With this Maple sheet we obtain the systems of nonlinear algebraic equations -and their jacobians- that determine the forced response curves and modal backbones of a mechanical system, with parameter ω, by applying the harmonic balance method.

Single degree of freedom case:

Equation of the one degree of freedom oscillator:

$$eq := M\cdot \frac{d^2}{d\,t^2}\,x(t) + C\cdot\frac{d}{d\,t}\,x(t) + K\cdot x(t) + \beta\cdot x(t)^3 - F\cdot\sin(\omega\cdot t)$$

$$M\left(\frac{d^2}{dt^2}\,x(t)\right) + C\left(\frac{d}{dt}\,x(t)\right) + K\,x(t) + \beta\,x(t)^3 - F\sin(\omega\,t) \qquad (1)$$

We substitute a solution with one single fundamental harmonic, resulting in the residuals of the equations:

$$subs(x(t) = A\cdot\sin(\omega\cdot t) + B\cdot\cos(\omega\cdot t), eq)$$

$$M\left(\frac{\partial^2}{\partial t^2}\,(A\sin(\omega\,t) + B\cos(\omega\,t))\right) + C\left(\frac{\partial}{\partial t}\,(A\sin(\omega\,t) + B\cos(\omega\,t))\right) + K\,(A\sin(\omega\,t) + B\cos(\omega\,t)) + \beta\,(A\sin(\omega\,t) + B\cos(\omega\,t))^3 - F\sin(\omega\,t) \qquad (2)$$

$$eq := simplify(\%)$$

$$-M\,A\sin(\omega\,t)\,\omega^2 - M\,B\cos(\omega\,t)\,\omega^2 + C\,A\cos(\omega\,t)\,\omega - C\,B\sin(\omega\,t)\,\omega + K\,A\sin(\omega\,t) + K\,B\cos(\omega\,t) + \beta\,A^3\sin(\omega\,t) - \beta\,A^3\sin(\omega\,t)\cos(\omega\,t)^2 + 3\,\beta\,A^2\,B\cos(\omega\,t) - 3\,\beta\,A^2\,B\cos(\omega\,t)^3 + 3\,\beta\,A\sin(\omega\,t)\,B^2\cos(\omega\,t)^2 + \beta\,B^3\cos(\omega\,t)^3 - F\sin(\omega\,t) \qquad (3)$$

We apply the Galerkin method to ortogonalise the residual and to obtain the algebraic equations for the coefficients of the solution:

Their derivatives with respect to the residuals will determine the jacobian matrix of the algebraic system.

$$a := \int_0^{\frac{2\cdot \mathrm{Pi}}{\omega}} eq\cdot\sin(\omega\cdot t)\,dt$$

$$\frac{1}{4}\,\frac{\pi\left(3\,\beta\,A\,B^2 - 4\,F + 4\,K\,A - 4\,M\,A\,\omega^2 - 4\,C\,B\,\omega + 3\,\beta\,A^3\right)}{\omega} \qquad (4)$$

$$\frac{\partial}{\partial A}\, a$$

$$\frac{1}{4}\,\frac{\pi\left(3\,\beta\,B^2+4\,K-4\,\omega^2\,M+9\,\beta\,A^2\right)}{\omega} \tag{5}$$

$$\frac{\partial}{\partial B}\, a$$

$$\frac{1}{4}\,\frac{\pi\left(6\,\beta\,A\,B-4\,\omega\,C\right)}{\omega} \tag{6}$$

Setting B, the forcing and the damping coefficients to zero we obtain the equation of the modal backbone and its derivative.

