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Game Theory Based Bandwidth Allocation Scheme for Network Virtualization

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Abstract—Running multiple virtual networks over a shared physical network is a promising way to support diverse applications, consequently network virtualization is viewed as the key-stone of the next-generation architecture. However, decoupling the role of traditional ISPs into Infrastructure Providers (InPs) and Service Providers (SPs), also brings some new challenges to us. For example, how to fairly and efficiently share the sacred physical resources of InPs among multiple SPs is a key problem. The interaction between InPs and SPs, such as cooperation and competition, makes this topic even more complicated. In this paper, we develop a novel approach to encourage efficient behavior in solving the interaction between InPs and SPs by introducing economic incentives, in the form of Game Theory. Based on the non-cooperative game model, a bandwidth allocation scheme in the network virtualization environment is established, using the concept of the Nash Equilibrium. Then we propose an iterative algorithm to find the Nash Equilibrium and solve the bandwidth allocation problem. Finally, we demonstrate the convergence and the effectiveness of our scheme in the experiments.

I. INTRODUCTION

In the last few years, network virtualization has attracted a great deal of attention in the debate on how to model the next-generation Internet. Deploying new Internet services is difficult without the cooperation among all stakeholders, as radical changes of the Internet architecture is not allowed [1][2][3]. Researchers believe that network virtualization can overcome the ossification of the current Internet and stimulate innovation [4][5]. Furthermore, it is considered as the keystone of the next-generation architecture [4].

In network virtualization, the role of the traditional ISPs are decoupled into two parts: Infrastructure Providers (InPs) are in charge of physical networks, and Service Providers (SPs) deploy customized Virtual Networks (VNs) [5][6]. Then the deployment of VNs should be observed from two different perspectives: the former is the perspective of InPs, which mean to maximize their own revenue by allocating physical resources; while the later one regards to SPs, which mean to obtain the contracted resources.

As a result, network virtualization faces a fundamental challenge of fairly and efficiently sharing physical resources among multiple VNs. On one hand, InPs should focus on how to keep balance between ensuring the fairness and maximizing their own revenue. On the other hand, SPs just focus on how to gain enough resources, by competing with each other or

selecting from available InPs. In summary, SPs may cooperate or compete with each other to suffice their own requirements, especially when they are controlled by multiple parties. Due to the complex interaction between InPs and SPs, we introduce economic incentives into the resource allocation scheme in the network virtualization environment, with the hope of encouraging efficient behavior. In this paper, we consider a model that consists of an InP and multiple SPs, and focus on the interaction among multiple SPs in the framework of Game Theory. Game Theory is a field of applied mathematics that analyzes interactive decision situations, and provides analytical tools to predict the outcome of complex interactions among rational entities. We believe Game Theory will be applicable to analyze the network virtualization environment.

In this paper, we consider the bandwidth allocation scheme based on Game Theory. Moreover, we only focus on how InPs allocate the limited bandwidth among multiple VNs. It's obviously that the sharing of bandwidth has the most direct impact on the performance of VNs, as well as the total performance of the network virtualization environment. Therefore, our goal is to efficiently allocate the bandwidth among multiple VNs in the framework of the non-cooperative game model. The contributions of our work can be summarized as follows:

- We propose a bandwidth allocation scheme based on the non-cooperative game model to describe the interactions among multiple VNs. We are the first to develop a case for the application of Game Theory in the network virtualization environment.
- The non-cooperative game model we set satisfies the situation that the total bandwidth requirements of multiple VNs exceed the capacity of the physical network. The pricing scheme in our model associate the congestion control with their payments, encouraging the efficient behavior of VNs. Furthermore, the InP introduces differential pricing scheme according to the characteristic of different VNs.
- We prove that our scheme can achieve the Nash Equilibrium [7], and develop an iterative algorithm to implement our scheme. Besides, the effectiveness and the convergence of the Nash Equilibrium are also demonstrated in our experiments.

The remainder of this paper is organized as follows. After presenting the related work in section II, we describe the system model, propose our bandwidth allocation scheme, and develop an iterative algorithm in section III. Then we study the performance of our experiments in section IV. Finally, we conclude the paper in section V.

