Commodity Price Shocks and Production Networks in Small Open Economies*

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Abstract

We study the role of domestic production networks in the transmission of commodity price shocks in small open economies. We provide empirical evidence of strong propagation of commodity price shocks to quantities produced in domestic sectors that supply intermediate inputs to commodity sectors (*upstream* propagation) and muted propagation to sectors using commodities as intermediate inputs (*downstream* propagation). We develop a small open economy production network model to explain these transmission patterns. We show that the domestic production network is crucial for shaping the propagation of commodity prices. The two key mechanisms that rationalize the evidence are (i) the foreign demand channel and (ii) the input-output substitution channel. These two channels amplify the *upstream* propagation of commodity price changes, by increasing the demand for non-commodity inputs, and, at the same time, they mitigate the *downstream* cost channel by allowing firms to use relatively cheaper primary inputs in production.

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1 Introduction

This paper analyzes the propagation of commodity price shocks through domestic production networks in small open economies in terms of both quantities and prices. We take advantage of two stylized facts. First, the commodity sectors (mining, agriculture, and food sectors) are central sectors in small open economies, both as suppliers and buyers of intermediate inputs, which gives them a potential role as a source of supply and demand shock propagation. Second, commodity price shocks are only mildly correlated across sectors within a country. Therefore, as commodity prices are exogenous to the economies analyzed, we have an ideal scenario to study the propagation of sectoral commodity price shocks along the production chain.

First, we provide empirical evidence of a strong upstream propagation of quantities—to sectors providing intermediate inputs to commodity sectors—of commodity price shocks in a sample of nine small open economies for the period 1995-2009. In a production network setup, in which each sector buys intermediate inputs from other sectors, commodity price booms (busts) generate an increase (decrease) in intermediate input demand. We also show that while commodity price shocks increase the price of downstream sectors—that is, those sectors buying from the commodity sector to produce their output—they have no real effect on the output of downstream sectors. Thus, commodity price shocks appear to propagate mainly as a demand-side shock in small open economies.

We then developed a static small open economy model featuring a domestic production network to explain these empirical patterns. Our model features labor and capital that are supplied inelastically. While capital is specific to the commodity sector, labor is used for all sectors of the economy and can move cost-free. Both domestic factors and goods markets are competitive, and representative firms in each sector display constant returns to scale in production. Importantly, the commodity sector supplies goods to domestic firms and consumers at home and abroad and makes production decisions; that is, it uses labor, capital, and domestic intermediate inputs in production. The commodity price is exogenously determined in international markets and driven by foreign demand.

Our model highlights four mechanisms by which commodity price shocks propagate to non-commodity sectors via the domestic production network. There is one supply-side channel and three demand-side channels. We label the supply side component *cost push channel* because costs for non-commodity producers using the commodity as input increase following an increase in the commodity price. Consequently, the prices of these downstream sectors increase, generating further downstream propagation of the initial shock to other producers. The second is the *foreign demand channel*, where higher commodity exports, induced by an increase in foreign demand for the commodity sector, increase the commodity sector's demand for factors and intermediate inputs in production, pushing up production in non-commodity sectors. The key to this channel is the commodity sector's role as a buyer of non-commodity sectors, both directly and indirectly, via domestic production networks. The third is the *domestic demand channel*: changes in commodity prices affect the total available expenditure for domestic consumers, affecting demand for all non-commodity sectors, akin to the well-known wealth effect of commodity prices. The extent to which each non-commodity sector is affected by this channel depends on its exposure to domestic consumer expenditure changes, which considers both direct and indirect linkages through production networks. The final mechanism is the *input-output substitution channel*, where all sectors reallocate their demand towards (away from) other sectors in response to changes in good and factor prices. The latter channel crucially depends on the elasticities of substitution at the consumer and producer levels and the production network structure.

In the last part of the paper, we use a simplified version of our model to shed light on the qualitative and quantitative importance of these channels. In a simple calibration exercise using Australian sectoral data as a benchmark, we point to the essential role of demand-side channels in explaining the upstream effect on quantities that we find in the empirical results. The quantitative exercise can also feature a muted downstream propagation channel in quantities, which is in line with the empirical results. In this case, the elasticity of substitution between intermediates and labor plays a crucial role, as it allows industries to rely less on relatively more expensive intermediates.

Related Literature. This paper contributes to two strands of literature. We relate to the now extensive literature on the macroeconomic effects of commodity price shocks (e.g. Corden and Neary, 1982; Mendoza, 1995; Kose, 2002; Drechsel and Tenreyro, 2018; Benguria et al., 2020; Cao and Dong, 2020; Kohn et al., 2021; Romero, 2022; González, 2022). We contribute to this literature by providing empirical evidence on the role of domestic production networks in propagating commodity price shocks to non-commodity upstream sectors. On the theoretical front, we highlight the role of non-unitary production elasticities in amplifying the upstream propagation and dampening the downstream propagation of commodity price shocks. Moreover, we show that the well-known wealth effect of commodity price shocks has an important (upstream) network propagation component.

We also contribute to the literature on production networks and business cycles fluctuations (Horvath, 1998; Foerster et al., 2011; Acemoglu et al., 2012; Atalay, 2017; Baqaee and Farhi, 2019, 2021; Miranda-Pinto, 2021; vom Lehn and Winberry, 2020; Carvalho et al., 2021). Different from these studies, we show that commodity price shocks can have important real effects on output quantities, besides productivity and financial shocks, and are largely propagated through input-output linkages. In addition, our paper emphasizes the role that non-unitary elasticity of substitution between inputs plays in matching salient facts of the transmission of shocks via domestic production networks in line with recent literature (Boehm et al., 2019; Miranda-Pinto, 2021; Carvalho et al., 2021; Miranda-Pinto and Young, 2022).

2 Stylized Facts

In this section, we present two stylized facts regarding commodity sectors. First, commodity sectors are central in the domestic production network. Second, commodity price shocks strongly commove across countries but present a very small correlation across sectors within countries.

We first define what we mean by commodity sectors. To do so, we combine data on commodity goods' exports from Fernández et al. (2018) and input-output data from the WIOD. We use the WIOD data as, unlike the OECD input-output data, it contains sectoral information on production and prices, separately. For more details on data sources and definitions please refer to our Appendix A. We match each commodity good to one of the 34 industries in the World Input-Output Database (WIOD). Table B2 in our Appendix B provides a detailed mapping between goods and sectors in the WIOD data.[1] The three commodity sectors in the WIOD are Agriculture, Forestry, and Fishing; Mining and Quarrying;

[1] In our Appendix A we also provide information on the sample of countries we use from the WIOD and the definition of the variables.

and Food Products, Beverages, and Tobacco.

Fact 1: Commodity sectors are central sectors in the production network. We describe the network centrality of commodity sectors using standard centrality measures that capture how connected the sectors I am connected to and how connected the sectors that are connected to the sectors that I am connected to, etc. To that end, we analyze commodity sectors' customer and supplier centrality following Acemoglu et al. (2016).[2] We measure the supplier or *downstream* centrality of a given sector i as

$$Supplier_i = \sum_{j=1}^{N} \Psi_{ji}, \tag{1}$$

where Ψ_{ij} is an element of the Leontieff-Inverse matrix defined as

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}^s$$

where $\boldsymbol{I}$ is an identity matrix of size equal to the size of $\boldsymbol{\Omega}$. An element of $\boldsymbol{\Omega}$ is $\Omega_{ji} = P_i M_{ji}/P_j Q_j$. This represents the share of intermediates that sector i supplies to sector j ($P_i M_{ji}$) as a fraction of sector j's sales ($P_j Q_j$). This shows the direct importance of producer j as a supplier to producer i. An element Ψ_{ji} then records the importance of producer i as a supplier to producer j after considering both direct and indirect linkages. This intuition is precisely highlighted by the last equality in the equation above, where $\boldsymbol{\Psi}$ is an infinite sum of direct and indirect linkages across producers. Therefore, $Supplier_i$ adds across all buyers of good i and measures its importance as a *supplier* to the economy after taking into account direct and indirect linkages.

[2] These definitions are slightly different from the notions of downstreamness and upstreamness highlighted in the global value chains literature (see Antras and Chor, 2021). Their measure of upstreamness shows how important other sectors are as buyers to a given sector i. In our case, customer centrality comes from the importance of sector i as a buyer to other sectors. This difference is expected because we focus on how shocks propagate, as in Acemoglu et al. (2016), while Antras and Chor (2021) focuses on the distance of each sector to final demand and primary factors. Our concept is closer to the Katz-Bonacich centrality used in the production networks literature. See Carvalho (2014) for an overview, especially footnote 11.

We then measure the customer or *upstream* centrality of a sector i as

$$Customer_i = \sum_{j=1}^{N} \widetilde{\Psi}_{ij}, \tag{2}$$

where $\widetilde{\Psi}_{ij}$ is an element of the following matrix

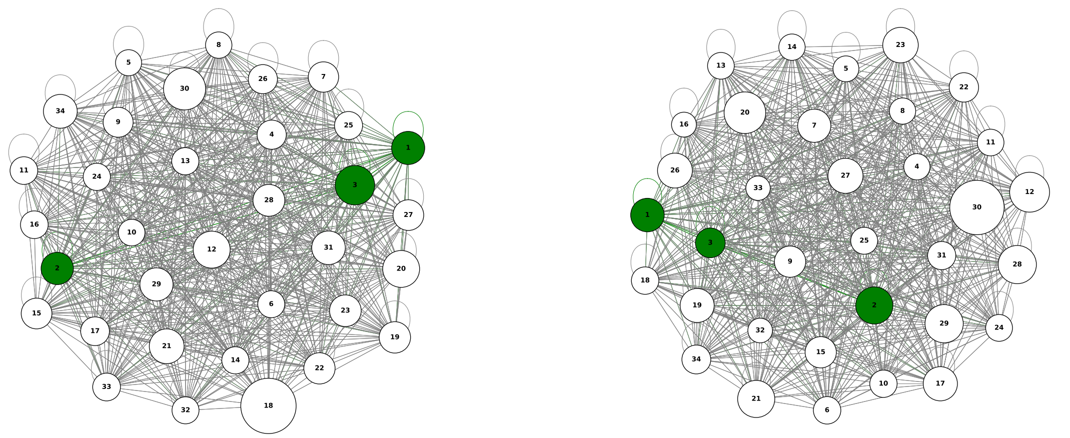
$$\widetilde{\boldsymbol{\Psi}} = (\boldsymbol{I} - \boldsymbol{M})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{M}^s$$

where $\boldsymbol{I}$ is an identity matrix of size equal to the size of $\boldsymbol{M}$. An element of $\boldsymbol{M}$ is $m_{ij} = P_j M_{ij}/P_j Q_j$. This represents the share of the sector's j sales that the sector i accounts for. This shows the direct importance of producer i as a buyer to producer j. An element $\widetilde{\Psi}_{ij}$ then records the importance of producer i as a *buyer* to producer j after considering both direct and indirect linkages. $Customer_i$ adds across all suppliers to sector i and measures sector i's importance as a *buyer* to the economy after considering direct and indirect linkages.

Figure 1 plots the domestic network structure of Australia in 1995, using input-output data from the WIOD database. Each node (circle) is a different sector in the economy, and the node's size represents how important that sector is in the network based on the network centralities defined above. Panel (a) shows the network in which each node's size describes the customer centrality of the sector—this is, how much output of other sectors a given sector uses, directly and indirectly—, while in panel (b), the node size is based on each sector's supplier centrality—how much of a given sector output is used as input by other sectors, directly and indirectly.

We observe in Figure 1 that commodity sectors were central sectors in the domestic production network of Australia in 1995. In particular, panel (a) shows that the food (3) sector is one of the sectors with the largest customer centrality. Panel (b) also shows that mining is one of the most central sectors in its direct and indirect supply of intermediates inputs.

