$e^- e^+ \to$ hadrons, order α_s corrections

Total Cross Section, My Calculations

$$\sigma = \sigma_B^d + \tilde{\delta}_V^d + \sigma_R^d$$

$$= \sigma_B^d + \sigma_B^d \left(Z_2{}^2 - 1 + 2\,\Lambda\left(\frac{1}{\epsilon_{UV}} - 1\right)\right) + \sigma_R^d$$

$$= \sigma_B^d + \sigma_B^d \left(1 + \frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) - 1 + 2\,\Lambda\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right)$$
$$+\sigma_{B.c.c}^d \left(1 + \frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) - 1 + 2\,\Lambda\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) + \sigma_R^d$$

Where:

$$\Lambda_\mu = \gamma_\mu \frac{g_0{}^2\,C_F}{(4\,\pi)^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{\left(\frac{d-4}{2}\right)} \Gamma\left(3 - \frac{d}{2}\right)$$
$$\int dx\,dy\,dz\,\delta(x + y + z - 1)\left\{\frac{(2-d)^2}{(4-d)}(x\,y)^{d/2-2} + \frac{(2-d)\,((1-x)\,(1-y)) - 2\,\epsilon\,(x\,y)}{(x\,y)^{3-d/2}}\right\}$$

$$\Lambda = (2)\frac{g_0{}^2\,C_F}{(4\,\pi)^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{\left(\frac{d-4}{2}\right)} \Gamma\left(3 - \frac{d}{2}\right) B\left(\frac{d}{2} - 1, \frac{d}{2}\right)$$

$$= \frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon} \Gamma(1+\epsilon)\,B(1-\epsilon, 2-\epsilon)$$

Expand : $\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon} \Gamma(1+\epsilon)\,B(1-\epsilon, 2-\epsilon)$ to ϵ^0 order equals $1/2$

$$\sigma = \sigma_B^d + \sigma_B^d \left(\frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) +$$
$$\sigma_{B.c.c}^d \left(\frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\,C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) + \sigma_R^d$$

UV divergence cancels and is replaced with IR pole and $\sigma_B^d = \sigma_{B.c.c}^d$

$$\sigma = \sigma_B^d + \sigma_B^d\left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) +$$
$$\sigma_{B.c.c}^d\left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) + \sigma_R^d$$

$$= \sigma_B^d + 2\,\sigma_B^d\left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon}\Gamma\,(1+\epsilon)\,B\,(1-\epsilon,\,2-\epsilon)\left(\frac{1}{\epsilon} - \frac{2}{\epsilon^2} - 2\right)\right) + \sigma_R^d$$

In[ ]:=
```
Series[Gamma[1 + ϵ] Beta[1 - ϵ, 2 - ϵ] (1/ϵ - 2/ϵ^2 - 2), {ϵ, 0, 0}] /. {EulerGamma → 0} // Expand //
  Normal // ExpandAll
```

Out[ ]= $-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} + \frac{\pi^2}{12} - 4$

Real Emissions Cross Section:

$$\sigma = \sigma_B^d + 2\,\sigma_B^d\left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{4\,\pi\,\mu^2}{-q^2}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} - 4 + \frac{\pi^2}{12}\right)\right) + \sigma_R^d$$

$$\sigma_R^d = \frac{8\,Q_f{}^2\,e^4\,g_0{}^2\,\mu^{2\left(\frac{4-d}{2}\right)}\,C_F}{s^2}\;\frac{(d-2)^2}{(d-1)}.\,Q^2.\,\frac{1}{8\,s}\;\frac{2^{1-2\,d}\,\pi^{1-d}\,s^{d-3}}{\Gamma\,(d-2)}$$
$$\int d x_1\, d x_2\, d x_3\,((1-x_1)\,(1-x_2)\,(1-x_3))^{\frac{d-4}{2}}\,\delta\,(x_1 - x_2 - x_3 - 2)\;\frac{x_1{}^2 + x_2{}^2 + \left(\frac{d-4}{2}\right)x_3{}^2}{(1-x_1)\,(1-x_2)}$$

$$= \frac{4\,Q_f{}^2\,e^4\,g_0{}^2\,C_F}{s}\left(\frac{4\,\pi\,\mu}{s}\right)^{2\,\epsilon}\,\frac{(1-\epsilon)^2}{(3-2\,\epsilon)}.\;\frac{2^{-7}\,\pi^{-3}}{\Gamma\,(2-2\,\epsilon)}.K$$

$$K = \int d x_1\, d x_2\, d x_3\,((1-x_1)\,(1-x_2)\,(1-x_3))^{\frac{d-4}{2}}\,\delta\,(x_1 - x_2 - x_3 - 2)\;\frac{x_1{}^2 + x_2{}^2 + \left(\frac{d-4}{2}\right)x_3{}^2}{(1-x_1)\,(1-x_2)}$$

$$= \int_0^1 d x_1 \int_{1-x_1}^1 d x_2\,((1-x_1)\,(1-x_2)\,(1-(2-x_1-x_2)))^{\frac{d-4}{2}}\;\frac{x_1{}^2 + x_2{}^2 - \left(\frac{d-4}{2}\right)(2-x_1-x_2)^2}{(1-x_1)\,(1-x_2)}$$

$$= \frac{(d-3)\left(d^2 - 4\,d + 8\right)}{d-2}\;\frac{\Gamma\left(\frac{d-4}{2}\right)^2\,\Gamma\left(\frac{d}{2}\right)}{\Gamma\left(\frac{3\,d-6}{2}\right)}$$

$$= -\frac{6\,(-1+2\,\epsilon)\,(2+(-2+\epsilon)\,\epsilon)\,\Gamma[2-\epsilon]\,\Gamma[-\epsilon]^2}{\Gamma[4-3\,\epsilon]}$$

