Inflation in Disaggregated Small Open Economies*

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Abstract

This paper studies inflation in small open economies with production networks. I show that the production network alters the elasticity of the consumer price index (CPI) to changes in sectoral technology, factor prices, and import prices. Sectors can import and export directly but also indirectly through domestic intermediate input-output linkages. Indirect exporting dampens the inflationary pressure from domestic forces, such as adverse sectoral technology shocks and increases in factor prices. In contrast, indirect importing increases the inflation sensitivity to import price changes. Computing these CPI elasticities requires knowledge of the production network structure as these do not coincide with typical sufficient statistics used in the literature, such as sectoral sales-to-GDP ratios (Domar weights), factor shares, or imported consumption shares. Using input-output tables, I provide empirical evidence that adjusting CPI elasticities for indirect exports and imports matters quantitatively for small open economies. I then use the model to illustrate the importance of production networks during the recent COVID-19 inflation in Chile and the United Kingdom.

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1 Introduction

In 2022, inflation reached 8 percent in the United States, its highest level in 40 years. The picture was similar on the other side of the Atlantic: Euro Area inflation was 8.4 percent, the highest since its creation. Explanations include shocks to commodity prices (Blanchard and Bernanke, 2023; Gagliardone and Gertler, 2023), sectoral demand changes (Ferrante et al., 2022), fiscal stimulus (di Giovanni et al., 2023b), and supply chain disruptions (di Giovanni et al., 2022; Comin et al., 2023). As shown in Figure 1, high inflation was not restricted to these two economies: the median small open economy experienced an inflation rate of around 10 percent in 2022. However, inflation in this group of countries has been less studied during the current episode. This paper attempts to fill this gap using both theory and data.

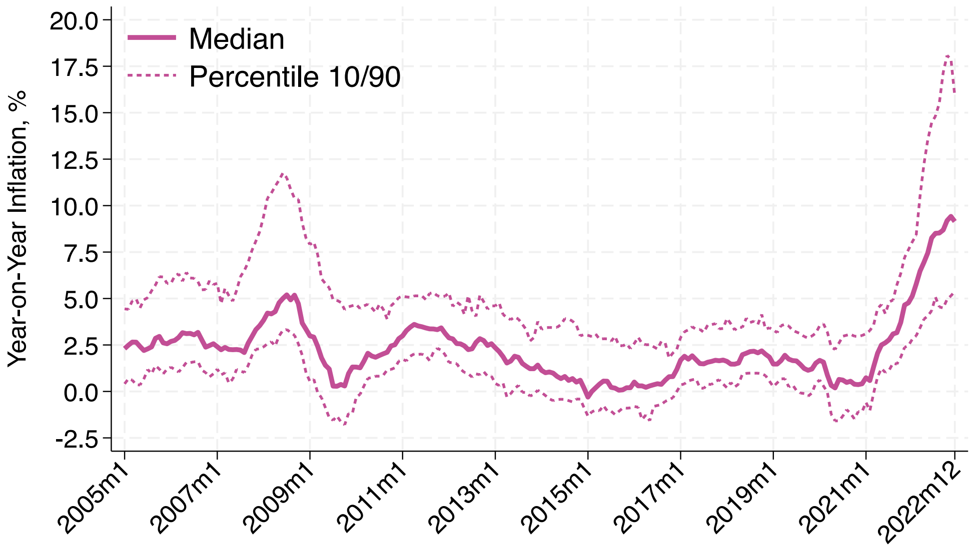
My starting point is the multi-sector and multi-factor production network closed economy model in Baqaee and Farhi (2019b). It provides a useful benchmark to analyze inflation during macroeconomic shocks such as COVID-19, a combination of sectoral and aggregate shocks. Given my focus on small open economies, I augment this model to feature imports and exports at the sectoral level, adapting the production network model to the small open economy case. I use the model to study how the consumer price index (CPI) reacts to changes in sectoral technology, factor prices, and import prices, going from the micro to the macro level.

I show that openness and production networks affect our understanding of inflation in small open economies via two distinct channels. On the one hand, exporting, either directly or indirectly through other economic producers, dampens the effect of sectoral technology shocks and factor price changes relative to a closed economy. On the other hand, direct importing gives rise to the problem of imported inflation as the domestic consumer's basket now contains imported goods. On top of this channel, production networks imply that domestic goods are manufactured using imported inputs indirectly. As a result, production networks amplify imported inflation.[1] Uncovering these effects and quantifying their importance is only possible when both openness and network linkages are explicitly considered.

The key economic intuition is that opening up the economy is one of the ways to break the link between what the country produces and what is consumed by domestic consumers. In an efficient closed economy — an economy without distortions — with intersectoral linkages and domestic final consumption only, everything produced is consumed by the domestic

[1]This channel is distinct from inflation resulting from imported intermediate goods, as models can have intermediate goods without intersectoral linkages. See Svensson (2000) for an early analysis of imported inflation via intermediate goods.

Figure 1. CPI Inflation in Small Open Economies.



Note: The figure shows the median inflation rate (solid lines) and 10th and 90th percentile (dashed lines). Small open economies are economies that represent less than 5 percent of world GDP and have a trade openness larger than 30 percent of GDP. See section 3.1 for more details. Plot shows an unbalanced panel of 46 small open economies over time. Source: Bank for International Settlements.

consumer. *Network-adjusted domestic consumption*, by which I mean domestic consumption adjusted by domestic production network linkages, is thus equivalent to sales in the closed economy.[2] That domestic households consume everything produced, directly or indirectly, is one of the key building blocks of why the production network structure is irrelevant to first-order for macroeconomic outcomes such as real GDP or welfare in closed economies. This irrelevance result is a useful benchmark. It allows us to use the ratios of sectoral sales to nominal GDP (the so-called "Domar weights") and factor payments to nominal GDP (factor shares) as sufficient statistics for the pass-through of sectoral technology changes or factor price changes to the CPI, respectively.[3] Increases in sectoral technology decrease consumer inflation by the Domar weight of the sector, while increases in factor prices increase inflation by the factor share.

I show that this irrelevance result no longer holds for consumer inflation in small open economies, without the need for second-order approximations, as in Baqaee and Farhi

[2]This definition is deliberately reminiscent of the network-adjusted labor share introduced in Baqaee (2015).

[3]As I show in the theory section, this can be thought of as a corollary of Hulten's theorem (Hulten, 1978) but for the CPI rather than for real GDP. Recall that for real GDP, Hulten's theorem states that in an efficient closed economy with inelastic factor supplies, the first-order effect of sectoral technology on real GDP is given by the Domar weights, and the first-order effect of changes in factor supply is given by its factor share.

(2019b), or distortions, as in Baqaee and Farhi (2020) and Bigio and La'o (2020).[4] The reason is as follows. Consider first the impact of sectoral technology shocks on the CPI. In a small open economy, there are two final uses for goods produced within borders: domestic consumption or exports. Unlike the closed economy case, sectoral sales do not map to the network-adjusted domestic consumption for two reasons: (i) direct exports and (ii) indirect exports through domestic production network linkages. Instead, sectoral sales map to network-adjusted domestic consumption *plus* network-adjusted exports. Since what matters for the CPI is network-adjusted domestic consumption, one must subtract network-adjusted exports from sectoral sales. Hence, relative to a closed economy, a consumer is weakly less exposed to changes in sectoral technology. Importantly, we require knowledge of the domestic production network structure to compute these network-adjusted domestic consumption measures.

A similar intuition holds for how factor price changes affect CPI. The relevant statistic here is the *network-adjusted domestic factor share*: how much of each factor is embedded in goods consumed domestically after considering domestic production network linkages (in the spirit of the domestic factor demand concept in Adao et al. (2022)). The total amount of a factor available in the economy can be "consumed" by domestic or foreign consumers,[5] with the production network potentially reshaping these patterns. While factor shares are sufficient statistics in the closed economy, in the small open economy with production networks we need to subtract from factor shares the fraction of each factor that is exported either directly or indirectly via production networks. This means that relative to a closed economy, the domestic consumer is weakly less exposed to changes in factor prices.

The effect of import prices on the CPI, on the other hand, is amplified in a small open economy with production networks relative to a small open economy without production networks. The relevant statistics here are *network-adjusted import consumption shares*. Since the domestic consumer directly imports, the direct consumption share captures part of the exposure to import price changes. However, if there are intersectoral linkages across producers, domestic good producers may end up importing intermediate inputs either directly, by buying from abroad, or indirectly, by buying from domestic sectors that buy from abroad or that buy from sectors that buy from abroad, and so on. This means that the imported con-

[4] Strictly speaking, those papers I cited sought ways to break Hulten's theorem in closed economies, which refer to quantities meaning the effect of sectoral technology changes or distortions on real GDP. However, since, as a corollary of Hulten's theorem, we can back out changes in CPI, I referenced them here.

[5] Here, consumers do not directly consume factors but goods. Given that goods are ultimately made of factors of production, we can think of consumers implicitly consuming them. This notion can be found in the reduced factor demand system proposed by Adao et al. (2017).

tent of domestically produced goods increases in the presence of production networks. To the extent that domestic goods increase their reliance on imported intermediate goods, so does the domestic consumer. Thus, the domestic consumer's exposure to import prices must account for both direct and indirect exposure, which are encapsulated in the *network-adjusted import consumption shares*.

Guided by the model, I turn to the data to measure the importance of these production network adjustments. I find that these adjustments matter quantitatively using data from the World Input-Output Tables. I illustrate these adjustments by focusing on the three sources of variation I have considered so far: sectoral technology, factor prices, and import prices.

First, consider the electricity sector in the United Kingdom (UK). The domar weight of this sector is around 5.95 percent. Once we consider direct exports (but not indirect exports), the relevant ratio for the pass-through to CPI decreases to 5.90 percent, a negligible change. This is expected, as the UK electricity sector hardly exports directly to other countries. Yet, when considering indirect exporting, the network-adjusted domestic consumption share decreases to 4.4 percent, a 25 percent decrease relative to the Domar weight benchmark. This is because other export-heavy sectors in the UK use electricity as a production input either directly or indirectly. Thus, Domar weights would overestimate the impact of a change in productivity in the UK electricity sector on the domestic CPI.

Second, consider the role of wage changes in the CPI. In a closed economy, the labor share is the relevant statistic for how wage changes pass through to the CPI. In the data, the labor share for the average small open economy is around 57 percent. However, the small open economy model with production networks suggests that we need to subtract from the labor share the portion that is exported directly or indirectly. After accounting for network-adjusted exports, this average labor share decreases to 39 percent. This means that the same increase in domestic wages has a 32 percent lower impact on inflation in a small open economy relative to a closed economy.

Finally, let me consider the role of import prices. In the data, the average small open economy exhibits a direct import consumption share of around 17 percent of its total expenditure. Yet, on average, the network-adjusted import consumption share is 30 percent. This implies that the impact of import prices on domestic inflation is (almost) twice what would be implied by a measure ignoring indirect linkages.

In the last section of the paper, I use the model to analyze the recent inflation in two small open economies: Chile and the United Kingdom. I chose these two countries as (i) they fit into the small open economy definition, (ii) they have experienced high inflation in recent years, and (iii) they allow me to compute and contrast between emerging and developed markets. Using these countries, I show that network adjustment on exports and imports provides a quantitative improvement in matching data moments over both closed economy models and small open economy models without production networks.

Between 2020 and 2022, the average annual inflation rate in Chile was 6.13 percent, with a standard deviation of 3.89. A quantitative closed economy model with production networks implies an average inflation rate of 0.98 percent with a standard deviation 9.69. A small open economy model without production networks delivers an average inflation rate of 1.45 percent with a standard deviation 6.88. Finally, the small open economy model with production networks delivers an average inflation rate of 2.41 with a standard deviation 6.67. Overall, the small open economy with production networks better matches the mean and the standard deviation.

For the United Kingdom, average inflation rate was 3.69 percent over the same period, with a standard deviation of 3.11. The closed economy model with production networks implies an average inflation rate of 2.27 percent with a standard deviation 2.57. The small open economy model without network adjustments exhibits an average inflation rate of 2.72 percent with a standard deviation of 2.64. The small open economy model with production networks shows an average inflation rate of 3.21 percent with a standard deviation 3.00. As in the Chilean case, the production network coupled with openness helps to get closer to the date moments of United Kingdom inflation.

The measurement and application sections illustrate that the required network adjustments in a small open economy matter for our understanding of inflation not only as a theoretical curiosity but also in practice.

Related Literature. This paper relates to several strands of the literature. The first one studies inflation in closed economies with production networks (Basu, 1995; La'o and Tahbaz-Salehi, 2022; Guerrieri et al., 2022; Baqaee and Farhi, 2022b; Luo and Villar, 2023; Afrouzi and Bhattarai, 2022; Ferrante et al., 2022; Rubbo, 2023; Minton and Wheaton, 2023; di Giovanni et al., 2023a,b; Lorenzoni and Werning, 2023).[6] These studies consistently find that the interaction between sectoral price/wage rigidities and production networks is key to understanding the behavior of inflation, which has implications for the conduct of

[6]There is also extensive literature on multisector models with sticky prices that do not necessarily feature a production network structure, so I omit them from the main text. For earlier contributions, see Woodford (2003) and the references therein.

monetary policy, such as what inflation rate to target. This paper focuses on how introducing production networks in a small open economy model helps us to understand the pass-through of different shocks to inflation. Although there is no price rigidity in the model, and thus I cannot speak about the optimal conduct of monetary policy, I contribute to this literature by showing that the production network can have a first-order impact on inflation beyond its role in the sales share distribution without the need for any distortions.

Second, this paper relates to the literature on inflation in small open economies. In the second part of the 20th century, Latin America experienced episodes of high and persistent inflation, a term later coined as "chronic inflation". In response, there was extensive literature during the 1990s on how to best control chronic inflation and the impact of different nominal and real policy rules in small open economies (see Calvo and Végh, 1995; Calvo et al., 1995; Calvo and Végh, 1999, and especially the last one, for an overview of this earlier literature). Modern treatments that introduce New Keynesian features such as sticky prices and monopolistic competition into small open economy models include Gali and Monacelli (2005) and Faia and Monacelli (2008)[7]. These models have been augmented to include intersectoral linkages and applied to understand the recent inflationary episode, focusing on the United States (Comin and Johnson, 2020; Comin et al., 2023). Relative to this literature, I explicitly analyze the role of production networks on inflation for small open economies. I show how the production network interacts with trade, affecting how domestic and foreign shocks ultimately affect CPI inflation both theoretically and quantitatively. Moreover, the fact that I use a first-order approximation allows me to consider unrestricted intersectoral linkages, in the sense of not needing to assume any functional forms for production or utility. Finally, the model also features multiple factors of production, while the previous models typically focus on only one factor (labor).

Finally, in focusing on the role of network-adjusted exports and imports, this paper contributes to the literature on indirect trade (Huneeus, 2018; Adao et al., 2022; Dhyne et al., 2021, 2023; Muñoz, 2023). This literature focuses on the firm-level consequences of indirect trade, which is equivalent to my trade network-adjustments. For example, Dhyne et al. (2021) use Belgian firm-to-firm level transaction data and find that the relevant concept for a firm sales' exposure to international markets is total exports (network-adjusted exports), while its exposure in costs is total imports (network-adjusted import share). My contribution

[7]There is also a large literature focusing on two or more countries. My work is not directly related to these models as I focus on small open economies. I refer the interested reader to Corsetti and Pesenti (2007) and Corsetti et al. (2010) for an overview of such models. Recent literature focusing on COVID-19 inflation using multi-country and multi-sector models include, for example, di Giovanni et al. (2022) and Ho et al. (2022).

to this literature is to embed indirect trade into a small open economy model to analyze how it matters for aggregate inflation.

Outline. The rest of this paper is structured as follows. Section 2 describes the model I use to understand inflation in small open economies. Section 3 measures the importance of production networks and trade for inflation comparing across sectors and countries. Section 4 applies the model to Chile and the United Kingdom. Finally, Section 5 concludes.