To describe the relative importance of commodity sectors in the domestic production network of small open economies, we report in Table 1 the ranking of the customer and supplier propagation centrality for the three commodity sectors, with respect to all the other sectors in the economy (a total of 34 in the WIOD data). The main takeaway from Table 1 is



(a) Customer Centrality (b) Supplier Centrality

Figure 1. Domestic Production Network Australia

Note: This figure shows the domestic production network of Australia (WIOD Input-Output data) for 2011 at the sector level (ISIC rev. 3). An arrow from sector j to sector i represents intermediate inputs flowing from j o i. Each node (circle) is a different sector in the economy, and the size of the node represents how important that sector is as a direct and indirect buyer (panel a) and supplier (panel b) of intermediate inputs. The labels in the nodes are linked to sectors in Table B1 of our Appendix.

that for all the countries in our sample, at least one of the commodity sectors (many times 2 of them) is a central customer and/or a central supplier (top-10) in the domestic production network.

Fact 2: Sectoral commodity price shocks are only mildly correlated across sectors, within a country. We first describe the process of constructing sectoral indexes of commodity prices.

(i) We use the export data Fernández et al. (2018) and calculate, for each country, the share of each commodity good in its sectoral group, be it agriculture, mining, or food sectors. Then, we multiply each sector-country weight by the monthly commodity price.

(ii) The outcome from step (i) is a matrix of country-specific monthly commodity price index that we deflate using the US Consumer Price Index (CPI).

Table 1. Ranking of Network Centrality of Commodity Sectors in 1995

	Customer Centrality			*Supplier Centrality*		
Country	Agric.	Mining	Food	Agric.	Mining	Food
Australia	10	11	3	13	6	17
Bulgaria	2	8	1	2	9	13
Brazil	14	25	2	7	14	10
Canada	6	18	3	4	10	15
Denmark	6	33	1	8	17	11
India	9	25	6	3	9	23
Lithuania	1	33	3	2	34	9
Mexico	10	18	1	7	1	15
Russia	3	6	2	5	3	14
Average	**7**	**20**	**2**	**6**	**11**	**4**

Note: This table presents, for each country and commodity sector, the customer and supplier network centrality. Source: WIOD Input-Output database, 1995.

(iii) We take the average across months within each quarter by year.

To measure the sectoral commodity shock we assume that commodity prices are exogenous to the commodity exporter. Thus, innovations to commodity prices are simply the log change of the sectoral commodity price index.

We now investigate the correlation between sectoral commodity price shocks within countries. As highlighted in Fernández et al. (2018), commodity shocks strongly commove across countries. Indeed, the cross-country correlation between commodity price shocks in Agriculture and Forestry, Mining and Quarrying, and Foods Products and Beverage sectors are 0.85, 0.65, and 0.5, respectively. However, as shown in Table 2, our estimated commodity shocks present a small correlation across sectors within countries. The average cross-country correlation between shocks to agriculture and mining is 0.57; the average cross-country correlation between shocks to agriculture and food products is 0.16; and the average cross-country correlation between shocks to mining and foods products is -0.13.[3]

[3] Figure B1 to Figure B3 in our Appendix depict our estimated sectoral commodity price shocks for countries in our sample. Besides confirming Fact 2 (low within-country correlation across commodity shocks),

Table 2. Average Pairwise Correlation across Commodity Prices

	Correlation
Agriculture/Mining	0.57
Agriculture/Food	0.16
Mining/Food	-0.13

Note: This table presents the cross-country average of the within-country pairwise correlations among the log change of sectoral commodity prices.

3 Commodity price shocks via production networks

In this section, we study the network effects of commodity price shocks. Our empirical specification follows Acemoglu et al. (2016) and Carvalho et al. (2021). In particular, we estimate

$$y_{ict} = \delta_t + \alpha_{i,c} + \delta_{c,t} + \phi_1 Upstream_{ict} + \phi_2 Downstream_{ict} + \boldsymbol{\nu}' \boldsymbol{X}_{ict-1} + \epsilon_{ict}, \tag{3}$$

where y_{ict} is a measure of sector i's performance in the country c at time t, which can be the logarithm of sectoral output, value-added, employment, or capital. δ_t represent year fixed effects, $\alpha_{i,c}$ are country-sector fixed-effects, and $\delta_{c,t}$ are a full set of country-time fixed effects. $Upstream_{ict}$ and $Downstream_{ict}$ are our network spillover measures, which we explain in the next subsection, and vary at the sector-country-year level. $\boldsymbol{X}_{ict-1}$ is a $H \times 1$ vector of lagged controls, including the dependent variable and our network spillover measures. Finally, ϵ_{ict} is an error term.

Theoretically, as shown in Acemoglu et al. (2016), supply-side shocks (e.g., productivity shocks) should mainly propagate to downstream industries through an intermediate input cost channel. In our case, if the Downstream shock reflects increased production cost from increased commodity prices, we expect $\phi_2 < 0$. On the other hand, an increase in commodity prices can boost exports and, therefore, increase demand for domestic inputs, implying $\phi_1 > 0$. We include a full set of fixed effects to account for different sources of variation. We first include a full set of year-fixed effects that account for any common differences across years.

these figures show substantial volatility of commodity price shocks over time.

Then, we use a set of country-sector fixed effects, to account for any time-invariant differences idiosyncratic to each country-pair ($\alpha_{i,c}$). Finally, we add country-time fixed effects to account for differences between countries over time ($\delta_{c,t}$).

In the next subsection, we define and briefly explain the network spillover measures that shape the effects of commodity price shocks on non-commodity sectoral output in a small open economy. These network measures are an application of the strategy in Acemoglu et al. (2016)—developed to understand the propagation of productivity shocks and government spending shocks in a closed economy—for the context of a small open economy subject to commodity price shocks. As we will show in Section 4, commodity price shocks, unlike productivity and government spending shocks, have a demand-side and a supply-side component.

3.1 Measuring Network Spillovers

We now outline our network spillover measures along the lines of Acemoglu et al. (2016). We denote a commodity sector by $k \in \mathcal{K}$ where $\mathcal{K}$ is the set of commodity sectors. We denote non-commodity sectors by either i or j, where $i, j = 1, ..., N$ with N the total number of non-commodity sectors.

The downstream effect of commodity price shocks that is to those *buying* from the commodity sector either directly or indirectly through input-output linkages, in sector i in the country c at time t is

$$Downstream_{ict} = \sum_{k \in \mathcal{K}} \left(\Psi_{ikc} - \mathbf{1}_{i=k} \right) \cdot \tilde{p}_{kct}, \tag{4}$$

where Ψ_{ikc} stands for the importance of the commodity sector k in supplying intermediate inputs to sector i both directly and indirectly through the domestic production network of country c, $\mathbf{1}_{i=k}$ is an indicator variable that takes the value of 1 when $i = k$ and zero otherwise. $\tilde{p}_{kct}$ corresponds to the identified commodity price shock for commodity sector k in the country c at time t.

The upstream effect of commodity price shocks, that is from the commodity sectors to that *selling* to it, from sector k to sector i in the country c at time t is measured as

$$Upstream_{ict} = \sum_{k \in \mathcal{K}} \left(\tilde{\Psi}_{kic} - \mathbf{1}_{i=k} \right) \cdot \tilde{p}_{kct}, \tag{5}$$

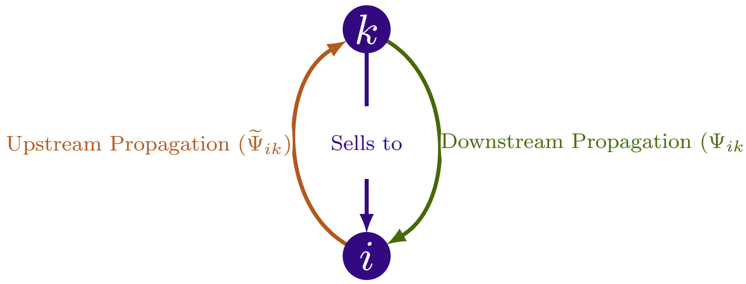


Figure 2. Upstream and Downstream Propagation

Note: This figure shows the propagation of shocks along the production network where we remove all other nodes and focus on total propagation (both direct and indirect). Downstream propagation from seller k to buyer i (Ψ_{ik}) and upstream propagation from buyer i to seller k ($\tilde{\Psi}_{ik}$). This illustrates the construction of measures in equations (4) and (5).

where $\tilde{\Psi}_{kic}$ stands for the direct and indirect importance of commodity sector k as a buyer to sector i.

As pointed out in Acemoglu et al. (2016), the production networks literature is usually ambiguous about what upstream or downstream means. In this paper, we strictly follow their approach in that upstream or downstream refers to how shocks are propagated throughout the network structure and not by the sectors' position. A graphical representation of this idea is in Figure 2 below, where we plot two sectors k and i where sector k supplies to sector i. Here, the shock to sector k (the supplier) propagates *downstream*, while a shock to sector i (the buyer) propagates *upstream*.

3.2 Network propagation

We now present empirical evidence on the transmission mechanism of commodity price shocks via production networks using the WIOD database. The WIOD database has an important advantage compared to the OECD database: it reports sectoral quantity and price indexes, allowing us to better study the channels in which commodity price shocks affect quantities and prices. Instead, the OECD data only reports nominal data (in US dollars) for sales, value-added and intermediate input use. To construct the Upstream and Downstream network effects defined in Equation (4) and Equation (5) we use the input-output structure in 1995.

Table 3 presents the results of estimating Equation (3) using quantity and price indexes for gross output. All regressions include one lag of the dependent variable. To ease the interpretation of our coefficients, we standardized our Upstream and Downstream measures to have a unit standard deviation. We first focus on the effects on sectors selling to the commodity sector ($Upstream_{ict}$). Columns (1) to (3) show that real commodity price shocks positively affect the gross output of non-commodity sectors. In particular, in column (3)—where we control for a year, country-sector, and country-year fixed effects—a one standard deviation increase in commodity prices generates a 0.72 percent (2 percent) increase in the sectoral gross output quantity index, on impact (cumulative). We find no evidence of downstream ($Downstream_{ict}$) effects on quantities of commodity price shocks. Columns (4) to (6) show that, despite the muted downstream effect on quantities, we observe a strong downstream propagation of commodity prices to the price of non-commodity sectors, with no upstream propagation.A one standard deviation increase in commodity prices generates a 0.82 percent (2.8 percent) increase in the sectoral gross output price index, on impact (cumulative).

The empirical evidence in this section points to strong upstream propagation of commodity prices, alongside a muted downstream propagation, on the quantity produced by non-commodity industries. At the same time, we find a strong increase in the price of industries that are downstream from the commodity sector with no effect on upstream industries. In the next section, we build a theoretical model of a small open economy with production networks and a commodity sector that rationalizes the findings we document in this section. In particular, we ask how can a small open economy model with production networks, in which the commodity sector is a central supplier and user of intermediate inputs (as documented in Table 1), rationalize the large upstream propagation and muted downstream propagation of commodity price shocks.

4 Theory

Setup.

Our model features a representative consumer that consumes $N+1$ goods in a static setting.