$$subs(B=0,\,C=0,\,F=0\,,\,a)$$

$$\frac{1}{4}\,\frac{\pi\left(4\,K\,A-4\,M\,A\,\omega^2+3\,\beta\,A^3\right)}{\omega} \tag{7}$$

$$subs\left(B=0,\,C=0,\,F=0,\,\frac{\partial}{\partial A}\,a\right)$$

$$\frac{1}{4}\,\frac{\pi\left(4\,K-4\,\omega^2\,M+9\,\beta\,A^2\right)}{\omega} \tag{8}$$

$$a := \int_0^{\frac{2\cdot \mathrm{Pi}}{\omega}} eq\cdot\cos(\omega\cdot t)\,\mathrm{d}t$$

$$\frac{1}{4}\,\frac{\pi\left(4\,K\,B-4\,M\,B\,\omega^2+4\,\omega\,C\,A+3\,\beta\,B^3+3\,\beta\,A^2\,B\right)}{\omega} \tag{9}$$

$$\frac{\partial}{\partial A}\, a$$

$$\frac{1}{4}\,\frac{\pi\left(4\,\omega\,C+6\,\beta\,A\,B\right)}{\omega} \tag{10}$$

$$\frac{\partial}{\partial B}\, a$$

$$\frac{1}{4}\,\frac{\pi\left(4\,K-4\,\omega^2\,M+9\,\beta\,B^2+3\,\beta\,A^2\right)}{\omega} \tag{11}$$

Two degrees of freedom case:

Equations of the 2dofs oscillator:

$$eq1 := M1\cdot\frac{d^2}{dt^2}x1(t) + C1\cdot\frac{d}{dt}x1(t) - C2\cdot\frac{d}{dt}(x2(t) - x1(t)) + K1\cdot x1(t) - K2\cdot(x2(t) - x1(t)) - \beta\cdot(x2(t) - x1(t))^3 - F11\cdot\sin(\omega\cdot t) - F12\cdot\cos(\omega\cdot t)$$

$$M1\left(\frac{d^2}{dt^2}x1(t)\right) + C1\left(\frac{d}{dt}x1(t)\right) - C2\left(\frac{d}{dt}x2(t) - \left(\frac{d}{dt}x1(t)\right)\right) + K1\,x1(t) - K2\,(x2(t) - x1(t)) - \beta\,(x2(t) - x1(t))^3 - F11\sin(\omega t) - F12\cos(\omega t) \quad \textbf{(12)}$$

$$eq2 := M2\cdot\frac{d^2}{dt^2}x2(t) + C2\cdot\frac{d}{dt}(x2(t) - x1(t)) + K2\cdot(x2(t) - x1(t)) + \beta\cdot(x2(t) - x1(t))^3 - F21\cdot\sin(\omega\cdot t) - F22\cdot\cos(\omega\cdot t)$$

$$M2\left(\frac{d^2}{dt^2}x2(t)\right) + C2\left(\frac{d}{dt}x2(t) - \left(\frac{d}{dt}x1(t)\right)\right) + K2\,(x2(t) - x1(t)) + \beta\,(x2(t) - x1(t))^3 - F21\sin(\omega t) - F22\cos(\omega t) \quad \textbf{(13)}$$

We substitute the single-harmonic solutions for the displacement of each degree of freedom, obtaining the residuals:

$$eq1 := subs(x1(t) = A1\cdot\sin(\omega\cdot t) + B1\cdot\cos(\omega\cdot t), eq1)$$

$$M1\left(\frac{\partial^2}{\partial t^2}(A1\sin(\omega t) + B1\cos(\omega t))\right) + C1\left(\frac{\partial}{\partial t}(A1\sin(\omega t) + B1\cos(\omega t))\right) - C2\left(\frac{d}{dt}x2(t) - \left(\frac{\partial}{\partial t}(A1\sin(\omega t) + B1\cos(\omega t))\right)\right) + K1\,(A1\sin(\omega t) + B1\cos(\omega t)) - K2\,(x2(t) - A1\sin(\omega t) - B1\cos(\omega t)) - \beta\,(x2(t) - A1\sin(\omega t) - B1\cos(\omega t))^3 - F11\sin(\omega t) - F12\cos(\omega t) \quad \textbf{(14)}$$

$$eq1 := subs(x2(t) = A2\cdot\sin(\omega\cdot t) + B2\cdot\cos(\omega\cdot t), eq1)$$

