Commodity Price Shocks and Production Networks in Small Open Economies*

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Abstract

We study the role of domestic production networks in the transmission of commodity price shocks in small open economies. We provide empirical evidence of a strong propagation of commodity shocks to domestic sectors that supply intermediate inputs to commodity sectors (*upstream* propagation) and a muted propagation to sectors using commodities as intermediate inputs (*downstream* propagation). We develop a small open economy production network model to explain these transmission patterns. We show that the domestic production network amplifies the well-known *wealth effect* of commodity price shocks and, thus, accounts for an important fraction of the *upstream* propagation of commodity price shocks observed in the data. Moreover, the elasticity of substitution between intermediates amplifies the *upstream* channel by increasing the demand for non-commodity sectors. In contrast, the elasticity of substitution between intermediates and value-added mitigates the *downstream* cost channel by allowing firms to use relatively cheaper primary inputs in production.

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1 Introduction

This paper analyzes the propagation of commodity price shocks through domestic production networks in small open economies. We take advantage of two stylized facts. First, commodity sectors—namely mining, agriculture, and food sectors—are central sectors in small open economies, both as suppliers and buyers of intermediate inputs, which gives them a potential role as a source of supply and demand shocks' propagation. Second, commodity price shocks are only mildly correlated across sectors within a country. Therefore, as commodity prices are exogenous to the economies we analyze, we have an ideal scenario to study the propagation of sectoral commodity price shocks along the production chain.

We first provide empirical evidence of a strong upstream propagation—to sectors providing intermediate inputs to commodity sectors—of commodity price shocks. In a production network setup where each sector buys intermediate inputs from other sectors, commodity price booms (busts) generate an increase (decrease) in intermediate input demand. We also show that, while commodity price shocks increase the price of downstream sectors—i.e., those sectors buying from the commodity sector to produce their output—they have no real effect on the output of downstream sectors. Commodity price shocks thus appear to propagate mainly as a demand-side shock in small open economies.

We then develop a small open economy model featuring a domestic production network to explain these empirical patterns. In our model, the commodity sector supplies goods to domestic firms and consumers at home and abroad. Importantly, firms in the commodity sector use labor and domestic intermediate inputs in production. The commodity price is exogenously determined in international markets and driven by foreign demand. Our baseline model assumes inelastic labor supply, competitive domestic factor, and good markets and that non-commodity sector firms display constant returns to scale in production. Instead, the commodity sector shows decreasing returns to scale in production, and its profits are rebated back to the household.

Our model highlights three mechanisms in which commodity price shocks propagate to non-commodity sectors through the domestic production network. The first one is a standard *wealth channel*. Since profits of the commodity sector are rebated back to the household, an increase in the price of the commodity sector increases households' income and thus boosts demand, which then increases demand for domestic intermediate inputs. The second is a *pure downstream channel*: after an increase in the commodity price, the costs of every non-commodity producer that uses commodities as input increase. These downstream sectors also increase their price, which generates further downstream propagation of the shock. The third channel that we label *buyers' substitution effect* stems from the fact that after an increase in the commodity price, all sectors can substitute away from the rise in the commodity price towards other intermediate inputs or primary inputs, thus providing another channel of upstream and downstream propagation.

Finally, we use a simplified version of our model to shed some light on the quantitative and qualitative importance these channels. In a simple calibration exercise using Australian sectoral data as a benchmark, we point to the essential role of the wealth and the buyers' substitution channel to explain the upstream effect we find in the empirical results. The quantitative exercise is also able to feature a muted downstream propagation channel, in line with the empirical results. In this case, the elasticity of substitution between intermediates and labor plays a crucial role as it allows industries to rely less on relatively more expensive intermediates.

Related Literature. This paper contributes to two strands of literature. We contribute to the extensive literature on the macroeconomic effects of commodity price shocks in Mendoza (1995), Kose (2002), Drechsel and Tenreyro (2018) Benguria, Saffie, and Urzúa (2020), Cao and Dong (2020), Kohn, Leibovici, and Tretvoll (2021), Romero (2022) and González (2022) by providing empirical evidence on the role of domestic production networks in propagating commodity price shocks to non-commodity upstream sectors. On the theoretical front, we contribute by highlighting the role of non-unitary production elasticities in amplifying the upstream propagation and dampening the downstream propagation of commodity price shocks. Moreover, we show that the the well-known wealth effect of commodity price shocks has an important (upstream) network propagation component.

We also contribute to the literature on production networks and business cycles fluctuations (Horvath, 1998; Foerster et al., 2011; Acemoglu et al., 2012; Atalay, 2017; Baqaee and Farhi, 2019, 2021; Miranda-Pinto, 2021; vom Lehn and Winberry, 2020; Carvalho et al., 2021). Different from these studies, we show that commodity price shocks can have important real effects on output quantities, besides productivity and financial shocks, and are largely propagated through input-output linkages. In addition, as in Miranda-Pinto (2021), Miranda-Pinto and Young (2021) and Carvalho et al. (2021), our paper emphasizes the role that non-unitary elasticity of substitution between inputs plays in matching salient facts of the transmission of shocks via domestic production networks.

2 Stylized Facts

In this section, we present two stylized facts regarding commodity sectors. First, commodity sectors are central in the domestic production network. Second, commodity price shocks strongly commove across countries but present a very small correlation across sectors within countries.

We first define what we mean by commodity sectors. To do so, we combine data on commodity goods' exports from Fernández, González, and Rodríguez (2018) and input-output data from the WIOD and OECD. For more details on data sources and definitions please refer to our Appendix A. We match each commodity good to one of the 34 industries in the World Input-Output Database (WIOD) and one of the 45 industries in the OECD input-output tables. Table 8 in our Appendix B provides a detailed mapping between goods and sectors in the WIOD data, which is our benchmark dataset.[1] The three commodity sectors in the WIOD are Agriculture, Forestry, and Fishing;

[1] In our Appendix A we also provide information on the sample of countries we use from the WIOD and OECD and the variables definition.

Mining and Quarrying; and Food Products, Beverages, and Tobacco.

Fact 1: Commodity sectors are central sectors in the production network.

We describe the network centrality of commodity sectors using standard centrality measures that capture how connected are the sectors I am connected to and how connected are the sectors that are connected to the sectors that I am connected to, etc. To that end, we analyze commodity sectors' customer and supplier centrality following Acemoglu, Akcigit, and Kerr (2016).[2] We measure the supplier or *downstream* centrality as

$$\textbf{Supplier} = (\boldsymbol{I} - \boldsymbol{\Omega}_F')^{-1}\mathbf{1},$$

in which an element Ω_{ij} is the share of intermediates that sector j supplies to sector i as a fraction of sector i's gross output, i.e. how much sector i buys from sector j as a fraction of sector i's output. $\boldsymbol{I}$ is an identity matrix of dimension equals to the dimension of $\boldsymbol{\Omega}_F$. The matrix $\boldsymbol{\Omega}_F$ collects all Ω_{ij} and includes all sectors in the economy. $\mathbf{1}$ is an $N \times 1$ vector of ones. The vector **Supplier**, with dimension $N \times 1$, summarizes the direct and indirect importance of sectors as suppliers of intermediate inputs to other sectors.

We then measure the customer or *upstream* centrality as

$$\textbf{Customer} = (\boldsymbol{I} - \boldsymbol{\Omega}_F \odot \tilde{\boldsymbol{S}})^{-1}\mathbf{1}$$

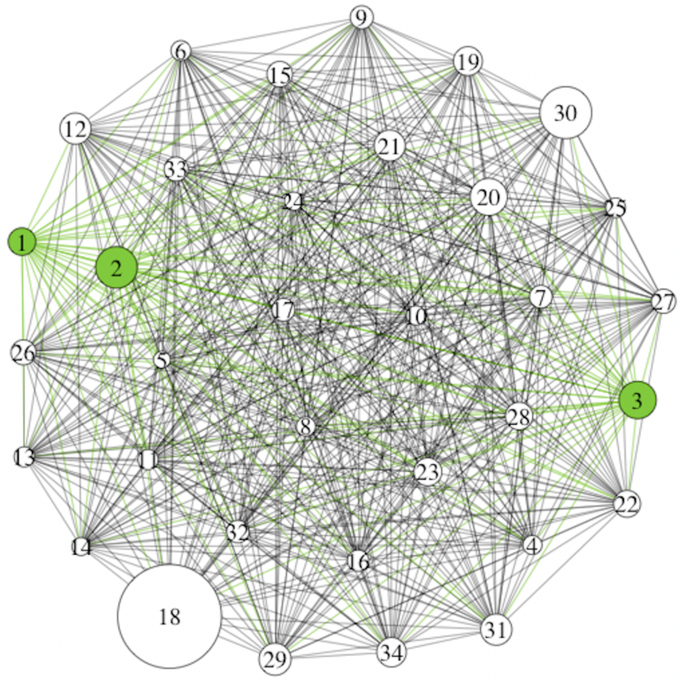
where $\tilde{\boldsymbol{S}} = \{S_{ij}\}$ is an $N \times N$ matrix with the typical element satisfying $S_{ij} = S_i/S_j$ with $S_k = P_k Q_k$ for all k. This is a matrix that records the ratio of sales across all producers. $\odot$ is the Haddamard product and implies multiplication of the matrices element-by-element. The vector **Customer**, of dimension $N \times 1$, summarizes the direct and indirect importance of sectors as users of intermediate inputs from other sectors.

Figure 1 plots the domestic network structure of Australia in 2011, using input-output data from the WIOD database. Each node (circle) is a different sector in the economy, and the node's size represents how important that sector is in the network, based on the network centralities defined above. Panel (a) shows the network in which each node's size describes the customer centrality of the sector—this is, how much output of other sectors a given sector uses, directly and indirectly—, while in panel (b), the node size is based on each sector's supplier centrality—how much of a given sector output is used as input by other sectors, directly and indirectly.

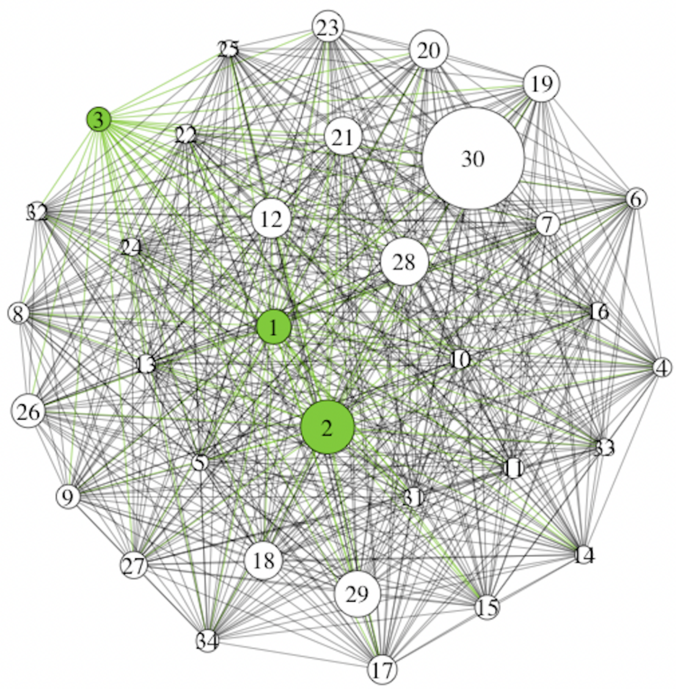
We observe in Figure 1 that commodity sectors were central sectors in the domestic production network of Australia in 2011. In particular, panel (a) shows that mining (2) and food (3) sectors

[2] These definitions are slightly different from the notions of downstreamness and upstreamness highlighted in the global value chains literature (see Antras and Chor, 2021). Their measure of upstreamness shows how important are other sectors as buyers to a given sector i. In our case, customer centrality comes from the importance of sector i as a buyer to other sectors. This difference is expected because we focus on how shocks propagate, as in Acemoglu et al. (2016), while they focus on the distance of each sector to final demand and primary factors. Our concept is closer to the Katz-Bonacich centrality used in the production networks literature. See Carvalho (2014) for an overview, especially footnote 11.

Figure 1. Domestic Production Network Australia



(a) Customer centrality



(b) Supplier centrality

Note: This figure shows the domestic production network of Australia (WIOD Input-Output data) for 2011 at the sector level (ISIC rev. 3). An arrow from sector j to sector i represents intermediate inputs flowing from j o i. The intensity of the arrow (darkness and width) indicates how much sector i is buying from j as a fraction of total intermediate input expenses. Each node (circle) is a different sector in the economy, and the size of the node represents how important is that sector as a direct and indirect buyer (panel a) and user (panel b) of intermediate inputs. The labels in the nodes are linked to sectors in Table 7 of our Appendix.

are among the sectors with the largest customer centrality. Panel (b) also shows that mining and agriculture are central sectors in their direct and indirect supply of intermediates inputs.

To describe the relative importance of commodity sectors in the domestic production network of small open economies, we report in Table 1 the ranking of the customer and supplier propagation centrality for the three commodity sectors, with respect to all the other sectors in the economy (a total of 34 in the WIOD data), for the years 1995, 2005, and 2011. There are two main takeaways from Table 1. First, for all the countries in the WIOD sample, at least one of the commodity sectors (many times 2 of them) is a central customer and/or a central supplier in the domestic production network. Second, there is meaningful variation in centrality measures over time. For example, the mining sector in Australia and Brazil were ranked 11th and 14th in 1995, but in 2011 they ranked 3rd and 2nd in terms of customer centrality, respectively.

Fact 2: Sectoral commodity price shocks are only mildly correlated across sectors, within a country.

We first describe the process of constructing sectoral indexes of commodity prices. We then describe the approach we follow to estimate commodity price shocks.

(i) We use the export data Fernández, González, and Rodríguez (2018) and calculate, for each country, the share of each commodity good in its sectoral group, be agriculture, mining, or food sectors. Then, we multiply each sector-country weight by the monthly commodity price.

(ii) The outcome from step (i) is a matrix of country-specific monthly commodity price index that we deflate using the US Consumer Price Index (CPI).

(iii) We take the average across months within each quarter by year.

