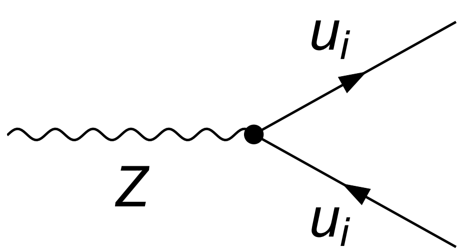
Feynman Diagram

In[6]:=
```
ZtoQQbar = InsertFields[CreateTopologies[0, 1 → 2],
    {V[2]} → {F[3, {i}], -F[3, {i}]}, InsertionLevel → {Classes}, Model → "SMQCD"];

Paint[ZtoQQbar, ColumnsXRows → {2, 1},
  Numbering → None, SheetHeader → None, ImageSize → {512, 256}];
```



Amplitude Squared

$$|M|^2 = \frac{1}{3}\left(\frac{g_2}{2\cos\theta_w}\right)^2 \epsilon_\mu(p_1)\,\epsilon_\nu{}^*(p_1)$$
$$\mathrm{Tr}\left[\,\gamma^\mu\left(g_V^{q-Z} - g_A^{q-Z}\,\gamma^5\right)(p_{2\,\mathrm{slash}} + m_q)\,\gamma^\nu\left(g_V^{q-Z} - g_A^{q-Z}\,\gamma^5\right)(p_{3\,\mathrm{slash}} - m_{\bar{q}})\right]$$

Trace Calculation

In[8]:=
```
Tr[GA[μ].(gV - gA GA5).(GS[p2] + m2).GA[ν].(gV - gA GA5).(GS[p3] - m3)];
ChangeDimension[%, D]
% // StandardForm;
```

Out[9]=
$$4\left(\mathrm{m2}\,\mathrm{m3}\,g_A^2\,g^{\mu\nu} + g_A^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu + g_A^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu - g_A^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3}) + 2\,i\,g_A\,g_V\,\epsilon^{\mu\nu\,\mathrm{p2}\,\mathrm{p3}} - \mathrm{m2}\,\mathrm{m3}\,g_V^2\,g^{\mu\nu} + g_V^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu + g_V^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu - g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})\right)$$

Getting rid of imaginary/tensor term $i\,\epsilon^{\mu\nu\sigma\rho}$

In[11]:=
```
4 (m2 m3 Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] gA^2 +
   Pair[LorentzIndex[μ, D], Momentum[p3, D]] × Pair[LorentzIndex[ν, D], Momentum[p2, D]] gA^2 +
   Pair[LorentzIndex[μ, D], Momentum[p2, D]] × Pair[LorentzIndex[ν, D], Momentum[p3, D]] gA^2 -
   Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] × Pair[Momentum[p2, D], Momentum[p3, D]] gA^2 -
   m2 m3 Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] gV^2 +
   Pair[LorentzIndex[μ, D], Momentum[p3, D]] × Pair[LorentzIndex[ν, D], Momentum[p2, D]] gV^2 +
   Pair[LorentzIndex[μ, D], Momentum[p2, D]] × Pair[LorentzIndex[ν, D], Momentum[p3, D]] gV^2 -
   Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] × Pair[Momentum[p2, D], Momentum[p3, D]] gV^2)
```

Out[11]= $4\left(\mathrm{m2\,m3}\,g_A^2\,g^{\mu\nu}+g_A^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu+g_A^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu-g_A^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})-\mathrm{m2\,m3}\,g_V^2\,g^{\mu\nu}+g_V^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu+g_V^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu-g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})\right)$

In[12]:=
```
% // Simplify
```

Out[12]= $4\left(g^{\mu\nu}\left(g_A^2\,(\mathrm{m2\,m3}-\mathrm{p2}\cdot\mathrm{p3})-g_V^2\,(\mathrm{m2\,m3}+\mathrm{p2}\cdot\mathrm{p3})\right)+\left(g_A^2+g_V^2\right)(\mathrm{p2}^\nu\,\mathrm{p3}^\mu+\mathrm{p2}^\mu\,\mathrm{p3}^\nu)\right)$

Expanding and simplifying $g^{\mu\nu}\left(g_A^2\,(\mathrm{m2\,m3}-\mathrm{p2.p3})-g_V^2\,(\mathrm{m2\,m3}+\mathrm{p2.p3})\right)$
and recombining with previous calculation

In[13]:=
```
Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]]
  ((m2 m3 - Pair[Momentum[p2, D], Momentum[p3, D]]) gA^2 -
    (m2 m3 + Pair[Momentum[p2, D], Momentum[p3, D]]) gV^2) // Expand
```

Out[13]= $\mathrm{m2\,m3}\,g_A^2\,g^{\mu\nu}-g_A^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})-\mathrm{m2\,m3}\,g_V^2\,g^{\mu\nu}-g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})$

In[14]:=
```
TraceCalc == 4 HoldForm[(gA^2 + gV^2) (p2^μ p3^ν + p2^ν p3^μ - g^μν (p2.p3)) + (gA^2 - gV^2) g^μν m2 m3]
```