II. RELATED WORK

To the best of our knowledge, there is no existing work in the literature that tries to solve the bandwidth allocation problem in non-cooperative game model. But there are several proposals that have motivated as well as influenced our work.

Resource allocation in the network virtualization environment aims to fairly and efficiently share physical resource among VNs. For the static resource allocation scheme could cause worse performance than existing solutions, the authors in [8] propose an adaptive network virtualization, with dynamic resource allocation scheme. In this architecture, resource are periodically reassigned among VNs, aided by optimization theory. As mentioned in [8], if any of the VNs exhibit greedy or malicious behaviors, unfair allocation results.

Then this problem is discussed in [9], the authors analyze the problem of allocating bandwidth among multiple VNs, especially when there are some greedy applications. Moreover, the mechanism to offer a fair bandwidth distribution is introduced. Though the authors propose a strategy that tries to fairly distribute the bandwidth among competing VNs, the scheme itself is static and only applied to VNs with only one service class. In order to efficiently use physical resources, the authors in [10] propose a runtime, distributed, local view approach to manage physical resources. An associated self-organizing algorithm is used to reallocate resources of VNs along different physical nodes.

The authors in [11] introduce three branches of Game Theory, leader-follower, cooperative, and two-person nonzero sum games, to the study of the Internet pricing issue. In addition, both non-cooperative and cooperative game are applied to the Internet pricing framework, especially the resource allocation problem. Recently, many researchers have used game theoretical methods to analyze the resource allocation problem in computer networks, especially wireless network. The authors in [12] proposed a non-cooperating power control game based on a specific energy efficient utility function that is common to all users, and proved that the game has a unique Nash Equilibrium. Then in [7], the concept of Pareto efficiency was introduced in the game to gain better overall performance. Based on the model mentioned above, the authors in [13] generalized the game model to consider quality-of-service constraints. In [14], the authors summarize the game theoretical approaches used for energy efficient resource allocation in wireless network. Hence we believe Game Theory's ability to simplify analysis of interaction among different players can help handling problems from network virtualization, for example the resource allocation problem.

III. RESOURCE ALLOCATION SCHEME BASED ON NON-COOPERATIVE GAME MODEL

A. System Model

In this section, we introduce the system model of the network virtualization environment. The basic entity is the VN, a collection of virtual nodes and virtual links. Hence, here comes the problem of allocating the physical resource among multiple VNs. In this paper we don't consider how to embed VNs onto the physical network, and we assume that the mapping results are already known. Furthermore, we ignore the admission control of VNs, assuming that a number of VNs are already allowed to use the physical network. With all these assumptions we can set up the model of our network virtualization environment.

We model the physical network managed by the only InP as a weighed undirected graph and denote it as $G = (N, L)$, where N is the set of physical nodes and L is the set of physical links, denoted by $L = \{1, 2, \dots, n\} (n \geq 2)$. The InP uses $C = \{c_1, c_2, \dots, c_n\}$ to represent the bandwidth capacity on every physical link. In order to obtain revenue from VNs, the InP also denotes the price vector as $p = \{p_1, p_2, \dots, p_n\}$. Another vector $\beta = \{\beta_1, \beta_2, \dots, \beta_n\}$ is introduced to state the importance of every physical link to the InP. We let $\forall 1 \leq i \leq n, 0 \leq \beta_i \leq 1$ and set the most important link's congestion price equal to 1.

