General Cross Section d-dimension (2 initial)

$$\sigma == \frac{1}{2\sqrt{s\,(s-4\,m_1\,m_2)}}\sum_{\text{spins}}\frac{1}{4}\int\prod_{i=1}^{n}\frac{d^{d-1}k_i}{(2\,\pi)^{d-1}\,2\,k_i{}^0}\,(2\,\pi)^d\,\delta^{\,d}\left(\sum_{i=1}^{n}k_i-p_1-p_2\right)M^\dagger\,M$$

Born Cross Section d-dimension (2→2 final)

$$\sigma_B == \frac{1}{8\sqrt{s\,(s-4\,m_e\,m_e)}}\,(2\,\pi)^{2-d}\int\frac{d^{d-1}k_1}{2\,k_1{}^0}\,\frac{d^{d-1}k_2}{2\,k_2{}^0}\,\delta^{\,d-3}\left(k_1{}^0-k_2{}^0-\sqrt{s}\right)\delta^{\,d-1}\,(k_1-k_2)\,M^\dagger\,M$$

$$\sigma_B == \frac{1}{8\sqrt{s\,(s-4\,m_e\,m_e)}}\,(2\,\pi)^{2-d}$$

$$\int\frac{d^{d-1}k_1}{2\sqrt{k_1{}^2+M_q{}^2}\;2\sqrt{k_1{}^2+M_{qb}{}^2}}\,\delta^{\,d-3}\left(\sqrt{k_1{}^2+M_q{}^2}-\sqrt{k_1{}^2+M_{qb}{}^2}-\sqrt{s}\right)M^\dagger\,M$$

$$\sigma_B == \frac{1}{8\sqrt{s\,(s-4\,m_e\,m_e)}}\,(2\,\pi)^{2-d}\int\frac{k^{d-2}\,dk\,d\Omega_{d-1}}{2\sqrt{k^2+M_q{}^2}\;2\sqrt{k^2+M_{qb}{}^2}}\,\delta^{\,d-3}\left(\sqrt{k^2+M_q{}^2}-\sqrt{k^2+M_{qb}{}^2}-\sqrt{s}\right)M^\dagger\,M$$

$\delta\left(\sqrt{s}-\sqrt{k^2+m_1{}^2}-\sqrt{k^2+m_2{}^2}\right)$

In[∘]:= $\left(\sqrt{s}\right)^2 == \left(\sqrt{k^2+m_1{}^2}+\sqrt{k^2+m_2{}^2}\right)^2$ // Expand

Out[∘]= $s == 2\,k^2+m_1^2+m_2^2+2\sqrt{k^2+m_1^2}\,\sqrt{k^2+m_2^2}$

In[∘]:= $\left(s-m_1{}^2-m_2{}^2-2\,k^2\right)^2 == \left(2\sqrt{m_1{}^2+k^2}\,\sqrt{m_2{}^2+k^2}\right)^2$ // Expand

Out[∘]= $4\,k^4-4\,k^2\,s+s^2+4\,k^2\,m_1^2-2\,s\,m_1^2+m_1^4+4\,k^2\,m_2^2-2\,s\,m_2^2+2\,m_1^2\,m_2^2+m_2^4 == 4\,k^4+4\,k^2\,m_1^2+4\,k^2\,m_2^2+4\,m_1^2\,m_2^2$

In[∘]:= $m_1{}^4+2\,m_2{}^2\,m_2{}^2+m_2{}^4+4\,m_1{}^2\,k^2+4\,m_2{}^2\,k^2+4\,k^4-$
$2\,m_1{}^2\,s-2\,m_2{}^2\,s-4\,k^2\,s+s^2-4\,m_1{}^2\,m_2{}^2-4\,m_1{}^2\,k^2-4\,m_2{}^2\,k^2-4\,k^4 == 0$

Out[∘]= $-4\,k^2\,s+s^2-2\,s\,m_1^2+m_1^4-2\,s\,m_2^2-4\,m_1^2\,m_2^2+3\,m_2^4 == 0$