Out[14]= $\mathrm{TraceCalc}=4\left(\left(g_A^2+g_V^2\right)(p_2^\mu\,p_3^\nu+p_2^\nu\,p_3^\mu-g^{\mu\nu}\,p_2.p_3)+\left(g_A^2-g_V^2\right)g^{\mu\nu}\,m_2\,m_3\right)$

$m_2=m_3=m_q$

In[15]:=
```
TraceCalc == 4 HoldForm[(gA^2 + gV^2) (p2^μ p3^ν + p2^ν p3^μ - g^μν (p2.p3)) + (gA^2 - gV^2) g^μν mq^2]
TraceCalc == 4 HoldForm[(gA^2 + gV^2) (p2^μ p3^ν + p2^ν p3^μ - g^μν (p2.p3)) - (gV^2 - gA^2) g^μν mq^2]
```

Out[15]= $\mathrm{TraceCalc}=4\left(\left(g_A^2+g_V^2\right)(p_2^\mu\,p_3^\nu+p_2^\nu\,p_3^\mu-g^{\mu\nu}\,p_2.p_3)+\left(g_A^2-g_V^2\right)g^{\mu\nu}\,m_q^2\right)$

Out[16]= $\mathrm{TraceCalc}=4\left(\left(g_A^2+g_V^2\right)(p_2^\mu\,p_3^\nu+p_2^\nu\,p_3^\mu-g^{\mu\nu}\,p_2.p_3)-\left(g_V^2-g_A^2\right)g^{\mu\nu}\,m_q^2\right)$

Multiply by Full Propagator

$$\epsilon_\mu(p_1)\,\epsilon_\nu(p_1)=\left(\frac{p_{1\,\mu}\,p_{1\,\nu}}{M_z{}^2}-g_{\mu\nu}\right)$$

$$\mathrm{A}=\left(g_A^2+g_V^2\right)$$

$$\left(\frac{p_{1\,\mu}\,p_{1\,\nu}}{M_z{}^2}-g_{\mu\nu}\right)\left(p_2^\mu\,p_3^\nu+p_2^\nu\,p_3^\mu-g^{\mu\nu}\,p_2.p_3\right)$$

In[∘]:=
```
(FV[p1, μ].FV[p1, ν]/Mz^2 - MT[μ, ν]) * A *
    (FV[p2, μ].FV[p3, ν] + FV[p2, ν].FV[p3, μ] - MT[μ, ν].SP[p2, p3]) // Expand // Contract;
ChangeDimension[%, D]
% /. {Pair[Momentum[p1, D], Momentum[p1, D]] → Mz^2}
```

Out[∘]= $\frac{2 A (\text{p1} \cdot \text{p2}) (\text{p1} \cdot \text{p3})}{M_z^2} - \frac{A\, \text{p1}^2 (\text{p2} \cdot \text{p3})}{M_z^2} + 2 A (\text{p2} \cdot \text{p3})$

Out[∘]= $\frac{2 A (\text{p1} \cdot \text{p2}) (\text{p1} \cdot \text{p3})}{M_z^2} + A (\text{p2} \cdot \text{p3})$

$$\frac{2\,\text{A} (\text{p1} \cdot \text{p2}) (\text{p1} \cdot \text{p3})}{M_z^2} + A\, (\text{p2} \cdot \text{p3}), \quad (\text{ Equation 1 })$$

$$\text{B } = \left(g_V^2 - g_A^2\right)$$

$$\left(\frac{p_{1\mu} p_{1\nu}}{M_z{}^2} - g_{\mu\nu}\right) \left(g^{\mu\nu} m_q^2\right)$$

In[∘]:=
```
(FV[p1, μ].FV[p1, ν]/Mz^2 - MT[μ, ν]) * B * (MT[μ, ν].mq^2) // Expand // Contract;
ChangeDimension[%, D]
% /. {Pair[Momentum[p1, D], Momentum[p1, D]] → Mz^2}
```

Out[∘]= $\frac{B\, \text{p1}^2 m_q^2}{M_z^2} - 4 B m_q^2$

Out[∘]= $-3 B m_q^2$

$$-3 B m_q^2, \ (\text{ Equation 2 })$$

Kinematics: $p_1 = (M_z\,, 0\,), \quad p_2 = \left(\frac{M_z}{2}, \vec{p}\right), \quad p_3 = \left(\frac{M_z}{2}, -\vec{p}\right)$

In[∘]:=
```
one = {Mz , 0};
two = {Mz/2, p};
three = {Mz/2, p};
one.two
one.three
two.three
```

Out[∘]= $\frac{M_z^2}{2}$

Out[∘]= $\frac{M_z^2}{2}$

Out[∘]= $\frac{M_z^2}{4} + p^2$

Putting Equation 1 and Equation 2 back into TraceCalc:

In[∘]:=
```
4 (A (2 (p1.p2).(p1.p3)/Mz^2 + (p2.p3)) - B.(-3 mq^2)) ==
  4 ((gA^2 + gV^2) (2 (one.two) (one.three)/Mz^2 + (two.three)) - (gV^2 - gA^2).(-3 mq^2))
```

Out[∘]= $$4\left(A\left(\frac{2\,p_1.p_2.p_1.p_3}{M_Z^2}+p_2.p_3\right)-B.\left(-3\,m_q^2\right)\right)=4\left(\left(g_A^2+g_V^2\right)\left(\frac{3\,M_z^2}{4}+p^2\right)-\left(g_V^2-g_A^2\right).\left(-3\,m_q^2\right)\right)$$

