Lecture 1: Introduction

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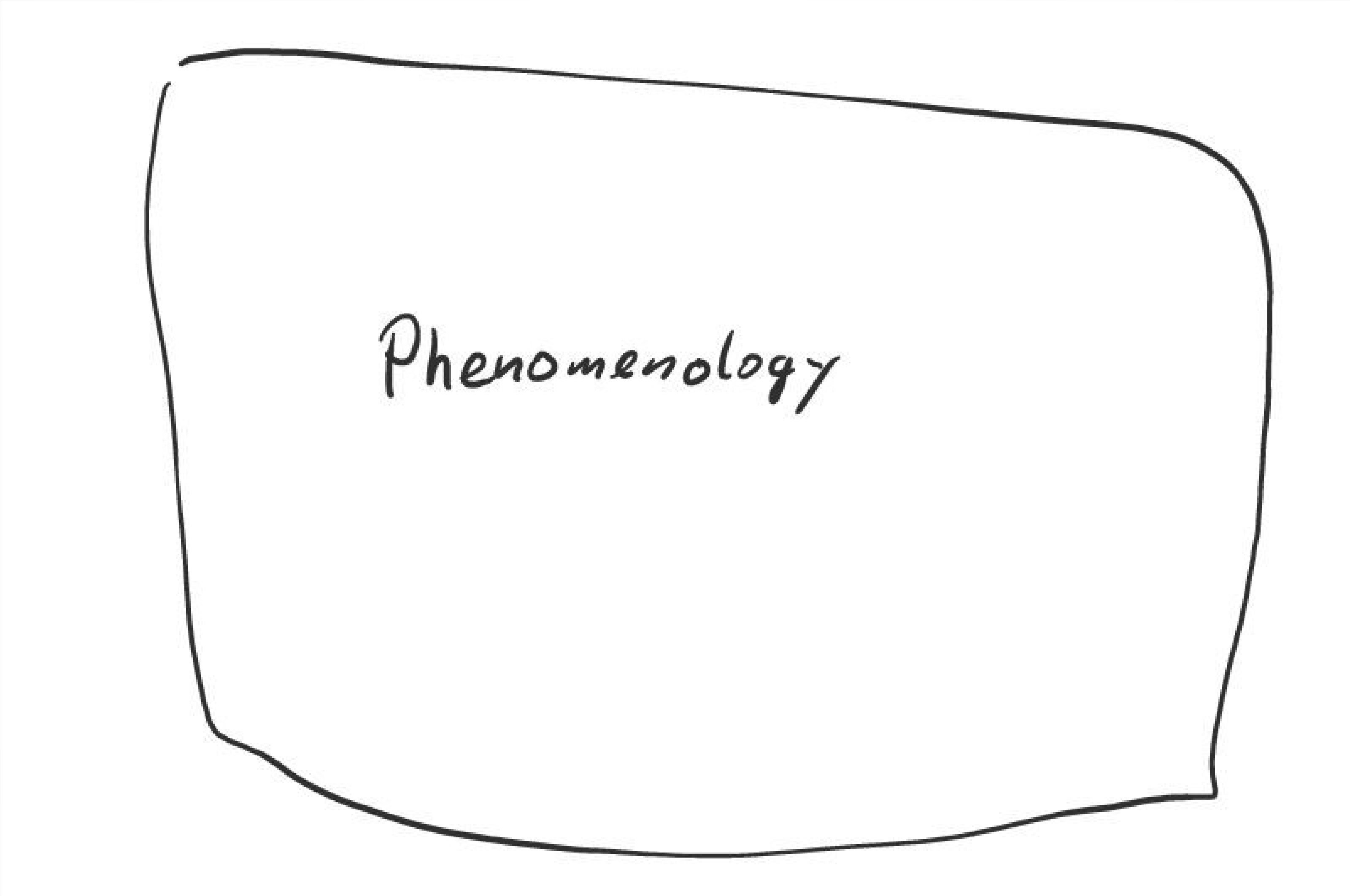
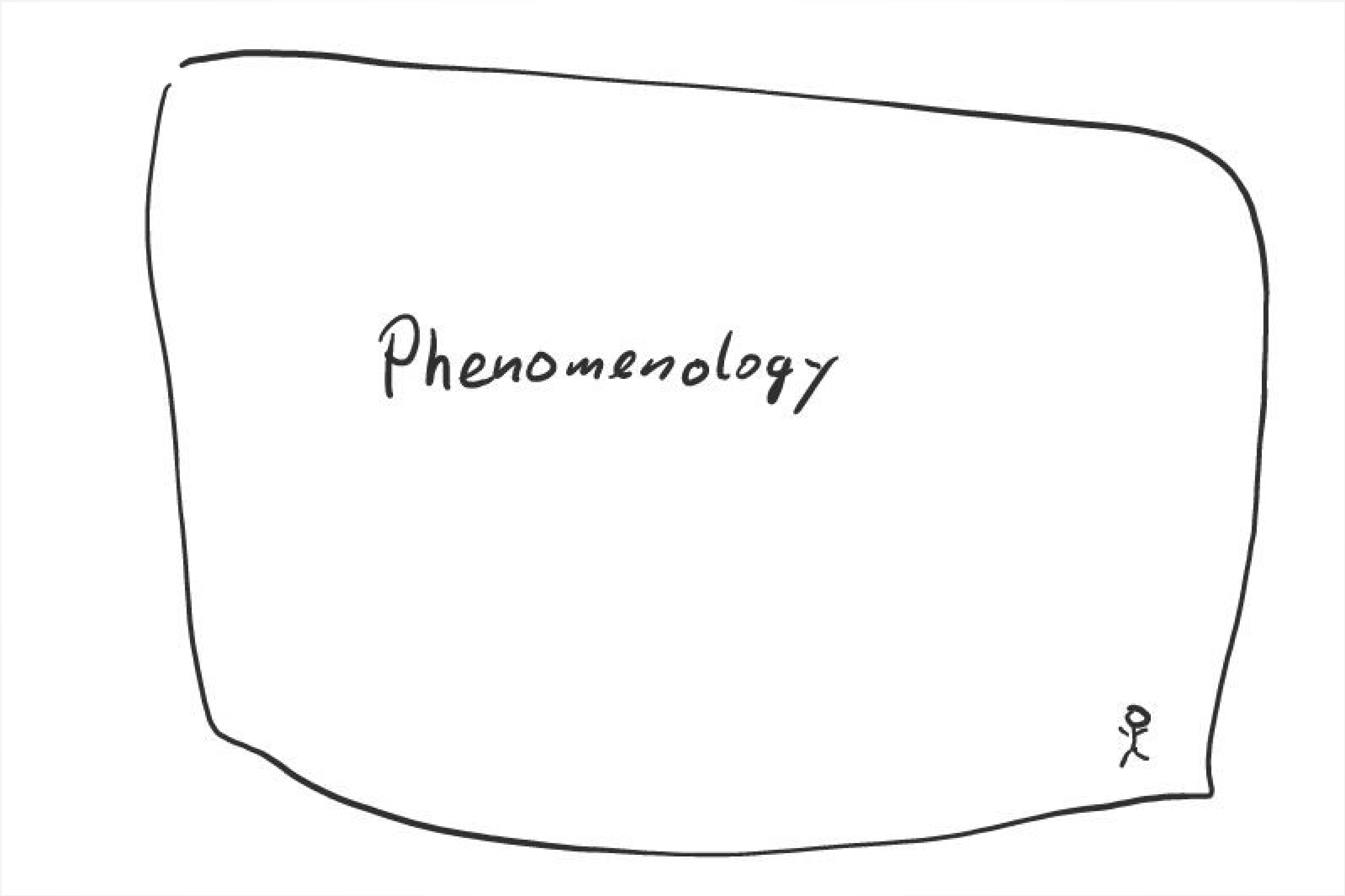
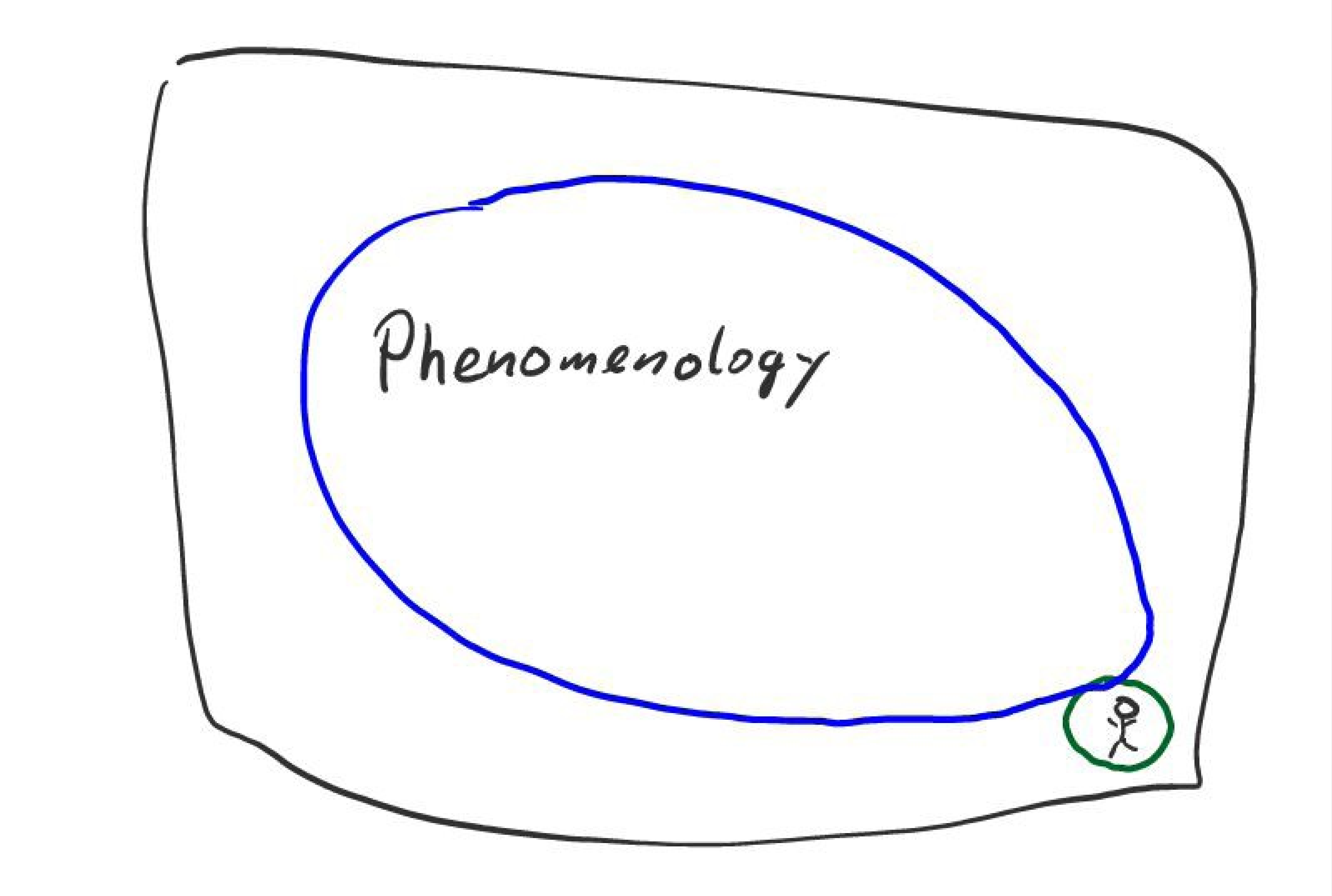
Phenomenology



Phenomenology



Phenomenology



Introduction

- This set of lectures have been adapted from those given by Peter Richardson in the past (which probably were inspired by other earlier lectures), but
  - Added my own typos, english mistakes
  - Changed the emphasis on some topics
- Don't hesitate to ask questions

Outline of the lectures

- Introduction
- QCD (perturbative calculations)
- QCD (jets)
- QCD (Monte Carlo)
- Electro-Weak Physics
- Higgs Boson
- BSM
- Flavour physics/neutrino (?)

Literature

- **Quarks and Leptons** Halzen and Martin
- **Collider Physics** Barger and Philips
- **QCD and Collider Physics** Ellis, Stiling, Webber
- **Quantum Chromodynamics** Dissertori, Knowles Schmelling
- **The Higgs Hunter's Guide** Gunion, Haber, Kane and Dawson
- **Towards Jetography** arXiv:0906.1833 Salam
- **General-purpose event generators for LHC physics** arXiv:1101.2599 (15 authors)

Outline of lecture 1

- Cross section calculation
- Electron-positron annihilation
- Deep inelastic scattering
- Hadron colliders
- Higher order calculation

Cross section calculation

- $2 \to n$ cross section

$$\mathrm{d}\sigma = \frac{(2\pi)^4}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} \mathrm{d}\Phi_n(p_a + p_b; p_1 \ldots p_n) |\overline{\mathcal{M}}|^2$$

- Where the *n*-particle phase-space element is given by

$$\mathrm{d}\Phi_n(p_a + p_b; p_1 \ldots p_n) =$$
$$\delta^4\left(p_a + p_b - p_1 - p_2\right) \frac{\mathrm{d}^3 p_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2}$$

- The invariant mass of the initial state particles is denoted with $s = (p_a + p_b)^2$
- For massless particles $4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2} = 2s$

Cross section calculation

- For 2 → 2 processes:

$$
\begin{aligned}
&\mathrm{d}\Phi_n(p_a+p_b;p_1\ldots p_n)\\
&= \quad \delta^4\left(p_a+p_b-p_1-p_2\right)\frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2},\\
&= \quad \delta\left(E_a+E_b-E_1-E_2\right)\frac{1}{(2\pi)^6 4E_1E_2}|p_1|^2\mathrm{d}|p_1|\mathrm{d}\cos\theta\mathrm{d}\phi,\\
&= \quad \frac{1}{8\pi(2\pi)^4}\frac{|p_1|}{\sqrt{s}}\mathrm{d}\cos\theta.
\end{aligned}
$$

$2 \to 2$ Cross section

- The cross section is:

$$\mathrm{d}\sigma = \frac{1}{16\pi s}\frac{|p_1|}{\sqrt{s}}\mathrm{d}\cos\theta|\overline{\mathcal{M}}|^2.$$

- It is common to use the Mandelstam variables *s,t,u* (only two are independent)

$$s = (p_a + p_b)^2, \quad t = (p_a - p_1)^2, \quad u = (p_a - p_2)^2$$

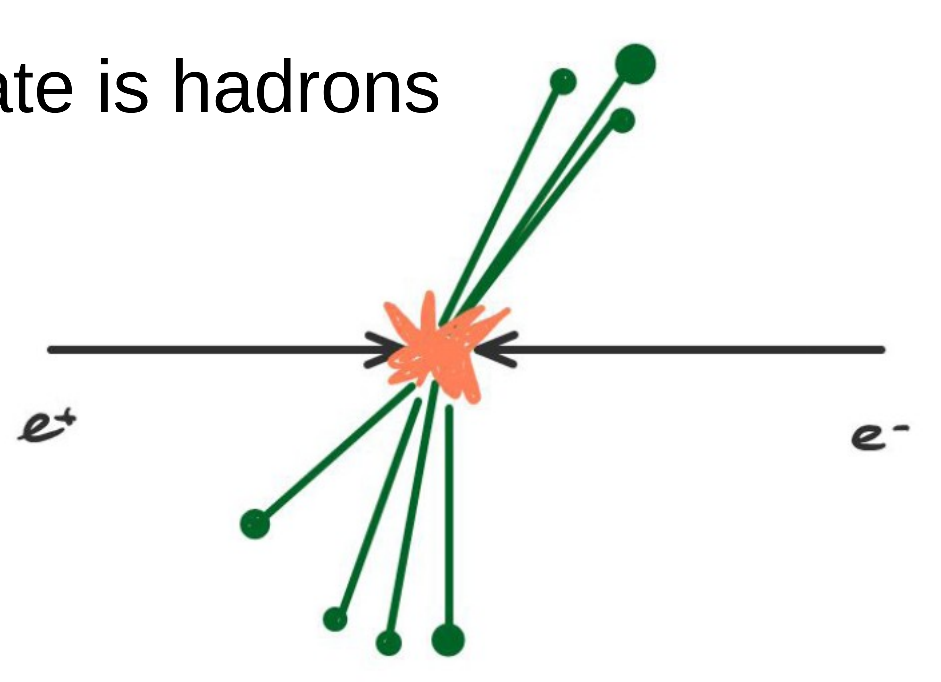
$$s + t + u = m_1^2 + m_2^2 + m_a^2 + m_b^2 \xrightarrow{\text{massless}} 0$$

- Using these variables the cross section is:

$$\mathrm{d}\sigma = \frac{1}{16\pi s^2}\mathrm{d}t|\overline{\mathcal{M}}|^2$$

Electron positron annihilation

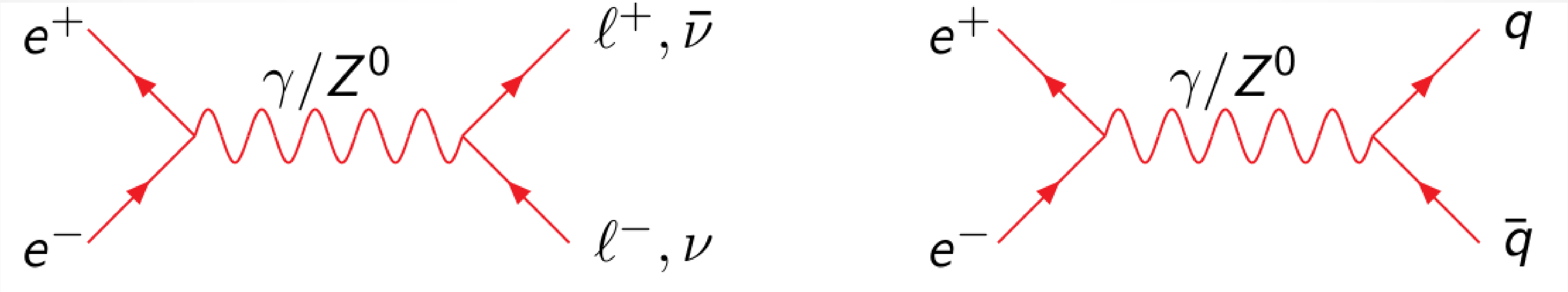
- LEP results are still important
- Theoretical description is easier than hadron colliders
- Most likely final state is hadrons



- There are no Feynman rules for electrons and hadrons, only with quarks and gluons (how to relate to the hadrons is going to be covered later)

Electron positron annihilation

- The Feynman diagrams for the annihilation involve an off-shell photon or a Z boson decaying in a lepton or quark pair



- The quarks will produce hadrons with probability one, so the total cross section for hadrons can be computed from the total cross section for $e^+e^- \to q\bar{q}$
- In general we want observables for which we can compute things using partons, this is the easiest example

Electron positron annihilation

- From the other lectures we know how to compute the total cross section for the annihilation
- To reduce experimental uncertainties it is useful to compute the ratio

$$R \equiv \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

- At low energies the contribution from the *Z* boson can be neglected, leaving only the photon contribution. We get:

$$\sigma(\mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

- We get the same cross section for each quark of each color (assuming we can neglect the mass)

Electron positron annihilation

- The cross section for the production of quarks is

$$\sigma(\text{hadrons}) = \frac{4\pi\alpha^2}{3s}\sum_q e_q^2 N_C$$

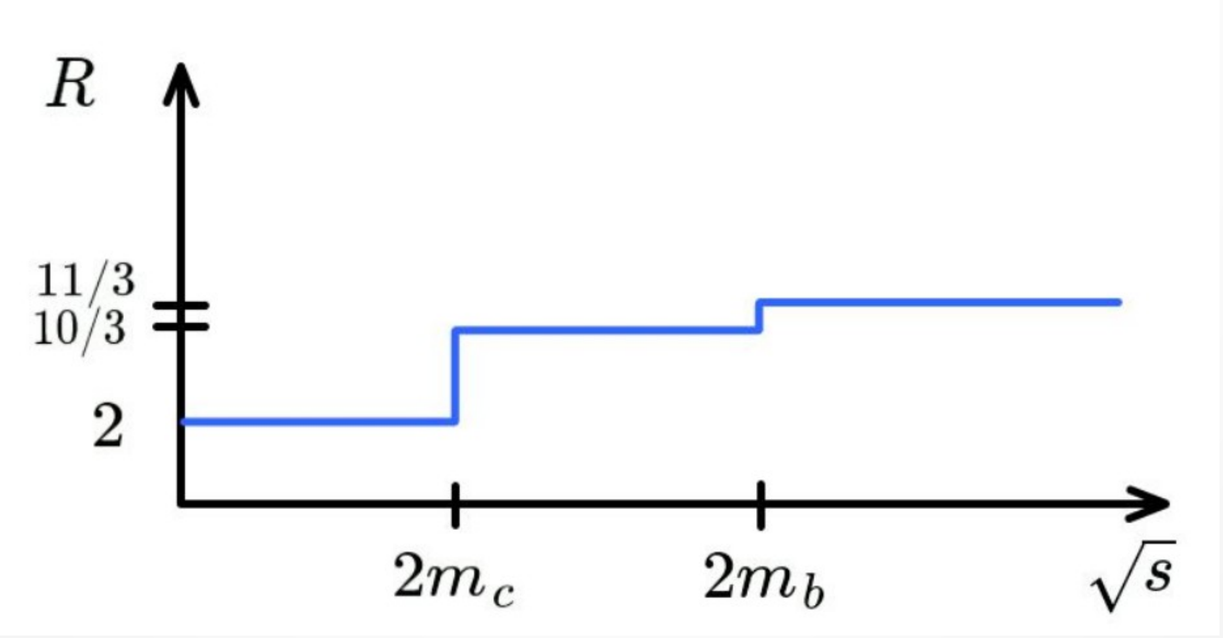
where e_q is the charge of the quark in units of the positron charge and the sum runs over all quarks above the production threshold $\sqrt{s} > 2m_q$

- The factor of N_c comes from the fact that we need to sum over all non-observed quantum numbers
- So we get for the R ratio:

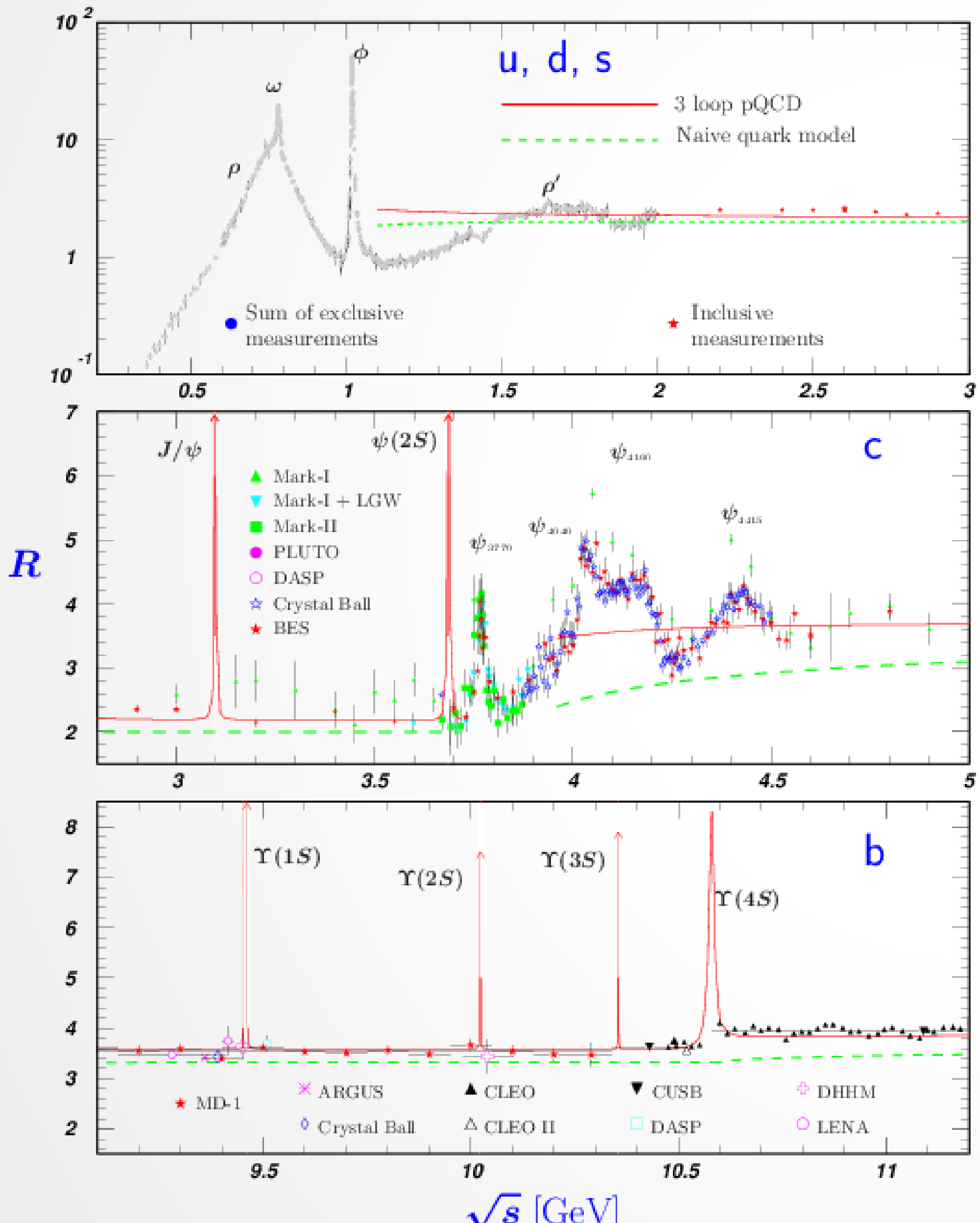
$$R = \sum_q e_q^2 N_C$$

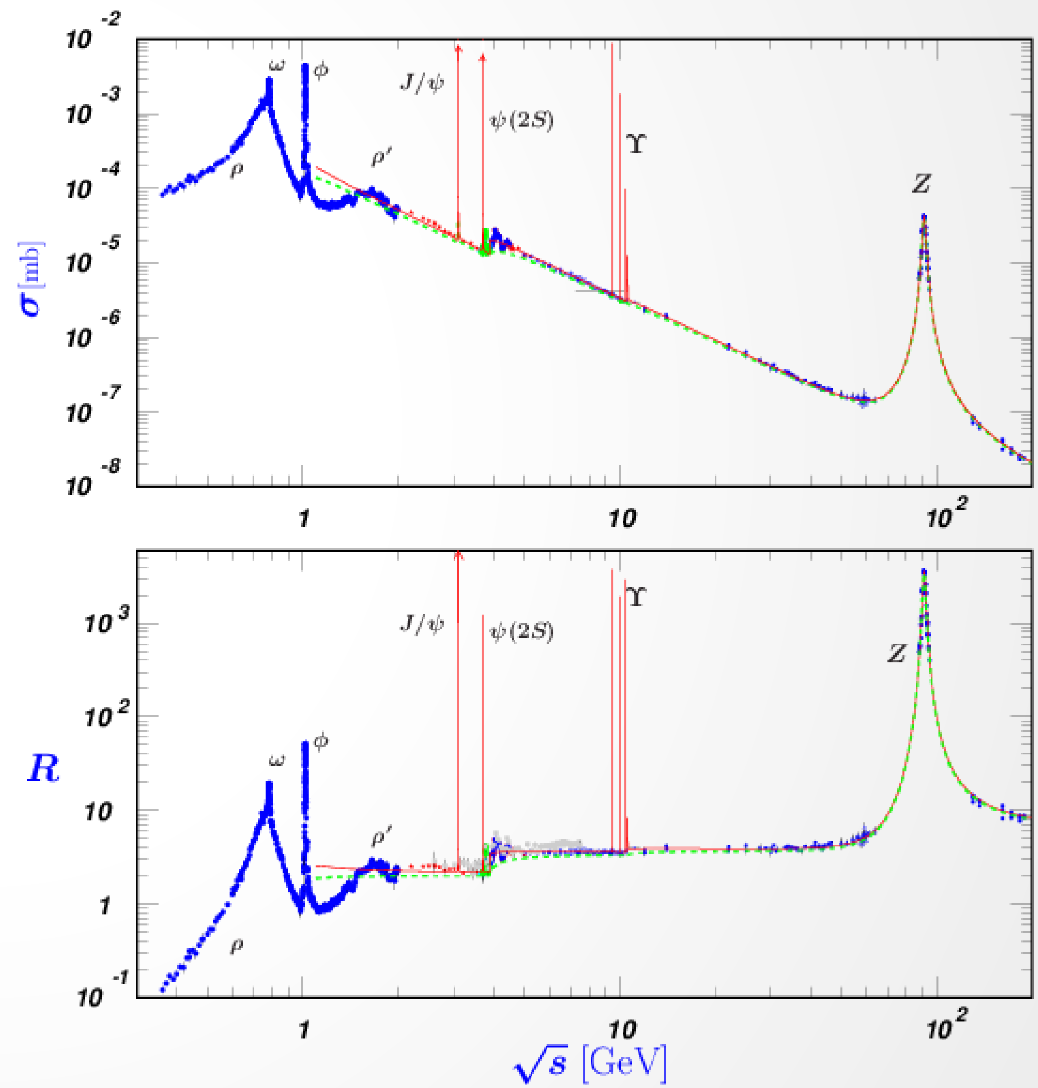
R ratio

$$R = 3\left(\underbrace{\underbrace{\underbrace{\frac{4}{9}+\frac{1}{9}+\frac{1}{9}}_{u,d,s}+\frac{4}{9}}_{u,d,s,c}+\frac{1}{9}}_{u,d,s,c,b}\right)$$



Measurement of R





Taken from PDG

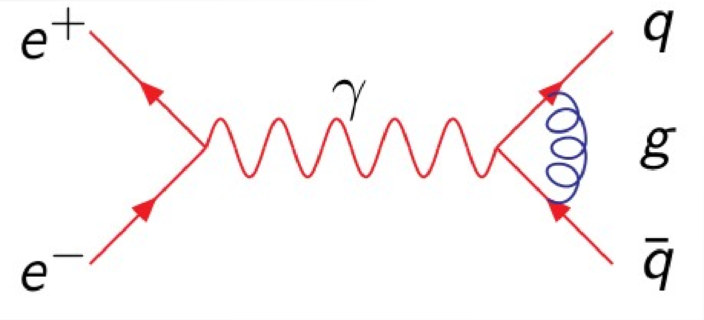
Higher Order Corrections

- The process we computed is only the first order in the perturbative expansion
- The strong coupling constant is not very small:

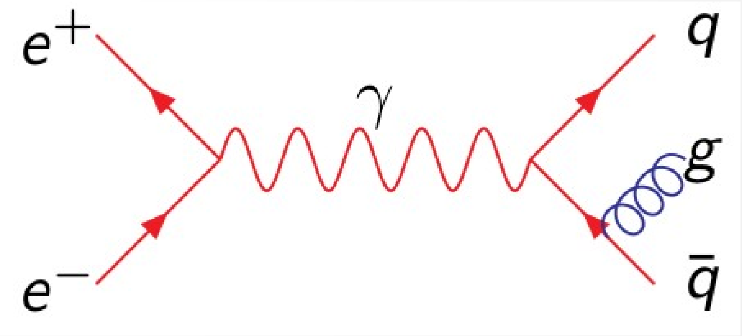
$$\alpha_s(M_Z) \simeq 0.118$$

- To obtain reliable results we need to computer higher orders in the perturbative expansion
- There are two types of corrections

Virtual:



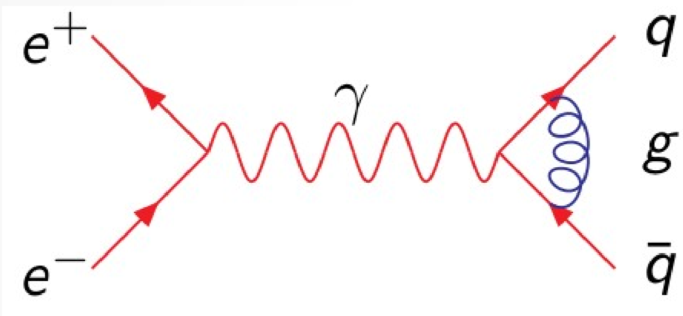
Real:



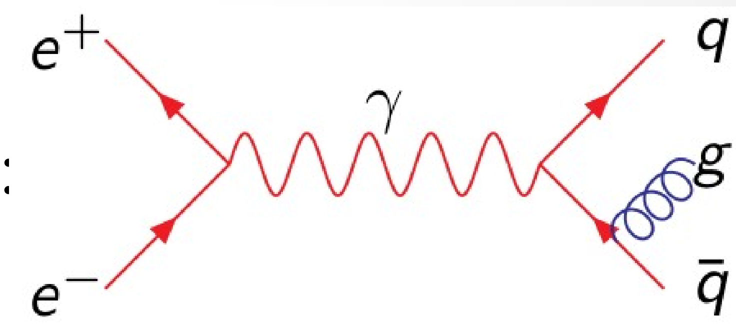
Quiz questions

- The virtual correction diagram has two additional vertices, while the real correction has only one, why are they of the same order?

Virtual:

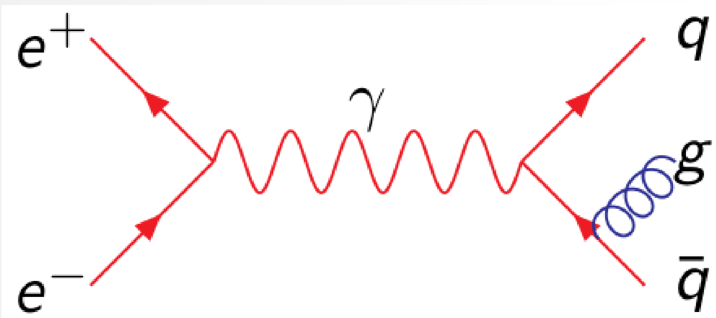


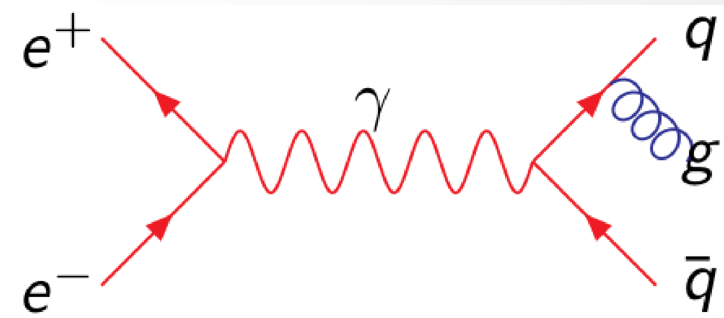
Real:



Real Emission

- There are two real emission diagrams:





- The matrix element is given by:

$$\begin{aligned}\mathcal{M} &= e^2 e_q g_s t^a_{ij} \bar{v}(p_b)\gamma_\mu u(p_a)\frac{-g^{\mu\nu}}{q^2} \\ & \bar{u}_i(p_1)\left[\gamma_\sigma \frac{\not{p}_1 + \not{k}}{(p_1+k)^2}\gamma_\nu - \gamma_\nu \frac{\not{p}_2 + \not{k}}{(p_2+k)^2}\gamma_\sigma\right] v_j(p_2)\epsilon^\sigma_a(k)\end{aligned}$$

Real Emission

- Using the color algebra

$$\sum_a^{N_C^2-1} t_{ij}^a \left(t_{ij}^a\right)^* = t_{ij}^a t_{ji}^a = \frac{1}{2}\delta^{aa} = \frac{1}{2}(N_C^2-1) = N_C C_F,$$

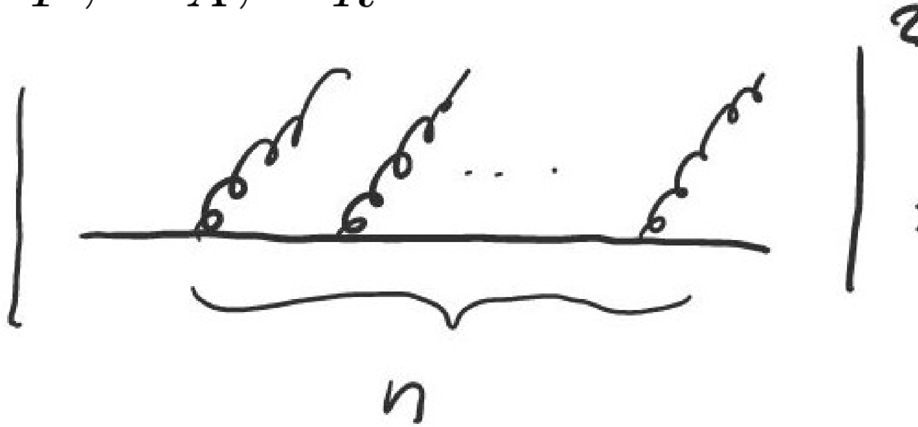
$$C_F \equiv \frac{1}{2N_C}(N_C^2-1), \quad C_A \equiv N_C, \quad T_R \equiv \frac{1}{2}$$

- and summing/averaging over final/initial state spins and colors we get:

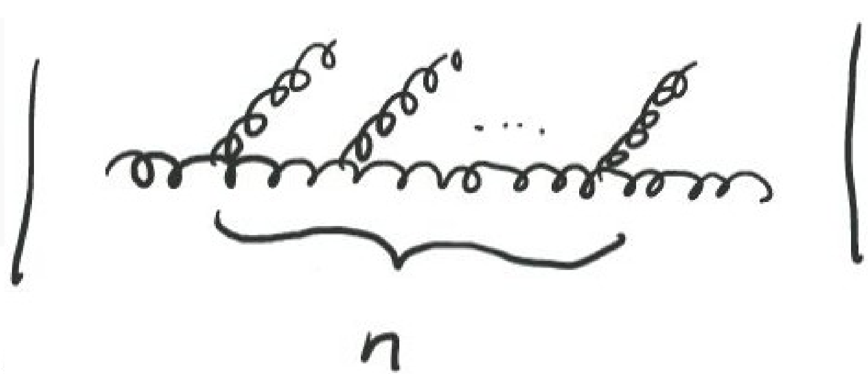
$$|\overline{\mathcal{M}}|^2 = \frac{4e^2e_q^2g_s^2N_c}{s} C_F \frac{(p_1\cdot p_a)^2+(p_1\cdot p_b)^2+(p_2\cdot p_a)^2+(p_2\cdot p_b)^2}{p_1\cdot k\; p_2\cdot k}$$

Color algebra

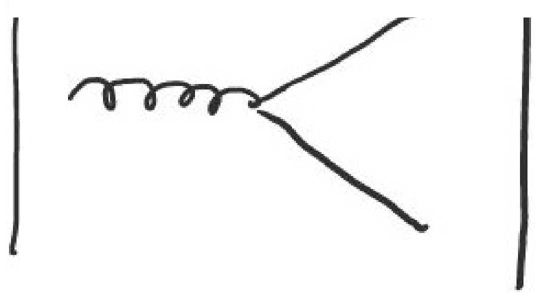
- What are $C_F,\ C_A,\ T_R$?



$$= N_c C_F^n$$



$$= \left(N_c^2 - 1\right) C_A^n$$

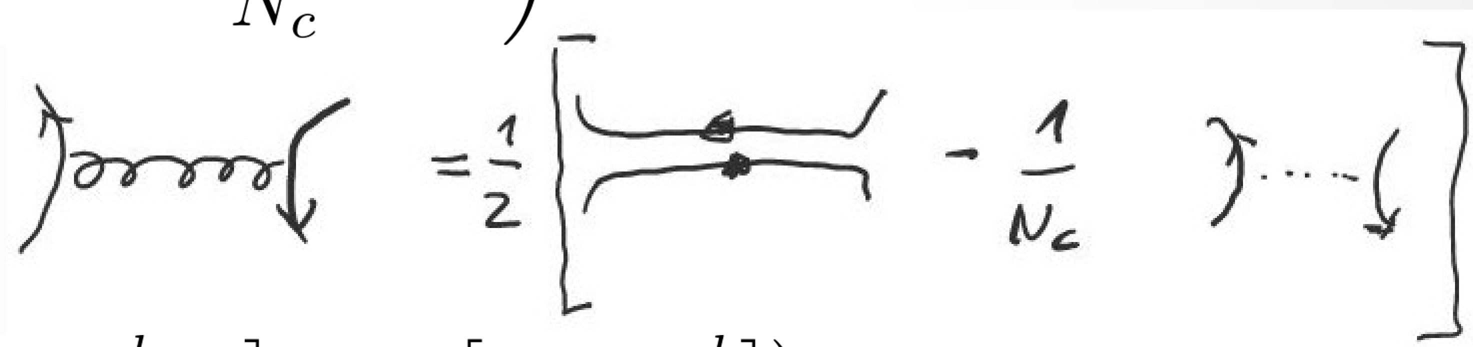


$$= \left(N_c^2 - 1\right) T_R$$

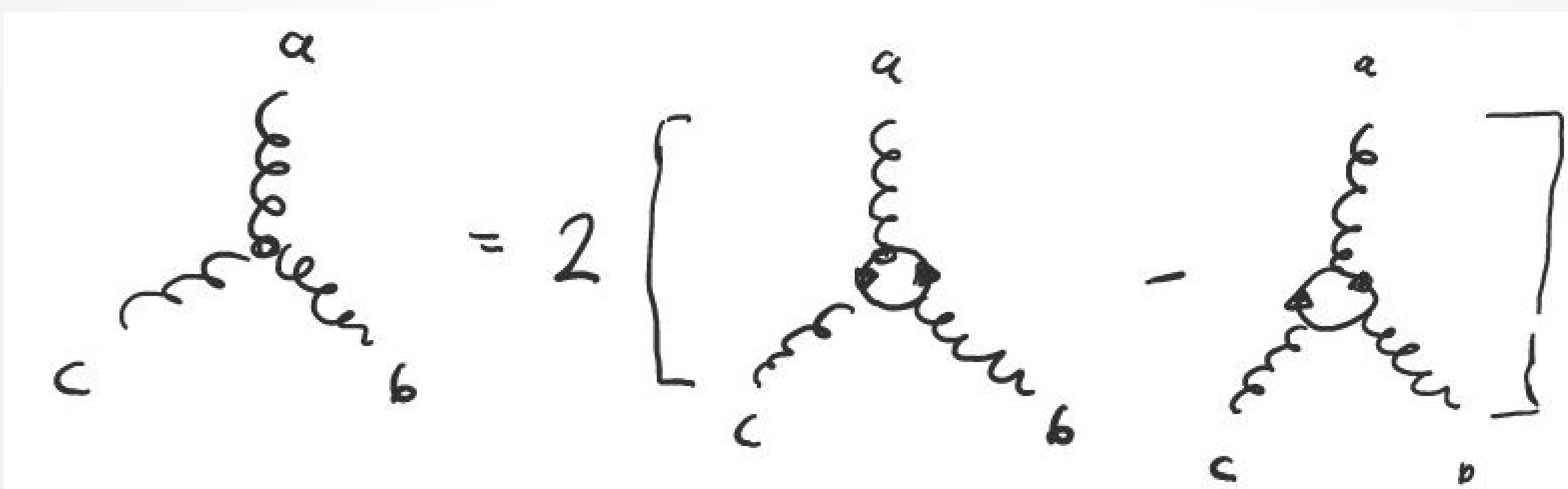
Color algebra

- We can convert identities for T matrices and f tensors into pictorial rules

$$T^a_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$

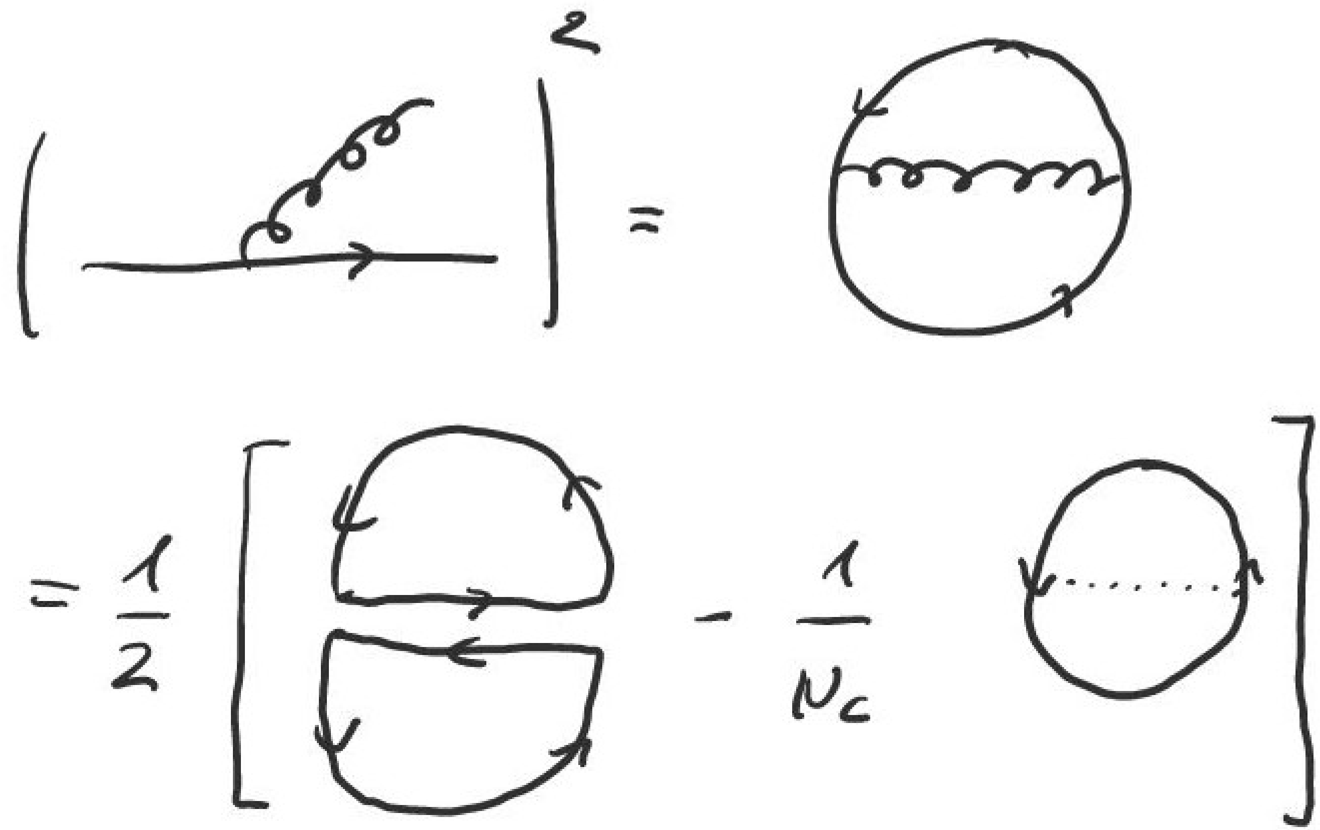


$$if^{abc} = 2\left(\mathrm{Tr}\left[T^aT^bT^c\right] - \mathrm{Tr}\left[T^aT^cT^b\right]\right)$$