In[∘]:= Simplify$\left[m_1{}^4-2\,m_1{}^2\,m_2{}^2+m_2{}^4\right]$ + Factor$\left[-2\,m_1{}^2\,s-2\,m_2{}^2\,s\right]-4\,k^2\,s+s^2 == 0$

Out[∘]= $-4\,k^2\,s+s^2+\left(m_1^2-m_2^2\right)^2-2\,s\left(m_1^2+m_2^2\right) == 0$

In[∘]:= MultiplySides$\left[-4\,k^2\,s == -s^2+2\left(m_1{}^2+m_2{}^2\right)s-\left(m_1{}^2-m_2{}^2\right)^2,\,-1\right]$

Out[∘]= $4\,k^2\,s == s^2+\left(m_1^2-m_2^2\right)^2-2\,s\left(m_1^2+m_2^2\right)$

In[∘]:= `Divide[4 k^2 s, s] == Divide[(m_1^2 - m_2^2)^2 - 2 (m_1^2 + m_2^2) s + s^2, s] // Expand`

Out[∘]= $$4\,k^2 == s - 2\,m_1^2 + \frac{m_1^4}{s} - 2\,m_2^2 - \frac{2\,m_1^2\,m_2^2}{s} + \frac{m_2^4}{s}$$

In[∘]:= $$4\,k^2 == \mathrm{Factor}\left[-2\,m_1{}^2 - 2\,m_2{}^2\right] + \mathrm{Simplify}\left[\frac{m_1{}^4}{s} - \frac{2\,m_1{}^2\,m_2{}^2}{s} + \frac{m_2{}^4}{s}\right] + s$$

Out[∘]= $$4\,k^2 == s + \frac{\left(m_1^2 - m_2^2\right)^2}{s} - 2\,\left(m_1^2 + m_2^2\right)$$

In[∘]:= $$\mathrm{MultiplySides}\left[4\,k^2 == s.\left(1 - \frac{2\,\left(m_1{}^2 + m_2{}^2\right)}{s} + \frac{\left(m_1{}^2 - m_2{}^2\right)^2}{s^2}\right),\ 1/4\right]$$

Out[∘]= $$k^2 == \frac{1}{4}\,s.\left(1 + \frac{\left(m_1^2 - m_2^2\right)^2}{s^2} - \frac{2\,\left(m_1^2 + m_2^2\right)}{s}\right)$$

In[∘]:= $$\mathrm{PowerExpand}\left[\left(k^2\right)^{1/2}\right] == \mathrm{PowerExpand}\left[\left(\frac{s}{4}\right)^{1/2}\right].\left(1 - \frac{2\,\left(m_1{}^2 + m_2{}^2\right)}{s} + \frac{\left(m_1{}^2 - m_2{}^2\right)^2}{s^2}\right)^{1/2}$$

`% // TraditionalForm`

Out[∘]= $$k == \frac{\sqrt{s}}{2}.\sqrt{1 + \frac{\left(m_1^2 - m_2^2\right)^2}{s^2} - \frac{2\,\left(m_1^2 + m_2^2\right)}{s}}$$

Out[∘]//TraditionalForm=

$$k = \frac{\sqrt{s}}{2}.\sqrt{\frac{\left(m_1^2 - m_2^2\right)^2}{s^2} - \frac{2\left(m_1^2 + m_2^2\right)}{s} + 1}$$

Rewrite integral with new delta solution

$$\sigma_B == \frac{1}{8\sqrt{s(s-4m_e m_e)}}(2\pi)^{2-d}$$

$$\int \frac{k^{d-2}\,dk\,d\Omega_{d-1}}{2\sqrt{k^2+M_q^2}\;2\sqrt{k^2+M_{qb}^2}}\,\delta\left(k-\frac{\sqrt{s}}{2}.\sqrt{1-\frac{2\left(M_q^2+M_{qb}^2\right)}{s}+\frac{\left(M_q^2-M_{qb}^2\right)^2}{s^2}}\right)M^\dagger M$$

$$\sigma_B == \frac{1}{8\sqrt{s(s-4m_e m_e)}}(2\pi)^{2-d}\int \frac{k^{d-2}\,dk\,d\Omega_{d-1}}{4\sqrt{k^2+M_q^2}\;\sqrt{k^2+M_{qb}^2}}\left(\frac{k}{E_1}+\frac{k}{E_2}\right)^{-1}\delta\left(k-\frac{\sqrt{s}}{2}.\beta\right)M^\dagger M$$