We compute sectoral commodity price shocks following Schmitt-Grohé and Uribe (2018). We follow the original treatment of data series as in Schmitt-Grohé and Uribe (2018) and apply a quadratic detrending procedure to the series (the original paper reports that results are not sensitive to the filtering procedure). We then estimate a structural vector autoregression (SVAR, hereafter) model for each country and commodity sector at an individual level.

We define the vector x_t as

$$x_t = \left[\hat{p}_t^{c_i} \quad \hat{tb}_t \quad \hat{y}_t \quad \hat{c}_t \quad \hat{i}_t \quad r\hat{e}r_t,\right]' \tag{1}$$

in which $\hat{p}_t^{c_i}, \hat{y}_t, \hat{c}_t, \hat{i}_t, r\hat{e}r_t$ are the log-deviations of the commodity price for sector i, real output per capita, real private consumption, real gross investment per capita and real exchange rate from their time trends, respectively. At the same time, $\hat{tb}_t$ is the deviation of the ratio between trade balance and output from its trend.[3]

[3] In our Appendix A we provide detailed information on the sources and definition of each variable.

Table 1. Ranking of Network Centrality of Commodity Sectors

		Customer Centrality			Supplier Centrality		
Country	**Year**	**Agric.**	**Mining**	**Food**	**Agric.**	**Mining**	**Food**
Australia	1995	10	11	3	13	6	17
Australia	2005	15	6	3	12	4	18
Australia	2011	13	3	4	10	2	17
Bulgaria	1995	2	8	1	2	9	13
Bulgaria	2005	3	21	4	3	11	18
Bulgaria	2011	6	22	3	10	12	16
Brazil	1995	14	25	2	7	14	10
Brazil	2005	9	12	1	9	7	11
Brazil	2011	10	7	2	7	2	14
Canada	1995	6	18	3	4	10	15
Canada	2005	8	14	3	8	4	16
Canada	2011	7	19	4	4	14	16
Denmark	1995	6	33	1	8	17	11
Denmark	2005	9	32	3	14	8	12
Denmark	2011	9	27	3	11	5	13
India	1995	9	25	6	3	9	23
India	2005	9	24	4	2	11	17
India	2011	9	24	4	5	11	19
Lithuania	1995	1	33	3	2	34	9
Lithuania	2005	3	31	1	8	23	14
Lithuania	2011	3	33	1	9	28	12
Mexico	1995	10	18	1	7	1	15
Mexico	2005	13	17	2	10	2	15
Mexico	2011	11	14	2	8	1	15
Russia	1995	3	6	2	5	3	14
Russia	2005	11	4	6	10	5	19
Russia	2011	10	8	5	9	5	22
Average		**8**	**18**	**3**	**7**	**10**	**15**

Note: This table presents, for each country and commodity sector, the customer and supplier network centrality. Source: WIOD Input-Output database, 1995-2011.

The econometric model has the following specification:

$$A_0 x_t = A_1 x_{t-1} + \mu_t, \tag{2}$$

where the matrices A_0 and A_1 of dimension 6×6 contain the parameters of the SVAR model. Furthermore, we assume that A_0 is lower triangular, with 1's on the main diagonal. Finally, μ_t is a vector of dimensions 6×1 with mean zero and a diagonal variance-covariance matrix denoted by Σ. We can further write that:

$$x_t = A x_{t-1} + \Pi \epsilon_t. \tag{3}$$

We continue to use the original notation. Here $A = A_0^{-1} A_1$, $\Pi = A_0^{-1} \Sigma^{1/2}$, $\epsilon_t = \Sigma^{-1/2} \mu_t$. ϵ_t is a random variable characterized by a zero mean and an identity variance-covariance matrix. Since countries in the sample are all small open economies, a key assumption is that a given country takes the commodity price as given (they are exogenous). Imposing that the commodity price follows an univariate process, we obtain that the first equation of the SVAR model (see Equation (3)) gives us the dynamics of the commodity price, namely that:

$$\hat{p}_t^c = a_{11} \hat{p}_{t-1}^c + \pi_{11} \epsilon_t^1. \tag{4}$$

We use a_{11} and π_{11} to denote the first elements in the matrices A and Π. Thus, the corresponding first element of vector ϵ_t, e.g., ϵ_t^1, can be interpreted as the commodity price shocks. To obtain this shock, we use an OLS approach to estimate the SVAR model in Equation (3) for each country individually.

We study the conditional properties of the shocks we identify by looking at the median impulse response function from sectoral commodity price shocks to aggregate variables. In particular, Figures 5- 7 show the response of output, trade balance, consumption, investment, and exchange rate to a one standard deviation shock to commodity prices. We observe that the shocks have the expected effect. Increases in commodity prices tend to increase aggregate output, consumption, and investment.

We now investigate the correlation between sectoral commodity price shocks within countries. As highlighted in Fernández, González, and Rodríguez (2018), commodity shocks strongly commove across countries. Indeed, the cross-country correlation between commodity price shocks in Agriculture and Forestry, Mining and Quarrying, and Foods Products and Beverage sectors are 0.92, 0.88, and 0.62, respectively. However, as shown in Table 2 (and Table 9 in our Appendix), our estimated commodity shocks present a small correlation across sectors within countries. The average cross-country correlation between shocks to agriculture and mining is 0.47; the average cross-country correlation between shocks to agriculture and food products is 0.14; and the average cross-country correlation between shocks to mining and foods products is 0.07.[4]

[4]Figure 2 to Figure 4 in our Appendix depict our estimated sectoral commodity price shocks for countries in our sample. Besides confirming Fact 2 (low within-country correlation across commodity shocks), these figures show

Table 2. Average Pairwise Correlation Across Commodity Shocks

	Agriculture	Mining	Food
Agriculture	1		
Mining	0.47	1	1
Food	0.14	0.07	1

Note: This table presents the cross-country average of the within country pairwise correlations among the estimated sectoral commodity price shocks.

3 Commodity price shocks via production networks

In this section, we study the network effects of commodity price shocks. Our empirical specification is the following

$$y_{i,t}(c) = \alpha_i(c) + \delta_t(c) + \gamma_{i,t}(c) + \sum_{k=1}^{K} y_{i,t-k}(c) + \sum_{k=1}^{K} \phi^{shock} shock_{i,t-k}(c) + \epsilon_{i,t}(c), \tag{5}$$

where $y_{i,t}(c)$ is a measure of sector i performance in country c at time t, which can be the logarithm of sectoral output, value-added, employment, or capital. $\alpha_i(c)$ is a country-sector fixed-effect, $\delta_t(c)$ considers country time-varying controls (country's real GDP growth and credit spreads), and $\gamma_{i,t}(c)$ represents sector-country time-varying controls (log sectoral final consumption from households). $shock^{C}_{i,t-k}(c)$ measures the network spillover of sectoral commodity price shock at time $t-k$. The shock can describe upstream or downstream spillovers through the domestic production network (more details below).

In the next section, we define and briefly explain the network spillover measures that shape the effects of commodity price shocks on non-commodity sectoral output in a small open economy. These network measures are derived and described in more detail in Section 4. We basically extend the measures in Acemoglu et al. (2016)—developed to understand the propagation of productivity shocks and government spending shocks in a closed economy—for the context of a small open economy subject to commodity price shocks. As we will see in the next section, commodity price shocks, unlike productivity and government spending shocks, have a demand-side and a supply-side component.

3.1 Measuring Network Spillovers

We denote a commodity sector by $k \in \mathcal{K}$ where $\mathcal{K}$ is the set of commodity sectors. We denote non-commodity sectors by either i or j, where $i, j = 1, ..., N$ with N the total number of non-commodity sectors.

The downstream effect of commodity price shocks from k sectors to sector i in country c at

substantial volatility of commodity price shocks over time.

time $t-h$ is

$$\text{Down}_{i,t-h}(c) = \sum_{k \in \mathcal{K}} \sum_{j=1}^{N} \Psi_{ij}(c) \Omega_{jk}(c) \cdot \tilde{p}^{C}_{k,t-h}(c) \tag{6}$$

where $\Psi_{ij}(c)$ stands for the importance of sector j in supplying intermediate inputs to sector i both directly and indirectly through the domestic production network of country c. The element $\Omega_{jk}(c)$ measures the direct exposure of sector j to a change in the price of commodity sector k. Thus, $\sum_{j=1}^{N} \Psi_{ij}\Omega_{jk}$ measures the *network-adjusted commodity exposure* of sector i to commodity sector k. $\tilde{p}^{C}_{k,t-h}(c)$ corresponds to the identified commodity price shock for commodity sector k in country c at time $t-h$.

The matrix $\mathbf{\Psi}$ contains the direct and indirect *domestic* downstream network connections and is defined as

$$\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1}$$

where $\mathbf{\Omega}$ is the matrix of input-output shares collected from the OECD or WIOD databases. An element Ω_{ij} is the share of intermediates from sector j in gross output of sector i. Importantly, this matrix considers only domestic connections.

The upstream effect of commodity price shocks from sectors k to sector i in country c at time $t-h$ is measured as

$$\text{Up}_{i,t-h}(c) = \sum_{k \in \mathcal{K}} \sum_{k=1}^{N} \Psi^{U}_{ij}(c) m_{kj}(c) \cdot \tilde{p}^{C}_{k,t-h}(c) \tag{7}$$

where $\Psi^{U}_{ij}(c)$ stands for the direct and indirect importance of sector j in using intermediate inputs from sector i. Note that the element $m_{kj} = P_k M_{jk}/P_k Q_k$ measures the importance of sector j as a *buyer* to commodity sector k i.e. how much of the sales of the commodity sector are bought by sector j. Thus the element $\sum_{k=1}^{N} \Psi^{U}_{ij}(c) m_{kj}$ measures the importance of each sector j as a buyer to sector i in reaction to a change in the commodity price k.

The matrix $\mathbf{\Psi}^{U}$ is defined as

$$\mathbf{\Psi}^{U} = (\boldsymbol{I} - \boldsymbol{M}')^{-1}$$

where $\boldsymbol{M} = m_{ij} = P_j M_{ij}/P_j Q_j$ measures the importance of sector i as a buyer to sector j.

3.2 Network propagation

We now present empirical evidence on the transmission mechanism of commodity price shocks via production networks using the WIOD database. The WIOD database has an important advantage compared to the OECD database: it reports sectoral quantity and price indexes, allowing us to better study the channels in which commodity price shocks affect quantities and prices. Instead, the OECD data only reports nominal data (in US dollars) for sales, value-added and intermediate input use. To construct the Up and $Down$ network effects defined in Equation (6) and Equation (7) we use the input-output structure in 1995.

Table 3 presents the results of estimating Equation (5) using quantity indexes for value-added and gross output, number of employees, quantity index for fixed capital stock, and sectoral price indexes. All regressions include one lag of the dependent variable, one lag of the upstream and downstream shocks, country-sector fixed effects and controls such as country credit spreads, real GDP growth, and the log of final sectoral consumption. To ease the interpretation of our coefficients, we standardized our estimated commodity price shocks and expressed them with unit standard deviation. We first focus on the effects on upstream sectors (Up). Column 1 shows that real commodity price shocks positively affect the gross output of non-commodity sectors. In particular, a one standard deviation increase in commodity prices generates a 7.9 percent increase in the sectoral gross output quantity index. Columns 2 to 4 also show that a one standard deviation increase in commodity prices increases value-added (by 6.5 percent), employment (by 7.3 percent), and fixed capital (by 4.5 percent) at the sectoral level. We observe that commodity price shocks do not affect the sectoral prices of upstream industries.

Regarding the downstream ($Down$) effects of commodity price shocks, columns 1 to 4 show no statistically significant effect on gross output, value-added, employment, and capital of industries downstream to commodity sectors. We observe that a one standard deviation increase in commodity prices generates a 2 percent increase in prices of downstream sectors.

Table 3. Network Effects of Commodity Price Shocks on Non-Commodity Sectors (IO 1995)

	(1) GO	(2) VA	(3) E	(4) K	(5) P
Up	0.079***	0.065***	0.073***	0.045***	0.022
	(0.017)	(0.018)	(0.012)	(0.007)	(0.015)
Down	0.019	0.015	0.005	0.005	0.020**
	(0.013)	(0.014)	(0.006)	(0.004)	(0.009)
Obs.	2790	2790	2790	2790	2790
Adj. R^2	0.95	0.94	0.99	0.99	0.99
Country-Sector FE	✓	✓	✓	✓	✓
Controls	✓	✓	✓	✓	✓

Note: This table presents an OLS regression using sectoral log value-added quantity index and log sales quantity index as the dependent variables. The independent variables are one lag of the dependent variable, the indirect commodity price shocks propagated to upstream sectors (Up) and its one period lag, and the indirect commodity price shocks propagated to downstream industries (Down) and its one period lag. The regressions also control for country-sector fixed effects, log real sectoral consumption from households, GDP growth, and country spreads. Standard errors clustered at the country-sector level in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

In Table 10 we provide the results of estimating Equation (5) using the network measures in Acemoglu et al. (2016). These network measures are constructed for a closed economy and to understand the propagation of productivity (supply-side) and government spending (demand-side) shocks. We observe that the effects on non-commodity sectors are very similar to those in Table 10.

We also find similar results using the OECD input-output tables. Table 11 shows that, consistent with the WIOD evidence, commodity price shocks have positive effects on upstream and sectors' sales and value-added. Unlike the WIOD evidence, the OECD results show a positive effect on downstream sectors' output and value-added. This discrepancy is because the OECD dataset uses nominal values. Therefore, the positive results for downstream sectors might be driven by increases in sectoral prices (as column suggested by column 5 in Table 3.

The empirical evidence in this section points to strong upstream propagation of commodity prices, alongside a muted downstream propagation. In the next section, we build a theoretical model of a small open economy with production networks and a commodity sector that rationalizes the findings we document in this section. In particular, we ask how can a small open economy model with production networks, in which the commodity sector is a central supplier and user of intermediate inputs (as documented in Table 1), generate large upstream propagation but muted downstream propagation on quantities from commodity price shocks?

4 Theory

We consider a small open economy model at the steady state. This economy features a representative consumer and $N+1$ sectors/firms. Sectors up to N produce non-commodity goods, while sector $N+1$ produces a commodity. There is only one factor of production labor, which is inelastically supplied by the household and used to produce all goods. The representative consumer has income payments/debts from assets equal to rB. Both r and B are parameters in our model, and they will allow us to model the direct wealth effect. We now describe each block in turn.