Color Algebra

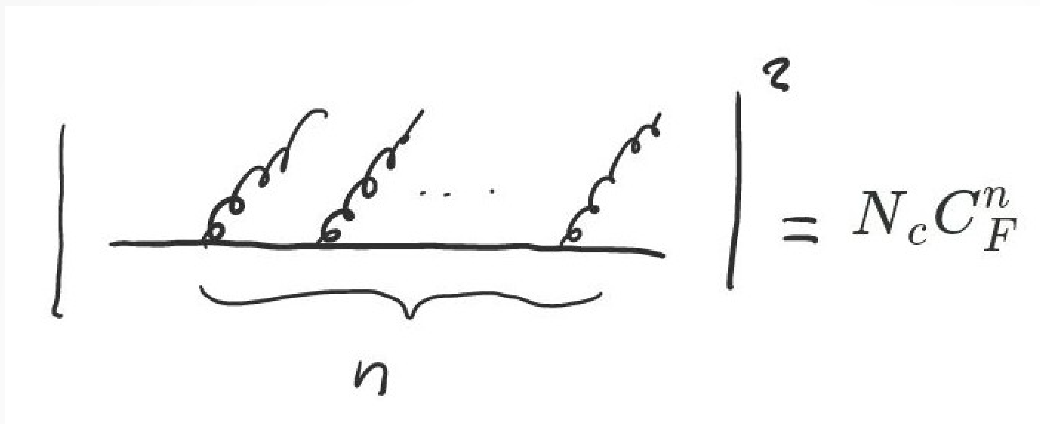
- Example

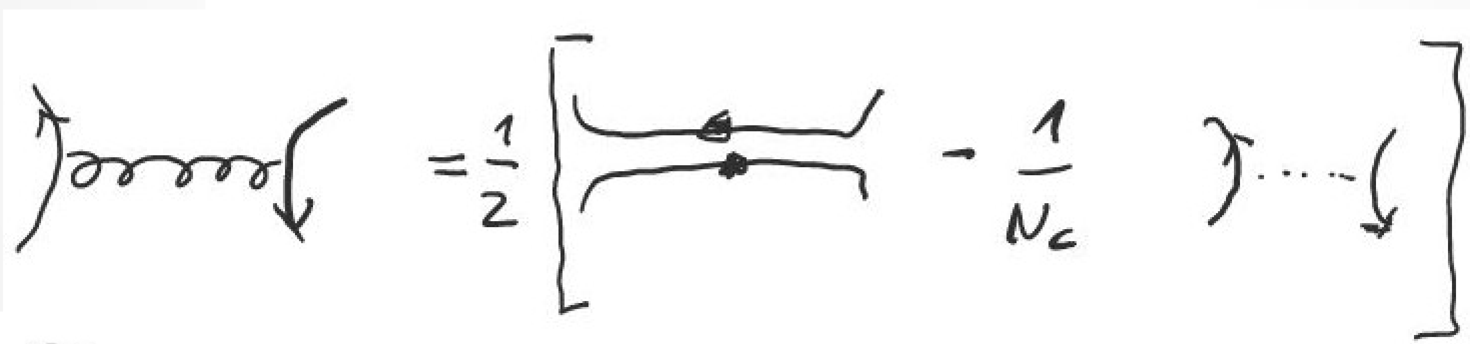


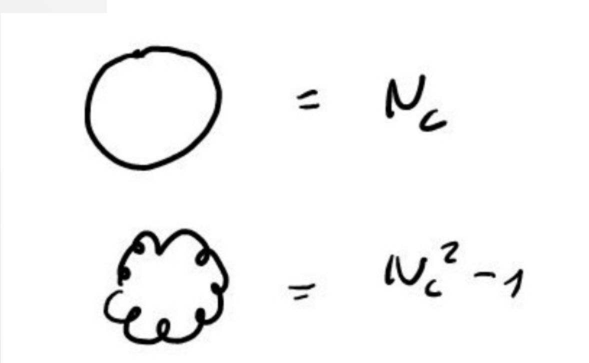
$$= \frac{1}{2}\left(N_c^2 - \frac{1}{N_c} N_c \right) = N_c C_F$$

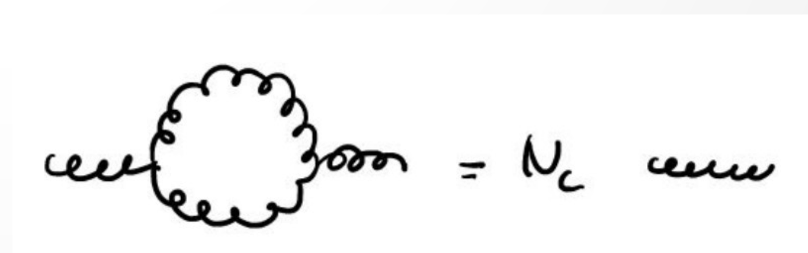
Color algebra

- Can you prove:









Three-body phase-space

- The three-particle phase-space is given by:

$$
\begin{aligned}
&\mathrm{d}\Phi_n(p_a+p_b;p_1,p_2,p_3)\\
&= \quad \delta^4\left(p_a+p_b-p_1-p_2-p_3\right)\frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3p_3}{(2\pi)^3 2E_3}\\
&= \quad \frac{1}{8(2\pi)^9}p_1\mathrm{d}p_1\mathrm{d}\cos\theta\mathrm{d}\phi p_2\mathrm{d}p_2\mathrm{d}\cos\beta\mathrm{d}\alpha\frac{1}{p_3}\delta(\sqrt{s}-p_1-p_2-p_3)
\end{aligned}
$$

- Polar coordinates *θ,φ* for p_1 defined with *z* axis along beam
- Polar coordinate *β,α* of p_2 defined with z axis in the p_1 direction
- Integrate the vectorial part of p_3 using the delta function

Three body phase-space

- Use the identity

$$p_3 = |\vec{p_3}| = |\vec{p_1} + \vec{p_2}| = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\beta}$$

  and integrate the energy delta function over β

$$\int \mathrm{d}\cos\beta\,\delta(\sqrt{s} - p_1 - p_2 - p_3) = \frac{p_3}{p_1p_2}$$

- So we get (using $x_i \equiv 2p_i/\sqrt{s}$):

$$\begin{aligned} \mathrm{d}\Phi_n(p_a+p_b;p_1,p_2,p_3) &= \frac{1}{8(2\pi)^9}\mathrm{d}p_1\mathrm{d}\cos\theta\mathrm{d}\phi\mathrm{d}p_2\mathrm{d}\alpha, \\ &= \frac{s}{16(2\pi)^7}\mathrm{d}x_1\mathrm{d}x_2\frac{\mathrm{d}\cos\theta\mathrm{d}\phi\mathrm{d}\alpha}{2(2\pi)^2} \end{aligned}$$

Real Emission

- Back to the cross section:

$$\sigma = \frac{1}{2s}\frac{s}{16(2\pi)^3}\int \mathrm{d}x_1\mathrm{d}x_2\frac{\mathrm{d}\cos\theta\mathrm{d}\phi\mathrm{d}\alpha}{2(2\pi)^2}|\overline{M}|^2,$$

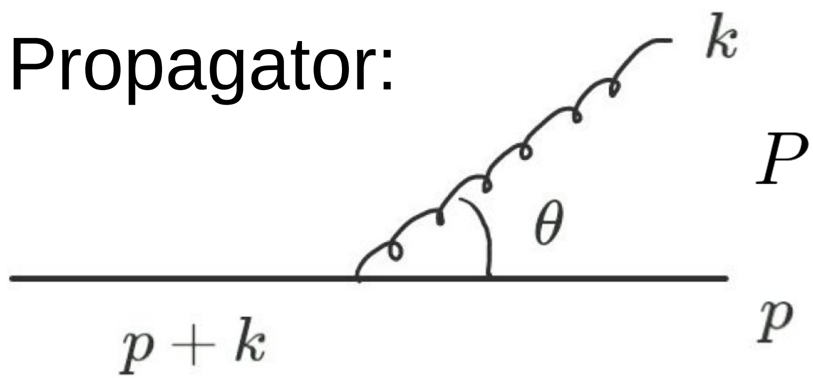
$$= \frac{4\pi\alpha^2 e_q^2 N_c}{3s} C_F\frac{\alpha_S}{2\pi}\int \mathrm{d}x_1\mathrm{d}x_2\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}.$$

Born cross section

- This is divergent at the edge of phase space!
- This is a common feature of perturbative QCD calculations

Real Emission

- Configurations that are undistinguishable from a leading order configuration are divergent
- Propagator:

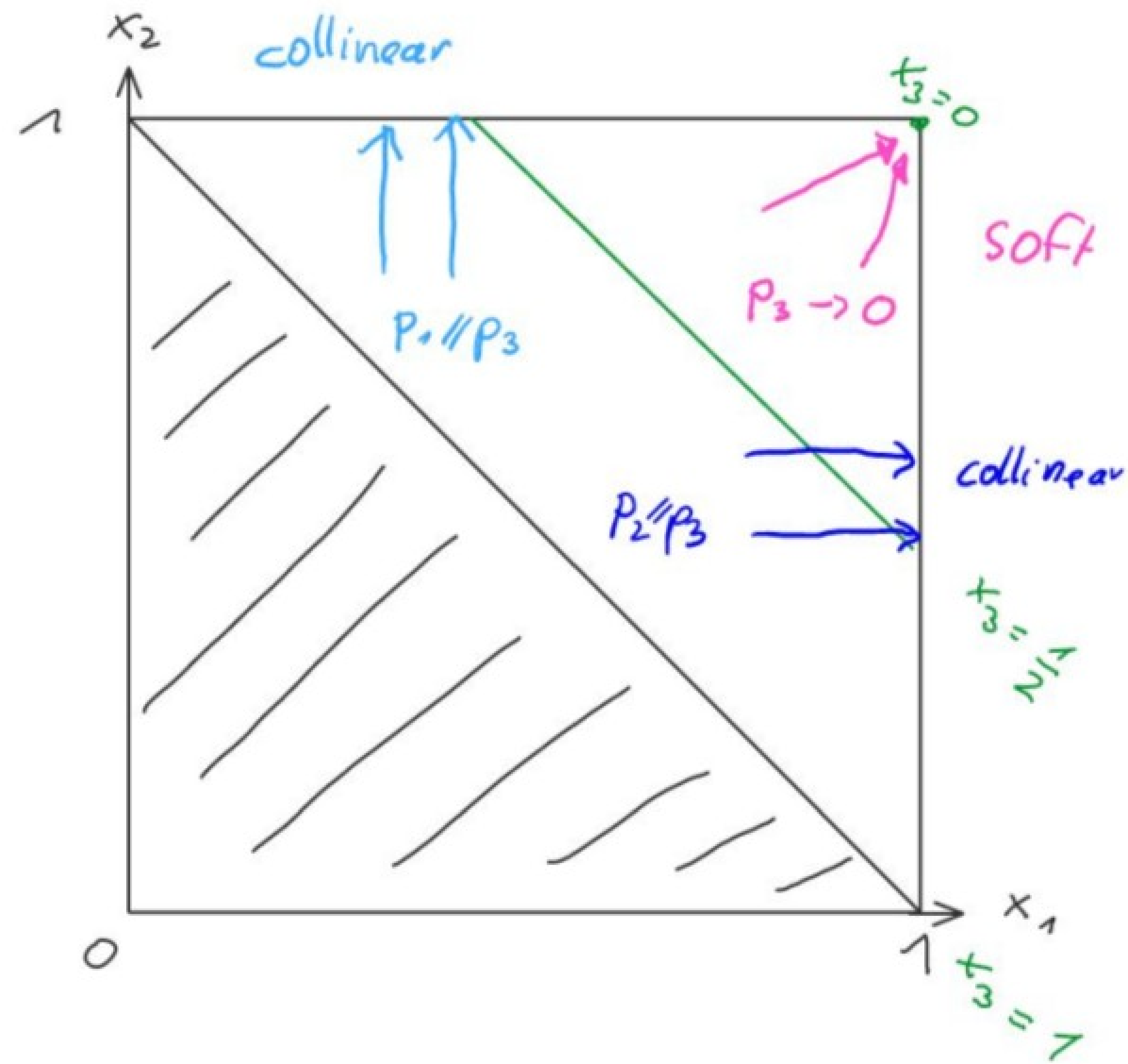


$$P^2 = (k+p)^2 = 2|k||p|(1-\cos\theta)$$

- There are two regions where this happens:
  - Collinear limit: cos$\rightarrow$*1*
  - Soft limit: $E \rightarrow 0$

Soft and collinear singularities

- Looking back at our 3 particle phase-space we have the regions
- Collinear: fixed x_1 or x_2 and $x_2/x_1 \rightarrow 1$
- Soft: x_1 and $x_2 \rightarrow 1$ with fixed $(1-x_1)/(1-x_2)$



Collinear limit

- If we take p_3 parallel to p_2 we can define

$$p_2 = (1-z)\bar{p}_2, p_3 = z\bar{p}_2, \text{with} \quad \bar{p}_2^2 = 0$$

- In the collinear limit the matrix element factorizes

$$|\mathcal{M}_{q\bar{q}g}|^2 \to |\mathcal{M}_{q\bar{q}}|^2 \times \frac{g_s^2}{p_2 \cdot p_3} \times C_F \frac{1+(1-z)^2}{z}$$

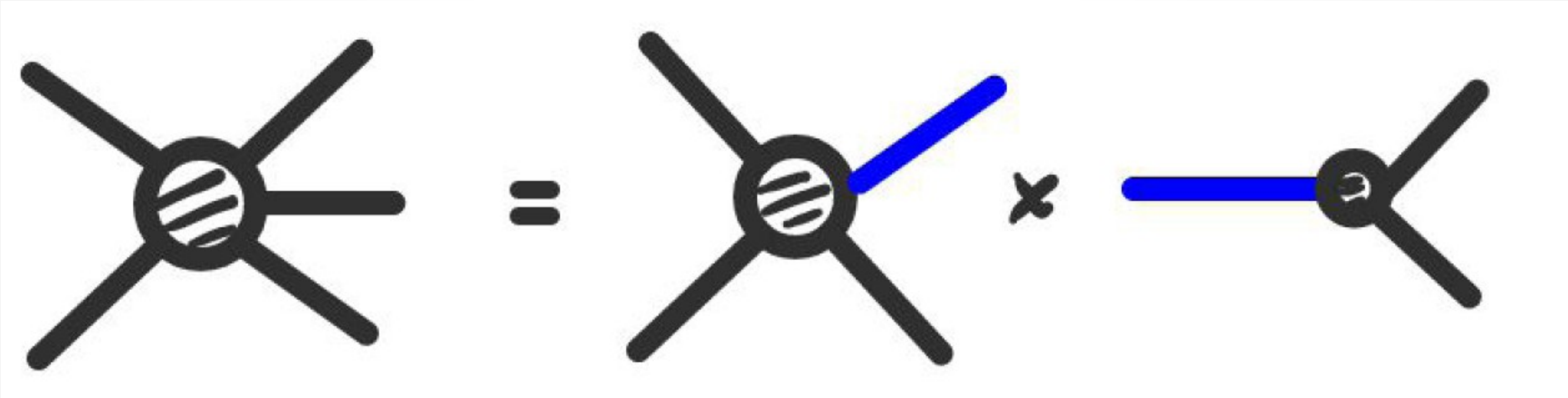
- The phase space also factorizes

$$\mathrm{d}\cos\theta \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\alpha \longrightarrow \mathrm{d}\cos\theta \, \frac{1}{4} z(1-z) \mathrm{d}z \mathrm{d}\theta_{23}^2 \mathrm{d}\phi$$

Two-particle phase space

Collinear limit

- Can use the same picture to symbolize both phase-space and matrix element factorisation



Collinear limit

- The cross section in the collinear limit becomes:

$$\sigma = \sigma_0 \int \frac{\mathrm{d}\theta_{23}^2}{\theta_{23}^2} dz \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z}$$

DGLAP splitting function

- The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting function is a universal probability distribution for the radiation of a collinear gluon in any process producing a quark

Soft limit

- The amplitude also factorizes in the soft limit

$$\mathcal{M}_{q\bar{q}g} = \mathcal{M}_{q\bar{q}} g_s t^a_{ij} \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right) \cdot \epsilon_A(k)$$

- The term in parenthesis is called the eikonal current
- In this case the squared matrix element factorizes

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 g_s^2 C_F \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k}$$

- So does the phase space

$$\mathrm{d}x_1 \mathrm{d}x_2 \longrightarrow \frac{2}{s} E_g \mathrm{d}E_g \mathrm{d}\cos\theta$$

Soft limit

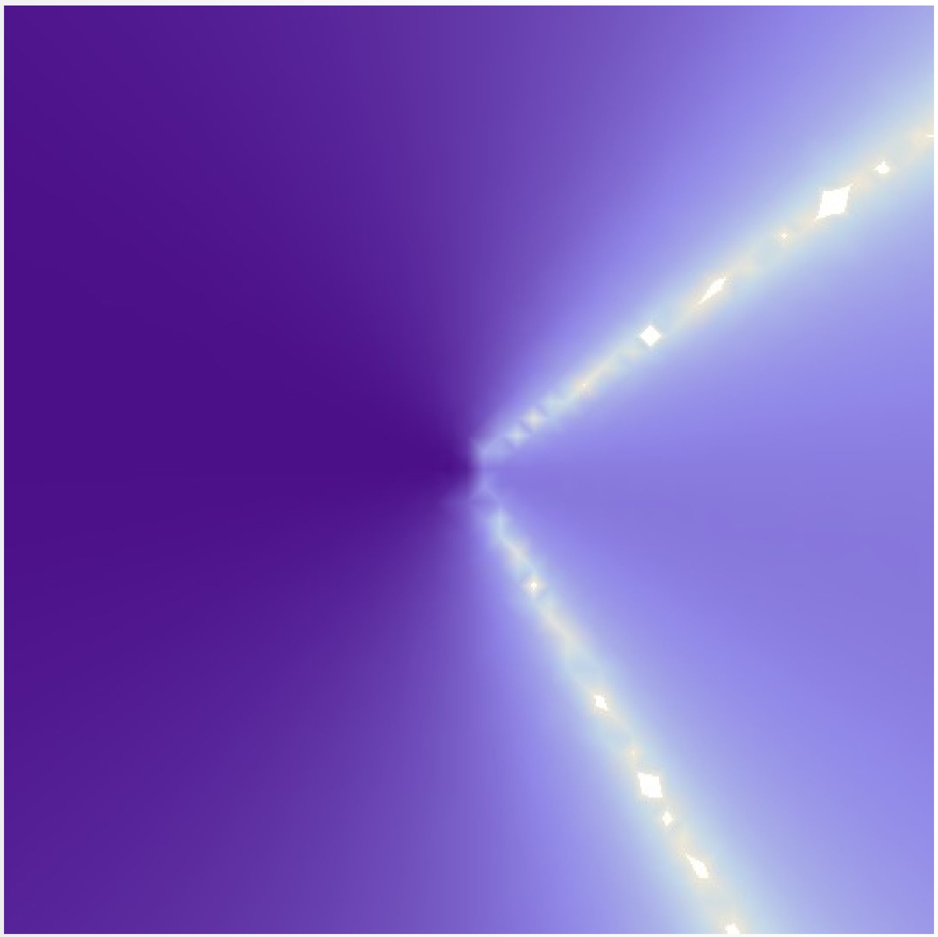
- We get for the cross section in the soft limit

$$\sigma = \sigma_0 \int C_F \frac{\alpha_S}{2\pi} \frac{\mathrm{d}E_g}{E_g} \mathrm{d}\cos\theta \frac{2(1-\cos\theta_{q\bar{q}})}{(1-\cos\theta_{qg})(1-\cos\theta_{\bar{q}g})}$$

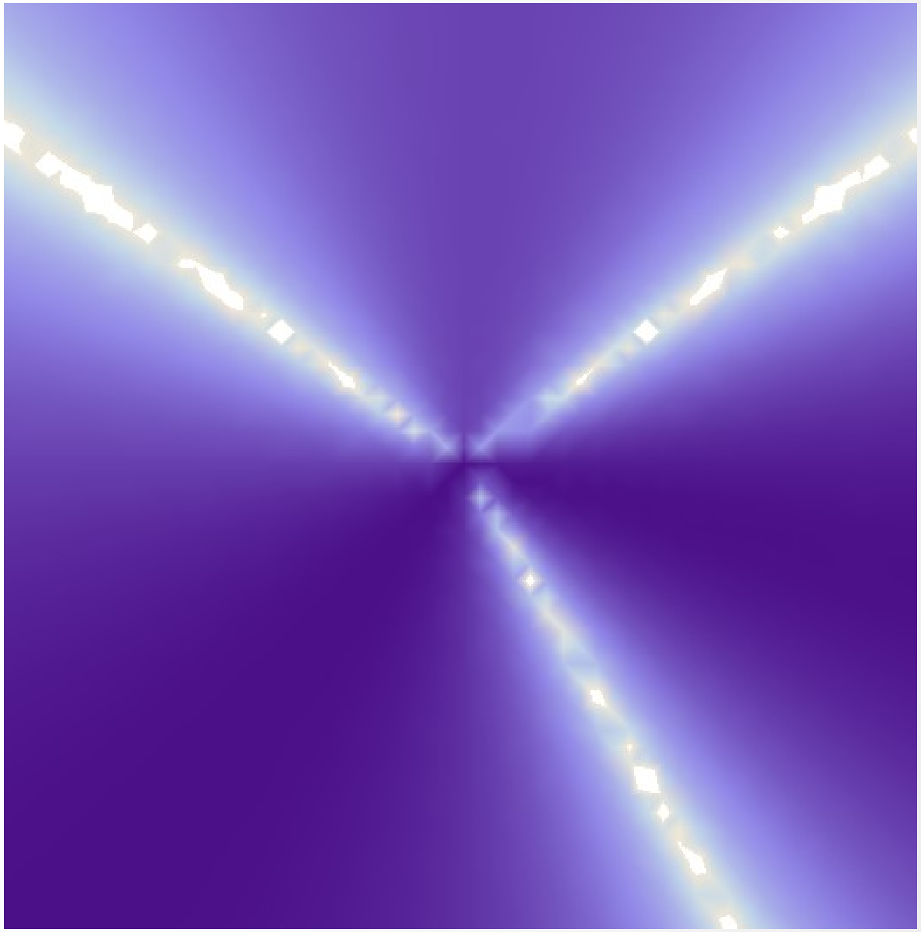
- Called the dipole radiation pattern
- One for each pair of colored particle
- Factorisation is only universal at the amplitude level

Soft emissions

$e^+e^- \to q\bar{q}\gamma$

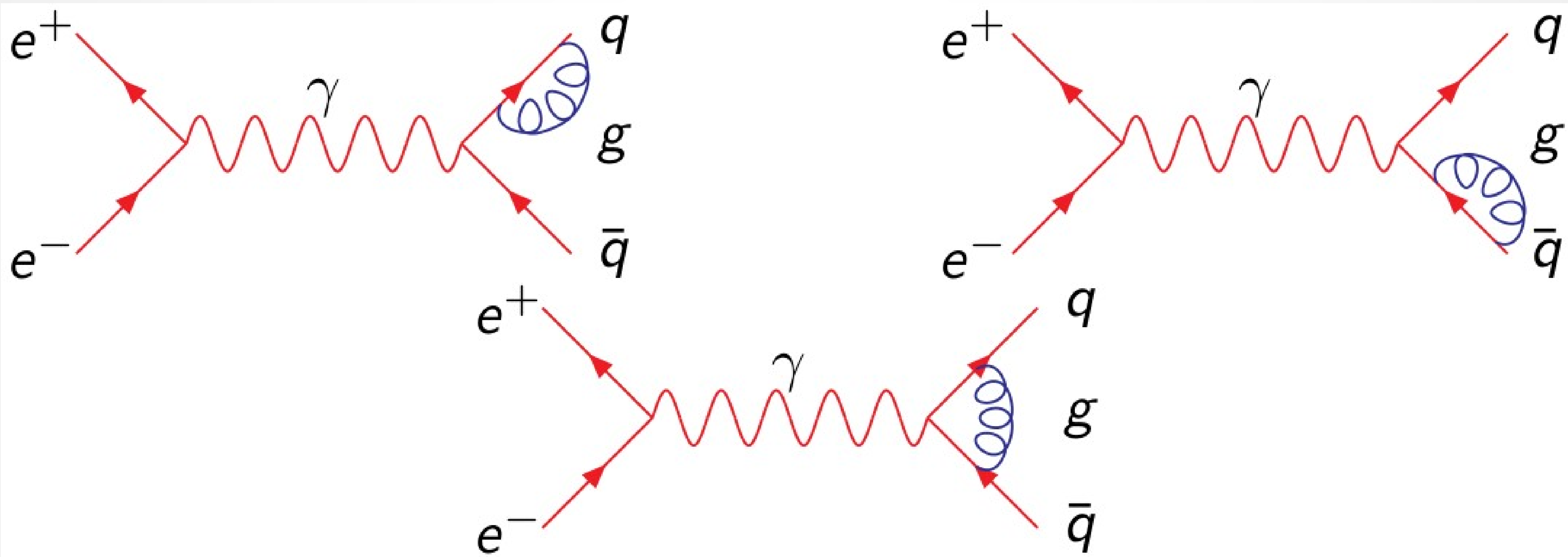


$e^+e^- \to q\bar{q}g$



Virtual Corrections

- At the same order there are also one-loop virtual diagrams



- They are also divergent, and typically negative
- These divergences will cancel the real emission divergencies

