First and Second Order Circuits

Experiment 4

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February 29th 2016

Section 01: Monday 9:00am – 11:00 am

Partner: None (TA Armando request)

Introduction

The overall purpose of this lab is to allow students to familiarize themselves with First-Order and Second-Order circuits. In this experiment, I compared theoretical values against my experimental values that I obtained on certain values, such as the time constant in terms of the voltage value. The first order circuits, the theoretical aspect was such that, I used a differential equation with an initial condition based on the form $dy(t)\ dt + \tau^{-1}\ y(t) = K$. Once solving $f(t) = Ae^{-t/\tau} + B$, where $t = \tau$ will be 36.8%, the time constant = RC and we use Kirchhoff's voltage law for such that Vs = Ri(t) + 1/c di(t)/dt. Thus, Vc(t) = 1- 2e^-1 = 0.26 which should give us our time constant Tao. Through this, we can determine the experimental value Tao and compare the results with our theoretical value.

For part 2, we are faced with a second-ordered Circuit. Here, our second order circuits have more complex behavior; where the circuit has two energy storage elements. Thus, there is a damping behavior associated with the response which depends on wo and alpha. Through the circuit, we can use Kirchhoff's Voltage Law to determine the voltage across the resistor. Here, we first change the circuit to phasor form, thus giving us 0 = ~VL + ~Vc + ~Vr. ~Vr then equals to ~I (R + j2πfl + (1/j2πfc), simplifying to ~Vr = (~V1*R)/ (R^2+(2πfl – 1/(2πfc) ^2) ^ (1/2).

Another learning experience that was not added to this lab was RL circuits. These circuits are simple circuits containing dc sources, resistances, and one energy storage element, mostly inductances. To find the solution to RL circuits there is an approach taken. We use Kirchhoff's current and voltage law and write the equation. If the equation has integrals or differentiation, we produce a pure differential equation. Substitute the solution K1 + K2e^st to solve the equation and solve k1 and s. Then we rewrite the solution. However, much like RC circuits, RL circuits generally consist of two solutions, the particular solution that satisfies the equation, tao dx(t)/dt + x(t) = f(t) and the homogeneous equation that sets the function to 0. For this lab, we aren't dealt with a RL circuit however, this would be the approach for a theoretical result.

In the last part of the lab, we will measure the voltage across the resistor and compare the results with the theoretical value of the amplitude ratio of the voltage.

Part 1: First Order RC-Circuit

Create the circuit displayed in **Figure 1** on the circuit board using the source voltage at 1 Volts amplitude, frequency of 500 Hz, a resistor of 2 kilo ohms, and capacitor of 0.1 microfarads.

Note: The resistor that I used was at 1.975 kilo ohms, therefore our result may vary.

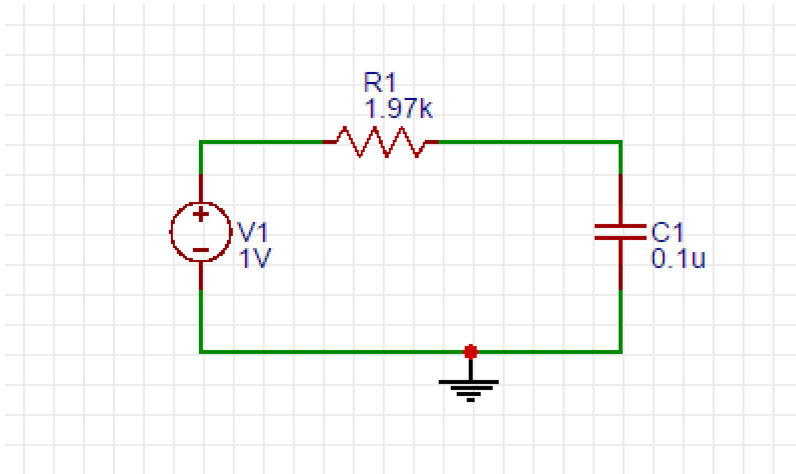


Figure 1

RC circuit Schematic for Lab 4, Part 1

Using the scope, I shifted the image on the oscilloscope so that the period began with the input voltage transitioning from the minimum to the maximum value. Using the measurement tools on the scope, I placed the vertical cursor (a) at the time where the voltage begins increasing and cursor b to find the time where the voltage difference is ΔV is at 63.2% from its maximum value. One can observe from **Figure 2.**