Each $N+1$ sectors produce using constant returns to scale production function. Sectors up to sector N produce using labor and intermediate inputs. Sector $N+1$ produces using labor, intermediate inputs, and capital. Importantly, sector $N+1$ good price is *exogenously*

Table 3. Network Effects of Commodity Price Shocks on Non-Commodity Sectors

	Panel (a): Quantity			Panel (b): Prices		
	(1)	(2)	(3)	(4)	(5)	(6)
Upstream_{ict}	0.0067**	0.0080**	0.0072***	0.0004	0.0067	0.0019
	(0.0031)	(0.0031)	(0.0022)	(0.0067)	(0.0075)	(0.0024)
Upstream_{ict-1}	0.0027	0.0055	0.0058***	-0.0171	-0.0008	-0.0003
	(0.0035)	(0.0037)	(0.0020)	(0.0137)	(0.0070)	(0.0018)
Downstream_{ict}	0.0022	0.0018	-0.0007	0.0104*	0.0099**	0.0082***
	(0.0017)	(0.0016)	(0.0012)	(0.0054)	(0.0049)	(0.0026)
$\text{Downstream}_{ict-1}$	-0.0020	-0.0020	-0.0024**	0.0074***	0.0090**	0.0115***
	(0.0015)	(0.0015)	(0.0011)	(0.0058)	(0.0039)	(0.0023)
Accumulated Upstream	0.016**	0.021***	0.020***	-0.016	0.0125	0.003
	(0.007)	(0.007)	(0.005)	(-0.020)	(0.015)	(0.006)
Accumulated Downstream	0.002	0.002	-0.004	0.028**	0.029**	0.028***
	(0.004)	(0.004)	(-0.003)	(0.014)	(0.012)	(0.006)
Observations	3906	3906	3906	3906	3906	3906
Within R^2	0.924	0.777	0.766	0.959	0.737	0.694
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Country $\times$ Sector F.E.		Yes	Yes		Yes	Yes
Country $\times$ Year F.E.			Yes			Yes

Note: This table presents OLS regressions using sectoral log quantity (columns 1 to 3) and log price index (columns 4 to 6) as the dependent variable. The independent variables also include one lag of the dependent variable. Double clustered country-year standard errors in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

given. All factor payments are rebated back to the household.

Notation and Definitions.

We use **bold** to denote vectors and matrices. For any matrix $\boldsymbol{X}$, we use $\boldsymbol{X}^T$ for its transpose.

We now define some objects that are going to be key for the analysis.

We let $\boldsymbol{\Omega}$ to be the *input-output matrix* of this economy, with typical element

$$\boldsymbol{\Omega} = \{\Omega_{ij}\} = \frac{P_j M_{ij}}{P_i Q_i} \quad \text{for all } i, j = 1, ..., N+1.$$

This typical element states how much producer i spend on good j, $P_j M_{ij}$, as a fraction of i's sales, $P_i Q_i$. Here P_i is the price of good i, Q_i is the quantity sold of good i, and M_{ij} is how much producer i buys of the quantity of good j.

With some abuse of notation, we also define producer's i expenditure on factor $f = \{L, K\}$ i.e. expenditure on labor and capital, respectively as

$$\Omega_{iL} = \frac{WL_i}{P_i Q_i}; \qquad \Omega_{iK} = \frac{RK_i}{P_i Q_i}.$$

We define $\boldsymbol{\Psi}$ as the *Leontieff-Inverse matrix* that satisfy

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}^s \text{ with typical element } \{\Psi_{ij}\}.$$

Notice that this is defined over the $N+1$ goods and does not incorporate spending on factors. This matrix captures both the direct and indirect linkages across producers. For instance, Ψ_{ij} denotes how important producer j as a *direct and indirect* supplier to producer i.[4]

On the consumption side, we define the vectors of final domestic consumption, $\boldsymbol{b}$, as follows

$$\boldsymbol{b} = \{b_i\} = \frac{P_i C_i}{GDP},$$

where C_i represents home consumption of good i.

Since there are two factors of production, capital, and labor, we define their shares on *Nominal Gross Domestic Product* (GDP) as

$$\Lambda_L = \frac{WL}{GDP}; \quad \Lambda_K = \frac{RK}{GDP}; \quad \Lambda_L + \Lambda_K = 1.$$

[4]There are some regularity conditions that $\boldsymbol{\Omega}$ must satisfy to be able to write in this way. We note, however that they are seldom satisfied since $\sum_{j=1}^{N+1} \Omega_{ij} < 1$, and so is a sub-stochastic matrix. This implies its spectral radii are less than one. Then by the *Neumann Series Lemma*, the result follows. See Sargent and Stachurski (2022), pp 12-16, for a more formal discussion on these issues.

where W and R are the wage rate (labor price) and the rental rate (capital price), L is the equilibrium labor quantity, and K is the capital equilibrium quantity. The last result above follows from the fact that everything in this economy is produced out of factors, and therefore total value added (GDP) should equal factor payments.

Finally, we let λ_i to denote the *Domar weight* of producer i on total value added i.e.

$$\lambda_i = \frac{P_i Q_i}{GDP}.$$

In the presence of intermediate goods in production, this is the relevant size statistic of each producer on total value added.

4.1 Representative Consumer.

We assume a representative consumer that owns both factors of production that it supplies inelastically to producers in the economy. We denote these inelastic supplies as $\bar{L}$ and $\bar{K}$. It has a utility function defined over the $N+1$ goods, $U(\boldsymbol{C})$. We introduce an additional income source in the budget constraint of the consumer that we label $\bar{Y}$, a net transfer to the rest of the world as in Baqaee and Farhi (2021). The purpose of this is two-fold. First, it allows the economy to exhibit exports in equilibrium. Second, it will allow us to break the relationship between expenditure and production side. Later, we will make this object a function of the commodity price. From the household perspective, though, this is taken as given.

Taking good and factor prices, $(\boldsymbol{P}, W, R)$, together with $\bar{Y}$ as given, the representative consumer solves the following program

$$\max_{\boldsymbol{C}} \quad U(\boldsymbol{C}) \quad \text{s.t.} \quad \sum_{i=1}^{N+1} P_i C_i \leq W\bar{L} + R\bar{K} - \bar{Y} = E, \tag{6}$$

where we use E as a short-cut for *total expenditure*. The solution to this program delivers consumption schedules that are a function of prices and the additional income source i.e. $\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{P}, W, R, \bar{Y})$.

4.2 Non-Tradable Sectors: $1, 2, ..., N$

Gross output in sector i, Q_i, is produced according to the following production function

$$Q_i = Z_i F_i(L_i, \{M_{ij}\}_{j=1}^{N+1}), \tag{7}$$

where Z_i is a producer-specific shock, $F_i(.)$ is a constant-returns to scale function. We use a subscript i to index this production function to allow for the possibility of different production functions across producers. L_i is labor demand of producer i and M_{ij} is intermediate demand for good j by producer i.

Cost-minimization implies that the marginal cost of production, MC_i can be written as

$$P_i = MC_i(W, \boldsymbol{P}; Z_i). \tag{8}$$

This implies that the marginal cost is a function of the wage rate, the price of all goods $\boldsymbol{P} = (P_1, P_2, ..., P_{N+1})$ and its own productivity, Z_i. Its equality to its good price, P_i, then follows from profit maximization. In a nutshell, this is just another way to write the zero-profit condition of each producer $i = 1, 2, ..., N$.

To get conditional demands for labor and each intermediate input, we can simply differentiate the marginal cost function

$$L_i = Q_i \frac{\partial MC_i(.)}{\partial W},$$
$$M_{ij} = Q_i \frac{\partial MC_i(.)}{\partial P_j}.$$

4.3 Commodity Sector: $N + 1$

The commodity sector gross output Q_{N+1} is produced according to the following production function

$$Q_{N+1} = Z_{N+1} F_{N+1}(L_{N+1}, K_{N+1}, \{M_{N+1,j}\}_{j=1}^{N+1}).$$

Similar to non-tradable sectors, cost minimization delivers the commodity sector's marginal costs, which in equilibrium must coincide with the commodity sector price

$$P_{N+1} = MC_{N+1}(W, R, \boldsymbol{P}; Z_{N+1}), \tag{9}$$

which is exogenously given for the small open economy.

4.4 Equilibrium.

4.4.1 Quantities.

The following conditions characterize the equilibrium in our model

$$\begin{aligned}
Q_i &= C_i + \sum_{j=1}^{N+1} M_{ji} \qquad \forall i = 1, ..., N, \\
P_{N+1}Q_{N+1} &= P_{N+1}C_{N+1} + \sum_{j=1}^{N+1} P_{N+1}M_{j,N+1} + \bar{Y}, \\
\bar{L} &= \sum_{i=1}^{N+1} L_i, \\
\bar{K} &= K_{N+1},
\end{aligned}$$

The first row shows the market clearing condition in non-tradable goods markets. The second row shows the aggregate resource constraint in this economy that follows by combining the consumer's budget constraint with the non-tradable market clearing conditions. The last two rows refer to the labor market and capital market clearing respectively.

Multiplying each non-tradable goods market clearing condition by each good price and dividing by GDP, we arrive at a market clearing condition in terms of observables

$$\lambda_i = b_i + \sum_{j=1}^{N+1} \Omega_{ji}\lambda_j.$$

We can also divide by GDP the aggregate resource constraint to get

$$\lambda_{N+1} = b_{N+1} + \sum_{j=1}^{N+1} \Omega_{j,N+1}\lambda_j + \frac{\bar{Y}}{GDP} \qquad \text{for } i = 1, ..., N.$$

Stacking all these conditions in a vector and inverting the system delivers the following

$$\boldsymbol{\lambda} = \boldsymbol{\Psi}^T \left(\boldsymbol{b} + \boldsymbol{e}_{N+1} \frac{\bar{Y}}{GDP} \right),$$

where $\boldsymbol{e}_{N+1}$ is a unit vector with its $N+1$ element equal to one and all the rest equal to zero.

Similarly, we can express the factor markets clearing conditions conveniently in terms of aggregate factor shares as

$$\Lambda_L = \sum_{i=1}^{N+1} \Omega_{i,L} \lambda_i,$$

$$\Lambda_K = \Omega_{N+1,K} \lambda_{N+1},$$

$$\Lambda_L + \Lambda_K = 1.$$

4.5 Comparative Statics.

In what follows, we consider a perturbation of the commodity price, $d \log P_{N+1}$, and study how this affects good and factor prices, $(\boldsymbol{P}, W, R)$ together with quantities produced, $\boldsymbol{Q}$. In all our comparative exercises below, we assume that technology and factor supplies are fixed. Therefore, $d \log \boldsymbol{Z} = \boldsymbol{0}$ and $d \log \bar{K} = d \log \bar{L} = 0$.

4.5.1 Prices.

Totally differentiating Equation (8) and Equation (9) and using *Shephard's lemma*, yields that up to a first-order approximation price changes should satisfy

$$d \log P_i = \Omega_{iL} d \log W + \sum_{j=1}^{N+1} \Omega_{ij} d \log P_j - d \log Z_i \quad \text{for all } i = 1, 2, ..., N,$$

$$d \log P_{N+1} = \Omega_{N+1,L} d \log W + \Omega_{N+1,K} d \log R + \sum_{j=1}^{N+1} \Omega_{N+1,j} d \log P_j - d \log Z_{N+1},$$

where

$$\Omega_{iL} = \frac{WL_i}{P_i Q_i} = \frac{WL_i}{TC_i}, \qquad \Omega_{iK} = \frac{RK_i}{P_i Q_i} = \frac{RK_i}{TC_i} \qquad \text{for all } i = 1, 2, ..., N+1,$$

is how much producer i spends on either labor and capital as a fraction of its sales, P_iQ_i, which due to the constant returns to scale assumption of the production function equals total costs, $P_iQ_i = TC_i$.

The above system of equations is $N+1$ equations in $N+2$ unknowns ($d\log \boldsymbol{P}, d\log W, d\log R$). Up to choosing a numeraire, we can solve for domestic price changes as a function of commodity price changes. We let the nominal wage W be the numeraire. The following proposition characterizes these responses as we show in Appendix C.