$$M1\left(-A1\sin(\omega t)\,\omega^2 - B1\cos(\omega t)\,\omega^2\right) + C1\,(A1\cos(\omega t)\,\omega - B1\sin(\omega t)\,\omega) - C2\left(\frac{\partial}{\partial t}(A2\sin(\omega t) + B2\cos(\omega t)) - A1\cos(\omega t)\,\omega + B1\sin(\omega t)\,\omega\right) + K1\,(A1\sin(\omega t) + B1\cos(\omega t)) - K2\,(A2\sin(\omega t) + B2\cos(\omega t) - A1\sin(\omega t) - B1\cos(\omega t)) - \beta\,(A2\sin(\omega t) + B2\cos(\omega t) - A1\sin(\omega t) - B1\cos(\omega t))^3 - F11\sin(\omega t) - F12\cos(\omega t) \quad \textbf{(15)}$$

$$eq2 := subs(x1(t) = A1\cdot\sin(\omega\cdot t) + B1\cdot\cos(\omega\cdot t), eq2)$$

$$M2\left(\frac{d^2}{dt^2}x2(t)\right) + C2\left(\frac{d}{dt}x2(t) - \left(\frac{\partial}{\partial t}(A1\sin(\omega t) + B1\cos(\omega t))\right)\right) + K2\,(x2(t) - A1\sin(\omega t) - B1\cos(\omega t)) + \beta\,(x2(t) - A1\sin(\omega t) - B1\cos(\omega t))^3 - F21\sin(\omega t) - F22\cos(\omega t) \quad \textbf{(16)}$$

$eq2 := subs(x2(t) = A2 \cdot \sin(\omega \cdot t) + B2 \cdot \cos(\omega \cdot t), eq2)$

$$M2 \left(\frac{\partial^2}{\partial t^2} (A2 \sin(\omega t) + B2 \cos(\omega t)) \right) + C2 \left(\frac{\partial}{\partial t} (A2 \sin(\omega t) + B2 \cos(\omega t)) - A1 \cos(\omega t)\, \omega + B1 \sin(\omega t)\, \omega \right) + K2\, (A2 \sin(\omega t) + B2 \cos(\omega t) - A1 \sin(\omega t) - B1 \cos(\omega t)) + \beta\, (A2 \sin(\omega t) + B2 \cos(\omega t) - A1 \sin(\omega t) - B1 \cos(\omega t))^3 - F21 \sin(\omega t) - F22 \cos(\omega t) \tag{17}$$

$eq1 := simplify(eq1)$

$$6\, \beta\, A2 \sin(\omega t)\, B2 \cos(\omega t)^2 B1 - 6\, \beta\, B2 \cos(\omega t)^2 A1 \sin(\omega t)\, B1 - 3\, \beta\, A2 \sin(\omega t)\, B2^2 \cos(\omega t)^2 - 3\, \beta\, A2 \sin(\omega t)\, B1^2 \cos(\omega t)^2 + 3\, \beta\, B2^2 \cos(\omega t)^2 A1 \sin(\omega t) + 3\, \beta\, A1 \sin(\omega t)\, B1^2 \cos(\omega t)^2 - 6\, \beta\, A2\, A1\, B1 \cos(\omega t) + 6\, \beta\, A2\, B2 \cos(\omega t)\, A1 - 3\, \beta\, A2^2 A1 \sin(\omega t) \cos(\omega t)^2 + 3\, \beta\, A2\, A1^2 \sin(\omega t) \cos(\omega t)^2 + 6\, \beta\, A2\, A1\, B1 \cos(\omega t)^3 - 6\, \beta\, A2\, B2 \cos(\omega t)^3 A1 + 3\, \beta\, A2^2 A1 \sin(\omega t) - 3\, \beta\, A2\, A1^2 \sin(\omega t) + K1\, A1 \sin(\omega t) + K1\, B1 \cos(\omega t) - K2\, A2 \sin(\omega t) - K2\, B2 \cos(\omega t) + K2\, A1 \sin(\omega t) + K2\, B1 \cos(\omega t) - \beta\, B2^3 \cos(\omega t)^3 + \beta\, B1^3 \cos(\omega t)^3 - \beta\, A2^3 \sin(\omega t) + 3\, \beta\, A2^2 B2 \cos(\omega t)^3 - 3\, \beta\, A2^2 B1 \cos(\omega t)^3 + 3\, \beta\, B2 \cos(\omega t)^3 A1^2 - 3\, \beta\, A1^2 B1 \cos(\omega t)^3 - \beta\, A1^3 \sin(\omega t) \cos(\omega t)^2 + \beta\, A2^3 \sin(\omega t) \cos(\omega t)^2 + \beta\, A1^3 \sin(\omega t) - F11 \sin(\omega t) - F12 \cos(\omega t) + 3\, \beta\, A1^2 B1 \cos(\omega t) - M1\, A1 \sin(\omega t)\, \omega^2 - M1\, B1 \cos(\omega t)\, \omega^2 + C1\, A1 \cos(\omega t)\, \omega - C1\, B1 \sin(\omega t)\, \omega - C2\, A2 \cos(\omega t)\, \omega + C2\, B2 \sin(\omega t)\, \omega + C2\, A1 \cos(\omega t)\, \omega - C2\, B1 \sin(\omega t)\, \omega + 3\, \beta\, B2^2 \cos(\omega t)^3 B1 - 3\, \beta\, B2 \cos(\omega t)^3 B1^2 - 3\, \beta\, B2 \cos(\omega t)\, A1^2 + 3\, \beta\, A2^2 B1 \cos(\omega t) - 3\, \beta\, A2^2 B2 \cos(\omega t) \tag{18}$$