4.1 Consumer's Preferences

Since we want to focus on the role of firms and how their linkages affect the propagation of commodity price shocks, we keep the demand side as simple as possible. We assume that the representative consumer at home has Cobb-Douglas preferences over all $N+1$ goods. In particular,

$$U(\{C_i\}_{i=1}^{N+1}) = \prod_{i=1}^{N+1} C_i^{\beta_i}. \tag{8}$$

The consumer receives income from labor and from firms' profits. Its budget constraint thus reads

$$\sum_{i=1}^{N+1} P_i C_i = W\bar{L} + \sum_{i=1}^{N+1} \Pi_i + rB, \tag{9}$$

where P_i denotes the price of good i, W is the wage, which we will set as the numeraire, $\bar{L}$ is labor supply, Π_i is firm i's profits and rB are assets at the steady state.

Taking prices of goods and labor as given, together with $r\bar{B}$, utility maximization subject to the budget constraint yields the usual first-order conditions

$$P_i C_i = \beta_i \left(W\bar{L} + \sum_{i=1}^{N+1} \Pi_j + rB \right) \qquad \text{for all } i = 1, ..., N+1. \tag{10}$$

We model both foreign demand for the commodity sector C^*_{N+1} and its price as a function of the rest of the world's income (Y^*)

$$C^*_{N+1} = \left(\frac{P_{N+1}}{P^*} \right)^{-\sigma^*_D} \frac{Y^*}{P^*} = \eta_C (Y^*)^{\frac{\sigma^*_S}{\sigma^*_S + \sigma^*_D}}, \tag{11}$$

$$P_{N+1} = (Y^*)^{\frac{1}{\sigma^*_S + \sigma^*_D}} (P^*)^{(\sigma^*_D - 1)\frac{\sigma^*_S}{\sigma^*_D + \sigma^*_S}} = \eta_{N+1} (Y^*)^{\frac{1}{\sigma^*_S + \sigma^*_D}}, \tag{12}$$

where Y^* is the nominal income of the rest of the world, P^* is its price index, σ^*_S and σ^*_D are the elasticity of *world* supply and demand of commodity sector. The constants $\eta_C = \eta_{N+1}^{-\sigma^*_D} (P^*)^{\sigma^*_D - 1}$ and $\eta_{N+1} = (P^*)^{(\sigma^*_D - 1)\frac{\sigma^*_S}{\sigma^*_D + \sigma^*_S}}$ depend on the price index of the rest of the world but are constant for our practical purposes in the small open economy.

This formulation allows us to explicitly talk about an increase in foreign income that induces an increase in both foreign consumption (exports) and the price of the commodity sector, consistent with the evidence in Section 3.

4.2 Firms.

We assume that there N non-commodity goods and one commodity sector (sector $N+1$). The main difference between domestic and commodity sectors is that the commodity sector features decreasing returns to scale in production. In contrast, the remaining N non-commodity sectors feature constant returns to scale. We describe the problem of each type of firm in turn.

4.2.1 Non-Commodity Sectors: $i = 1, 2, ..., N$

A representative firm in each i sector produces using a CES production function that combines labor and intermediate goods from other sectors. In particular, sector i operates using the following production function

$$Q_i = Z_i \left(a_i^{\frac{1}{\sigma_i^{VA}}} L_i^{\frac{\sigma_i^{VA}-1}{\sigma_i^{VA}}} + (1-a_i)^{\frac{1}{\sigma_i^{VA}}} M_i^{\frac{\sigma_i^{VA}-1}{\sigma_i^{VA}}} \right)^{\frac{\sigma_i^{VA}}{\sigma_i^{VA}-1}}, \tag{13}$$

where Q_i is gross output, Z_i is physical productivity, L_i is labor, and M_i is a composite of intermediate goods specified below. Importantly, σ_i^{VA} represents the elasticity of substitution between labor and the intermediate input bundle that we allow to vary across sectors.

Firm i aggregates intermediate inputs from all sectors using a CES aggregator

$$M_i = \left(\sum_{j=1}^{N+1} \omega_{ij}^{\frac{1}{\sigma_i^I}} M_{ij}^{\frac{\sigma_i^I-1}{\sigma_i^I}} \right)^{\frac{\sigma_i^I}{\sigma_i^I-1}},$$

where M_{ij} is the demand of firm i for inputs of firm j, σ_i^I is the elasticity of substitution across intermediates and ω_{ij} is a parameter. As usual, cost minimization of the intermediate input bundle defines its price index

$$P_i^I = \left(\sum_{j=1}^{N+1} \omega_{ij} P_j^{1-\sigma_i^I} \right)^{\frac{1}{1-\sigma_i^I}}.$$

Using a similar logic, we can compute the marginal cost of producing good i

$$MC_i = \frac{1}{Z_i} \left(a_i W^{1-\sigma_i^{VA}} + (1-a_i)(P_i^I)^{1-\sigma_i^{VA}} \right)^{\frac{1}{1-\sigma_i^{VA}}}.$$

Because this is a perfectly competitive economy, marginal costs should equal prices in all non-commodity sectors, then

$$P_i = MC_i = \frac{1}{Z_i} \left(a_i W^{1-\sigma_i^{VA}} + (1-a_i)(P_i^I)^{1-\sigma_i^{VA}} \right)^{\frac{1}{1-\sigma_i^{VA}}} \qquad \text{for all } i = 1, ..., N.$$

4.2.2 Commodity Sector: $i = N+1$

We model the commodity sector with the following production function

$$Q_{N+1} = Z_{N+1} \left(a_{N+1}^{\frac{1}{\sigma_{N+1}^{VA}}} L_{N+1}^{\frac{\sigma_{N+1}^{VA}-1}{\sigma^{VA}}} + (1-a_{N+1})^{\frac{1}{\sigma_{N+1}^{VA}}} M_{N+1}^{\frac{\sigma_{N+1}^{VA}-1}{\sigma^{VA}}} \right)^{\delta_{N+1} \frac{\sigma_{N+1}^{VA}}{\sigma_{N+1}^{VA}-1}} = Z_{N+1} B_{N+1}^{\delta_{N+1}},$$

$$B_{N+1} = \left(a_{N+1}^{\frac{1}{\sigma_{N+1}^{VA}}} L_{N+1}^{\frac{\sigma_{N+1}^{VA}-1}{\sigma^{VA}}} + (1-a_{N+1})^{\frac{1}{\sigma_{N+1}^{VA}}} M_{N+1}^{\frac{\sigma_{N+1}^{VA}-1}{\sigma^{VA}}} \right)^{\frac{\sigma_{N+1}^{VA}}{\sigma_{N+1}^{VA}-1}},$$

the same as in the other sectors but with decreasing returns to scale governed by $0 < \delta_{N+1} < 1$. We can think of B_{N+1} as a composite good made of labor and intermediate inputs, with corresponding price P^B_{N+1} that is then aggregated again with a Cobb-Douglas production function of the form $Z_{N+1} B_{N+1}^{\delta_{N+1}}$.

The commodity sector aggregates the intermediate input bundle in the same way as the other sectors i.e.

$$M_{N+1} = \left(\sum_{j=1}^{N+1} \omega_{N+1,j}^{\frac{1}{\sigma^I_{N+1}}} M_{N+1,j}^{\frac{\sigma^I_{N+1}-1}{\sigma^I_{N+1}}} \right)^{\frac{\sigma^I_{N+1}}{\sigma^I_{N+1}-1}} .$$

We let P^I_{N+1} to be the price of the intermediate input bundle defined below.

As before, cost minimization implies the following marginal cost curve for the commodity sector

$$P_{N+1} = \frac{1}{\delta_{N+1}} \left(\frac{1}{Z_{N+1}} \right)^{\frac{1}{\delta_{N+1}}} Q_{N+1}^{\frac{1-\delta_{N+1}}{\delta_{N+1}}} P^B_{N+1}$$

with

$$P^B_{N+1} = \left(a_{N+1} W^{1-\sigma^{VA}_{N+1}} + (1-a_{N+1})(P^I_{N+1})^{1-\sigma^{VA}_{N+1}} \right)^{\frac{1}{1-\sigma^{VA}_{N+1}}} ,$$
$$P^I_{N+1} = \left(\sum_{j=1}^{N+1} \omega_{N+1,j} P_j^{1-\sigma^I_{N+1}} \right)^{\frac{1}{1-\sigma^I_{N+1}}} .$$

Since the commodity sector features decreasing returns to scale in production, its marginal cost curve is increasing in quantity produced. This *scale effect* is absent in the non-commodity sector, where the marginal cost is independent of the scale of production due to the constant returns to scale production function.

4.3 Equilibrium

The resource constraints of sector i, with $i = 1, ..., N+1$, is:

$$Q_i = C_i + C_i^* + \sum_{j=1}^{N+1} M_{ji}, \tag{14}$$

where C_i is domestic consumption demand, M_{ji} is demand from firm j, and C_i^* is the external demand. Since the commodity sector is the only one that exports in this economy, we assume $C_i^* = 0$ for all $i \neq N+1$ but include it everywhere for completeness.

The labor market clearing reads

$$\sum_{i=1}^{N+1} L_i = \bar{L}. \tag{15}$$

A competitive equilibrium in this economy is defined in the usual way. Taking prices as given consumers and firms maximize utility and profits and all markets clear.

4.4 Characterization of Commodity Price Shocks Propagation

Our goal in this section is to show how changes in the commodity price P_{N+1} propagate throughout the economy and affects the sectoral output. Before fully characterizing it, we need to define some notation that we will use repeatedly.

Notation. We use **bold** letters to refer to vectors and matrices. We use a hat notation to denote changes relative to a given equilibrium i.e. $\hat{X} = d\log X = \log X - \log X^*$. We define our notation for matrices and vectors in the following table.

Table 4. Notation

Notation	Typical Element	Comment
Matrices ($N \times N$, domestic sectors only)		
$\boldsymbol{\Omega}$	$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$	Expenditure of goods from j by sector i
$\boldsymbol{\Psi} = (I - \boldsymbol{\Omega})^{-1}$	Ψ_{ij}	Importance of j as a direct and indirect supplier to i
$\boldsymbol{M}$	$m_{ij} = \frac{M_{ij}}{Q_j}$	How much of good j is allocated to sector i
$\boldsymbol{\Psi}^U = (I - \boldsymbol{M}')$	Ψ^U_{ij}	Importance of j as a direct and indirect buyer to i
Vectors ($N \times 1$)		
$\boldsymbol{\Omega}_0$	$\Omega_{0i} = \frac{P_i C_i}{GDP}$	
$\boldsymbol{\Omega}_{N+1}(b)$	$\Omega_{N+1,j} = \frac{P_j M_{N+1,j}}{P_{N+1} Q_{N+1}}$	Importance of good j as a direct supplier to sector $N+1$
$\boldsymbol{\Omega}_{N+1}(s)$	$\Omega_{i,N+1} = \frac{P_{N+1} M_{i,N+1}}{P_i Q_i}$	Importance of sector $N+1$ as a direct supplier to sector i
$\boldsymbol{\lambda} = \boldsymbol{\Psi}'\boldsymbol{\Omega}_0 + \boldsymbol{\Psi}'(\boldsymbol{\Omega}_{N+1}(b))\lambda_{N+1}$	$\lambda_i = \frac{P_i Q_i}{GDP}$	Domar Weight of Sector i

4.4.1 Change in Domestic Prices.

The following proposition characterizes changes in domestic prices up to a first-order that are going to be useful later.

Proposition 1 *Consider a differential change in the commodity price, $\hat{P}_{N+1}$. Up to a first-order, the change in domestic prices satisfy*

$$\hat{P}_i = \left(\sum_{k=1}^{N} \Psi_{ik} \Omega_{k,N+1} \right) \hat{P}_{N+1}. \tag{16}$$

Proof. See Appendix C.1. ■

This proposition captures both the direct and indirect exposure of sector i to changes in the commodity price $\hat{P}_{N+1}$. This exposure depends on how sectors react to the initial change in commodity price. Consider, for example, a given producer $k \neq i$ within the domestic production network. The direct effect of a change in commodity price on its own price is given by $\Omega_{k,N+1}$. To understand how this effect affects producer's i, we need to track all the many rounds it propagates throughout the domestic production network, an effect captured by Ψ_{ik}. The *network-adjusted* exposure to the commodity sector is a weighted average of these differential effects of each producer k.

4.4.2 Changes in quantity of commodity sector : Q_{N+1}

The following proposition characterizes changes in quantities in the commodity sector

Proposition 2 *Consider a differential change in the commodity price, $\hat{P}_{N+1}$. Up to a first-order, $\hat{Q}_{N+1}$ satisfies*

$$\hat{Q}_{N+1} = \phi^{N+1}_{P_{N+1}} \hat{P}_{N+1}, \tag{17}$$

where

$$\begin{aligned}
\phi^{N+1}_{P_{N+1}} &= \frac{\delta_{N+1}}{1-\delta_{N+1}} \left(1 - b_{N+1}\tilde{\omega}_{N+1,N+1}\right) - \frac{\delta_{N+1}}{1-\delta_{N+1}} b_{N+1} \left(\sum_{i=1}^{N} \tilde{\omega}_{N+1,i} \left(\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1} \right) \right), \\
b_{N+1} &= \frac{P^I_{N+1} M^I_{N+1}}{P^B_{N+1} B_{N+1}}, \\
\tilde{\omega}_{ij} &= \frac{P_j M_{ij}}{P^I_i M_i}.
\end{aligned}$$

Proof. See Appendix C.1. ■

The above proposition highlights that the commodity sector's output change is a function of the price of the commodity sector $\phi^{N+1}_{P_{N+1}}$, which can be either positive or negative. The intuition for this result is as follows. At an optimum, marginal costs and prices should equalize for the commodity sector. An exogenous increase in its price rises the commodity sector revenues forcing it to increase its marginal cost along the marginal cost curve, which, all else equals, pressures the commodity sector to produce more. Yet, the commodity price increase also shifts the marginal cost curve upwards since its cost of production is going up, for any given production level, as all sectors (including the commodity sector) experience a cost shift in their input. The effect of the commodity price increase in the commodity sector quantity depends on the strength of these two effects. Although theoretically ambiguous, the empirically relevant case is when this effect is positive.