In[ ]:=
```
Series[-(6 (-1 + 2 ϵ) (2 + (-2 + ϵ) ϵ) Gamma[2 - ϵ] Gamma[-ϵ]^2)/Gamma[4 - 3 ϵ], {ϵ, 0, 0}] // Expand // Normal //
  ExpandAll
```

Out[ ]= $\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}$

$$\sigma_R^d = \frac{4\, Q_f^2\, e^4\, g_0^2\, C_F}{128\,\pi^3\, s}\left(\frac{4\,\pi\,\mu}{s}\right)^{2\,\epsilon}\frac{(1-\epsilon)^2}{(3-2\,\epsilon)\,\Gamma\,(2-2\,\epsilon)}.\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}\right)$$

Born Cross Section in d-dimension:

$$\sigma_B^d \;= \frac{Q_f^2\, e^4}{4\,(3)\,\pi\, s}\left(\frac{s}{4\,\pi}\right)^{\frac{d-4}{2}}\frac{1}{2}\,\frac{(3)\,(d-2)}{d-1}\,\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$= \frac{Q_f^2\, e^4}{4\,(3)\,\pi\, s}\left(\frac{4\,\pi}{s}\right)^{\epsilon}\frac{1}{2}\,\frac{(3)\,2\,(1-\epsilon)}{3-2\,\epsilon}\,\frac{\Gamma(2-\epsilon)}{\Gamma(2-2\,\epsilon)}$$

$$\sigma_R^d = \frac{(2)}{(2)}\,\frac{4\, g_0^2\, C_F}{32\,\pi^2}\,\mu^{2\,\epsilon}\left(\frac{4\,\pi}{s}\right)^{\epsilon}\left(\frac{(1-\epsilon)}{\Gamma\,(2-\epsilon)}.\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}\right)\right)\frac{Q_f^2\, e^4}{4\,(3)\,\pi\, s}\left(\frac{4\,\pi}{s}\right)^{\epsilon}\frac{(3)\,(1-\epsilon)\,\Gamma\,(2-\epsilon)}{(3-2\,\epsilon)\,\Gamma\,(2-2\,\epsilon)}$$

$$= \frac{g_0^2\, C_F}{4\,\pi^2}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(\frac{(1-\epsilon)}{\Gamma\,(2-\epsilon)}.\left(\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}+\frac{19}{4}-\frac{\pi^2}{2}\right)\right)\sigma_B^d$$

In[ ]:=
```
Series[(1 - ϵ)/Gamma[2 - ϵ] (1/ϵ^2 + 3/(2 ϵ) + 19/4 - π^2/2), {ϵ, 0, 0}] /. {EulerGamma → 0} // Expand // Normal //
  ExpandAll
```

Out[ ]= $\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}-\frac{7\,\pi^2}{12}+\frac{19}{4}$

$$\sigma = \;\sigma_B^d+\sigma_B^d\left(\frac{g_0^2\, C_F}{4\,\pi^{\,2}}\left(\frac{4\,\pi\,\mu^2}{-q^2}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2}-\frac{3}{2\,\epsilon}-4+\frac{\pi^2}{12}\right)\right)+$$
$$\sigma_B^d\left(\frac{g_0^2\, C_F}{4\,\pi^2}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}+\frac{19}{4}-\frac{7\,\pi^2}{12}\right)\right)$$

$$= \left(1+\frac{g_0^2\, C_F}{4\,\pi^{\,2}}\left(\left(\frac{4\,\pi\,\mu^2}{-q^2}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2}-\frac{3}{2\,\epsilon}-4+\frac{\pi^2}{12}\right)\right)+\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}+\frac{19}{4}-\frac{7\,\pi^2}{12}\right)\right)\sigma_B^d$$

Expand: Let $A = 4\,\pi\mu^2$

In[ ]:=
```
Series[(A/(-q^2))^ϵ, {ϵ, 0, 2}]
ϵ (Log[A] - Log[-q^2]) // PowerExpand // ExpandAll
1/2 ϵ^2 (Log[A]^2 - Log[-q^2]^2) // PowerExpand // ExpandAll
```

Out[ ]= $1+\epsilon \log\left(-\frac{A}{q^2}\right)+\frac{1}{2}\epsilon^2\log^2\left(-\frac{A}{q^2}\right)+O\left(\epsilon^3\right)$

Out[ ]= $\epsilon\log(A)-2\epsilon\log(q)-i\pi\epsilon$

Out[ ]= $\frac{1}{2}\epsilon^2\log^2(A)-2\epsilon^2\log^2(q)-2i\pi\epsilon^2\log(q)+\frac{\pi^2\epsilon^2}{2}$

In[ ]:=
```
(1 + ϵ Log[(A)] - ϵ Log[(q^2)] + 1/2 ϵ^2 Log[(A)]^2 - 1/2 ϵ^2 Log[(q^2)]^2 + (π^2 ϵ^2)/2) *
  (-1/ϵ^2 - 3/(2 ϵ) - 4 + π^2/12) // Expand
```