Proposition 1 (Price Responses to a Commodity Price Change)**.** Consider a perturbation of the commodity price, $d\log P_{N+1}$. Up to a first-order approximation, changes in good prices satisfy

$$d\log P_i = \frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}} d\log P_{N+1} = \frac{\Psi_{i,N+1}}{\Psi_{N+1,N+1}} d\log P_{N+1}, \tag{10}$$

where $\widetilde{\Omega}_{i,K} = \sum_{j=1}^{N+1} \Psi_{ij}\Omega_{jK} = \Psi_{i,N+1}\Omega_{N+1,K}$ are the network-adjusted usage of capital of producer i and Ψ_{ij} represents how important is sector j as a supplier, both directly and indirectly, to sector i.

Proof. See Appendix C. ■

This equation states that all prices increase proportionally to their exposure to the commodity sector. Intuitively, a rise in the commodity sector price raises the marginal cost of all producers. The relevant exposure to the commodity sector in the presence of intermediate input linkages is $\Psi_{i,N+1}$. It measures how important the commodity sector is as a *supplier* to sector i after considering both direct and indirect linkages. As with productivity shocks in the production network literature, commodity price shocks propagate downstream to other prices in the economy.

4.5.2 Quantities.

To solve for changes in gross output, $d\log Q_i$, we use the definition of the Domar weight, λ_i, and totally differentiate it to get

$$d\log Q_i = d\log \lambda_i + d\log GDP - d\log P_i. \tag{11}$$

We already know $d \log P_i$, so we are left to determine changes in nominal GDP, GDP, and the Domar weights.

Since labor is the numeraire, we can write changes in nominal GDP as

$$\Lambda_L d \log W + \Lambda_K d \log R = d \log GDP \Longrightarrow \Lambda_K d \log R = d \log GDP.$$

Therefore, nominal GDP rises in proportion to changes in the rental rate R. We can then link changes in nominal GDP to changes in the commodity price shocks by using the zero profit condition in the commodity sector that links the rental rate with the commodity price shock

$$d \log P_{N+1} = \widetilde{\Omega}_{N+1,K} d \log R.$$

Hence, nominal GDP changes depend on the change in the commodity price as

$$d \log GDP = \frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} d \log P_{N+1}. \tag{12}$$

We are left to determine changes in the Domar weight. Before doing so, we need to introduce a version of the *Input-Output Substitution Operator* first introduced in Baqaee and Farhi (2019) applied directly to our context. In particular, in response to a change in the commodity price $\mathrm{d} \log P_{N+1}$, producer j substitutes to/away from sector i as

$$\begin{aligned} \Phi_j(i, N+1) = {} & \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \Psi_{ki} \Omega_{jk} \widetilde{\Omega}_{h,K}, \\ & + \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \Psi_{ki} \Omega_{jk} (\theta^j_{kK} - 1) \Omega_{jK}, \end{aligned} \tag{13}$$

where $\delta_{kh} = 1$, whenever $k = h$, and zero otherwise. θ^j_{kh} is the Allen-Uzawa elasticity of substitution for producer j between any two pair of inputs (k, h), where k is an intermediate input and h can be other intermediate inputs or factors, defined as

$$\theta^j_{kh} = \frac{\frac{\partial \log M_{jk}}{\partial \log P_h}}{\Omega_{jh}}. \tag{14}$$

We discuss the intuition for this operator below. At this point, the substitution operator is helpful because it allows us to write changes in Domar weights in response to a change in the commodity price compactly, as the following proposition shows.

Proposition 2 (Changes in Domar Weights.)**.** Up to a first-order approximation, changes in the Domar weight of sector i, $d\log\lambda_i$, following a commodity price shock $\mathrm{d}\log P_{N+1}$, satisfy

$$\frac{\mathrm{d}\log\lambda_i}{\mathrm{d}\log P_{N+1}} = \frac{1}{\lambda_i}\left(\sum_{j=1}^{N+1}\lambda_j\Phi_j(i,N+1) + \sum_{k=1}^{N+1} b_k\Psi_{ki}\frac{\mathrm{d}\log E}{\mathrm{d}\log P_{N+1}} + \sum_{k=1}^{N+1}\Psi_{ki}\delta_{k,N+1}\frac{\bar{Y}}{GDP}\frac{\mathrm{d}\log\bar{Y}}{\mathrm{d}\log P_{N+1}}\right) - \frac{d\log GDP}{\mathrm{d}\log P_{N+1}}. \tag{15}$$

Proof. See Appendix C. ■

The last proposition follows by just differentiating the market clearing condition of each good. To get this result, we impose Cobb-Douglas preferences for domestic consumers. Four terms govern changes in Domar weights. The first three terms are related to changes in sales of good i, while the last term is a mechanical effect of an increase in aggregate *nominal* value added for given sales of producer i. The first term on the right-hand side represents *substitution* that occurs at the level of the firm/sector that is then propagated *upstream* to other sectors, i.e., from buyers to suppliers. These complicated substitution patterns are captured by the input-substitution operator $\Phi_j(i,N+1)$, which can be thought of as a measure of *expenditure switching*. This input-output substitution operator comprises three steps that occur when a commodity price shock hits the economy that we explain next.

The first step is that following a positive shock to the commodity sector price, the price of good h increases by $\widetilde{\Omega}_{h,K}/\widetilde{\Omega}_{N+1,K}$.[5] This initial price change occurs because of downstream propagation on costs, as we already showed in Proposition 1.

In response to a change in the price of good h, each producer j may substitute away/towards other intermediate goods or to factors of production. If, for example, it substitutes away/towards to some other intermediate good k, it does so by $\Omega_{jk}(\delta_{kh}+(\theta^j_{kh}-1)\Omega_{jh})$, which represents how much the expenditure share of producer j on k respond to a change in the price of good h ($\partial\Omega_{jk}/\partial\log P_h$). This depends on the direct exposure of producer j to both k and h, and also on the elasticity of substitution between k and h, a term captured by the

[5]When good h is capital (K), this measure is simply $1/\widetilde{\Omega}_{N+1,K}$ as the network-adjusted capital share of the commodity sector is enough to pin down the rental rate changes in general equilibrium.

Allen-Uzawa elasticity of substitution, θ^j_{kh}. If goods have a high degree of substitutability, meaning that $\theta^j_{kh} > 1$, an increase in the price of good h increases the expenditure share of producer j on good k: producer j thus substitute away from producer h to k in such a way that its expenditure share on k increase. If goods k and h have a low degree of substitutability ($\theta^j_{kh} < 1$), then producer j cannot reallocate its input demand by that much, which in turn decreases its expenditure share in good k, i.e., it cannot get away from the price increase in good h, and it is forced to decrease is expenditure share on good k as a result. This is the second step.

How does this ultimately affect producer i? The third step answers this question by tying the substitution that each producer j is doing towards/away other producers k in the economy. The key term in this final step is the element of the Leontief-inverse Ψ_{ki} that represents how important producer i is as a supplier to producer k. This last term shows the *upstream propagation* of substitution since it goes from k (buyer) to i (seller). While the initial shock in this economy was propagated downstream in the network structure, these different demand substitution patterns propagated upstream in the production network.

The second and third terms in the equation represent how changes in total expenditure, induced by changes in domestic expenditure $\mathrm{d}\log E$ and $\mathrm{d}\log \bar{Y}$, propagate upstream to sector i. Since the logic is the same for both, we explain it here for domestic demand. An increase in domestic expenditure raises, up to first order, consumption of all goods that the consumer buys directly. These increases in demand translate into increases in intermediate input demand by all sectors. In turn, this means that the relevant statistic for exposure to final changes in demand is not just the direct exposure of sector i to final demand (b_i) but rather a network-adjusted measure, $\tilde{b}_i$, that consider the fact that changes in demand for other sectors also affect sector i via input-output linkages.

The last term, $\mathrm{d}\log GDP/\mathrm{d}\log P_{N+1}$ enters with a negative sign and captures the idea that keeping everything else constant, if total value added in this economy goes up, the Domar weight of each sector should decline proportionally. This follows by the definition of the Domar weights.

We are now ready to characterize changes in gross output.

Proposition 3 (Changes in Gross Output, $d\log Q_i$)**.** Up to a first-order approximation, changes in gross output following a commodity price shock, $\mathrm{d}\log P_{N+1}$, satisfy

$$\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_{N+1}} = \underbrace{\sum_{j=1}^{N+1} \frac{\lambda_j}{\lambda_i} \Phi_j(i, N+1)}_{\text{Input-Output Substitution}} + \underbrace{\frac{\tilde{b}_i}{\lambda_i} \left(\frac{GDP}{E} \frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi \right)}_{\text{Domestic Demand}} + \underbrace{\frac{\Psi_{N+1,i}}{\lambda_i} \frac{\bar{Y}}{GDP}\phi}_{\text{Foreign Demand}} - \underbrace{\frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}}}_{\text{Cost Push}} \tag{16}$$

where

$$\tilde{b}_i = \sum_{k=1}^{N+1} \Psi_{ki} b_k,$$

$$\tilde{b}_i + \Psi_{N+1,i} = \lambda_i, \quad \text{for all } i = 1, 2, ..., N+1$$

Proof. See Appendix C. ■

Intuitively, changes in the gross output of producer i can be inferred from changes in its sales and its price. The first three terms on the right-hand side correspond to the change in sales and represent how consumers, both final and intermediates, of good i react to an exogenous change in the commodity price. We already discussed these terms in a general way when stating Proposition 2. Here, we impose more structure that allows us to separate changes in domestic expenditure from changes in net transfers to the rest of the world.

Let us first focus on the domestic demand component. When the commodity price increases, nominal GDP increases due to higher demand by the commodity sector. This change in nominal GDP then propagates upstream throughout the network structure affecting all sectors. In a way, this effect captures a form of the well-known *wealth effect*: keeping everything else equal, the economy now has more income to spend. This idea is captured by the first term in the domestic demand component $\frac{\widetilde{b}_i}{\lambda_i} \frac{GDP}{E} \frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}}$, which depends on how much of total sales by producer i ended up meeting final domestic demand $(\widetilde{b}_i/\lambda_i)$, how important is nominal GDP in total domestic expenditure at the initial equilibrium (GDP/E), and the relative importance of the commodity sector in the economy $(\Lambda_K/\widetilde{\Omega}_{N+1,K} = \lambda_{N+1}/\Psi_{N+1,N+1})$. At the same time, the increase in the commodity price increases net transfers to the rest of the world, $\bar{Y}$. Everything else equal, this effectively reduces the domestic economy's expenditure on the goods it produces, pushing down domestic demand as a whole. There are many possible interpretations for this reduced-form device of net transfers to the rest of the world. One intuitive explanation is to think of $\bar{Y}$ as interest rate payments on foreign assets at the steady state of a dynamic small open economy model. To support exports at a steady state, the economy is effectively borrowing from the rest of the world. For the current account to be balanced, exports and interest rate payments on these assets must go hand in hand. By increasing exports, interest rate payments go up because the economy is now borrowing more from abroad in the new equilibrium, thus reducing expenditure on domestically produced goods, a form of expenditure switching towards the foreign economy.[6] This is why the second term on the domestic demand component $(\frac{\widetilde{b}_i}{\lambda_i}\frac{\bar{Y}}{E}\phi)$ enters with a negative sign. What effect dominates is, in the end, a quantitative matter.

We label the third term *foreign demand* since it follows from changes in net transfers to the rest of the world, which we can think of as exports for intuition purposes. With increases in the commodity price, everything else equal, the economy would like to export more. The only sector that can export is the commodity sector in our model. To produce more, the commodity sector requires domestically produced intermediate inputs, pushing their demand up and thus increasing gross output in these sectors. How much this channel matters for each producer, i, depends on how much it, directly and indirectly, supplies to the commodity sector $\Psi_{N+1,i}$, highlighting the upstream property of this channel.

The final term, the price of good i, fully characterizes the goods supply side of this market. Since the marginal cost of sector i pins down the price of good i in general equilibrium, conditional on factor prices and technology, it encompasses all relevant information on the supply of good i.

