We present a few examples of some possible phenomenological interest. For future convenience and for a direct comparison with the literature on the Standard Model, we rewrite here the interaction Lagrangian of quarks and leptons with the neutral gauge bosons Z and Z_I' in 4-component form. We obtain for the leptons

$$
\begin{aligned}
\mathcal{L}_l \;=\;& e\bar{e}_i\gamma^\mu e_i - \frac{g_2}{2\sqrt{2}}W^+\bar{e}_j\gamma^\mu(1-\gamma_5)\nu_i\,\mathcal{U}^\nu_{ji} - \frac{g_2}{2\sqrt{2}}W^-\bar{\nu}_j\gamma^\mu(1-\gamma_5)e_i\,\mathcal{U}^{\nu\dagger}_{ji} \\
& -\frac{g_2}{2\cos\theta_W}Z^\mu\bar{\nu}_i\left(g_V^{\nu-Z}\gamma^\mu - g_A^{\nu-Z}\gamma^\mu\gamma^5\right)\nu_i - \frac{g_2}{2\cos\theta_W}Z^\mu\bar{e}_i\left(g_V^{e-Z}\gamma^\mu - g_A^{e-Z}\gamma^\mu\gamma^5\right)e_i \\
& -g_I Z_I'^{\mu}\bar{\nu}_i\left(g_V^{\nu-Z_I'}\gamma^\mu - g_A^{\nu-Z_I'}\gamma^\mu\gamma^5\right)\nu_i - g_I Z_I'^{\mu}\bar{e}_i\left(g_V^{e-Z_I'}\gamma^\mu - g_A^{e-Z_I'}\gamma^\mu\gamma^5\right)e_i
\end{aligned}
\tag{5.35}
$$

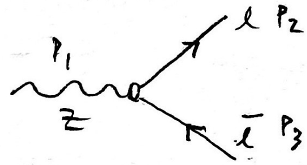
and

$$
\begin{aligned}
\mathcal{L}_q \;=\;& -e\left(\frac{2}{3}\bar{u}_i\gamma^\mu u_i - \frac{1}{3}\bar{d}_i\gamma^\mu d_i\right)A_\mu \\
& -\frac{g_2}{2\sqrt{2}}W^+\bar{u}_j\gamma^\mu(1-\gamma_5)d_i\,\mathcal{U}^q_{ji} - \frac{g_2}{2\sqrt{2}}W^-\bar{d}_j\gamma^\mu(1-\gamma_5)u_i\,\mathcal{U}^{q\dagger}_{ji} \\
& -\frac{g_2}{2\cos\theta_W}Z^\mu\bar{u}_i\left(g_V^{u-Z}\gamma^\mu - g_A^{u-Z}\gamma^\mu\gamma^5\right)u_i - \frac{g_2}{2\cos\theta_W}Z^\mu\bar{d}_i\left(g_V^{d-Z}\gamma^\mu - g_A^{d-Z}\gamma^\mu\gamma^5\right)d_i \\
& -g_I Z_I'^{\mu}\bar{u}_i\left(g_V^{u-Z_I'}\gamma^\mu - g_A^{u-Z_I'}\gamma^\mu\gamma^5\right)u_i - g_I Z_I'^{\mu}\bar{d}_i\left(g_V^{d-Z_I'}\gamma^\mu - g_A^{d-Z_i'}\gamma^\mu\gamma^5\right)d_i
\end{aligned}
\tag{5.36}
$$

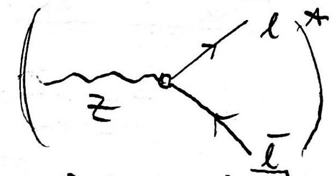
for Leptons, Z^{μ}

$$\mathcal{L}_{\ell} \supset -\frac{g_z}{2\cos\theta_W} Z^{\mu}\bar{\ell}\gamma^{\mu}\left(g_V^{\ell-Z} - g_A^{\ell-Z}\gamma^5\right)\ell$$

$$M = \left(\frac{-i g_z}{2\cos\theta_W}\right)\varepsilon_{\mu}(p_1)\,\bar{u}(p_2)\,\gamma^{\mu}\left(g_V - g_A\gamma^5\right)v(p_3) =$$



$$M^{\dagger} = \frac{i g_z}{2\cos\theta_W}\,\varepsilon_{\nu}^{*}(p_1)\,\bar{v}(p_3)\,\gamma^{\nu}\left(g_V^{*} - g_A^{*}\gamma^5\right)u(p_2) =$$



$$M^{\dagger}M = |M|^2 = \frac{g_z^2}{4\cos^2\theta_W}\,\varepsilon_{\mu}(p_1)\varepsilon_{\nu}^{*}(p_1)\,\mathrm{Tr}\left[\gamma^{\mu}(g_V - g_A\gamma^5)(\not{p}_2 + m_{\ell})\gamma^{\nu}(g_V - g_A\gamma^5)(\not{p}_3 - m_{\bar{\ell}})\right]$$

$$\text{Trace} = 4(g_V^2 + g_A^2)\left(p_2^{\mu}p_3^{\nu} + p_2^{\nu}p_3^{\mu} - g^{\mu\nu}(p_2\cdot p_3)\right) - 4(g_V^2 - g_A^2)\,g^{\mu\nu}m_{\ell}m_{\bar{\ell}}$$