4.4.3 Changes in quantity of domestic sectors: Q_i

We are now ready to define changes in the production of domestic sectors. We sum up these changes in the following proposition, the main proposition of this paper.

Proposition 3 *Consider a differential change in the commodity price, $\hat{P}_{N+1}$. Up to a first-order, $\hat{Q}_i$, for $i \neq N+1$, satisfies*

$$\hat{Q}_i = (\zeta_i^{N+1} + \zeta_i^Q \phi_{P_{N+1}}^{N+1})\hat{P}_{N+1}, \tag{18}$$

where

$$\begin{aligned}
\zeta_i^{N+1} &= \underbrace{-\alpha_{N+1}\alpha_B \sum_{k=1}^{N} \Psi_{ik}^U \frac{\Omega_{0k}}{\lambda_k}}_{\textit{Wealth Effect}} + \underbrace{\left(\alpha_{N+1} + \left(\sum_{k=1}^{N} \Psi_{ik}^U \left(\xi_{ki}^{N+1} m_{ki} + m_{N+1,k}\left[\xi_{N+1,k}^{N+1} + (1-\alpha_{N+1})\right]\right)\right)\right)}_{\textit{Buyers' Substitution}} \\
&\quad - \underbrace{\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1}}_{\textit{Pure Downstream Effect}} \\
\zeta_i^Q &= -\alpha_{N+1}\alpha_B \sum_{k=1}^{N} \Psi_{ik}^U \frac{\Omega_{0k}}{\lambda_k} + \left(\alpha_{N+1} + \left(1 - \alpha_{N+1} - \xi_{N+1}^Q\right)\left(\sum_{k=1}^{N} \Psi_{ik}^U m_{N+1,k}\right)\right) \\
\alpha_{N+1} &= \frac{\Pi_{N+1}}{GDP} \\
\alpha_B &= \frac{rB}{\bar{L} + \Pi_{N+1} + rB} \\
\xi_{ji}^{N+1} &= (1-\sigma_j^I)\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1} + (\sigma_j^I - \sigma_j^{VA})\left(\sum_{i=1}^{N} \tilde{\omega}_{ji}\left(\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{j,N+1}\right) \\
&\quad + (\sigma_j^{VA} - 1)\sum_{k=1}^{N} \Psi_{jk}\Omega_{k,N+1} \quad \textit{for all } j = 1, ..., N \\
\xi_{N+1}^Q &= \frac{(1-\delta_{N+1})(\sigma_{N+1}^{VA} - 1)}{\delta_{N+1}} \\
\xi_{N+1,i}^{N+1} &= (1-\sigma_{N+1}^I)\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1} + (\sigma_{N+1}^I - \sigma_{N+1}^{VA})\left(\sum_{i=1}^{N} \tilde{\omega}_{N+1,i}\left(\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{N+1,N+1}\right) \\
&\quad + (\sigma_{N+1}^{VA} - 1).
\end{aligned}$$

Proof. See Appendix C.1. ■

The commodity price shock affects quantities in all sectors through three different channels captured by the term ζ_i^{N+1}, which is dampened/reinforced by the second term $\zeta_i^Q \phi_{P_{N+1}}^{N+1}$. The first one, which we label *wealth effect*, is standard in the small open economy literature. It hinges on how the increase in income of the domestic economy that follows the increase in commodity prices allows the consumer to increase its consumption relative to what is produced within borders, GDP. The strength of this effect is governed by the size of profits of the commodity sector in value-added, α_{N+1} and, more importantly, its direction depends on whether at the initial equilibrium, the economy is a net debtor or net creditor, $\alpha_B \neq 0$. For example, an indebted economy at the steady-state, $\alpha_B < 0$, implies that expenditure is lower than GDP. As a result, the share of the commodity sector's profits on expenditure is higher than its share of GDP. In this case, the commodity sector's profits are more important for expenditure than for GDP, thus allowing the consumer to increase its share of consumption over GDP for all goods i in response to an increase in the commodity price. Note that this effect is propagated upstream from consumers to sellers and is thus a demand shock for these producers.

The *buyers' substitution effect* captures how intermediate inputs' buyers try to avoid the increase in the price of the commodity sector and redirect their demands towards other inputs. On the one hand, buyers substitute away from commodity inputs and demand more intermediate inputs from other sectors. This effect is the upstream effect highlighted in Carvalho et al. (2021), where substitution of producers away from some inputs to others leads to an upstream propagation of shocks. Yet, this effect is not coming from a productivity shock but rather from a change in the commodity price. On the other hand, the *buyers' substitution effect* implies a muted downstream cost-channel propagation of commodity price shocks. One can more clearly see this channel in the last term of ξ_{ki}^{N+1}

$$\begin{aligned}\xi_{ki}^{N+1} = &(1-\sigma_k^I)\sum_{l=1}^{N}\Psi_{il}\Omega_{l,N+1} + (\sigma_k^I - \sigma_k^{VA})\left(\sum_{i=1}^{N}\tilde{\omega}_{ki}\left(\sum_{l=1}^{N}\Psi_{il}\Omega_{l,N+1}\right) + \tilde{\omega}_{k,N+1}\right)\\ &+ (\sigma_k^{VA}-1)\sum_{l=1}^{N}\Psi_{kl}\Omega_{l,N+1}.\end{aligned}$$

The muted downstream propagation occurs indirectly in two cases. When buyers of sector i (sector k) are less vulnerable to shocks to their suppliers, say sector l. Or when sector k can easily substitute intermediates for labor (high σ^{VA}), shocks to its supplier l that permeate the network through the term $\Psi_{kl}\Omega_{l,N+1}$ generate smaller reductions in sector k's demand for sector i's goods.

The third and final effect that we label *pure downstream effect*, captures the increase in marginal cost experienced by sector i after a change in the commodity price. This effect depends on how prices in the other k sectors react in the economy to the change in commodity price, which in this model is captured by the terms $\Psi_{ik}\Omega_{k,N+1}$. This is a pure downstream effect since it is passed through marginal costs. In this sense, commodity price shocks are akin to the micro productivity shocks extensively studied in the production networks literature (Acemoglu et al., 2012; Baqaee and Farhi, 2019). Nevertheless, as shown above, with CES technologies, this downstream effect is contaminated, be muted, or amplified by the *buyers' substitution effect*.

5 Quantitative exploration

Our theoretical model in the previous section highlighted three channels by which a change in the commodity price affects the groos output of non-commodities. The sign and importance of these channels depend on the presence of assets/debts (α_B), the degree of decreasing returns to scale in the commodity sector (δ_{N+1}), the importance of profits on domestic production (α_{N+1}), the elasticities of substitution among intermediates (σ^I) and the elasticities of substitution between labor and the intermediate input bundle (σ^{VA}). In this section, we calibrate the model to study whether it can match the results of Section 3.

The empirical results in Section 3 show that following an increase in commodity prices, there is positive upstream propagation and a null downstream propagation on quantities produced in non-commodity sectors. While our model predicts a positive upstream effect and a negative downstream effect, these effects are muted or amplified by the model's structural parameters. In particular, the wealth effect of commodity price and substitutability between intermediate inputs amplify the upstream propagation channel. On the other hand, the downstream propagation channel can be muted by a high elasticity of substitution between intermediates and labor. This high elasticity implies that downstream sectors to commodities only pass a small fraction of the increased production cost to their customer as these sectors can use less expensive labor.

To better understand the quantitative importance of these different channels, we calibrate our model to match the Australian production structure in 1995. We calibrate the input-output parameters ω_{ij} and a_j assuming the economy starts at a symmetric equilibrium with $P_j = 1$ for all j. This way, we have that $\omega_{ij} = \frac{P_j M_{ij}}{P_i^M M_i}$ and $1 - a_j = \frac{P_j^M M_j}{P_j Q_j}$ equal the observed intermediate input shares. We assume $\delta_{N+1} = 0.9$, the parameter that governs the decreasing returns to scale in the commodity sector, and vary the values of α_B and α_{N+1}. The version of the model that shuts down the wealth effect assumes $\alpha_B = 0$ and $\alpha_{N+1} = 0$, while the version with wealth effect assumes $\alpha_B = -0.65$ (debt to GDP ratio) and $\alpha_{N+1} = 0.053$ (commodity profits to GDP ratio).

We simulate series from the model hitting the economy with sequences of commodity price shocks. We use these series to estimate Equation (5). Table 5 and Table 6 report the model implied coefficients on the effect of commodity price shocks on gross output when we shut down the wealth effect and when we allow it to exist, respectively. We start by looking at how upstream and downstream propagation affects quantities, allowing the elasticity between intermediates and labor to move but fixing the elasticity among intermediate inputs.[5] Looking at Panel (a) of Table 5, we observe that the upstream effect on quantities declines as we increase σ^{VA}. The intuition for this result comes from intermediate-labor substitution. As buyers of a given sector i experience an increase in their intermediate input bundle due to the commodity price shock, they can substitute away from intermediates toward labor, effectively reducing their demand for the output of intermediate sectors. The latter mechanism explains the reduction of the Up

[5]Several papers have estimated elasticities of substitution between inputs (see for example, Huneeus (2020), Miranda-Pinto (2021), Carvalho (2014), and Miranda-Pinto and Young (2021)). Our goal here is to characterize the mechanisms of propagation using values of the elasticity that lie in the range of values estimated in the literature.

coefficient and is statistical significance. We also observe that the downstream effect moves in the opposite direction but is not statistically significant in all cases, consistent with our empirical results.

Turning to Panel (b) of Table 5 highlights the relevance of complementarities across intermediate inputs. When $\sigma^I < 1$ and is also lower than σ^{VA}, we get a negative, although insignificant upstream coefficient. The upstream effect becomes negative because of complementarities: one input in the bundle is becoming more expensive, and since all other goods are complement in production, the demand for all sectors declines. Of course, the upstream effect turns positive when we move from complements to substitutes in the intermediate input bundle as these effects are reversed. In contrast to Panel (A), we find a negative downstream effect on quantities when we allow for substitution among intermediate inputs. Since substitution across intermediate inputs is now easier, demand for each input goes up by other producers, which drives prices of other non-commodity sectors up, ultimately impacting a given sector's i marginal costs through the production network.

Table 5. Network effects of commodity price shocks on non-mining sectors' output. No wealth effect ($\alpha_B = 0$ and $\alpha_{N+1} = 0$)

Dependent variable: $\log Q_i$							
	Panel A: σ^I = 1.1				*Panel B:* σ^{VA} = 1.1		
	(1)	(2)	(3)		(4)	(5)	(6)
σ^{VA}	1.2	1.4	1.6	σ^I	0.8	1.2	1.4
Up	0.096***	0.040	0.039	Up	-0.011	0.083***	0.115***
	(3.733)	(1.471)	(1.447)		(-0.487)	(4.125)	(6.173)
Down	-0.007	0.012	0.014	Down	0.015	-0.036**	-0.032**
	(-0.257)	(0.435)	(0.521)		(0.68)	(-1.812)	(-1.725)
N	4,950	4,950	4,950	N	4,950	4,950	4,950

Note: This table presents an OLS regression using the model implied logarithm of sectoral output as the dependent variable. The independent variables are the indirect commodity price shocks propagated to upstream sectors (Up) and the indirect commodity price shocks propagated to downstream industries (Down). T-statistics in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

We now turn to Table 6. Overall, we find the same patterns: the upstream coefficients (i) decline for higher σ^{VA} given σ^I and (ii) increase for higher σ_I given σ^{VA}. We also find the same results for the downstream coefficients i.e. no effect on Panel (A) and negative and significant effects in Panel (B). However, our upstream coefficients are now an order of magnitude higher when we include the *wealth effect.* This result, although expected, is quite important quantitatively, as it increases the upstream coefficient by around 40 percent (from 0.096 to 0.134 in the first column of Panel (A) in both tables). As we discussed in the previous section, the fact that the representative consumer gets the profits from the commodity sector allows it to consume more when the commodity price goes up. This effect is important, but it is amplified by the presence of production networks that transmit these shocks further upstream to other producers.

Finally, note that the magnitudes of the upstream coefficients in our calibration exercise are roughly in line with those of Section 3. Indeed, in Table 3 we find an upstream coefficient equals to 0.069 (first column), while our estimates in Table 5 and Table 6 range from 0.069 and 0.115. However, we take these results with a grain of salt, as our model is highly stylized.

Overall, these exercises suggest that our simplified calibration can qualitatively, and to some extent quantitatively, match the results in Section 3. Importantly, all three channels we highlighted in Section 3 are important to understand how commodity price shocks propagate to non-commodity sectors domestically.

Table 6. Network effects of commodity price shocks on non-mining sectors' output. Positive wealth effect ($\alpha_B = 0$ and $\alpha_{N+1} = 0$)

Dependent variable: $\log Q_i$							
Panel A: $\sigma^I = 1.1$				*Panel B:* $\sigma^{VA} = 1.1$			
	(1)	(2)	(3)		(4)	(5)	(6)
σ^{VA}	1.2	1.4	1.6	σ^I	0.8	1.2	1.4
Up	0.134***	0.070**	0.069**	Up	0.020	0.106***	0.155***
	(5.32)	(2.568)	(2.553)		(0.929)	(5.277)	(8.330)
Down	-0.007	0.011	0.013	Down	0.014	-0.041**	-0.037**
	(-0.286)	(0.387)	(0.473)		(0.627)	(-2.026)	(-1.982)
N	4,950	4,950	4,950	N	4,950	4,950	4,950

Note: This table presents an OLS regression using the model implied logarithm of sectoral output as the dependent variable. The independent variables are the indirect commodity price shocks propagated to upstream sectors (Up) and the indirect commodity price shocks propagated to downstream industries (Down). T-statistics in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

6 Conclusion

We study how sectoral commodity price shocks propagate through domestic production networks in small open economies. We provide empirical evidence and a theoretical model that highlight the role of the well-known wealth effect and the role of production elasticities in shaping the propagation of commodity price shocks along the production chain. We first show that commodity sectors are central sectors, both as sellers and buyers, in the domestic production network of small open economies. We then show, empirically and theoretically, that the propagation of sectoral commodity price shocks to non-commodity sectors' output has an important production network component. We find that gross output, value-added, employment, and capital of non-commodity upstream sectors, those sectors supplying intermediate inputs to commodity sectors, largely respond to commodity price shocks. In contrast, we find evidence of muted downstream propagation, to those buying intermediate inputs from commodity sectors.