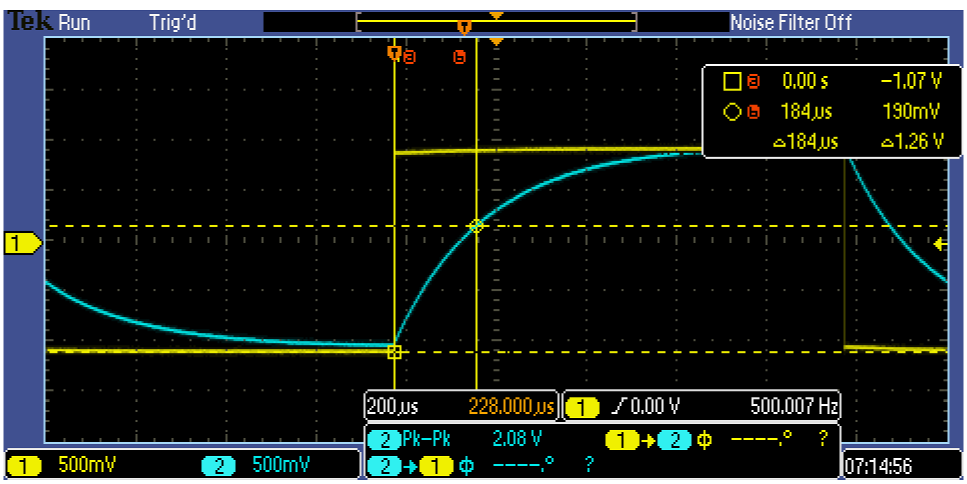


Figure 2

The Voltage difference between voltage in and capacitor at τ=1

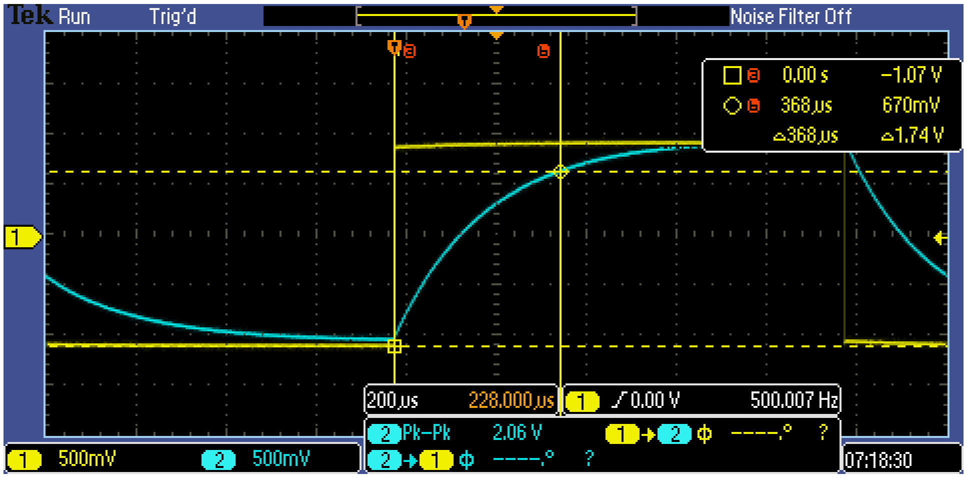


Figure 3

The Voltage difference between voltage in and capacitor at τ=2

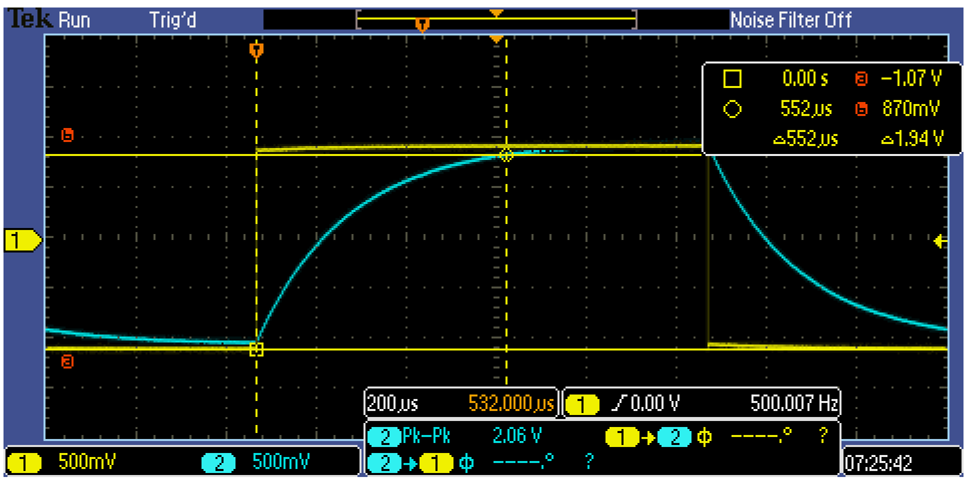


Figure 4

The Voltage difference between voltage in and capacitor at τ=3