$m_{\ell} = m_{\bar{\ell}}$

$$|M|^2 = \frac{g_z^2}{4\cos^2\theta_W}\left[\frac{p_{1\mu}p_{1\nu}}{M_Z^2} - g_{\mu\nu}\right]4\left[(g_V^2 + g_A^2)\left(p_2^{\mu}p_3^{\nu} + p_2^{\nu}p_3^{\mu} - g^{\mu\nu}(p_2\cdot p_3)\right) - (g_V^2 - g_A^2)\,g^{\mu\nu}m_{\ell}^2\right]$$

$$\Rightarrow \left(\frac{p_{1\mu}p_{1\nu}}{M_Z^2} - g_{\mu\nu}\right)\left(p_2^{\mu}p_3^{\nu} + p_2^{\nu}p_3^{\mu} - g^{\mu\nu}(p_2\cdot p_3)\right) = \frac{2(p_1\cdot p_2)(p_1\cdot p_3)}{M_Z^2} + (p_2\cdot p_3)$$

$$\Rightarrow \left(\frac{p_{1\mu}p_{1\nu}}{M_Z^2} - g_{\mu\nu}\right)g^{\mu\nu}m_{\ell}^2 = \frac{p_1^2}{M_Z^2}m_{\ell}^2 - 4m_{\ell}^2 = -3m_{\ell}^2, \quad p_1^2 = M_Z^2$$

Kinematics: $p_1 = (M_Z, 0)$ $p_2 = \left(\frac{M_Z}{2}, \vec{P}_2\right)$ $p_3 = \left(\frac{M_Z}{2}, -\vec{P}_2\right)$

General $E^2 - \vec{p}^{\,2} = m^2 \Rightarrow \vec{p}^{\,2} = E^2 - m_f^2$, $\vec{p}^{\,2} = \left(\frac{M_f}{2}\right)^2 - m_f^2$

for Massless case: f = final $\Rightarrow m_l$, i = initial $\Rightarrow M_Z$, $m_l = 0$

$$|M|^2 = \frac{1}{3}\frac{g_Z^2}{\cos^2\theta_W}(g_V^2 + g_A^2)\left(\frac{2(p_1\cdot p_2)(p_1\cdot p_3)}{M_Z^2} + (p_2\cdot p_3)\right),$$

$$(p_1\cdot p_2) = \frac{M_Z^2}{2}, \quad (p_1\cdot p_3) = \frac{M_Z^2}{2}, \quad (p_2\cdot p_3) = \frac{M_Z^2}{4} + \vec{P}_2^{\,2}$$

$$\vec{P}_2^{\,2} = \vec{p}^{\,2} = \left(\frac{M_Z}{2}\right)^2 - \cancel{m_l^2}^{\,0}, \quad (p_2\cdot p_3) = \frac{M_Z^2}{4} + \frac{M_Z^2}{4} = \frac{M_Z^2}{2}$$

$$\Rightarrow \cancel{2}\left(\frac{M_Z^{\cancel{2}}}{\cancel{4}}\right)\left(\frac{M_Z^2}{2}\right)/M_Z^{\cancel{2}} + \frac{M_Z^2}{2} = M_Z^2, \quad \boxed{\therefore |M|^2 = \frac{g_Z^2 M_Z^2}{3\cos^2\theta_W}(g_V^2 + g_A^2)}$$

Decay $\Gamma = \frac{|\vec{p}|}{8\pi M_Z^2}|M|^2$, $\vec{p}^{\,2} = \frac{M_Z^2}{4}$, $|\vec{p}| = \frac{M_Z}{2}$

$$\boxed{(5.43)\ \therefore \Gamma(Z\to l\bar{l}) = \frac{g_Z^2 M_Z}{48\pi\cos^2\theta_W}\left((g_V^{l-Z})^2 + (g_A^{l-Z})^2\right)}$$

Massive Cases, Z^μ (quarks) $\quad \mathcal{L}_q \supset -\frac{g_z}{2\cos\theta_W} Z^\mu \bar{q}\gamma^\mu(g_V^{q-Z} - g_A^{q-Z}\gamma^5)q$