In[∘]:=
```
% /. {p^2 → Mz^2/4 - mq^2}
```

Out[∘]= $$4\left(A\left(\frac{2\,p_1.p_2.p_1.p_3}{M_Z^2}+p_2.p_3\right)-B.\left(-3\,m_q^2\right)\right)=4\left(\left(g_A^2+g_V^2\right)\left(M_z^2-m_q^2\right)-\left(g_V^2-g_A^2\right).\left(-3\,m_q^2\right)\right)$$

In[∘]:=
```
(gA^2 + gV^2) (Mz^2 - mq^2) - (gV^2 - gA^2) (-3 mq^2) // Expand
```

Out[∘]= $$-4\,g_A^2\,m_q^2+g_A^2\,M_z^2+2\,g_V^2\,m_q^2+g_V^2\,M_z^2$$

In[∘]:=
```
Collect[%, Mz^2]
```

Out[∘]= $$-4\,g_A^2\,m_q^2+M_z^2\left(g_A^2+g_V^2\right)+2\,g_V^2\,m_q^2$$

In[∘]:=
```
Collect[%, mq^2]
```

Out[∘]= $$m_q^2\left(2\,g_V^2-4\,g_A^2\right)+M_z^2\left(g_A^2+g_V^2\right)$$

In[∘]:=
```
4 ((gA^2 + gV^2).Mz^2 + 2 mq^2 (gV^2 - 2 gA^2)) == 4 Mz^2 HoldForm[((gA^2 + gV^2) + 2 mq^2/Mz^2 (gV^2 - 2 gA^2))]
4 Mz^2 HoldForm[((gA^2 + gV^2) + 2 mq^2/Mz^2.(gV^2 - 2 gA^2))] ==
  4 Mz^2 HoldForm[(gA^(q-z))^2 + (gV^(q-z))^2 + 2 mq^2/Mz^2.((gV^(q-z))^2 - 2 (gA^(q-z))^2)]
```

Out[∘]= $$4\left(2\,m_q^2\left(g_V^2-2\,g_A^2\right)+\left(g_A^2+g_V^2\right).M_z^2\right)=4\,M_z^2\left(\left(g_A^2+g_V^2\right)+\frac{2\,m_q^2\left(g_V^2-2\,g_A^2\right)}{M_z^2}\right)$$

Out[∘]= $$4\,M_z^2\left(\left(g_A^2+g_V^2\right)+2\,\frac{m_q^2}{M_z^2}.\left(g_V^2-2\,g_A^2\right)\right)=4\,M_z^2\left(\left(g_A^{q-z}\right)^2+\left(g_V^{q-z}\right)^2+2\,\frac{m_q^2}{M_z^2}.\left(\left(g_V^{q-z}\right)^2-2\left(g_A^{q-z}\right)^2\right)\right)$$

$$|M|^2=\frac{1}{3}\left(\frac{g_2}{2\cos\theta_w}\right)^2 4\,M_z^2\left(\left(g_A^{q-z}\right)^2+\left(g_V^{q-z}\right)^2+2\,\frac{m_q^2}{M_z^2}.\left(\left(g_V^{q-z}\right)^2-2\left(g_A^{q-z}\right)^2\right)\right)$$

$$|M|^2=\frac{1}{3}\,\frac{g_2{}^2}{\cos^2\theta}\,M_z^2\left(\left(g_A^{q-z}\right)^2+\left(g_V^{q-z}\right)^2+2\,\frac{m_q^2}{M_z^2}.\left(\left(g_V^{q-z}\right)^2-2\left(g_A^{q-z}\right)^2\right)\right)$$

Decay (Initial → Final (2 Body))

$$\Gamma=\frac{|\vec{p}|}{8\,\pi\,M_z{}^2}\,|M|^2$$

$$|\overrightarrow{\mathrm{p}}| = \sqrt{(E)^2 - m_2\, m_3} = \sqrt{\left(\frac{M_Z}{2}\right)^2 - m_2\, m_3} = \frac{M_Z}{2}\sqrt{1 - 4\,\frac{m_2\, m_3}{M_Z{}^2}}$$

$m_2 = m_3 = m_q$

$$\Gamma\,(\mathrm{Z} \to q\,\overline{q}) = \frac{\mathrm{g_2}^2}{24\,\pi\,\cos^2\theta\, M_z^2}\,\frac{M_Z}{2}\,\sqrt{1 - 4\,\frac{m_q{}^2}{M_Z{}^2}}\, M_z^2.\left(\left(g_V^f\right)^2 + \left(g_A^f\right)^2 + \frac{2\, m_q{}^2\left(\left(g_V^f\right)^2 - 2\left(g_A^f\right)^2\right)}{M_z^2}\right)$$

$$\therefore\ \Gamma\,(\mathrm{Z} \to q\,\overline{q}) = \frac{\mathrm{g_2}^2\, M_Z}{48\,\pi\,\cos^2\theta}\left(\left(g_V^{q-Z}\right)^2 + \left(g_A^{q-Z}\right)^2 + 2\,\frac{m_q{}^2}{M_Z{}^2}\left(\left(g_V^{q-Z}\right)^2 - 2\left(g_A^{q-Z}\right)^2\right)\right)\sqrt{1 - 4\,\frac{m_q{}^2}{M_Z{}^2}}\,,$$