2 A Small Open Economy Model with Production Networks

Environment. There is a set of domestically produced goods that I denote by N with typical element i. These goods can be consumed domestically, used as intermediate inputs by other domestic sectors, and exported. I denote the imported goods set by M, with typical element m. These imported goods can be used as intermediate inputs to produce domestic goods or as final consumption. Finally, there is a set F of factors with typical element f.

Notation. I denote matrices and vectors using **bold** i.e., $\boldsymbol{Y}$. I denote the transpose of such a matrix as $\boldsymbol{Y}^T$. Unless otherwise noted, vectors are always column vectors. For example, the vector of Domar weights, defined below, is $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_N)^T$. Log-changes are expressed with a hat as $\mathrm{d}\log Y = \hat{Y}$.

Table 1 shows the different shares and matrices that are key for the analysis. I reserve a bar over a variable for shares based on *total expenditure*, while GDP-based measures do not contain a bar.

2.1 Households

There is a representative household that consumes domestically produced goods and foreign goods. It has an instantaneous utility function that I denote by $U(\boldsymbol{C}_D, \boldsymbol{C}_M)$, where $\boldsymbol{C}_D = \{C_i\}_{i\in N}$ denotes the vector of domestically-produced goods consumption and $\boldsymbol{C}_M = \{C_m\}_{m\in M}$ is the vector of foreign goods consumption. These consumption vectors have associated vector prices $\boldsymbol{P}_D = \{P_i\}_{i\in N}$ and $\boldsymbol{P}_M = \{P_m\}_{m\in M}$. Unless otherwise stated, all prices are denominated in local currency. I assume the utility function $U(.)$ is homogeneous of degree one in its arguments. The representative consumer also owns all factors of production and supplies them inelastically at the given factor prices.

Table 1. Definitions

Name	Notation	Expression	Goods/Factors
GDP-based			
Domar Weight	λ_i	$\frac{P_i Q_i}{GDP}$	for $i \in N$
Consumption Share	b_i	$\frac{P_i C_i}{GDP}$	for $i \in N$
Imported Consumption Share	b_m	$\frac{P_m C_m}{GDP}$	for $m \in M$
Export Share	x_i	$\frac{P_i X_i}{GDP}$	for $i \in N$
Factor Shares	$\boldsymbol{\Lambda}$	$\Lambda_f = \frac{W_f L_f}{GDP}$	for $f \in F$
Expenditure-based			
Domar Weight	$\bar{\lambda}_i$	$\frac{P_i Q_i}{E}$	for $i \in N$
Consumption Share	$\bar{b}_i$	$\frac{P_i C_i}{E}$	for $i \in N$
Imported Consumption Share	$\bar{b}_m$	$\frac{P_m C_m}{E}$	for $m \in M$
Export Share	$\bar{x}_i$	$\frac{P_i X_i}{E}$	for $i \in N$
Factor Shares	$\bar{\boldsymbol{\Lambda}}$	$\bar{\Lambda}_f = \frac{W_f L_f}{E}$	for $f \in F$
Sector-level Shares			
Input-Output Matrix	$\boldsymbol{\Omega}$	$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$	$j \in N$
Leontieff-Inverse Matrix	$\boldsymbol{\Psi}_D = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$	$\boldsymbol{\Psi}_{ij} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}_{ij}^s$	$i, j \in N$
Factor Spending Matrix	$\boldsymbol{A}$	$a_{if} = \frac{W_f L_{if}}{P_i^D Q_i}$	$i \in N; f \in F$
Intermediate Import Spending Matrix	$\boldsymbol{\Gamma}$	$\Gamma_{im} = \frac{P_m M_{im}}{P_i Q_i}$	$i \in N; m \in M$

Given a vector of prices, both domestically produced and foreign goods, the cost-minimization problem satisfies

$$PC = \min_{\boldsymbol{C}_D, \boldsymbol{C}_M} \sum_{i \in N} P_i C_i + \sum_{m \in M} P_m C_m \text{ subject to } U(\boldsymbol{C}_D, \boldsymbol{C}_M) \geq \bar{U}. \tag{1}$$

Solving this problem delivers a price index that is a function of good prices. I denote this price index by $P = P(\boldsymbol{P}_D, \boldsymbol{P}_M)$. As a reminder, notice that up to a first-order approximation, changes in this price index satisfy

$$\widehat{P} = \bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D + \bar{\boldsymbol{b}}_M^T \widehat{\boldsymbol{P}}_M, \tag{2}$$

where

$$\bar{\boldsymbol{b}}_D = \{\bar{b}_i\} = \frac{P_i C_i}{E}; \quad \bar{\boldsymbol{b}}_M = \{\bar{b}_m\} = \frac{P_m C_m}{E}; \quad E = \boldsymbol{P}_D^T \boldsymbol{C}_D + \boldsymbol{P}_M^T \boldsymbol{C}_M = PC,$$

are the expenditure share on domestically produced goods ($\bar{b}_i$), imported goods ($\bar{b}_m$), and total expenditure (E), respectively.

The consumer budget constraint reads

$$PC + T = \sum_{f\in F} W_f L_f + \sum_{i\in N} \Pi_i,$$

where T is an *exogenous* net transfer to the rest of the world as in Baqaee and Farhi (2022b). In Appendix B, I provide a justification for having such a force in the current model using a two-period model without changing the main results.

2.2 Sectors

There is a representative firm in each i sector that produces according to the following production function

$$Q_i = Z_i F^i \left(\{L_{if}\}_{f\in F}, \{M_{ij}\}_{j\in N}, \{M_{im}\}_{m\in M}\right), \tag{3}$$

where Z_i is a sector-specific productivity, L_{if} is demand for factor f by firm i, M_{ij} represents intermediate input demand for good $j \in N$ by firm i, and M_{im} represents input demand for imported good $m \in M$. We can write cost-minimization firm i as

$$TC_i = \min_{\{L_{if}\}_{f=1}^F, \{M_{ij}\}_{j\in N}, \{M_{im}\}_{m\in M}} \sum_{f\in F} W_f L_{if} + \sum_{j\in N} P_j M_{ij} + \sum_{m\in M} P_m^M M_{im}^M$$
$$\text{subject to } Z_i F^i \left(\{L_{if}\}_{f\in F}, \{M_{ij}\}_{j\in N}, \{M_{im}\}_{m\in M}\right) \geq \bar{Q}_i.$$

This delivers a marginal cost function that only depends on prices and technology due to the constant returns to scale assumption. In particular,

$$MC_i = MC_i(Z_i, \boldsymbol{P}_D, \boldsymbol{P}_M, \boldsymbol{W}), \tag{4}$$

where $\boldsymbol{W} = \{W_f\}_{f \in F}$ is a vector of factor prices.

We can get conditional factor and intermediate input demand by applying Shephard's lemma to the optimized total cost, TC_i, such that

$$\frac{\partial MC_i}{\partial W_f} Q_i = L_{if} \quad \text{for each } f \in F, \tag{5}$$

$$\frac{\partial MC_i}{\partial P_j} Q_i = M_{ij} \text{ for each } j \in N, \tag{6}$$

$$\frac{\partial MC_i}{\partial P_m} Q_i = M_{im} \text{ for each } m \in M. \tag{7}$$

Due to constant returns to scale and perfectly competitive good and factor markets, each firm i makes zero profit:

$$P_i Q_i = \sum_{f \in F} W_f L_{if} + \sum_{j \in N} P_j M_{ij} + \sum_{m \in M} P_m M_{im} \quad \text{for all } i \in N. \tag{8}$$

2.3 Equilibrium

Market clearing conditions for good and factor markets satisfy

$$Q_i = C_i + X_i + \sum_{j \in N} M_{ji} \qquad \text{for each } i \in N. \tag{9}$$

Equation (9) is the good market clearing condition. I assume X_i is exogenous as in Adao et al. (2022) so that a price clearing the market always exists for each domestically produced good even if it is exported.

Since this is a real model, nominal prices are indeterminate unless I supplement one additional equation. To do so, I impose the following

$$PC \leq \mathcal{M} = E,$$

where $\mathcal{M}$ is the money supply that I take as exogenous in what follows. This is a cash-in-advance constraint used, for example, in La'o and Tahbaz-Salehi (2022) and Afrouzi and

Bhattarai (2022).[8] We can think of this restriction as the small open economy's central bank effectively pinning down total nominal expenditure (E), providing an exogenous nominal anchor. It is apparent that the central bank, conditional on knowing C, which is determined by *real variables*, can implement any price level, P, that it desires consistent with C. This model features the classical dichotomy, where real variables are determined independently of the nominal side. Under these assumptions, one should interpret the results as highlighting the role of production networks for the consumer price index, conditional on an exogenous central bank monetary policy.

Similar to Baqaee and Farhi (2019a), I define an equilibrium in this economy using a dual approach in which feasible and equilibrium allocations are found by taking as given factor prices $\boldsymbol{W}$ and a level of expenditure, E, as follows

1. Given sequences $(\boldsymbol{W}, \boldsymbol{P}_D, \boldsymbol{P}_M, \boldsymbol{\Pi})$ and exogenous parameters (T), the household chooses $(\boldsymbol{C}_D, \boldsymbol{C}_M)$ to maximize its utility subject to its budget constraint.

2. Given $(\boldsymbol{W}, \boldsymbol{P}_D, \boldsymbol{P}_M)$ and production technologies, firms choose $(\boldsymbol{L}_i, \boldsymbol{M}_i)$ to minimize their cost of production.

3. Given $\boldsymbol{X}$, goods markets clears.

4. The cash-in-advance constraint holds with equality $PC = \mathcal{M} = E$

2.4 Characterizing Changes in the Price Index

Having defined the environment, optimality, and equilibrium conditions, I can now study changes in the consumer price index, $\widehat{CPI}$. Inflation here consists of a log-linear approximation around the initial price level equilibrium. The purpose of the model is to distill whether and how the production network may matter for inflation, which in the model is a cross-sectional statement rather than a dynamic statement. "Inflation" in this context can thus be understood in the space rather than the time dimension. This concept has been used, for example, to study inflation in the US during the COVID-19 period (Baqaee and Farhi, 2022b; di Giovanni et al., 2022), and the role of sticky prices in production networks (La'o and Tahbaz-Salehi, 2022; Baqaee and Rubbo, 2023).

In Appendix B, I provide a detailed two-period model of a small open economy to show that the simplified model shares the same intuition. The key idea follows Baqaee and Farhi

[8] It can be shown that this "constraint" is isomorphic to a model with money in the utility function that is separable from aggregate consumption. The cash-in-advance constraint thus serves no other purpose than pinning down nominal variables without affecting real allocations in this model.

(2022b), who in turn build on Krugman (1998) and Eggertsson and Krugman (2012), where we can separate a dynamic problem into two sub-periods: the present and the future. All action happens in the present, while the future can be taken as given. Shocks occur during the present and last only for that period, wherein the "future", the economy returns to its initial no-shock equilibrium. In deriving such a model, I contribute to the literature by effectively implementing this concept in a small open economy setup with production networks. Conditional on this interpretation, the simplified model is isomorphic to a multi-period model.

The following result characterizes how the consumer price index reacts to changes in exogenous variables.

Proposition 1. *Consider a perturbation* $(\widehat{\boldsymbol{Z}}, \widehat{\boldsymbol{W}}, \widehat{\boldsymbol{P}}_M)$ *around some initial equilibrium. Up to a first order, changes in the aggregate price index,* $\widehat{CPI}$*, satisfy*

$$\widehat{CPI} = \left(\bar{\boldsymbol{\lambda}}^T - \widetilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} + \left(\bar{\boldsymbol{\Lambda}}^T - \widetilde{\boldsymbol{\Lambda}}^T\right)\widehat{\boldsymbol{W}} + (\bar{\boldsymbol{b}}_M^T + \widetilde{\boldsymbol{b}}_M^T)\widehat{\boldsymbol{P}}_M, \tag{10}$$

where

$$\widetilde{\boldsymbol{\lambda}}^T = \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D; \qquad \widetilde{\boldsymbol{\Lambda}}^T = \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D\boldsymbol{A}; \qquad \widetilde{\boldsymbol{b}}_M^T = \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}$$

Proof. See Appendix A.1. □

The above expression highlights how opening up the economy to goods trade and introducing a production network structure alter the usual prediction of closed economy models. I now proceed with some illustrations that provide intuition for this expression.

Illustration 1: Closed economy. The following proposition characterizes CPI in a closed economy.

Proposition 2. *In a closed economy, equation (10) reduces to*

$$\widehat{CPI} = -\boldsymbol{\lambda}^T\widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T\widehat{\boldsymbol{W}},$$

Proof. See Appendix A.2. □

Proposition 2 is the exact form of changes in CPI in a closed economy (see Baqaee and Farhi, 2022b). Intuitively, CPI changes are a weighted average of changes in productivity (weighted by the Domar weights, $\boldsymbol{\lambda}$) and factor prices (weighted by the factor shares, $\boldsymbol{\Lambda}$). Equation (10) extends this for small open economies.

Three differences exist between the closed economy expression and equation (10). Of course, I will ignore import price changes in this comparison as they do not appear by definition in a closed economy.

First, domar weights and factor shares in equation (10) are based on expenditure rather than on nominal GDP. This distinction arises in small open economies that feature any trade imbalance where the income from what they produce does not need to equal what they consume. Since what matters for CPI is what the domestic consumer is spending, nominal expenditure is the relevant object for dividing sales and factor payments.

Second, the effect of sectoral productivity changes on CPI is dampened relative to a closed economy or a small open economy without production networks. To see this, note that the relevant statistic for the effect of sectoral productivity changes on CPI is $\bar{\boldsymbol{\lambda}}^T - \widetilde{\boldsymbol{\lambda}}^T$ and thus the Domar weight, $\bar{\boldsymbol{\lambda}}^T$, is no longer the sufficient statistic for understanding how sectoral productivity changes affect CPI. Importantly, the relevant object requires an adjustment to the expenditure-based Domar weight, $\bar{\boldsymbol{\lambda}}$ according to $\widetilde{\boldsymbol{\lambda}}^T = \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. This adjustment comes from the fact that what matters is the domestic consumer's exposure to changes in sectoral productivity.

To be precise, let me write the price index as a function of domestic and imported goods prices, i.e., $P = \mathcal{P}(\boldsymbol{P}_D, \boldsymbol{P}_M)$. Suppose there is a change in the productivity of sector k, $\widehat{Z}_k$, with no changes in factor or import prices. This shock impacts all domestic goods prices due to input-output linkages. Its propagation to CPI is a tale of two elasticities. First, how exposed is the consumer to changes in domestic good prices $\frac{\partial \log \mathcal{P}}{\partial \log P_i}$, for all i. By the envelope theorem, this is simply the consumption share on the good at the initial equilibrium, $\bar{b}_i$. Second, how productivity passes through to each domestic goods price, $\frac{\partial \log P_i}{\partial \log Z_k}$. This last term is simply given by $-\Psi_{ik}$, which measures the sensitivity of the price of good i to a change in productivity of sector k after taking into account all direct and indirect linkages via the production network. Collecting all these pieces, we can write:

$$\widehat{P} = \sum_{i \in N} \underbrace{\frac{\partial \log \mathcal{P}}{\partial \log P_i}}_{=\bar{b}_i} \underbrace{\frac{\partial \log P_i}{\partial \log Z_k}}_{=-\Psi_{ik}} \widehat{Z}_k = -\bar{b}_D^T \boldsymbol{\Psi}_{(:,\,k)} \widehat{Z}_k,$$

where $\boldsymbol{\Psi}_{(:,\,k)}$ is the kth column of the Leontieff-inverse matrix $\boldsymbol{\Psi}$. Note that again, the reason why $\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}$ is not equivalent to the sales share is precisely because this is not the relevant exposure of the domestic consumer.

Third, the effect of factor prices on CPI is also dampened relative to the closed economy benchmark or a small open economy without production networks. Although the logic is similar to that of how productivity changes pass through CPI, I analyze the factor price case in detail in the next example, as it also allows me to relate to a well-known concept in the trade literature: the factor content of exports.