The VNs are denoted by $V = \{1, 2, \dots, m\} (m \geq 2)$, which means that m VNs co-exist in the model. For the k -th VN, the bandwidth allocated on physical links is described with the vector $y^k = \{y_1^k, y_2^k, \dots, y_n^k\}$, which naturally follows the constraint of $\forall 1 \leq i \leq n, 0 \leq y_i^k \leq c_i$. Moreover, the InP sets the price-weighted vector $w^k = \{w_1^k, w_2^k, \dots, w_n^k\}$ for each VN, in order to realize differential pricing scheme. For example, InP may set small price-weighted factor to encourage bandwidth on low-propagation-delay links to be allocated to VNs with delay-sensitive traffic class. In order to further describe the workload of physical links, the path rate used by the k -th VN is denoted as $z^k = \{z_1^k, z_2^k, \dots, z_n^k\}$, with the constraint of $\forall 1 \leq i \leq n, 0 \leq z_i^k \leq y_i^k$. In order to represent both allocated bandwidth and path rate of each VN, we also denote the vector x^k as $x^k = \{y_1^k, y_2^k, \dots, y_n^k, z_1^k, z_2^k, \dots, z_n^k\}$. In Table I, the key notations used through the paper are summarized.

In our model, the payoff function of the k -th VN includes the following three parts:

- The Utility Function: we use $U^k(y^k, z^k)$ to represent the utility function of the k -th VN. As we can see, U^k depends on both the path rate and the bandwidth allocated. We follow the common practice in study of Internet that utility function is defined as a convex function.
- The Pricing Function: we use $P_k(y^k)$ to represent the pricing function of the k -th VN. The k -th VN should pay $p_j w_j^k$ as total price for the assigned bandwidth on the j -th link. So we present the pricing function as the following

TABLE I
LIST OF KEY NOTATION IN THE NETWORK VIRTUALIZATION MODEL.

Variable	Description
n	Index of physical links
m	Index of VNs
c_l	Bandwidth capacity of the physical link l
p_l	Price of the bandwidth on the physical link l
β_l	Congestion price of the physical link l
x^k	Bandwidth allocated and path rate of the k -th VN
y_l^k	Bandwidth assigned to the k -th VN on physical link l
z_l^k	Path rate of the k -th VN on physical link l
w_l^k	Price-weighted factor of the k -th VN on physical link l

equation:

$$P_k(y^k) = \sum_{j=1}^n p_j w_j^k y_j^k. \quad (1)$$

- The Congestion Function: we use $C_k(y^k, z^k)$ to represent the congestion function of the k -th VN, measuring the congestion cost according to the assigned bandwidth and actual path rate. If the total path rate of all VNs are less than the capacity of the physical link, no congestion cost will be charged. Otherwise, we denote the congestion price as the following equation:

$$C_k(y^k, z^k) = \sum_{j=1}^n \beta_j y_j^k \left[\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \right]^+ \left(-\frac{z_j^k}{c_j} \right). \quad (2)$$

In Equation (2), $\left[\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \right]^+ \left(-\frac{z_j^k}{c_j} \right)$ is defined as follows:

if $\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 < -\frac{z_j^k}{c_j}$, then $\left[\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \right]^+ \left(-\frac{z_j^k}{c_j} \right) = 0$, which implies that no congestion cost is charged; if $\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \geq -\frac{z_j^k}{c_j}$, then $\left[\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \right]^+ \left(-\frac{z_j^k}{c_j} \right) = \frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1$, which implies that the congestion cost is charged. We charge every VN assigned with the bandwidth on the congested link. In our model, when we consider the congestion function of the k -th VN, we assume that z_j^k is not the reason that causes the congestion, because we already pay for y_j^k when we calculate the pricing function.

With these three parts, we can obtain the payoff function Φ_k of the k -th VN, denoted by:

$$\Phi_k(y^k, z^k) = U_k(y^k, z^k) - P_k(y^k) - C_k(y^k, z^k). \quad (3)$$

B. The Resource Allocation Scheme

A non-cooperative game is one in which players are unable to make enforceable contracts outside of those specifically modeled in the game. In our model, individual VN does not communicate with others to modify its own strategy. Then the only InP play the role of enforcing VNs to modify their strategies in the form of pricing scheme. To analyze

the outcome of the game, we adopt the well-known Nash Equilibrium concept, in which every player will select a utility-maximizing strategy given the strategies of other players.