$eq2 := simplify(eq2)$

$$-6\, \beta\, A2 \sin(\omega t)\, B2 \cos(\omega t)^2 B1 + 6\, \beta\, B2 \cos(\omega t)^2 A1 \sin(\omega t)\, B1 + 3\, \beta\, A2 \sin(\omega t)\, B2^2 \cos(\omega t)^2 + 3\, \beta\, A2 \sin(\omega t)\, B1^2 \cos(\omega t)^2 - 3\, \beta\, B2^2 \cos(\omega t)^2 A1 \sin(\omega t) - 3\, \beta\, A1 \sin(\omega t)\, B1^2 \cos(\omega t)^2 + 6\, \beta\, A2\, A1\, B1 \cos(\omega t) - 6\, \beta\, A2\, B2 \cos(\omega t)\, A1 + 3\, \beta\, A2^2 A1 \sin(\omega t) \cos(\omega t)^2 - 3\, \beta\, A2\, A1^2 \sin(\omega t) \cos(\omega t)^2 - 6\, \beta\, A2\, A1\, B1 \cos(\omega t)^3 + 6\, \beta\, A2\, B2 \cos(\omega t)^3 A1 - 3\, \beta\, A2^2 A1 \sin(\omega t) + 3\, \beta\, A2\, A1^2 \sin(\omega t) + K2\, A2 \sin(\omega t) + K2\, B2 \cos(\omega t) - K2\, A1 \sin(\omega t) - K2\, B1 \cos(\omega t) + \beta\, B2^3 \cos(\omega t)^3 - \beta\, B1^3 \cos(\omega t)^3 + \beta\, A2^3 \sin(\omega t) - 3\, \beta\, A2^2 B2 \cos(\omega t)^3 + 3\, \beta\, A2^2 B1 \cos(\omega t)^3 - 3\, \beta\, B2 \cos(\omega t)^3 A1^2 \tag{19}$$

$$+3\,\beta\,A1^2\,B1\cos(\omega\,t)^3+\beta\,A1^3\sin(\omega\,t)\cos(\omega\,t)^2-\beta\,A2^3\sin(\omega\,t)\cos(\omega\,t)^2$$
$$-M2\,A2\sin(\omega\,t)\,\omega^2-M2\,B2\cos(\omega\,t)\,\omega^2-\beta\,A1^3\sin(\omega\,t)-3\,\beta\,A1^2\,B1\cos(\omega\,t)$$
$$+C2\,A2\cos(\omega\,t)\,\omega-C2\,B2\sin(\omega\,t)\,\omega-C2\,A1\cos(\omega\,t)\,\omega+C2\,B1\sin(\omega\,t)\,\omega$$
$$-3\,\beta\,B2^2\cos(\omega\,t)^3\,B1+3\,\beta\,B2\cos(\omega\,t)^3\,B1^2+3\,\beta\,B2\cos(\omega\,t)\,A1^2-F21\sin(\omega\,t)$$
$$-F22\cos(\omega\,t)-3\,\beta\,A2^2\,B1\cos(\omega\,t)+3\,\beta\,A2^2\,B2\cos(\omega\,t)$$