(5.44)

Multiply by Full Propagator (Flip)

$$\epsilon_\mu(p_1)\,\epsilon_\nu(p_1) = \left(g_{\mu\nu} - \frac{p_{1\,\mu}\, p_{1\,\nu}}{M_z{}^2}\right)$$

$$\mathrm{A} = \left(g_A^2 + g_V^2\right)$$

$$\left(\mathrm{g}_{\mu\nu} - \frac{\mathrm{p}_{1\,\mu}\,\mathrm{p}_{1\,\nu}}{\mathrm{M_z}^2}\right)\left(p_2^\mu\, p_3^\nu + p_2^\nu\, p_3^\mu - g^{\mu\nu}\, p_2.p_3\right)$$

In[36]:=
```
(MT[μ, ν] - FV[p1, μ].FV[p1, ν]/Mz^2) * A *
    (FV[p2, μ].FV[p3, ν] + FV[p2, ν].FV[p3, μ] - MT[μ, ν].SP[p2, p3]) // Expand // Contract;
ChangeDimension[%, D]
% /. {Pair[Momentum[p1, D], Momentum[p1, D]] → Mz^2}
```

Out[37]= $-\frac{2\,A\,(\mathrm{p1}\cdot\mathrm{p2})\,(\mathrm{p1}\cdot\mathrm{p3})}{M_z^2} + \frac{A\,\mathrm{p1}^2\,(\mathrm{p2}\cdot\mathrm{p3})}{M_z^2} - 2\,A\,(\mathrm{p2}\cdot\mathrm{p3})$

Out[38]= $-\frac{2\,A\,(\mathrm{p1}\cdot\mathrm{p2})\,(\mathrm{p1}\cdot\mathrm{p3})}{M_z^2} - A\,(\mathrm{p2}\cdot\mathrm{p3})$

$$-\frac{2\,\mathrm{A}\,(\mathrm{p1}\cdot\mathrm{p2})\,(\mathrm{p1}\cdot\mathrm{p3})}{M_z^2} - A\,(\mathrm{p2}\cdot\mathrm{p3}), \qquad (\text{ Equation } 1\ )$$

$$\mathrm{B} = \left(g_V^2 - g_A^2\right)$$

$$\left(\mathrm{g}_{\mu\nu} - \frac{\mathrm{p}_{1\,\mu}\,\mathrm{p}_{1\,\nu}}{\mathrm{M_z}^2}\right)\left(g^{\mu\nu}\, m_q^2\right)$$

In[42]:=
```
(MT[μ, ν] - FV[p1, μ].FV[p1, ν]/Mz^2) * B * (MT[μ, ν].mq^2) // Expand // Contract;
ChangeDimension[%, D]
% /. {Pair[Momentum[p1, D], Momentum[p1, D]] → Mz^2}
```

Out[43]= $4\,B\,m_q^2 - \dfrac{B\,\text{p1}^2\,m_q^2}{M_z^2}$

Out[44]= $3\,B\,m_q^2$

$3\,B\,m_q^2$, (Equation 2)

Kinematics: $p_1 = (\,M_z\,,\,0\,)$, $p_2 = \left(\,\frac{M_z}{2},\,\vec{p}\,\right)$, $p_3 = \left(\,\frac{M_z}{2},\,-\vec{p}\,\right)$

In[45]:=
```
one = {Mz , 0};
two = {Mz/2, p};
three = {Mz/2, p};
one.two
one.three
two.three
```

Out[48]= $\dfrac{M_z^2}{2}$

Out[49]= $\dfrac{M_z^2}{2}$

Out[50]= $\dfrac{M_z^2}{4} + p^2$

Putting Equation 1 and Equation 2 back into TraceCalc:

In[51]:=
```
4 (A (2 (p1.p2).(p1.p3)/MZ^2 + (p2.p3)) - B.(3 mq^2)) ==
  4 ((gA^2 + gV^2) (-2 (one.two) (one.three)/Mz^2 - (two.three)) - (gV^2 - gA^2).(3 mq^2))
```

Out[51]= $4\left(A\left(\dfrac{2\,p_1.p_2.p_1.p_3}{M_Z^2} + p_2.p_3\right) - B.\left(3\,m_q^2\right)\right) = 4\left(\left(g_A^2 + g_V^2\right)\left(-\dfrac{3\,M_z^2}{4} - p^2\right) - \left(g_V^2 - g_A^2\right).\left(3\,m_q^2\right)\right)$

In[52]:=
```
% /. {p^2 → Mz^2/4 - mq^2}
```

Out[52]= $4\left(A\left(\dfrac{2\,p_1.p_2.p_1.p_3}{M_Z^2} + p_2.p_3\right) - B.\left(3\,m_q^2\right)\right) = 4\left(\left(g_A^2 + g_V^2\right)\left(m_q^2 - M_z^2\right) - \left(g_V^2 - g_A^2\right).\left(3\,m_q^2\right)\right)$