Total cross section

- To show the cancellation we need to regularize both the real and the virtual calculation
- The standard regularisation is dimensional regularisation. We perform the entire calculation in *d=4-2ε* space-time dimensions and we take the limit of four dimensions after the divergences have cancelled.
- For our calculation:

$$\begin{aligned} \sigma_{\rm real} &= \sigma_0 C_F \frac{\alpha_S}{2\pi} H(\epsilon) \left(\frac{4}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right) \\ \sigma_{\rm virtual} &= \sigma_0 C_F \frac{\alpha_S}{2\pi} H(\epsilon) \left(-\frac{4}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right) \end{aligned}$$

- The sum is finite and we can take the $\varepsilon \to 0$ limit

Total cross section

- So we get for the R ratio:

$$R(e^+e^-) = R_0(e^+e^-)\left(1 + \frac{\alpha_s}{\pi}\right)$$

- We can extract a value of the strong coupling constant from the measurement of *R*

$$\alpha_S(m_Z) = 0.1226 \pm 0.0038$$

From the PDG

- The R ratio is know to higher orders (NNNLO for massless quarks) and the NLO calculation is available with massive quarks

Event shapes

- The definition of thrust is:

$$T = \max_{\vec{\hat{n}}} \frac{\sum_i |\vec{p}_i \cdot \vec{\hat{n}}|}{\sum_i |p_i|}.$$

- For events with back-to-back jets (pencil-like) we have *T=1*
- For perfectly spherical events *T*=1/2
- For three partons:

$$T = \max(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3)$$

Differential cross section

- To obtain a differential cross section with respect to an observable *O*, we need to compute

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} = \frac{\mathrm{d}}{\mathrm{d}\mathcal{O}}\mathcal{N}\int \mathrm{d}\Phi\, |\mathcal{M}|^2$$

- This is easy if we can write

$$\mathrm{d}\Phi = \mathrm{d}\mathcal{O}\mathrm{d}\mathrm{x}_1 \ldots \mathrm{d}\mathrm{x}_\mathrm{n}$$

- We the get:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} = \mathcal{N}\int \mathrm{d}\mathrm{x}_1 \ldots \mathrm{d}\mathrm{x}_\mathrm{n}\, |\mathcal{M}|^2$$

Differential cross section

- If not we introduce a delta function

$$1 = \int \mathrm{d}\mathcal{O}\delta\left(\mathcal{O} - \mathcal{O}(\Phi)\right)$$

into the phase-space integration of the final state Φ so that we now have the explicit dependence on the observable

$$\sigma = \mathcal{N} \int \mathrm{d}\mathcal{O}\mathrm{d}\Phi \left|\mathcal{M}\right|^2 \delta(\mathcal{O} - \mathcal{O}(\Phi))$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} = \mathcal{N} \int \mathrm{d}\Phi \left|\mathcal{M}\right|^2 \delta(\mathcal{O} - \mathcal{O}(\Phi))$$

- In most cases we cannot compute it analytically and we need to do it numerically

Thrust

- Let's look at the differential cross section with respect to the thrust variable
- For the born and virtual part we know *T=1*
- For the real emission we use:

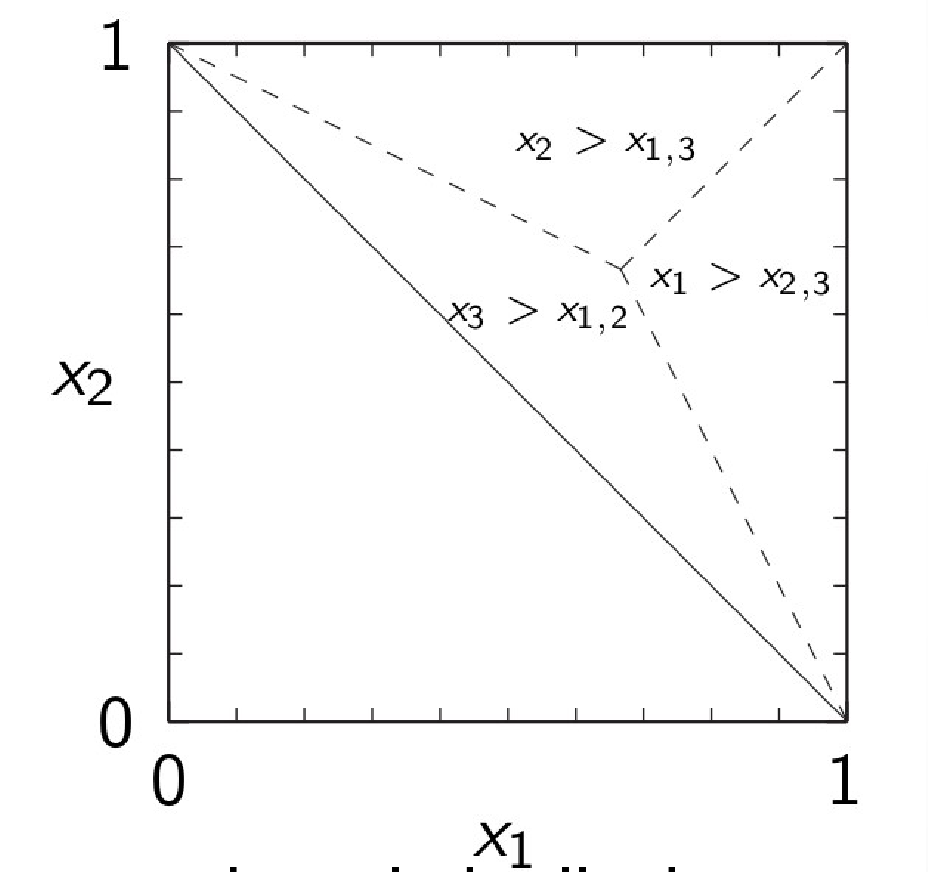
$$1 = \int \mathrm{d}T \delta(T - \max\{x_1, x_2, x_3\})$$

- And get:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}T} = \sigma_0 C_F \frac{\alpha_S}{2\pi} \int \mathrm{d}x_1 \mathrm{d}x_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \delta(T - \max\{x_1, x_2, x_3\})$$

Thrust

- In our 3 particle phase space we have 3 regions:



- Can be computed analytically, but we will only look at the limit $T \rightarrow 1$

Thrust

- We look at the region where x_1 is largest.

$$
\begin{aligned}
\left.\frac{\mathrm{d}\sigma}{\mathrm{d}T}\right|_{x_1>x_{2,3}} &= \sigma_0 C_F \frac{\alpha_S}{2\pi} \int_{2(1-T)}^{T} \mathrm{d}x_2 \frac{T^2+x_2^2}{(1-T)(1-x_2)} \\
&\approx \sigma_0 C_F \frac{\alpha_S}{2\pi} \frac{1}{1-T} \int_0^T \mathrm{d}x_2 \frac{1+x_2^2}{(1-x_2)} \\
&\approx \sigma_0 C_F \frac{\alpha_S}{2\pi} \frac{1}{1-T} \int_0^T \mathrm{d}x_2 \frac{x_2^2-1+2}{(1-x_2)} \\
&\approx \sigma_0 C_F \frac{\alpha_S}{2\pi} \frac{1}{1-T} \int_0^T \mathrm{d}x_2 \frac{2}{(1-x_2)} \\
&\approx -\sigma_0 C_F \frac{\alpha_S}{2\pi} \frac{2}{1-T} \ln(1-T),
\end{aligned}
$$

Thrust

- This is divergent as $T \to 1$
- We get the same from the region where x_2 is largest
- The third region x_3 largest is less divergent
- The most divergent part of the large thrust distribution is then:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}T} \approx -C_F \frac{\alpha_S}{2\pi}\frac{4}{1-T}\ln(1-T)$$

Thrust: Two jet limit

- We can use thrust to separate events in 2-jet and 3-jets:

$$R_3(\tau) = \int_{1/2}^{1-\tau} \mathrm{d}T \frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}T} \overset{\tau\to 0}{\sim} C_F \frac{\alpha_S}{2\pi} 2\ln^2\tau$$

$$R_2(\tau) \equiv 1 - R_3(\tau) \overset{\tau\to 0}{\sim} 1 - C_F \frac{\alpha_S}{2\pi} 2\ln^2\tau + \left(C_F \frac{\alpha_S}{2\pi}\right)^2 2\ln^4\tau.$$

- For each order we get an additional term $\ln^2(\tau)$

- When the log becomes of the same order than the coupling constant, the perturbation series breaks down

Thrust: jet rate

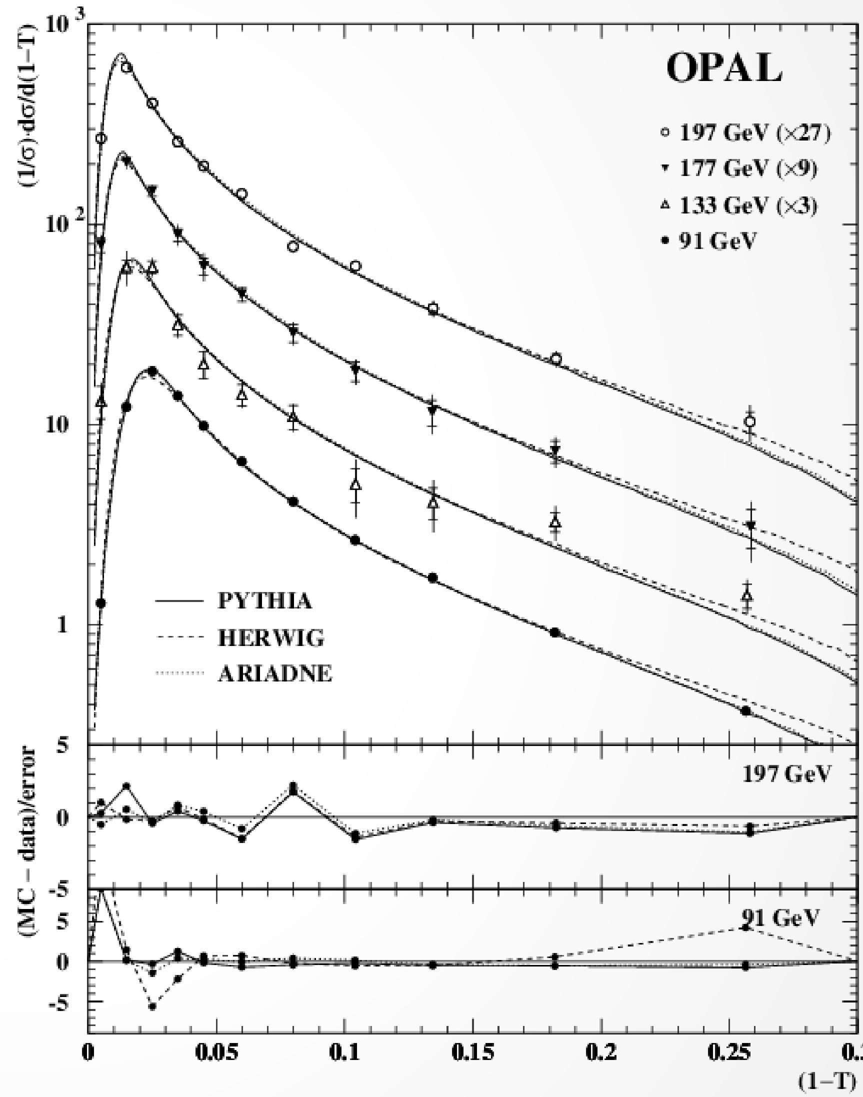
- The solution is the re-sum these terms to all order:

$$R_2(\tau) \stackrel{\tau\to 0}{\sim} \exp\left[-C_F \frac{\alpha_S}{2\pi} 2 \ln^2 \tau\right] + \cdots$$

- This is an example of a Sudakov Form Factor
- This is finite for $\tau \to 0$ (it is in fact 0 which means that the probability of not emitting is 0)

Thrust

- *1-T* distribution
- Large *1-T* is described by fixed order
- Small *1-T* is described by resummation



Running coupling

- Virtual corrections are also ultraviolet divergent. These divergences are removed through renormalisation. As an effect, the coupling constant becomes scale dependent.
- Alphas → alphas(mu)
- Can compute the way the coupling is changing

$$\mu^2 \frac{\mathrm{d}\alpha_S}{\mathrm{d}\mu^2} \equiv \beta(\alpha_S) = -\beta_0 \alpha_S^2 + \ldots \qquad \beta_0 = \frac{11 N_c - 4 T_R N_f}{12\pi}$$

- The fact that β_0 is negative leads to asymptotic freedom

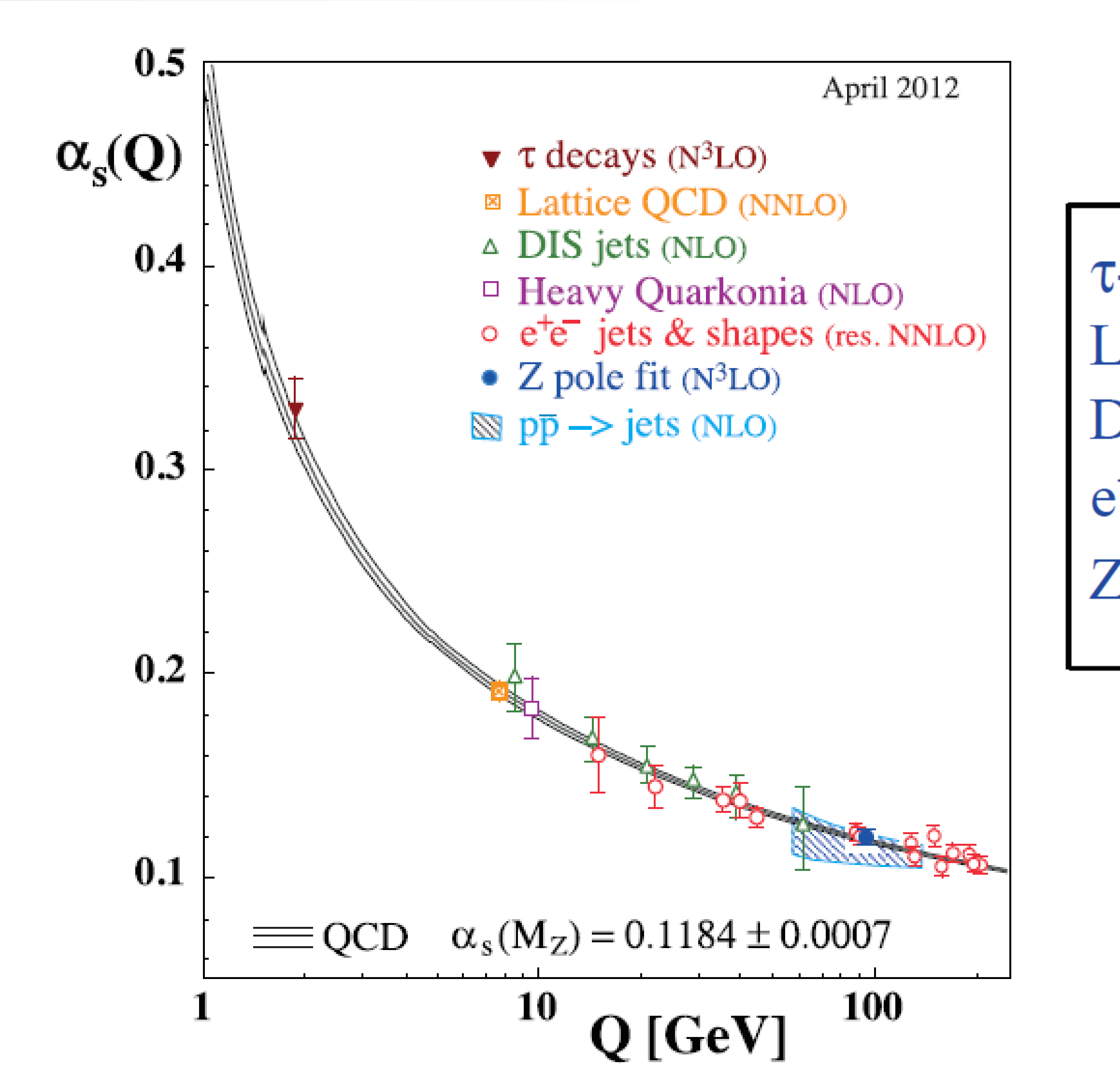
Running coupling

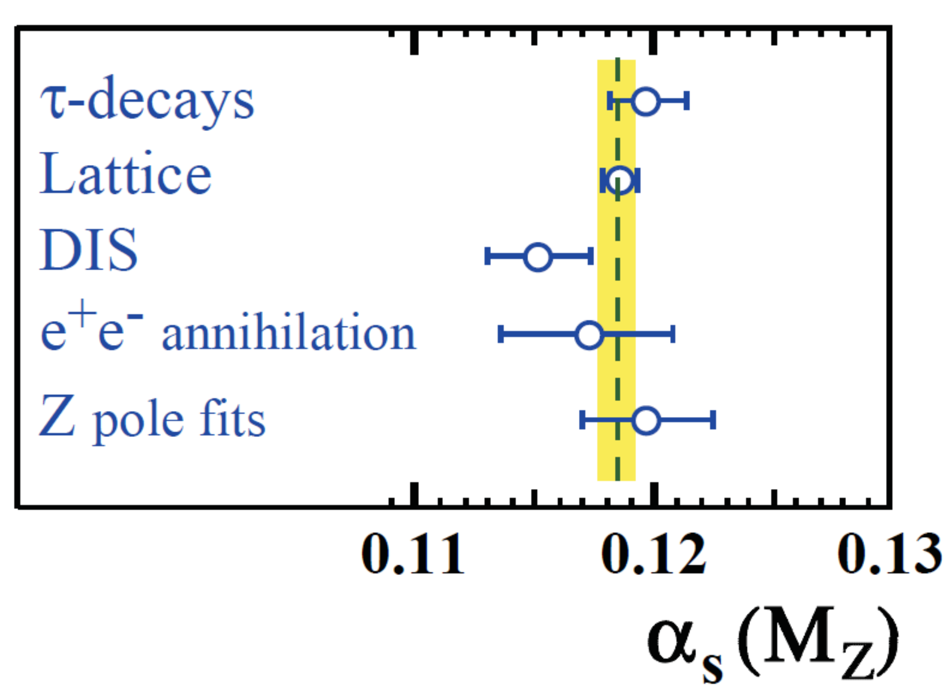
- The values of the strong coupling are usually quoted at the Z mass
- The numerical value at other scales can be computed by solving the differential equation

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log\left(\frac{\mu^2}{\Lambda^2_{\mathrm{QCD}}}\right)}\left[1 + \ldots\right]$$

- This defines Λ_{qcd}
- There is some freedom in the renormalisation process, which leads to different renormalisation schemes
- Dependence on the renormalisation scale and scheme for a physical prediction is of a higher order than the calculation

Strong coupling measurement





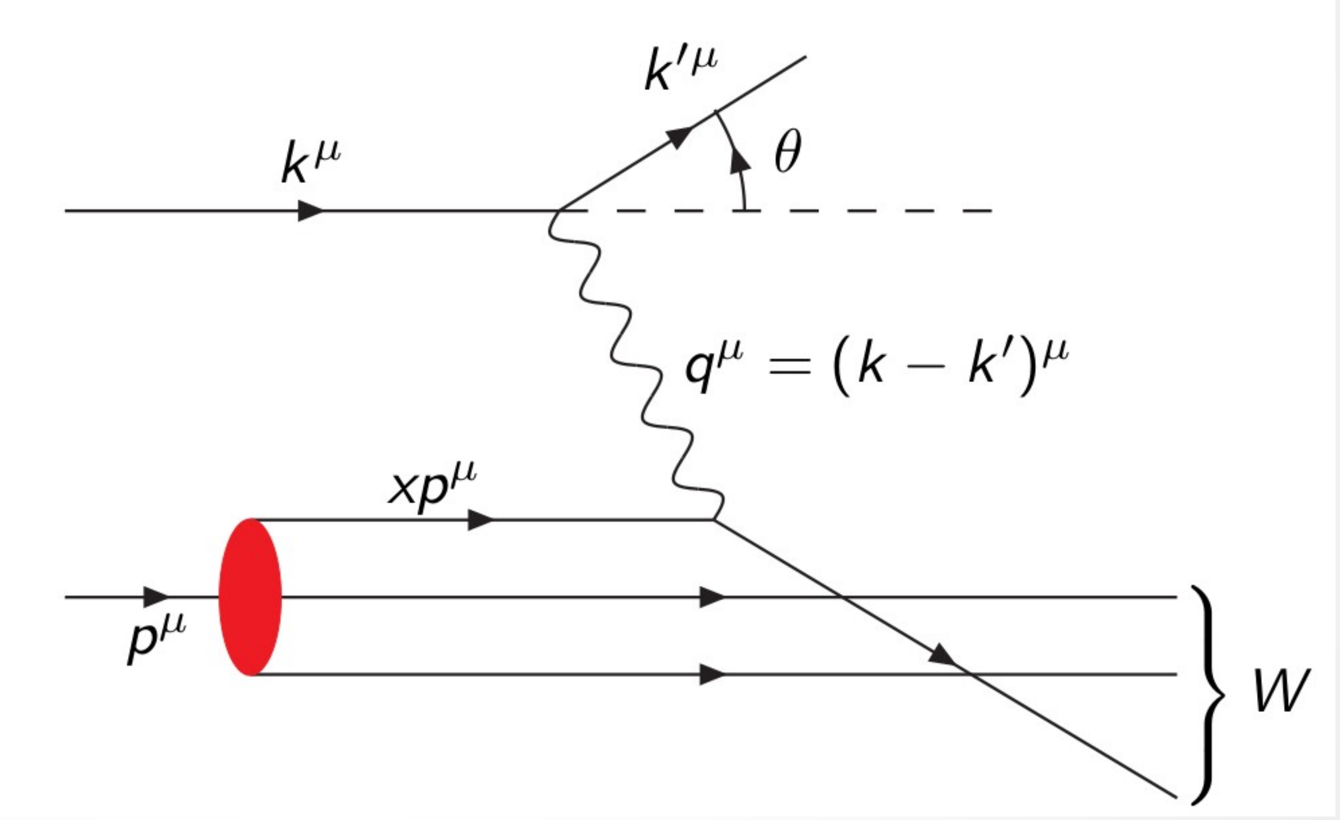
From PDG 2012

Deep Inelastic Scattering

- Proton electron scattering
- DIS was a key piece of evidence for quarks
- Used to measure parton distribution functions

DIS Kinematics

- $Q^2 = -q^2$
- $x = \frac{Q^2}{2p \cdot q}$
- $W^2 = (p+q)^2$
- $y = \frac{p \cdot q}{p \cdot k} = \frac{Q^2}{xs}$



- Deep means Q^2 large and inelastic means W^2 large (compared to the Proton mass)

DIS

- We can parametrize the cross section in terms of two unknown structure functions

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{xQ^4}\left[y^2 x F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right]$$

- If we consider the proton as a bound state of partons, we can calculate these structure function in terms of the probability $f_q(\eta)$ of a quark to carry a given fraction η of the proton's momentum

$$\frac{\mathrm{d}^2\sigma(e+\mathrm{proton})}{\mathrm{d}x\mathrm{d}Q^2} = \sum_q \int_0^1 \mathrm{d}\eta f_q(\eta)\frac{\mathrm{d}^2\sigma(e+q(\eta p))}{\mathrm{d}x\mathrm{d}Q^2}.$$

DIS

- If we take the outgoing parton to be on-shell we can relate η to our kinematic variables

$$(q+\eta p)^2 = 2\eta p\cdot q - Q^2 = 0,\ \eta = x$$

- So we get:

$$\frac{\mathrm{d}^2\sigma(e+\mathrm{proton})}{\mathrm{d}x\mathrm{d}Q^2} = \sum_q f_q(x)\frac{\mathrm{d}^2\sigma(e+q(xp))}{\mathrm{d}Q^2}$$

- With the partonic cross section:

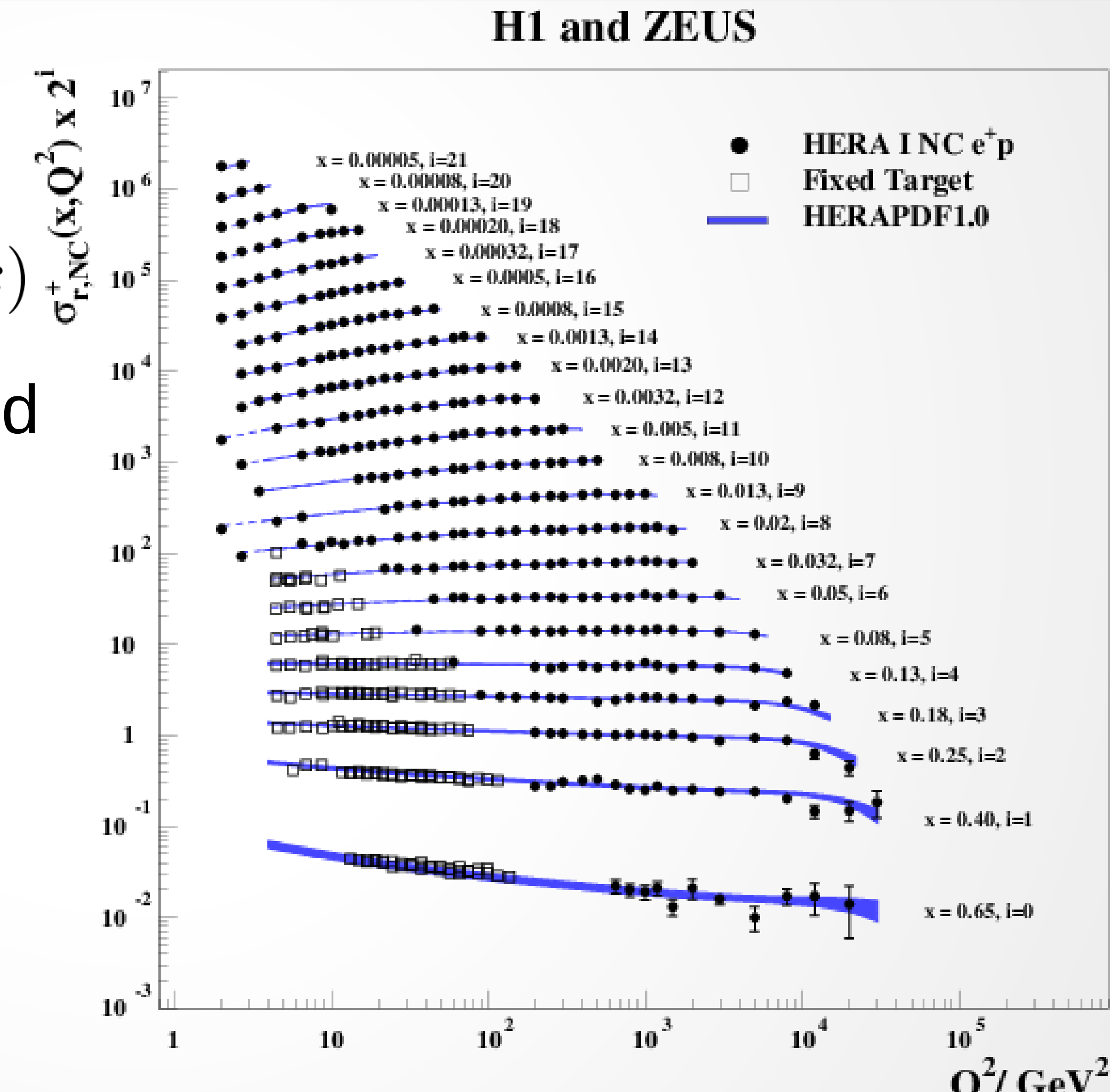
$$\frac{\mathrm{d}^2\sigma(e+q(xp))}{\mathrm{d}Q^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1+(1-y)^2\right]$$

DIS

- So we get

$$
\begin{aligned}
2xF_1(x) &= F_2(x), \\
F_2(x) &= \sum_q e_q^2 x f_q(x)
\end{aligned}
$$

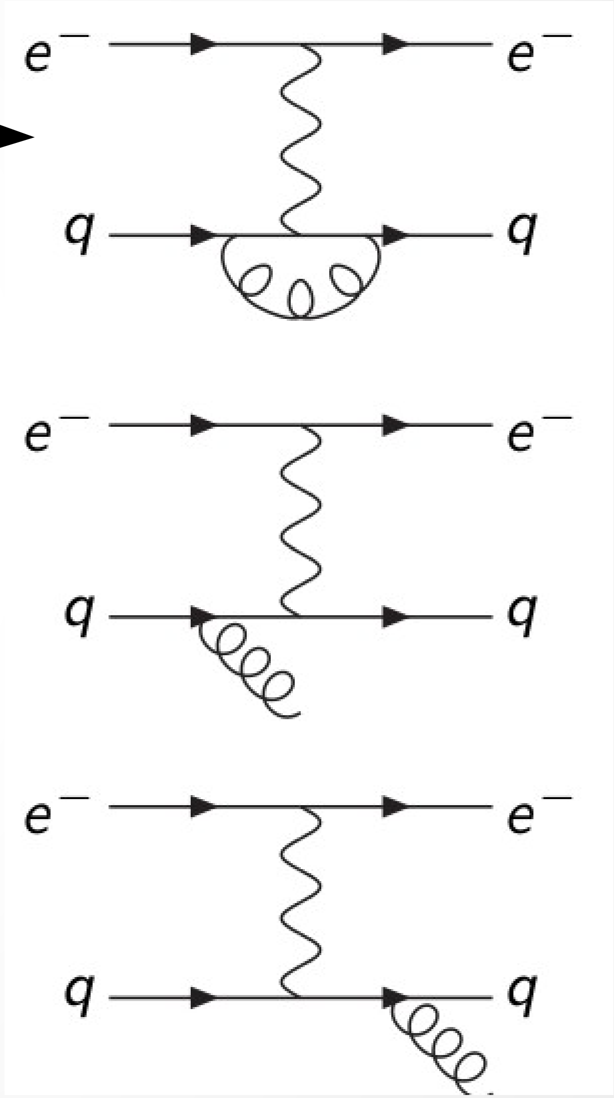
- Only functions of *x* and not *Q* (referred to as Bjorken scaling)
- Quite a good description, but quantum corrections lead to scaling violations



From arXiv:1001:109

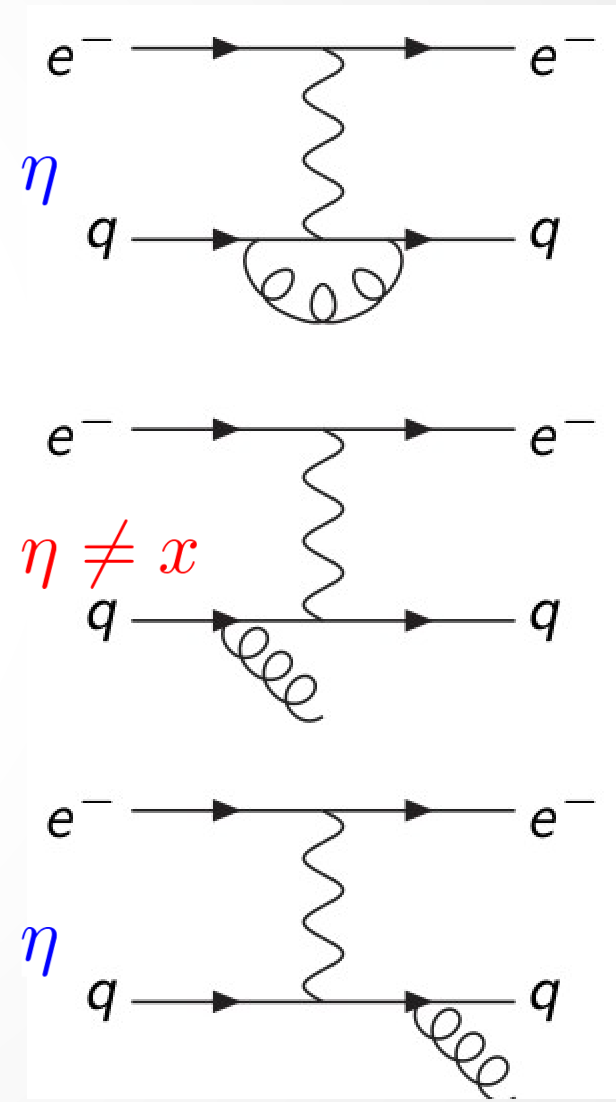
Radiative corrections

- We want to add radiative corrections
- Virtual correction
- Real correction:
  - Initial state radiation
  - Final state radiation
- As always we have divergencies:
  - Gluon soft or collinear to either initial or final state quark
  - Negative divergence in the virtual part



Radiative corrections

- In the collinear and soft limits, the final state radiation and the virtual part have the same momentum fraction so the cancellation can happen
- The initial state radiation has a different momentum fraction $\eta > x$



Initial state collinear limit

- In the collinear limit we have a factorisation similar to that we have seen in the final state

$$\mathrm{d}\sigma_{q\to qg} \to \mathrm{d}\sigma_{q\to q} \times \frac{\alpha_S}{2\pi} C_F \hat{P}_{qq}(z) \frac{\mathrm{d}t}{t}\frac{\mathrm{d}z}{z} \qquad \hat{P}_{qq}(z)\frac{1+z^2}{1-z}$$

- This is the unregularised DGLAP splitting function, which is singular as z-->1
- The virtual contribution contains a compensating singularity at z=1
- We can define regularised splitting functions to be the sum of the real part and the virtual contribution

Initial-state collinear limit

- The regularized splitting function is given by:

$$
\begin{aligned}
P_{qq}(z) &= C_F \frac{1+z^2}{1-z} + C_F \delta(1-z) \left\{ \frac{3}{2} - \int_0^1 \mathrm{d}z' \frac{2}{1-z'} \right\} \\
&\equiv C_F \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right)
\end{aligned}
$$

- With the "+" distribution

$$
\int_0^1 g(x) \left(\frac{f(x)}{1-x} \right)_+ \mathrm{d}x = \int_0^1 (g(x) - g(1)) \frac{f(x)}{1-x} \mathrm{dx}
$$

Initial state Collinear limit

- We get the following result for F_2

$$F_2(x,Q^2) = \sum_q e_q^2 \int_x^1 \frac{\mathrm{d}\eta}{\eta} f_q(\eta) \left[\delta\left(1-\frac{x}{\eta}\right) + \frac{\alpha_S}{2\pi} P_{qq}\left(\frac{x}{\eta}\right) \int_0 \frac{\mathrm{d}t}{t} + \bar{R}_{qq}\left(\frac{x}{\eta}\right)\right],$$

  where $\bar{R}_{qq}$ is a calculable finite correction.
- We still have a divergence as $t \to 0$
- This divergence comes from long time scales, for which perturbative QCD is not valid
- These singularities should be absorbed in the pdfs

Factorization

- We introduce a factorization scale and absorb the infrared divergence below that scale into the definition of the pdf (same as a renormalization procedure, we don't measure the 'bare' pdf)

$$
\begin{aligned}
F_2(x,Q^2) &= x\sum_q e_q^2 \int_x^1 \frac{\mathrm{d}\eta}{\eta} f_q(\eta,\mu_F^2)\left[\delta\left(1-\frac{x}{\eta}\right)\right. \\
&\left. +\frac{\alpha_S}{2\pi} P_{qq}\left(\frac{x}{\eta}\right)\log\frac{Q^2}{\mu_F^2} + R_{qq}\left(\frac{x}{\eta}\right)\right].
\end{aligned}
$$

- The pdf now depend on the factorization scale (and scheme)
- Physical observables would be independent on this scale if we could compute all orders

Evolution equation

- We cannot compute the coupling constant, but we can compute its evolution

$$\mu^2 \frac{\mathrm{d}\alpha_S}{\mathrm{d}\mu^2} = -\beta_0 \alpha_S^2 + \dots$$

- The same applies to the PDF:

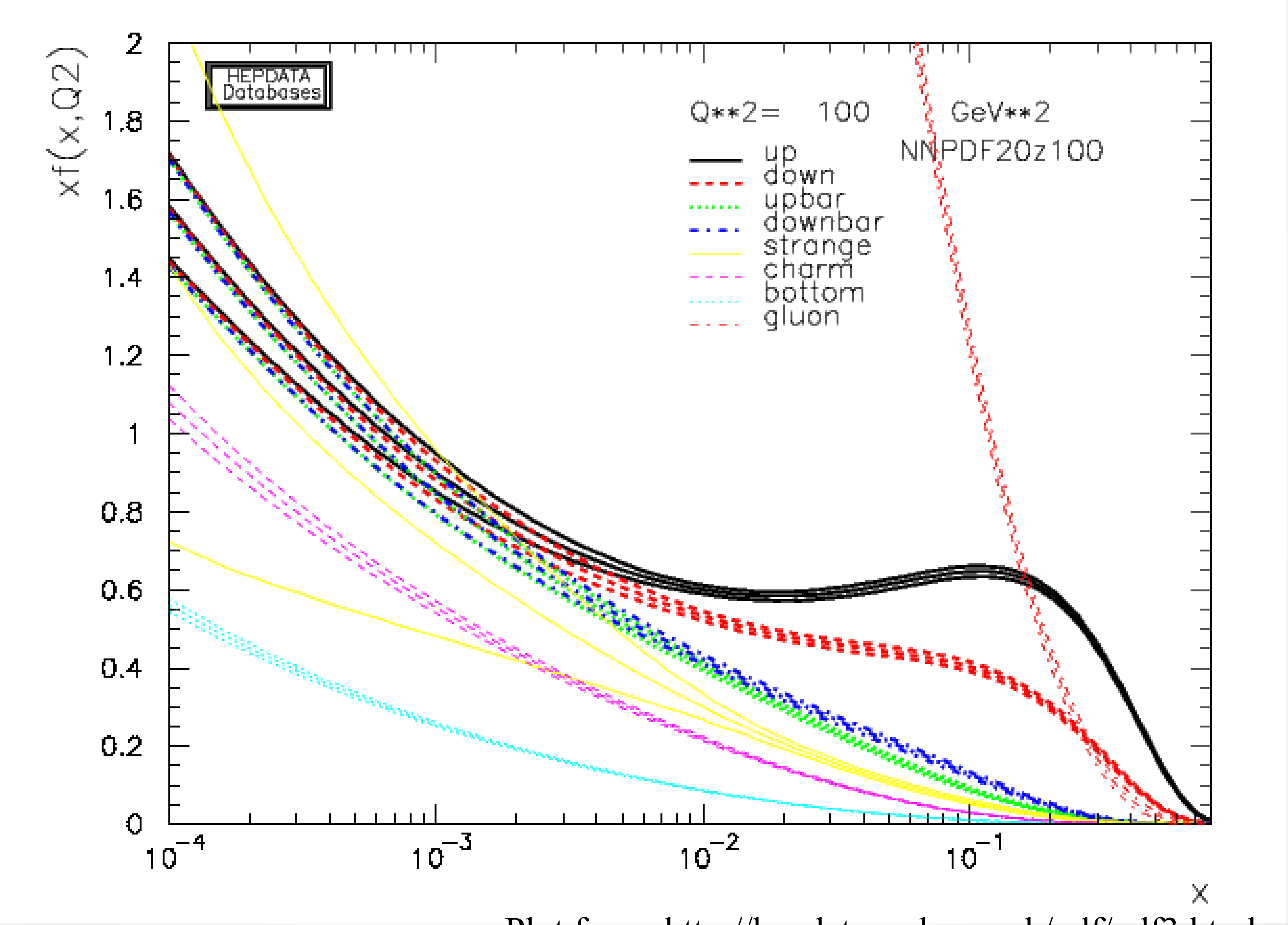
$$\mu_F^2 \frac{\partial f_q(x, mu_F^2)}{\partial \mu_F^2} = \frac{\alpha_S(\mu_F^2)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} f_q(y, \mu_F^2) P_{qq}\left(\frac{x}{y}\right) + \dots$$

- These are the DGLAP evolution equations

Measuring the PDFs

- Universal: measure in DIS and use in hadron collider
- Measure at one scale and evolve to other scales
- Several pdf sets that distinguish themselves through
  - Methodology
    - MSTW, ABM, Cteq, CT10, ... : Fit parameters of ansatz
    - NNDPF : neural network (no parametrisation)
  - Choice of data sets
  - Treatment of quark masses, spectrum
  - Order: LO,NLO,NNLO (you have to take the pdf set that matches the accuracy of your calculation)

PDF sets



Plot from: http://hepdata.cedar.ac.uk/pdf/pdf3.html

Hadron Colliders

Hadron colliders

- Hadron colliders are more complicated:

$$\mathrm{d}\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_{a/A}(x_1, \mu_F^2) f_{b/B}(x_2, \mu_F^2) \hat{\sigma}_{ab}(\hat{s}, \mu_F^2, \mu_R^2)$$

Momentum fractions

Parton density functions

Partonic cross section

- Partonic variables are often denoted with ^
- Factorization is formally proven only for few processes and observables

Kinematics

- At hadron colliders we normally use transverse momenta and rapidity *y* or pseudorapidity *η* to describe the kinematic

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} \qquad \eta = -\ln\tan\frac{\theta}{2} \qquad \eta = y \text{ for massless p}$$

- Using these variables we can write momenta as:

$$p^\mu = (E,\, p_x,\, p_y,\, p_z) = (m_\perp \cosh y, p_\perp \sin\phi, p_\perp \cos\phi, m_\perp \sinh y),$$

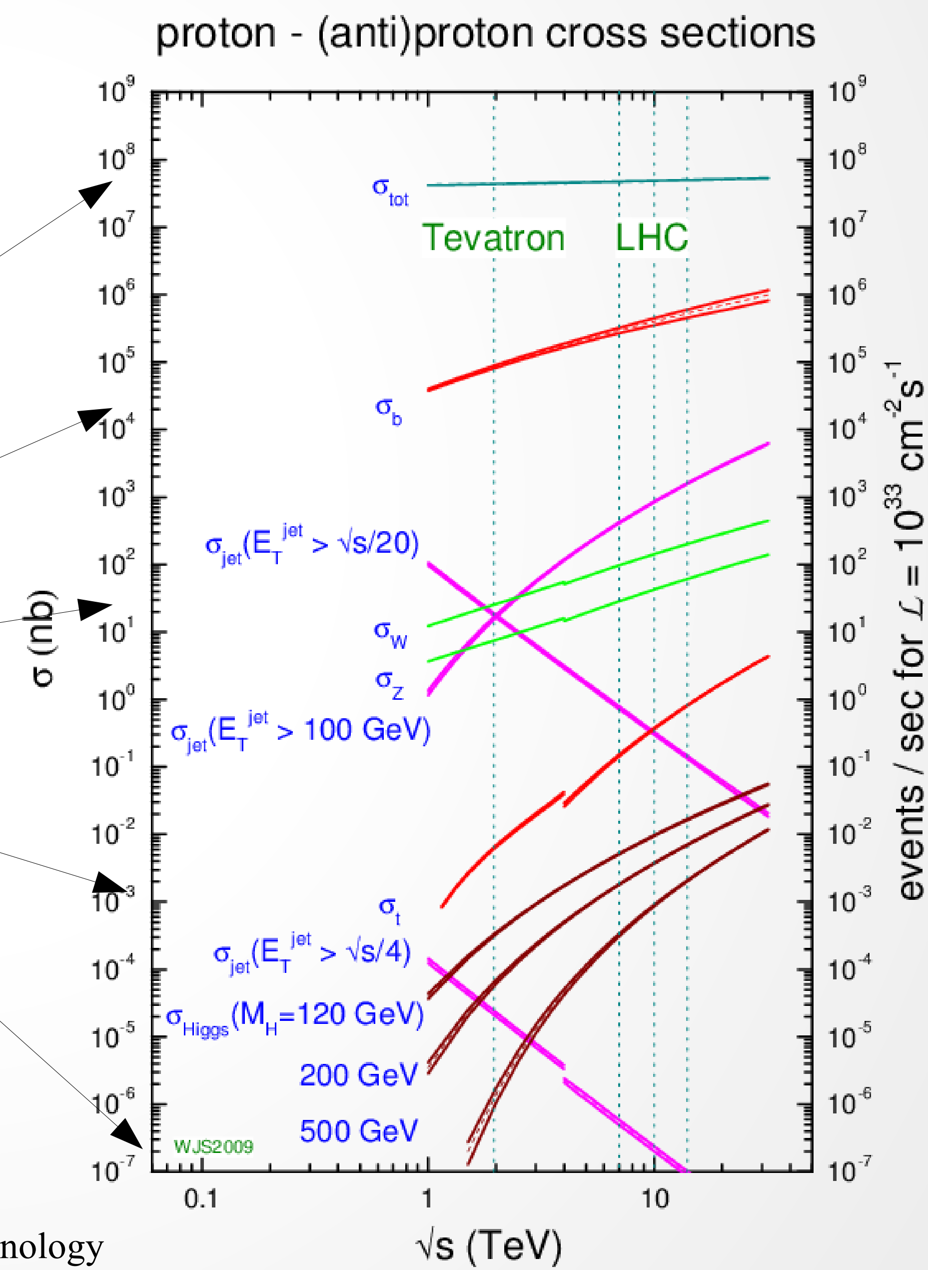
$$m_\perp^2 = p_\perp^2 + m^2$$

- In terms of these variables the phase-space element can be written as

$$\frac{\mathrm{d}^4 p}{(2\pi)^4}\delta(p^2 - m^2)\theta(E) = \frac{\mathrm{d}^3 p}{(2\pi)^2 2E} = \frac{\mathrm{d}y\mathrm{d}^2 p_\perp}{2(2\pi)^3}.$$

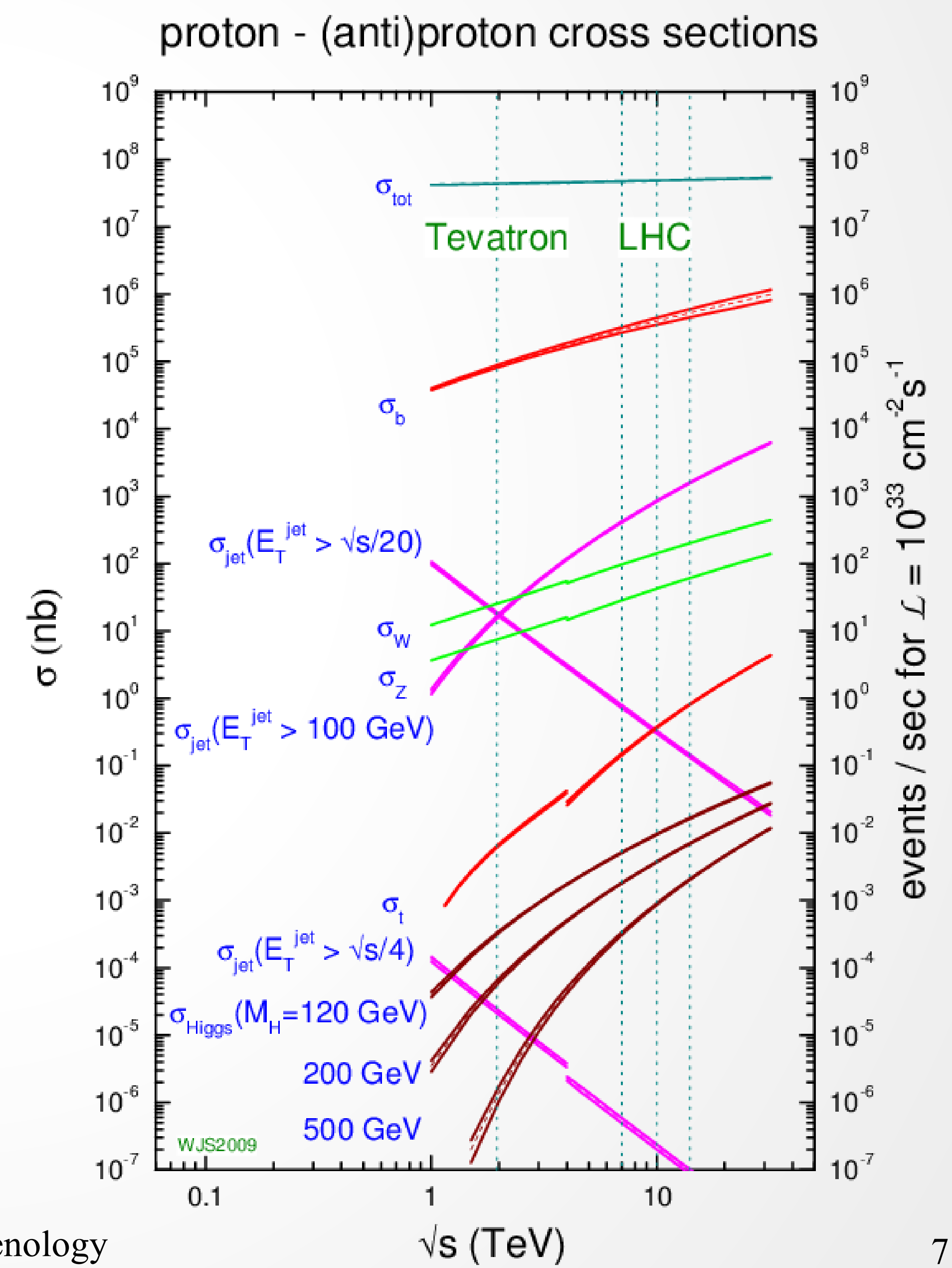
Hadron collider cross sections

- QCD processes dominate
- Orders of magnitude between the different processes
  - 1 time per second
  - 1 time per hour
  - 1 time per month
  - 1 time per century
  - 1 time per million year



Quizz question

- Why are there jumps?
- Why not always?