Out[ ]= $\frac{1}{24}\pi^2\epsilon^2\log^2(A)-2\epsilon^2\log^2(A)-\frac{3}{4}\epsilon\log^2(A)+\frac{1}{12}\pi^2\epsilon\log(A)-4\epsilon\log(A)-\frac{\log(A)}{\epsilon}-\frac{\log^2(A)}{2}-\frac{3\log(A)}{2}-\frac{1}{24}\pi^2\epsilon^2\log^2\left(q^2\right)+2\epsilon^2\log^2\left(q^2\right)+\frac{3}{4}\epsilon\log^2\left(q^2\right)-\frac{1}{12}\pi^2\epsilon\log\left(q^2\right)+4\epsilon\log\left(q^2\right)+\frac{\log\left(q^2\right)}{\epsilon}+\frac{1}{2}\log^2\left(q^2\right)+\frac{3\log\left(q^2\right)}{2}+\frac{\pi^4\epsilon^2}{24}-2\pi^2\epsilon^2-\frac{1}{\epsilon^2}-\frac{3\pi^2\epsilon}{4}-\frac{3}{2\epsilon}-\frac{5\pi^2}{12}-4$

In[ ]:=
```
(1 + ϵ Log[(A)] - ϵ Log[(q^2)] + 1/2 ϵ^2 Log[(A)]^2 - 1/2 ϵ^2 Log[(q^2)]^2 + (π^2 ϵ^2)/2) * (-1/ϵ^2) // Expand
(1) * (-1/ϵ^2 - 3/(2 ϵ) - 4 + π^2/12) // Expand
```

Out[ ]= $-\frac{\log(A)}{\epsilon}-\frac{1}{2}\log^2(A)+\frac{\log\left(q^2\right)}{\epsilon}+\frac{1}{2}\log^2\left(q^2\right)-\frac{1}{\epsilon^2}-\frac{\pi^2}{2}$

Out[ ]= $-\frac{1}{\epsilon^2}-\frac{3}{2\epsilon}+\frac{\pi^2}{12}-4$

The $\log^2(-q^2)$ expansion gives a $-\frac{\pi^2}{2}$ term, so add $+\frac{\pi^2}{2}$ to the original expanded terms of the Virtual Calculation:
For Logic see Schwartz (20.A.101) Proof Below

In[ ]:=
```
% + π^2/2
```

Out[ ]= $-\frac{1}{\epsilon^2}-\frac{3}{2\epsilon}+\frac{7\pi^2}{12}-4$

$$\sigma = \left(1 + \frac{g_0{}^2\, C_F}{4\,\pi^2}\left(\left(\frac{4\,\pi\,\mu^2}{q^2}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} - 4 + \frac{7\,\pi^2}{12}\right)\right) + \left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(\frac{1}{\epsilon^2} + \frac{3}{2\,\epsilon} + \frac{19}{4} - \frac{7\,\pi^2}{12}\right)\right)\sigma_B^d$$

$$= \left(1 + \frac{g_0{}^2\, C_F}{4\,\pi^2}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} - 4 + \frac{7\,\pi^2}{12} + \frac{1}{\epsilon^2} + \frac{3}{2\,\epsilon} + \frac{19}{4} - \frac{7\,\pi^2}{12}\right)\right)\sigma_B^d$$

$$= \left(1 + \frac{g_0{}^2\, C_F}{4\,\pi^2}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(-4 + \frac{19}{4}\right)\right)\sigma_B^d$$

$$= \left(1 + \frac{3\,\alpha_s\, C_F}{4\,\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\right)\sigma_B^d$$

When $\epsilon \to 0$,

$$\left(\frac{4\,\pi\,\mu^2}{s}\right)^0 = 1$$

$$\sigma_B^d = \sigma_B = \frac{Q_f{}^2\, e^4}{4\,(3)\,\pi\, s}\left(\frac{4\,\pi}{s}\right)^0 \frac{(3)\,(1-0)\,\Gamma\,(2-0)}{(3-2\,(0))\,\Gamma\,(2-2\,(0))} = \frac{4\,\pi\,\alpha^2}{3\,s}\, Q_f{}^2$$

$$\therefore\ \sigma = \left(1 + \frac{3\,\alpha_s\, C_F}{4\,\pi}\right)\sigma_B$$

Total Cross Section, Muta Calculations Comparison

$$\sigma = \sigma_B^d + \tilde{o}_V^d + \sigma_R^d$$

$$= \sigma_B^d + \sigma_B^d\left(Z_2{}^2 - 1 + 2\,\Lambda\left(\frac{1}{\epsilon_{UV}} - 1\right)\right) + \sigma_R^d$$

$$= \sigma_B^d + \sigma_B^d\left(1 + \frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) - 1 + 2\,\Lambda\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right)$$
$$+\sigma_{B.c.c}^d\left(1 + \frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) - 1 + 2\,\Lambda\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) + \sigma_R^d$$

Where:

$$\Lambda_\mu = \gamma_\mu \frac{g_0{}^2\, C_F}{(4\,\pi)^2} \left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{\left(\frac{d-4}{2}\right)} \Gamma\left(3 - \frac{d}{2}\right)$$

$$\int d x\, d y\, d z\, \delta(x + y + z - 1) \left\{\frac{(2-d)^2}{(4-d)} (x\,y)^{d/2-2} + \frac{(2-d)\,((1-x)(1-y)) - 2\,\epsilon\,(x\,y)}{(x\,y)^{3-d/2}}\right\}$$

$$\Lambda = (2) \frac{g_0{}^2\, C_F}{(4\,\pi)^2} \left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{\left(\frac{d-4}{2}\right)} \Gamma\left(3 - \frac{d}{2}\right) B\left(\frac{d}{2} - 1, \frac{d}{2}\right)$$

$$= \frac{g_0{}^2\, C_F}{8\,\pi^2} \left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon} \Gamma(1+\epsilon)\, B(1-\epsilon, 2-\epsilon)$$