We develop a small open economy model featuring domestic production networks and characterize the transmission channels of commodity price shocks through production chains. We highlight two upstream channels (from input demands and the wealth effect of increased household income) and one downstream channel (increased input costs). We show that the wealth effect and the elasticities of substitution between inputs are crucial in amplifying or dampening the upstream and downstream channels. In particular, the wealth effect operates as follows. An increase in commodity prices rises income of the household and, therefore, increases domestic consumption, which then increases demand for domestic intermediate inputs. The role of production elasticities works as follows. When sectors present high substitutability between intermediate inputs, a higher commodity price increases demand for non-commodity intermediates, which generates an increase in the production of upstream sectors. On the other hand, when sectors present high substitutability between intermediates and labor, increases in commodity prices have smaller effects on marginal costs of sectors downstream to commodities. Thus, there exists a dampened effect on quantities of industries downstream to commodity sectors.

All in all, our results highlight the importance of the production network in propagating commodity price shocks throughout the economy.

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A Data sources and Definitions

Macroeconomic Aggregates

The data for the estimation of commodity price shocks are obtained from the following sources: OECD Quarterly National Accounts database and IMF-IFS for the macroeconomic aggregates; FRED for the Real Effective Exchange Rate (REER); World Bank, Bloomberg and Eurostat for the EMBI index (interest rate 3-month period was used when there was missing data on the spread measure); finally, Fernández et al. (2018) is used for the sectoral commodity price index. The aggregate macroeconomic variables are: nominal GDP, Private Consumption, Gross Fixed Capital Formation and Trade Balance, all given in nominal local currency. The variables are transformed to real terms using the 2010 GDP deflator as base year. The REER also uses 2010 as base year.

Input-Output Table Database

WIOD Data. Our main database is the World Input-Output database (Timmer et al., 2015), release 2013. It provides information on intersectoral and across country final and intermediate flows for 40 countries and 35 sectors classified according to the International Standard Industrial Classification revision 3 (ISIC Rev. 3). These tables match the 1993 version of the SNA. We use the sectoral data on quantities (gross output, value added, number of employees, and capital) and price indexes for the period 1995-2011(2009) in the National IO tables. The sample of small open economies with data on commodity prices and WIOD input-output data includes the following countries: Australia, Bulgaria, Brazil, Canada, Denmark, India, Lithuania, Mexico, and Russia.

This dataset is freely available here https://www.rug.nl/ggdc/valuechain/wiod/wiod-2013-release.

OECD Input-Output Structural Analysis (STAN). We obtained input-output data for 45 sectors and the period 1995-2018 from the OECD 2021ed. database. We collected the following variables measured in millions of USD: sales, value added, and private consumption. The sample of small open economies with data on commodity prices and OECD input-output data includes the following countries: Argentina, Australia, Brazil, Bulgaria, Canada, Chile, Colombia, Costa Rica, Croatia, Denmark, India, Indonesia, Lithuania, Malaysia, Mexico, Philippines, Russia, South Africa, and Thailand. This dataset is freely available here https://stats.oecd.org/Index.aspx?DataSetCode=IOTS$_2$021.

Commodity Data

To measure sectoral linkages to the *commodity sector* we use detailed information on each country's commodity bundle composition from Fernández et al. (2018). There is a total of 44 commodities classified according to the Harmonized System (HS) 1992 – 4 digits. We separate commodities into 3 groups: Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying;and Food Products, Beverages and Tobacco.

B Additional Tables and Figures

Table 7. Sectors in WIOD Database

Sector Number	Sector Name
1	Agriculture, Hunting, Forestry and Fishing
2	Mining and Quarrying
3	Food, Beverages and Tobacco
4	Textiles and Textile Products
5	Leather, Leather and Footwear
6	Wood and Products of Wood and Cork
7	Pulp, Paper, Paper , Printing and Publishing
8	Coke, Refined Petroleum and Nuclear Fuel
9	Chemicals and Chemical Products
10	Rubber and Plastics
11	Other Non-Metallic Mineral
12	Basic Metals and Fabricated Metal
13	Machinery, Nec
14	Electrical and Optical Equipment
15	Transport Equipment
16	Manufacturing, Nec; Recycling
17	Electricity, Gas and Water Supply
18	Construction
19	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles
20	Wholesale Trade and Commission Trade
21	Retail Trade
22	Hotels and Restaurants
23	Inland Transport
24	Water Transport
25	Air Transport
26	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
27	Post and Telecommunications
28	Financial Intermediation
29	Real Estate Activities
30	Renting of M&Eq and Other Business Activities
31	Public Admin and Defence; Compulsory Social Security
32	Education
33	Health and Social Work
34	Other Community, Social and Personal Services

Table 8. Commodities and WIOD Industries.

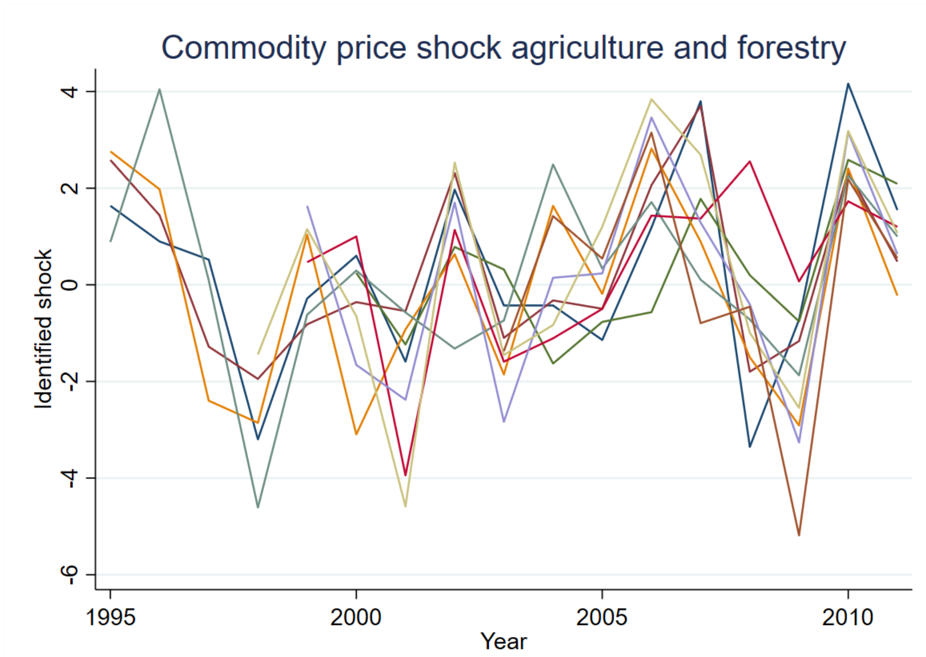
Commodity	HS Code	Industry
Beef	201	Agriculture, hunting, forestry and fishing
Pork	203	Agriculture, hunting, forestry and fishing
Lamb	204	Agriculture, hunting, forestry and fishing
Chicken	207	Agriculture, hunting, forestry and fishing
Fish	301	Agriculture, hunting, forestry and fishing
Fish Meal	304	Agriculture, hunting, forestry and fishing
Shrimp	306	Agriculture, hunting, forestry and fishing
Bananas	803	Agriculture, hunting, forestry and fishing
Coffee	901	Agriculture, hunting, forestry and fishing
Tea	902	Agriculture, hunting, forestry and fishing
Wheat	1001	Agriculture, hunting, forestry and fishing
Barley	1003	Agriculture, hunting, forestry and fishing
Corn	1005	Agriculture, hunting, forestry and fishing
Rice	1006	Agriculture, hunting, forestry and fishing
Soybeans	1201	Agriculture, hunting, forestry and fishing
Groundnuts	1202	Agriculture, hunting, forestry and fishing
Wool	1505	Agriculture, hunting, forestry and fishing
Sugar	1701	Agriculture, hunting, forestry and fishing
Cocoa	1801	Agriculture, hunting, forestry and fishing
Natural Rubber	4001	Agriculture, hunting, forestry and fishing
Hides	4101	Agriculture, hunting, forestry and fishing
Hard Log	4401	Agriculture, hunting, forestry and fishing
Soft Log	4403	Agriculture, hunting, forestry and fishing
Hard Swan	4407	Agriculture, hunting, forestry and fishing
Soft Swan	4408	Agriculture, hunting, forestry and fishing
Cotton	5201	Agriculture, hunting, forestry and fishing
Iron	2601	Mining and quarrying
Copper	2603	Mining and quarrying
Nickel	2604	Mining and quarrying
Aluminum	2606	Mining and quarrying
Lead	2607	Mining and quarrying
Zinc	2608	Mining and quarrying
Tin	2609	Mining and quarrying
Coal	2701	Mining and quarrying
Crude Oil	2709	Mining and quarrying
NatGas	2711	Mining and quarrying

Continuation of Table 8

Commodity	HS Code	Industry
Uranium	2844	Mining and quarrying
Gold	7108	Mining and quarrying
Soybean Meal	1208	Food products, beverages and tobacco
Soy Oil	1507	Food products, beverages and tobacco
Olive Oil	1509	Food products, beverages and tobacco
Palm Oil	1511	Food products, beverages and tobacco
Sun Oil	1512	Food products, beverages and tobacco
Coconut Oil	1513	Food products, beverages and tobacco

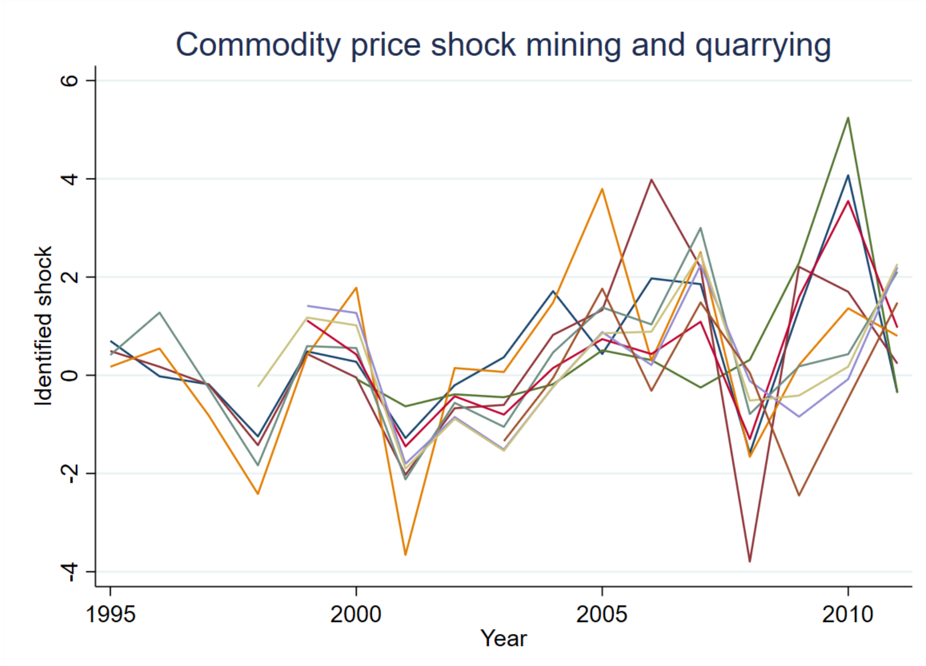
Extra Figures

Figure 2. Identified Commodity Price Shock: Agriculture and Forestry



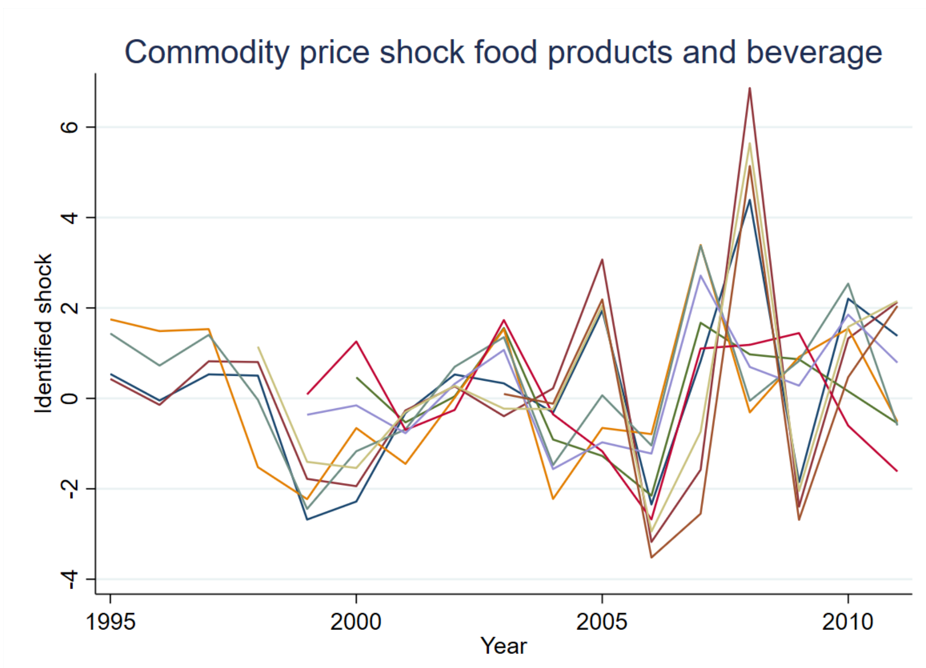
Note: This figure plots our SVAR identified commodity price shock for agriculture and forestry for our sample of 9 countries in the WIOD database. The shocks are estimated at the quarterly level and we convert them to annual by taking averages.

Figure 3. Identified Commodity Price Shock: Mining and Quarrying



Note: This figure plots our SVAR identified commodity price shock for mining and quarrying for our sample of 9 countries in the WIOD database. The shocks are estimated at the quarterly level and we convert them to annual by taking averages.