In[53]:=
```
(gA^2 + gV^2) (-Mz^2 + mq^2) - (gV^2 - gA^2) (3 mq^2) // Expand
```

Out[53]= $4\,g_A^2\,m_q^2 - g_A^2\,M_z^2 - 2\,g_V^2\,m_q^2 - g_V^2\,M_z^2$

In[54]:= `Collect[%, Mz^2]`

Out[54]= $4\, g_A^2\, m_q^2 + M_z^2 \left(-g_A^2 - g_V^2\right) - 2\, g_V^2\, m_q^2$

In[55]:= `Collect[%, mq^2]`

Out[55]= $m_q^2 \left(4\, g_A^2 - 2\, g_V^2\right) + M_z^2 \left(-g_A^2 - g_V^2\right)$

In[58]:= $4\left(-\left(g_A{}^2 + g_V{}^2\right).M_z{}^2 + 2\, m_q{}^2 \left(2\, g_A{}^2 - g_V{}^2\right)\right) == 4\, M_z{}^2\, \text{HoldForm}\left[\left(-\left(g_A{}^2 + g_V{}^2\right) + 2\, \frac{m_q{}^2}{M_z{}^2} \left(2\, g_A{}^2 - g_V{}^2\right)\right)\right]$

$-4\, M_z{}^2\, \text{HoldForm}\left[\left(\left(g_A{}^2 + g_V{}^2\right) - 2\, \frac{m_q{}^2}{M_z{}^2}.\left(2\, g_A{}^2 - g_V{}^2\right)\right)\right] ==$

$-4\, M_z{}^2\, \text{HoldForm}\left[\left(g_A^{q-z}\right)^2 + \left(g_V^{q-z}\right)^2 - 2\, \frac{m_q{}^2}{M_z{}^2}.\left(2\left(g_A^{q-z}\right)^2 - \left(g_V^{q-z}\right)^2\right)\right]$

Out[58]= $4\left(2\, m_q^2 \left(2\, g_A^2 - g_V^2\right) - \left(g_A^2 + g_V^2\right).M_z^2\right) = 4\, M_z^2 \left(-\left(g_A^2 + g_V^2\right) + \frac{2\, m_q^2 \left(2\, g_A^2 - g_V^2\right)}{M_z^2}\right)$

Out[59]= $-4\, M_z^2 \left(\left(g_A^2 + g_V^2\right) - 2\, \frac{m_q^2}{M_z^2}.\left(2\, g_A^2 - g_V^2\right)\right) = -4\, M_z^2 \left(\left(g_A^{q-z}\right)^2 + \left(g_V^{q-z}\right)^2 - 2\, \frac{m_q^2}{M_z^2}.\left(2\left(g_A^{q-z}\right)^2 - \left(g_V^{q-z}\right)^2\right)\right)$

$$|M|^2 = -\frac{1}{3}\left(\frac{g_2}{2\cos\theta_w}\right)^2 4\, M_z^2 \left(\left(g_A^{q-z}\right)^2 + \left(g_V^{q-z}\right)^2 - 2\, \frac{m_q^2}{M_z^2}.\left(2\left(g_A^{q-z}\right)^2 - \left(g_V^{q-z}\right)^2\right)\right)$$

$$|M|^2 = -\frac{1}{3}\,\frac{g_2{}^2}{\cos^2\theta}\, M_z^2 \left(\left(g_A^{q-z}\right)^2 + \left(g_V^{q-z}\right)^2 + 2\, \frac{m_q^2}{M_z^2}.\left(\left(g_V^{q-z}\right)^2 - 2\left(g_A^{q-z}\right)^2\right)\right)$$

Decay (Initial → Final (2 Body)) (Flip Propagator)

$$\Gamma = \frac{|\vec{p}|}{8\,\pi\, M_z{}^2}\, |M|^2$$

$$|\vec{p}| = \sqrt{(E)^2 - m_2\, m_3} = \sqrt{\left(\frac{M_Z}{2}\right)^2 - m_2\, m_3} = \frac{M_Z}{2}\sqrt{1 - 4\,\frac{m_2\, m_3}{M_Z{}^2}}$$

$m_2 = m_3 = m_q$

$$\Gamma\left(Z \to q\,\overline{q}\right) = -\frac{g_2{}^2}{24\,\pi\cos^2\theta\, M_z^2}\,\frac{M_Z}{2}\sqrt{1 - 4\,\frac{m_q{}^2}{M_Z{}^2}}\; M_z^2.\left(\left(g_V^f\right)^2 + \left(g_A^f\right)^2 + \frac{2\, m_q{}^2\left(\left(g_V^f\right)^2 - 2\left(g_A^f\right)^2\right)}{M_z^2}\right)$$

$$\therefore\;\; \Gamma\left(Z \to q\,\overline{q}\right) = -\frac{g_2{}^2\, M_Z}{48\,\pi\cos^2\theta}\left(\left(g_V^{q-Z}\right)^2 + \left(g_A^{q-Z}\right)^2 + 2\,\frac{m_q{}^2}{M_Z{}^2}\left(\left(g_V^{q-Z}\right)^2 - 2\left(g_A^{q-Z}\right)^2\right)\right)\sqrt{1 - 4\,\frac{m_q{}^2}{M_Z{}^2}},$$