5 Quantitative exploration

Our theoretical model in the previous section highlighted four channels by which a change in the commodity price affects the gross output of non-commodity sectors. All these channels depend on the direct and indirect network linkages between non-commodity sectors and the

[6]Another explanation is to interpret $\bar{Y}$ as imports of final goods. To preserve the trade balance and allow the possibility of exporting in equilibrium, the increase in exports implies that imports must also rise, but this deviates demand from domestically produced goods to foreign-produced goods, again a form of expenditure switching.

commodity sector. Moreover, these channels are a function of the degree of substitutability among production inputs and the price elasticity of demand for commodity goods.

In this section, we provide a quantitative illustration of the model. To do so, we must take a stand on the different production functions of each producer. We assume that each producer has a nested CES function. They substitute among intermediate inputs with an elasticity of substitution equal to ϵ, which can potentially be different across producers. They also substitute between the intermediate input bundle and value-added with an elasticity of substitution equal to σ. These production elasticities play a crucial role in our results. Rather than use one value for each, we will illustrate the importance of different combinations of σ and ϵ in shaping the propagation of commodity price shocks to non-commodity sectors output. We provide a detailed description of the model structure in Appendix D.

Our goal is to understand the empirical evidence in Section 3, which highlights a strong positive upstream propagation and a muted downstream propagation of commodity price shocks on the gross output.

5.1 The Case of Non-Unitary Elasticities in Production

To better understand the role of production elasticities in shaping the output responses to commodity prices, we re-write the key terms of the input-output substitution operator in Proposition 3, based on Equation (13), as a function of CES production elasticities as follows

$$\left(\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_{N+1}}\right)^{\text{Term 1}} = \sum_{j=1}^{N+1} \frac{\lambda_j}{\lambda_i} \left(\sum_{h=1}^{N+1} \sum_{k=1}^{N+1} \Omega_{jk} \left[(\varepsilon_j - 1) \left(\Omega^M_{jh} - \delta_{kh} \right) - (\sigma_j - 1) \Omega^M_{jh} \Omega_{jL} \right] \Psi_{ki} \frac{\Psi_{h,N+1}}{\Psi_{N+1,N+1}} \right). \tag{17}$$

where

$$\Omega_{jk} = \frac{P_k M_{jk}}{P_j Q_j}; \quad \Omega_{jL} = \frac{W L_j}{P_j Q_j}; \quad \Omega^M_{jh} = \frac{P_h M_{jh}}{P^M_j M_j}.$$

When the term in Equation (17) is negative, firms in sector j substitute away from sector h's intermediates because firms in sector h use commodity goods, now more expensive, as inputs in production. These effects are mediated by $\Psi_{h,N+1}$, which describes the importance of the commodity sector as a supplier of intermediate inputs to sector h and by $\frac{\lambda_j}{\lambda_i}\Psi_{ki}$ which is the importance of a given sector k as a buyer to sector i (upstream propagation). All else equal, this generates a decline in intermediate input demand for good i, thus decreasing its gross output.

On top of the network structure, production elasticities are important in determining the strength of the input substitutability channel in amplifying or dampening the upstream and downstream propagation of commodity prices on non-commodity sectors' output. To analyze the role of production elasticities, we study the two possible cases for the term in brackets in Equation (17). When $k \neq h$, so $\delta_{kh} = 0$, the term in brackets becomes

$$\Omega^M_{jh}\big((\varepsilon_j - 1) - (\sigma_j - 1)\Omega_{jL}\big).$$

In this case, when flexibility between intermediates is high and either (i) flexibility between intermediates and labor is low or (ii) the labor share is small, increases in commodity prices tend to increase the demand for non-commodity output i. When $k = h$, so $\delta_{kh} = 1$, the term in brackets becomes

$$-(\varepsilon_j - 1)(1 - \Omega^M_{jh}) - (\sigma_j - 1)\Omega^M_{jh}\Omega_{jL}.$$

In this case, demand for sector i's output can increase when ϵ_j and σ_j are low or when ϵ_j is low but $\Omega_{jL} \approx 0$. In both cases, the importance of sector h (the commodity client) as a supplier to sector j (Ω^M_{jh}) is crucial for the quantitative importance of these propagation channels.

5.2 A Calibrated Example

We calibrate our model to match the Australian production structure in 1995. We calibrate the input-output parameters Ω^M_{ji} and Ω_{jL} assuming the economy starts at a symmetric equilibrium with $P_j = 1$ for all j. This way, we have that $\Omega^M_{ji} = \frac{P_i M_{ji}}{P^M_j M_j}$ and $1 - \Omega_{jL} = \frac{P^M_j M_j}{P_j Q_j}$ equal the observed intermediate input shares. We calibrate the consumption shares $\boldsymbol{b}$ and $\boldsymbol{b}^*$ using observed data on sectoral consumption expenditure in 1995 and exports, respectively. Finally, we assume that the commodity price is exogenous and $\log \bar{Y} = \phi \log P_{N+1}$, where ϕ is the sensitivity of the transfer to the rest of the world to the change in the commodity price. We assume that $\phi = 2$. In our calibrated example, we treat all sectors as non-tradable sectors, except the commodity sector. We calibrate our model using mining as the only commodity sector to highlight how input-output linkages interact with production elasticities for different commodity industries.

We first investigate how production elasticities shape the relationship between sectoral output response to commodity price shocks and each sector's upstream and downstream exposure to the commodity sector. Figure 3 plots in the vertical axis the model implied log change in non-commodities sectoral output from a change in the commodity price. The horizontal axis in panel (a) shows the upstream distance of each sector to the commodity sector, while panel (b) shows the downstream distance of each sector to the commodity sector. Regarding the upstream exposure to commodities, we observe that high substitutability between inputs (high σ or ε) can amplify the upstream propagation of commodity prices (panel (a) of Figure 3). For example, a high ε makes the output of two important upstream sectors to mining, sectors 13 (Machinery, Nec) and 24 (Water Transport), increase even more. Interestingly, the opposite occurs with sector 8 (Coke, Refined Petroleum, and Nuclear Fuel), which sees a larger reduction in output when ε is large. The reason is that while sector 8 is a relatively important supplier of mining, it is also, by far, the most important client of mining. Therefore, as we observe in panel (b), for the case of $\varepsilon > 1$, sector 8 is the sector with the largest output decline. The increase in mining price strongly propagates downstream to sector 8, increasing its marginal cost and reducing its production. Here we can see how the interaction between production elasticities and network connections to the commodity sector are crucial in shaping the transmission of commodity price shocks.

In Panel (b) of Figure 3, we observe that production elasticities are very important in shaping the relationship between downstreamness to commodity and the output response to an increase in commodity prices. As we discussed above, when intermediate inputs are highly substitutable (high ε), the downstream propagation of commodity prices can be further amplified. Nevertheless, when firms are flexible in substituting labor and intermediates (high σ), the downstream propagation of commodity prices is completely muted. Intuitively, a high σ allow firms that are downstream to commodities to use more labor which is now relatively cheaper. Hence, deviating from Cobb-Douglas production technologies is important to understand the empirical observations on the transmission of commodity prices in small open economies.

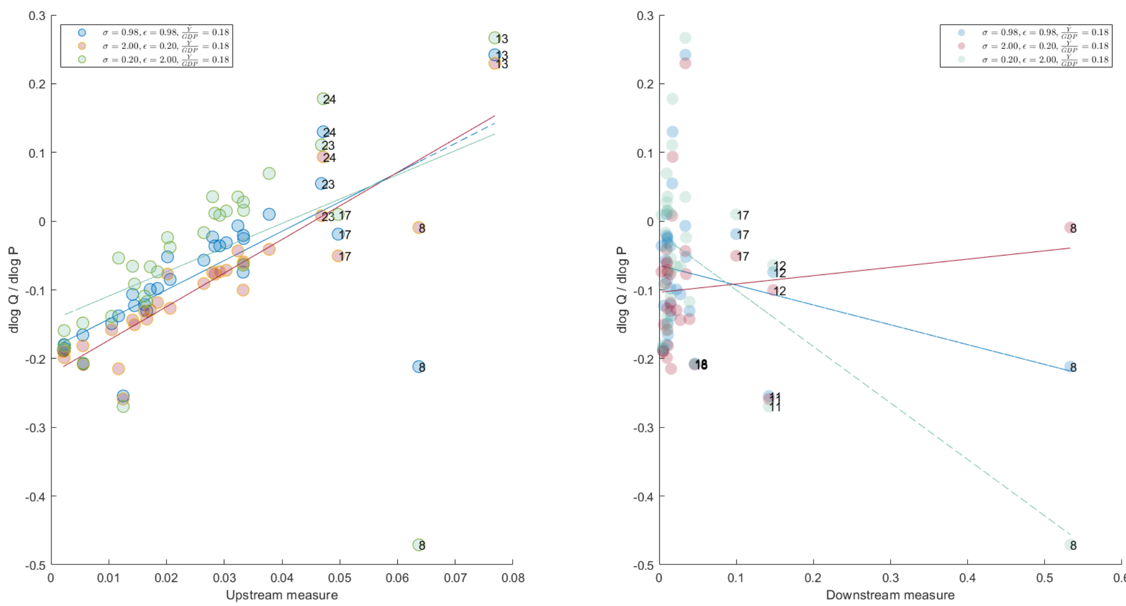


Figure 3. Sectoral Propagation of a Mining Price Shock: The Role of Elasticities

Note: This figure shows the model-implied relationship between sectors' Downstream (right-panel) and Upstream (left-panel) exposure to the mining sector, using Equation (1) and (2), respectively. The vertical axis measures the output response to an increase in the commodity price based on Proposition 3. The size of each node indicates the magnitude of the cost-push channel (right panel) and the input-output substitution channel (left panel). Labels in each node are linked to sectors in Table B1 of our Appendix.

In Figure 4, we study the importance of the foreign demand channel in rationalizing our evidence. We solve our model for different values of $\frac{\bar{Y}}{GDP}$, the importance of commodity exports for the small open economy. In this case, we fix production elasticities to be close to Cobb-Douglas. We observe that low values of $\frac{\bar{Y}}{GDP}$ would generate a counterfactual relationship between the upstreamness and downstreamness of the commodity sector and the sectoral output effect of commodity prices. In particular, that calibration would generate muted upstream and strong downstream propagation. A large value of $\frac{\bar{Y}}{GDP}$ can rationalize the evidence of strong upstream propagation and muted downstream propagation.[7]

[7] Tables B3 and B4 in Appendix B.1 provide regressions results for different exercises using log output change as the dependent variable and upstreamness to commodity (Table B3) or downstreamness to commodity (Table B4) as independent variables.