Figure 4. Identified Commodity Price Shock: Food Products and Beverages



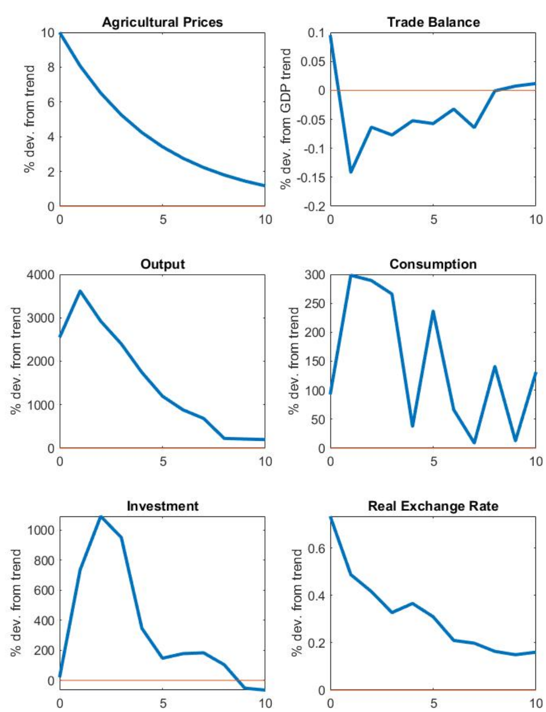
Note: This figure plots our SVAR identified commodity price shock for foods sectors for our sample of 9 countries in the WIOD database. The shocks are estimated at the quarterly level and we convert them to annual by taking averages.

Table 9. Average Pairwise Correlation Across Commodity Shocks (OECD sectors)

	Agriculture	Fishing	Mining	Food
Agriculture	1			
Fishing	0.38	1		
Mining	0.42	0.38	1	
Food	0.32	0.12	0.26	1

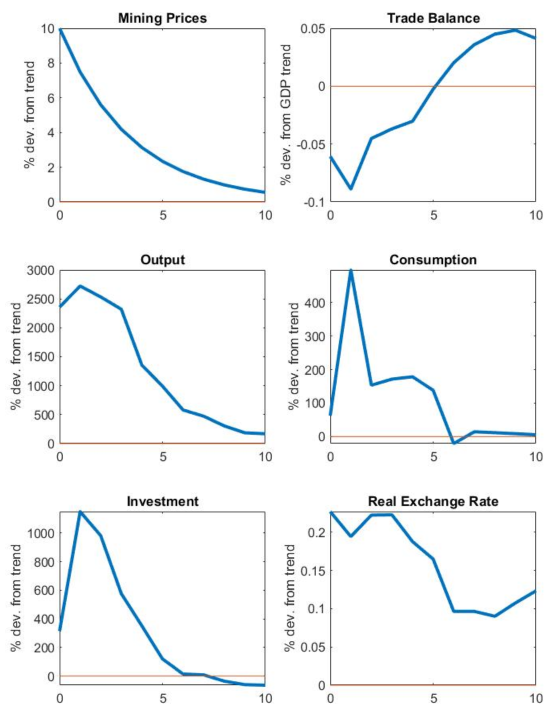
Note: This table presents the cross-country average of the within country pairwise correlations among the estimated sectoral commodity price shocks for the OECD sectoral classification.

Figure 5. IRFs Identified Commodity Price Shock to Agriculture



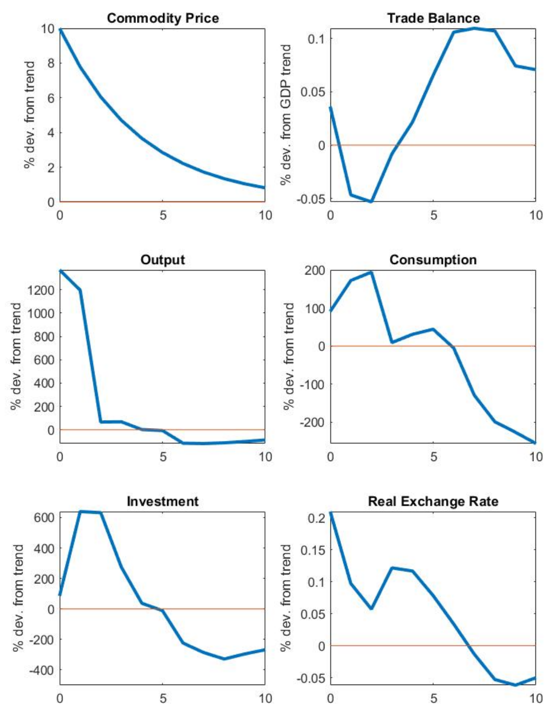
Note: This figure plots the median IRF from our SVAR to an identified commodity price shock to agriculture.

Figure 6. IRFs Identified Commodity Price Shock to Mining



Note: This figure plots the median IRF from our SVAR to an identified commodity price shock to mining.

Figure 7. IRFs Identified Commodity Price Shock to Food Products and Beverages



Note: This figure plots the median IRF from our SVAR to an identified commodity price shock for food products and beverages.

Additional evidence on the network effects of commodity price shocks

Table 10. Network Effects of Commodity Price Shocks on Non-Commodity Sectors (network measures in Acemoglu et al. (2016), WIOD data)

	(1) GO	(2) VA	(3) E	(4) K	(5) P
Up	0.070***	0.064***	0.061***	0.046**	0.010
	(0.019)	(0.021)	(0.018)	(0.020)	(0.030)
Down	-0.003	-0.015	-0.023	-0.002	0.003
	(0.016)	(0.023)	(0.018)	(0.017)	(0.014)
Obs.	2790	2790	2790	2790	2790
Adj. R^2	0.84	0.83	0.99	0.99	0.92
Country-Sector FE	✓	✓	✓	✓	✓
Controls	✓	✓	✓	✓	✓

Note: This table presents an OLS regression using sectoral logarithm of gross output and value-added, the number of employees, the logarithm of capital stock and sectoral prices as the dependent variables, for non-commodity sectors. The independent variables are: one lag of the dependent variable, the indirect commodity price shocks propagated to upstream sectors (Up) and its one period lag, and the indirect commodity price shocks propagated to downstream industries ($Down$) and its one period lag. The regressions also control for country-sector fixed effects, log sectoral real consumption from households, GDP growth, and country spreads. Standard errors clustered at the country-sector level are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 11. Network Effects of Commodity Price Shocks on Non-Commodity Sectors (OECD data)

	(1) VA	(2) GO
Up	0.007***	0.007***
	(0.002)	(0.002)
Down	0.009***	0.008***
	(0.002)	(0.001)
Obs.	12840	12840
Adj. R^2	0.99	0.99
Country-Sector FE	✓	✓
Controls	✓	✓

Note: This table presents an OLS regression using sectoral log value-added and log sales as the dependent variable, all denominated in USD dollars, for non-commodity sectors. The independent variables are: one lag of the dependent variable, the indirect commodity price shocks propagated to upstream sectors (Up) and its one period lag, and the indirect commodity price shocks propagated to downstream industries (Down) and its one period lag. The regressions also control for country-sector fixed-effects, log sectoral real consumption from households, GDP growth, and country spreads. Standard errors clustered at the country-sector level are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

C Theory

In this appendix, we provide a detailed derivation of the model's propositions we derived in Section 4.

C.1 Proofs.

Proof of Proposition 1. Using Shephard's lemma the change in domestic prices, $i = 1, ..., N$, to changes in productivity, the commodity price and other prices, can be written as

$$\hat{P}_i = -\hat{Z}_i + \sum_{j=1}^{N} \Omega_{ij} \hat{P}_j + \Omega_{i,N+1} \hat{P}_{N+1}$$

Solving for domestic prices yield

$$\hat{P}_i = -\sum_{k=1}^{N} \Psi_{ik} \hat{Z}_k + \sum_{k=1}^{N} \Psi_{ik} \Omega_{k,N+1} \hat{P}_{N+1} \tag{19}$$

Setting $\hat{Z}_k = 0$ for all k yields the expression in the proposition. ■

Proof of Proposition 2. Before proceeding, recall that the change in the intermediate input price index for each sector j is

$$\hat{P}_j^I = \sum_{i=1}^{N+1} \omega_{ji} \left(\frac{P_i}{P_j^I} \right)^{1-\sigma_j^I} \hat{P}_i = \sum_{i=1}^{N+1} \tilde{\omega}_{ji} \hat{P}_i$$

where by Shephard's Lemma

$$\tilde{\omega}_{ji} = \omega_{ji} \left(\frac{P_i}{P_j^I} \right)^{1-\sigma_j^I} = \frac{P_i M_{ji}}{P_j^I M_j}$$

The marginal cost curve of the commodity sector is such that

$$\begin{aligned}
P_{N+1} &= MC_{N+1} \\
&= \frac{1}{\delta_{N+1}} \left(\frac{1}{Z_{N+1}} \right)^{\frac{1}{\delta_{N+1}}} Q_{N+1}^{\frac{1-\delta_{N+1}}{\delta_{N+1}}} P_{N+1}^B \\
&= \frac{1}{\delta_{N+1}} \left(\frac{1}{Z_{N+1}} \right)^{\frac{1}{\delta_{N+1}}} Q_{N+1}^{\frac{1-\delta_{N+1}}{\delta_{N+1}}} \left(a_{N+1} W^{1-\sigma_{N+1}^{VA}} + (1-a_{N+1})(P_{N+1}^I)^{1-\sigma_{N+1}^{VA}} \right)^{\frac{1}{1-\sigma_{N+1}^{VA}}}
\end{aligned}$$

Log-differentiating this expression, remember that we set $W = 1$ as the numeraire, and rearranging yields

$$\hat{Q}_{N+1} = \frac{\delta_{N+1}}{1-\delta_{N+1}}\hat{P}_{N+1} + \frac{1}{1-\delta_{N+1}}\hat{Z}_{N+1} - \frac{\delta_{N+1}}{1-\delta_{N+1}}\frac{P^I_{N+1}M^I_{N+1}}{P^B_{N+1}B_{N+1}}\hat{P}^I_{N+1}$$

$$\hat{Q}_{N+1} = \frac{\delta_{N+1}}{1-\delta_{N+1}}\hat{P}_{N+1} + \frac{1}{1-\delta_{N+1}}\hat{Z}_{N+1} - \frac{\delta_{N+1}}{1-\delta_{N+1}}b_{N+1}\hat{P}^I_{N+1} \qquad \text{with } b_{N+1} = \frac{P^I_{N+1}M^I_{N+1}}{P^B_{N+1}B_{N+1}}$$

$$\begin{aligned}\hat{Q}_{N+1} = {} & \frac{\delta_{N+1}}{1-\delta_{N+1}}\hat{P}_{N+1} + \frac{1}{1-\delta_{N+1}}\hat{Z}_{N+1} \\ & - \frac{\delta_{N+1}}{1-\delta_{N+1}}b_{N+1}\left(-\sum_{i=1}^N \tilde{\omega}_{N+1,i}\sum_{k=1}^N \Psi_{ik}\hat{Z}_k + \left(\sum_{i=1}^N \tilde{\omega}_{N+1,i}\left(\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{N+1,N+1}\right)\hat{P}_{N+1}\right)\end{aligned}$$

$$\begin{aligned}\hat{Q}_{N+1} = {} & \frac{\delta_{N+1}}{1-\delta_{N+1}}\hat{P}_{N+1} - \frac{\delta_{N+1}}{1-\delta_{N+1}}b_{N+1}\left(\sum_{i=1}^N \tilde{\omega}_{N+1,i}\left(\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{N+1,N+1}\right)\hat{P}_{N+1} \\ & + \frac{\delta_{N+1}}{1-\delta_{N+1}}b_{N+1}\left(\sum_{i=1}^N \tilde{\omega}_{N+1,i}\sum_{k=1}^N \Psi_{ik}\hat{Z}_k\right) + \frac{1}{1-\delta_{N+1}}\hat{Z}_{N+1}\end{aligned}$$

$$\begin{aligned}\hat{Q}_{N+1} = {} & \underbrace{\frac{\delta_{N+1}}{1-\delta_{N+1}}\left(1 - b_{N+1}\left(\tilde{\omega}_{N+1,N+1} + \sum_{i=1}^N \tilde{\omega}_{N+1,i}\left(\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1}\right)\right)\right)}_{=\phi^{N+1}_{P_{N+1}}}\hat{P}_{N+1} \\ & + \underbrace{\frac{\delta_{N+1}}{1-\delta_{N+1}}b_{N+1}\left(\sum_{i=1}^N \tilde{\omega}_{N+1,i}\sum_{k=1}^N \Psi_{ik}\hat{Z}_k\right) + \frac{1}{1-\delta_{N+1}}\hat{Z}_{N+1}}_{=\phi^{N+1}_Z}\end{aligned}$$

$$\hat{Q}_{N+1} = \phi^{N+1}_{P_{N+1}}\hat{P}_{N+1} + \phi^{N+1}_Z$$

This determines changes in the quantity produced in the commodity sector as a function of the commodity price and exogenous productivity changes. With no changes in productivities, $\hat{Z}_k = 0$ for all $k = 1, ..., N+1$, $\phi^{N+1}_Z = 0$ and we get the expression in the text. ■

Before proving Proposition 3, we first need to prove a few intermediate results. The following lemma characterizes changes in intermediate input spending on good i for all $j = 1, ..., N$ and the commodity sector.