$$\sigma_B == \frac{1}{8\sqrt{s(s-4m_e m_e)}}(2\pi)^{2-d}\int \frac{k^{d-2}\,dk\,d\Omega_{d-1}}{4\sqrt{k^2+M_q^2}\;\sqrt{k^2+M_{qb}^2}}\frac{1}{k}\left(\frac{E_1 E_2}{E_1+E_2}\right)\delta\left(k-\frac{\sqrt{s}}{2}.\beta\right)M^\dagger M$$

$$\sigma_B == \frac{Q_f^2 e^4}{s^2}\;\frac{1}{8\sqrt{s(s-4m_e m_e)}}(2\pi)^{2-d}$$

$$\int \frac{\left(\frac{\sqrt{s}}{2}.\beta\right)^{d-3} d\Omega_{d-1}}{4(E_1+E_2)}\mathrm{Tr}[\gamma^\mu\,(p_{2\,\mathrm{slash}}-m_e)\,\gamma^\nu\,(p_{1\,\mathrm{slash}}+m_e)]\times\mathrm{Tr}\left[\gamma_\mu\left(k_{1\,\mathrm{slash}}+M_q\right)\gamma_\nu\left(k_{2\,\mathrm{slash}}-M_{qb}\right)\right]$$

$$\sigma_B == \frac{Q_f^2 e^4}{32\,s^2(E_1+E_2)}\;\frac{1}{\sqrt{s(s-4m_e m_e)}}(2\pi)^{2-d}\left(\frac{\sqrt{s}}{2}.\beta\right)^{d-3}L^{\mu\nu}\int d\Omega_{d-1}\,K_{\mu\nu}$$

Contract $K_{\mu\nu}$

In[∘]:=
```
Tr[(GSD[k1] + m1).GAD[μ].(GSD[k2] - m2).GAD[ν]]
```

Out[∘]= $-4\left(g^{\mu\nu}(k1\cdot k2)+m1\,m2\,g^{\mu\nu}-k1^\nu k2^\mu-k1^\mu k2^\nu\right)$

In[∘]:=
```
Tr[(GSD[p2] - m1).GAD[μ].(GSD[p1] + m2).GAD[ν]] ×
 Tr[(GSD[k1] + M1).GAD[μ].(GSD[k2] - M2).GAD[ν]]
```

Out[∘]= $16\left(g^{\mu\nu}(k1\cdot k2)+M1\,M2\,g^{\mu\nu}-k1^\nu k2^\mu-k1^\mu k2^\nu\right)\left(m1\,m2\,g^{\mu\nu}+g^{\mu\nu}(p1\cdot p2)-p1^\nu p2^\mu-p1^\mu p2^\nu\right)$

In[∘]:=
```
Expand[%43] // Contract
```

Out[∘]= $16D\,m1\,m2\,(k1\cdot k2)+16D\,(k1\cdot k2)(p1\cdot p2)+16D\,m1\,M1\,m2\,M2+16D\,M1\,M2\,(p1\cdot p2)-32\,m1\,m2\,(k1\cdot k2)+32\,(k1\cdot p2)(k2\cdot p1)+32\,(k1\cdot p1)(k2\cdot p2)-64\,(k1\cdot k2)(p1\cdot p2)-32\,M1\,M2\,(p1\cdot p2)$

In[∘]:=
```
% // Factor
```

Out[∘]= $16\left(D\,m1\,m2\,(k1\cdot k2)+D\,(k1\cdot k2)(p1\cdot p2)+D\,m1\,M1\,m2\,M2+D\,M1\,M2\,(p1\cdot p2)-2\,m1\,m2\,(k1\cdot k2)+2\,(k1\cdot p2)(k2\cdot p1)+2\,(k1\cdot p1)(k2\cdot p2)-4\,(k1\cdot k2)(p1\cdot p2)-2\,M1\,M2\,(p1\cdot p2)\right)$