Expand : $\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon} \Gamma(1+\epsilon)\, B(1-\epsilon, 2-\epsilon)$ to ϵ^0 order equals $1/2$

$$\sigma = \sigma_B^d + \sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) +$$
$$\sigma_{B.c.c}^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) + \sigma_R^d$$

UV divergence cancels and is replaced with IR pole and $\sigma_B^d = \sigma_{B.c.c}^d$

$$\sigma = \sigma_B^d + \sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) +$$
$$\sigma_{B.c.c}^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right) + 2\,\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{1}{2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}{}^2} - 2\right)\right) + \sigma_R^d$$

$$= \sigma_B^d + 2\,\sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon} \Gamma(1+\epsilon)\, B(1-\epsilon, 2-\epsilon)\left(\frac{1}{\epsilon} - \frac{2}{\epsilon^2} - 2\right)\right) + \sigma_R^d$$

In[ ]:=
```
Series[Gamma[1 + ϵ] Beta[1 - ϵ, 2 - ϵ] (1/ϵ - 2/ϵ^2 - 2), {ϵ, 0, 0}] /. {EulerGamma → 0} // Expand //
  Normal // ExpandAll
Series[Beta[1 - ϵ, 2 - ϵ] (1/ϵ - 2/ϵ^2 - 2), {ϵ, 0, 0}] /. {EulerGamma → 0} // Expand // Normal //
  ExpandAll
```

Out[ ]= $-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} + \frac{\pi^2}{12} - 4$

Out[ ]= $-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} + \frac{\pi^2}{6} - 4$

Choices:

$$\sigma = \sigma_B^d + 2\,\sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} + \frac{\pi^2}{12} - 4\right)\right) + \sigma_R^d$$

$$\sigma = \sigma_B^d + 2\,\sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon}\Gamma(1+\epsilon)\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} + \frac{\pi^2}{6} - 4\right)\right) + \sigma_R^d$$

Muta Writes:

$$A_V = \frac{\alpha_s\, C_F}{\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\frac{\mathrm{Cos}(\pi\,\epsilon)}{\Gamma(1-\epsilon)}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} - 4\right)$$

$$\tilde{o}_V = A_V\,\sigma_B$$

$$\sigma = \sigma_B^d + 2\,\sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{-q^2}{4\,\pi\,\mu^2}\right)^{-\epsilon}\frac{\mathrm{Cos}(\pi\,\epsilon)}{\Gamma(1-\epsilon)}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} - 4\right)\right) + \sigma_R^d$$

In[ ]:=
```
Series[Cos[π ϵ]/Gamma[1 - ϵ] (-1/ϵ^2 - 3/(2 ϵ) - 4), {ϵ, 0, 0}] /. {EulerGamma → 0} // Expand // Normal //
  ExpandAll
```

Out[ ]=
$$-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} + \frac{7\,\pi^2}{12} - 4$$

Real Emissions Cross Section:

$$\sigma = \sigma_B^d + 2\,\sigma_B^d \left(\frac{g_0{}^2\, C_F}{8\,\pi^2}\left(\frac{4\,\pi\,\mu^2}{-q^2}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2} - \frac{3}{2\,\epsilon} - 4\right)\right) + \sigma_R^d$$

$$\sigma_R^d = \frac{8\,Q_f{}^2\, e^4\, g_0{}^2\, \mu^{2\left(\frac{4-d}{2}\right)}\, C_F}{s^2}\,\frac{(d-2)^2}{(d-1)}.\,Q^2.\frac{1}{8\,s}\,\frac{2^{1-2\,d}\,\pi^{1-d}\,s^{d-3}}{\Gamma(d-2)}$$

$$\int d x_1\, d x_2\, d x_3\, ((1-x_1)(1-x_2)(1-x_3))^{\frac{d-4}{2}}\,\delta(x_1 - x_2 - x_3 - 2)\,\frac{x_1{}^2 + x_2{}^2 + \left(\frac{d-4}{2}\right) x_3{}^2}{(1-x_1)(1-x_2)}$$

$$= \frac{32\,Q_f{}^2\, e^4\, g_0{}^2\, \mu^{2\,\epsilon}\, C_F}{s}\,\frac{(1-\epsilon)^2}{(3-2\,\epsilon)}.\frac{1}{8\,s}\,\frac{2^{-7+4\,\epsilon}\,\pi^{-3+2\,\epsilon}\,s^{1-2\,\epsilon}}{\Gamma(2-2\,\epsilon)}$$

$$\int_0^1 d x_1\, d x_2\, d x_3\, ((1-x_1)(1-x_2)(1-x_3))^{-\epsilon}\,\delta(x_1 - x_2 - x_3 - 2)\,\frac{x_1{}^2 + x_2{}^2 - (\epsilon)\, x_3{}^2}{(1-x_1)(1-x_2)}$$

$$= \frac{4\,Q_f{}^2\, e^4\, g_0{}^2\, C_F}{s}\left(\frac{4\,\pi\,\mu}{s}\right)^{2\,\epsilon}\frac{(1-\epsilon)^2}{(3-2\,\epsilon)}.\,\frac{2^{-7}\,\pi^{-3}}{\Gamma(2-2\,\epsilon)}.K$$

$$= \frac{2\,Q_f{}^2\, \alpha^2\, \alpha_s\, C_F}{s}\left(\frac{4\,\pi\,\mu}{s}\right)^{2\,\epsilon}\frac{(1-\epsilon)^2}{(3-2\,\epsilon).\,\Gamma(2-2\,\epsilon)}.K$$