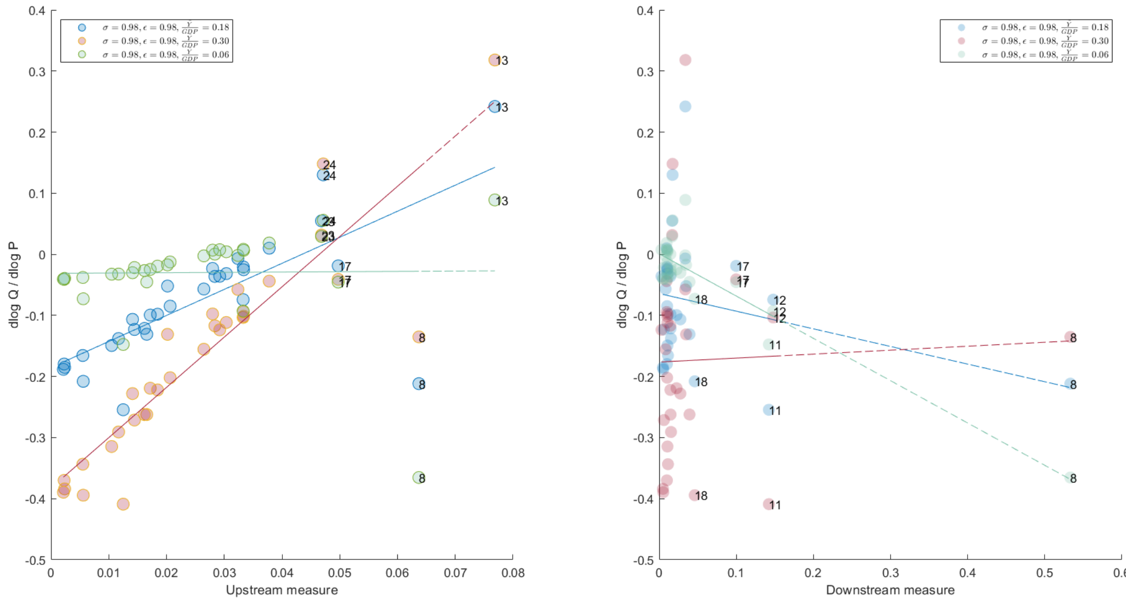


Figure 4. Sectoral Propagation of a Mining Price Shock: the Role of the Export Share

Note: This figure shows the model-implied relationship between sectors' Downstream (right-panel) and Upstream (left-panel) exposure to the mining sector, using Equation (1) and (2), respectively. The vertical axis measures the output response to an increase in the commodity price based on Proposition 3. The size of each node indicates the magnitude of the cost-push channel (right panel) and the input-output substitution channel (left panel). Labels in each node are linked to sectors in Table B1 of our Appendix.

6 Conclusion

We study how sectoral commodity price shocks propagate through domestic production networks in small open economies. We provide empirical evidence and a theoretical model that highlight the role of production elasticities and commodity export intensity in shaping the propagation of commodity price shocks along the production chain. We first show that commodity sectors are central sectors, both as sellers and buyers, in the domestic production network of small open economies. We then show, empirically and theoretically, that the propagation of sectoral commodity price shocks to non-commodity sectors' output has an important production network component. We find that the gross output of non-commodity upstream sectors, those sectors supplying intermediate inputs to commodity sectors, largely respond to commodity price shocks. In contrast, we find evidence of muted downstream propagation, to those buying intermediate inputs from commodity sectors.

We develop a small open economy model featuring domestic production networks and characterize the transmission channels of commodity price shocks through production chains. We highlight three upstream channels (from intermediate input demand, domestic household income, and foreign demand) and one downstream channel (increased input costs). We show that the elasticities of substitution between inputs and the importance of commodity exports on total GDP are crucial in amplifying or dampening the upstream and downstream channels. In particular, the demand-side effects operate as follows. An increase in commodity price can increase demand for domestic intermediate inputs (upstream propagation) from changes in foreign demand or domestic demand ("wealth effect"). The role of production elasticities works as follows. When sectors present high substitutability between intermediate inputs, a higher commodity price increases demand for non-commodity intermediates, which generates an increase in the production of upstream sectors. On the other hand, when sectors present high substitutability between intermediates and labor, increases in commodity prices have smaller effects on the marginal costs of sectors downstream of commodities. Thus, there exists a dampened effect on quantities of industries downstream to commodity sectors. The

All in all, our results highlight the importance of the production network in propagating commodity price shocks throughout the economy.

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ONLINE APPENDIX

A Data sources and Definitions

Macroeconomic Aggregates

The data for the estimation of commodity price shocks are obtained from the following sources: Fernández et al. (2018) is used for the sectoral commodity price index.

Input-Output Table Database

WIOD Data. Our main database is the World Input-Output database (Timmer et al., 2015), release 2013. It provides information on intersectoral and cross-country final and intermediate flows for 40 countries and 35 sectors classified according to the International Standard Industrial Classification Revision 3 (ISIC Rev. 3). These tables match the 1993 version of the SNA. We use the sectoral data on quantities (gross output, value-added, number of employees, and capital) and price indexes for the period 1995-2011(2009) in the National IO tables. The sample of small open economies with data on commodity prices and WIOD input-output data includes the following countries: Australia, Bulgaria, Brazil, Canada, Denmark, India, Lithuania, Mexico, and Russia.

This dataset is freely available here https://www.rug.nl/ggdc/valuechain/wiod/wiod-2013-release.

Commodity Data

To measure sectoral linkages to the *commodity sector* we use detailed information on each country's commodity bundle composition from Fernández et al. (2018). There is a total of 44 commodities classified according to the Harmonized System (HS) 1992 – 4 digits. We separate commodities into 3 groups: Agriculture, Hunting, Forestry, and Fishing; Mining and Quarrying; and Food Products, Beverages, and Tobacco.

B Additional Tables and Figures

B.1 Tables

Table B1. Sectors in WIOD Database

Sector Number	Sector Name
1	Agriculture, Hunting, Forestry and Fishing
2	Mining and Quarrying
3	Food, Beverages, and Tobacco
4	Textiles and Textile Products
5	Leather, Leather, and Footwear
6	Wood and Products of Wood and Cork
7	Pulp, Paper, Paper, Printing, and Publishing
8	Coke, Refined Petroleum and Nuclear Fuel
9	Chemicals and Chemical Products
10	Rubber and Plastics
11	Other Non-Metallic Mineral
12	Basic Metals and Fabricated Metal
13	Machinery, Nec
14	Electrical and Optical Equipment
15	Transport Equipment
16	Manufacturing, Nec; Recycling
17	Electricity, Gas and Water Supply
18	Construction
19	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles
20	Wholesale Trade and Commission Trade
21	Retail Trade
22	Hotels and Restaurants
23	Inland Transport
24	Water Transport
25	Air Transport
26	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
27	Post and Telecommunications
28	Financial Intermediation
29	Real Estate Activities
30	Renting of M&Eq and Other Business Activities
31	Public Admin and Defence; Compulsory Social Security
32	Education
33	Health and Social Work
34	Other Community, Social and Personal Services

Table B2. Commodities and WIOD Industries.

Commodity	HS Code	Industry
Beef	201	Agriculture, hunting, forestry and fishing
Pork	203	Agriculture, hunting, forestry and fishing
Lamb	204	Agriculture, hunting, forestry and fishing
Chicken	207	Agriculture, hunting, forestry and fishing
Fish	301	Agriculture, hunting, forestry and fishing
Fish Meal	304	Agriculture, hunting, forestry and fishing
Shrimp	306	Agriculture, hunting, forestry and fishing
Bananas	803	Agriculture, hunting, forestry and fishing
Coffee	901	Agriculture, hunting, forestry and fishing
Tea	902	Agriculture, hunting, forestry and fishing
Wheat	1001	Agriculture, hunting, forestry and fishing
Barley	1003	Agriculture, hunting, forestry and fishing
Corn	1005	Agriculture, hunting, forestry and fishing
Rice	1006	Agriculture, hunting, forestry and fishing
Soybeans	1201	Agriculture, hunting, forestry and fishing
Groundnuts	1202	Agriculture, hunting, forestry and fishing
Wool	1505	Agriculture, hunting, forestry, and fishing
Sugar	1701	Agriculture, hunting, forestry and fishing
Cocoa	1801	Agriculture, hunting, forestry and fishing
Natural Rubber	4001	Agriculture, hunting, forestry, and fishing
Hides	4101	Agriculture, hunting, forestry and fishing
Hard Log	4401	Agriculture, hunting, forestry and fishing
Soft Log	4403	Agriculture, hunting, forestry and fishing
Hard Swan	4407	Agriculture, hunting, forestry and fishing
Soft Swan	4408	Agriculture, hunting, forestry and fishing
Cotton	5201	Agriculture, hunting, forestry and fishing
Iron	2601	Mining and quarrying
Copper	2603	Mining and quarrying
Nickel	2604	Mining and quarrying
Aluminum	2606	Mining and quarrying
Lead	2607	Mining and quarrying
Zinc	2608	Mining and quarrying
Tin	2609	Mining and quarrying
Coal	2701	Mining and quarrying
Crude Oil	2709	Mining and quarrying
NatGas	2711	Mining and quarrying
Uranium	2844	Mining and quarrying
Gold	7108	Mining and quarrying
Soybean Meal	1208	Food products, beverages and tobacco
Soy Oil	1507	Food products, beverages and tobacco
Olive Oil	1509	Food products, beverages and tobacco
Palm Oil	1511	Food products, beverages and tobacco
Sun Oil	1512	Food products, beverages and tobacco
Coconut Oil	1513	Food products, beverages and tobacco

Table B3. Calibration Exercises: The Role of Upstreamness

	Calibration: $(\sigma, \epsilon, \phi, \frac{\bar{Y}}{GDP})$				
	(0.98, 0.98, 2, 0.18)	(2, 0.2, 2, 0.18)	(0.2, 2, 2, 0.18)	(0.98, 0.98, 2, 0.30)	(0.98, 0.98, 2, 0.06)
Upstream	4.26***	4.89***	3.53***	8.23***	0.05
	(0.7205)	(0.4083)	(1.2552)	(0.7278)	(0.7805)
Observations	31	31	31	31	31

Note: This table shows a cross-sectional regression of the form $\mathrm{d}\log Q_i/\mathrm{d}\log P_{N+1} = \beta_0 + \beta_1 Upstream_i + \varepsilon_i$. The dependent variable is the change in quantities implied by Proposition 3, where $Upstream_i$ refers to how upstream and upstream sector i is from the commodity sector.

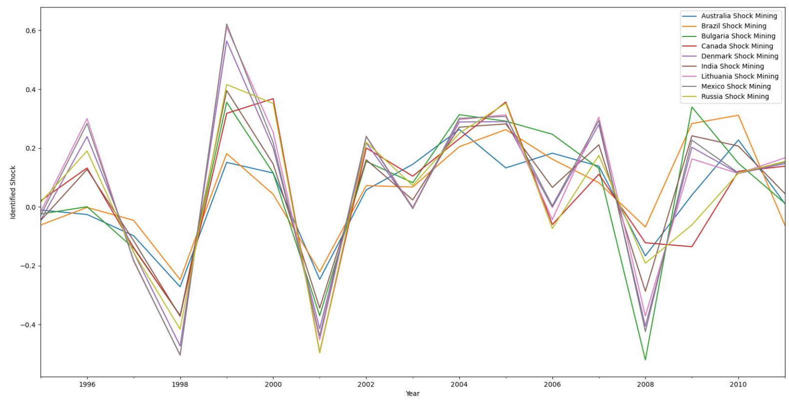
Table B4. Calibration Exercises: The Role of Downstreamness

	Calibration: $(\sigma, \epsilon, \phi, \frac{\bar{Y}}{GDP})$				
	(0.98, 0.98, 2, 0.18)	(2, 0.2, 2, 0.18)	(0.2, 2, 2, 0.18)	(0.98, 0.98, 2, 0.30)	(0.98, 0.98, 2, 0.06)
Downstream	-0.29	0.12	-0.82***	0.06	-0.69***
	(0.1889)	(0.1811)	(0.2101)	(0.3102)	(0.0628)
Observations	31	31	31	31	31

Note: This table shows a cross-sectional regression of the form $\mathrm{d}\log Q_i/\mathrm{d}\log P_{N+1} = \beta_0 + \beta_1 Downstream_i + \varepsilon_i$. The dependent variable is the change in quantities implied by Proposition 3, where $Downstream_i$ refers to how downstream sector i is from the commodity sector.