In[∘]:=
```
% /. {m1 → 0} /. {m2 → 0}
```

Out[∘]= $16\left(D\,(k1\cdot k2)(p1\cdot p2)+D\,M1\,M2\,(p1\cdot p2)+2\,(k1\cdot p2)(k2\cdot p1)+2\,(k1\cdot p1)(k2\cdot p2)-4\,(k1\cdot k2)(p1\cdot p2)-2\,M1\,M2\,(p1\cdot p2)\right)$

In[ ]:=
```
% /. {M1 → 0} /. {M2 → 0}
```
Out[ ]= $16\,(D\,(\mathrm{k1}\cdot\mathrm{k2})\,(\mathrm{p1}\cdot\mathrm{p2})+2\,(\mathrm{k1}\cdot\mathrm{p2})\,(\mathrm{k2}\cdot\mathrm{p1})+2\,(\mathrm{k1}\cdot\mathrm{p1})\,(\mathrm{k2}\cdot\mathrm{p2})-4\,(\mathrm{k1}\cdot\mathrm{k2})\,(\mathrm{p1}\cdot\mathrm{p2}))$

In[ ]:=
```
MTD[μ, ν].Tr[(GSD[k1] + m1).GAD[μ].(GSD[k2] - m2).GAD[ν]] // Contract
% /. {m1 → 0}
```
Out[ ]= $-4\,(D\,(\mathrm{k1}\cdot\mathrm{k2})+D\,\mathrm{m1}\,\mathrm{m2}-2\,(\mathrm{k1}\cdot\mathrm{k2}))$

Out[ ]= $-4\,(D\,(\mathrm{k1}\cdot\mathrm{k2})-2\,(\mathrm{k1}\cdot\mathrm{k2}))$

In[ ]:=
```
% // StandardForm;
```
In[ ]:=
```
g^{μ ν}.K_{μν} == -4 .Pair[Momentum[k1, D], Momentum[k2, D]].(D - 2)
```
Out[ ]= $g^{\mu\,\nu}.K_{\mu\nu}=-4.(\mathrm{k1}\cdot\mathrm{k2}).(D-2)$

Contract $L_{\mu\nu}$

In[ ]:=
```
-MTD[μ, ν] × Tr[(GSD[p1]).GAD[μ].(GSD[p2]).GAD[ν]] // Expand // Contract // Factor
```
Out[ ]= $4\,(D-2)\,(\mathrm{p1}\cdot\mathrm{p2})$

In[ ]:=
```
(FVD[q, μ] × FVD[q, ν] / Q^2).Tr[(GSD[p1]).GAD[μ].(GSD[p2]).GAD[ν]] // Expand // Contract //
 Factor
```
Out[ ]= $$\frac{4\left(2\,(\mathrm{p1}\cdot q)\,(\mathrm{p2}\cdot q)-q^2\,(\mathrm{p1}\cdot\mathrm{p2})\right)}{Q^2}$$

Massless Electrons, Massless Quarks

$\beta=\sqrt{1-\frac{2\left(\mathrm{M_q}^2+\mathrm{M_{qb}}^2\right)}{\mathrm{s}}+\frac{\left(\mathrm{M_q}^2-\mathrm{M_{qb}}^2\right)^2}{\mathrm{s}^2}}$, then $\beta=1$

$$\sigma_B == \frac{Q_f^2\, e^4}{32\, s^2\, (E_1 + E_2)}\ \frac{1}{\sqrt{s\,(s - 4\, m_e\, m_e)}}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}.\beta\right)^{d-3} L^{\mu\nu}\int d\Omega_{d-1}\, K_{\mu\nu}$$

$$\sigma_B == \frac{Q_f^2\, e^4}{32\, s^3\, (E_1 + E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-3}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\, L^{\mu\nu}\left(\frac{q_\mu\, q_\nu}{Q^2} - g_{\mu\nu}\right) g^{\mu\nu}.K_{\mu\nu}$$

$$\sigma_B == -\frac{4\, Q_f^2\, e^4}{32\, s^3\, (E_1 + E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-3}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{d-2}{d-1}.Q^2\, g^{\mu\nu}.K_{\mu\nu}$$

