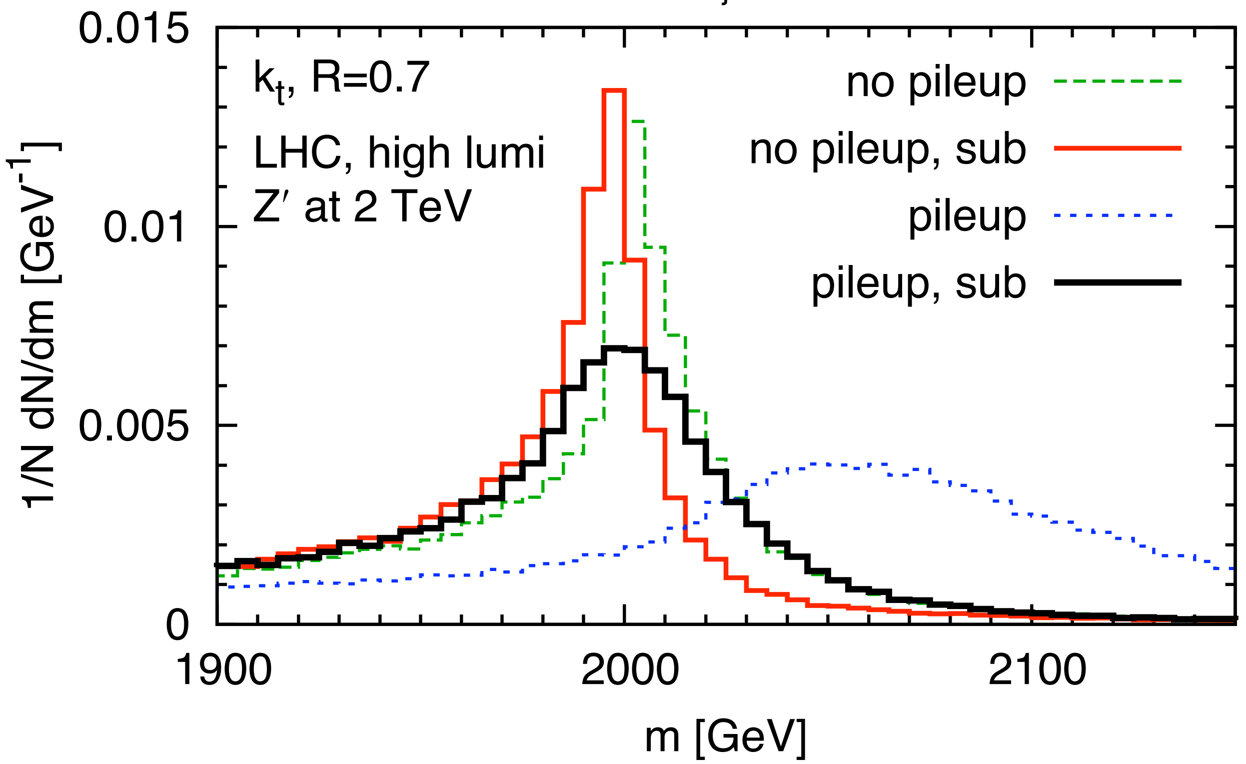
Pileup = generic p-p interaction (hard, soft, single-diffractive...) overlapping with hard scattering

Sample 2 TeV mass reconstruction



Sample 2 TeV mass reconstruction





Cacciari et al. '07

Quality measures of jets

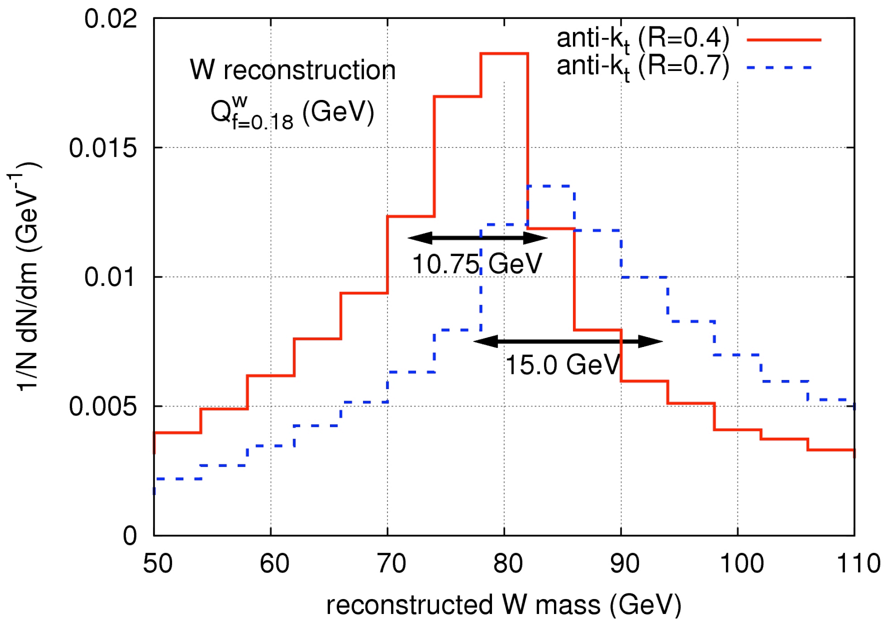
Suppose you are searching for a heavy state (H→gg, Z'→qq, ...)