B.2 Extra Figures

Figure B1. Identified Commodity Price Shock: Agriculture and Forestry

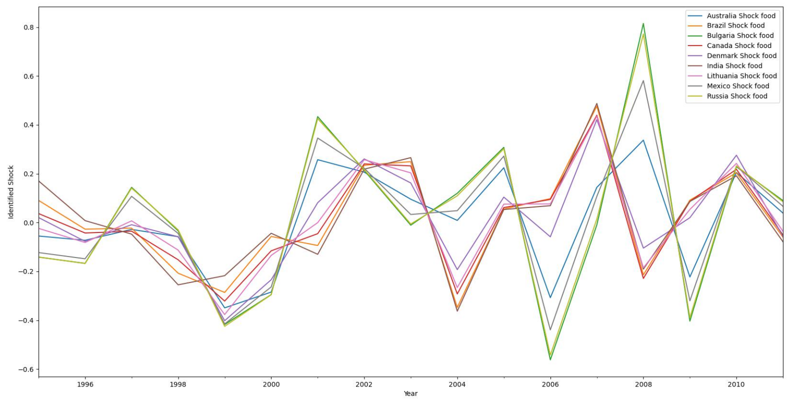


Note: This figure plots commodity price shock for mining and quarrying for our sample of 9 countries in the WIOD database. The shocks are estimated at the quarterly level and we convert them to annual by taking the sum.

Figure B2. Identified Commodity Price Shock: Mining and Quarrying

Note: This figure plots commodity price shock for agriculture and forestry for our sample of 9 countries in the WIOD database. The shocks are estimated at the quarterly level and we convert them to annual by taking the sum.

Figure B3. Identified Commodity Price Shock: Food Products and Beverages



Note: This figure plots commodity price shock for foods sectors for our sample of 9 countries in the WIOD database. The shocks are estimated at the quarterly level and we convert them to annual by taking the sum.

C Proofs

Proof of Proposition 1. Starting from price changes, we have

$$d\log P_i = \Omega_{iL} d\log W + \sum_{j=1}^{N+1} \Omega_{ij} d\log P_j - d\log Z_i \qquad \text{for all } i = 1, 2, ..., N$$

$$d\log P_{N+1} = \Omega_{N+1,L} d\log W + \Omega_{N+1,K} d\log R + \sum_{j=1}^{N+1} \Omega_{N+1,j} d\log P_j - d\log Z_{N+1}$$

where we define $\boldsymbol{\Omega}_K = (0, 0, ..., \Omega_{N+1,K})$ is a $(N+1) \times 1$ vector where its first N elements are 0 because non-tradable sectors do not use capital directly.

Using the wage as the numeraire, $d\log W = 0$, and stacking the system into matrix/vector form, we have

$$d\log \boldsymbol{P} = \boldsymbol{\Omega} d\log \boldsymbol{P} + \boldsymbol{\Omega}_K d\log R - d\log \boldsymbol{Z}$$

Setting $d\log \boldsymbol{Z} = \boldsymbol{0}$ and inverting the system we arrive

$$d\log \boldsymbol{P} = \boldsymbol{\Psi}\boldsymbol{\Omega}_K d\log R \Longrightarrow d\log P_i = \Psi_{i,N+1}\Omega_{N+1,K} d\log R \text{ for all } i = 1, 2, ..., N+1 \quad (18)$$

Note that we can write the above expression as

$$d \log \boldsymbol{P} = \widetilde{\boldsymbol{\Omega}}_K d \log R$$

where we define the typical element of $\widetilde{\boldsymbol{\Omega}}_K = \{\widetilde{\Omega}_{iK}\} = \{\Psi_{i,N+1}\Omega_{N+1,K}\}$, that represents the *network-adjusted* capital share of producer i.

We now make use of the fact that $d \log P_{N+1}$ is exogenously given to express changes in the rental rate, $d \log R$, as an explicit function of it since

$$d \log P_{N+1} = \widetilde{\Omega}_{N+1,K} d \log R \Longrightarrow d \log R = \frac{1}{\widetilde{\Omega}_{N+1,K}} d \log P_{N+1}$$

Replacing this expression into Equation (18), we get

$$d \log P_i = \frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}} d \log P_{N+1} \tag{19}$$

which completes the proof. ■

Proof of Proposition 2. Start from the market clearing condition and aggregate resource constraints in terms of Domar weights and matrix form

$$\boldsymbol{\lambda} = \boldsymbol{\Psi}^T \left(\boldsymbol{b} + \boldsymbol{e}_{N+1} \frac{\bar{Y}}{GDP} \right) \tag{20}$$

Totally differentiating this expression

$$d\boldsymbol{\lambda} = \underbrace{d\boldsymbol{\Psi}^T \left(\boldsymbol{b} + \boldsymbol{e}_{N+1} \frac{\bar{Y}}{GDP} \right)}_{\text{Changes in IO matrix given Demand Shares}} + \underbrace{\boldsymbol{\Psi}^T \left(d\boldsymbol{b} + \boldsymbol{e}_{N+1} d \left(\frac{\bar{Y}}{GDP} \right) \right)}_{\text{Changes in Demand Shares given IO linkages}} \tag{21}$$

We now totally differentiate the definition of the Leontieff-Inverse, $\boldsymbol{\Psi}$, to map its changes to changes in the IO matrix, $\boldsymbol{\Omega}$

$$
\begin{aligned}
\boldsymbol{\Psi}^T &= (\boldsymbol{I} - \boldsymbol{\Omega}^T)^{-1} \\
\boldsymbol{\Psi}^T(\boldsymbol{I} - \boldsymbol{\Omega}^T) &= \boldsymbol{I} \\
\boldsymbol{\Psi}^T - \boldsymbol{\Psi}^T\boldsymbol{\Omega}^T &= \boldsymbol{I} \\
d\boldsymbol{\Psi}^T - d\boldsymbol{\Psi}^T\boldsymbol{\Omega}^T - \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T &= \boldsymbol{0} \\
d\boldsymbol{\Psi}^T(\boldsymbol{I} - \boldsymbol{\Omega}^T) &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \\
d\boldsymbol{\Psi}^T &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \boldsymbol{\Psi}^T \\
d\boldsymbol{\Psi}^T(\boldsymbol{b} + \boldsymbol{b}^*) &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}' \underbrace{\boldsymbol{\Psi}^T(\boldsymbol{b} + \boldsymbol{b}^*)}_{=\boldsymbol{\lambda}} \\
d\boldsymbol{\Psi}^T(\boldsymbol{b} + \boldsymbol{b}^*) &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \boldsymbol{\lambda}
\end{aligned}
$$

Using this expression into Equation (21)

$$
d\boldsymbol{\lambda} = \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \boldsymbol{\lambda} + \boldsymbol{\Psi}^T \left(d\boldsymbol{b} + \boldsymbol{e}_{N+1} d \left(\frac{\bar{Y}}{GDP} \right) \right) \tag{22}
$$

For a given producer i, we have

$$
d\lambda_i = \sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1} \Psi_{ki} \left(b_k d\log b_k + \delta_{k,N+1} \frac{\bar{Y}}{GDP} (\mathrm{d}\log\bar{Y} - \mathrm{d}\log GDP) \right) \tag{23}
$$

We now focus on the last term on the right-hand side of the above equation. To focus on the propagation mechanisms from intermediate inputs, we assume that the home consumer has Cobb-Douglas preferences over goods. This means that consumption of each good k as a share of *total expenditure* is constant and independent of quantities and prices. Note, however, that b_k is a ratio with respect to the nominal GDP of the home country. As a result, they may respond to changes in both expenditure and GDP. The Cobb-Douglas preferences do not imply that these ratios are constant. To be more transparent, write

$$
b_k = \frac{P_k C_k}{GDP} = \frac{P_k C_k}{E} \frac{E}{GDP}
$$

where E represents total expenditure at home.

Log-differentiating the above expression

$$\mathrm{d}\log b_k = \underbrace{d\log\frac{P_kC_k}{E}}_{\text{=0 due to Cobb-Douglas Preferences}} + \mathrm{d}\log\frac{E}{GDP} = \mathrm{d}\log E - \mathrm{d}\log GDP$$

Therefore, b_k will change, provided that there is a difference between changes in expenditure and changes in nominal GDP. Intuitively, under Cobb-Douglas preferences, expenditure in good k raises proportionally to changes in total expenditure. This raises the numerator in $d\log E$ for all good k, while the denominator raises as changes in nominal GDP $d\log GDP$. Therefore, if expenditure raises more than nominal GDP, the expenditure share on good k as a fraction of nominal GDP, $d\log b_k$, will raise.

Using these results into Equation (23), we get

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}(b_k d\log b_k + \delta_{k,N+1}\frac{\bar{Y}}{GDP}(d\log\bar{Y} - d\log GDP))$$

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}\left(b_k(d\log E - d\log GDP) + \delta_{k,N+1}\frac{\bar{Y}}{GDP}(d\log\bar{Y} - d\log GDP)\right)$$

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y}) - \underbrace{\sum_{k=1}^{N+1}\Psi_{ki}\left(b_k + \delta_{k,N+1}\frac{\bar{Y}}{GDP}\right)}_{=\lambda_i} d\log GDP$$

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}\left(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y}\right) - \lambda_i d\log GDP$$

Upon rearranging

$$d\log\lambda_i = \frac{1}{\lambda_i}\left(\sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y})\right) - d\log GDP \tag{24}$$

We are now ready to construct the input-output substitution operator. Write the changes in expenditure shares

$$
\begin{aligned}
\mathrm{d}\log\Omega_{jk} &= \mathrm{d}\log P_k + \sum_{h=1}^{N+1+F}(\theta^j_{kh}-1)\Omega_{jh}\mathrm{d}\log P_h \\
&= \delta_{kh}\mathrm{d}\log P_h + \sum_{h=1}^{N+1+F}(\theta^j_{kh}-1)\Omega_{jh}\mathrm{d}\log P_h \\
\mathrm{d}\log\Omega_{jk} &= \sum_{h=1}^{N+1+F}(\delta_{kh}+(\theta^j_{kh}-1)\Omega_{jh})\mathrm{d}\log P_h \qquad (25)
\end{aligned}
$$

where δ_{kh} is the Kronecker delta, equal to 1 if $k=h$ and zero otherwise. In that expression, we also define

$$\theta^j_{kh} = \frac{\varepsilon^j_{kh}}{\Omega_{jh}} \qquad (26)$$

$$\varepsilon^j_{kh} = \frac{\partial\log M_{jk}}{\partial\log P_h} \qquad (27)$$

where Equation (26) is the Allen-Uzawa elasticity for producer j between input k and h. Note that input h can be either a factor or an intermediate input, while input k is always an intermediate good. Equation (27) represents the constant-output elasticity of input demand of producer j of good k with respect to a change in the price of good/factor h.