$$\sigma_B == \frac{16\, Q_f^2\, e^4}{32\, s^3\, (E_1 + E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-3}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}.Q^2\,(k_1.k_2)$$

$$\sigma_B == \frac{Q_f^2\, e^4}{2\, s^3\, (E_1 + E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-3}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}.\frac{Q^4}{2}$$

$$\sigma_B == \frac{Q_f^2\, e^4}{2\, s^3\, (E_1 + E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)\left(\frac{\sqrt{s}}{2}\right)^{d-4}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}.\frac{s^2}{2}$$

$$\sigma_B == \frac{Q_f^2\, e^4}{4\, s}\,(2\,\pi)^{-1}\,(2\,\pi)^{-1}\,(2\,\pi)^{4-d}\left(\frac{\sqrt{s}}{2}\right)^{d-4}\frac{\pi^{d/2}}{\sqrt{\pi}\,\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}$$

$$\sigma_B == \frac{Q_f^2\, e^4}{4\, s}\,2^{-2}\left(\frac{\sqrt{s}}{4\,\pi}\right)^{d-4}\frac{(\pi)^{-2}\,\pi^{d/2}}{\sqrt{\pi}\,\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}$$

$$\sigma_B == \frac{Q_f^2\, e^4}{4\, s}\,2^{-2}\left(\frac{1}{2}\right)^{d-4}\left(\frac{\sqrt{s}}{2\,\pi}\right)^{d-4}\frac{(\pi)^{-2}\,\pi^{d/2}}{\sqrt{\pi}\,\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}$$

$$\sigma_B == \frac{Q_f^2\, e^4}{4\, s}\,2^{2}\left(\frac{s}{4\,\pi^2}\right)^{\frac{d-4}{2}}\frac{2^{-d}\,(\pi)^{-2}\,\pi^{d/2}}{\sqrt{\pi}\,\Gamma\left[\frac{d-1}{2}\right]}\ \frac{1}{2}\ \frac{(d-2)^2}{d-1}$$

$$\sigma_B == \frac{2^2\, Q_f^2\, e^4}{4\, s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\ \frac{(d-2)^2}{d-1}\ \frac{2^{-d}}{\sqrt{\pi}\,\Gamma\left[\frac{d-1}{2}\right]}$$

Gamma Simplifications and Identities

Using Legendre Duplication Formula:

$\Gamma(z)\Gamma(z{+}1) = 2^{1-2z}\sqrt{\pi}\,\Gamma(2z), \quad z = \frac{d-2}{2}$

In[ ]:=
```
Clear[d]
```

In[∘]:= $\dfrac{\text{Gamma}\left[\frac{d-2}{2}\right]}{2^{3-d}\sqrt{\pi}\ \text{Gamma}[d-2]}$ // FullSimplify

Out[∘]= $\dfrac{1}{\Gamma\left(\frac{d-1}{2}\right)}$

Using General Recursive Formula:
$(z)\Gamma(z) = \Gamma(z+1)$

In[∘]:= $\left(\dfrac{d-2}{2}\right)\ \text{Gamma}\left[\dfrac{d-2}{2}\right]$ // FullSimplify

Out[∘]= $\Gamma\left(\dfrac{d}{2}\right)$

In[∘]:= $\dfrac{1}{\text{Gamma}\left[\frac{d-1}{2}\right]} == \dfrac{\text{Gamma}\left[\frac{d-2}{2}\right]}{2^{3-d}\sqrt{\pi}\ \text{Gamma}[d-2]}$

$\text{Gamma}\left[\dfrac{d-2}{2}\right] == \left(\dfrac{2}{d-2}\right).\text{Gamma}\left[\dfrac{d}{2}\right]$

Out[∘]= $\dfrac{1}{\Gamma\left(\frac{d-1}{2}\right)} = \dfrac{2^{d-3}\,\Gamma\left(\frac{d-2}{2}\right)}{\sqrt{\pi}\ \Gamma(d-2)}$

Out[∘]= $\Gamma\left(\dfrac{d-2}{2}\right) = \dfrac{2}{d-2}.\Gamma\left(\dfrac{d}{2}\right)$