$$K = \int dx_1\, dx_2\, dx_3\, ((1-x_1)(1-x_2)(1-x_3))^{-\epsilon}\, \delta(x_1-x_2-x_3-2)\, \frac{x_1{}^2+x_2{}^2-\epsilon\, x_3{}^2}{(1-x_1)(1-x_2)}$$

$$= \int_0^1 dx_1 \int_{1-x_1}^1 dx_2\, ((1-x_1)(1-x_2)(1-(2-x_1-x_2)))^{-\epsilon}\, \frac{x_1{}^2+x_2{}^2-\epsilon(2-x_1-x_2)^2}{(1-x_1)(1-x_2)}$$

$$= B(1-\epsilon, 2-2\epsilon)\, B(1-\epsilon, 1-\epsilon)\left(\frac{4}{\epsilon^2}-\frac{12}{\epsilon}+10-4\epsilon\right)$$

$$= \frac{(d-3)\left(d^2-4d+8\right)}{d-2}\, \frac{\Gamma\left(\frac{d-4}{2}\right)^2 \Gamma\left(\frac{d}{2}\right)}{\Gamma\left(\frac{3d-6}{2}\right)}$$

$$= -\frac{6(-1+2\epsilon)(2+(-2+\epsilon)\epsilon)\,\Gamma(2-\epsilon)\,\Gamma(-\epsilon)^2}{\Gamma(4-3\epsilon)}$$

In[ ]:=
```
Series[- 6 (-1 + 2 ϵ) (2 + (-2 + ϵ) ϵ) Gamma[2 - ϵ] Gamma[-ϵ]^2 / Gamma[4 - 3 ϵ], {ϵ, 0, 0}] // Expand // Normal //
  ExpandAll
Series[Beta[1 - ϵ, 2 - 2 ϵ] Beta[1 - ϵ, 1 - ϵ] (4/ϵ^2 - 12/ϵ + 10 - 4 ϵ), {ϵ, 0, 0}] // Expand //
   Normal // ExpandAll
```

Out[ ]= $\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}$

Out[ ]= $\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}$

$$\sigma_R^d = \frac{2\, Q_f{}^2\, \alpha^2\, \alpha_s\, C_F}{s}\left(\frac{4\pi\mu}{s}\right)^{2\epsilon} \frac{(1-\epsilon)^2}{(3-2\epsilon).\,\Gamma(2-2\epsilon)}.\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}\right)$$

Muta Writes:

$$A_R = \frac{\alpha_s\, C_F}{\pi}\left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \frac{\mathrm{Cos}(\pi\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{19}{2}\right)$$

$$\sigma_R^d = A_R\, \sigma_B^d$$

Born Cross Section in d-dimension:

$$\sigma_B^d = \frac{Q_f{}^2\, e^4}{4(3)\pi s}\left(\frac{s}{4\pi}\right)^{\frac{d-4}{2}} \frac{1}{2}\, \frac{(3)(d-2)}{d-1}\, \frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)}$$

$$= \frac{Q_f{}^2\, 16\pi^2\alpha^2}{4(3)\pi s}\left(\frac{4\pi}{s}\right)^{\epsilon} \frac{1}{2}\, \frac{(3)\, 2(1-\epsilon)}{3-2\epsilon}\, \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)}$$

$$\sigma_R^d = \frac{(2)}{(2)}\frac{4\,g_0^2\,C_F}{32\,\pi^2}\mu^{2\epsilon}\left(\frac{4\,\pi}{s}\right)^{\epsilon}\left(\frac{(1-\epsilon)}{\Gamma(2-\epsilon)}.\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+\frac{19}{2}\right)\right)\frac{Q_f^2\,e^4}{4\,(3)\,\pi\,s}\left(\frac{4\,\pi}{s}\right)^{\epsilon}\frac{(3)\,(1-\epsilon)\,\Gamma(2-\epsilon)}{(3-2\,\epsilon)\,\Gamma(2-2\,\epsilon)}$$

$$= \frac{(2)}{(2)}\frac{16\,\alpha_s\,C_F}{32\,\pi^2}\mu^{2\epsilon}\left(\frac{4\,\pi}{s}\right)^{\epsilon}\left(\frac{\mathrm{Cos}(\pi\,\epsilon)}{\Gamma(1-\epsilon)}.\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{19}{2}\right)\right)\frac{4\,\pi\,\alpha^2}{(3)\,s}Q_f^2\left(\frac{4\,\pi}{s}\right)^{\epsilon}\frac{(3)\,(1-\epsilon)\,\Gamma(2-\epsilon)}{(3-2\,\epsilon)\,\Gamma(2-2\,\epsilon)}$$

$$= \frac{\alpha_s\,C_F}{\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(\frac{\mathrm{Cos}(\pi\,\epsilon)}{\Gamma(1-\epsilon)}.\left(\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}+\frac{19}{4}\right)\right)\sigma_B^d$$

In[ ]:=
```
Series[Cos[π ϵ]/Gamma[1 - ϵ] (1/ϵ^2 + 3/(2 ϵ) + 19/4), {ϵ, 0, 0}] /. {EulerGamma → 0} // Expand // Normal //
  ExpandAll
```