(5.44)

Amplitude Squared ($m_3 > 0$)

$$|M|^2 = \frac{1}{3}\left(\frac{g_2}{2\cos\theta_w}\right)^2 \epsilon_\mu(p_1)\,\epsilon_\nu{}^*(p_1)$$
$$\mathrm{Tr}\left[\gamma^\mu\left(g_V^{q-Z} - g_A^{q-Z}\gamma^5\right)(p_{2\,\mathrm{slash}} + m_q)\,\gamma^\nu\left(g_V^{q-Z} - g_A^{q-Z}\gamma^5\right)(p_{3\,\mathrm{slash}} + m_{\bar{q}})\right]$$

Trace Calculation ($m_3 > 0$)

In[82]:=
```
Tr[GA[μ].(gV - gA GA5).(GS[p2] + m2).GA[ν].(gV - gA GA5).(GS[p3] + m3)];
ChangeDimension[%, D]
% // StandardForm;
```

Out[83]= $-4\left(\mathrm{m2\,m3}\,g_A^2\,g^{\mu\nu} - g_A^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu - g_A^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu + g_A^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3}) - 2\,i\,g_A\,g_V\,\epsilon^{\mu\nu\,\mathrm{p2}\,\mathrm{p3}} - \mathrm{m2\,m3}\,g_V^2\,g^{\mu\nu} - g_V^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu - g_V^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu + g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})\right)$

Getting rid of imaginary/tensor term $i\,\epsilon^{\mu\nu\sigma\rho}$

In[85]:=
```
-4 (m2 m3 Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] gA^2 -
    Pair[LorentzIndex[μ, D], Momentum[p3, D]] × Pair[LorentzIndex[ν, D], Momentum[p2, D]] gA^2 -
    Pair[LorentzIndex[μ, D], Momentum[p2, D]] × Pair[LorentzIndex[ν, D], Momentum[p3, D]] gA^2 +
    Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] × Pair[Momentum[p2, D], Momentum[p3, D]] gA^2 -
    m2 m3 Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] gV^2 -
    Pair[LorentzIndex[μ, D], Momentum[p3, D]] × Pair[LorentzIndex[ν, D], Momentum[p2, D]] gV^2 -
    Pair[LorentzIndex[μ, D], Momentum[p2, D]] × Pair[LorentzIndex[ν, D], Momentum[p3, D]] gV^2 +
    Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]] × Pair[Momentum[p2, D], Momentum[p3, D]] gV^2)
```

Out[85]= $-4\left(\mathrm{m2\,m3}\,g_A^2\,g^{\mu\nu} - g_A^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu - g_A^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu + g_A^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3}) - \mathrm{m2\,m3}\,g_V^2\,g^{\mu\nu} - g_V^2\,\mathrm{p2}^\nu\,\mathrm{p3}^\mu - g_V^2\,\mathrm{p2}^\mu\,\mathrm{p3}^\nu + g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})\right)$

In[86]:=
```
% // Expand // Simplify
```

Out[86]= $4\left(g_A^2 + g_V^2\right)(\mathrm{p2}^\nu\,\mathrm{p3}^\mu + \mathrm{p2}^\mu\,\mathrm{p3}^\nu) - 4\,g^{\mu\nu}\left(g_A^2\,(\mathrm{m2\,m3} + \mathrm{p2}\cdot\mathrm{p3}) + g_V^2\,(\mathrm{p2}\cdot\mathrm{p3} - \mathrm{m2\,m3})\right)$

Expanding and simplifying $-4\,g^{\mu\nu}\left(g_A^2\,(\mathrm{m2\,m3} + \mathrm{p2.p3}) + g_V^2\,(\mathrm{p2.p3} - \mathrm{m2\,m3})\right)$
and recombining with previous calculation

In[87]:=
```
-4 Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]]
   ((m2 m3 + Pair[Momentum[p2, D], Momentum[p3, D]]) gA^2 +
     (-m2 m3 + Pair[Momentum[p2, D], Momentum[p3, D]]) gV^2) // Expand
```

Out[87]= $-4\,\mathrm{m2\,m3}\,g_A^2\,g^{\mu\nu} - 4\,g_A^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3}) + 4\,\mathrm{m2\,m3}\,g_V^2\,g^{\mu\nu} - 4\,g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})$

In[88]:=
```
Collect[%, gA^2]
```

Out[88]= $g_A^2\,(-4\,\mathrm{m2\,m3}\,g^{\mu\nu} - 4\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})) + 4\,\mathrm{m2\,m3}\,g_V^2\,g^{\mu\nu} - 4\,g_V^2\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})$

In[89]:=
```
Collect[%, gV^2]
```

Out[89]= $g_A^2\,(-4\,\mathrm{m2\,m3}\,g^{\mu\nu} - 4\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3})) + g_V^2\,(4\,\mathrm{m2\,m3}\,g^{\mu\nu} - 4\,g^{\mu\nu}\,(\mathrm{p2}\cdot\mathrm{p3}))$

In[90]:=
```
TraceCalc == 4 HoldForm[(gA^2 + gV^2) (p2^μ p3^ν + p2^ν p3^μ - g^μν (p2.p3)) + (gV^2 - gA^2) g^μν m2 m3]
```