Final Born Cross Section d-dimension

$$\sigma_B == \frac{2^2\,Q_f^{\,2}\,e^4}{4\,s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\,\frac{(d-2)^2}{d-1}\,\frac{2^{-d}}{\sqrt{\pi}}\,\frac{2^{d-3}\,\Gamma\left(\frac{d-2}{2}\right)}{\sqrt{\pi}\ \Gamma(d-2)}$$

$$\sigma_B == \frac{2^2\,Q_f^{\,2}\,e^4}{4\,\pi\,s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\,\frac{(d-2)^2}{d-1}\,\frac{2^{-d} * 2^{d-3}}{\Gamma(d-2)}\,\frac{2}{d-2}.\Gamma\left(\frac{d}{2}\right)$$

$$\sigma_B == \frac{2^2\,Q_f^{\,2}\,e^4}{4\,\pi\,s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\,\frac{(d-2)}{d-1}\,\frac{2^{-2}\,\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$\sigma_B == \frac{Q_f^{\,2}\,e^4}{4\,\pi\,s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\,\frac{(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

if $e^2 = 4\,\pi\,\alpha,\ \ e^4 = 16\,\pi^2\,\alpha^2$

$$\sigma_B == \frac{16\,\pi^2\,\alpha^2\,Q_f^2}{4\,\pi\,s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\,\frac{(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$\sigma_B == \frac{4\,\pi\,\alpha^2\,Q_f^2}{s}\left(\frac{4\,\pi}{s}\right)^{\frac{4-d}{2}}\frac{1}{2}\,\frac{(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$\therefore\ \sigma_B == \frac{4\,\pi\,\alpha^2\,Q_f^2}{3\,s}\left(\frac{4\,\pi}{s}\right)^{\frac{4-d}{2}}\frac{3\,(d-2)}{(2)\,d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

If writing $L^{\mu\nu}\left(\frac{q_\mu\,q_\nu}{Q^2}-g_{\mu\nu}\right)g^{\mu\,\nu}.K_{\mu\nu} = \frac{d-2}{d-1}.Q^2\,g^{\mu\,\nu}.K_{\mu\nu}$

instead of $L^{\mu\nu}\left(\frac{q_\mu\,q_\nu}{Q^2}-g_{\mu\nu}\right)g^{\mu\,\nu}.K_{\mu\nu} = \frac{1}{2}\,\frac{d-2}{d-1}.Q^2\,g^{\mu\,\nu}.K_{\mu\nu}$:

$$\sigma_B == -\frac{4\,Q_f^2\,e^4}{32\,s^3\,(E_1+E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-3}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\,\frac{d-2}{d-1}.Q^2\,g^{\mu\,\nu}.K_{\mu\nu}$$

$$\sigma_B == \frac{16\,Q_f^2\,e^4}{32\,s^3\,(E_1+E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-3}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\,\frac{(d-2)^2}{d-1}.Q^2\,(k_1.k_2)$$

$$\sigma_B == \frac{Q_f^2\,e^4}{2\,s^3\,(E_1+E_2)}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)\left(\frac{\sqrt{s}}{2}\right)^{d-4}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\,\frac{(d-2)^2}{d-1}.\frac{s^2}{2}$$

$$\sigma_B == \frac{Q_f^2\,e^4}{4\,s}\,(2\,\pi)^{2-d}\left(\frac{\sqrt{s}}{2}\right)^{d-4}\frac{2\,\pi^{\frac{d-1}{2}}}{\Gamma\left[\frac{d-1}{2}\right]}\,\frac{(d-2)^2}{d-1}$$

Same Gamma Simplifications:

$$\sigma_B == \frac{Q_f^2\,e^4}{4\,\pi\,s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$\sigma_B == \frac{4\,\pi\,\alpha^2\,Q_f^2}{s}\left(\frac{4\,\pi}{s}\right)^{\frac{4-d}{2}}\frac{(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$\therefore\ \sigma_B == \frac{4\,\pi\,\alpha^2\,Q_f^2}{3\,s}\left(\frac{4\,\pi}{s}\right)^{\frac{4-d}{2}}\frac{3\,(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$