Illustration 2: Domestic factor demand and the factor content of exports. This example helps to illustrate how the fact that exports used domestic factors lowers the sensitivity of prices to changes in domestic factor prices. Equation (10) already highlights a tension between domestic factor demand and the factor content of exports, in the spirit of Adao et al. (2022). When an economy exports, some of its factors of production end up meeting foreign demand, which everything else equals reduce *domestic factor demand*. These factors meet foreign demand because they are used to produce domestic goods. As a result, there is less pressure on the price index through this channel[9]. Moreover, this channel is in place whenever an economy exports to the rest of the world and does not rely on production networks. To see this, notice that factor payments to a given factor f can be written as

$$W_f L_f = \sum_{i \in N} W_f L_{if} = \sum_{i \in N} a_{if} \lambda_i. \tag{11}$$

Without intermediate inputs, the Domar weight of each sector, λ_i, is simply that sector's share in the total final demand

$$Q_i = C_i + X_i \Longrightarrow \lambda_i = b_i + x_i. \tag{12}$$

Combining Equation (11) and (12), I get

$$W_f L_f - \underbrace{\sum_{i \in N} a_{if} x_i}_{\text{Factor Content of Exports}} = \underbrace{\sum_{i \in N} a_{if} b_i}_{\text{Domestic Factor Demand}}. \tag{13}$$

This equation shows the tension: a rise in exports – higher x_i – must be balanced out by a fall in domestic factor demand on the right-hand side, conditional on aggregate payments to factor f being constant. This is one of the mechanisms that put less pressure on domestic prices.

[9]Though the factor content of exports is already well known in the trade literature, I am unaware of works linking this precise notion to inflation.

The role of the production network here is that some sectors that do not export much directly (have low x_i) could end up exporting via other producers. The case I analyzed above, without a production network, is a particular case where $\mathbf{\Omega} = \mathbf{0}_{N\times N}$ and thus $\mathbf{\Psi}_{\boldsymbol{D}} = \boldsymbol{I}$. What this suggests is that in the presence of intermediate input linkages, what matters for the price index changes is not just how much each sector exports directly, $\boldsymbol{x}^T$, but also how much it exports indirectly through intermediate input linkages, $\boldsymbol{x}^T\mathbf{\Psi}_D$ (see Dhyne et al., 2021). This logic also affects how much each factor ended up being exported abroad and how much factor price changes are passed through CPI, since $\boldsymbol{x}^T\mathbf{\Psi}_D\boldsymbol{A}$ represents the factor content of exports when there are intermediate inputs linkages across sectors and -$\left(\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T\mathbf{\Psi}_D\right)$ is the relevant "Domar weight" for passing through sectoral technology shocks to inflation.

Illustration 3: Import price changes with intersectoral linkages and the network-adjusted import consumption share. This example illustrates that intersectoral linkages amplify the influence of import price changes on inflation. In the presence of intermediate input linkages and imported intermediate inputs, $\bar{\boldsymbol{b}}_M^T$ — *direct* import consumption shares — are, in general, *not* a sufficient statistic for the effect of imports prices on the price index. To see this, fix factor prices and assume no productivity shocks, $\widehat{\boldsymbol{W}} = \mathbf{0}_F$ and $\widehat{\boldsymbol{Z}} = \mathbf{0}_N$, then

$$\widehat{P} = \underbrace{\left(\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T\mathbf{\Psi}_D\mathbf{\Gamma}\right)}_{\text{Network-adjusted import consumption share}} \widehat{\boldsymbol{P}}_M$$

Just as was the case for the factor content of exports, this equation shows the *network-adjusted import consumption share*: while the domestic consumer is consuming imports directly as final consumption ($\bar{\boldsymbol{b}}_M$), it is also consuming imports indirectly because it buys domestically produced goods that use intermediate inputs to be produced, which in turn may require imported intermediate inputs. This channel is captured by the second term in the right-hand side $\bar{\boldsymbol{b}}_D^T\mathbf{\Psi}_D\mathbf{\Gamma}$, which tells how much import content each domestically produced good has at the end when we account for intermediate input linkages. The intuition for this is the typical downstream propagation of prices in closed economies with production networks. A rise in the import price of good m raises the marginal cost of a given producer h by Γ_{hm}. The rise in the marginal cost implies that P_h raises. This increase in P_h, through intermediate input linkages, raises the price of other goods, say i, by Ψ_{ih}, which denotes the exposure of sector i to changes in the price of sector h after taking into account intermediate input linkages. This increase in the price of good i, in turn, is passed through the consumer price index via $\bar{b}_i$.

2.5 An alternative representation of factor markets: from factor prices to factor supplies

Factor price changes on the right-hand side of the equation (10) are exogenous and thus can be considered primitives in my exercise. However, typical neoclassical models treat factor prices as endogenous and factor supply as exogenous. Writing the problem considering factor prices as given simplified the intuition for the main result of this paper.[10] I now show that the same intuition holds if we reverse the ordering, treating factor prices as endogenous outcomes and factor supply as exogenous objects.

2.5.1 Solving in terms of factor supply quantities

The key difference when solving for factor prices as endogenous objects is that we need to introduce factor market clearing conditions. I introduce them below,

$$\sum_{i \in N} L_{if} = \bar{L}_f \quad \forall f \in F,$$

where the left-hand side is factor demand, and the right-hand side is factor supply. In what follows, I assume that factor supplies, $\bar{L}_f$, are exogenous.

Recall that the definition of the expenditure-based factor share f on total expenditure, $E = \mathcal{M}$, is

$$\bar{\Lambda}_f = \frac{W_f \bar{L}_f}{\mathcal{M}},$$

where I already introduced the factor market clearing condition.

Thus, changes in factor prices can be written as

$$\widehat{W}_f = \widehat{\bar{\Lambda}}_f + \widehat{\mathcal{M}} - \widehat{\bar{L}}_f, \tag{14}$$

which in vector form is

$$\widehat{\boldsymbol{W}} = \widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}.$$

[10]This is also the route followed by Baqaee and Farhi (2019a) when studying aggregation in disaggregated economies via *aggregate cost functions* rather than aggregate production functions.

Intuitively, factor prices can go up because (i) demand is *reallocated* toward that factor, as captured by $\widehat{\bar{\boldsymbol{\Lambda}}}$, (ii) aggregate demand is going up ($\widehat{\mathcal{M}}$), and (iii) there is a decrease in (inelastic) factor supply ($\widehat{\bar{\boldsymbol{L}}}$). As shown in the proposition below, this decomposition allows me to write changes in the price index as a function of sectoral and aggregate shocks and also changes in these expenditure-based factor shares.

Proposition 3. *Consider a perturbation* $(\widehat{\mathcal{M}}, \mathrm{d}T, \widehat{\boldsymbol{Z}}, \widehat{\boldsymbol{P}}_M, \widehat{\boldsymbol{X}}, \widehat{\bar{\boldsymbol{L}}})$ *around some initial equilibrium. Up to a first order, changes in the aggregate price index,* $\widehat{P}$*, satisfy*

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \tilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} - \underbrace{\tilde{\boldsymbol{\Lambda}}^T\widehat{\bar{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \tilde{\boldsymbol{\Lambda}}^T\right)\widehat{\bar{\boldsymbol{L}}} + \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \tilde{\boldsymbol{\Lambda}}^T\mathbf{1}_F\right)\widehat{\mathcal{M}}}_{\textit{Factor Price Changes}}$$
$$+ \left(\bar{\boldsymbol{b}}_M^T + \tilde{\boldsymbol{b}}_M^T\right)\widehat{\boldsymbol{P}}_M, \tag{15}$$

Proof. See Appendix A.3. □

Proposition 3 is an *ex-post* sufficient statistics results in the spirit of Baqaee and Farhi (2022a) since there is still one endogenous vector equation that needs to be solved for, namely, $\widehat{\bar{\boldsymbol{\Lambda}}}$. Conditional on knowing this vector, we can compute the response of CPI to changes in the other primitives. Note that in a closed economy, this term would be zero, since $\tilde{\boldsymbol{\Lambda}} = \mathbf{0}_F$ and thus factor share reallocation would not have any first-order effect on inflation.

Note that the only difference relative to the model with exogenous factor prices changes is that we are now mapping factor price changes to other exogenous objects $(\widehat{\mathcal{M}}, \mathrm{d}T, \widehat{\bar{\boldsymbol{L}}})$. As in Baqaee and Farhi (2022b), decreases in factor supply are inflationary because, conditional on factor demands, this increases factor prices, which are then passed through CPI. Money supply changes, $\widehat{\mathcal{M}}$, are dampened relative to the closed economy because of the same reason why factor prices pass less through inflation. Increases in net transfers to the rest of the world, $\mathrm{d}T$, also increase CPI inflation because, conditional on money supply, they increase nominal GDP and, via this channel, increase factor prices. In this sense, factor price changes were "sufficient statistics" for how money supply and net transfer changes affected CPI.

A few additional remarks regarding Proposition 3 are in order. First, note that sectoral export demands, $\boldsymbol{X}$, do not appear directly in this equation. It means that it can only affect inflation through its effect on $\widehat{\bar{\boldsymbol{\Lambda}}}$. Second, as I show in Appendix C, $\widehat{\bar{\boldsymbol{\Lambda}}}$ can be found by solving a linear system of equations. This system of equations depends on primitives, the production network structure, and the elasticities of substitution by producers and consumers. Thus, we would be required to take a stand on the values of elasticities of substitution of producers across their different inputs and of consumers across their different consumption goods.

Perhaps more important than this is the fact that through this endogenous vector, elasticities of substitution matter to a first-order for CPI in small open economies. Hence, even with a simple, sufficient statistics framework, the common idea that elasticities of substitution do not matter, to a first-order, for inflation simply does not hold in small open economies.

3 The empirical relevance of adjustments of CPI elasticity

This section shows the quantitative relevance of the proposed production network adjustments across small open economies. I start by describing the data sources and how I classify countries as small open economies. I then discuss three different results. First, I focus on the network-adjusted domestic consumption shares, which are the relevant elasticities for the pass-through of sectoral technology to inflation. Second, I show the adjustment to labor shares once we account for indirect exports. Finally, I compare direct and network-adjusted import consumption shares.

3.1 Data

Although network-adjusted shares do require more information than sales and factor shares, they are still easily computable from available data. In this subsection, I briefly describe the necessary data to compute them.

Input-Output Tables. The ideal information to compute the statistics requires the following objects $(\boldsymbol{\Omega}, \bar{\boldsymbol{x}}, \bar{\boldsymbol{b}}_D, \bar{\boldsymbol{b}}_M, \bar{\boldsymbol{\lambda}})$. All this information is readily available in the domestic input-output tables from the World Input-Output database release 2016, the latest available.

Penn World Tables (PWT). I use version PWT 10.01. It contains income, input, output, and productivity information between 1950 and 2019. It covers around 183 countries. This database is freely available to download at https://www.rug.nl/ggdc/productivity/pwt/?lang=en.

Using this database, I construct two measures. First, I denote the share of world GDP accounted for by country c as α_c. Formally,

$$\alpha_c = \frac{nGDP_c}{nGDP_W}, \quad nGDP_W = \sum_{c \in C} nGDP_c$$

I measure $nGDP_c$ using the series *cgdpo*, which corresponds to the Output-side real GDP at current PPP's (in 2017 US$ millions).

To measure trade openness, I resort to series *csh_x* and *csh_m* in the PWT. The first corresponds to the share of merchandise exports over nominal GDP, while the second corresponds to imports over nominal GDP at current PPP's. I define the trade openness of country c as

$$\text{Openness}_c = \frac{\text{Exports}_c + \text{Imports}_c}{nGDP_c} = csh\_x_c - csh\_m_c,$$

where the last line follows since in the data $csh\_m_c = -\frac{\text{Imports}_c}{nGDP_c}$

Classifying Small Open Economies. I apply two criteria to separate countries into small and non-small open economies according to the data

Criterion (1). An economy is *small* if $\alpha_c \leq 5\%$.

Criterion (2). An economy is *open* if $\text{Openness}_c \geq 30\%$

Definition 3.1. An economy is "small and open" if it jointly satisfies Criteria (1) and (2). Countries not satisfying *both* criteria are labeled as non small open economies.

3.2 Results

Across this subsection, I compare the network-adjusted objects with their closed economy and no-network adjustment counterparts whenever possible. All cross-sectional plots, be it sector or country level, are based on the year 2014 unless stated otherwise.

3.2.1 Network-adjusted domestic consumption shares

I start the analysis by showing results for the network-adjusted domestic consumption shares $\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. Figure 2 shows three scenarios for the average sector in small open economies in panel (a) and for non-small open economies in panel (b). The x-axis shows the unadjusted domar weights, while the y-axis shows the adjusted objects. Light squares in these figures refer to the export adjustment, while dark points include the network export adjustment. Thus, the dark points in these plots are the network-adjusted domestic consumption shares. As we can see from these figures, the adjustments are stronger for small open economies than non-small open economies. Moreover, we can see that the average domar weight in small open economies is around 4 percent; it decreases to around 2.84 percent with the export adjustment and to nearly 2.31 percent when adjusting for network-adjusted exports. This is a non-negligible change. It suggests that the same-sized sectoral productivity shock in the average-sized sector will be dampened by around 50 percent for the average small open economy relative to the closed economy benchmark.

To provide a more concrete example, Figure 3 shows the three sectors where the network-export adjustment is the largest in the United Kingdom. These sectors include administrative support, legal and accounting, and electricity, gas, and water. The last sector is illustrative. Its domar weight is around 5.95 percent. This number goes down to 5.9 percent when we subtract direct exports. For all practical purposes, this means this sector is non-tradable. Once we consider indirect exports, the network-adjusted consumption share decreases to 4.4 percent. This illustrates how indirect linkages are quantitatively relevant and can go beyond the direct export share.

Regression framework. What kind of sectors and countries are most affected by these changes? To answer this question, I conduct a simple empirical exercise running the following cross-sectional regression

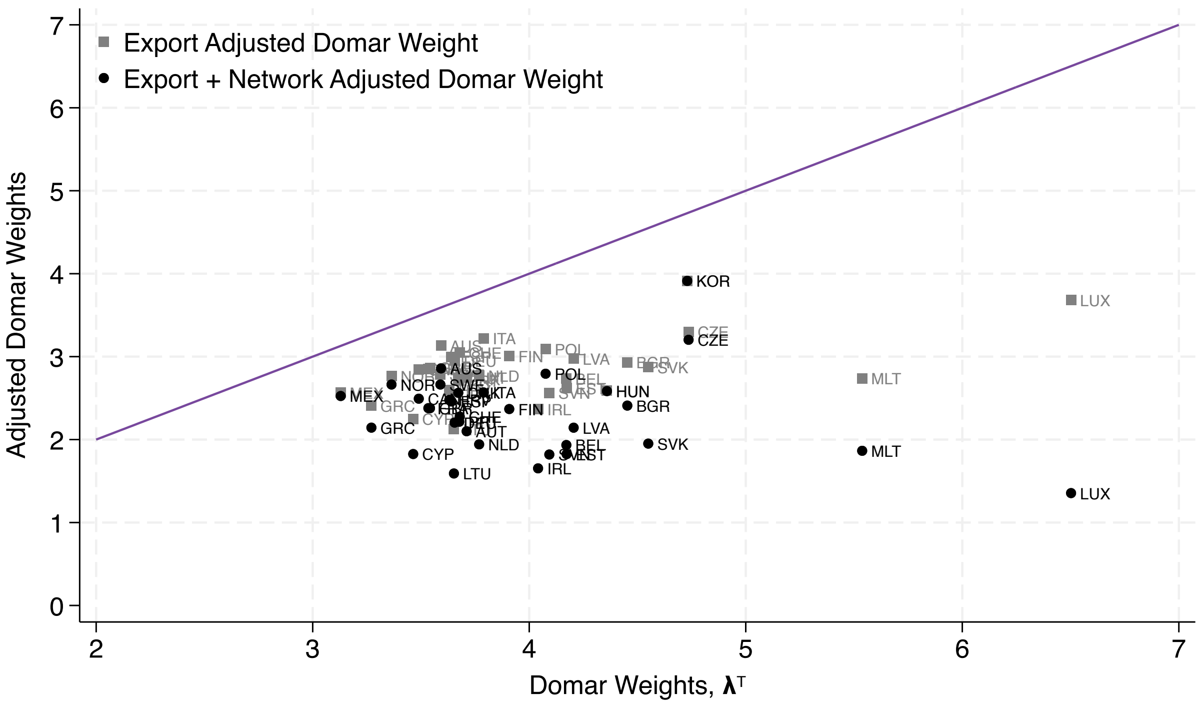
$$y_{sc} = \alpha_s + \alpha_c + \varepsilon_{cs}, \tag{16}$$

where y_{sc} represents the difference between a measure for the small open economy with a production network relative to the small open economy without networks, for a given country c and sector s. α_s is a sector-specific fixed effect, α_c is a country-specific fixed effect, and ε_{cs} is an error term. From this regression, I get estimates of sector and country-specific fixed effects. Notice that these are identified up to a normalization, which in my case is that $\sum_{s\in S} \hat{\alpha}_s = 0$ and $\sum_{c\in C} \hat{\alpha}_c = 0$. All fixed effects are interpreted as deviations from the average fixed effects.