The object is reconstructed through its decay products
⇒ *Which jet algorithm (JA) is best ? Does the choice of R matter?*

Define: Q_f^w(JA, R) ≡ width of the smallest mass window that contains a fraction f of the generated massive objects

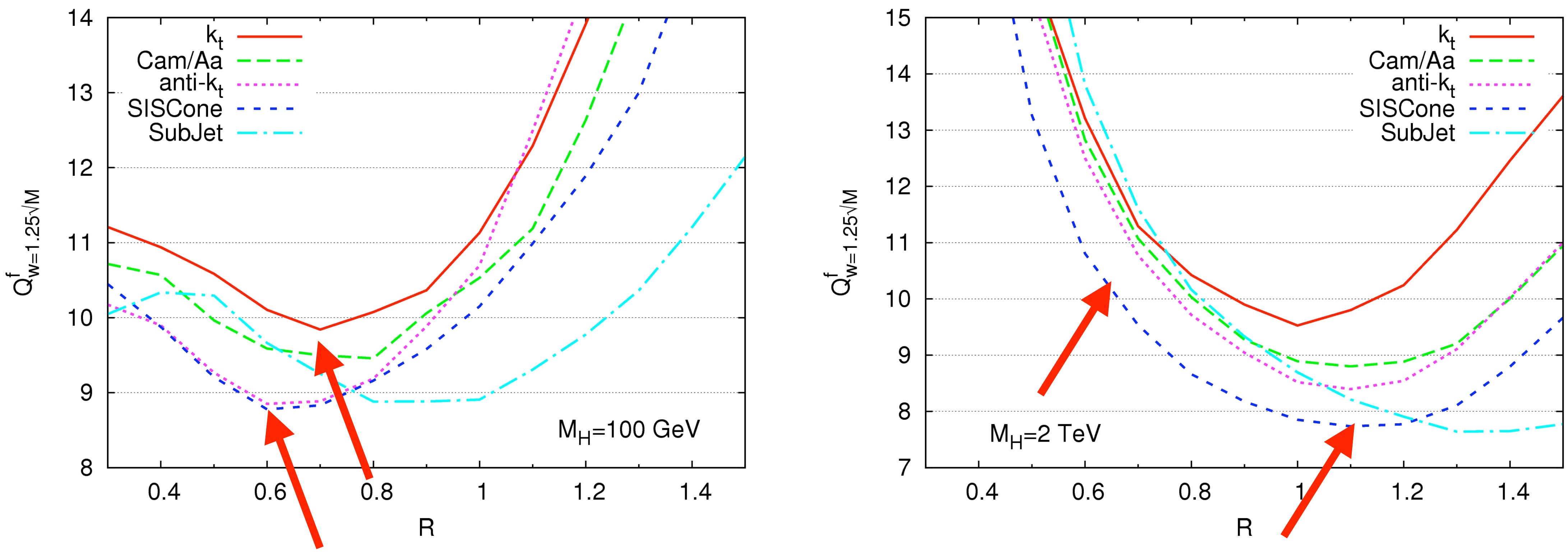
- good algo ⇔ small Q_f^w(JA, R)
- ratios of Q_f^w(JA,R): mapped to ratios of effective luminosity (with same $S/\sqrt{B}$)

$$\mathcal{L}_2 = \rho_{\mathcal{L}} \mathcal{L}_1 \qquad \rho_{\mathcal{L}} = \frac{Q_z^f(JA_2, R_2)}{Q_z^f(JA_1, R_1)}$$