Again, applying Galerkin on each residual:

$$a:=\int_0^{\frac{2\cdot \mathrm{Pi}}{\omega}} eq1\cdot\sin(\omega\cdot t)\,\mathrm{d}t$$

$$\frac{1}{4}\,\frac{1}{\omega}\Big(\pi\,\big(3\,\beta\,A1^3-3\,\beta\,A2^3+4\,K2\,A1+4\,K1\,A1-4\,K2\,A2-4\,F11+6\,\beta\,A2\,B2\,B1-6\,\beta\,B2\,A1\,B1+3\,\beta\,B2^2\,A1+3\,\beta\,A1\,B1^2-3\,\beta\,A2\,B2^2-3\,\beta\,A2\,B1^2+9\,\beta\,A2^2\,A1-9\,\beta\,A2\,A1^2-4\,M1\,A1\,\omega^2-4\,C1\,B1\,\omega-4\,C2\,B1\,\omega+4\,C2\,B2\,\omega\big)\Big) \tag{20}$$

The derivatives of each nonlinear algebraic equation with respect to the coefficients determine the jacobian:

$$\frac{\partial}{\partial A1}\,a$$

$$\frac{1}{4}\,\frac{1}{\omega}\Big(\pi\,\big(9\,\beta\,A1^2+4\,K2+4\,K1-6\,\beta\,B2\,B1+3\,\beta\,B2^2+3\,\beta\,B1^2+9\,\beta\,A2^2-18\,\beta\,A2\,A1-4\,M1\,\omega^2\big)\Big) \tag{21}$$

$$\frac{\partial}{\partial B1}\,a$$

$$\frac{1}{4}\,\frac{\pi\,(6\,\beta\,A2\,B2-6\,\beta\,B2\,A1+6\,\beta\,A1\,B1-6\,\beta\,A2\,B1-4\,C1\,\omega-4\,C2\,\omega)}{\omega} \tag{22}$$

$$\frac{\partial}{\partial A2}\,a$$

$$\frac{1}{4}\,\frac{\pi\,(-9\,\beta\,A2^2-4\,K2+6\,\beta\,B2\,B1-3\,\beta\,B2^2-3\,\beta\,B1^2+18\,\beta\,A2\,A1-9\,\beta\,A1^2)}{\omega} \tag{23}$$

$$\frac{\partial}{\partial B2}\,a$$

$$\frac{1}{4}\,\frac{\pi\,(6\,\beta\,A2\,B1-6\,\beta\,A1\,B1+6\,\beta\,B2\,A1-6\,\beta\,A2\,B2+4\,C2\,\omega)}{\omega} \tag{24}$$

Equally to the SDOF case, setting the B coefficients, dampings and forcings to zero results in the equations for the modal backbones:

$$subs(B1=0, B2=0, C1=0, C2=0, F11=0, F12=0, F21=0, F22=0, a)$$

$$\frac{1}{4}\,\frac{1}{\omega}\left(\pi\left(3\,\beta\,A1^3-3\,\beta\,A2^3+4\,K2\,A1+4\,K1\,A1-4\,K2\,A2+9\,\beta\,A2^2\,A1-9\,\beta\,A2\,A1^2-4\,M1\,A1\,\omega^2\right)\right) \tag{25}$$

$$subs\left(B1=0, B2=0, C1=0, C2=0, F11=0, F12=0, F21=0, F22=0, \frac{\partial}{\partial A1}\,a\right)$$

$$\frac{1}{4}\,\frac{\pi\left(9\,\beta\,A1^2+4\,K2+4\,K1+9\,\beta\,A2^2-18\,\beta\,A2\,A1-4\,M1\,\omega^2\right)}{\omega}$$

$$subs\left(B1=0, B2=0, C1=0, C2=0, F11=0, F12=0, F21=0, F22=0, \frac{\partial}{\partial A2}\,a\right)$$