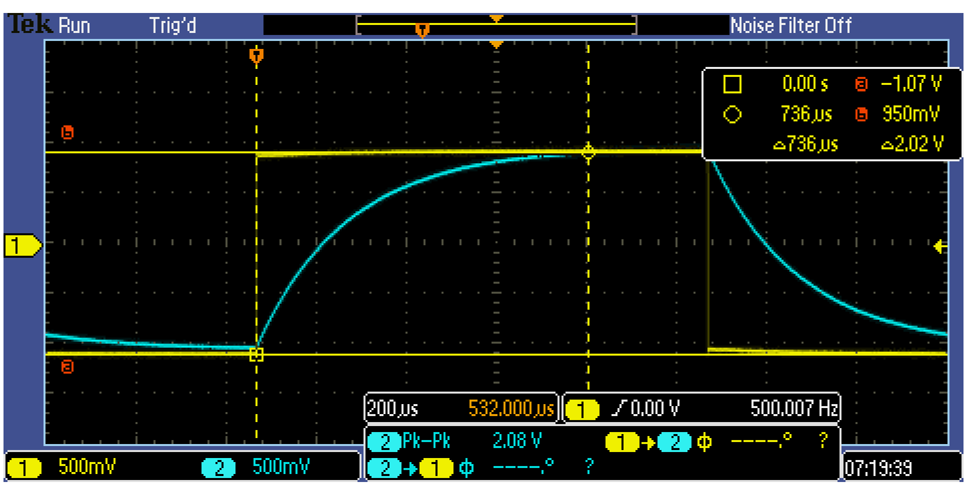


Figure 5

The Voltage difference between voltage in and capacitor at τ=4

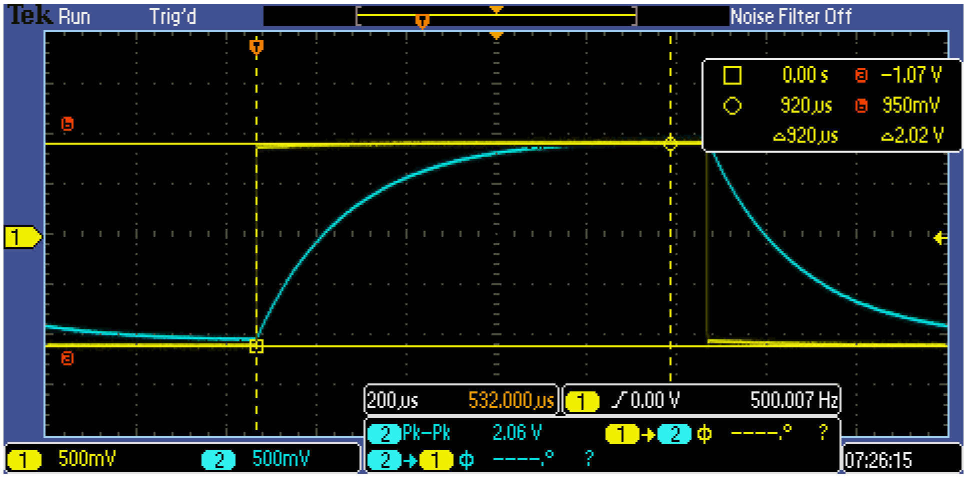


Figure 6

The voltage difference between voltage in and capacitor at τ=5

The following table has the record of τ along with the difference in time from cursor "a" to "b" as the time constant τ, and the difference in voltage:

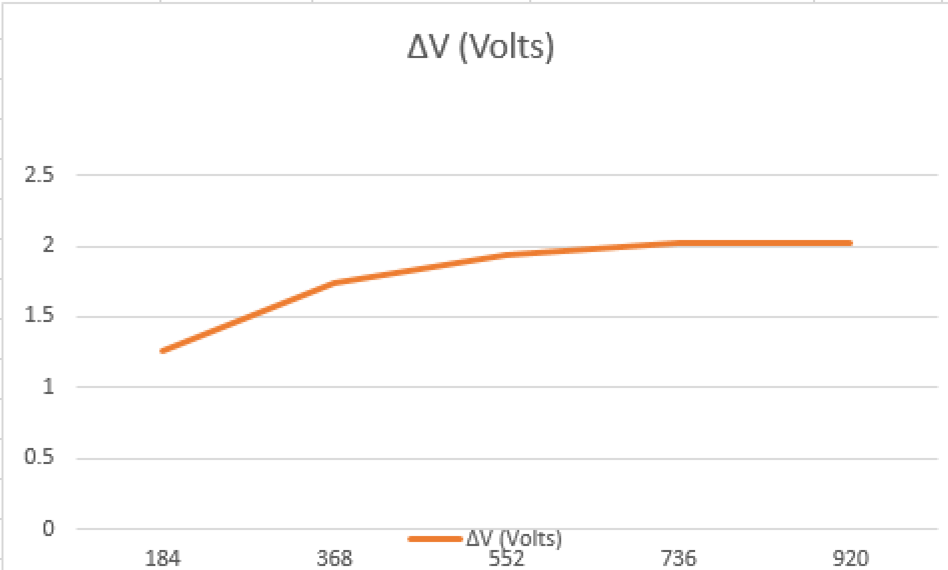
Δt (µs)	ΔV (Volts)
184	1.26
368	1.74
552	1.94
736	2.02
920	2.02

Figure 7

The table for the differences corresponding to τ=0 to 5

As one can see here, as τ increases over time, the voltage rate decreases and becomes constant. Therefore, over time we can see that the difference of voltage over time becomes constant.

Δt (μs)	ΔV (Volts)	difference τ (μs)	difference V(Volts)
184	1.26	16	0
368	1.74	32	0.02
552	1.94	48	0.04
736	2.02	64	0.06
920	2.02	80	0.04