Lemma 1 (Changes in Shares) *The change in spending of goods from sector i for domestic and the commodity sector satisfies*

$$\begin{aligned}\hat{\Omega}_{ji} = {} & (\sigma^{VA}_j - 1)\left(\hat{Z}_j - \sum_{k=1}^N \Psi_{jk}\hat{Z}_k\right) + (\sigma^I_j - 1)\sum_{k=1}^N \Psi_{ik}\hat{Z}_k + (\sigma^{VA}_j - \sigma^I_j)\sum_{i=1}^N \tilde{\omega}_{ji}\sum_{k=1}^N \Psi_{ik}\hat{Z}_k \\ & + \left((1-\sigma^I_j)\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1} + (\sigma^I_j - \sigma^{VA}_j)\left(\sum_{i=1}^N \tilde{\omega}_{ji}\left(\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{j,N+1}\right) + (\sigma^{VA}_j - 1)\sum_{k=1}^N \Psi_{jk}\Omega_{k,N+1}\right)\hat{P}_{N+1} \\ & \textit{for } j = 1, ..., N; i = 1, ..., N\end{aligned}$$

$$\hat{\Omega}_{N+1,i} = (\sigma^I_{N+1} - 1)\sum_{k=1}^N \Psi_{ik}\hat{Z}_k + (\sigma^{VA}_{N+1} - \sigma^I_{N+1})\left(\sum_{i=1}^N \tilde{\omega}_{N+1,i}\sum_{k=1}^N \Psi_{ik}\hat{Z}_k\right) + \frac{(\sigma^{VA}_{N+1} - 1)}{\delta_{N+1}}\hat{Z}_{N+1}$$

$$+\left((1-\sigma^I_{N+1})\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1} + (\sigma^I_{N+1}-\sigma^{VA}_{N+1})\left(\sum_{i=1}^N \tilde{\omega}_{N+1,i}\left(\sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1}\right)+\tilde{\omega}_{N+1,N+1}\right)+(\sigma^{VA}_{N+1}-1)\right)\hat{P}_{N+1}$$
$$-\frac{(1-\delta_{N+1})(\sigma^{VA}_{N+1}-1)}{\delta_{N+1}}\hat{Q}_{N+1}$$
for $i = 1, ..., N+1$

Proof of Lemma 1. Using the CES production function, we get that shares are

$$\Omega_{ji} = (1-a_j)\omega_{ji}P_i^{1-\sigma^I_j}(P^I_j)^{\sigma^I_j-\sigma^{VA}_j}P_j^{\sigma^{VA}_j-1}Z_j^{\sigma^{VA}_j-1} \quad \text{for } i=1,...,N+1; j=1,...,N$$
$$\Omega_{N+1,i} = (1-a_{N+1})\omega_{N+1,i}\delta_{N+1}^{\sigma^{VA}_{N+1}}P_i^{1-\sigma^I_{N+1}}(P^I_{N+1})^{\sigma^I_{N+1}-\sigma^{VA}_{N+1}}P_{N+1}^{\sigma^{VA}_{N+1}-1}Z_{N+1}^{\frac{1}{\delta_{N+1}}(\sigma^{VA}_{N+1}-1)}Q_{N+1}^{-(\sigma^{VA}_{N+1}-1)\frac{(1-\delta_{N+1})}{\delta_{N+1}}}$$
$$\text{for } i=1,...,N+1$$

Log differentiating the above expressions

$$\hat{\Omega}_{ji} = (1-\sigma^I_j)\hat{P}_i + (\sigma^I_j-\sigma^{VA}_j)\hat{P}^I_j + (\sigma^{VA}_j-1)\hat{P}_j + (\sigma^{VA}_j-1)\hat{Z}_j$$
$$\hat{\Omega}_{N+1,i} = (1-\sigma^I_{N+1})\hat{P}_i + (\sigma^I_j-\sigma^{VA}_j)\hat{P}^I_{N+1} + (\sigma^{VA}_{N+1}-1)\hat{P}_{N+1} + \frac{(\sigma^{VA}_{N+1}-1)}{\delta_{N+1}}\hat{Z}_{N+1}$$
$$-\frac{(1-\delta_{N+1})(\sigma^{VA}_{N+1}-1)}{\delta_{N+1}}\hat{Q}_{N+1}$$

Using the results in Proposition 1, we can write the changes in Ω_{ji}

$$\hat{\Omega}_{ji} = (1-\sigma^I_j)\left(-\sum_{k=1}^N \Psi_{ik}\hat{Z}_k + \sum_{k=1}^N \Psi_{ik}\Omega_{k,N+1}\hat{P}_{N+1}\right)$$
$$+(\sigma^I_j-\sigma^{VA}_j)\left(-\sum_{i=1}^N\tilde{\omega}_{ji}\sum_{k=1}^N\Psi_{ik}\hat{Z}_k + \left(\sum_{i=1}^N\tilde{\omega}_{ji}\left(\sum_{k=1}^N\Psi_{ik}\Omega_{k,N+1}\right)+\tilde{\omega}_{j,N+1}\right)\hat{P}_{N+1}\right)$$
$$+(\sigma^{VA}_j-1)\left(-\sum_{k=1}^N \Psi_{jk}\hat{Z}_k + \sum_{k=1}^N \Psi_{jk}\Omega_{k,N+1}\hat{P}_{N+1}\right)$$
$$+(\sigma^{VA}_j-1)\hat{Z}_j$$

Grouping terms, we arrive at an expression for the change in shares in terms of the exogenous variables.

$$\hat{\Omega}_{ji} = \underbrace{(\sigma^{VA}_j-1)\left(\hat{Z}_j-\sum_{k=1}^N\Psi_{jk}\hat{Z}_k\right)+(\sigma^I_j-1)\sum_{k=1}^N\Psi_{ik}\hat{Z}_k+(\sigma^{VA}_j-\sigma^I_j)\sum_{i=1}^N\tilde{\omega}_{ji}\sum_{k=1}^N\Psi_{ik}\hat{Z}_k}_{=\xi^Z_{ji}}$$
$$+\underbrace{\left((1-\sigma^I_j)\sum_{k=1}^N\Psi_{ik}\Omega_{k,N+1}+(\sigma^I_j-\sigma^{VA}_j)\left(\sum_{i=1}^N\tilde{\omega}_{ji}\left(\sum_{k=1}^N\Psi_{ik}\Omega_{k,N+1}\right)+\tilde{\omega}_{j,N+1}\right)+(\sigma^{VA}_j-1)\sum_{k=1}^N\Psi_{jk}\Omega_{k,N+1}\right)\hat{P}_{N+1}}_{=\xi^{N+1}_{ji}}$$

Therefore

$$\hat{\Omega}_{ji} = \xi_{ji}^{Z} + \xi_{ji}^{N+1}\hat{P}_{N+1}$$

This proves the first equation of the domestic sectors $j = 1, ..., N$. We can do the same for the spending of the commodity sector since

$$\begin{aligned}
\hat{\Omega}_{N+1,i} &= (1-\sigma_{N+1}^{I})\hat{P}_i + (\sigma_{N+1}^{I} - \sigma_{N+1}^{VA})\hat{P}_{N+1}^{I} + (\sigma_{N+1}^{VA} - 1)\hat{P}_{N+1} + \frac{(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}}\hat{Z}_{N+1} - \frac{(1-\delta_{N+1})(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}}\hat{Q}_{N+1} \\
&= (1-\sigma_{N+1}^{I})\left(-\sum_{k=1}^{N}\Psi_{ik}\hat{Z}_k + \sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1}\hat{P}_{N+1}\right) \\
&+ (\sigma_{N+1}^{I} - \sigma_{N+1}^{VA})\left(-\sum_{i=1}^{N}\tilde{\omega}_{N+1,i}\sum_{k=1}^{N}\Psi_{ik}\hat{Z}_k + \left(\sum_{i=1}^{N}\tilde{\omega}_{N+1,i}\left(\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{N+1,N+1}\right)\hat{P}_{N+1}\right) \\
&+ (\sigma_{N+1}^{VA} - 1)\hat{P}_{N+1} + \frac{(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}}\hat{Z}_{N+1} - \frac{(1-\delta_{N+1})(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}}\hat{Q}_{N+1} \\
\hat{\Omega}_{N+1,i} &= \underbrace{(\sigma_{N+1}^{I} - 1)\sum_{k=1}^{N}\Psi_{ik}\hat{Z}_k + (\sigma_{N+1}^{VA} - \sigma_{N+1}^{I})\left(\sum_{i=1}^{N}\tilde{\omega}_{N+1,i}\sum_{k=1}^{N}\Psi_{ik}\hat{Z}_k\right) + \frac{(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}}\hat{Z}_{N+1}}_{=\xi_{N+1,i}^{Z}} \\
&+ \underbrace{\left((1-\sigma_{N+1}^{I})\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1} + (\sigma_{N+1}^{I} - \sigma_{N+1}^{VA})\left(\sum_{i=1}^{N}\tilde{\omega}_{N+1,i}\left(\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{N+1,N+1}\right) + (\sigma_{N+1}^{VA}-1)\right)}_{=\xi_{N+1,i}^{N+1}}\hat{P}_{N+1} \\
&- \underbrace{\frac{(1-\delta_{N+1})(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}}}_{=\xi_{N+1}^{Q}}\hat{Q}_{N+1}
\end{aligned}$$

Therefore

$$\hat{\Omega}_{N+1,i} = \xi_{N+1,i}^{Z} + \xi_{N+1,i}^{N+1}\hat{P}_{N+1} - \xi_{N+1}^{Q}\hat{Q}_{N+1}$$

that proves the second equation. ■

The following lemma characterizes changes in *Nominal GDP* in this economy

Lemma 2 (Changes in Nominal GDP) *Changes in Nominal GDP, $G\hat{D}P$, can be written as*

$$G\hat{D}P = \alpha_{N+1}(\hat{P}_{N+1} + \hat{Q}_{N+1})$$

where $\alpha_{N+1} = \Pi_{N+1}/GDP$ is the share of commodity sector's profits on nominal GDP at the initial equilibrium.

Proof of Lemma 2. Nominal GDP in this economy can be written as

$$GDP = W\bar{L} + \Pi_{N+1}$$

Log-differentiating the above expression

$$G\hat{D}P = \frac{W\bar{L}}{GDP}(\hat{W} + \hat{\bar{L}}) + \frac{\Pi_{N+1}}{GDP}\hat{\Pi}_{N+1} = \frac{\Pi_{N+1}}{GDP}\hat{\Pi}_{N+1}$$

where the last line follows since W is the numeraire and the labor supply is fixed.

To get an expression for the change in profits in the commodity sector, recall that the production function of the commodity sector is

$$\begin{aligned}
Q_{N+1} &= Z_{N+1}B_{N+1}^{\delta_{N+1}} \\
B_{N+1} &= \left(a_{N+1}^{\frac{1}{\sigma_{N+1}^{VA}}} L_{N+1}^{\frac{\sigma_{N+1}^{VA}-1}{\sigma^{VA}}} + (1-a_{N+1})^{\frac{1}{\sigma_{N+1}^{VA}}} M_{N+1}^{\frac{\sigma_{N+1}^{VA}-1}{\sigma^{VA}}} \right)^{\frac{\sigma_{N+1}^{VA}}{\sigma_{N+1}^{VA}-1}}
\end{aligned}$$

The production function can thus be written as

$$Q_{N+1} = Z_{N+1}B_{N+1}^{\delta_{N+1}} \implies B_{N+1} = \left(\frac{Q_{N+1}}{Z_{N+1}}\right)^{\frac{1}{\delta_{N+1}}}$$

Let P_{N+1}^{B} be the price of the input bundle B_{N+1}. It satisfies

$$P_{N+1}^{B} = \left(a_{N+1}W^{1-\sigma_{N+1}^{VA}} + (1-a_{N+1})(P_{N+1}^{I})^{1-\sigma_{N+1}^{VA}} \right)^{\frac{1}{1-\sigma_{N+1}^{VA}}}$$

Total cost of the commodity sector is

$$TC_{N+1} = P_{N+1}^{B}B_{N+1} = P_{N+1}^{B}\left(\frac{Q_{N+1}}{Z_{N+1}}\right)^{\frac{1}{\delta_{N+1}}}$$

Marginal costs

$$MC_{N+1} = \frac{\partial TC_{N+1}}{\partial Q_{N+1}} = \frac{1}{\delta_{N+1}}P_{N+1}^{B}(Q_{N+1})^{\frac{1-\delta_{N+1}}{\delta_{N+1}}} Z_{N+1}^{-\frac{1}{\delta_{N+1}}} = \frac{1}{\delta_{N+1}}\frac{TC_{N+1}}{Q_{N+1}}$$

We can get profits from the above expression since

$$\begin{aligned}
\Pi_{N+1} &= P_{N+1}Q_{N+1} - TC_{N+1} \\
&= P_{N+1}Q_{N+1} - \delta_{N+1}MC_{N+1}Q_{N+1} \\
\Pi_{N+1} &= (1-\delta_{N+1})P_{N+1}Q_{N+1}
\end{aligned}$$

where the last line follows by the equilibrium condition that the exogenous price P_{N+1} should equals the marginal cost of the commodity sector i.e. $P_{N+1} = MC_{N+1}$.

