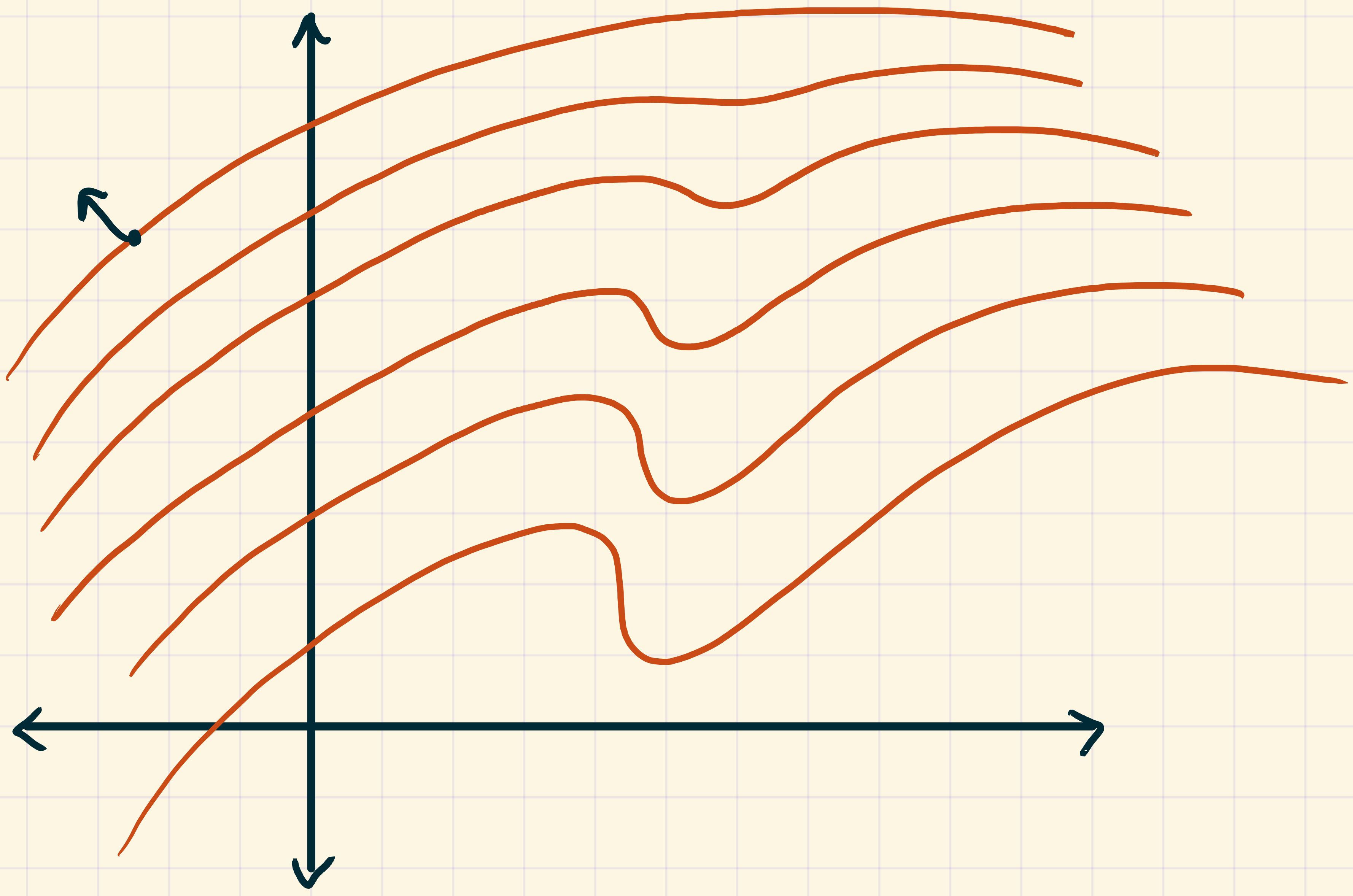
LAGRANGE MULTIPLIERS

MATH 1014

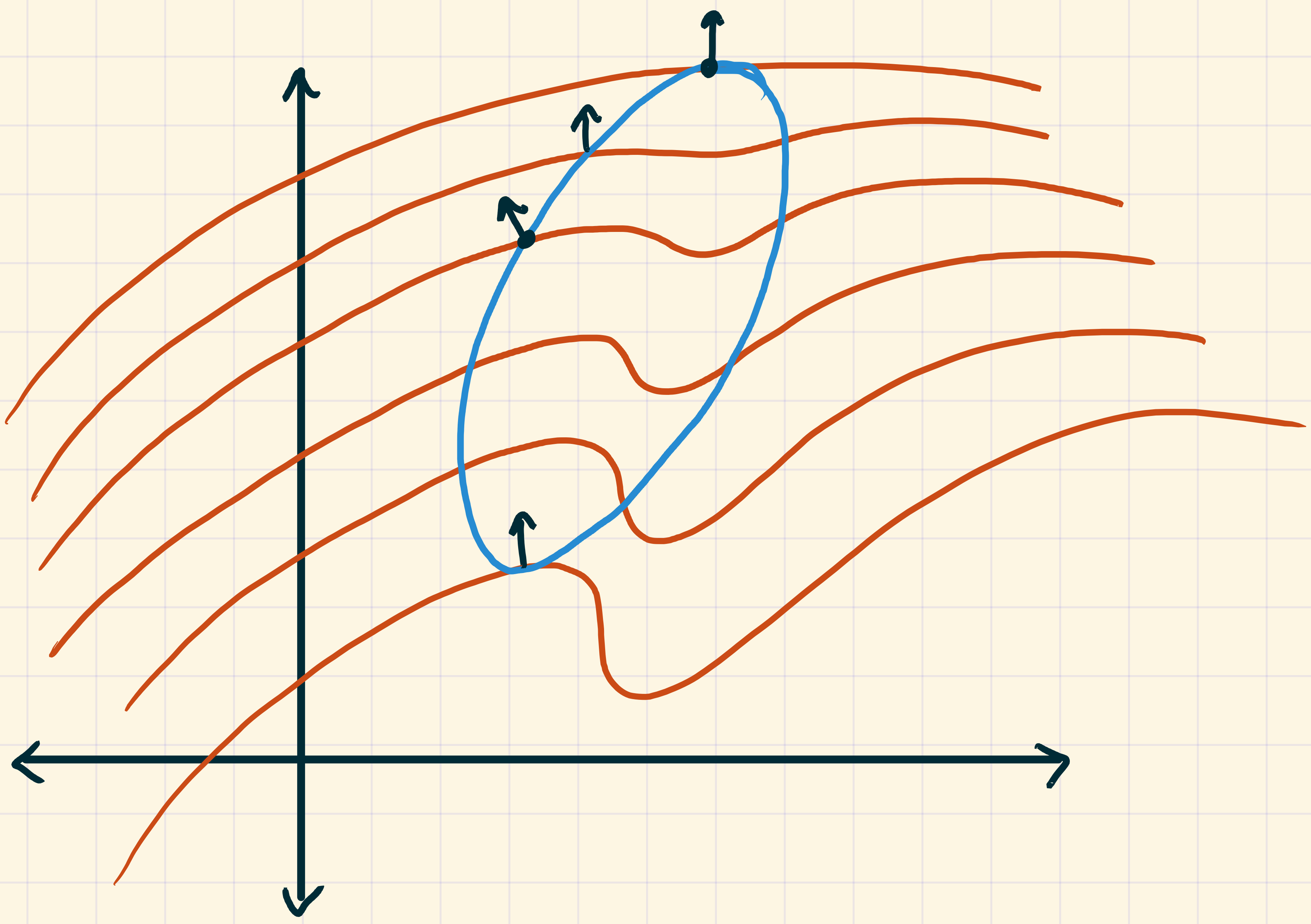
(11.8)

Level curves of $f(x,y)$



∇f perpendicular to level curve.

$f(x,y)$



$g(x,y)=c$

METHOD OF LAGRANGE MULTIPLIERS

To find max/min values of $f(x,y)$ subject to the constraint that $g(x,y)=c$ [assuming they exist & $\nabla g \neq 0$ on the curve $g(x,y)=c$], we solve

$$\boxed{\nabla f = \lambda \nabla g \quad \& \quad g(x,y)=c}$$

$$f_x(x,y)=\lambda g_x(x,y), \quad f_y(x,y)=\lambda g_y(x,y), \quad g(x,y)=c$$

METHOD OF LAGRANGE MULTIPLIERS (MULTIVARIABLE)

To find max/min values of $f(\vec{x})$ subject to the constraint that $g(\vec{x}) = c$ [assuming they exist & $\nabla g \neq 0$ on $g(\vec{x}) = c$], we solve

$$\boxed{\nabla f = \lambda \nabla g \quad \& \quad g(\vec{x}) = c}$$

$$\left.\frac{\partial f}{\partial x_i} = \lambda \frac{\partial g}{\partial x_i} \quad \& \quad g(\vec{x}) = c \right\} \text{ n+1 eqns}$$

n+1 unknowns

LAGRANGE MULTIPLIERS (MORE CONSTRAINTS)

To maximise/minimise $f(\vec{x})$ subject to $g(\vec{x}) = c$ & $h(\vec{x}) = d$, we solve

$$\nabla f = \lambda(\nabla g) + \mu(\nabla h) \quad \& \quad g(\vec{x}) = c, \; h(\vec{x}) = d.$$