Quality measures: sample results

NB: Here "fake Higgs" = narrow resonance decaying to gluons

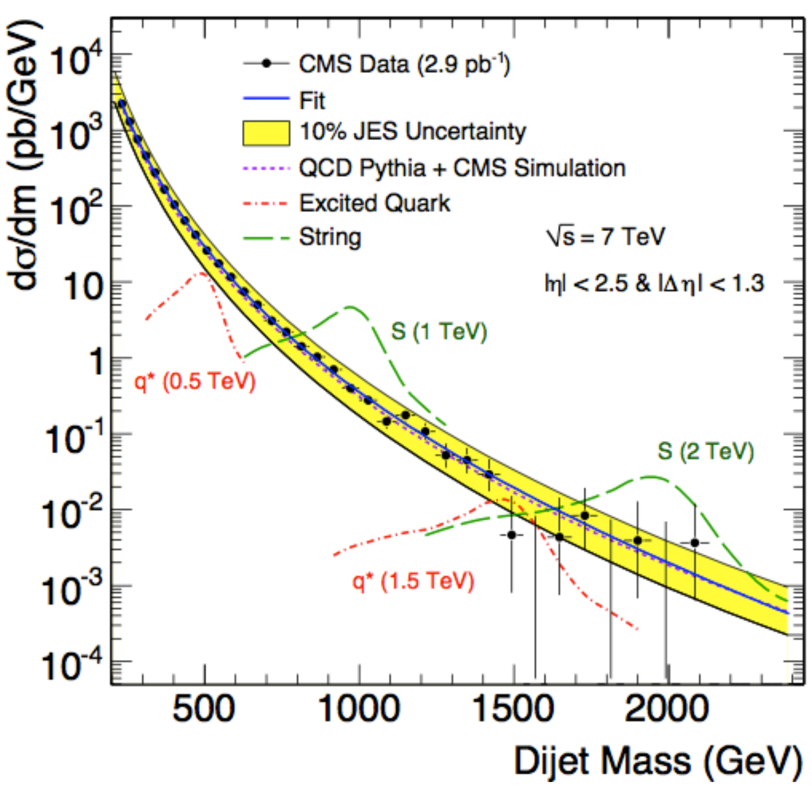


- At 100GeV: use a Tevatron standard algo (k_t, R=0.7) instead of best choice (SISCone, R=0.6 $\Rightarrow$ lose $\rho_{\mathcal{L}} = 0.8$ in effective luminosity
- At 2 TeV: use $M_{Z'}$=100GeV Tevatron best choice instead SIScone, R=1.1 $\Rightarrow$ lose $\rho_{\mathcal{L}} = 0.6$ in effective luminosity

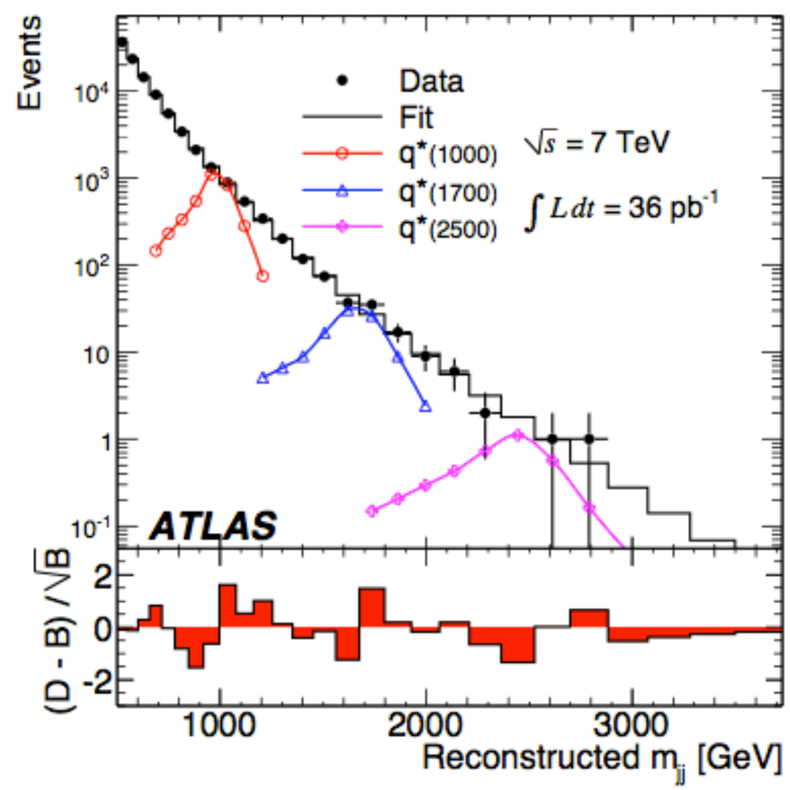
A good choice of jet-algorithm can make the difference
Bad choice of jet-algorithm $\Leftrightarrow$ loose in discrimination power

Jets today at the LHC

ATLAS and CMS adopted as default jet-algorithm: anti-k_t , unfortunately with different default R 0.4 & 0.6 [ATLAS] 0.5 & 0.7 [CMS]



CMS PRL 105 (2010)



ATLAS New J. Phys 13 (2011)