From the perspective of each VN, there are two ways to increase its own revenue: firstly, each VN is assigned with certain bandwidth to run its own path rate, the more the better; secondly, since congestion on certain links will absolutely decrease the revenue of all VNs on those links, VNs have to avoid congestion. Formally, we model our proposed scheme as the following non-cooperative game:

- Players: $\mathcal{K} = \{0, 1, 2, \dots, m\}$, where the 0-th player is the only InP and $k = 1, 2, \dots, m$ stands for the k -th VN.
- Action space: $\mathcal{P} = \mathcal{Q} \times \mathcal{P}_1 \times \mathcal{P}_2 \dots \times \mathcal{P}_m$, where $\mathcal{Q} = [0, \bar{Q}]$ and \mathcal{P}_k represents the action space of the k -th VN. We denote \bar{Q} as $\bar{Q} = \{c_1, c_2, \dots, c_n\}$ to represent the bandwidth capacity on every physical link, and $\mathcal{P}_k = \{y_1^k, y_2^k, \dots, y_n^k, z_1^k, z_2^k, \dots, z_n^k\}$ to represent the resource allocated to the k -th VN.
- Payoff function: we use $\Phi_k, \forall k = 1, 2, \dots, m$ to represent the final revenue of the k -th VN. They compete with each other for the shared physical resources, aiming to obtain more resources to increase their revenue as well as avoid congestion cost when unnecessary.

In our scheme, we try to solve the problem by finding an Nash Equilibrium, and we prove the existence of the Nash Equilibrium in Theorem 1.

Theorem 1: There exists a Nash Equilibrium for the problem stated in Equation (3).

Proof: First we rewrite the Equation (3) as follows:

$$\begin{aligned} \Phi_k(y^k, z^k) &= U_k(y^k, z^k) - P_k(y^k) - C_k(y^k, z^k) \\ &= U_k(y^k, z^k) - \sum_{j=1}^n p_j w_j^k y_j^k - \sum_{j=1}^n \beta_j y_j^k \left[\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \right]^+ \left(-\frac{z_j^k}{c_j} \right). \end{aligned} \quad (4)$$

In Equation (4), if there is no congestion, the congestion function will be equal to 0, which simplifies the problem. However, we will consider the most complicated situation, which means congestion occurs in every link and VN. Hence we can simplify the Equation (4) as follows:

$$\Phi_k(y^k, z^k) = U_k(y^k, z^k) - \sum_{j=1}^n p_j w_j^k y_j^k - \sum_{j=1}^n \beta_j y_j^k \left(\frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1 \right). \quad (5)$$

The action space of the only InP is a closed subset of R^n , and with the constraints of $\forall i, \forall k, 0 \leq z_i^k \leq y_i^k \leq c_i$, the action space of every VN is also a closed subset of R^n . It is obvious that all three parts of Φ_k are continuous, thus Φ_k is continuous in the action space. Furthermore, we mean to prove the convexity of the Φ_k in Equation (5). Because the utility function is a convex function, and the pricing function is a linear function, the convexity of Φ_k depends on the congestion function. When we consider the congestion function, we only focus on the path rate caused by the other VNs, and we denote

$\alpha_j^k = \frac{\sum_{i=1, i \neq k}^m z_j^i}{c_j} - 1$. We notice that α_j^k is a constant as far as the k -th VN is concerned, then it is obviously that $\sum_{j=1}^n \beta_j y_j^k \alpha_j^k$ is also a linear function. In a word, the payoff function Φ_k is a convex function, which means it is also quasi-convex. According to the Ref. [7], the Nash Equilibrium exists. ■

Then we develop the Algorithm 1 to implement the scheme mentioned above and reach the Nash Equilibrium, aided by the concept of iteration.

