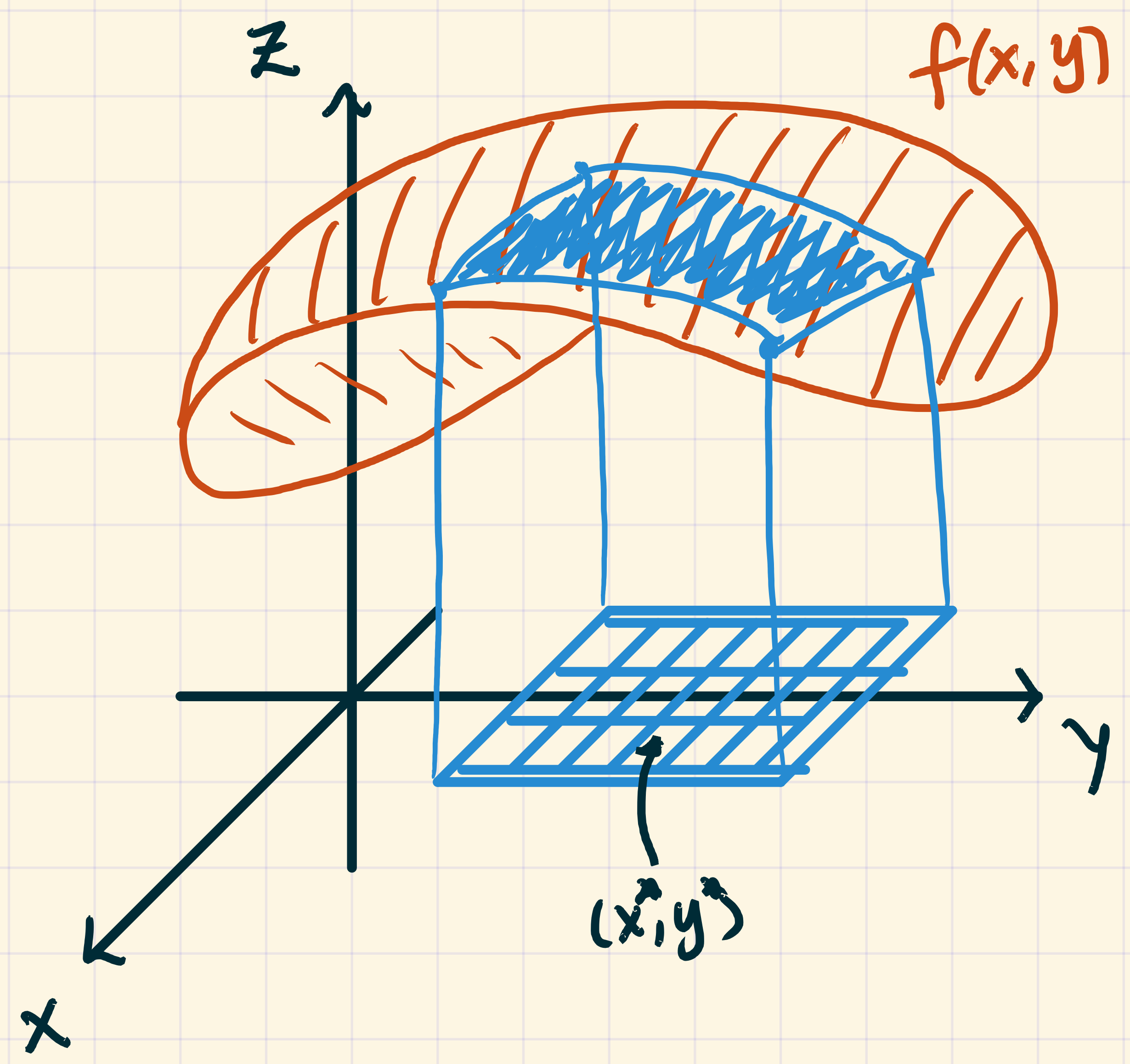
DOUBLE INTEGRALS OVER RECTANGLES

MATH 1014

(12.1)



$\iint_R f(x,y)\,dA$ for

$R = [a,b] \times [c,d]$

* Approximate with Riemann sums over rectangles.

$$\iint_R f(x,y)\,dA \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x^*, y^*)\,\Delta A_{ij}$$

DEFINITION

The double integral

$$\iint_R f(x,y)\,dA = \lim_{\substack{\max \Delta x_i,\\ \Delta y_j \to 0}} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x^*, y^*)\,\Delta A_{ij},$$

if it exists

* Note: $dA = dx\,dy = dy\,dx$

* We can express $\iint_R f(x,y)\,dA$ as

$$\int_a^b \left(\int_c^d f(x,y)\,dy \right) dx.$$

THEOREM (FUBINI'S THEOREM)

If f is continuous on the rectangle $[a,b]\times[c,d]$ then

$$\iint_R f(x,y)\,dA = \int_a^b\left(\int_c^d f(x,y)\,dy\right)dx = \int_c^d\left(\int_a^b f(x,y)\,dx\right)dy.$$

PROPERTIES

1) $$\iint_R [f(x,y)+g(x,y)]\,dA = \iint_R f(x,y)\,dA + \iint_R g(x,y)\,dA$$

2) $$\iint_R c\cdot f(x,y)\,dA = c\cdot\iint_R f(x,y)\,dA$$ (where c is constant)

3) If $f(x,y)\geq g(x,y)$ for all $(x,y)\in R$ then

$$\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$$