Out[90]= $\text{TraceCalc} = 4\left(\left(g_A^2+g_V^2\right)\left(p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu}\, p_2.p_3\right)+\left(g_V^2-g_A^2\right)g^{\mu\nu}\, m_2\, m_3\right)$

$m_2 = m_3 = m_q$

In[91]:=
```
TraceCalc == 4 HoldForm[(gA^2 + gV^2) (p2^μ p3^ν + p2^ν p3^μ - g^μν (p2.p3)) + (gV^2 - gA^2) g^μν mq^2]
TraceCalc == 4 HoldForm[(gA^2 + gV^2) (p2^μ p3^ν + p2^ν p3^μ - g^μν (p2.p3)) - (gA^2 - gV^2) g^μν mq^2]
```

Out[91]= $\text{TraceCalc} = 4\left(\left(g_A^2+g_V^2\right)\left(p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu}\, p_2.p_3\right)+\left(g_V^2-g_A^2\right)g^{\mu\nu}\, m_q^2\right)$

Out[92]= $\text{TraceCalc} = 4\left(\left(g_A^2+g_V^2\right)\left(p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu}\, p_2.p_3\right)-\left(g_A^2-g_V^2\right)g^{\mu\nu}\, m_q^2\right)$

Multiply by Full Propagator ($m_3 > 0$)

$\epsilon_\mu(p_1)\,\epsilon_\nu(p_1) = \left(\frac{p_{1\,\mu}\,p_{1\,\nu}}{M_z{}^2} - g_{\mu\nu}\right)$

$\mathrm{A} = \left(g_A^2 + g_V^2\right)$

$\left(\frac{p_{1\,\mu}\,p_{1\,\nu}}{M_z{}^2} - g_{\mu\nu}\right)\left(p_2^\mu\, p_3^\nu + p_2^\nu\, p_3^\mu - g^{\mu\nu}\, p_2.p_3\right)$

In[93]:=
```
(FV[p1, μ].FV[p1, ν]/Mz^2 - MT[μ, ν]) * A *
    (FV[p2, μ].FV[p3, ν] + FV[p2, ν].FV[p3, μ] - MT[μ, ν].SP[p2, p3]) // Expand // Contract;
ChangeDimension[%, D]
% /. {Pair[Momentum[p1, D], Momentum[p1, D]] → Mz^2}
```

Out[94]= $\frac{2\,A\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p3})}{M_z^2} - \frac{A\,\text{p1}^2\,(\text{p2}\cdot\text{p3})}{M_z^2} + 2\,A\,(\text{p2}\cdot\text{p3})$

Out[95]= $\frac{2\,A\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p3})}{M_z^2} + A\,(\text{p2}\cdot\text{p3})$

$\frac{2\,\mathrm{A}\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p3})}{M_z^2} + A\,(\text{p2}\cdot\text{p3}),\qquad (\text{ Equation 1 })$

$\mathrm{B} = \left(g_A^2 - g_V^2\right)$

$\left(\frac{p_{1\,\mu}\,p_{1\,\nu}}{M_z{}^2} - g_{\mu\nu}\right)\left(g^{\mu\nu}\, m_q^2\right)$

In[96]:=
```
(FV[p1, μ].FV[p1, ν]/Mz^2 - MT[μ, ν]) * B * (MT[μ, ν].mq^2) // Expand // Contract;
ChangeDimension[%, D]
% /. {Pair[Momentum[p1, D], Momentum[p1, D]] → Mz^2}
```

Out[97]= $\frac{B\,\text{p1}^2\,m_q^2}{M_z^2} - 4\,B\,m_q^2$

Out[98]= $-3\,B\,m_q^2$

$-3\,B\,m_q^2$, (Equation 2)

Kinematics: $p_1 = (M_z\,,\, 0\,)$, $p_2 = \left(\frac{M_z}{2}, \vec{p}\right)$, $p_3 = \left(\frac{M_z}{2}, -\vec{p}\right)$

In[99]:=
```
one = {M_z , 0};
two = {M_z/2, p};
three = {M_z/2, p};
one.two
one.three
two.three
```

Out[102]= $\frac{M_z^2}{2}$

Out[103]= $\frac{M_z^2}{2}$

Out[104]= $\frac{M_z^2}{4} + p^2$

Putting Equation 1 and Equation 2 back into TraceCalc:

In[105]:=
```
4 (A (2 ((p1.p2).(p1.p3))/Mz^2 + (p2.p3)) - B.(-3 mq^2)) ==
  4 ((gA^2 + gV^2) (2 ((one.two) (one.three))/Mz^2 + (two.three)) - (gA^2 - gV^2).(-3 mq^2))
```

Out[105]= $4\left(A\left(\frac{2\, p_1.p_2.p_1.p_3}{M_Z^2} + p_2.p_3\right) - B.\left(-3\, m_q^2\right)\right) = 4\left(\left(g_A^2 + g_V^2\right)\left(\frac{3\, M_z^2}{4} + p^2\right) - \left(g_A^2 - g_V^2\right).\left(-3\, m_q^2\right)\right)$