Algorithm 1 Bandwidth allocation scheme based on the non-cooperative game model

*State*₀ represents the initial bandwidth allocation
while *State*_{*i*} ≠ *State*_{*i*−1} (The Nash Equilibrium isn't arrived) **do**
 Compute the *i*+1-th iteration
 for Every VN in the model **do**
 Find the best response by solving the problem stated in Equation (6)
 end for
 The result of the *i*+1-th iteration is denoted as *State*_{*i*+1}
end while

In our iterative algorithm, we use the vector *State*_{*i*} = {*x*_{*i*}¹, *x*_{*i*}², ..., *x*_{*i*}^{*m*}} to represent the bandwidth allocation of all VNs after the *i*-th iteration. Initially, we assign resources to all VNs, denotes by *State*₀. In the *i*-th iteration, we update the response of each VN one by one, based on the existing bandwidth allocation situation. For example, we suppose the bandwidth allocation of all the other VNs are fixed, and try to find the best response of the k -th VN, denoted by *x*_{*i*+1}^{*k*}. The following equation describes our work in detail:

$$x_{i+1}^k = \max_{x^k \in \mathcal{P}_k} (x^k, x_i^{-k}), \quad (6)$$

where *x*_{*i*}^{−*k*} = {*x*_{*i*}¹, *x*_{*i*}², *x*_{*i*}^{*k*−1}, *x*_{*i*}^{*k*+1}, ..., *x*_{*i*}^{*m*}}. After we figure out the best response of all VNs, we finish the *i*-th iteration and the bandwidth allocation of all VNs can be updated to *State*_{*i*+1}. Finally, we can get an convergent state, which stands for the final results of our scheme. Now, we have proposed a bandwidth allocation scheme based on the non-cooperative game model, and presented an iterative algorithm to implement our scheme.

IV. THE PERFORMANCE OF THE RESOURCE ALLOCATION SCHEME

Based on the algorithm mentioned above, we study our experiments in the MATLAB environment and focus on the convergence of the Nash Equilibrium, which matters the convergence of our scheme. In our experiments, we use the topology in Figure 1, in which *Link*1 has low bandwidth as well as small propagation-delay, and *Link*2 has high bandwidth as well as large propagation-delay. Though the topology we used here is simple, it can distinguish the preference of the following two traffic classes: delay-sensitive traffic and

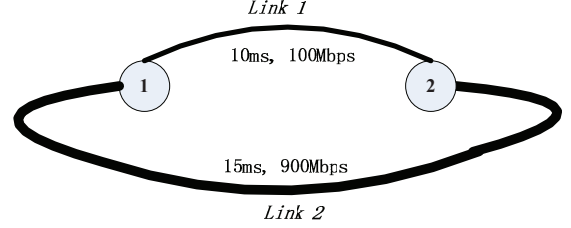


Fig. 1. Topology with two links and two nodes.

throughput-sensitive traffic. Furthermore, the authors in [8] also used it to evaluate the performance of the architecture proposed. Then we employ two VNs with different objectives on our topology. Virtual Network 1 (VN1) is used to run delay-sensitive traffic, and Virtual Network 2 (VN2) is used to run throughput-sensitive traffic. It is clearly that VN1 prefers *Link*1 and VN2 prefers *Link*2.

Following Ref. [15], VN1 tries to minimize average end-to-end delay:

$$\sum_{j=1}^n z_j^k (l_j + l_0 \exp(\frac{z_j^k}{y_j^k})), \quad (7)$$

where *l*_{*j*} is the propagation delay on link *l*, *l*₀ = 1ms is the fixed delay on every link, and *l*₀exp(*z*_{*j*}^{*k*}/*y*_{*j*}^{*k*}) stands for the queueing delay as a function of the link utilization. Following Ref. [15], VN2 tries to minimize:

$$\sum_{j=1}^n \log(z_j^k) - q \sum_{j=1}^n \exp(\frac{z_j^k}{y_j^k}), \quad (8)$$

where VN2 is maximizing its utility as a logarithmic function of its path rate, and *q* = 0.5 keeps a balance between maximal utility and minimal congestion. Moreover, we define the utility functions of these two VNs as follows:

$$U_1(y^1, z^1) = - \sum_{j=1}^2 z_j^1 (l_j + l_0 \exp(\frac{z_j^1}{y_j^1})), \quad (9)$$

$$U_2(y^2, z^2) = \sum_{j=1}^2 \log(z_j^2) - q \sum_{j=1}^2 \exp(\frac{z_j^2}{y_j^2}). \quad (10)$$

*U*₁(*y*¹, *z*¹) and *U*₂(*y*², *z*²) are both convex functions, so according to the *Theorem 1* the Nash Equilibrium exists.