$$d_{ij} = \frac{1}{\max(k_{ti}^2, k_{tj}^2)} \frac{\Delta R_{ij}}{R}$$

Cacciari, Salam, Soyez '08

So far, at the LHC jets could probe the highest energy scales

~ 4 TeV

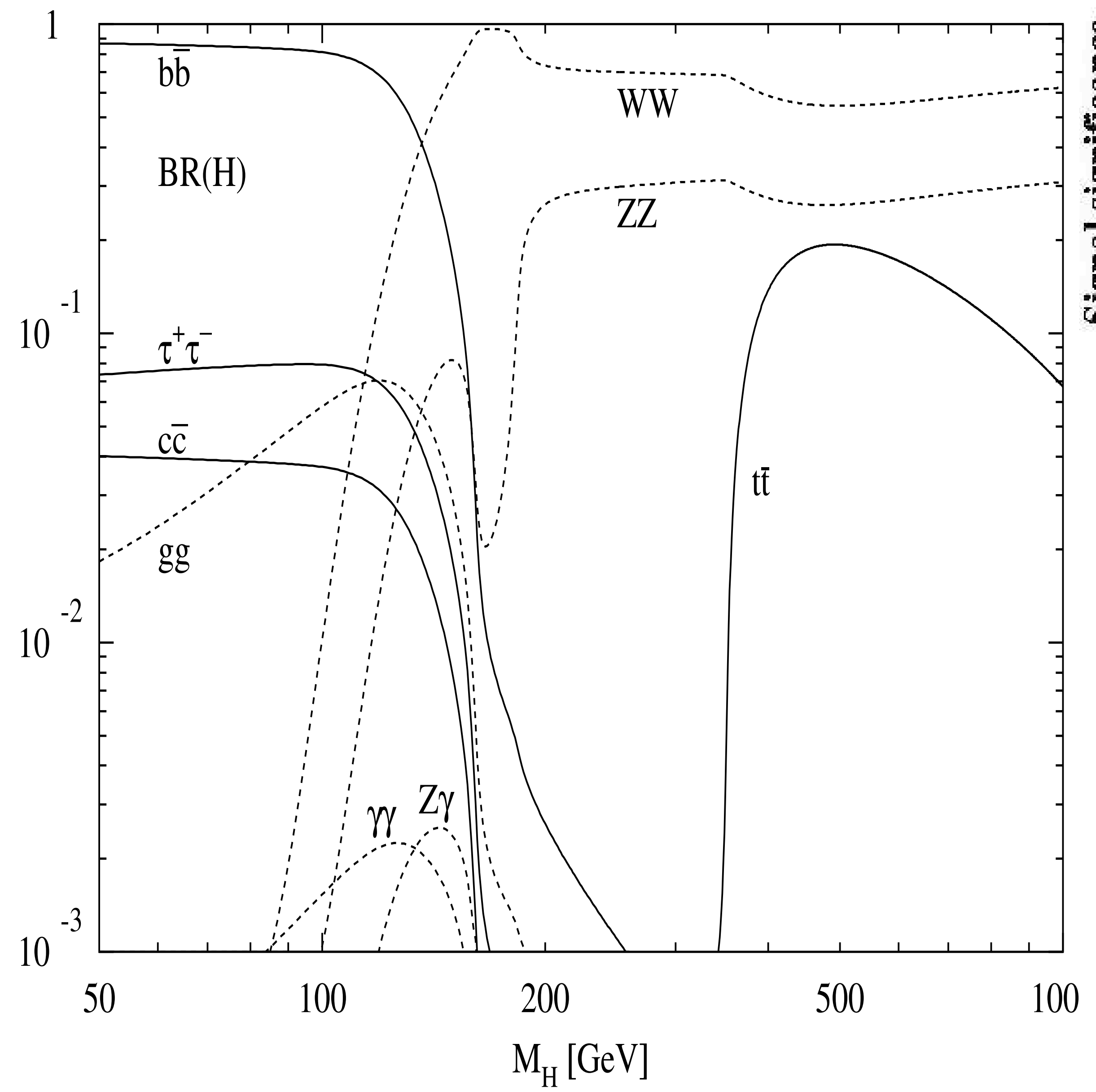
[Tevatron ~ 1 TeV]