Resonance production

- We take the initial state partons as massless and the centre of mass energy of the hadron collision to be $s=4E^2$
- The cross section for the Z production is

$$\hat{\sigma}_{q\bar{q}\to Z^0\to\mu^+\mu^-} = \frac{1}{N_C^2}\frac{12\pi\hat{s}}{M_Z^2}\frac{\Gamma_{q\bar{q}}\Gamma_{\mu^+\mu^-}}{(\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

- If the width is much smaller than the mass, we can use the narrow-width approximation

$$\frac{1}{(\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2} \approx \frac{\pi}{M_Z\Gamma_Z}\delta(\hat{s}-M_Z^2)$$

Z boson is on-shell

Resonance production

- We define:

$$\hat{s} = x_1 x_2 s \quad , \qquad \hat{y} = \frac{1}{2}\ln\frac{x_1 + x_2 + x_1 - x_2}{x_1 + x_2 - x_1 + x_2} = \frac{1}{2}\ln\frac{x_1}{x_2}$$

- In terms of these variables we have

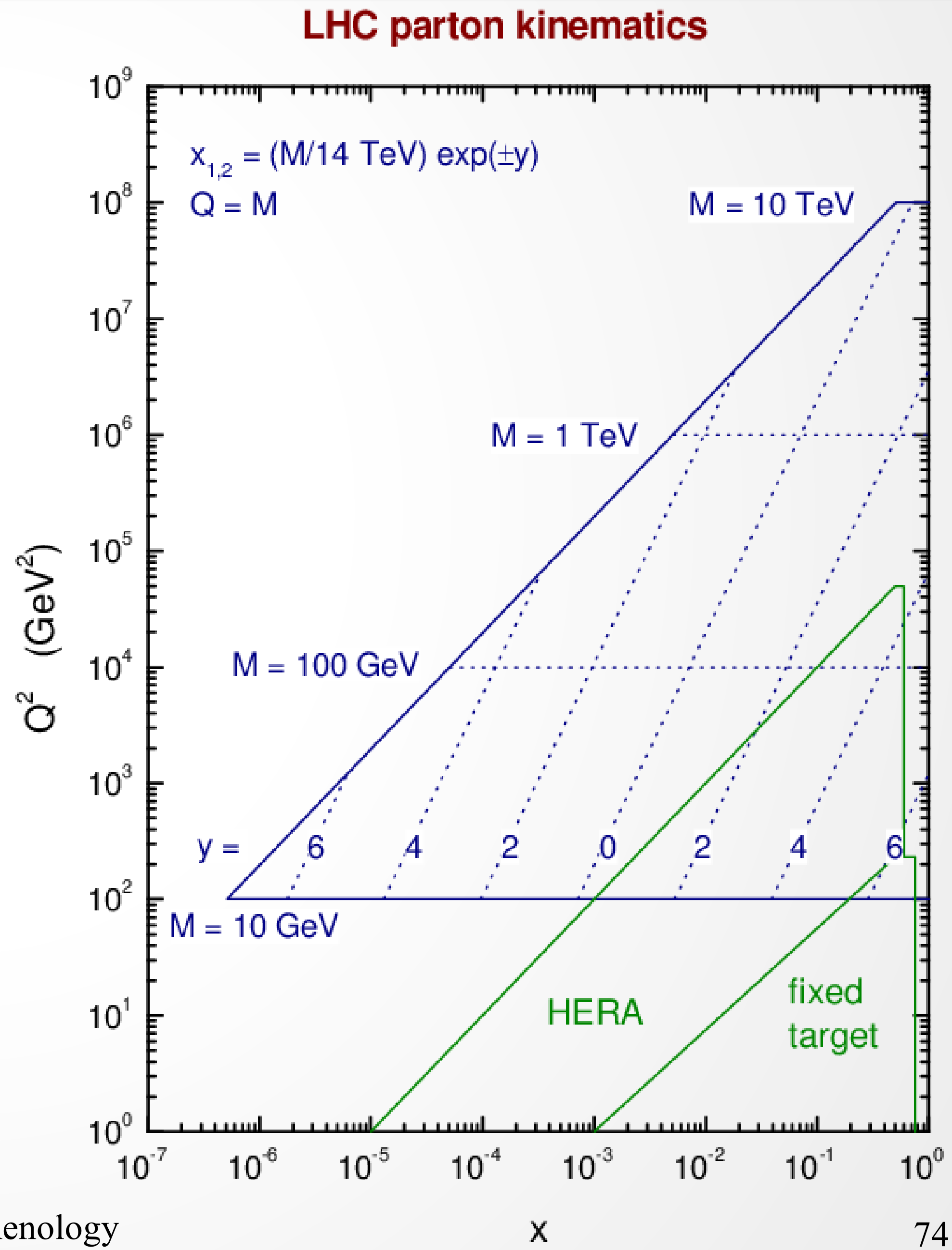
$$x_{1,2} = \sqrt{\frac{\hat{s}}{s}}\, e^{\pm\hat{y}} \;, \quad \hat{y} = \frac{1}{2}\ln\frac{x_1^2 s}{\hat{s}} \le \ln\frac{2E}{\sqrt{\hat{s}}} = \hat{y}_{\mathrm{max}} \;, \quad s\mathrm{d}x_1\mathrm{dx}_2 = \mathrm{d\hat{s}d\hat{y}}$$

- So we get:

$$\sigma_{AB\to Z^0\to\mu^+\mu^-} = \sum_{q\bar{q}} \int_{-\hat{y}_{\mathrm{max}}}^{\hat{y}_{\mathrm{max}}} \mathrm{d}\hat{y}\, x_1 f_{q/A}(x_1,\, \mu_F^2)\, x_2 f_{\bar{q}/B}(x_2,\, \mu_F^2)\, \frac{12\pi^2}{N_C^2 M_Z^3}\, \Gamma_{q\bar{q}} B_{\mu^+\mu^-}$$

Resonance production

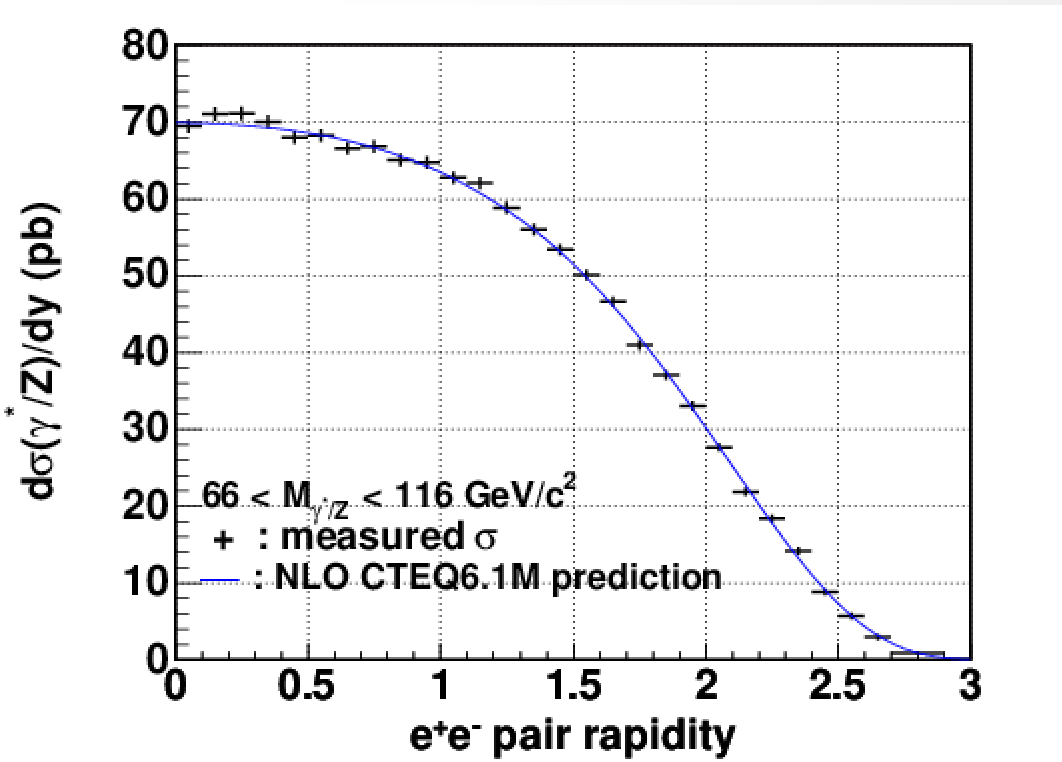
- In this case the dependence on the rapidity is only through the PDFs
- Can generalize for other resonances
- The higher the mass, the more central



Plot from the MSTW homepage

Z boson rapidity

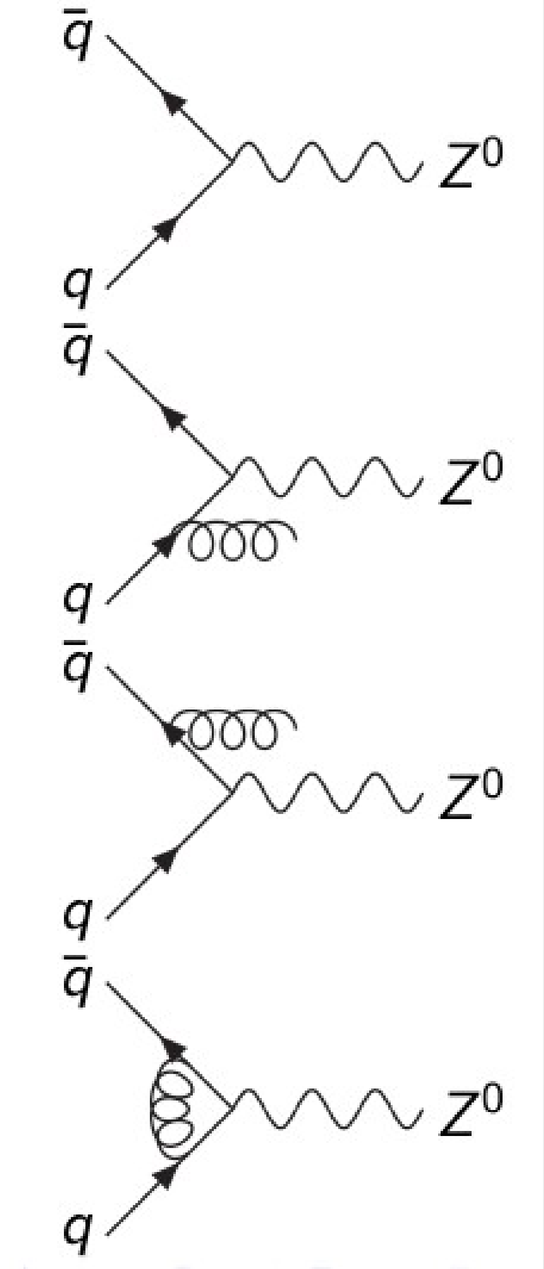
- Because the rapidity only enters the cross section through the pdf (at leading order and without cuts) it is a good observable to extract information about the pdf



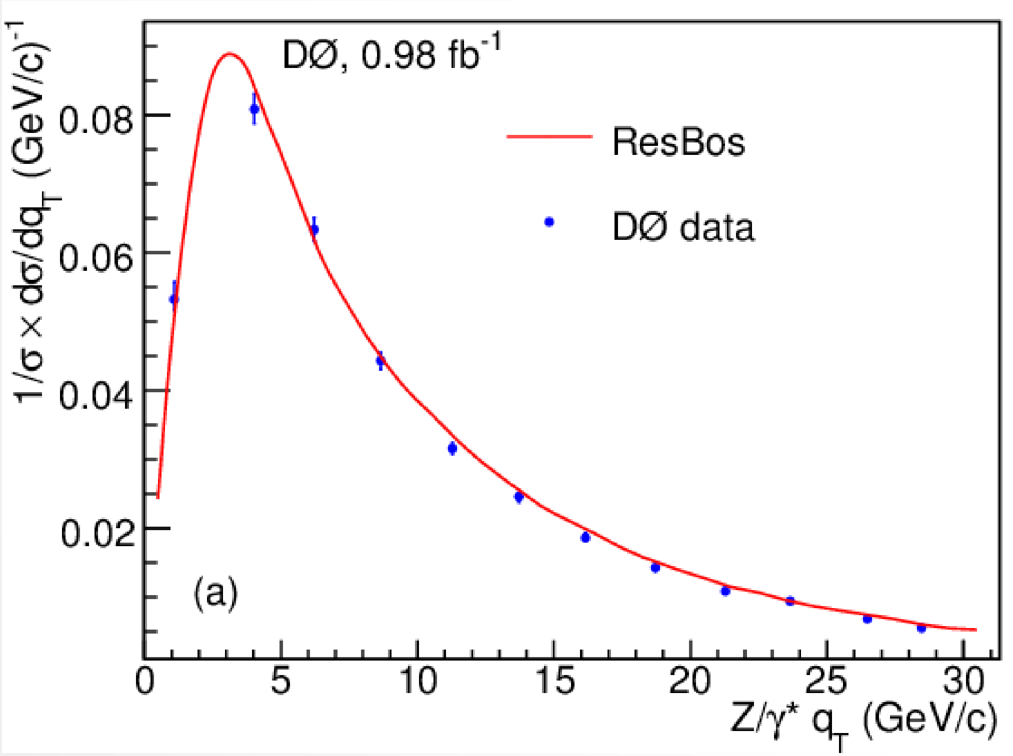
CDF phys. Lettr. B692:232-239. 2010

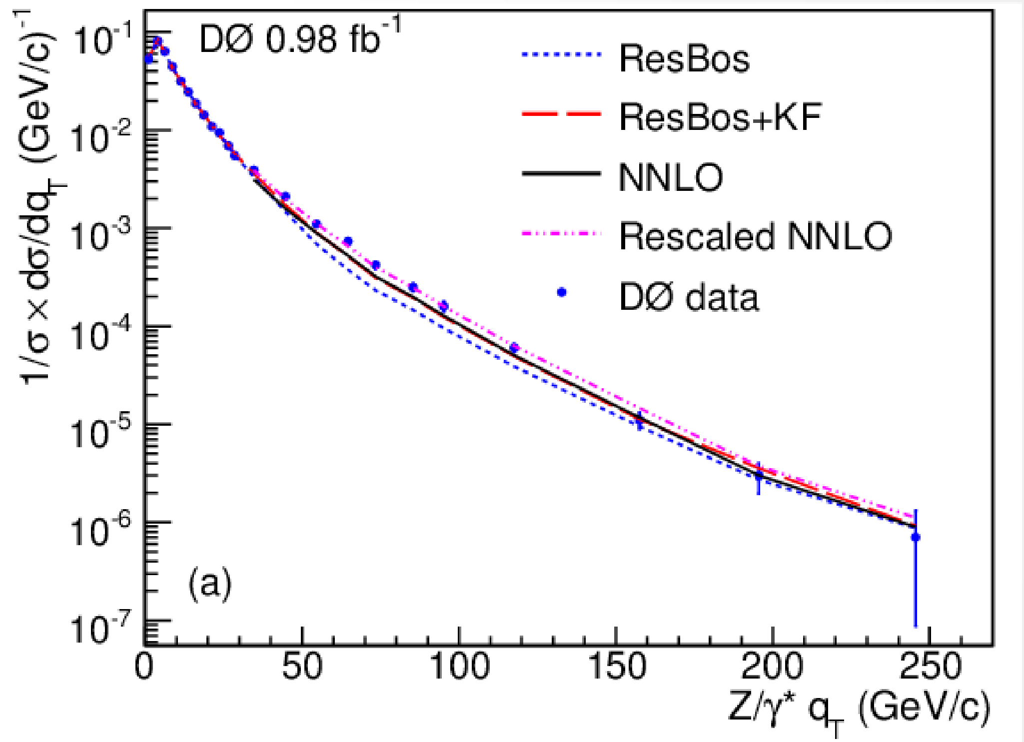
Higher order corrections

- At leading order the transverse momentum of the gauge boson is zero
- When adding real radiation we get non-vanishing transverse momentum
- As before we need to absorb the infrared singularities in the pdf (but it is the same way as in DIS)
- There is a divergence for small transverse momentum, as for the thrust, so we need to re-sum

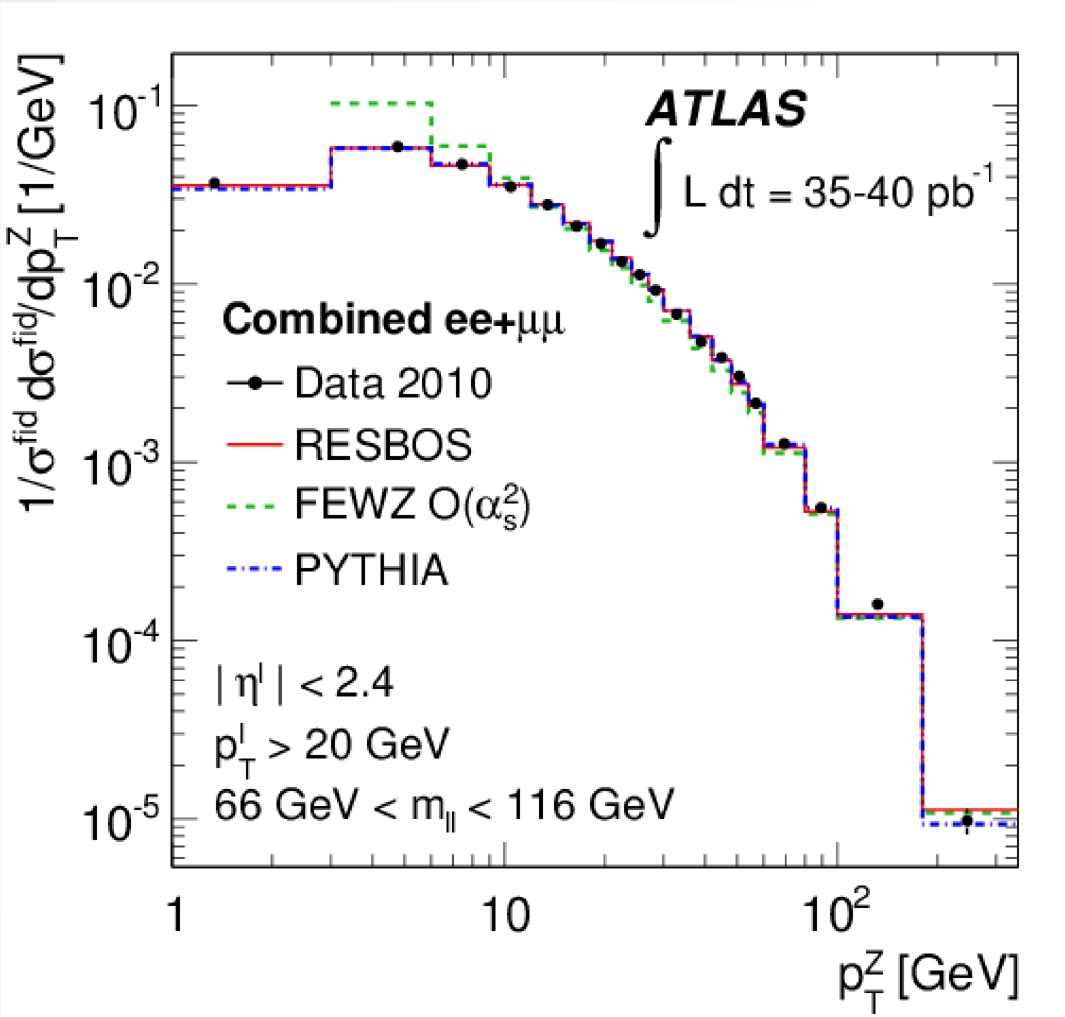


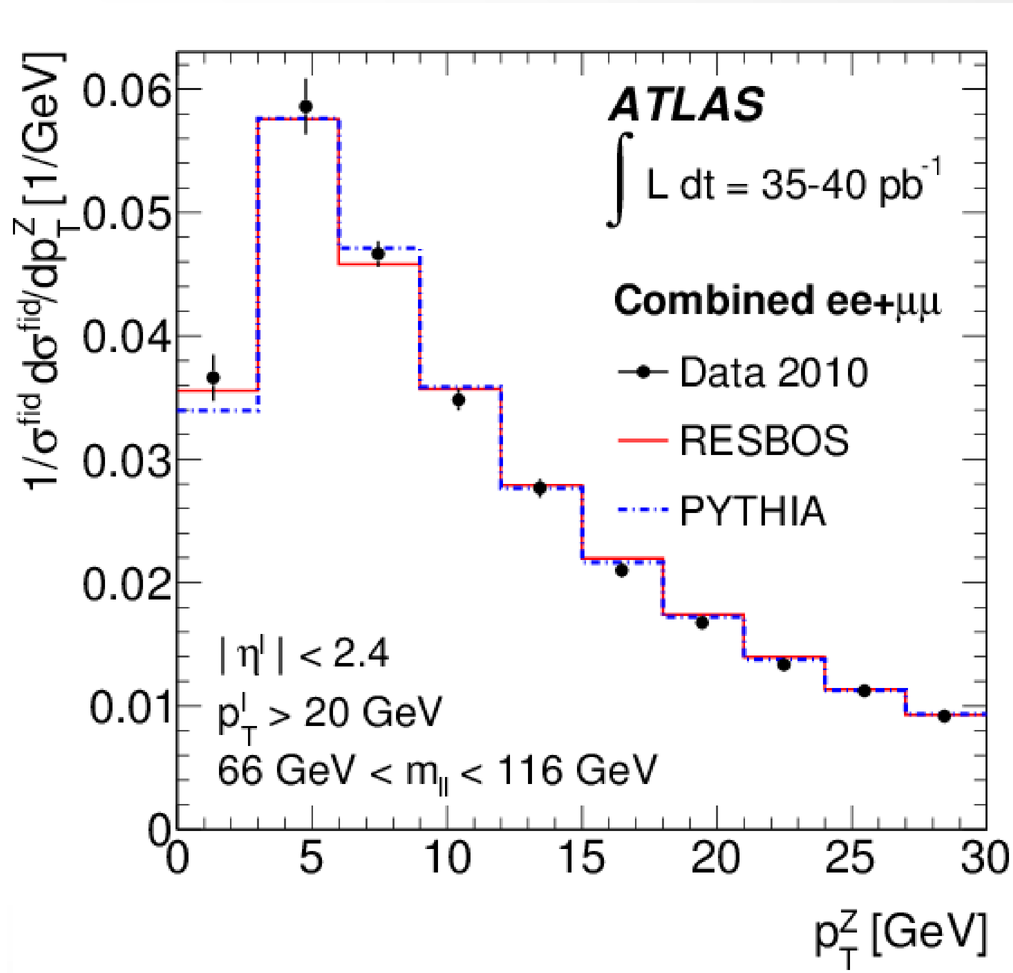
Transverse momentum of the Z





Transverse momentum of the Z



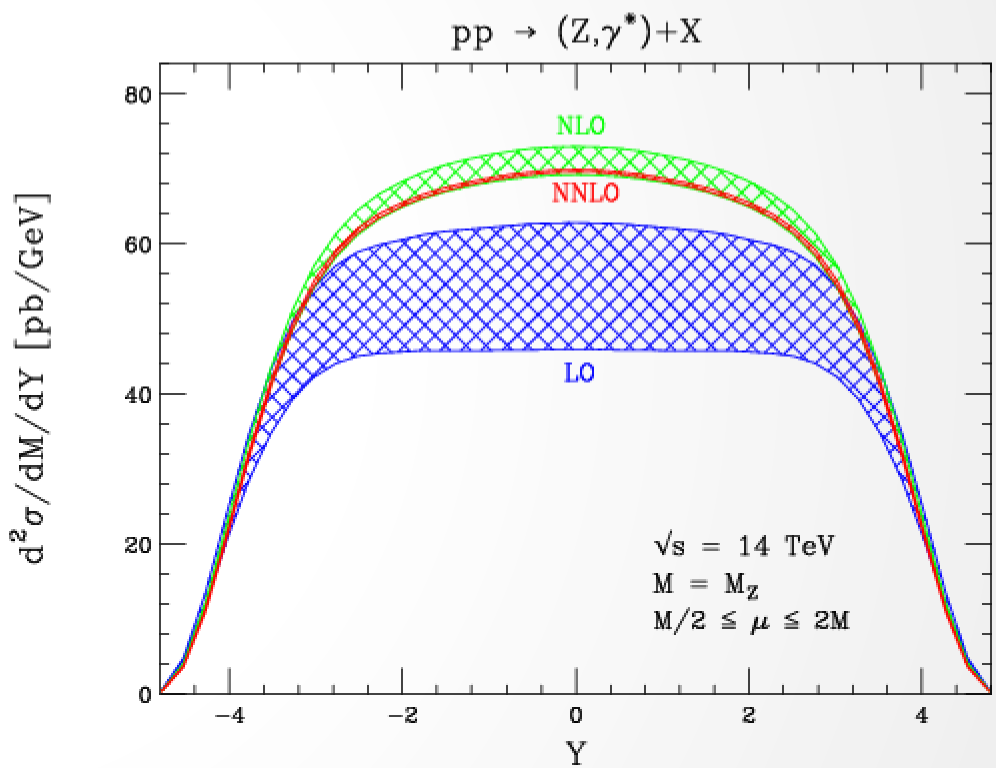


Higher order calculation

- For hadron collider phenomenology we would like to have at least NLO calculations
- It is the first order at which we have a reliable calculation of the cross section
- NNLO would be better, but it is much harder!
- We only have NNLO for Drell-Yan, Higgs production through gluon fusion as well as top pair production.