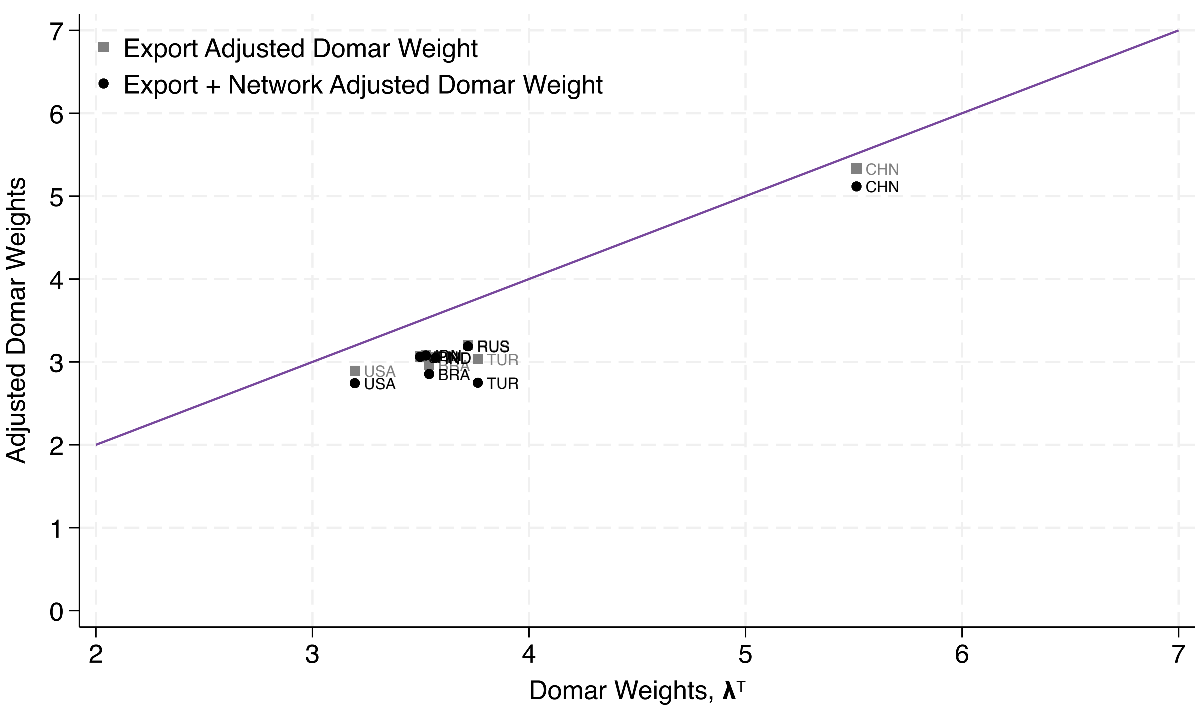
In Panel (a) of Figure 4, I show the country-fixed effect estimates when the left-hand side variable is the difference between the network-adjusted export share and the direct export share. Note that these country-fixed estimates tell us the average difference between these shares — as a fraction of aggregate expenditure — across sectors within a country. We can see that the countries with the largest adjustments are Luxembourg, Slovak Republic, Malta, and Latvia. Among countries with the lower average production network adjustment are Korea, Hungary, and Mexico. These numbers indicate that the latter set of economies is one where the exporting sector does not rely much on inputs from the domestic economy; it does not export much indirectly. Remember that it does not need to be confused with a

Figure 2. Export and Network-Export adjusted Domar weights.

(a) Small Open Economies

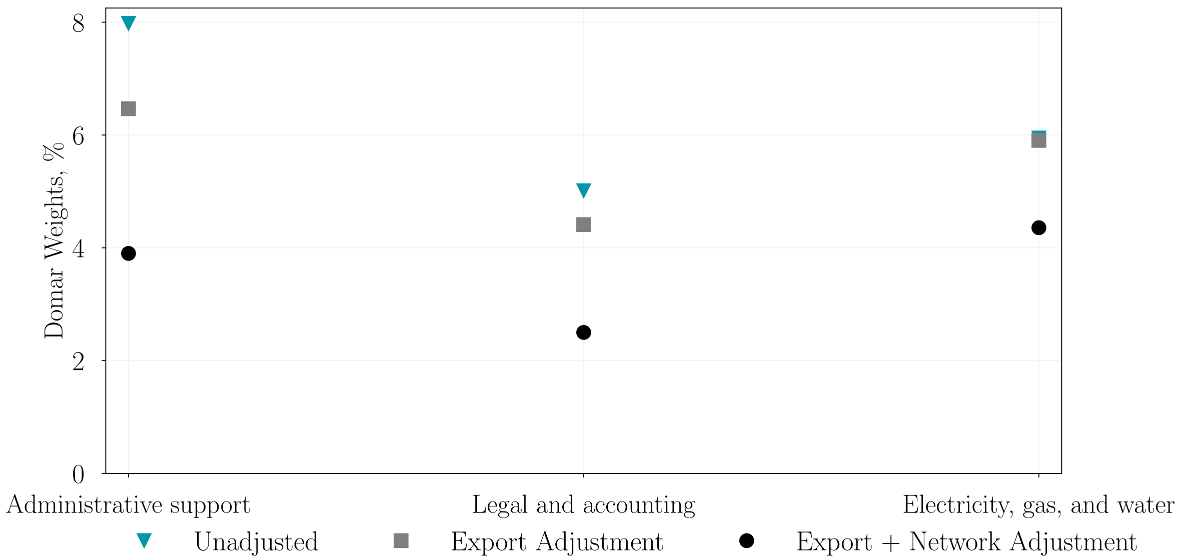


(b) Non-Small Open Economies



Note: This figure shows the average domar weight for each country. The x-axis corresponds to the average domar weight computed as in the closed economy model, $\boldsymbol{\lambda}^T$. The gray squares subtract only exports i.e. $\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T$. The black circles further consider the production network structure, $\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. Panel (a) shows the results for small open economies, while Panel (b) shows the results for non-small open economies.

Figure 3. Three sectors with largest adjustments: United Kingdom.



Note: This figure shows the three sectors with the largest export network-adjusted share for the United Kingdom.

country that does not export at all.

Panel (b) does the same exercise for the sector-fixed effect estimates. Similarly, these estimates tell us the average difference between shares across countries within a sector. Note that Electricity, Gas, and Water (EGSA) is the seventh sector with the largest difference. Legal and accounting is also the sixth sector with the largest difference. This means that the above sectoral examples of the United Kingdom are not just specific to the United Kingdom but something that happens consistently across countries. In terms of intuition, this tells us that these sectors are important suppliers for sectors that either export directly or indirectly. Financial services are the sector with the largest average production network adjustment. This means that the financial sector is an important supplier to sectors that export directly or indirectly.

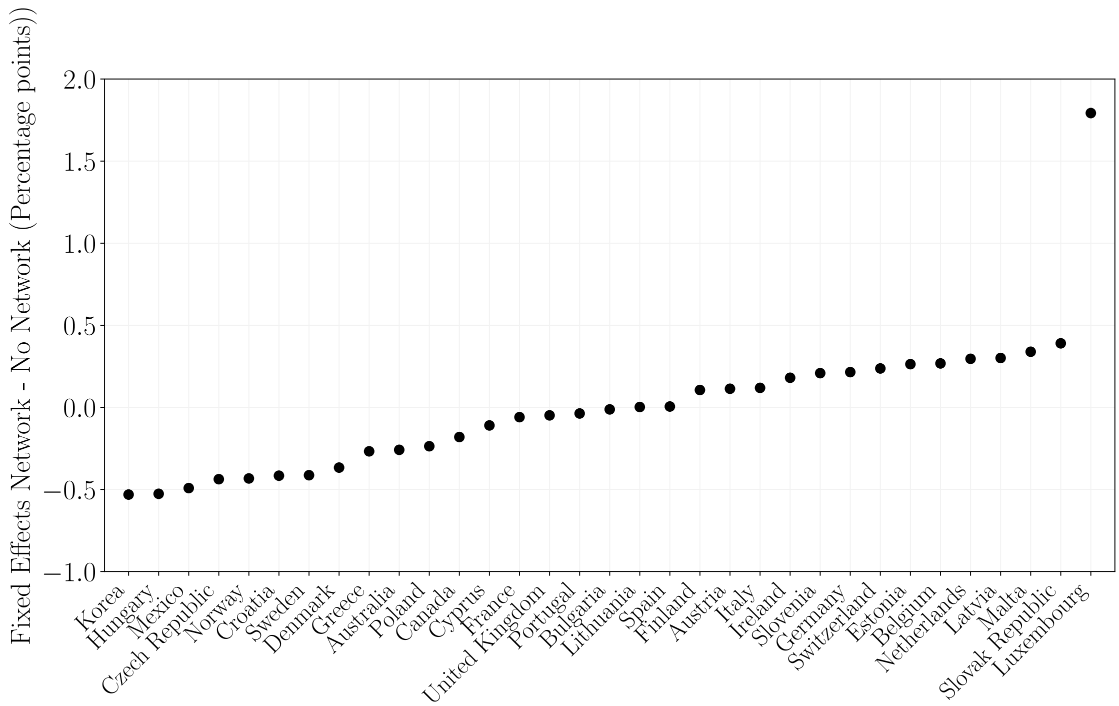
These exercises suggest that accounting for the production network is important in computing these shares and varies substantially across countries and sectors.

3.2.2 Network-adjusted domestic factor demand

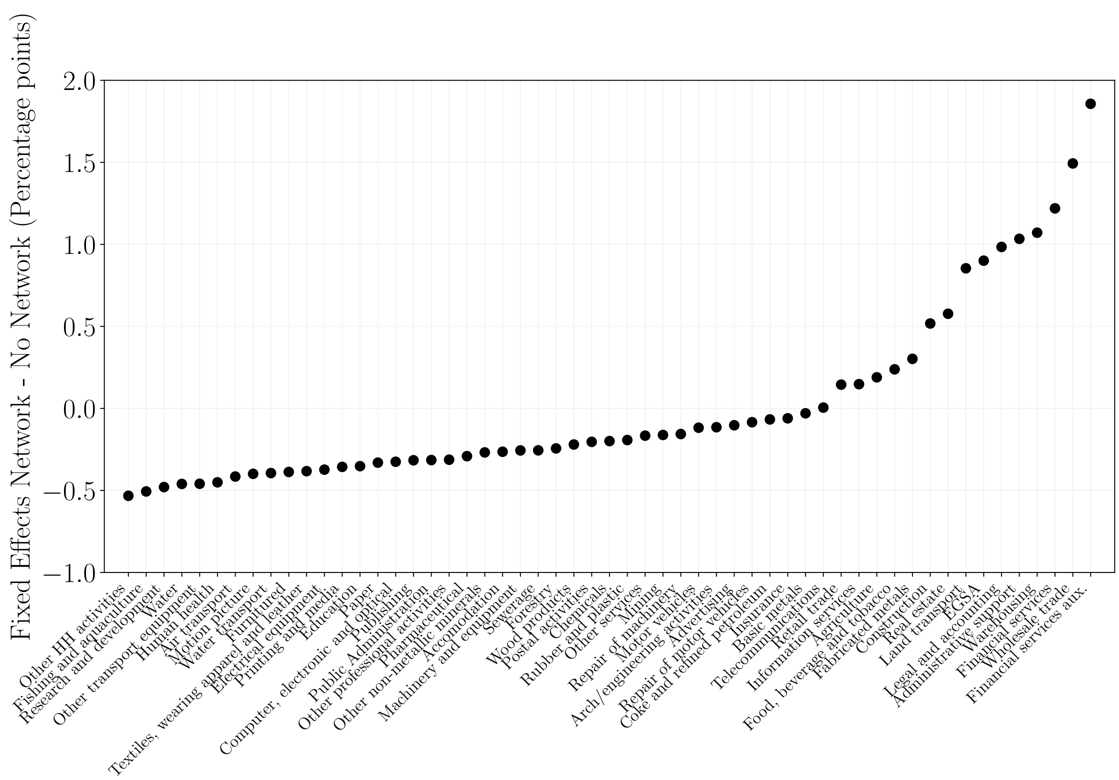
I now conduct a similar exercise to the one in the previous section. First, I study the aggregate labor share across countries and how it varies depending on the export and network export adjustment. I then consider the more general case of sector-specific labor shares and how they vary when considering direct and indirect exports.

Figure 4. Country and sector fixed effects: export-network adjusted - export adjusted.

(a) Country Fixed Effects



(b) Sector Fixed Effects



Note: This figure shows the fixed effects when the dependent variable is the difference between the network-adjusted export share and the direct export share. Panel (a) shows this difference for the country fixed effects estimates, while Panel (b) does the same for sector fixed effects.

Labor share. Figure 5 shows the labor share across small open economies on the x-axis and the network export-adjusted labor share on the y-axis. As we can see, the adjustments are, again, not minor. The average labor share in small open economies is around 57 percent, while the network export-adjusted labor share decreases to 39 percent. This means the same aggregate wage increase will be 32 percent lower in a small open economy with production networks relative to an otherwise equal closed economy.

Sector-specific labor shares. I now conduct a similar exercise to that of export and network-adjusted export measures above. Here, I consider the dependent variable to be the difference between the network-adjusted labor content of exports relative to the non-network-adjusted labor content of exports.

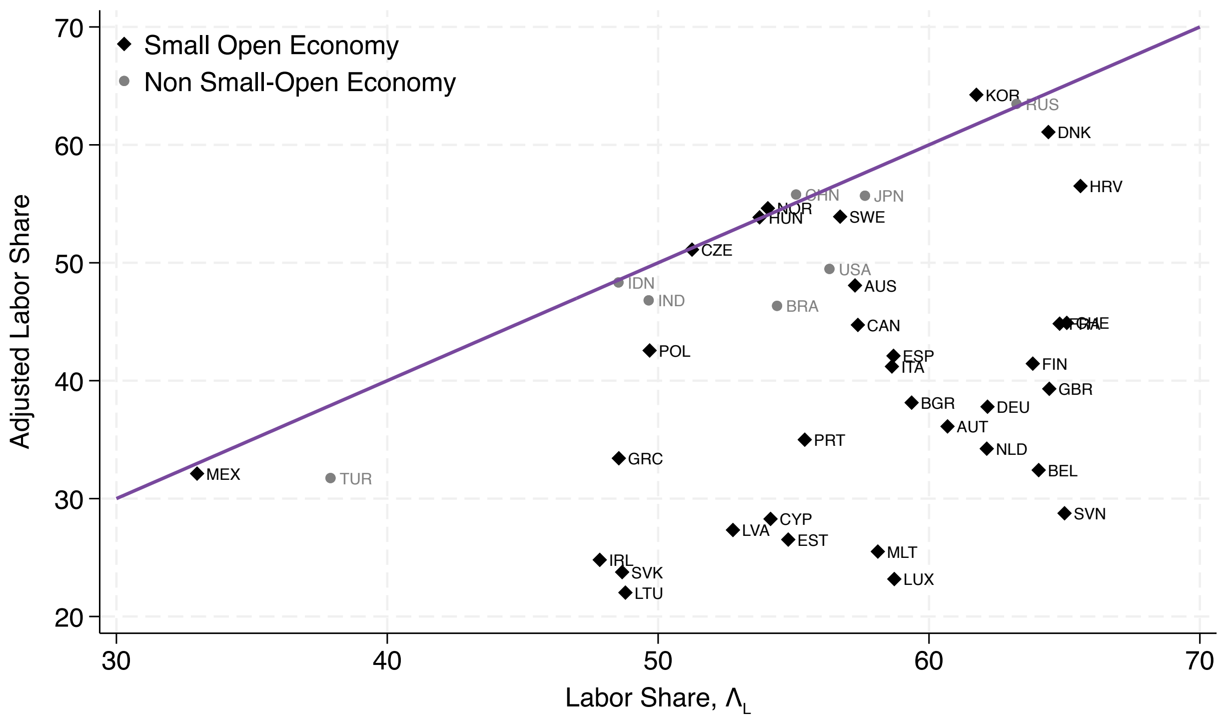
Doing this exercise is important as it will illustrate the heterogeneity across sectoral labor markets. Before, I considered the aggregate labor share. However, this aggregate labor share is, in the end, a weighted average of what happens at the sectoral level and thus can be a misleading statistic for certain questions, especially in an environment such as COVID-19, where different sectoral labor markets were hit differently.