Out[ ]= $\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}-\frac{7\,\pi^2}{12}+\frac{19}{4}$

$$\sigma = \sigma_B^d + \sigma_B^d\left(\frac{\alpha_s\,C_F}{\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2}-\frac{3}{2\,\epsilon}-4+\frac{7\,\pi^2}{12}\right)\right)$$

$$+\sigma_B^d\left(\frac{\alpha_s\,C_F}{\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}+\frac{19}{4}-\frac{7\,\pi^2}{12}\right)\right)$$

$$= \left(1+\frac{\alpha_s\,C_F}{\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(-\frac{1}{\epsilon^2}-\frac{3}{2\,\epsilon}-4+\frac{7\,\pi^2}{12}+\frac{1}{\epsilon^2}+\frac{3}{2\,\epsilon}+\frac{19}{4}-\frac{7\,\pi^2}{12}\right)\right)\sigma_B^d$$

$$= \left(1+\frac{\alpha_s\,C_F}{\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\left(-4+\frac{19}{4}\right)\right)\sigma_B^d$$

$$= \left(1+\frac{3\,\alpha_s\,C_F}{4\,\pi}\left(\frac{4\,\pi\,\mu^2}{s}\right)^{\epsilon}\right)\sigma_B^d$$

When $\epsilon \to 0$,

$$\left(\frac{4\,\pi\,\mu^2}{s}\right)^0 = 1$$

$$\sigma_B^d = \sigma_B = \frac{4\,\pi\,\alpha^2}{(3)\,s}Q_f^2\left(\frac{4\,\pi}{s}\right)^0\frac{(3)\,(1-0)\,\Gamma(2-0)}{(3-2\,(0))\,\Gamma(2-2\,(0))} = \frac{4\,\pi\,\alpha^2}{3\,s}Q_f^2$$

$$\therefore\ \sigma = \left(1+\frac{3\,\alpha_s\,C_F}{4\,\pi}\right)\sigma_B$$

Schwartz (20.A.101) and (20.A.102) pg. 377

In[∘]:= `d = 4 - ϵ;`

In[∘]:= $f\left[Q^2\right] == 4\, e_R{}^2 . (16\,\pi)^{\frac{1-d}{2}} . \left(\frac{\mu^2}{-Q^2}\right)^{\frac{4-d}{2}} . \frac{\Gamma\left[\frac{4-d}{2}\right].\Gamma\left[\frac{d}{2}\right]}{\Gamma\left[\frac{d-1}{2}\right]} . \frac{\left(d^2 - 7\,d + 16\right)}{\left(d^2 - 6\,d + 8\right)}$

Out[∘]= $f\left(Q^2\right) = 4\, e_R^2 . \left(4^{\epsilon-3}\,\pi^{\frac{\epsilon-3}{2}}\right) . \left(-\frac{\mu^2}{Q^2}\right)^{\epsilon/2} . \frac{\Gamma\left(\frac{\epsilon}{2}\right).\Gamma\left(\frac{4-\epsilon}{2}\right)}{\Gamma\left(\frac{3-\epsilon}{2}\right)} . \frac{(4-\epsilon)^2 - 7\,(4-\epsilon) + 16}{(4-\epsilon)^2 - 6\,(4-\epsilon) + 8}$

In[∘]:= $\text{Series}\left[\frac{\text{Gamma}\left[\frac{4-d}{2}\right]\,\text{Gamma}\left[\frac{d}{2}\right]}{\text{Gamma}\left[\frac{d-1}{2}\right]}\,\frac{\left(d^2 - 7\,d + 16\right)}{\left(d^2 - 6\,d + 8\right)}, \{\epsilon, 0, 0\}\right]$ `// Expand // Normal // ExpandAll // FullSimplify // ExpandAll`

Out[∘]= $-\frac{8}{\sqrt{\pi}\,\epsilon^2} + \frac{4\,\gamma}{\sqrt{\pi}\,\epsilon} - \frac{6}{\sqrt{\pi}\,\epsilon} + \frac{2\log(16)}{\sqrt{\pi}\,\epsilon} + \frac{\pi^{3/2}}{6} - \frac{\gamma^2}{\sqrt{\pi}} + \frac{3\,\gamma}{\sqrt{\pi}} - \frac{8}{\sqrt{\pi}} - \frac{\log^2(4)}{\sqrt{\pi}} - \frac{2\,\gamma\log(4)}{\sqrt{\pi}} + \frac{3\log(4)}{\sqrt{\pi}}$

In[∘]:= `% /. {EulerGamma → 0}`

Out[∘]= $-\frac{8}{\sqrt{\pi}\,\epsilon^2} - \frac{6}{\sqrt{\pi}\,\epsilon} + \frac{2\log(16)}{\sqrt{\pi}\,\epsilon} + \frac{\pi^{3/2}}{6} - \frac{8}{\sqrt{\pi}} - \frac{\log^2(4)}{\sqrt{\pi}} + \frac{3\log(4)}{\sqrt{\pi}}$

$$f\left(Q^2\right) =$$

$$\left(-\frac{8}{\pi^{1/2}}\right) 4\, e_R^2 . \left(4^{\epsilon-3}\,\pi^{\frac{\epsilon-3}{2}}\right) . \left(-\frac{\mu^2}{Q^2}\right)^{\epsilon/2} . \left(\frac{1}{\epsilon^2} + \frac{6}{8\,\epsilon} - \frac{2\log(16)}{8\,\epsilon} - \frac{\pi^{3/2}\,\pi^{1/2}}{(8)\,6} + \frac{8}{8} + \frac{\log^2(4)}{8} - \frac{3\log(4)}{8}\right)$$

$$=$$

$$\left(-\frac{2*4}{\pi^{1/2}\,\pi^{3/2}}\right) \frac{4}{4^3}\, e_R^2 . \left(\frac{16^{\epsilon/2}\,\pi^{\epsilon/2}\,\mu^{2\,(\epsilon/2)}}{-Q^2}\right) . \left(\frac{1}{\epsilon^2} + \frac{3}{4\,\epsilon} - \frac{\pi^2}{48} + 1 - \frac{\log(16)}{4\,\epsilon} + \frac{\log^2(4)}{8} - \frac{3\log(4)}{8}\right)$$