Higher orders

- As we have seen fixed order calculations have an artificial dependence on the factorization and renormalization scales
- It gets better at NLO and even better at NNLO



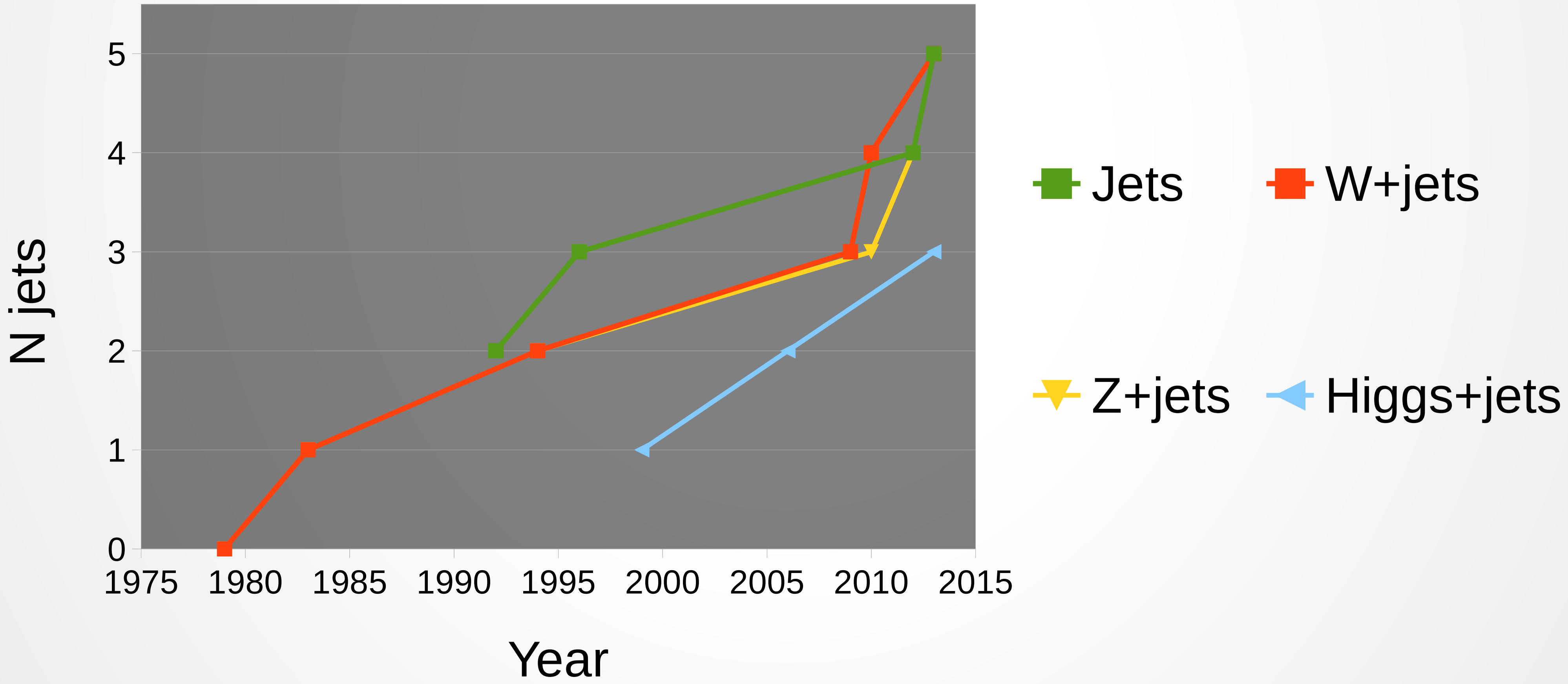
Anastasiou, Dixon, Melnikov, Petriello, Phys.Rev.D69:094008,2004

Higher jet multiplicities

- Many standard model and BSM models have signals which involve charged leptons, or missing energy from neutrinos and a large number of jets
- It is therefore important to understand these processes and have both good simulations and higher order calculations
- It has been the bottleneck for many years
- Tools are becoming more automated and we will get many more results at NLO in the near future

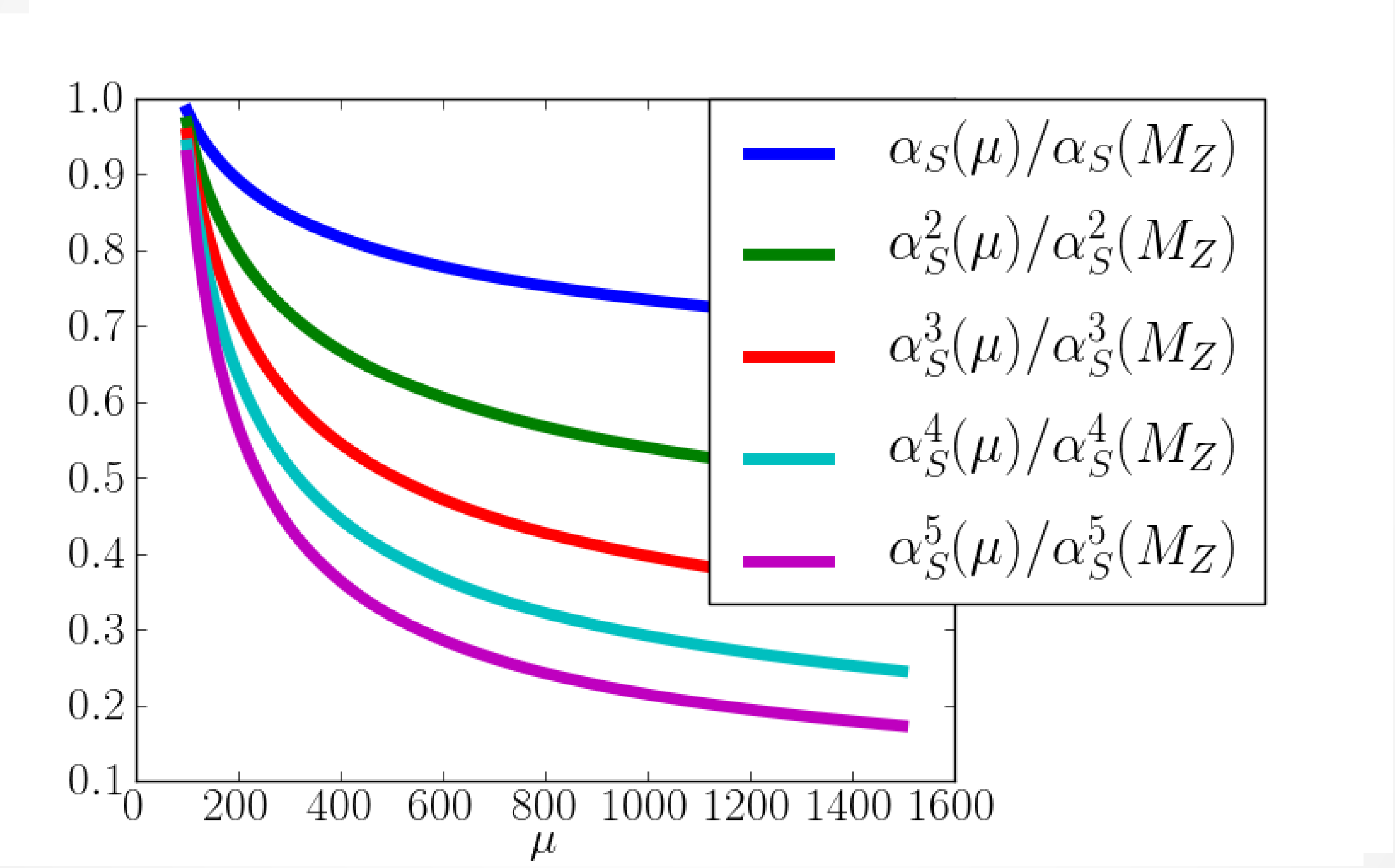
Higher jet multiplicities

- There has been a number of breakthroughs in the calculating processes at NLO with higher jet multiplicities.



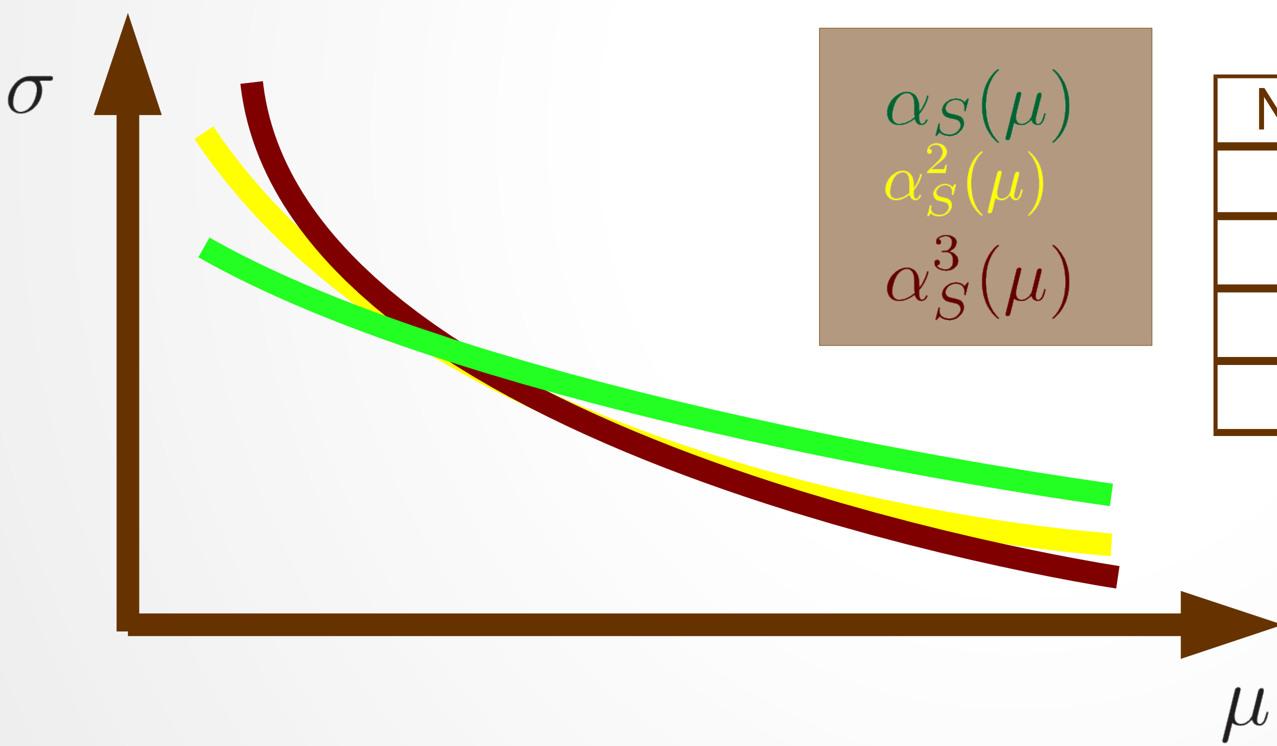
Why NLO

- Renormalisation scale dependence



Why NLO

- Scale dependence increases with the number of jets

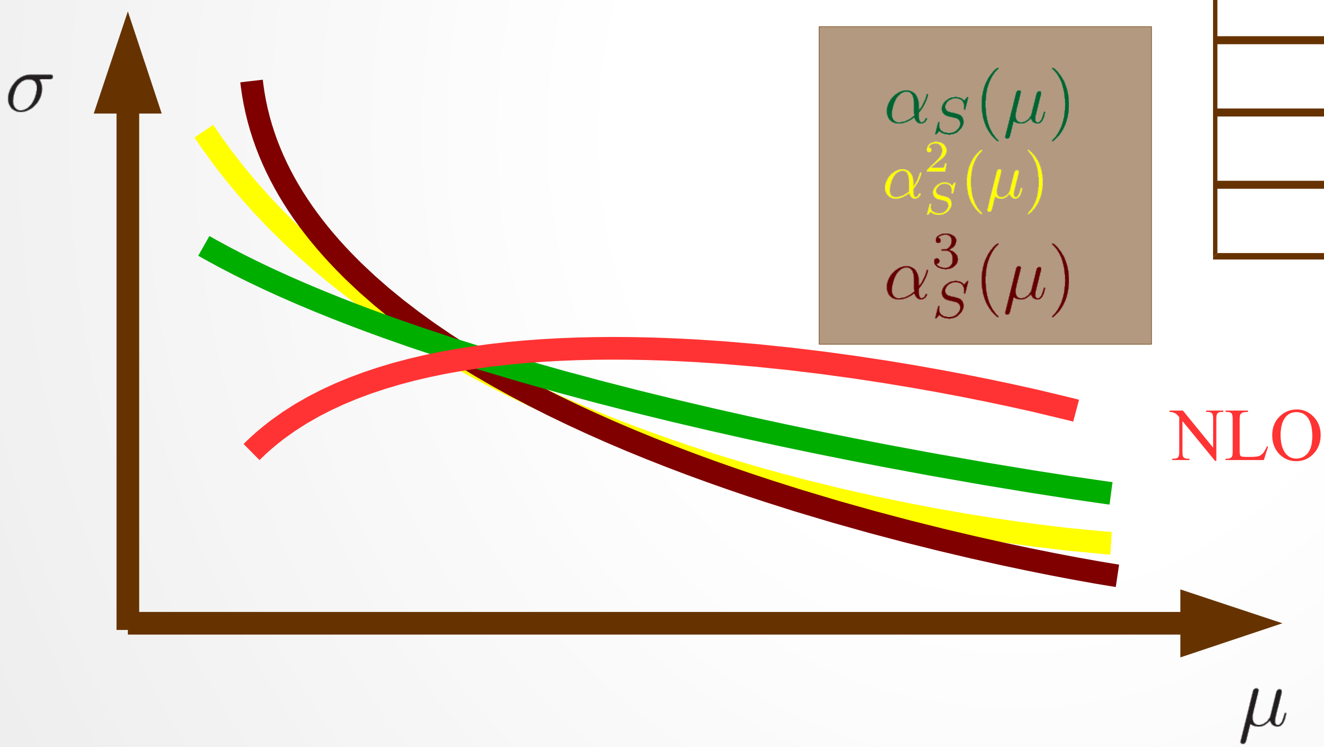


Number of jets	LO
1	9%
2	28%
3	47%
4	64%

[from table I in arXiv:1009.2338]

Why NLO

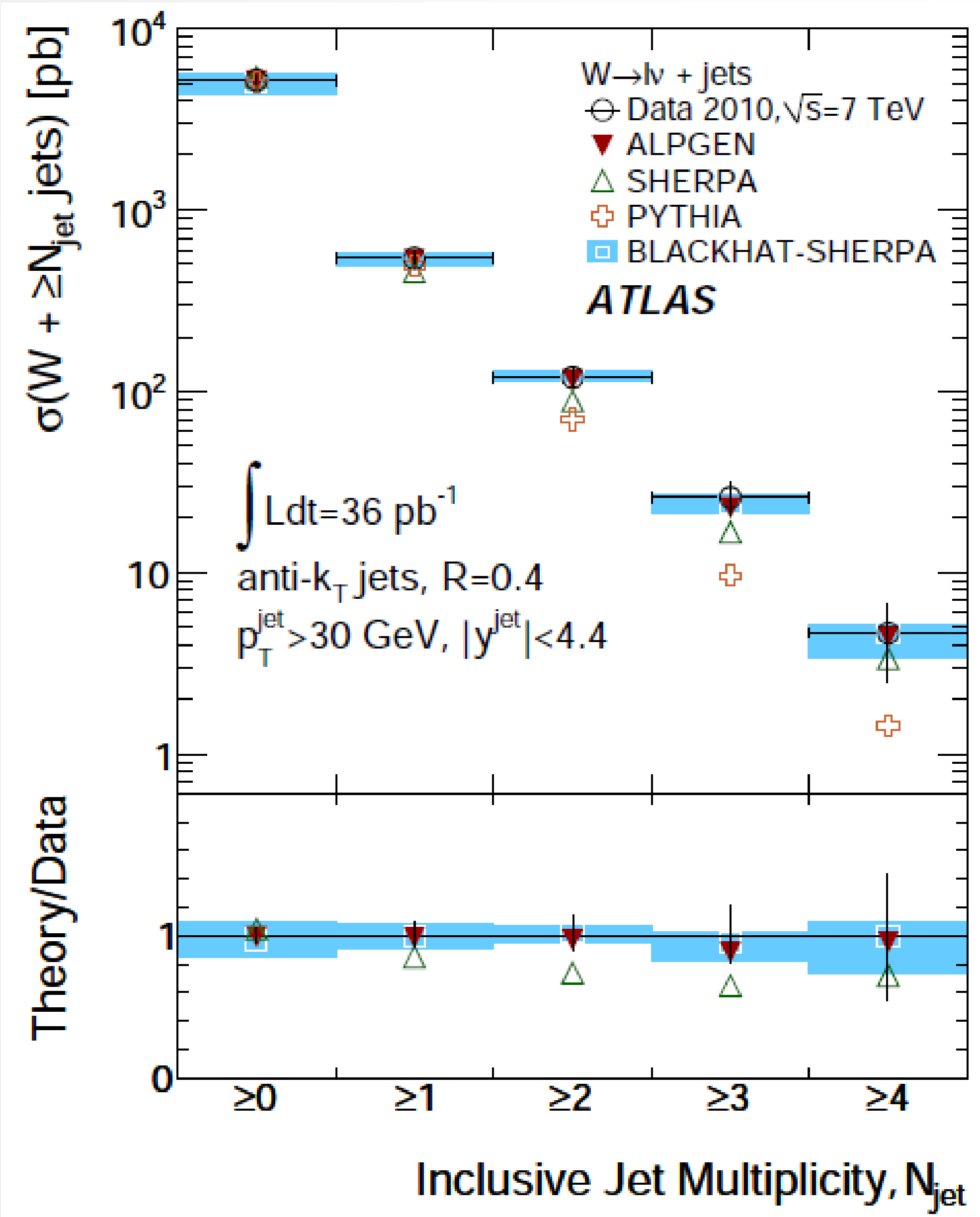
- Scale dependence increases with the number of jets



Number of jets	LO	NLO
1	9%	4,5%
2	28%	5,2%
3	47%	7,8%
4	64%	8,4%

[from table I in arXiv:1009.2338]

Why NLO



- W + jets at ATLAS
- Good agreement for all multiplicities
- Small scale dependence

How to compute NLO

- We have seen that we need two parts to compute NLO corrections, the real and the virtual contributions
- Both are divergent but the sum is finite for infrared safe observables
- In our examples we performed the integration over the real phase-space analytically, this is possible only in special situations, in the presence of cuts and for complicated processes it is in general not possible.