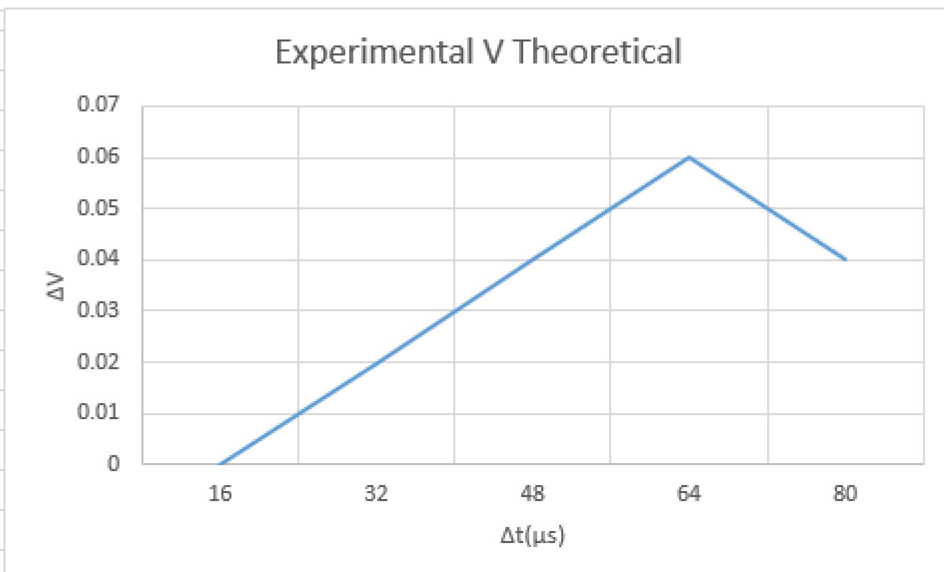


Figure 8

Plotted points between our experimental and the theoretical difference

In **Figure 8**, one may see the difference between my experimental and theoretical values. Here, the difference was slightly off; perhaps due to adjusting the cursors and "eyeballing" when the cursor is in line with the slope of the image. As well, maybe due to the resistor value being 1.975 kilo ohms instead of 2 kilo-ohms, my values became skewed.

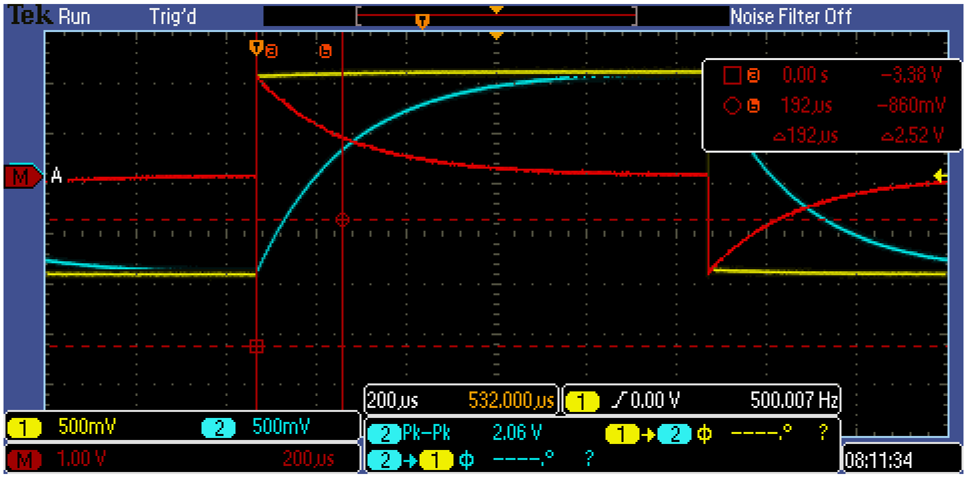


Figure 9

Math function difference between voltage in and capacitor

The next part of the lab was to analyze the difference of the voltage and the capacitor to determine why this waveform is the waveform of the resistor. Using Kirchhoff's law on **Figure 1**, we get the equation, Vs = Ri(t) + (1/c) di(t)/dt, and we obtain that Vs - RC dVc/dt – Vc = 0, in which R = Vs – Vc / dVc/dt. When the rate of capacitor voltage increases, the resistor decreases; and vice versa. The waveform seems to display that and such of a resistor on **Figure 1**. Lastly, we are display the circuit on **Figure 10.**

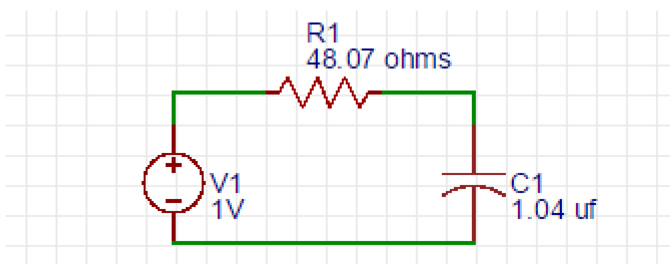


Figure 10

Circuit Diagram of Second RC circuit

8. RC = τ = 0.5 ms. Therefore, R= 0.5 ms/ (0.000104 micro farads) = 48.07 ohms, theoretically. We see that we obtained that value and build circuit from **Figure 10.** Because of choosing to change for the resistor and capacitor, we would not have to make changes on the input signals to the circuit.