Again, this exercise aims to examine the heterogeneity of these differences across countries and sectors. Panel (a) of Figure 6 shows the results for the country-fixed effects, while Panel (b) shows the same but for sector-fixed effects.

Note that apart from Luxembourg, the ranking differs from the network-adjusted domestic consumption share in Figure 4. Interestingly, the countries where the sector-specific labor shares adjusted the most due to the domestic production network adjustment are the Netherlands, Slovenia, and Germany, while the ranking at the bottom stays the same. This says that Germany exhibits an average production network adjustment of sector-specific factor shares, which is 0.15 percentage points larger than the adjustment for the average country.

Turning to the sector fixed-effects results, the sectors with the largest production network adjustment are Legal and Accounting, Wholesale Trade, and Administrative support. Legal and accounting, for example, has an average adjustment that is 0.6 percentage points larger than that of the average sector. This means that ignoring the production network adjustment would significantly overstate how much wage changes in the Legal and Accounting sector pass-through CPI since we need to subtract the adjustments from the factor shares. Since the 0.6 percentage point is an average across all countries, we can consider this sector in Germany, just to provide an example. The share of this sector's labor on nominal GDP is around 2.6 percent of GDP, while it goes down to 2.3 percent when we subtract exports, and to 0.8 percent when we consider the domestic production network structure.

Figure 5. Labor share adjustments across small open economies.



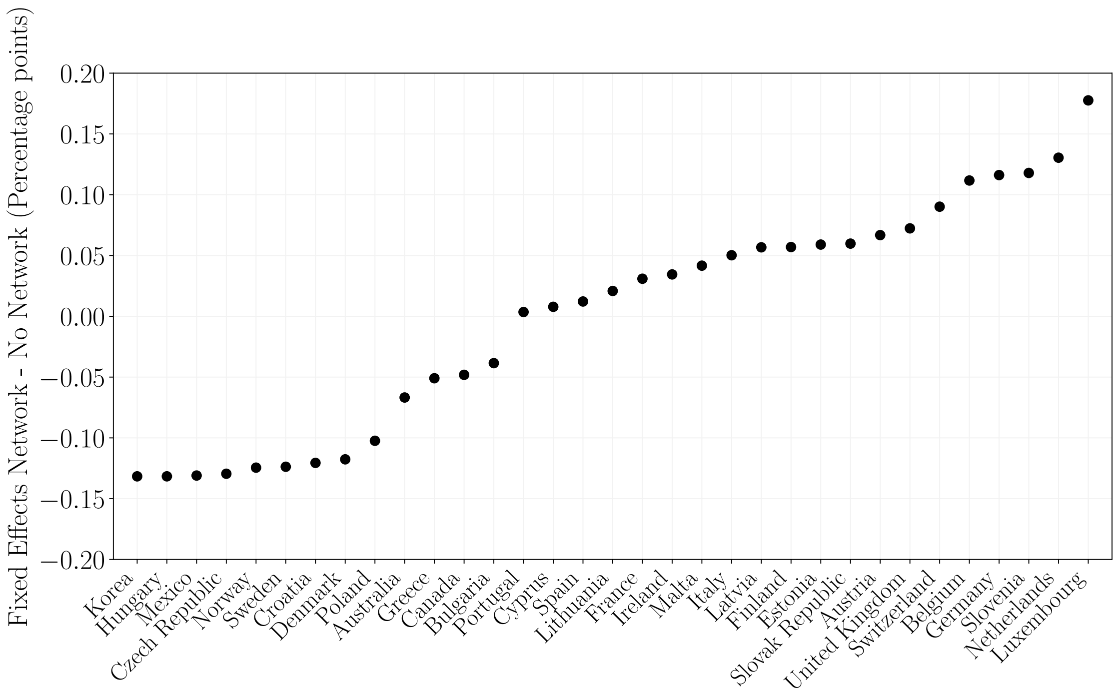
Note: This figure shows the average labor share on the x-axis and the export-network adjusted labor share for small open economies in black diamonds and non-small open economies in gray circles.

3.2.3 Network-adjusted import consumption shares

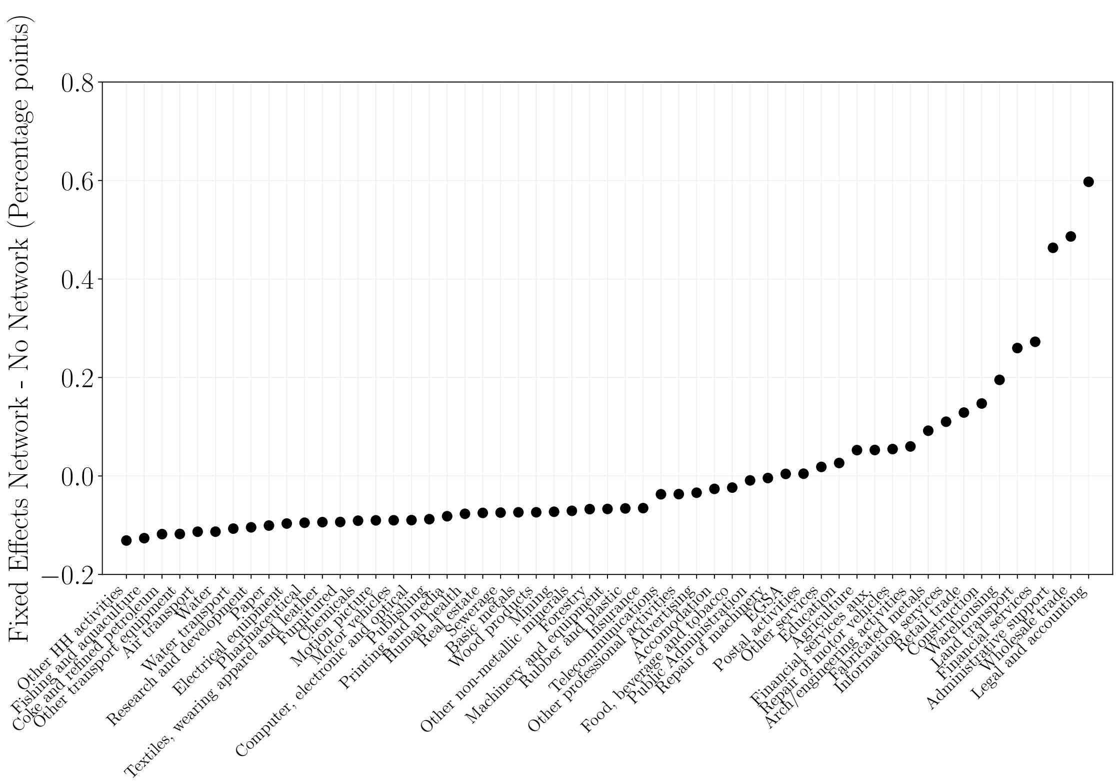
As a final empirical exercise, let me consider the import consumption shares. In Figure 7, I provide a scatterplot of these across economies. On the x-axis, I show the direct import consumption share, while on the y-axis, I show the network-adjusted import consumption share. The average direct import consumption share across small open economies is around 17 percent. This number goes up to 30 percent when considering the production network structure. As before, these adjustments are quantitatively relevant. They suggest that the importance of import price changes for inflation (almost) doubles when we introduce intersectoral linkages, meaning the same-sized import price changes double their impact on CPI when considering production networks.

Figure 6. Country and sector fixed effects: export-network adjusted sector-specific factor shares - export adjusted sector-specific factor shares.

(a) Country Fixed Effects

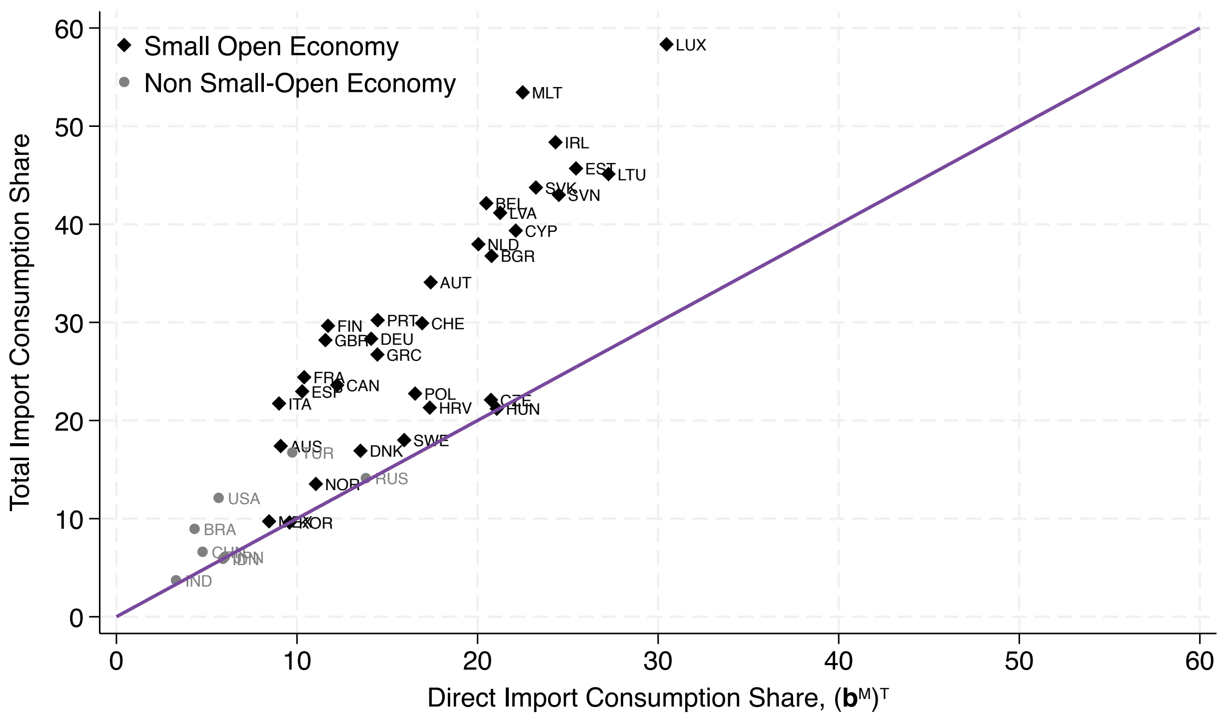


(b) Sector Fixed Effects



Note: This figure shows the fixed effects when the dependent variable is the difference between the network-adjusted sector-specific factor shares and the direct export share adjusted sector-specific factor shares. Panel (a) shows this difference for the country fixed effects estimates, while Panel (b) does the same for sector fixed effects.

Figure 7. Direct and Network-Adjusted import consumption shares.



Note: This figure shows the direct import consumption share on the x-axis and the network-adjusted import consumption share on the y-axis. Small open economies are the black diamonds, and non-small open economies are the gray circles.

4 The evolution of inflation in Chile and United Kingdom during COVID-19

In this section, I use the model to study the recent COVID-19 inflation in Chile and the United Kingdom.

This empirical examination requires more data relative to the previous section. While the earlier section showed information on the CPI elasticities and compared these across countries and sectors using the WIOT alone, this section requires taking a stand on the processes $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$, which are not readily available for most countries worldwide. Therefore, I picked Chile and the United Kingdom, countries with all the necessary information to construct $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$ that also belong to the small open economy category.

The exercise I provide in this section is an ex-post exercise using existing data to analyze the past behavior of inflation between 2020 and 2022. Yet, Proposition 1 is also useful for forecasting inflation and is thus a valuable tool for policymakers in small open economies. Provided that we have forecast information on the processes $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$, we can combine this information with input-output tables to get an estimate of inflation. The accuracy of this exercise will depend on the accuracy of elasticities and that of the forecasted series. Throughout this section, I focus on the former, as it is the main point of this paper.

In what follows, I first describe the data. Then, I show how I map the model to the data. Finally, I discuss the results for Chile and the United Kingdom.

4.1 Data

4.1.1 Chile

Input-Output Tables. Since Chilean data is unavailable from the WIOT, I resort to Chilean National Accounts. I use *Compilacion de Referencia* for year 2013. The structure is similar to that of the WIOT, having information on input-output linkages, final uses, and factor payments, among others. Moreover, it is quite disaggregated, containing information for up to 171 industries. However, I collapsed this data to a 17-sector classification due to data availability on sectoral wages. This 17-sector classification is equivalent to SIC2.

Sectoral Productivity. The ideal measure of productivity from the model is total factor productivity (TFP). However, TFP measures are hard to come by, especially at frequencies higher than yearly and more so at the sectoral level. To circumvent this problem, I proxy sectoral TFP using sectoral labor productivity. I collect data on real GDP for the same 17 sectors and divide it by total sectoral employment. Real GDP and sectoral employment data come from the Central Bank of Chile (CBCh) and are available quarterly from 1996 to 2022.

Sectoral Wages. I sourced sectoral nominal wages from the Chilean National Institute of Statistics (INE) series *Indice de Remuneraciones Nominal*. This database is available monthly from January 2016 to December 2022. To be consistent with the productivity data, I collapsed this data to a quarterly frequency.

Import Prices. I use the import price index available from the CBCh quarterly from 2013 to 2022.

4.1.2 United Kingdom

Input-Output Tables. I sourced data from the WIOT domestic tables as in the previous empirical section. I collapsed these input-output tables into 20 industries to be consistent with the data on sectoral wages.

Sectoral Productivity. I sourced data from the Office for National Statistics (ONS) of the United Kingdom. I downloaded quarterly estimates of labor productivity from the *Flash productivity* report.[11] This contains information for up to 17 industries.

Sectoral Wages. I sourced this data from the ONS of the United Kingdom as well. In particular, I resort to the dataset `EARN03`. This contains monthly information on average weekly earnings for around 20 industries. This dataset is available from 2000 to 2022.

Import Prices. I took the import price index from the ONS (series `GD74`, dataset: `MM22`). This series is available at different frequencies. I use quarterly information from 2009 until 2022.

4.2 Mapping the model to the data

Before showing the results, a few remarks are in order. Since the model is static, all inherent inflation dynamics will combine the dynamics of exogenous variables and their interaction with the CPI exposures.

[11] This data can be downloaded freely from the "*Flash productivity by section*" section at the ONS here: https://www.ons.gov.uk/economy/economicoutputandproductivity/productivitymeasures/datasets/flashproductivitybysecti

First, I take all series and compute their level deviations from their value in 2018Q4. Formally, the sources of variation I feed in to construct implied inflation from the different models take the following form

$$\hat{y}_t = y_t - y_{2018Q4},$$

where y_t represents (the log) of any time series and y_{2018Q4} is its value in 2018Q4. Notice that each vector now has a t subscript as they are deviations from 2018Q4 at each time t.