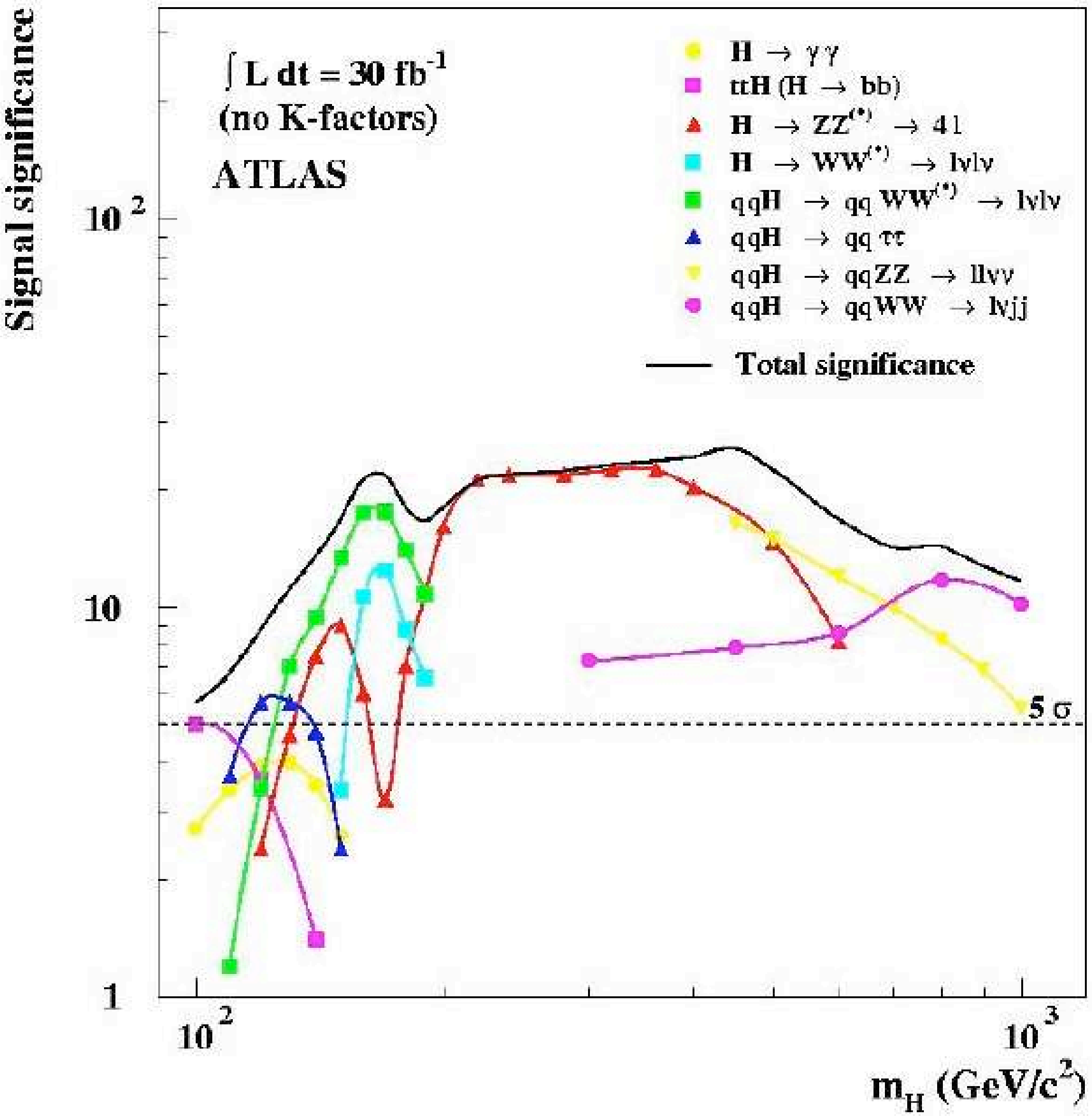
Also used: Cambridge-Aachen (CA), k_t algorithm and SISCone

Catani et al. '92-'93; Ellis and Soper '93; Dokshitzer et al. '97; Salam and Soyez '08

First time only infrared-safe algorithms are used systematically at a collider!

Z/W+ H (→bb) rescued ?





⇒ Light Higgs hard: Higgs mainly produced in association with Z/W, decay H→bb is dominant, but overwhelmed by QCD backgrounds

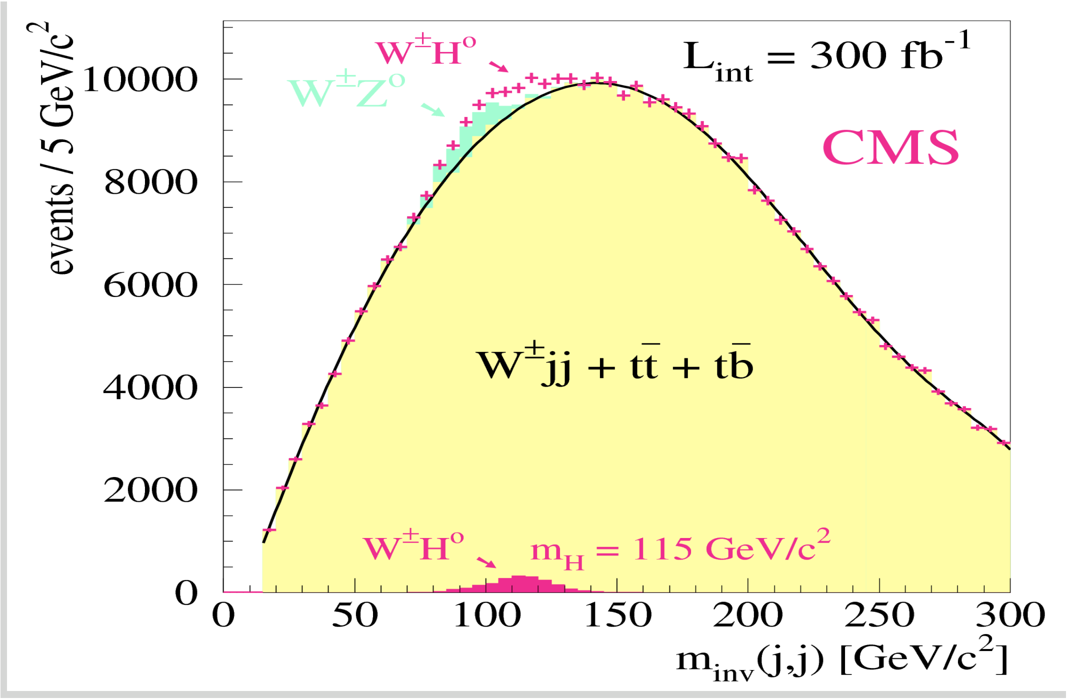
Z/W+ H (→bb) rescued ?