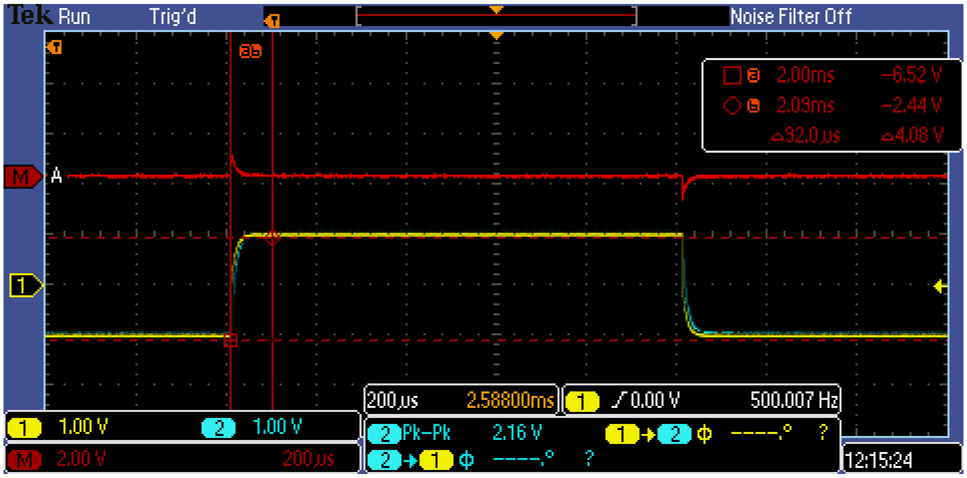


Figure 11

The voltage difference between voltage in and capacitor at τ=0.5

Here, I've adjusted my circuit as the described above; where I have a 0.000104 micro farad in series with a 47.1-ohm resistor. Thus as seen in **Figure 11**, one may see that the difference of voltage is 4V with a 2V input and where Tao = 0.5.

Part 2: Second Order RLC-Circuit

Create a circuit on the bread board using a voltage source of 2V amplitude, 10 mH inductor, 0.1 microfarad capacitor, and 1 kilo ohm resistor; but in my case 995 ohms' resistor. The circuit should resemble **Figure 12**.

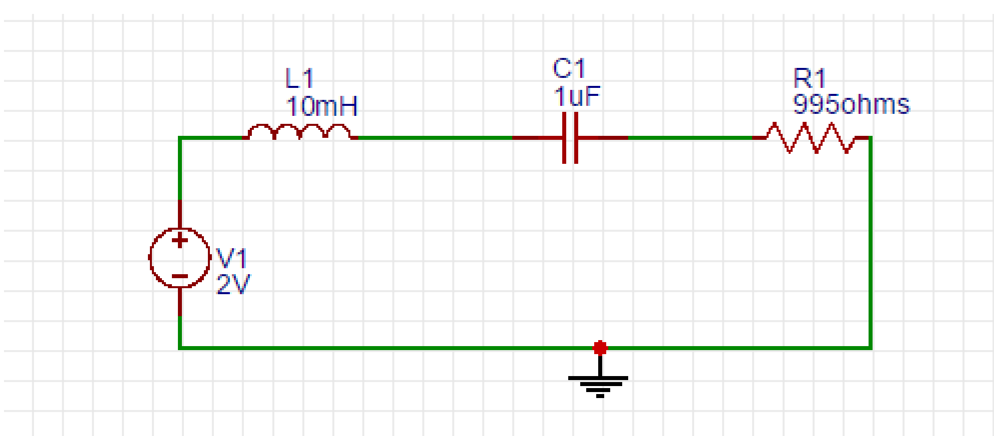


Figure 12

RLC circuit schematic for Lab 4, part 2

Here from **Figure 10**, we can change the inputs to be represented as phasors and take Kirchhoff's Voltage law. Then, we obtain by solving ~Vr = ~V1 * R/ (R^2 + (2πfl – 1/2πfc) ^2) ^ (1/2). When the frequency is in resonance, we get that ~Vr = ~V1, where ~V1 is voltage in, at 2cos(2πft) = 2V. Therefore, the voltage at this frequency should be the resonance. Similar, when we solve for frequency of the formula ~Vr = ~V1 * R/ (R^2 + (2πfl – 1/2πfc) ^2) ^ (1/2), we obtain frequency to equal 1.6 KHz