I call the deviation $\hat{y}_t$ in the above equation a "shock". This differs from a structurally identified shock because here, I feed some variation directly from the data, taking it as given. With this caveat in mind and throughout this section, I refer to these $\hat{y}_t$ simply as shocks.

Using this procedure I construct counterparts to $\boldsymbol{\theta}_t = (\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$ in the model. Here, I assume that factor prices map to sector-specific wages. Also, note that I assume segmented labor markets such that there are different wages across sectors to better capture the behavior of labor markets during the COVID-19 episode, as highlighted in the recent literature (Baqaee and Farhi, 2022b; di Giovanni et al., 2022, 2023a,b). Since I cannot observe other factor prices, such as the sector-specific capital prices or land, I assume those other factor prices did not change throughout the analyzed period.

CPI inflation in the data π_t, when t refers to a quarter, is

$$\pi_t = \log P_t - \log P_{t-4}$$

Combining the model and "shocks", we have $\widehat{P}_t^{\text{Model}}$ as

$$\widehat{P}_t^{\text{Model}} = -\sum_{i \in N} \mathcal{R}_i^{CPI,Z} \widehat{Z}_{it} + \sum_{f \in F} \mathcal{R}_f^{CPI,W} \widehat{W}_{ft} + \mathcal{R}_M^{CPI,M} \widehat{P}_{Mt}.$$

Note that here $(\mathcal{R}_i^{CPI,Z}, \mathcal{R}_f^{CPI,W}, \mathcal{R}_M^{CPI,M})$ stand for the responses of CPI to changes in sectoral technology, factor prices, and import price, respectively. These objects are *model-dependent* and thus will be different when considering the closed economy model, the small open economy model without production networks, and the small open economy with production networks.

Inflation *from the model* thus can be simply written as

$$\pi_t^{\text{Model}} = \widehat{P}_t^{\text{Model}} - \widehat{P}_{t-4}^{\text{Model}}.$$

I think this approach of taking log differences relative to some initial point is the most transparent in the sense of not modifying the data much and still being able to say something useful about inflation.

4.3 Results

In this subsection, I compare inflation implied by the models, π_t^{Model}, and that in the data.

Figures 8 and 9 show inflation in the data and the one implied by the model for Chile and the United Kingdom for 2020–2022, respectively. To highlight the distinct role of production networks and openness, I consider three models: a closed economy model (Closed, pink triangles), a small open economy without production networks (SOE no Network, green $*$), and a small open economy with production networks (SOE - Network, orange circles). Note that I choose to plot the model's numbers using symbols rather than lines to emphasize the absence of dynamics within the model apart from those generated by the shocks I am feeding in.

Although the empirical exercise is fairly simple, it captures the data patterns well and more significantly for the small open economy with a production network in Chile and the United Kingdom.

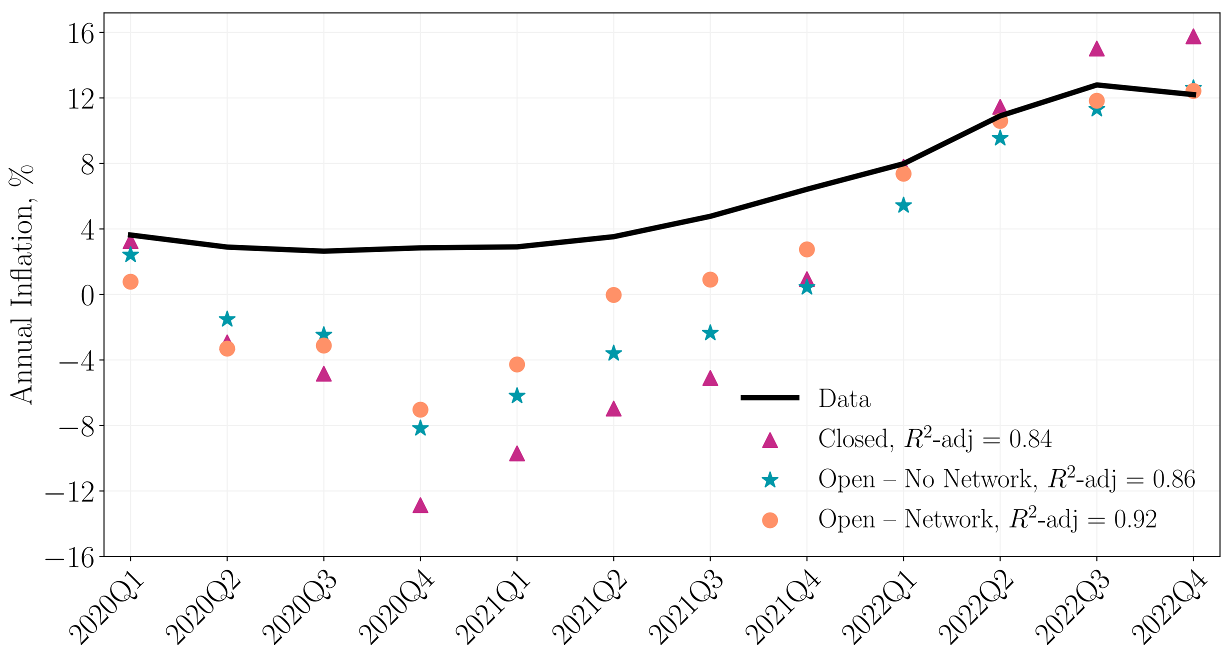
As pointed out, the model has no intrinsic dynamics: all the action over time comes from the dynamics in $\boldsymbol{\theta}_t$. A more meaningful comparison is to compare the moments implied by the model and those in the data. Table 2 does precisely this and shows the first two moments of inflation in the data and the model. Panel (a) is for Chile, while Panel (b) shows the United Kingdom.

The average annual inflation in Chile between 2020 and 2022 was 6.13 percent, with a standard deviation of 3.89. The closed economy model delivers substantially lower mean inflation (0.98) and higher standard deviation (9.69) relative to the data. We can see that the sole introduction of a small open economy aspect, without production networks, gets us in the right direction as it exhibits a larger mean relative to the closed economy benchmark (1.45) and a lower standard deviation (6.88). The small open economy with production networks gets us closer to the data, with an average inflation of 2.41 and a standard deviation of 6.67.

In the United Kingdom, the average inflation was 3.69 percent, almost half that of Chile during the same period, with a standard deviation of 3.11. The closed economy benchmark generates again too little average inflation (2.27 vs. 3.31) but now too low a standard deviation (2.57 vs. 3.11). As was the case for Chile, introducing the small open economy aspect put us in the right direction: inflation is higher on average (2.72) and has a higher standard deviation (2.64). Considering production networks again improves the results: the model exhibits an even higher mean (3.21) and standard deviation (3.00).

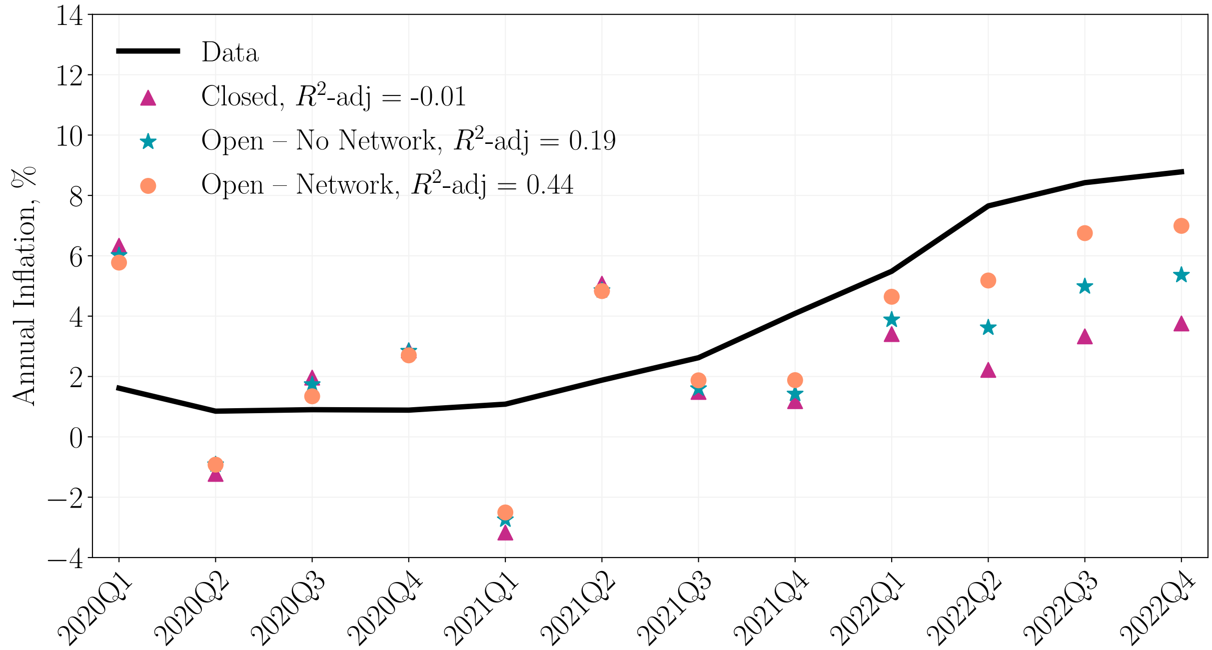
To sum up, this exercise suggests that a small open economy model with production networks better matches inflation moments during 2020 – 2022 than a closed economy model and a small open economy model without production networks. To be fair, the small open economy with production networks should indeed do better than the other two as it adds a piece of realism missing from these other models, namely, intersectoral linkages. The question is how much. The results here are suggestive evidence that this is quantitatively relevant. Of course, the stylized model has many missing parts, but remarkably, such a stylized exercise does well in matching these inflation data moments.

Figure 8. Chile Inflation under different models.



Note: This figure shows inflation in the data and the one implied by the different models. The black line is the data. The pink triangles correspond to the closed economy model. The green ∗ are the small open economy model without production networks, and the orange circles correspond to the small open economy model with production networks.

Figure 9. United Kingdom Inflation under different models.



Note: This figure shows inflation in the data and the one implied by the different models. The black line is the data. The pink triangles correspond to the closed economy model. The green $*$ are the small open economy model without production networks, and the orange circles correspond to the small open economy model with production networks.

Table 2. Average Inflation, 2020 – 2022.

	Panel (a): Chile		Panel (b): United Kingdom	
	Mean	Std. Dev.	Mean	Std. Dev
Data	6.13	3.89	3.69	3.11
Model				
Closed	0.98	9.69	2.27	2.57
SOE no Network	1.45	6.88	2.72	2.64
SOE - Network	2.41	6.67	3.21	3.00

Note: This table shows the mean and standard deviation of inflation for the data and the different models. The Closed model uses the implied elasticities as if we consider the economies as closed. The SOE no network model considers the elasticities in a small open economy that does not feature any production network. Finally, the SOE - Network model accounts for network linkages.

5 Conclusion

I study inflation in small open economies with production networks. Theoretically and empirically, I show that production networks matter for the effect of sectoral technology shocks, factor prices, and import prices on CPI inflation. I argue that it matters because opening up the economy is one of the ways to break what is produced within borders and what is consumed by the domestic consumer, the one that "creates" the consumption basket underlying the CPI. Once we break that relationship, the production network amplifies this discrepancy via indirect linkages: Non-exporters become indirect exporters, while non-importers become indirect importers. This ultimately affects the exposure of the domestic consumer to a different set of changes. The production network thus matters to first order on inflation, with sales and factor shares no longer being sufficient statistics to study how changes in sectoral technology or factor prices pass through inflation, as they were in a closed economy.

In a small open economy, indirect exporting dampens domestic shocks relative to an otherwise equivalent closed economy. Foreign shocks, such as import price shocks, are amplified relative to an otherwise equivalent small open economy without production networks. What effect dominates at the aggregate level depends on the production network structure and is, in the end, a quantitative question. I show that the production network adjustments on both the export and import side matter quantitatively across a set of small open economies. I apply the small open economy model with production networks to the recent inflationary episode in two small open economies: Chile and the United Kingdom. I show that including production networks helps better match the mean and variance of inflation of these countries between 2020 and 2022. Moreover, I argued that the main result of this paper, that the mapping of different sources of variations on CPI is different in small open economies, can also be used for forecasting inflation, provided we have good enough information about the future behavior of sectoral technology, factor prices, and import prices.

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A Proofs

A.1 Proof of Proposition 1

By definition, changes in the aggregate price index satisfy

$$\widehat{P} = \bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D + \bar{\boldsymbol{b}}_M^T \widehat{\boldsymbol{P}}_M \tag{17}$$

By Shephard's lemma the vector of domestic prices can be written as

$$\widehat{\boldsymbol{P}}_D = -\widehat{\boldsymbol{Z}} + \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Omega}\widehat{\boldsymbol{P}}_D + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M,$$

where this result follows since in equilibrium $MC_i = P_i$ for all $i = 1, 2, ..., N$.

Inverting this system yields

$$\widehat{\boldsymbol{P}}_D = -\boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M$$

Using the definitions and equilibrium results

$$\bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D = \bar{\boldsymbol{b}}_D^T \left[-\boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M \right] \tag{18}$$

Thus CPI changes can be written as

$$\widehat{P} = -\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + (\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{\Gamma}) \widehat{\boldsymbol{P}}_M \tag{19}$$

To get the expression in the text, simply note that goods market clearing condition and the definition of factor shares can be written as

$$\begin{aligned} \bar{\boldsymbol{\lambda}}^T &= (\bar{\boldsymbol{b}}_D^T + \bar{\boldsymbol{x}}^T)\boldsymbol{\Psi}_D \Longrightarrow \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D = \bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \\ \bar{\boldsymbol{\Lambda}}^T &= \bar{\boldsymbol{\lambda}}^T \boldsymbol{A} \Longrightarrow \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{A} = \bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}. \end{aligned}$$

Replacing these expressions with changes in the CPI index yields the result.

A.2 Proof of Proposition 2

To prove this proposition, notice that Hulten's theorem in an efficient closed economy with inelastic factor supplies implies that changes in real GDP must satisfy

$$\widehat{Y} = \boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T \widehat{\boldsymbol{L}}.$$

In turn, changes in factor supply have to comply $\widehat{\boldsymbol{L}} = -\widehat{\boldsymbol{W}} + \widehat{nGDP}\mathbf{1}_F + \widehat{\boldsymbol{\Lambda}}$, which upon replacing this expression above we get

$$\widehat{Y} = \boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \widehat{nGDP} - \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}}.$$

The definition of changes in nominal GDP has a quantity and a price component. The quantity component refers to $\widehat{Y}$ (real GDP). The price component is the GDP deflator, which is equivalent to CPI in a closed economy[12]. This allows us to write changes in nominal GDP as

$$\widehat{nGDP} = \widehat{Y} + \widehat{CPI} \Longrightarrow \widehat{CPI} = \widehat{nGDP} - \widehat{Y}$$

Combining the above with the changes in real GDP as a function of productivity, factor prices, and nominal GDP, we get

$$\widehat{CPI} = \widehat{nGDP} - (\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \widehat{nGDP} - \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}}) = -\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}},$$

which was the desired expression.