Recall why searching for pp →WH(bb) is hard:

$\sigma(\mathrm{pp} \to WH(bb)) \sim \text{few pb}$ $\sigma(\mathrm{pp} \to Wbb) \sim \text{few pb}$

$\sigma(\mathrm{pp} \to tt) \sim 800\mathrm{pb}$ $\sigma(\mathrm{pp} \to Wjj) \sim \text{few } 10^4\mathrm{pb}$ $\sigma(\mathrm{pp} \to bb) \sim 400\mathrm{pb}$

⇒ signal extraction very difficult



Conclusion [ATLAS TDR]:

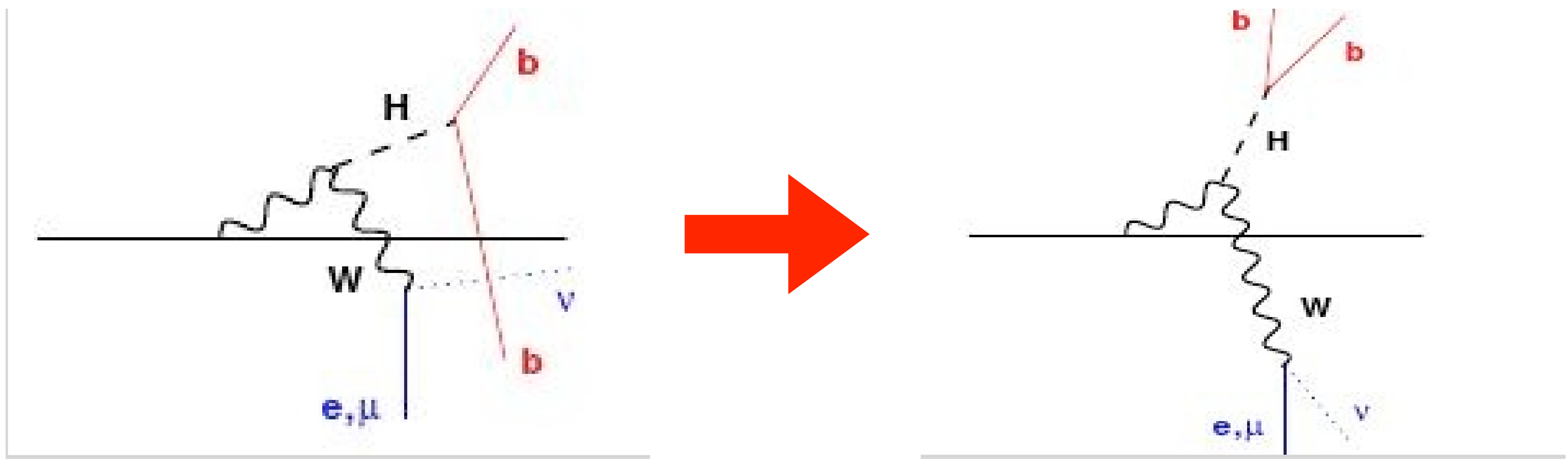
The extraction of a signal from H → bb decays in the WH channel will be very difficult at the LHC even under the most optimistic assumptions [...]

Z/W+ H (→bb) rescued ?

But ingenious suggestions open up to window of opportunity

Central idea: require high-p_T W and Higgs boson in the event

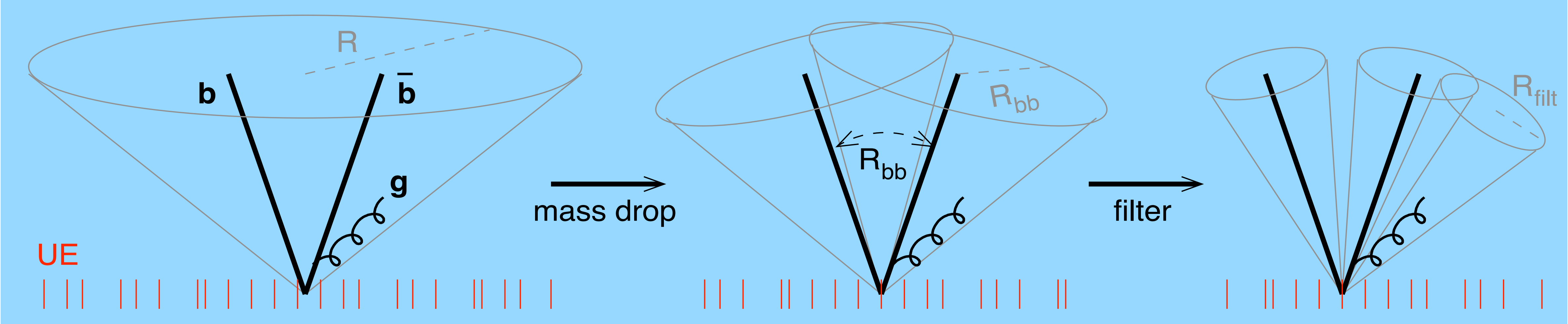
- leads to back-to-back events where two b-quarks are contained within the same jet
- high p_T reduces the signal but reduces the background much more
- improve acceptance and kinematic resolution



Z/W+ H (→bb) rescued ?