$$= \left(-\frac{2}{\pi^2}\right) e_R^2 . \left(\frac{(4*4)^{\epsilon/2}\,\pi^{\epsilon/2}\,\mu^{2\,(\epsilon/2)}}{-Q^2}\right) . \left(\frac{1}{\epsilon^2} + \frac{3}{4\,\epsilon} - \frac{\pi^2}{48} + 1 - \frac{\log(16)}{4\,\epsilon} + \frac{\log^2(4)}{8} - \frac{3\log(4)}{8}\right)$$

$$= -\frac{e_R^2}{2\,\pi^2} . \left(\frac{4\,\pi e^{-\gamma_E}\,\mu^2}{-Q^2}\right)^{\epsilon/2} . \left(\frac{1}{\epsilon^2} + \frac{3}{4\,\epsilon} - \frac{\pi^2}{48} + 1\right)$$

Expand: Let A = $4\,\pi e^{-\gamma_E}\,\mu^2$ and $Q^2 = \Lambda$

In[ ]:=
```
Series[(A/(-Λ))^(ϵ/2), {ϵ, 0, 2}]
1/2 (Log[A] - Log[-Λ]) // PowerExpand // ExpandAll
1/8 (Log[A]^2 - Log[-Λ]^2) // PowerExpand // ExpandAll
```

Out[ ]= $1+\frac{1}{2}\epsilon\log\left(-\frac{A}{\Lambda}\right)+\frac{1}{8}\epsilon^2\log^2\left(-\frac{A}{\Lambda}\right)+O\left(\epsilon^3\right)$

Out[ ]= $\frac{\log(A)}{2}-\frac{\log(\Lambda)}{2}-\frac{i\pi}{2}$

Out[ ]= $\frac{\log^2(A)}{8}-\frac{\log^2(\Lambda)}{8}-\frac{1}{4}i\pi\log(\Lambda)+\frac{\pi^2}{8}$

In[ ]:=
```
Series[(A/(-Q^2))^(ϵ/2), {ϵ, 0, 2}] (1/ϵ^2 + 3/(4 ϵ) - π^2/48 + 1) // Expand // Normal
```

Out[ ]= $\frac{\frac{1}{2}\log\left(-\frac{A}{Q^2}\right)+\frac{3}{4}}{\epsilon}+\frac{1}{8}\log^2\left(-\frac{A}{Q^2}\right)+\frac{3}{8}\log\left(-\frac{A}{Q^2}\right)+\frac{1}{\epsilon^2}-\frac{\pi^2}{48}+1$

In[ ]:=
```
1/ϵ (1/2 Log[A] - 1/2 Log[Λ] - ⅈ π/2 + 3/4) + 3/8 (Log[A] - Log[Λ] - ⅈ π) +
  1/8 Log[A]^2 - 1/8 Log[Λ]^2 - 1/4 ⅈ π Log[Λ] + π^2/8 + HoldForm[1/ϵ^2 - π^2/48 + 1] // Expand
```

Out[ ]= $\frac{\log(A)}{2\epsilon}+\frac{\log^2(A)}{8}+\frac{3\log(A)}{8}-\frac{\log^2(\Lambda)}{8}-\frac{1}{4}i\pi\log(\Lambda)-\frac{3\log(\Lambda)}{8}+\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{48}+1\right)-\frac{\log(\Lambda)}{2\epsilon}-\frac{i\pi}{2\epsilon}+\frac{3}{4\epsilon}+\frac{\pi^2}{8}-\frac{3i\pi}{8}$

```
% // StandardForm;
```

In[ ]:=
```
-3 ⅈ π/8 + 3 ⅈ π/8 + HoldForm[3 ⅈ π/8] + π^2/8 - π^2/8 + 3/(4 ϵ) - ⅈ π/(2 ϵ) + ⅈ π/(2 ϵ) + HoldForm[ⅈ π/(2 ϵ)] + (1/ϵ^2 - π^2/48 - π^2/8 + 1) +
  3 Log[A]/8 + Log[A]/(2 ϵ) + Log[A]^2/8 - 3 Log[Λ]/8 - 1/4 ⅈ π Log[Λ] - Log[Λ]/(2 ϵ) - Log[Λ]^2/8
```

Out[ ]= $\frac{\log(A)}{2\epsilon}+\frac{\log^2(A)}{8}+\frac{3\log(A)}{8}-\frac{\log^2(\Lambda)}{8}-\frac{1}{4}i\pi\log(\Lambda)-\frac{3\log(\Lambda)}{8}+\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{48}-\frac{\pi^2}{8}+1\right)-\frac{\log(\Lambda)}{2\epsilon}+\frac{i\pi}{2\epsilon}+\frac{3}{4\epsilon}+\frac{3i\pi}{8}$

```
% // StandardForm;
```

In[ ]:=
```
(1/ϵ^2) + 3/(4 ϵ) + ⅈ π/(2 ϵ) + 3/8 Log[A/Λ] + 1/(2 ϵ) Log[A/Λ] + 1/8 Log[A/Λ]^2 - 1/4 ⅈ π Log[Λ] - 7 π^2/48 + 1 + 3 ⅈ π/8
```