A.3 Proof of Proposition 3

As stated in the text

$$\widehat{\boldsymbol{W}} = \widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}.$$

Multiplying the above expression by the weight on wages in CPI from equation (10), I get

$$\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\boldsymbol{W}} = \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \left(\widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}\right)$$

Now note that, in general, the budget constraint of a consumer in a small open economy can be written as

$$\mathcal{M} + T = GDP \Longrightarrow \widehat{nGDP} = \frac{\mathcal{M}}{nGDP} \widehat{\mathcal{M}} + \frac{\mathrm{d}T}{nGDP}.$$

By definition,

$$\Lambda_f = \bar{\Lambda}_f \frac{\mathcal{M}}{nGDP} \Longrightarrow \widehat{\bar{\Lambda}}_f = \widehat{\Lambda}_f - \widehat{\mathcal{M}} + \widehat{nGDP},$$

$$\bar{\boldsymbol{\Lambda}}^T \mathbf{1}_F = \sum_{f \in F} \bar{\Lambda}_f = \frac{nGDP}{\mathcal{M}} = \frac{\mathcal{M} + T}{\mathcal{M}} \Longrightarrow \bar{\boldsymbol{\Lambda}}^T \widehat{\bar{\boldsymbol{\Lambda}}} = \sum_{f \in F} \mathrm{d}\bar{\Lambda}_f = \mathrm{d}\left(1 + \frac{T}{\mathcal{M}}\right) = \frac{\mathrm{d}T}{\mathcal{M}} - \frac{T}{\mathcal{M}} \widehat{\mathcal{M}}$$

Then,

$$\begin{aligned}
\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\boldsymbol{W}} &= \bar{\boldsymbol{\Lambda}}^T \widehat{\bar{\boldsymbol{\Lambda}}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \frac{T}{\mathcal{M}} \widehat{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F - \frac{T}{\mathcal{M}}\right) \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\frac{\mathcal{M} + T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \mathbf{1}_F - \frac{T}{\mathcal{M}}\right) \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \mathbf{1}_F\right) \widehat{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}}
\end{aligned}$$

[12] This provided that we interpret CPI broadly to include all final uses different from intermediate inputs, such as investment, government expenditure, and so on.

Replacing this expression into equation (10), I get

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \tilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} - \underbrace{\tilde{\boldsymbol{\Lambda}}^T\widehat{\bar{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \tilde{\boldsymbol{\Lambda}}^T\right)\widehat{\bar{\boldsymbol{L}}} + \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \tilde{\boldsymbol{\Lambda}}^T\boldsymbol{1}_F\right)\widehat{\mathcal{M}}}_{\text{Factor price changes}}$$
$$+ \left(\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}\right)\widehat{\boldsymbol{P}}_M, \tag{20}$$

which is the expression in the main text.

B Two period model

In this section, I justify using the one-period model in the main text. The intuition from the main text is unchanged in this more complicated model.

Suppose there are two periods, 0 and 1. There is no uncertainty. The consumer has preferences over the consumption bundle in both periods according to some utility function $U(C)$. The consumer can also access an internationally traded bond that pays a nominal interest rate i_t^*. This nominal interest rate is *exogenous* from the perspective of the small open economy.

The consumer's budget constraint at time 0 and 1 reads

$$P_0C_0 + \mathcal{E}_0B_0 = (1 + i^*_{-1})\mathcal{E}_0B_{-1} + nGDP_0,$$
$$P_1C_1 + \mathcal{E}_1B_1 = (1 + i^*_0)\mathcal{E}_1B_0 + nGDP_1,$$

where P_t is the price index at time t, C_t is consumption at time t, B_t denotes asset holdings in foreign currency at time t, $\mathcal{E}_t$ is the nominal exchange rate at time t defined as local currency per unit of foreign currency, and $nGDP_t$ denotes nominal GDP at time t.

Combining the two budget constraints gives the intertemporal budget constraint

$$P_0C_0 + \frac{P_1C_1}{\frac{\mathcal{E}_1}{\mathcal{E}_0}(1 + i^*_0)} = (1 + i^*_{-1})\mathcal{E}_0B_{-1} + nGDP_0 + \frac{nGDP_1}{\frac{\mathcal{E}_1}{\mathcal{E}_0}(1 + i^*_0)}$$

Under perfect mobility of capital flows, we have the *no-arbitrage condition*[13]

$$(1 + i_0) = (1 + i^*_0)\frac{\mathcal{E}_1}{\mathcal{E}_0}. \tag{21}$$

Where given perfect foresight, there is no expectation regarding the future level of the exchange rate. I can also get this condition by adding a domestic bond in zero net supply. This does not change any of the conclusions below.

The no-arbitrage condition is important for the small open economy as it clearly illustrates the fact that the Central bank has two instruments to set the nominal interest rate i_0: it can either choose i_0 directly and let the exchange rate $\mathcal{E}_0$ adjust. Or it can pick $\mathcal{E}_0$ and let the nominal interest rate accommodate to comply with that rule.

We can thus rewrite the maximization problem as solving the following program

$$\max_{C_0,C_1} \quad U(C_0) + \beta U(C_1) \quad \text{s.t} \quad P_0C_0 + \frac{P_1C_1}{(1 + i_0)} = (1 + i^*_{-1})\mathcal{E}_0B_{-1} + nGDP_0 + \frac{nGDP_1}{(1 + i_0)} \tag{22}$$

[13] Under uncertainty, this is simply the uncovered interest parity condition (UIP).

Letting λ be the multiplier on the intertemporal budget constraint, we have

$$U'(C_0) = \lambda P_0$$
$$\beta U'(C_1) = \lambda \frac{P_1}{(1+i_0)}$$

Combining both equations

$$\frac{U'(C_0)}{P_0} = \beta \frac{U'(C_1)}{P_1}(1+i_0)$$

Assume $U(C) = \log C$, as in Golosov and Lucas (2007) and Baqaee and Farhi (2022b), so that the intertemporal elasticity of substitution is unitary. Then

$$\beta P_0 C_0 (1+i_0) = P_1 C_1$$

As argued in Baqaee and Farhi (2022b), who in turn relied on an argument made in Krugman (1998) and Eggertsson and Krugman (2012), we can understand this model as assuming that anything happening at $t = 1$ is labeled as "future". The assumption here is isomorphic to an infinite horizon model where an unexpected shock happens at $t = 0$, and the economy returns to the long-run equilibrium from $t = 1$ onwards. For all practical purposes, this means we assume that any variables at $t = 1$ are exogenously given. Then, from the Euler equation, we have

$$P_0 C_0 = \frac{P_1 C_1}{\beta(1+i_0)} = \frac{P_1 C_1}{\beta(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1}$$

Therefore, since $(\beta, i_0^*, P_1 C_1, \mathcal{E}_1)$ are exogenous, the nominal exchange rate $\mathcal{E}_0$ is determined via the no-arbitrage condition, equation 21. This equation provides a value for current expenditure in local currency, $P_0 C_0$.

For simplicity, suppose $B_{-1} = 0$. Replacing the Euler equation in the intertemporal budget constraint.

$$P_0 C_0 + \frac{P_1 C_1}{(1+i_0)} = nGDP_0 + \frac{nGDP_1}{(1+i_0)}$$
$$P_0 C_0 + \frac{\beta P_0 C_0 (1+i_0)}{(1+i_0)} = nGDP_0 + \frac{nGDP_1}{(1+i_0)}$$
$$P_0 C_0 = \frac{1}{(1+\beta)} \left(nGDP_0 + \frac{nGDP_1}{(1+i_0)} \right)$$
$$P_0 C_0 = \frac{1}{(1+\beta)} \left(nGDP_0 + \frac{nGDP_1}{(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1} \right).$$

Note that given $(\mathcal{E}_0, P_0 C_0, nGDP_1, i_0)$ from the Euler equation and no-arbitrage condition, the latter equation pins down $nGDP_0$.

B.1 Solving for consumption at time 0, C_0.

Before solving for consumption, let me introduce the real exchange rate, $\mathcal{Q}_0$ as

$$\mathcal{Q}_0 = \frac{\mathcal{E}_0 P_0^F}{P_0},$$

where P_0^F is the rest of the world price index, which is exogenous from the perspective of the small open economy. Following, Schmitt-Grohé et al. (2022), we can write this foreign price index as $P_0^F = \mathcal{P}^F(\{P_{m,0}^*\}_{m\in M}, \{P_{j,0}^*\}_{j\in N^*})$, where M is the same set of goods that are imported by the small open economy and N^* is the set of all other goods consumed abroad by the foreign economy. Since I assume that all prices P_k^* for $k \in M \cup N^*$ are exogenous from the perspective of the small open economy, this allows me to write

$$\frac{P_0^F}{P_{m_0,0}^*} = \mathcal{P}^F(1, \{P_{m,0}^*/P_{0,0}^*\}_{m\in M \ 0}, \{P_{j,0}^*/P_{0,0}^*\}_{j\in N^*})$$

Using this in the definition of the real exchange rate and the law of one price for imported goods $P_{m,0} = \mathcal{E}_0 P_{m,0}^*$ for all $m \in M$, then

$$\mathcal{Q}_0 = \frac{P_0^F/P_{m_0,0}^*}{P_0/P_{m_0,0}}$$

Since the numerator is exogenous, the real exchange rate can be written as the relative price of CPI to one of the imported goods $P_{m_0,0}$. Let set $P_0^F/P_{m_0,0}^* = 1$, then

$$\mathcal{Q}_0 = \frac{P_{m_0,0}}{P_0}$$

Note that consumption at time 0 is only a function of this real exchange rate since from the Euler equation

$$C_0 = \frac{P_1 C_1}{\beta \mathcal{E}_1 (1+i_0^*)} \frac{\mathcal{E}_0}{P_0} = \frac{P_1 C_1}{\beta \mathcal{E}_1 (1+i_0^*)} \mathcal{Q}_0 = \frac{E_1}{\beta \mathcal{E}_1 (1+i_0^*)} \mathcal{Q}_0$$

Changes in the real exchange rate represent changes in all prices relative to the imported good m_0.

$$\begin{aligned}
\widehat{P}_0 - \widehat{P}_{m_0,0} &= \sum_{i\in N} \frac{P_i C_i}{PC}(\widehat{P}_i - \widehat{P}_{m_0,0}) + \sum_{m\in M} \frac{P_m C_m}{PC}(\widehat{P}_m - \widehat{P}_{m_0,0}) = \bar{\boldsymbol{b}}_D^T(\widehat{\boldsymbol{P}}_D - \mathbf{1}_N \widehat{P}_{m_0,0}) + \bar{\boldsymbol{b}}_M^T(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*) \\
\widehat{P}_0 - \widehat{P}_{m_0,0} &= \bar{\boldsymbol{b}}_D^T(-\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + \boldsymbol{\Psi}\boldsymbol{\Gamma}(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*)) + \bar{\boldsymbol{b}}_M^T(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*) \\
\widehat{P}_0 - \widehat{P}_{m_0,0} &= -\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*) \\
\widehat{Q}_0 &= -(\widehat{P}_0 - \widehat{P}_{m_0,0}) = \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\widehat{\boldsymbol{Z}} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) - (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*)
\end{aligned}$$

It follows that consumption changes satisfy

$$\begin{aligned}
\widehat{C}_0 &= \widehat{E}_1 - \widehat{\beta} - \widehat{\mathcal{E}}_1 - (\widehat{1+i_0^*}) + \widehat{Q}_0 \\
&= \widehat{E}_1 - \widehat{\beta} - \widehat{\mathcal{E}}_1 - (\widehat{1+i_0^*}) + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\widehat{\boldsymbol{Z}} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) - (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}\boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*),
\end{aligned}$$

this illustrates how consumption at time 0 is not pinned down by real GDP, Y_0, as is the case in the closed economy model. Rather, it is pinned down by the real exchange rate, $\mathcal{Q}_0$, which in turn depends on factor prices in units of good m_0.

Using the fact that $P_0 C_0 = E_0$ is given once we set either i_0 or $\mathcal{E}_0$, then changes in the price index satisfy

$$
\begin{aligned}
\widehat{P}_0 &= \widehat{E}_0 - \widehat{C}_0 \\
&= \widehat{E}_0 - \widehat{E}_1 + \widehat{\beta} + \widehat{\mathcal{E}}_1 + \widehat{(1+i_0^*)} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} (\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*) \\
&= \widehat{\mathcal{E}}_1 + \widehat{(1+i_0^*)} - \widehat{(1+i_0)} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} (\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*)
\end{aligned}
$$

From real wages to aggregate demand and inelastic labor supply changes. To solve the model in terms of factor quantities, let me define real wages, in term of imported good m_0, as a function of the these, factor shares, and the import price (denominated in foreign currency)

$$
\begin{aligned}
\widehat{W}_{f,0} - \widehat{P}_{m_0,0} &= \widehat{\bar{\Lambda}}_f - \widehat{\bar{L}}_{f,0} + \widehat{E}_0 - \widehat{P}_{m_0,0} \\
&= \widehat{\bar{\Lambda}}_f - \widehat{\bar{L}}_{f,0} + (\widehat{E}_0 - \widehat{\mathcal{E}}_0) - \widehat{P}_{m_0,0}^* \\
\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0} &= \widehat{\bar{\boldsymbol{\Lambda}}} - \widehat{\bar{\boldsymbol{L}}} + \mathbf{1}_F ((\widehat{E}_0 - \widehat{\mathcal{E}}_0) - \widehat{P}_{m_0,0}^*),
\end{aligned}
$$

where the second line follows from the law of one price.

Note that expenditure denominated in foreign currency is exogenous from the euler equation

$$
\frac{E_0}{\mathcal{E}_0} = \frac{E_1}{\mathcal{E}_1} \frac{1}{\beta(1+i_0^*)}.
$$

Thus $(\widehat{E}_0 - \widehat{\mathcal{E}}_0)$ represents an *aggregate demand shifter*. It increases if the consumer becomes more impatient (β declines), expenditure in the future increases ($E_1 = P_1 C_1$), the interest rate in foreign currency goes up, $(1+i_0^*)$, or the exchange rate in the future goes up, $\mathcal{E}_1$. These two last terms induce the consumer to move away from consumption towards saving in the traded bond, thus decreasing aggregate consumption at $t = 0$.

Set $\widehat{P}_{m0,0}^* = 0$ to simplify the exposition. Combining expenditure at time 0 in foreign currency with the expression for CPI changes we get

$$
\widehat{P}_0 = \widehat{\mathcal{E}}_0 - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \left(\widehat{\bar{\boldsymbol{\Lambda}}} - \widehat{\bar{\boldsymbol{L}}} + \mathbf{1}_F (\widehat{E}_0 - \widehat{\mathcal{E}}_0) \right) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) \widehat{\boldsymbol{P}}_M^*,
$$

which can be written as

$$
\begin{aligned}
\widehat{P}_0 = {} & \underbrace{\widehat{\mathcal{E}}_0}_{\text{Nominal Anchor}} + \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \mathbf{1}_F (\widehat{E}_0 - \widehat{\mathcal{E}}_0)}_{\text{Aggregate demand shifter}} - \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}}}_{\text{Technology effects}} + \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}}}_{\text{Factor share reallocation}} \\
& - \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \widehat{\bar{\boldsymbol{L}}}}_{\text{Factor supplies}} + \underbrace{(\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) \widehat{\boldsymbol{P}}_M^*}_{\text{Import price channel}}, \qquad (23)
\end{aligned}
$$

where I used the fact the no-arbitrage condition, implies that changes in the nominal exchange rate at time 0 can be written as

$$
\widehat{(1+i_0)} = \widehat{(1+i_0^*)} + \widehat{\mathcal{E}}_1 - \widehat{\mathcal{E}}_0 \Longrightarrow \widehat{\mathcal{E}}_0 = \widehat{(1+i_0^*)} + \widehat{\mathcal{E}}_1 - \widehat{(1+i_0)}.
$$

This provides a nominal anchor at time 0.

B.2 Mapping the net transfer, T.

We can use the two-period model above to justify the exogenous net transfer in the static setup T. To see this, write

$$T = nGDP_0 - P_0C_0 = (1+\beta)P_0C_0 - \frac{nGDP_1}{(1+i_0^*)}\frac{\mathcal{E}_0}{\mathcal{E}_1} - P_0C_0$$
$$T = \beta P_0C_0 - \frac{nGDP_1}{(1+i_0^*)}\frac{\mathcal{E}_0}{\mathcal{E}_1} = \beta P_0C_0 - \beta\frac{nGDP_1}{P_1C_1}P_0C_0 = \beta P_0C_0\left(1 - \frac{nGDP_1}{P_1C_1}\right),$$

this net transfer is positive or negative depending on whether nominal GDP in the future is higher or lower than future expenditure. This ultimately hinges on the difference between future consumption and income since if $nGDP_1/P_1C_1 > 1$; this ratio is negative, meaning $T < 0$. A negative net transfer means the economy receives resources at time 0 that do not come from their own production at time 0. In an intertemporal model, this comes from future resources. In a static model, this should come from the rest of the world. The converse also holds. Of course, if the steady state features no assets holding, this equation collapses to $T = 0$. To see why, note that the budget constraint at the steady state satisfies

$$PC\left(1+\frac{1}{(1+i)}\right) = \mathcal{E}i^*B + nGDP\left(1+\frac{1}{(1+i)}\right),$$

If $B = 0$, then

$$PC = nGDP,$$

and therefore $T = 0$.