Then use a jet-algorithm geared to exploit the specific pattern of H → bb vs g → gg, q → gg

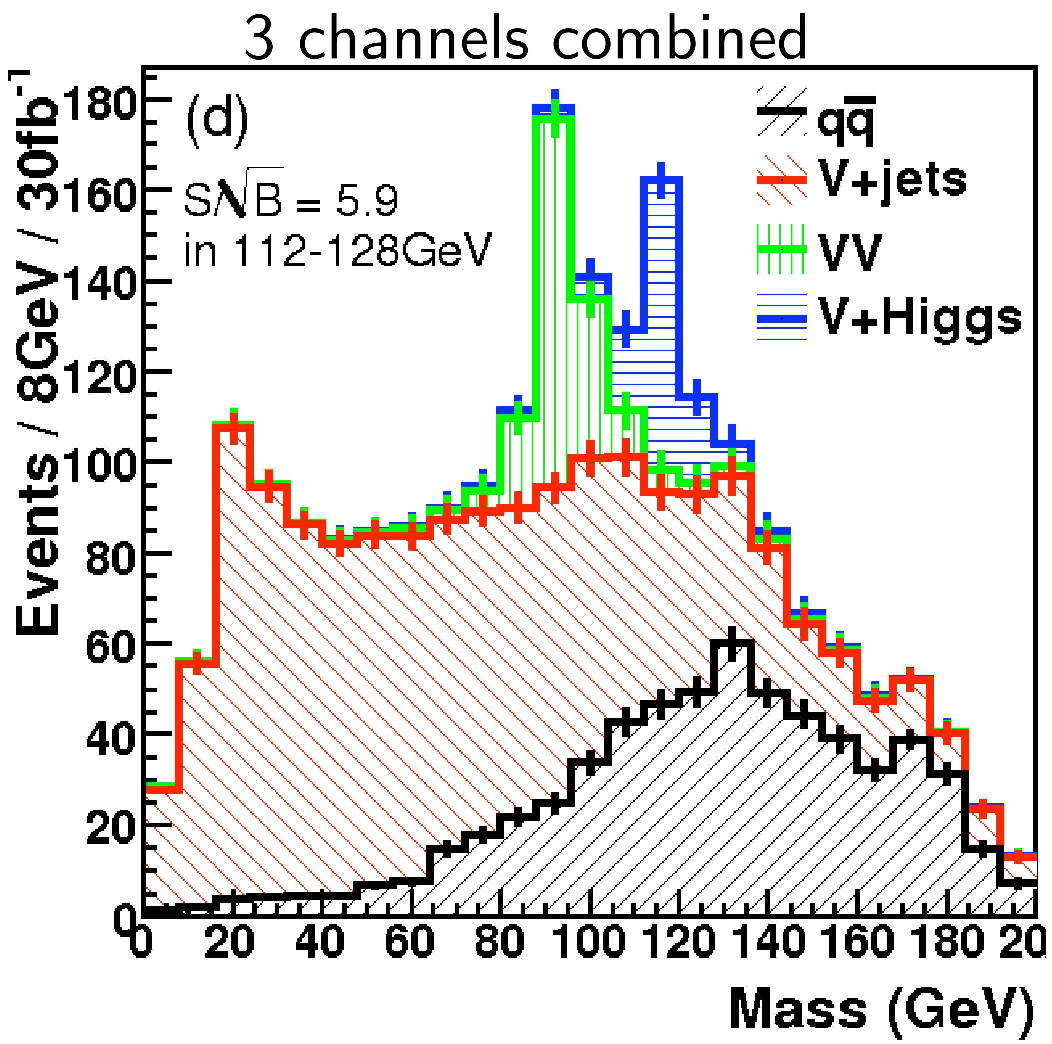
- QCD partons prefer soft emissions (hard → hard + soft)
- Higgs decay prefers symmetric splitting
- try to beat down contamination from underlying event
- try to capture most of the perturbative QCD radiation



1. cluster the event with e.g. CA algo and large-ish R
2. undo last recomb: large mass drop + symmetric + b tags
3. filter away the UE: take only the 3 hardest sub-jets

Z/W+ H (→bb) rescued ?