In[106]:=
```
% /. {p^2 → Mz^2/4 - mq^2}
```

Out[106]= $4\left(A\left(\frac{2\, p_1.p_2.p_1.p_3}{M_Z^2} + p_2.p_3\right) - B.\left(-3\, m_q^2\right)\right) = 4\left(\left(g_A^2 + g_V^2\right)\left(M_z^2 - m_q^2\right) - \left(g_A^2 - g_V^2\right).\left(-3\, m_q^2\right)\right)$

In[107]:=
```
(gA^2 + gV^2) (Mz^2 - mq^2) - (gA^2 - gV^2) (-3 mq^2) // Expand
```

Out[107]= $2\, g_A^2\, m_q^2 + g_A^2\, M_z^2 - 4\, g_V^2\, m_q^2 + g_V^2\, M_z^2$

In[108]:=
```
Collect[%, Mz^2]
```

Out[108]= $2\, g_A^2\, m_q^2 + M_z^2\left(g_A^2 + g_V^2\right) - 4\, g_V^2\, m_q^2$

In[109]:=
```
Collect[%, mq^2]
```

Out[109]= $m_q^2\left(2\, g_A^2 - 4\, g_V^2\right) + M_z^2\left(g_A^2 + g_V^2\right)$

In[110]:=
```
4 ((gA^2 + gV^2).Mz^2 + 2 mq^2 (gA^2 - 2 gV^2)) == 4 Mz^2 HoldForm[((gA^2 + gV^2) + 2 mq^2/Mz^2 (gA^2 - 2 gV^2))]
4 Mz^2 HoldForm[((gA^2 + gV^2) + 2 mq^2/Mz^2 . (gA^2 - 2 gV^2))] ==
4 Mz^2 HoldForm[(gA^(q-z))^2 + (gV^(q-z))^2 + 2 mq^2/Mz^2 . ((gA^(q-z))^2 - 2 (gV^(q-z))^2)]
```

Out[110]= $4\left(2\, m_q^2\left(g_A^2-2\, g_V^2\right)+\left(g_A^2+g_V^2\right).M_z^2\right)=4\, M_z^2\left(\left(g_A^2+g_V^2\right)+\frac{2\, m_q^2\left(g_A^2-2\, g_V^2\right)}{M_z^2}\right)$

Out[111]= $4\, M_z^2\left(\left(g_A^2+g_V^2\right)+2\,\frac{m_q^2}{M_z^2}.\left(g_A^2-2\, g_V^2\right)\right)=4\, M_z^2\left(\left(g_A^{q-z}\right)^2+\left(g_V^{q-z}\right)^2+2\,\frac{m_q^2}{M_z^2}.\left(\left(g_A^{q-z}\right)^2-2\left(g_V^{q-z}\right)^2\right)\right)$

$$|M|^2=\frac{1}{3}\left(\frac{g_2}{2\cos\theta_w}\right)^2 4\, M_z^2\left(\left(g_A^{q-z}\right)^2+\left(g_V^{q-z}\right)^2+2\,\frac{m_q^2}{M_z^2}.\left(\left(g_A^{q-z}\right)^2-2\left(g_V^{q-z}\right)^2\right)\right)$$

$$|M|^2=\frac{1}{3}\,\frac{{g_2}^2}{\cos^2\theta}\, M_z^2\left(\left(g_A^{q-z}\right)^2+\left(g_V^{q-z}\right)^2+2\,\frac{m_q^2}{M_z^2}.\left(\left(g_A^{q-z}\right)^2-2\left(g_V^{q-z}\right)^2\right)\right)$$

Decay (Initial → Final (2 Body)) ($m_3 > 0$)

$$\Gamma=\frac{|\vec{\mathrm{p}}|}{8\,\pi\, {M_z}^2}\,|M|^2$$

$$|\vec{\mathrm{p}}|=\sqrt{(E)^2-m_2\, m_3}=\sqrt{\left(\frac{M_Z}{2}\right)^2-m_2\, m_3}=\frac{M_Z}{2}\sqrt{1-4\,\frac{m_2\, m_3}{{M_Z}^2}}$$

$$m_2=m_3=m_q$$

$$\Gamma\,(\mathrm{Z}\to q\,\overline{q})=\frac{{g_2}^2}{24\,\pi\cos^2\theta\, M_z^2}\,\frac{M_Z}{2}\sqrt{1-4\,\frac{{m_q}^2}{{M_Z}^2}}\; M_z^2.\left(\left(g_V^f\right)^2+\left(g_A^f\right)^2+\frac{2\,{m_q}^2\left(\left(g_A^f\right)^2-2\left(g_V^f\right)^2\right)}{M_z^2}\right)$$

$$\therefore\;\Gamma\,(\mathrm{Z}\to q\,\overline{q})=\frac{{g_2}^2\, M_Z}{48\,\pi\cos^2\theta}\left(\left(g_A^{q-Z}\right)^2+\left(g_V^{q-Z}\right)^2+2\,\frac{{m_q}^2}{{M_Z}^2}\left(\left(g_A^{q-Z}\right)^2-2\left(g_V^{q-Z}\right)^2\right)\right)\sqrt{1-4\,\frac{{m_q}^2}{{M_Z}^2}},$$

(5.44)