We set aggregate bandwidth requirement as 600Mbps for both VNs, but the capacity of the physical network is 1000Mbps. Taking into account of the fairness, we assume that each VN will be allocated 500Mbps in the end. Initially, we set β₁ = 1 and β₂ = 0.8, which implies that *Link*1 is more important. Furthermore, we set the price-weighted factor for VN1 and VN2 as follows: *w*¹ = {1, 10} and *w*² = {10, 1}. Then we used our algorithm to demonstrate the convergence of the Nash Equilibrium.

Figure 2 illustrates the situation when congestion happens on both links. The VN1 is initially assigned 130Mbps on

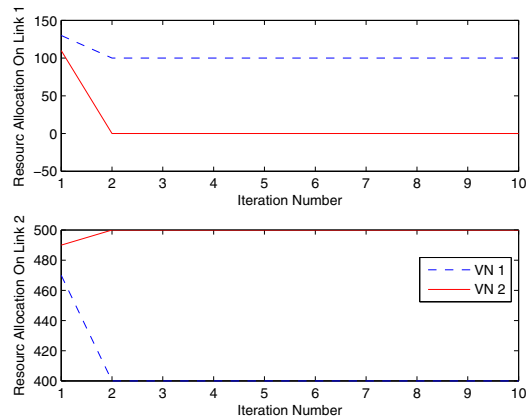


Fig. 2. Convergence of the Nash Equilibrium when congestion happens on both links.

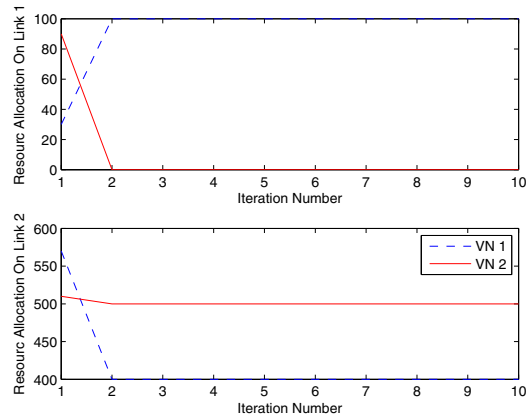


Fig. 3. Convergence of the Nash Equilibrium when congestion happens only on the second link.

Link1 and 470Mbps on *Link2*, while the VN2 is initially assigned 110Mbps on *Link1* and 490Mbps on *Link2*. This setup leads to the congestion on both links. As seen in Figure 2, after one iteration VN1 is allocated with all the bandwidth on *Link1*, and also 400Mbps on *Link2*. This is due to the large difference between the price-weight factor and the delay properties of the two links. Meanwhile, VN2 is assigned 500Mbps on *Link2*. The convergence of the Nash Equilibrium is demonstrated and our algorithm can quickly find the Nash Equilibrium Point. Similar behavior is observed in Figure 3. We set the bandwidth assigned on VN1 to be 30Mbps, and VN2 is allocated with 90Mbps as the initial state. Again our algorithm easily find the Nash Equilibrium Point, and confirms the fact that the existence of the Nash Equilibrium is independent of initial values.

V. CONCLUSION

In this paper, we introduce the economic incentives to analyze the complicated interaction between InPs and SPs, aiming to encourage the efficient behavior among multiple

VNs. As far as we know, we are the first to develop a case for the application of Game Theory in network virtualization environment. In our model, an InP and multiple SPs achieve efficient bandwidth allocation scheme in the framework of the non-cooperative game. We propose an algorithm to solve the scheme above, and demonstrate the convergence and the effectiveness of our scheme with the experiments based on Figure 1. In future, we will focus on the model with multiple InPs and SPs, in which SPs can obtain physical resources from multiple InPs, leading to the competition among multiple InPs. Of course, the model mentioned above requires more complicated bandwidth allocation scheme.

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