After displaying the circuit board as above, I used the oscilloscope channel 2 wire to measure the resistor voltage and measured the peak to peak voltage across the resistor for the following frequencies:

f = 80, 90, 100, 200, . . ., 1k, 2k, . . ., 10k, 20k, 30k, 40k Hz.

From **Figure 13**, one can observe that the values measured from the resistor's voltage that we can compare the experimental voltages.

Frequency	Peak to Peak (Volts)
80	0.04
90	0.048
100	2.24
200	3.12
300	3.44
400	3.6
500	3.68
600	3.76
700	3.76
800	3.76
900	3.8
1000	3.84
2000	3.84
3000	3.76
4000	3.72
5000	3.68
6000	3.6
7000	3.52
8000	3.4
9000	3.36
10000	3.28
20000	2.52
30000	1.96
40000	1.56

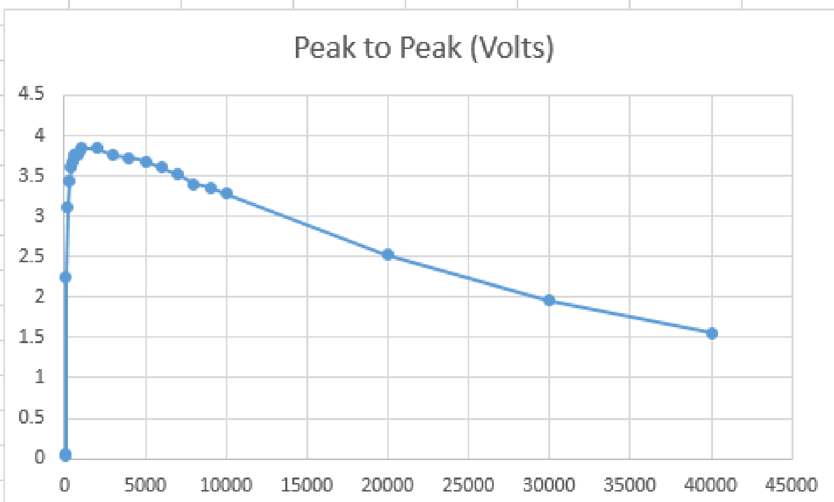


Figure 13

The plotted chart of Frequency Vs Peak to Peak Voltage

Here we can see that the maximum peak was when the frequency was at 1kHz-2kHz. Comparing it to my theoretical value, 1.6 KHz frequency, I was in range with the calculations, where the peak is that range. Thus, we are able to conclude that by taking the Kirchhoff's Voltage Law of a complex circuit, we are able to obtain the frequency values.

Next, I take the values I obtained from Figure 13, and plot 20log(Vr/Vin) vs frequency (Vr = resistor voltage peak to peak, Vin = input voltage peak to peak).

