# Analysis I, oral exam topics

1. The real and extended real numbers
2. Bounded and unbounded sets. Sup, inf, max, min of sets, and their various characterizations
3. The triangle inequalities. Means: arithmetic, geometric, harmonic means, the root mean square. The binomial theorem
4. Real functions, domain, range, invertibility, inverse, composition
5. Sequences. Subsequences. Monotonicity, boundedness, convergence, divergence, and the relationship among these concepts. Examples
6. The definition of sequence limit in the extended sense
7. Arithmetic operations and limits, the extension of the arithmetic operations to the extended reals
8. Ordering and limits. Limits of monotone sequences. The sandwich theorem
9. Cauchy sequences and convergence
10. The Bolzano–Weierstrass theorem
11. Infinite series. Partial sums. Convergence and divergence
12. Geometric series, harmonic series and their convergence/divergence. Telescopic summation
13. Tests for convergence: a necessary condition for convergence. Absolute convergence. Alternating series. The ratio test. The root test.
14. Cauchy product of series, sum of the Cauchy product
15. Power series, set of convergence, radius of convergence. Examples
16. Analytic functions: the definition of exp, cos, sin, cosh, sinh together with the radius of convergence
17. Various definitions of the number $e$ (sequence limits/infinite series)
18. Inverse trigonometric and inverse hyperbolic functions (including arctan and artanh) – domains, ranges, graphs, monotonicity
19. The proof of $\exp(x + y) = \exp(x)\exp(y)$. The proof of Euler's formula
20. The relation between cos, sin and the complex exponential function. The trigonometric and hyperbolic Pythagorean theorems with proofs. The proofs of the multiple angle formulae $\cos(2x)$ and $\sin(2x)$
21. Neighborhoods of real and extended real numbers. Accumulation points and isolated points of sets
22. The definition of the limit of a real function. 9 special cases (with quantifiers and examples)
23. Sequential limits
24. Algebraic operations (sum, difference, product, ratio) and limits of functions
25. One-sided limits
26. Limits of monotone functions
27. Various equivalent definitions of continuity
28. Classification of discontinuities
29. The IVP and Bolzano's theorem
30. The existence of extremal values, Weierstrass' theorem
31. Uniform continuity, Heine's theorem