Out[ ]= $\frac{1}{8}\log^2\left(\frac{A}{\Lambda}\right)+\frac{3}{8}\log\left(\frac{A}{\Lambda}\right)+\frac{\log\left(\frac{A}{\Lambda}\right)}{2\epsilon}-\frac{1}{4}i\pi\log(\Lambda)+\frac{1}{\epsilon^2}+\frac{i\pi}{2\epsilon}+\frac{3}{4\epsilon}+\frac{3i\pi}{8}-\frac{7\pi^2}{48}+1$

In[ ]:= Series$\left[\left(\frac{A}{\Lambda}\right)^{\epsilon/2}, \{\epsilon, 0, 2\}\right]\left(\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}+\frac{i\pi}{2\epsilon}-\frac{7\pi^2}{48}+1+\frac{3 i\pi}{8}\right)$ // Normal // ExpandAll

Out[ ]= $$\frac{1}{8}\log^2\left(\frac{A}{\Lambda}\right)+\frac{1}{4}i\pi\log\left(\frac{A}{\Lambda}\right)+\frac{3}{8}\log\left(\frac{A}{\Lambda}\right)+\frac{\log\left(\frac{A}{\Lambda}\right)}{2\epsilon}+\frac{1}{\epsilon^2}+\frac{i\pi}{2\epsilon}+\frac{3}{4\epsilon}-\frac{7\pi^2}{48}+\frac{3 i\pi}{8}+1$$

In[ ]:= RealIm = %;

In[ ]:= $f[Q^2] == -\frac{e_R^2}{2\pi^2}.(\text{RealIm})$

Out[ ]= $$f(Q^2)=-\frac{e_R^2}{2\pi^2}.\left(\frac{1}{8}\log^2\left(\frac{A}{\Lambda}\right)+\frac{1}{4}i\pi\log\left(\frac{A}{\Lambda}\right)+\frac{3}{8}\log\left(\frac{A}{\Lambda}\right)+\frac{\log\left(\frac{A}{\Lambda}\right)}{2\epsilon}+\frac{1}{\epsilon^2}+\frac{i\pi}{2\epsilon}+\frac{3}{4\epsilon}-\frac{7\pi^2}{48}+\frac{3 i\pi}{8}+1\right)$$

$$f\left(Q^2\right)=-\frac{e_R^2}{2\pi^2}.\left(\frac{4\pi e^{-\gamma_E}\mu^2}{Q^2}\right)^{\epsilon/2}.\left(\frac{1}{\epsilon^2}+\frac{i\pi}{2\epsilon}+\frac{3}{4\epsilon}-\frac{7\pi^2}{48}+1+\frac{3 i\pi}{8}\right); \qquad (20.A.101)$$

Drop Imaginary Terms:

% // StandardForm;

In[ ]:= $\text{RealL} = 1-\frac{7\pi^2}{48}+\frac{3}{4\epsilon}+\frac{1}{\epsilon^2}+\frac{3}{8}\text{Log}\left[\frac{A}{\Lambda}\right]+\frac{\text{Log}\left[\frac{A}{\Lambda}\right]}{2\epsilon}+\frac{1}{8}\text{Log}\left[\frac{A}{\Lambda}\right]^2;$

In[ ]:= $f[Q^2] == -\frac{e_R^2}{2\pi^2}.(\text{RealL})$

Out[ ]= $$f(Q^2)=-\frac{e_R^2}{2\pi^2}.\left(\frac{1}{8}\log^2\left(\frac{A}{\Lambda}\right)+\frac{3}{8}\log\left(\frac{A}{\Lambda}\right)+\frac{\log\left(\frac{A}{\Lambda}\right)}{2\epsilon}+\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}-\frac{7\pi^2}{48}+1\right)$$

$$f\left(Q^2\right)=-\frac{e_R^2}{2\pi^2}.\left(\frac{4\pi e^{-\gamma_E}\mu^2}{Q^2}\right)^{\epsilon/2}.\left(\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}-\frac{7\pi^2}{48}+1\right)$$

$$\sigma_V^d = 2\,\sigma_B^d\,\text{Re}\left(f\left(Q^2\right)\right)$$

$$= -\frac{Q_f^2 e^4}{12\pi s}\mu^{2\epsilon}\left(\frac{4\pi}{s}\right)^{\epsilon/2}\frac{(3)(2-\epsilon)}{2(3-\epsilon)}\frac{\Gamma(2-\epsilon/2)}{\Gamma(2-\epsilon)}\frac{e_R^2}{\pi^2}.\left(\frac{4\pi e^{-\gamma_E}\mu^2}{Q^2}\right)^{\epsilon/2}.\left(\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}-\frac{7\pi^2}{48}+1\right)$$

In[ ]:= Series$\left[\frac{3(2-\epsilon)\,\text{Gamma}[2-\epsilon/2]}{2(3-\epsilon)\,\text{Gamma}[2-\epsilon]}*\left(\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}+1-\frac{7\pi^2}{48}\right), \{\epsilon, 0, 0\}\right]$ /. {EulerGamma → 0} //
Expand // Normal // ExpandAll

Out[ ]= $$\frac{1}{\epsilon^2}+\frac{13}{12\epsilon}-\frac{5\pi^2}{24}+\frac{29}{18}$$

$$\sigma_V^d = -\frac{Q_f^2 e^4}{12\pi s}\frac{e_R^2}{\pi^2}\mu^{2\epsilon}\left(\frac{4\pi}{s}\right)^{\epsilon/2}.\left(\frac{4\pi e^{-\gamma_E}\mu^2}{s}\right)^{\epsilon/2}.\left(\frac{1}{\epsilon^2}+\frac{13}{12\epsilon}-\frac{5\pi^2}{24}+\frac{29}{18}\right); \qquad (20.A.102)$$

Expansion Trials 1

Expansion Trials 2