$$\frac{1}{4}\,\frac{\pi\left(-9\,\beta\,A2^2-4\,K2+18\,\beta\,A2\,A1-9\,\beta\,A1^2\right)}{\omega} \tag{27}$$

$$a := \int_0^{\frac{2\cdot \mathrm{Pi}}{\omega}} eq1\cdot\cos(\omega\cdot t)\,\mathrm{d}t$$

$$\frac{1}{4}\,\frac{1}{\omega}\left(\pi\left(3\,\beta\,B1^3-4\,K2\,B2-3\,\beta\,B2^3+4\,K2\,B1+4\,K1\,B1+4\,C2\,A1\,\omega-4\,M1\,B1\,\omega^2-4\,C2\,A2\,\omega+4\,C1\,A1\,\omega+3\,\beta\,A2^2\,B1-3\,\beta\,A2^2\,B2-3\,\beta\,B2\,A1^2+3\,\beta\,A1^2\,B1+9\,\beta\,B2^2\,B1-9\,\beta\,B2\,B1^2+6\,\beta\,A2\,B2\,A1-6\,\beta\,A2\,A1\,B1-4\,F12\right)\right) \tag{28}$$

$$\frac{\partial}{\partial A1}\,a$$

$$\frac{1}{4}\,\frac{\pi\left(4\,C2\,\omega+4\,C1\,\omega-6\,\beta\,B2\,A1+6\,\beta\,A1\,B1+6\,\beta\,A2\,B2-6\,\beta\,A2\,B1\right)}{\omega}$$

(30)

$$\frac{\partial}{\partial B1}\,a$$

$$\frac{1}{4}\,\frac{1}{\omega}\left(\pi\left(9\,\beta\,B1^2+4\,K2+4\,K1-4\,M1\,\omega^2+3\,\beta\,A2^2+3\,\beta\,A1^2+9\,\beta\,B2^2-18\,\beta\,B2\,B1-6\,\beta\,A2\,A1\right)\right) \tag{31}$$

$$\frac{\partial}{\partial A2}\,a$$

$$\frac{1}{4}\,\frac{\pi\left(-6\,\beta\,A1\,B1+6\,\beta\,B2\,A1-6\,\beta\,A2\,B2+6\,\beta\,A2\,B1-4\,C2\,\omega\right)}{\omega} \tag{32}$$

$$\frac{\partial}{\partial B2}\,a$$

$$\frac{1}{4}\,\frac{\pi\left(-4\,K2-9\,\beta\,B2^2-3\,\beta\,A2^2-3\,\beta\,A1^2+18\,\beta\,B2\,B1-9\,\beta\,B1^2+6\,\beta\,A2\,A1\right)}{\omega} \tag{33}$$

$$a := \int_0^{\frac{2\cdot \mathrm{Pi}}{\omega}} eq2\cdot\sin(\omega\cdot t)\,\mathrm{d}t$$

$$-\frac{1}{4}\,\frac{1}{\omega}\Big(\pi\big(4\,F21-3\,\beta\,A2^3+3\,\beta\,A1^3-4\,K2\,A2+4\,K2\,A1-4\,C2\,B1\,\omega+4\,\omega^2\,M2\,A2+4\,C2\,B2\,\omega+3\,\beta\,B2^2\,A1+3\,\beta\,A1\,B1^2-3\,\beta\,A2\,B2^2-3\,\beta\,A2\,B1^2+9\,\beta\,A2^2\,A1-9\,\beta\,A2\,A1^2+6\,\beta\,A2\,B2\,B1-6\,\beta\,B2\,A1\,B1\big)\Big) \tag{34}$$

$$\frac{\partial}{\partial A1}\,a$$

$$-\frac{1}{4}\,\frac{\pi\left(9\,\beta\,A1^2+4\,K2+3\,\beta\,B2^2+3\,\beta\,B1^2+9\,\beta\,A2^2-18\,\beta\,A2\,A1-6\,\beta\,B2\,B1\right)}{\omega}$$