Mass of the three hardest sub-jets:



- with common & channel specific cuts: p_{tV}, p_{tH} > 200GeV , ...
- real/fake b-tag rate: 0.7/0.01
- NB: very neat peak for WZ (Z →bb) Important for calibration

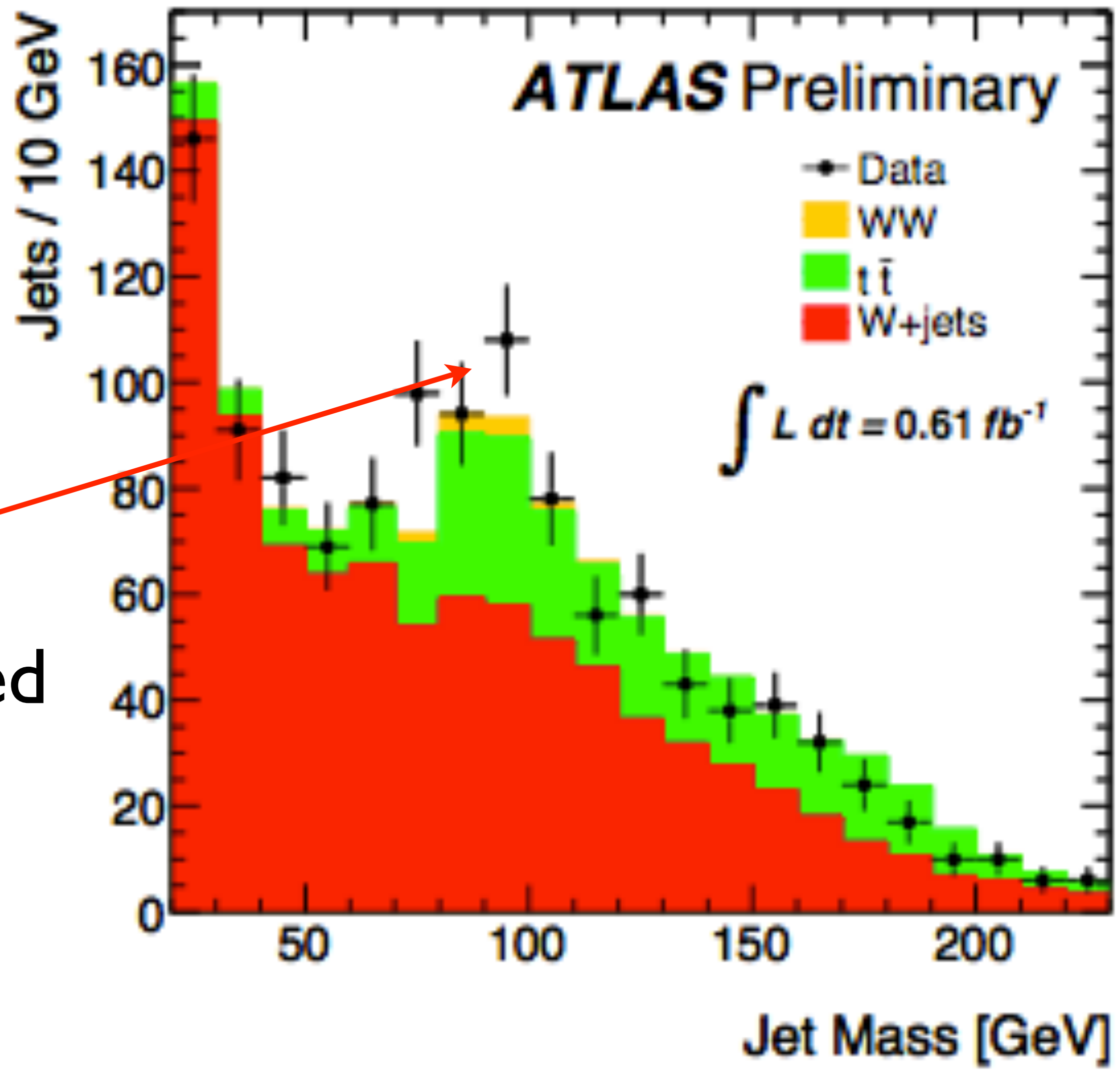
Butterworth, Davison, Rubin, Salam '08

5.9σ at 30 fb^{-1}:VH with H → bb recovered as one of the best discovery channels for light Higgs

Z/W+ H (→bb) rescued ?

These very recent techniques already in use at the LHC !

Presented at EPS 2011



Example relevant for WH(→bb):
single jet hadronic mass in W+1j

Z peak evident. Very promising

Expect many new results with boosted techniques at higher statistics soon

Recap on jets

- Two major jet classes: sequential (k_t, CA, ...) and cones (UA1, midpoint, ...)
- Jet algo is fully specified by: clustering + recombination + split merge or removal procedure + all parameters
- Standard cones based on seeds are IR unsafe
- SISCone is new IR safe cone algorithm (no seeds) and anti-kt a new sequential algorithm
- Using IRunsafe algos you can not use perturbative QCD calculations
- With IRsafe algo: sophisticated studies e.g. jet-area for pile-up subtraction
- Not all algorithms fare the same for BSM/Higgs searches: quality measures
- Recent applications using boosted techniques and jet substructure (Higgs example)