Log differentiating the last expression, we get

$$\hat{\Pi}_{N+1} = \hat{P}_{N+1} + \hat{Q}_{N+1}$$

Replacing this into the nominal GDP change, we have

$$G\hat{D}P = \frac{\Pi_{N+1}}{GDP}(\hat{P}_{N+1} + \hat{Q}_{N+1}) = \alpha_{N+1}(\hat{P}_{N+1} + \hat{Q}_{N+1})$$

which completes the proof. ■

Using the above result, the following lemma shows the change in the Domar weight of the commodity sector, $\lambda_{N+1} = P_{N+1}Q_{N+1}/GDP$,

Lemma 3 (Change in λ_{N+1}) *The change in Domar Weight of the commodity sector satisfies*

$$\hat{\lambda}_{N+1} = (1 - \alpha_{N+1})(\hat{P}_{N+1} + \hat{Q}_{N+1})$$

Proof of Lemma 3. Log-differentiating the expression for λ_{N+1}

$$\begin{aligned}\hat{\lambda}_{N+1} &= \hat{P}_{N+1} + \hat{Q}_{N+1} - G\hat{D}P \\ &= \hat{P}_{N+1} + \hat{Q}_{N+1} - \alpha_{N+1}(\hat{P}_{N+1} + \hat{Q}_{N+1}) \implies \hat{\lambda}_{N+1} = (1 - \alpha_{N+1})(\hat{P}_{N+1} + \hat{Q}_{N+1})\end{aligned}$$

where we used Lemma 2 for the second line. This completes the proof. ■

Lemma 4 (Changes in consumption shares over GDP) *The change in consumption shares over GDP, $\Omega_{0i} = P_iC_i/GDP$, can be written as*

$$\hat{\Omega}_{0i} = -\alpha_{N+1}\alpha_B(\hat{P}_{N+1} + \hat{Q}_{N+1})$$

where

$$\alpha_B = \frac{rB}{\bar{L} + \Pi_{N+1} + rB}$$

Proof of Lemma 4. With Cobb-Douglas preferences, we have that

$$\begin{aligned}\Omega_{0i} = \frac{P_iC_i}{\text{GDP}} &= \beta_i\frac{\text{Expenditure}}{\text{GDP}} \\ &= \beta_i\frac{W\bar{L} + \Pi_{N+1} + rB}{\text{GDP}}\end{aligned}$$

Totally differentiating the above expression

$$\begin{aligned}\hat{\Omega}_{0i} &= \frac{\Pi_{N+1}}{\text{Expenditure}}\hat{\Pi}_{N+1} - \text{G}\hat{\text{D}}\text{P} \\ &= \frac{\Pi_{N+1}}{\text{Expenditure}}\frac{\text{GDP}}{\text{GDP}}\hat{\Pi}_{N+1} - \text{G}\hat{\text{D}}\text{P} \\ &= \alpha_{N+1}\frac{\text{GDP}}{\text{Expenditure}}\left(\hat{P}_{N+1} + \hat{Q}_{N+1}\right) - \text{G}\hat{\text{D}}\text{P} \\ &= \alpha_{N+1}\frac{\text{Expenditure} - rB}{\text{Expenditure}}\left(\hat{P}_{N+1} + \hat{Q}_{N+1}\right) - \text{G}\hat{\text{D}}\text{P}\end{aligned}$$

$$
\begin{aligned}
&= \alpha_{N+1}\left(1-\frac{rB}{\text{Expenditure}}\right)\left(\hat{P}_{N+1}+\hat{Q}_{N+1}\right) - \hat{\text{GDP}} \\
&= \alpha_{N+1}\left(1-\frac{rB}{\text{Expenditure}}\right)\left(\hat{P}_{N+1}+\hat{Q}_{N+1}\right) - \alpha_{N+1}(\hat{P}_{N+1}+\hat{Q}_{N+1}) \\
&= -\alpha_{N+1}\frac{rB}{\text{Expenditure}}(\hat{P}_{N+1}+\hat{Q}_{N+1}) \\
\hat{\Omega}_{0i} &= -\alpha_{N+1}\alpha_B(\hat{P}_{N+1}+\hat{Q}_{N+1})
\end{aligned}
$$

with $\alpha_B = rB/\text{Expenditure}$ and we used the results found in Lemma 2. ■

The following lemma characterizes changes in Domar weights of domestic sectors i.e. $\lambda_i = P_iQ_i/GDP$ for $i = 1, ..., N$ as a function of changes in productivities and the commodity price.

Lemma 5 (Changes in Domar Weights) *Changes in the domestic Domar weights, λ_i for $i = 1, ..., N$, satisfy the following equations*

$$
\begin{aligned}
&\hat{\lambda}_i = \hat{P}_i + \hat{Q}_i - \alpha_{N+1}(\hat{P}_{N+1}+\hat{Q}_{N+1}) \\
&\hat{\lambda}_i = \sum_{k=1}^{N}\Psi^U_{ik}\frac{\Omega_{0k}}{\lambda_k}\hat{\Omega}_{0k} + \sum_{k=1}^{N}\Psi^U_{ik}\left(m_{ki}\xi^Z_{ki} + m_{N+1,k}\xi^Z_{N+1,k}\right) + \left(\sum_{k=1}^{N}\Psi^U_{ik}\left(\xi^{N+1}_{ki}m_{ki} + m_{N+1,k}\xi^{N+1}_{N+1,k}\right)\right)\hat{P}_{N+1} \\
&\quad + \left(\sum_{k=1}^{N}\Psi^U_{ik}m_{N+1,k}\right)\hat{\lambda}_{N+1} - \left(\sum_{k=1}^{N}\Psi^U_{ik}m_{N+1,k}\right)\xi^Q_{N+1}\hat{Q}_{N+1}
\end{aligned}
$$

Proof of Lemma 5. The first line of the lemma follows immediately by the definition of the Domar weight and by applying Lemma 2

$$\hat{\lambda}_i = \hat{P}_i + \hat{Q}_i - G\hat{D}P = \hat{P}_i + \hat{Q}_i - \alpha_{N+1}(\hat{P}_{N+1}+\hat{Q}_{N+1})$$

For completeness, note that this can also be written as

$$\hat{\lambda}_i = \hat{P}_i + \hat{Q}_i - G\hat{D}P = \hat{P}_i + \hat{Q}_i + \hat{\lambda}_{N+1} - (\hat{P}_{N+1}+\hat{Q}_{N+1})$$

The second requires more elaboration. The market clearing for each good i can be written in terms of Domar weights as follows

$$\lambda_i = \Omega_{0i} + \sum_{j=1}^{N}\Omega_{ji}\lambda_j + \Omega_{N+1,i}\lambda_{N+1}$$

Log-differentiating this expression

$$
\begin{aligned}
\hat{\lambda}_i &= \frac{\Omega_{0i}}{\lambda_i}\hat{\Omega}_{0i} + \sum_{j\in N}\frac{\Omega_{ji}\lambda_j}{\lambda_i}\left(\hat{\lambda}_j + \hat{\Omega}_{ji}\right) + \frac{\Omega_{N+1,i}\lambda_{N+1}}{\lambda_i}\left(\hat{\lambda}_{N+1} + \hat{\Omega}_{N+1,i}\right) \\
\hat{\lambda}_i - \sum_{j\in N}\frac{\Omega_{ji}\lambda_j}{\lambda_i}\hat{\lambda}_j &= \frac{\Omega_{0i}}{\lambda_i}\hat{\Omega}_{0i} + \sum_{j\in N}\frac{\Omega_{ji}\lambda_j}{\lambda_i}\hat{\Omega}_{ji} + \frac{\Omega_{N+1,i}\lambda_{N+1}}{\lambda_i}\left(\hat{\lambda}_{N+1} + \hat{\Omega}_{N+1,i}\right)
\end{aligned}
$$

In matrix form

$$\hat{\lambda} - \boldsymbol{M}'\hat{\lambda} = \Omega_0 \circ \hat{\Omega}_0 + (\hat{\Omega}' \circ M')e + m_{N+1}\hat{\lambda}_{N+1} + m_{N+1} \circ \hat{\Omega}_{N+1}$$
$$\hat{\lambda} = \Psi^U(\Omega_0 \circ \hat{\Omega}_0) + \Psi^U\left((\hat{\Omega}' \circ M')e\right) + \Psi^U(m_{N+1}\hat{\lambda}_{N+1}) + \Psi^U\left(m_{N+1} \circ \hat{\Omega}_{N+1}\right)$$

where $\circ$ is the Hadamard product and the matrices are defined as in Table 4.

Typical element

$$\hat{\lambda}_i = \sum_{k=1}^{N} \Psi^U_{ik}\frac{\Omega_{0k}}{\lambda_k}\hat{\Omega}_{0k} + \sum_{k=1}^{N} \Psi^U_{ik} m_{ki}\hat{\Omega}_{ki} + \sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\hat{\lambda}_{N+1} + \sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\hat{\Omega}_{N+1,k}$$

Recall that from Lemma 1, we have that changes in shares satisfy

$$\hat{\Omega}_{ji} = \xi^Z_{ji} + \xi^{N+1}_{ji}\hat{P}_{N+1}$$
$$\hat{\Omega}_{N+1,i} = \xi^Z_{N+1,i} + \xi^{N+1}_{N+1,i}\hat{P}_{N+1} - \xi^Q_{N+1}\hat{Q}_{N+1}$$

Hence, the change in the Domar weight of sector i is

$$\begin{aligned}
\hat{\lambda}_i &= \sum_{k=1}^{N} \Psi^U_{ik}\frac{\Omega_{0k}}{\lambda_k}\hat{\Omega}_{0k} + \sum_{k=1}^{N} \Psi^U_{ik} m_{ki}(\xi^Z_{ki} + \xi^{N+1}_{ki}\hat{P}_{N+1}) \\
&+ \sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\hat{\lambda}_{N+1} + \sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\left(\xi^Z_{N+1,k} + \xi^{N+1}_{N+1,k}\hat{P}_{N+1} + \xi^Q_{N+1}\hat{Q}_{N+1}\right) \\
\hat{\lambda}_i &= \sum_{k=1}^{N} \Psi^U_{ik}\frac{\Omega_{0k}}{\lambda_k}\hat{\Omega}_{0k} + \sum_{k=1}^{N} \Psi^U_{ik}\left(m_{ki}\xi^Z_{ki} + m_{N+1,k}\xi^Z_{N+1,k}\right) + \left(\sum_{k=1}^{N} \Psi^U_{ik}\left(\xi^{N+1}_{ki} m_{ki} + m_{N+1,k}\xi^{N+1}_{N+1,k}\right)\right)\hat{P}_{N+1} \\
&+ \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\right)\hat{\lambda}_{N+1} - \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\right)\xi^Q_{N+1}\hat{Q}_{N+1}
\end{aligned}$$

which completes the proof. ■

We are now ready to prove Proposition 3.

Proof of Proposition 3. Using Lemma 4 and Lemma 5, we can write

$$\begin{aligned}
\hat{P}_i + \hat{Q}_i + \hat{\lambda}_{N+1} - (\hat{P}_{N+1} + \hat{Q}_{N+1}) &= -\sum_{k=1}^{N} \Psi^U_{ik}\frac{\Omega_{0k}}{\lambda_k}\left(\alpha_{N+1}\alpha_B(\hat{P}_{N+1} + \hat{Q}_{N+1})\right) + \sum_{k=1}^{N} \Psi^U_{ik}\left(m_{ki}\xi^Z_{ki} + m_{N+1,k}\xi^Z_{N+1,k}\right) \\
&+ \left(\sum_{k=1}^{N} \Psi^U_{ik}\left(\xi^{N+1}_{ki} m_{ki} + m_{N+1,k}\xi^{N+1}_{N+1,k}\right)\right)\hat{P}_{N+1} \\
&+ \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\right)\hat{\lambda}_{N+1} - \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k}\right)\xi^Q_{N+1}\hat{Q}_{N+1} \\
\hat{Q}_i &= -\sum_{k=1}^{N} \Psi^U_{ik}\frac{\Omega_{0k}}{\lambda_k}\left(\alpha_{N+1}\alpha_B(\hat{P}_{N+1} + \hat{Q}_{N+1})\right) + \sum_{k=1}^{N} \Psi^U_{ik}\left(m_{ki}\xi^Z_{ki} + m_{N+1,k}\xi^Z_{N+1,k}\right) \\
&+ \left(\left(\sum_{k=1}^{N} \Psi^U_{ik}\left(\xi^{N+1}_{ki} m_{ki} + m_{N+1,k}\xi^{N+1}_{N+1,k}\right)\right) + 1\right)\hat{P}_{N+1}
\end{aligned}$$

$$
\begin{aligned}
&+ \left(\left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k} \right) - 1 \right) \hat{\lambda}_{N+1} + \left(- \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k} \right) \xi^Q_{N+1} + 1 \right) \hat{Q}_{N+1} \\
&- \hat{P}_i \\
\hat{Q}_i = &- \sum_{k=1}^{N} \Psi^U_{ik} \frac{\Omega_{0k}}{\lambda_k} \left(\alpha_{N+1} \alpha_B (\hat{P}_{N+1} + \hat{Q}_{N+1}) \right) + \sum_{k=1}^{N} \Psi^U_{ik} \left(m_{ki} \xi^Z_{ki} + m_{N+1,k} \xi^Z_{N+1,k} \right) \\
&+ \left(\left(\sum_{k=1}^{N} \Psi^U_{ik} \left(\xi^{N+1}_{ki} m_{ki} + m_{N+1,k} \xi^{N+1}_{N+1,k} \right) \right) + 1 \right) \hat{P}_{N+1} \\
&+ \left(\left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k} \right) - 1 \right) \hat{\lambda}_{N+1} + \left(- \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k} \right) \xi^Q_{N+1} + 1 \right) \hat{Q}_{N+1} \\
&+ \sum_{k=1}^{N} \Psi_{ik} \hat{Z}_k - \sum_{k=1}^{N} \Psi_{ik} \Omega_{k,N+1} \hat{P}_{N+1}
\end{aligned}
$$

Therefore, changes in quantities of the domestic sectors satisfy

$$
\hat{Q}_i = \zeta^Z_i + \zeta^{N+1}_i \hat{P}_{N+1} + \zeta^Q_i \hat{Q}_{N+1}
$$

where

$$
\begin{aligned}
\zeta^Z_i &= \sum_{k=1}^{N} \Psi^U_{ik} \left(m_{ki} \xi^Z_{ki} + m_{N+1,k} \xi^Z_{N+1,k} \right) + \sum_{k=1}^{N} \Psi_{ik} \hat{Z}_k \\
\zeta^{N+1}_i &= \underbrace{-\alpha_{N+1} \alpha_B \sum_{k=1}^{N} \Psi^U_{ik} \frac{\Omega_{0k}}{\lambda_k}}_{\text{Wealth Effect}} + \underbrace{\left(\alpha_{N+1} + \left(\sum_{k=1}^{N} \Psi^U_{ik} \left(\xi^{N+1}_{ki} m_{ki} + m_{N+1,k} \left[\xi^{N+1}_{N+1,k} + (1 - \alpha_{N+1}) \right] \right) \right) \right)}_{\text{Buyers' Substitution}} \\
&\quad - \underbrace{\sum_{k=1}^{N} \Psi_{ik} \Omega_{k,N+1}}_{\text{Pure Downstream Effect}} \\
\zeta^Q_i &= -\alpha_{N+1} \alpha_B \sum_{k=1}^{N} \Psi^U_{ik} \frac{\Omega_{0k}}{\lambda_k} + \left(\alpha_{N+1} + \left(1 - \alpha_{N+1} - \xi^Q_{N+1} \right) \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k} \right) \right)
\end{aligned}
$$

Without productivity shocks, $\zeta^Z_i = 0$. Using the result in Proposition 2, we arrive at

$$
\hat{Q}_i = (\zeta^{N+1}_i + \zeta^Q_i \phi^{N+1}_{P_{N+1}}) \hat{P}_{N+1}
$$

This completes the proof. ■