We now use the model's structure to simplify the above expression. In particular, notice that we only have one factor, which price is changing: capital. Therefore, $F=1$ and we can write Equation (25) as

$$\mathrm{d}\log\Omega_{jk} = \sum_{h=1}^{N+1}(\delta_{kh}+(\theta^j_{kh}-1)\Omega_{jh})\mathrm{d}\log P_h + (\theta^j_{kK}-1)\Omega_{jK}\mathrm{d}\log R$$

At this point, we can use the result in Proposition 1 that relates changes in prices to changes in the commodity price and the one that links changes in the price of capital to changes in the commodity price as well. We rewrite those below

$$\mathrm{d}\log P_h = \frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} \mathrm{d}\log P_{N+1} \tag{28}$$

$$\mathrm{d}\log R = \frac{1}{\widetilde{\Omega}_{N+1,K}} \mathrm{d}\log P_{N+1} \tag{29}$$

Plugging these expressions into the above expression, we get

$$\mathrm{d}\log \Omega_{jk} = \left(\sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} + \frac{(\theta^j_{kK} - 1)\Omega_{jK}}{\widetilde{\Omega}_{N+1,K}} \right) \mathrm{d}\log P_{N+1} \tag{30}$$

We can replace Equation (30) into the first term on the right-hand side of Equation (24), to get

$$\begin{aligned}
\sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk}\lambda_j d\log \Omega_{jk} &= \sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk}\lambda_j \left(\sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} + \frac{(\theta^j_{kK} - 1)\Omega_{jK}}{\widetilde{\Omega}_{N+1,K}} \right) \mathrm{d}\log P_{N+1} \\
&= \sum_{j=1}^{N+1} \lambda_j \left(\sum_{k=1}^{N+1} \sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \Psi_{ki}\Omega_{jk} \frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} + \sum_{k=1}^{N+1} \Psi_{ki}\Omega_{jk} \frac{(\theta^j_{kK} - 1)\Omega_{jK}}{\widetilde{\Omega}_{N+1,K}} \right) \mathrm{d}\log P_{N+1}
\end{aligned}$$

Let's define the following objects

$$\begin{aligned}
\Phi_j(\Psi_{(:,i)}, \widetilde{\mathbf{\Omega}}_K) &= \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \Psi_{ki}\Omega_{jk}\widetilde{\Omega}_{h,K} \\
\Phi_j(\Psi_{(:,i)}, \mathbf{\Omega}_K) &= \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \Psi_{ki}\Omega_{jk}(\theta^j_{kK} - 1)\Omega_{jK} \\
\Phi_j(i, N+1) &= \Phi_j(\Psi_{(:,i)}, \widetilde{\mathbf{\Omega}}_K) + \Phi_j(\Psi_{(:,i)}, \mathbf{\Omega}_K)
\end{aligned}$$

Hence, we have

$$\sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk}\lambda_j \mathrm{d}\log \Omega_{jk} = \sum_{j=1}^{N+1} \lambda_j \Phi_j(i, N+1) \mathrm{d}\log P_{N+1} \tag{31}$$

where $\Phi_j(i, N+1)$ is a version of the *Input-Substitution Operator* defined by Baqaee and Farhi (2019) applied to our small open economy environment. This operator captures how, in response to a change in the commodity price, producer j substitutes away/towards producer i both directly and indirectly through input-output linkages.

Therefore, changes in the Domar weights can be written as

$$d\log\lambda_i = \frac{1}{\lambda_i}\left(\sum_{j=1}^{N+1}\lambda_j\Phi_j(i,N+1)\mathrm{d}\log P_{N+1}\sum_{k=1}^{N+1}\Psi_{ki}\left(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y}\right)\right)$$
$$- d\log GDP, \tag{32}$$

which completes the proof. ■

Proof of Proposition 3. To prove this result, we rearrange the result in Proposition 2 to get

$$\begin{aligned} d\log Q_i &= d\log\lambda_i + d\log GDP - d\log P_i \\ &= \frac{1}{\lambda_i}\left(\sum_{j=1}^{N+1}\lambda_j\Phi_j(i,N+1)\mathrm{d}\log P_{N+1} + \sum_{k=1}^{N+1}\Psi_{ki}\left(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y}\right)\right) - d\log P_i \end{aligned}$$

Using Proposition 1, we can rewrite this as

$$\begin{aligned} d\log Q_i = &\left(\sum_{j=1}^{N+1}\frac{\lambda_j}{\lambda_i}\Phi_j(\Psi_{(:,i)},\Psi_{(:,N+1)}) - \frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}}\right)\mathrm{d}\log P_{N+1} \\ &+ \sum_{k=1}^{N+1}\frac{\Psi_{ki}}{\lambda_i}b_k\mathrm{d}\log E + \sum_{k=1}^{N+1}\frac{\Psi_{ki}}{\lambda_i}\frac{\bar{Y}}{GDP}\delta_{k,N+1}d\log\bar{Y} \end{aligned}$$

We are left to link $\mathrm{d}\log E$ and $\mathrm{d}\log\bar{Y}$ to $\mathrm{d}\log P_{N+1}$. For total expenditure, recall the following two equations

$$\mathrm{d}\log GDP = \frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}}\mathrm{d}\log P_{N+1},$$
$$E = GDP - \bar{Y}.$$

To proceed, we assume $\bar{Y}$ is a positive function of the commodity price. In particular,

$$\log\bar{Y} = \phi\log P_{N+1}.$$

Log-differentiating the expression for expenditure and using the above relationships, we get

$$\mathrm{d}\log E = \frac{GDP}{E}\mathrm{d}\log GDP - \frac{\bar{Y}}{E}\mathrm{d}\log \bar{Y} = \left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)\mathrm{d}\log P_{N+1}$$

Hence, changes in domestic expenditure satisfies

$$\mathrm{d}\log E = \left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)\mathrm{d}\log P_{N+1}$$

Replacing the expression for domestic expenditure and net transfers to the rest of the world as a function of the commodity price shock, we arrive at

$$\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_{N+1}} = \underbrace{\sum_{j=1}^{N+1}\frac{\lambda_j}{\lambda_i}\Phi_j(i, N+1)}_{\text{Input-Output Substitution}} + \underbrace{\frac{\tilde{b}_i}{\lambda_i}\left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)}_{\text{Domestic Demand}} + \underbrace{\frac{\Psi_{N+1,i}}{\lambda_i}\frac{\bar{Y}}{GDP}\phi}_{\text{Foreign Demand}} - \underbrace{\frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}}}_{\text{Cost Push}} \tag{33}$$

where

$$\tilde{b}_i = \sum_{k=1}^{N+1}\Psi_{ki}b_k,$$

$$\tilde{b}_i + \Psi_{N+1,i} = \lambda_i, \quad \text{for all } i = 1, 2, ..., N+1$$

This completes the proof. ■

D Model Structure

We assume that each producer possesses a nested CES structure.

D.1 Non-Tradable Producers: $j = 1, 2, ..., N$

Each non-tradable producer combines intermediate inputs and labor to produce gross output, Q_i, according to the following production function

$$Q_j = Z_j \left(a_j^{\frac{1}{\sigma_j}} L_j^{\frac{\sigma_j - 1}{\sigma_j}} + (1 - a_j)^{\frac{1}{\sigma_j}} M_j^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}$$

where Z_j is the productivity level, L_j is labor demand, M_j is an intermediate input bundle, and σ_j is the elasticity of substitution between labor and the intermediate input bundle. a_j is the share of labor in total sales of producer i.

The intermediate input bundle combines intermediate inputs from each producer j, such that

$$M_j = \left(\sum_{k=1}^{N+1} \omega_{jk}^{\frac{1}{\varepsilon_j}} M_{jk}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}}$$

where ω_{jk} is the share of intermediate input i on the cost of the intermediate input bundle and M_{jk} is the demand by producer j from producer k. ε_j is the elasticity of substitution among intermediate inputs by producer j.

Cost minimization implies the following conditional demands, marginal costs, and intermediate input price index

$$
\begin{aligned}
L_j &= a_j \left(\frac{W}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j-1} Q_j \\
M_j &= (1-a_j) \left(\frac{P_j^M}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j-1} Q_j \\
M_{jk} &= \omega_{jk} \left(\frac{P_k}{P_j^M}\right)^{-\varepsilon_j} M_j = \omega_{jk}(1-a_j) P_j^{-\varepsilon_j} (P_j^M)^{\varepsilon_j-\sigma_j} MC_j^{\sigma_j} Z_j^{\sigma_j-1} Q_j \\
MC_j &= Z_j \left(a_j W^{1-\sigma_i} + (1-a_j)(P_j^M)^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}} \\
P_j^M &= \left(\sum_{k=1}^{N+1} \omega_{jk} P_k^{1-\varepsilon_j}\right)^{\frac{1}{1-\varepsilon_j}}
\end{aligned}
$$

D.2 Commodity Sector: $j = N+1$

The only difference between the commodity sector and the rest of the sectors is that it also uses capital. Otherwise, the structure is the same. In particular, it produces according to the following production function

$$
Q_j = Z_j \left(a_j^{\frac{1}{\sigma_j}} V_j^{\frac{\sigma_j-1}{\sigma_j}} + (1-a_j)^{\frac{1}{\sigma_j}} M_j^{\frac{\sigma_j-1}{\sigma_j}}\right)^{\frac{\sigma_j}{\sigma_j-1}}
$$

where the intermediate input bundle is the same as before. This commodity sector also combines labor and capital according to the following aggregator

$$
V_j = \left(d_j^{\frac{1}{\nu_j}} L_j^{\frac{\nu_j-1}{\nu_j}} + (1-d_j)^{\frac{1}{\nu_j}} K_j^{\frac{\nu_j-1}{\nu_j}}\right)^{\frac{\nu_j}{\nu_j-1}}
$$

Here, K_j is capital demand, and ν_j is the elasticity of substitution between labor and capital for the commodity sector.

As before, conditional demands, marginal costs, and price indices satisfy

$$
\begin{aligned}
L_j &= d_j \left(\frac{W}{P_j^V}\right)^{-\nu_j} V_j \\
K_j &= d_j \left(\frac{W}{P_j^V}\right)^{-\nu_j} V_j \\
V_j &= a_j \left(\frac{P_j^V}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j-1} Q_j \\
M_j &= (1-a_j) \left(\frac{P_j^M}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j-1} Q_j \\
M_{jk} &= \omega_{jk} \left(\frac{P_k}{P_j^M}\right)^{-\varepsilon_j} M_j = \omega_{jk}(1-a_j) P_j^{-\varepsilon_j} (P_j^M)^{\varepsilon_j-\sigma_j} MC_j^{\sigma_j} Z_j^{\sigma_j-1} Q_j \\
MC_j &= Z_j \left(a_j (P_j^V)^{1-\sigma_i} + (1-a_j)(P_j^M)^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}} \\
P_j^M &= \left(\sum_{k=1}^{N+1} \omega_{jk} P_k^{1-\varepsilon_j}\right)^{\frac{1}{1-\varepsilon_j}} \\
P_j^V &= \left(d_j W^{1-\nu_i} + (1-d_j) R^{1-\nu_i}\right)^{\frac{1}{1-\nu_i}}
\end{aligned}
$$

D.3 Allen-Uzawa Elasticities of Substitution

From these conditional demands, we can compute each producer's Allen-Uzawa elasticities of substitution between any two pairs of inputs, (k, h), θ_{kh}^j, as follows

$$
\begin{aligned}
M_{jk} &= P_k^{-\varepsilon_j} (P_j^M)^{\varepsilon_j-\sigma_j} MC_j^{\sigma_j} Z_j^{\sigma_j-1} Q_j \\
\frac{\partial \log M_{jk}}{\partial \log P_h} &= -\varepsilon_j \delta_{kh} + (\varepsilon_j - \sigma_j) \frac{\partial \log P_j^M}{\partial \log P_h} + \sigma_j \frac{\partial \log MC_j}{\partial \log P_h} \\
\frac{\partial \log M_{jk}}{\partial \log P_h} &= -\varepsilon_j \delta_{kh} + (\varepsilon_j - \sigma_j) \underbrace{\frac{P_h M_{jh}}{P_j^M M_j}}_{=\Omega_{jh}^M} + \sigma_j \underbrace{\frac{P_h M_{jh}}{TC_j}}_{=\Omega_{jh}}
\end{aligned}
$$

$$
\theta_{kh}^j = \frac{\frac{\partial \log M_{jk}}{\partial \log P_h}}{\Omega_{jh}} = -\frac{\varepsilon_j \delta_{kh}}{\Omega_{jh}} + (\varepsilon_j - \sigma_j) \frac{\Omega_{jh}^M}{\Omega_{jh}} + \sigma_j
$$

where δ_{kh} is the Kronecker delta that equal 1 if $k = h$, and 0, otherwise. $h = 1, 2, ..., N + 1 \cup \{L, K\}$. Hence, P_h includes both good and factor prices to save on notation.

An object of interest is $\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}$ because this enters directly in the definition of the input-output substitution operator. We can write this term compactly using the above expression for the Allen-Uzawa elasticity of substitution

$$\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh} = (\varepsilon_j - 1)\left(\Omega^M_{jh} - \delta_{kh}\right) + (\sigma_j - 1)\left(\Omega_{jh} - \Omega^M_{jh}\right)$$