From Before:
$m_\ell \to m_q$ $\quad |M|^2 = \frac{g_z^2}{\cancel{4}\cos^2\theta_W}\left[\frac{P_{1\mu}P_{1\nu}}{M_Z^2} - g_{\mu\nu}\right]\cancel{4}\left[(g_V^2+g_A^2)(P_2^\mu P_3^\nu + P_2^\nu P_3^\mu - g^{\mu\nu}(P_2\cdot P_3)) - (g_V^2-g_A^2)g^{\mu\nu}m_q^2\right]$

$$|M|^2 = \frac{g_z^2}{\cos^2\theta_W}\left[(g_V^2+g_A^2)\left(\frac{2(P_1\cdot P_2)(P_1\cdot P_3)}{M_Z^2} + (P_2\cdot P_3)\right) - (g_V^2-g_A^2)(-3m_q^2)\right)\right)$$

Kinematics the same: $(P_1\cdot P_2) = \frac{M_Z^2}{2} = (P_1\cdot P_3)$ except $(P_2\cdot P_3) = \frac{M_Z^2}{4} + \vec{P}^2$

$\vec{P}^2 = \left(\frac{M_Z}{2}\right)^2 - m_q^2$, $\quad (P_2\cdot P_3) = \frac{M_Z^2}{4} + \frac{M_Z^2}{4} - m_q^2 = \frac{M_Z^2}{2} - m_q^2$

$$\Rightarrow \frac{2(P_1\cdot P_2)(P_1\cdot P_3)}{M_Z^2} + (P_2\cdot P_3) = \frac{\cancel{2}(M_Z^{\cancel{2}}/\cancel{2})(M_Z^2/2)}{\cancel{M_Z^2}} + \frac{M_Z^2}{2} - m_q^2 = M_Z^2 - m_q^2$$

$$|M|^2 = \frac{g_z^2}{3\cos^2\theta_W}\underbrace{\left[g_V^2M_Z^2 + g_A^2M_Z^2 - g_V^2m_q^2 - g_A^2m_q^2 + 3g_V^2m_q^2 - 3g_A^2m_q^2\right]}_{(g_V^2+g_A^2)M_Z^2 + 2m_q^2(g_V^2-2g_A^2)}$$

Decay: $\Gamma = \frac{|\vec{P}|}{8\pi M_Z^2}|M|^2$, $\quad |\vec{P}| = \left(\frac{M_Z^2}{4}\left(1-4\frac{m_q^2}{M_Z^2}\right)\right)^{1/2}$, $\quad |\vec{P}| = \frac{M_Z}{2}\sqrt{1-4\frac{m_q^2}{M_Z^2}}$

$$(5.44)\ \therefore\ \Gamma(Z\to q\bar{q}) = \frac{g_z^2 M_Z}{48\pi\cos^2\theta_W}\left((g_V^{q-Z})^2 + (g_A^{q-Z})^2 + 2\frac{m_q^2}{M_Z^2}\left((g_V^{q-Z})^2 - 2(g_A^{q-Z})^2\right)\right)\sqrt{1-4\frac{m_q^2}{M_Z^2}}$$

5.2 Z and Z_I' decays into fermions

In the case of the decay of the neutral gauge boson Z into leptons we obtain in the massless limit

$$\Gamma\left(Z \rightarrow l\bar{l}\right) = \frac{g_2^2 m_Z}{48\pi \cos^2\theta_W}\left((g_A^{l-Z})^2 + (g_V^{l-Z})^2\right) \tag{5.43}$$

and for its decay into massive (m_q) quarks

$$\Gamma\left(Z \rightarrow q\bar{q}\right) = \frac{g_2^2 m_Z}{48\pi \cos^2\theta_W}\left((g_A^{q-Z})^2 + (g_V^{q-Z})^2 + 2\frac{m_q^2}{m_Z^2}\left((g_A^{q-Z_I'})^2 - 2(g_V^{q-Z_I'})^2\right)\right)\left(1 - 4\frac{m_q^2}{m_Z^2}\right)^{1/2}. \tag{5.44}$$

Similar results hold for the Z_I' gauge boson

$$\Gamma\left(Z_I' \rightarrow l\bar{l}\right) = \frac{g_{PQ}^2 m_{Z_I'}}{12\pi}\left((g_A^{l-Z_I'})^2 + (g_V^{l-Z_I'})^2\right) \tag{5.45}$$

and for its decay into massive (m_q) quarks

$$\Gamma\left(Z_I' \rightarrow q\bar{q}\right) = \frac{g_{PQ}^2 m_{Z_I'}}{12\pi}\left((g_A^{q-Z_I'})^2 + (g_V^{q-Z_I'})^2 + 2\frac{m_q^2}{m_{Z_I'}^2}\left((g_A^{q-Z_I'})^2 - 2(g_V^{q-Z_I'})^2\right)\right)\left(1 - 4\frac{m_q^2}{m_{Z_I'}^2}\right)^{1/2}. \tag{5.46}$$

5.3 The Drell-Yan cross section

In e^+e^- annihilations and in $p\,p$ collisions there are some standard signatures for the new