Fixed order predictions

- NLO Formula

$$\sigma = \int B \mathrm{d}\phi_n + \int V \mathrm{d}\phi_n + \int R \mathrm{d}\phi_{n+1}$$

- Problem: the two last terms are divergent
  - V: explicit IR divergences from the loop integral
  - R: divergences when the integration over the phase-space is performed
- We need a way to combine these two divergences numerically

Subtraction term

- We introduce a subtraction term

$$\sigma = \int B \mathrm{d}\phi_n + \int \left(V + \int \mathrm{d}\phi_1 S \right) \mathrm{d}\phi_n + \int (R - S) \mathrm{d}\phi_{n+1}$$

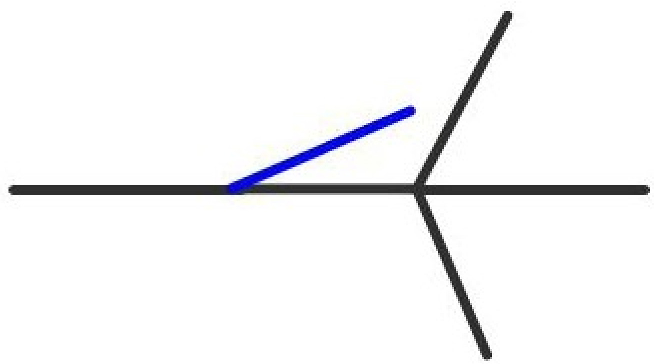
- The subtraction term is chosen such that
  - It has the same infrared pole structure than the real matrix element
  - It is easy enough to be integrated over the unresolved phase-space (here we use the factorisation properties of the matrix element in the soft and collinear limits)
- Now all terms are numerically finite and can be integrated with a MC generator

Subtraction term

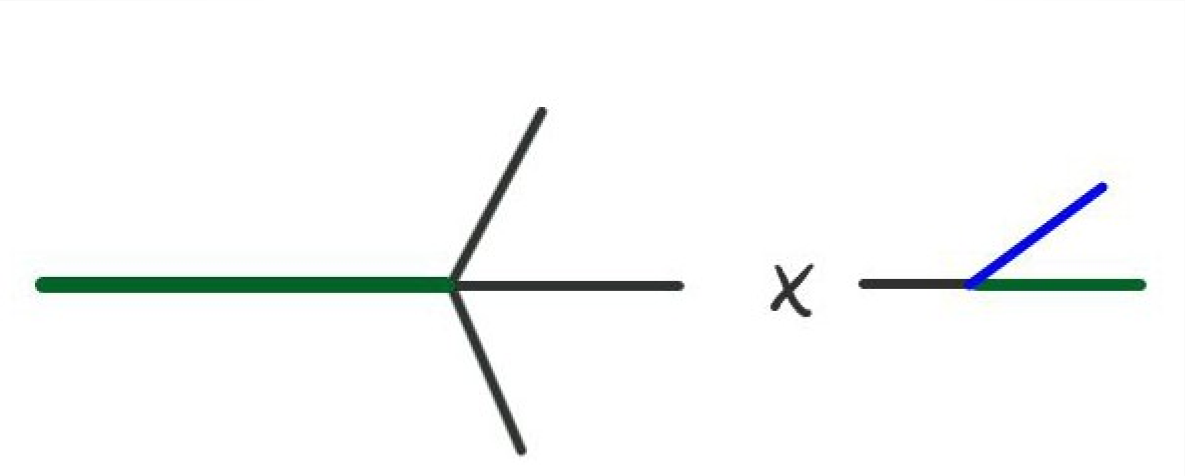
- There are two main methods to construct the subtraction term:
  - FKS (Frixione, Kunszt, Signer)
  - CS dipole subtraction (Catani-Seymour)
- These method can be automated (and they have been)

Subtraction term

- Initial state radiation:

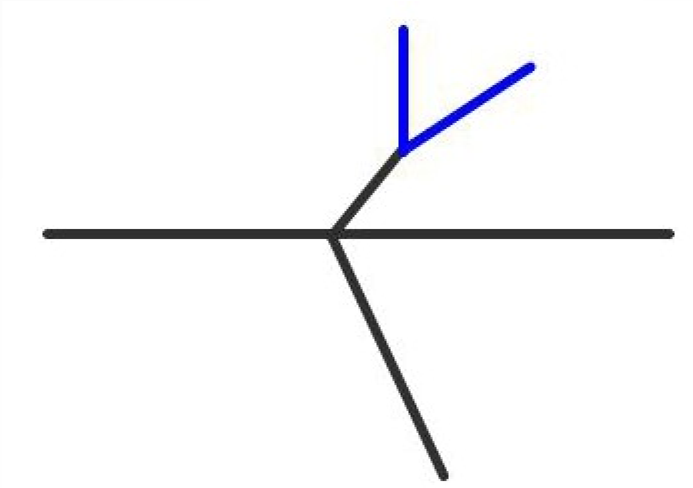


- Corresponding subtraction

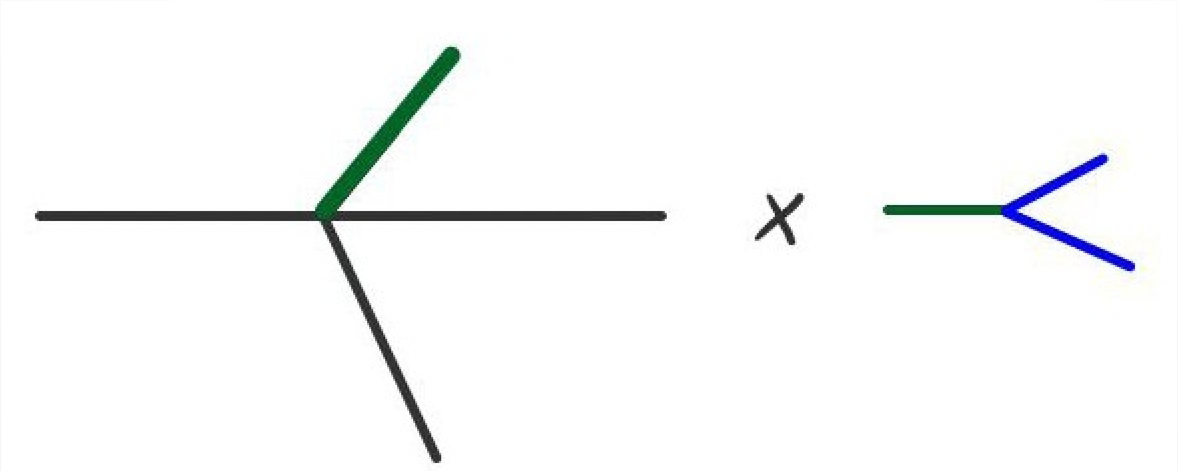


Subtraction term

- Final state radiation:

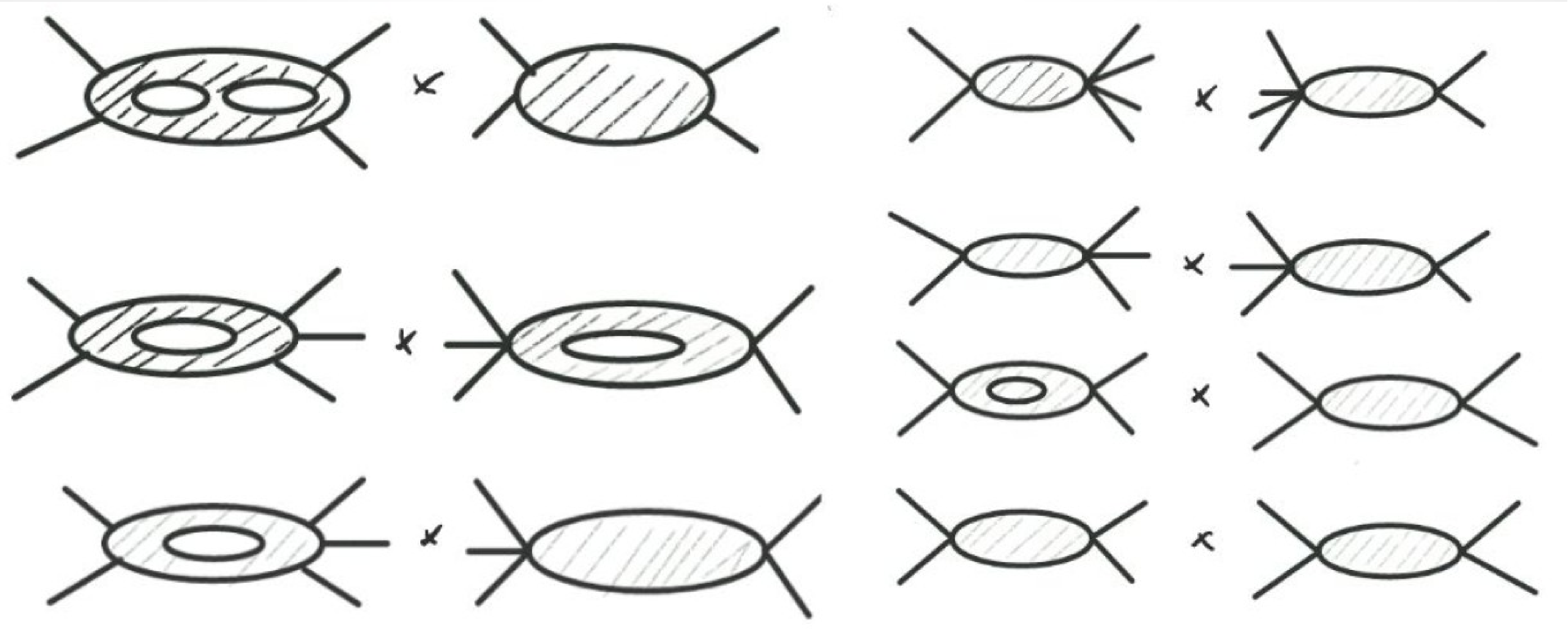


- Corresponding subtraction term

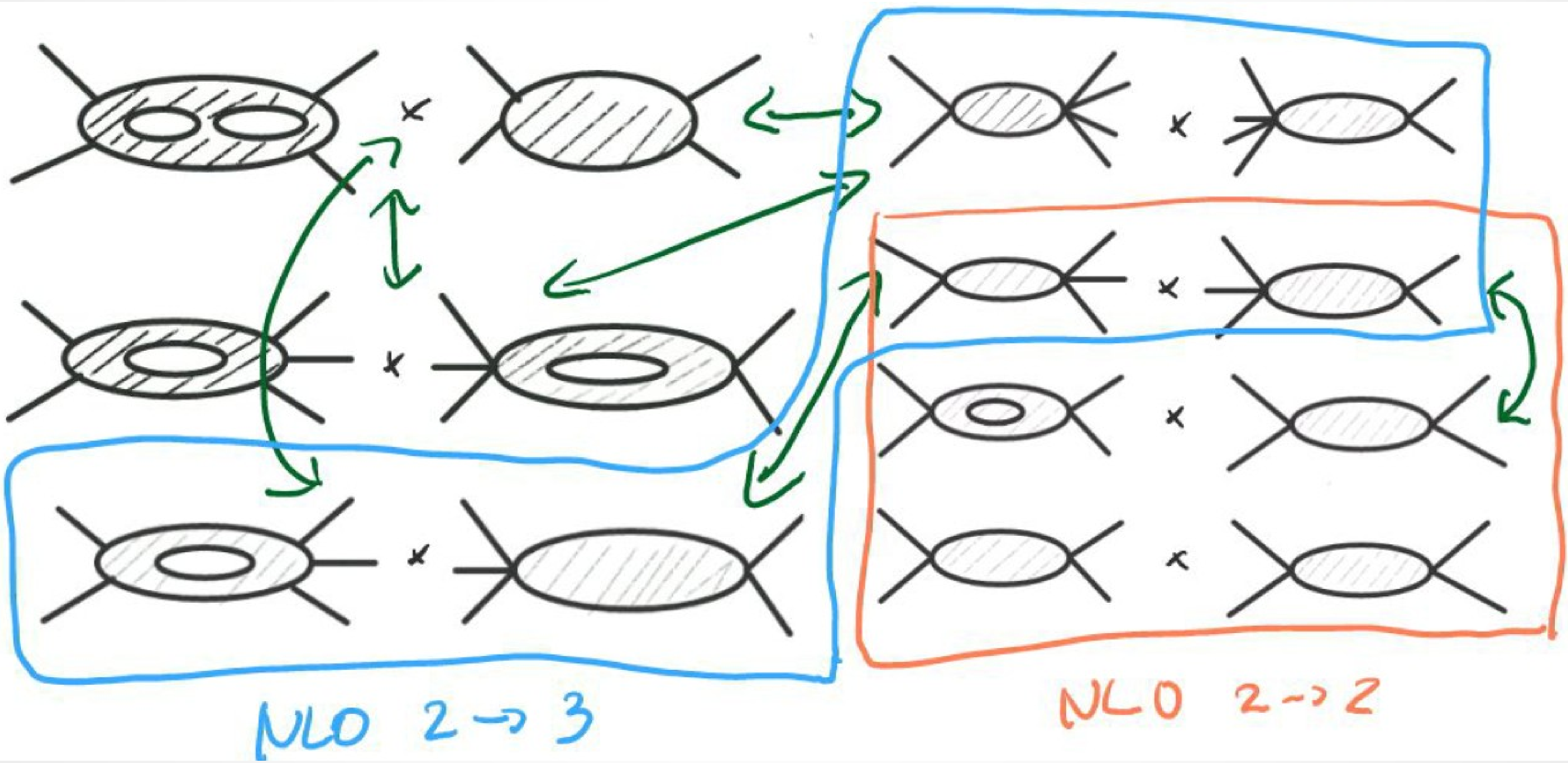


NNLO

- Many terms contribute to a NNLO calculation



NNLO



Infrared safety

- By infrared we mean that the observable does not change in the case of a soft or collinear emission
  - Collinear splitting

$$\mathcal{O}(p_1, ..., p_i, ..., p_n) = \mathcal{O}(p_1, ..., zp_i, (1-z)p_i, ..., p_n)$$

  - Soft emission

$$\begin{aligned} \mathcal{O}(p_1, ..., p_i, p_j, p_k, ..., p_n) &\rightarrow \mathcal{O}(p_1, ..., p_i, p_k, ..., p_n) \\ \text{for } p_j &\rightarrow 0 \end{aligned}$$

Infrared safety

- These are NOT infrared safe:
  - Number of partons
  - Observables using incoming parton momentum fractions
  - Older jet algorithms
  - Using infrared unsafe observable as a renormalisation or factorisation scale
- These are infrared safe
  - Number of jets above a transverse momentum threshold
  - Modern jet algorithms
- It is not always trivial to know whether an observable is infrared safe

Infrared safety

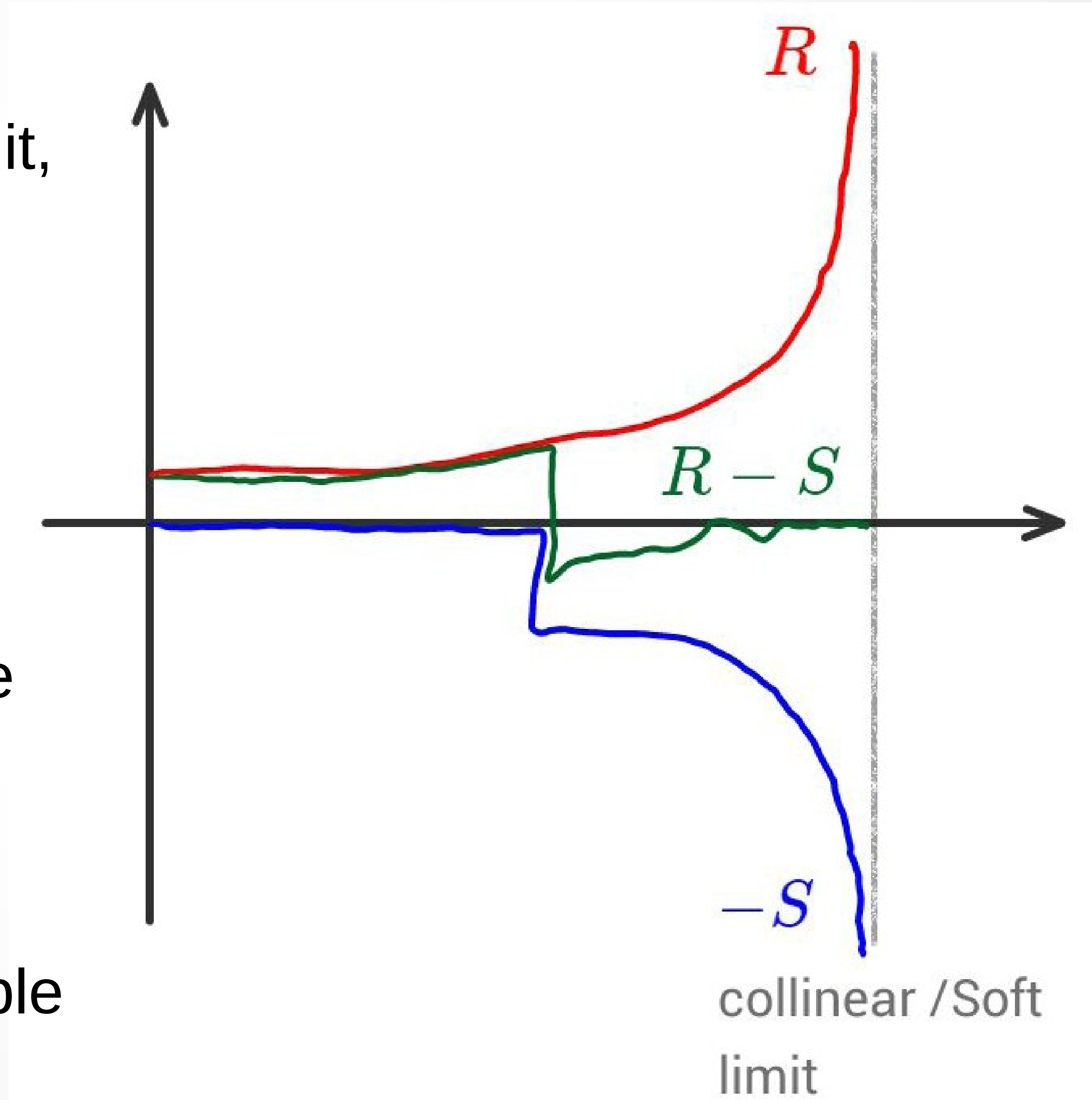
- We have seen that we get divergence free results at NLO for the total cross section
- We can get finite results when cuts are introduced and for expectation values of observable IF we don't spoil the cancellation between real and virtual part
- We need
  - Infrared safe observables
  - Infrared safe cuts
  - Infrared safe scales if you use dynamical scales

How to test infrared safety?

- You have to make sure that your observable does not change in these cases
  - Collinear splitting
  - soft emission
  - initial state splitting
- Can be tricky with jet algorithm for example
- Happens to the best... Consult your local theorist(s) if you are unsure
- One way to think about it is to remember about the subtraction method

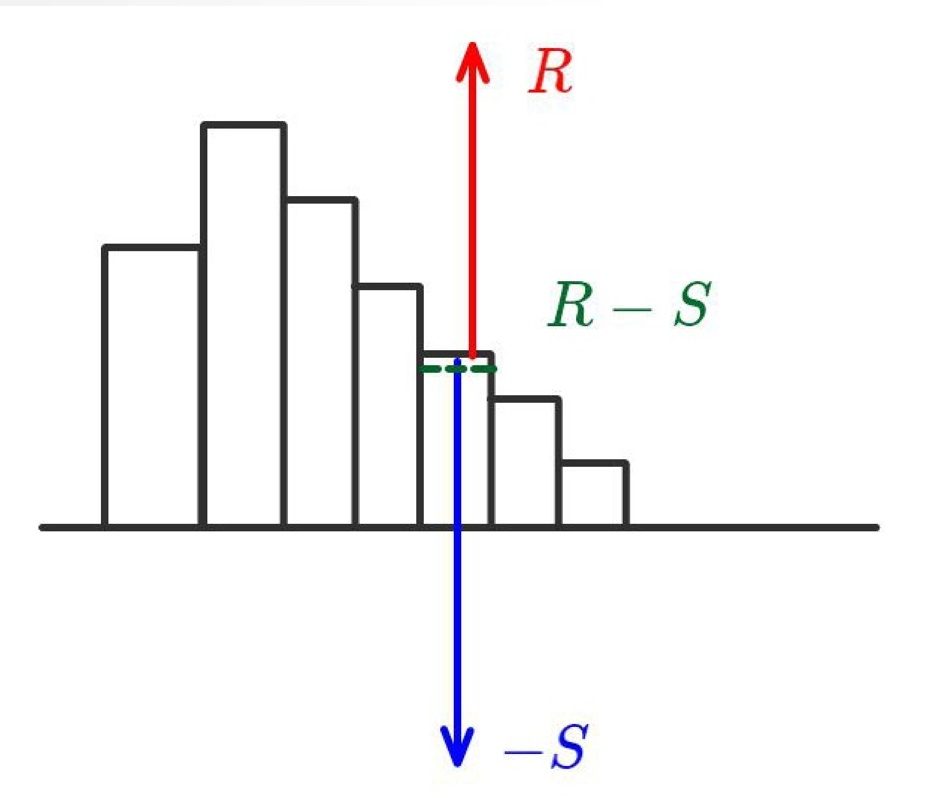
How to test infrared safety?

- As we approach a collinear or soft limit, the real matrix elements and the subtraction terms converge to the same value
- The weights associated with the two parts get very large
- They have to go in the same observable bin!

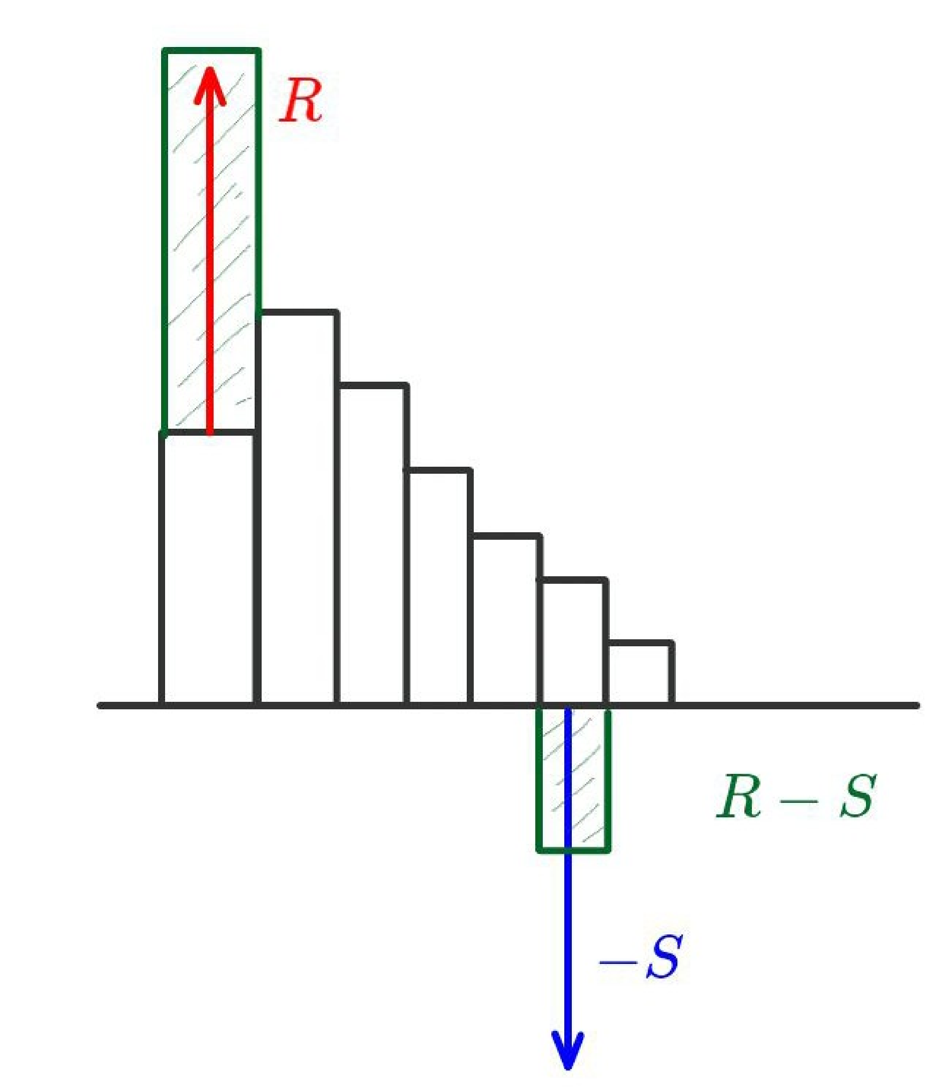


How to test infrared safety?

- Safe

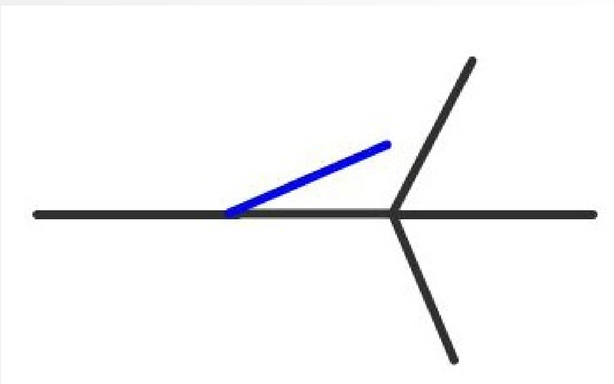


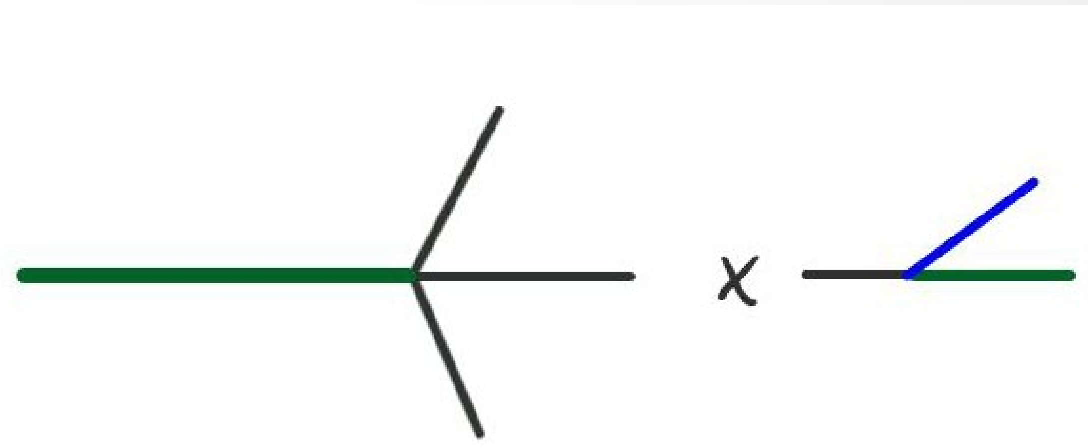
- unsafe



How to test infrared safety

- Make sure the observable does not change for
  - Initial state radiation:





  - Final state radiation