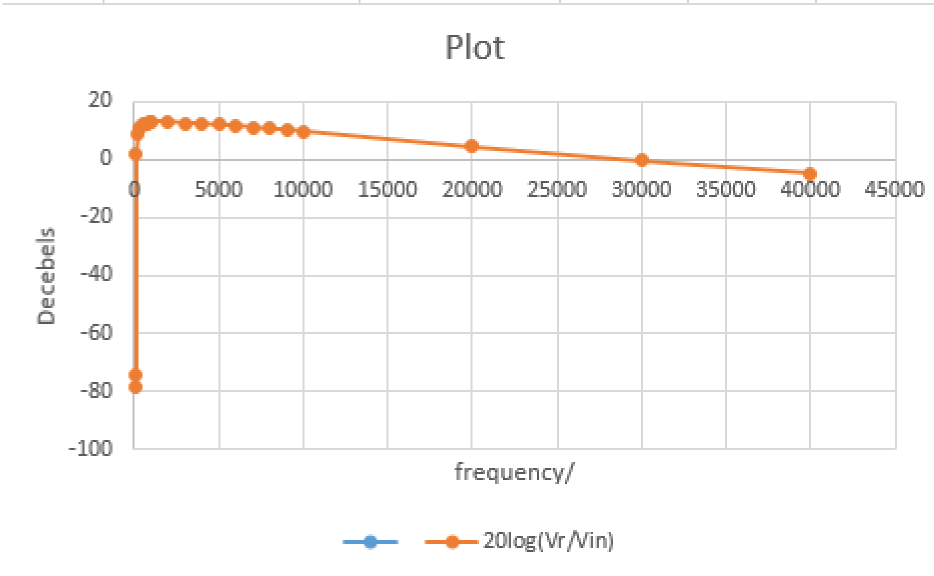


Figure 14

The plot of measuring resistor voltage peak to peak and taking 20 log (Vr/ Vin)

The theoretical amplitude ratio for the voltages above are given by the equation:

$$|Z| = R/\cos^{\circ}$$

Based upon the theoretical calculations, we can see that ~Vl is equal to ~I * j2pifL; here we can ideal suggest that the impedance of this inductor will cause a phase shift from the input voltage which would display a graph such like the resistance however delayed. Similarly, the impedance on the capacitor will have the delay, we will obtain that the capacitor will be such that Zc = 1/ jwc = -j/wc.

The way I would reconnect the circuit to verify the answer to my theoretical values of the step 7 would be have resistor, capacitor, and then inductor in series so that the resonant frequency of the circuit will cause the capacitor and inducer imaginary impedances to cancel out, see **Figure 15**.

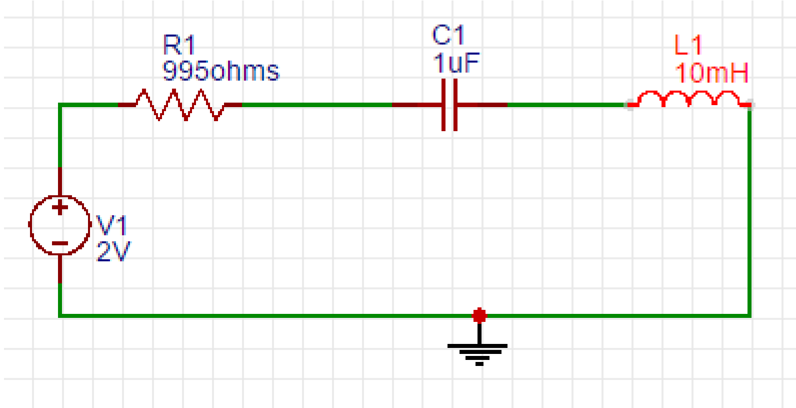


Figure 15

Rearrangement such that we will be able to plot the voltages across inductor and capacitor.

The plot of the capacitor would be that of a high pass filter in which that the cut off frequency point only passes signals above the selected cut off point eliminating low frequency signals. Calculating the cut-off would be from the equation fb = 1/ 2piRC. Next, the plot of the voltage on the inductor vs frequency would be a low pass filter allowing the cut off being the inverse of the capacitor. With the changes I made, it would negate the filter and allow the imaginary impedances to cancel out.

Conclusion

All in all, this lab was very tedious and demanded a lot of theoretical calculations. However, it did help us students familiarize ourselves with the calculations we were obtaining from first and second order circuits.

The first circuit was pretty simple, allowing us to see that with Kirchhoff's Voltage law, we can obtain an equation for the first order circuit in order to obtain values such as Tao, where the measurements of delta V were 64.2% of its maximum value. We also saw that taking the difference between channel 1 and 2 displays the resistor waveform, and we can verify using KVL as well.

As for the second circuit, a bit more of a thought process since we are taking values such that are complex. However, it was possible to obtain values that mirrored the theoretical results. Therefore, allowing us to see that these equations do satisfy the lab 4 experiments.

Revision

#2, 12, 14, 20 ,23, 24