(36)

$$\frac{\partial}{\partial B1}\,a$$

$$-\frac{1}{4}\,\frac{\pi\left(-4\,C2\,\omega+6\,\beta\,A1\,B1-6\,\beta\,A2\,B1+6\,\beta\,A2\,B2-6\,\beta\,B2\,A1\right)}{\omega} \tag{37}$$

$$\frac{\partial}{\partial A2}\,a$$

$$-\frac{1}{4}\,\frac{\pi\left(-9\,\beta\,A2^2-4\,K2+4\,\omega^2\,M2-3\,\beta\,B2^2-3\,\beta\,B1^2+18\,\beta\,A2\,A1-9\,\beta\,A1^2+6\,\beta\,B2\,B1\right)}{\omega} \tag{38}$$

$$\frac{\partial}{\partial B2}\,a$$

$$-\frac{1}{4}\,\frac{\pi\left(6\,\beta\,A2\,B1-6\,\beta\,A1\,B1+6\,\beta\,B2\,A1-6\,\beta\,A2\,B2+4\,C2\,\omega\right)}{\omega} \tag{39}$$

$$subs(B1=0, B2=0, C1=0, C2=0, F11=0, F12=0, F21=0, F22=0, a)$$

$$-\frac{1}{4}\,\frac{\pi\left(-3\,\beta\,A2^3+3\,\beta\,A1^3-4\,K2\,A2+4\,K2\,A1+4\,\omega^2\,M2\,A2+9\,\beta\,A2^2\,A1-9\,\beta\,A2\,A1^2\right)}{\omega} \tag{40}$$

$$subs\left(B1=0, B2=0, C1=0, C2=0, F11=0, F12=0, F21=0, F22=0, \frac{\partial}{\partial A1}\,a\right)$$

$$-\frac{1}{4}\frac{\pi\left(9\beta A1^2+4K2+9\beta A2^2-18\beta A2\,A1\right)}{\omega}$$

$$subs\left(B1=0, B2=0, C1=0, C2=0, F11=0, F12=0, F21=0, F22=0, \frac{\partial}{\partial A2}a\right)$$

$$-\frac{1}{4}\frac{\pi\left(-9\beta A2^2-4K2+4\omega^2 M2+18\beta A2\,A1-9\beta A1^2\right)}{\omega} \tag{42}$$

$$a := \int_0^{\frac{2\cdot \mathrm{Pi}}{\omega}} eq2\cdot\cos(\omega\cdot t)\,\mathrm{d}t$$

$$-\frac{1}{4}\frac{1}{\omega}\Big(\pi\big(-6\beta A2\,A1\,B1+6\beta A2\,B2\,A1-3\beta A2^2 B2+3\beta A1^2 B1-3\beta B2\,A1^2 +9\beta B2^2 B1+3\beta A2^2 B1-9\beta B2\,B1^2+4\omega^2 B2\,M2-3\beta B2^3-4K2\,B2+4K2\,B1 +3\beta B1^3+4C2\,A1\,\omega-4C2\,A2\,\omega+4F22\big)\Big) \tag{43}$$

$$\frac{\partial}{\partial A1}a$$

$$-\frac{1}{4}\frac{\pi\left(-6\beta A2\,B1+6\beta A2\,B2+6\beta A1\,B1-6\beta B2\,A1+4C2\,\omega\right)}{\omega} \tag{45}$$

$$\frac{\partial}{\partial B1}a$$

$$-\frac{1}{4}\frac{\pi\left(-6\beta A2\,A1+3\beta A1^2+9\beta B2^2+3\beta A2^2-18\beta B2\,B1+4K2+9\beta B1^2\right)}{\omega} \tag{46}$$

$$\frac{\partial}{\partial A2}a$$

$$-\frac{1}{4}\frac{\pi\left(-6\beta A1\,B1+6\beta B2\,A1-6\beta A2\,B2+6\beta A2\,B1-4C2\,\omega\right)}{\omega} \tag{47}$$

$$\frac{\partial}{\partial B2}a$$

$$-\frac{1}{4}\frac{\pi\left(6\beta A2\,A1-3\beta A2^2-3\beta A1^2+18\beta B2\,B1-9\beta B1^2+4\omega^2 M2-9\beta B2^2-4K2\right)}{\omega} \tag{48}$$