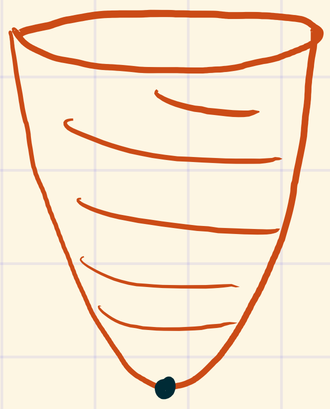
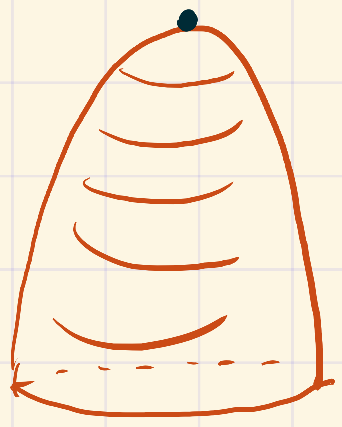
MAXIMA AND MINIMA

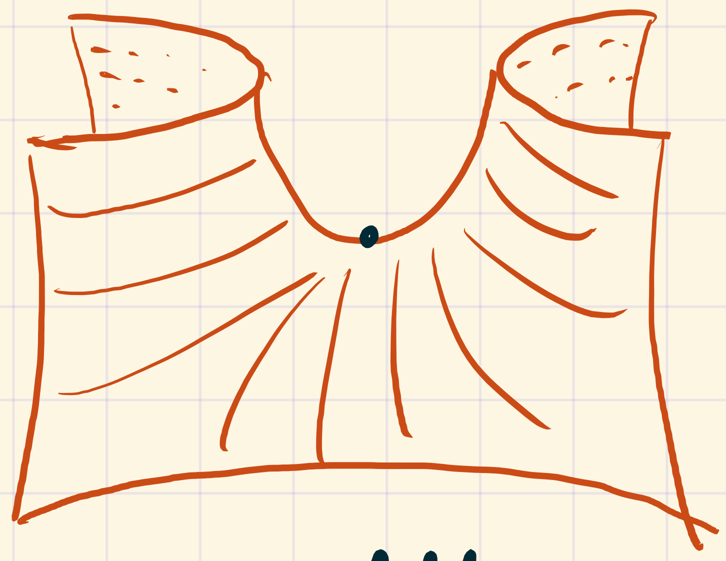
MATH 1014
(11.7)



local min



local max



saddle point

DEFINITION

Let $f(x,y)$ be a function of 2 variables

(1) (a,b) is a <u>local min</u> if for any (x,y) close to (a,b), we have $f(x,y) \geq f(a,b)$.

(2) (a,b) is a <u>local max</u> if for any (x,y) close to (a,b), we have $f(x,y) \leq f(a,b)$.

THEOREM

If f has a local max or a local min at (a,b), and f_x & f_y exist at (a,b), then

$$f_x(a,b) = f_y(a,b) = 0.$$

In other words, $(\nabla f)_{(a,b)} = 0$.

* If $f_x = f_y = 0$ at some point (a,b), then (a,b) is called a <u>critical point</u> of f

* THE HESSIAN

Let $f(x,y)$ have a critical point at (a,b).

DEFINITION

The Hessian of $f(x,y)$ is :

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

$$\det(H) = (f_{xx})(f_{yy}) - (f_{xy})(f_{yx}) = f_{xx} f_{yy} - f_{xy}^2 .$$

THEOREM (SECOND DERIVATIVES TEST)

Suppose that (a,b) is a critical point of f, such that the second partials are continuous around (a,b).

Let $D = \det(H)_{(a,b)} = f_{xx}(a,b) f_{yy}(a,b) - f_{xy}(a,b)^2$.

(i) If $D>0$ & $f_{xx}(a,b) > 0$ then local min.

(ii) If $D>0$ & $f_{xx}(a,b) < 0$ then local max.

(iii) If $D<0$ then saddle point

(iv) If $D=0$ then inconclusive

* A point (a,b) is an absolute maximum if $f(x,y) \leq f(a,b)$ for all (x,y) in the domain.

* A point (a,b) is an absolute minimum if $f(x,y) \geq f(a,b)$ for all (x,y) in the domain.

THEOREM (EXTREME VALUE THEOREM)

Let f be continuous on a closed & bounded set D in $\mathbb{R}^2$. Then f attains an absolute maximum value and an absolute minimum value on D.