C Shares

In this appendix I explicitly solve for $\widehat{\bar{\mathbf{\Lambda}}}$. Before doing so, I need to define several objects on the consumption and production sides.

C.1 Consumption

Let the consumer's share expenditure on good $i \in N \cup M$ be b_i and

$$\bar{b}_i = \frac{P_iC_i}{E}$$

Let the price elasticity of demand be $\varepsilon_{ik}^C = \frac{\partial \log C_i}{\partial \log P_k}$ and $\delta_{ik} = 1$ if $k = i$ and zero otherwise. This last element is usually called the Kronecker-delta. Log-differentiating the shares and using the homotheticity assumption, we have

$$\widehat{\bar{b}}_i = \sum_{k\in N\cup M}(\delta_{ik} + \varepsilon_{ik}^C - \bar{b}_i)\widehat{P}_k = \sum_{k\in N\cup M}\phi_{ik}^C\widehat{P}_k$$
$$\mathrm{d}\bar{b}_i = \bar{b}_i\sum_{k\in N\cup M}\phi_{ik}^C\widehat{P}_k \qquad \text{for } i \in N \cup M$$

where $\phi_{ik}^{C} = (\delta_{ik} + \varepsilon_{ik}^{C} - \bar{b}_k)$ represents the elasticity of consumption share on good i, b_i, in response to a change in the price of good k, P_k.

Note that we must have

$$\sum_{i \in N \cup M} \mathrm{d}\bar{b}_i = 0 \Longrightarrow \sum_{i \in N \cup M} \bar{b}_i \sum_{k \in N \cup M} \phi_{ik}^{C} \widehat{P}_k = 0,$$

for any changes in prices. It thus follows that

$$\sum_{i \in N \cup M} \bar{b}_i \phi_{ik}^{C} = 0 \quad \text{for all } k \in N \cup M$$

For further reference, it proves useful to define changes in domestic expenditure shares as

$$\mathrm{d}\bar{\boldsymbol{b}}_D = diag(\bar{\boldsymbol{b}}_D)(\boldsymbol{\Phi}_D^C \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M^C \widehat{\boldsymbol{P}}_M),$$

where $\boldsymbol{\Phi}_D^C$ is an $N \times N$ matrix with typical element ϕ_{ij}^{C} with $i, j \in N$, and $\boldsymbol{\Phi}_M^C$ is an $N \times M$ matrix with typical element ϕ_{im}^{C}, and $i \in N$, $m \in M$.

C.2 Production

On the production side, we have to define an operator similar to the above but for each $i \in N$ producer. In addition, the decision of the representative firms also depend on factor prices $\widehat{W}_f$. With this in mind, define the expenditure share of producer i on input $j \in N \cup M$ as

$$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$$

Define

$$\phi_{jk}^{i} = \frac{\partial \log \Omega_{ij}}{\partial \log P_k} \quad \text{for } i = 1, ..., N; \quad k = 1, ..., N \cup M$$

Then

$$\begin{aligned}
\widehat{\Omega}_{ij} &= \widehat{P}_j + \sum_{k \in N \cup M} \varepsilon_{jk}^{i} \widehat{P}_k + \sum_{f \in F} \varepsilon_{jf}^{i} \widehat{W}_f - \sum_{k \in N \cup M} \Omega_{ik} \widehat{P}_k - \sum_{f \in F} a_{if} \widehat{W}_f \\
&= \sum_{k \in N \cup M} (\delta_{jk} + \varepsilon_{jk}^{i} - \Omega_{ik}) \widehat{P}_k + \sum_{f \in F} (\varepsilon_{jf}^{i} - a_{if}) \widehat{W}_f \\
\widehat{\Omega}_{ij} &= \sum_{k \in N \cup M} \phi_{jk}^{i} \widehat{P}_k + \sum_{f \in F} \phi_{jf}^{i} \widehat{W}_f \\
\mathrm{d}\Omega_{ij} &= \Omega_{ij} \left(\sum_{k \in N \cup M} \phi_{jk}^{i} \widehat{P}_k + \sum_{f \in F} \phi_{jf}^{i} \widehat{W}_f \right)
\end{aligned}$$

where

$$\varepsilon^i_{jk} = \frac{\partial \log M_{ij}}{\partial \log P_k}$$
$$\phi^i_{jk} = \delta_{jk} + \varepsilon^i_{jk} - \Omega_{ik}$$
$$\phi^i_{jf} = \varepsilon^i_{jf} - a_{if}$$

represents the elasticity of expenditure share on good j by producer i, Ω_{ij}, when there is a change in either good $k \in N \cup M$ or factors $f \in F$.

By a similar logic, I can write the change in expenditure share of producer i on factor f as

$$\mathrm{d}a_{if} = a_{if} \left(\sum_{k \in N \cup M} (\varepsilon^i_{fk} - \Omega_{ik}) \widehat{P}_k + \sum_{f' \in F} (\delta_{ff'} + \varepsilon^i_{ff'} - a_{if'}) \widehat{W}_{f'} \right)$$
$$\mathrm{d}a_{if} = a_{if} \left(\sum_{k \in N \cup M} \phi^i_{fk} \widehat{P}_k + \sum_{f' \in F} \phi^i_{ff'} \widehat{W}_{f'} \right)$$

where again

$$\varepsilon^i_{fk} = \frac{\partial \log L_{if}}{\partial \log P_k}$$
$$\varepsilon^i_{ff'} = \frac{\partial \log L_{if}}{\partial \log W_{f'}}$$
$$\phi^i_{fk} = \varepsilon^i_{fk} - \Omega_{ik}$$
$$\phi^i_{ff'} = \delta_{ff'} + \varepsilon^i_{ff'} - a_{if'}$$

The first two rows represent the demand elasticity of factor f relative to a change in other good prices (first row) or factors of producton (second row).

The last two rows represent the elasticity of expenditure share of producer i on factor f relative to either good or factor price changes. When these are positive, then expenditure changes increase after a change in other input prices meaning that the producer substitute away from those price increases towards factor f. If this term is negative, then the expenditure share in factor f decline with a change in other input prices: it means that it has to move resources away from factor f towards those goods that are seeing an increase in their price. This is the complementarity in production case and it arises with low elasticities of substitution (low ε's).

For imported intermediate, we can construct the same as

$$\mathrm{d}\Gamma_{im} = \Gamma_{im} \left(\sum_{k \in N} (\varepsilon^i_{mk} - \Omega_{ik}) \widehat{P}_k + \sum_{m' \in M} (\delta_{mm'} + \varepsilon^i_{mm'} - \Gamma_{im'}) \widehat{P}_{m'} + \sum_{f \in F} (\varepsilon^i_{mf} - a_{if}) \widehat{W}_f \right)$$
$$\mathrm{d}\Gamma_{im} = \Gamma_{im} \left(\sum_{k \in N} \phi^i_{mk} \widehat{P}_k + \sum_{m' \in M} \phi^i_{mm'} \widehat{P}_{m'} + \sum_{f \in F} \phi^i_{mf} \widehat{W}_f \right)$$

where again

$$
\begin{aligned}
\varepsilon^i_{mk} &= \frac{\partial \log M_{im}}{\partial \log P_k} \\
\varepsilon^i_{mm'} &= \frac{\partial \log M_{im}}{\partial \log P_{m'}} \\
\varepsilon^i_{mf} &= \frac{\partial \log M_{im}}{\partial \log W_f} \\
\phi^i_{mk} &= (\varepsilon^i_{mk} - \Omega_{ik}) \\
\phi^i_{mm'} &= (\delta_{mm'} + \varepsilon^i_{mm'} - \Gamma_{im'}) \\
\phi^i_{mf} &= (\varepsilon^i_{mf} - a_{if})
\end{aligned}
$$

C.3 Market clearing conditions and substitution patterns

Goods market clearing conditions. Recall that the market clearing conditions for the N domestic goods can be written as

$$Q_i = C_i + X_i + \sum_{j \in N} M_{ji}$$

In nominal terms and dividing by expenditure, we have

$$\frac{P_i Q_i}{E} = \frac{P_i C_i}{E} + \frac{P_i X_i}{E} + \sum_{j \in N} \frac{P_i M_{ji}}{P_j Q_j} \frac{P_j Q_j}{E}$$

Define *expenditure-based* ratios with a bar i.e. $\bar{\lambda}_i = \frac{P_i Q_i}{E}$. Then,

$$\bar{\lambda}_i = \bar{b}_i + \bar{x}_i + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j$$

Differentiating this expression, we have

$$\mathrm{d}\bar{\lambda}_i = \mathrm{d}\bar{b}_i + \mathrm{d}\bar{x}_i + \sum_{j \in N} (\mathrm{d}\Omega_{ji} \bar{\lambda}_j + \Omega_{ji} \mathrm{d}\bar{\lambda}_j)$$

Now, recall from shares and making some changes of indices

$$\mathrm{d}\Omega_{ji} = \Omega_{ji} \left(\sum_{k \in N \cup M} \phi^j_{ik} \widehat{P}_k + \sum_{f \in F} \phi^j_{if} \widehat{W}_f \right)$$

Then the third term on the right-hand side can be written as

$$
\begin{aligned}
\sum_{j\in N} \mathrm{d}\Omega_{ji}\bar{\lambda}_j &= \sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\left(\sum_{k\in N\cup M}\phi^j_{ik}\widehat{P}_k + \sum_{f\in F}\phi^j_{if}\widehat{W}_f\right)\\
&= \sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\sum_{k\in N\cup M}\phi^j_{ik}\widehat{P}_k + \sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\sum_{f\in F}\phi^j_{if}\widehat{W}_f\\
&= \sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\sum_{k\in N}\phi^j_{ik}\widehat{P}_k + \sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\sum_{m\in M}\phi^j_{im}\widehat{P}_m + \sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\sum_{f\in F}\phi^j_{if}\widehat{W}_f\\
&= \sum_{k\in N}\underbrace{\left[\sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\phi^j_{ik}\right]}_{\equiv\phi_{ik}}\widehat{P}_k + \sum_{m\in M}\underbrace{\left[\sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\phi^j_{im}\right]}_{\equiv\phi_{im}}\widehat{P}_m + \sum_{f\in F}\underbrace{\left[\sum_{j\in N}\Omega_{ji}\bar{\lambda}_j\phi^j_{if}\right]}_{\equiv\phi_{if}}\widehat{W}_f\\
&= \sum_{k\in N}\phi_{ik}\widehat{P}_k + \sum_{m\in M}\phi_{im}\widehat{P}_m + \sum_{f\in F}\phi_{if}\widehat{W}_f
\end{aligned}
$$

A useful thing about writing this in this way, is that I can write this in matrix form

$$\mathrm{d}\boldsymbol{\Omega}^T\boldsymbol{\lambda} = \boldsymbol{\Phi}_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M\widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F\widehat{\boldsymbol{W}}$$

where $\boldsymbol{\Phi}$ represents *direct substitution matrices*. A version of these substitution matrices appears in Baqaee and Farhi (2019b), although they consider both direct and indirect substitution. Each column represents the changing price and the rows represent where intermediate input demand is going. This only takes into account *first-round effects* and does not consider any input-output linkages beyond the direct exposure (or path of order 1). The next step is to recompute these matrices using the Leontieff-inverse. To see this, note that the differentiated form of the market clearing condition write the problem as

$$
\begin{aligned}
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \boldsymbol{\Psi}^T(\mathrm{d}\bar{\boldsymbol{b}}_D + \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Phi}_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M\widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F\widehat{\boldsymbol{W}})\\
&= \boldsymbol{\Psi}^T\left(diag(\bar{\boldsymbol{b}}_D)(\boldsymbol{\Phi}^C_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}^C_M\widehat{\boldsymbol{P}}_M) + \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Phi}_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M\widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F\widehat{\boldsymbol{W}}\right)\\
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \boldsymbol{\Psi}^T\left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}^C_D + \boldsymbol{\Phi}_D\right)\widehat{\boldsymbol{P}}_D + \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}^C_M + \boldsymbol{\Phi}_M\right)\widehat{\boldsymbol{P}}_M + \boldsymbol{\Psi}^T\mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Psi}^T\boldsymbol{\Phi}_F\widehat{\boldsymbol{W}}
\end{aligned}
$$

Factor shares changes. We need F more equations, which come from the factor market clearing conditions

$$\bar{L}_f = \sum_{i\in N} L_{if}$$

Write this in share form

$$\bar{\Lambda}_f = \frac{W_f\bar{L}_f}{E} = \sum_{i\in N}\frac{W_fL_{if}}{P_iQ_i}\frac{P_iQ_i}{E} = \sum_{i\in N}a_{if}\bar{\lambda}_i$$

In differential form,

$$\mathrm{d}\Lambda_f = \sum_{i\in N}\mathrm{d}a_{if}\bar{\lambda}_i + \sum_{i\in N}a_{if}\mathrm{d}\bar{\lambda}_i$$

Using the expression for changes in factor usage at the producer level, I have

$$\mathrm{d}\Lambda_f = \sum_{i\in N}\left[a_{if}\left(\sum_{k\in N\cup M}\phi^i_{fk}\widehat{P}_k + \sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\right)\right]\bar{\lambda}_i + \sum_{i\in N}a_{if}\mathrm{d}\bar{\lambda}_i$$

$$\mathrm{d}\Lambda_f = \sum_{i\in N}\left[a_{if}\left(\sum_{k\in N}\phi^i_{fk}\widehat{P}_k + \sum_{m\in M}\phi^i_{fm}\widehat{P}_m + \sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\right)\right]\bar{\lambda}_i + \sum_{i\in N}a_{if}\mathrm{d}\bar{\lambda}_i$$

Taking each of the terms on the right-hand side, I can write

$$\sum_{i\in N}a_{if}\sum_{k\in N}\phi^i_{fk}\widehat{P}_k\bar{\lambda}_i = \sum_{k\in N}\underbrace{\left(\sum_{i\in N}a_{if}\phi^i_{fk}\bar{\lambda}_i\right)}_{\equiv\phi_{fk}}\widehat{P}_k = \sum_{k\in N}\phi_{fk}\widehat{P}_k$$

$$\sum_{i\in N}a_{if}\sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\bar{\lambda}_i = \sum_{f'\in F}\underbrace{\left(\sum_{i\in N}a_{if}\bar{\lambda}_i\phi^i_{ff'}\right)}_{\equiv\phi_{ff'}}\widehat{W}_{f'} = \sum_{f'\in F}\phi_{ff'}\widehat{W}_{f'}$$

$$\sum_{i\in N}a_{if}\sum_{m\in M}\phi^i_{fm}\widehat{P}_m\bar{\lambda}_i = \sum_{m\in M}\underbrace{\left(\sum_{i\in N}a_{if}\bar{\lambda}_i\phi^i_{fm}\right)}_{\equiv\phi_{fm}}\widehat{P}_m = \sum_{m\in M}\phi_{fm}\widehat{P}_m$$

Replacing this into the differential form for the factor share

$$\mathrm{d}\Lambda_f = \sum_{k\in N}\phi_{fk}\widehat{P}_k + \sum_{f'\in F}\phi_{ff'}\widehat{W}_{f'} + \sum_{m\in M}\phi_{fm}\widehat{P}_m + \sum_{i\in N}a_{if}\mathrm{d}\bar{\lambda}_i$$

In matrix form,

$$\mathrm{d}\bar{\boldsymbol{\Lambda}} = \boldsymbol{\Phi}^F_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}^F_F\widehat{\boldsymbol{W}} + \boldsymbol{\Phi}^F_M\widehat{\boldsymbol{P}}_M + \boldsymbol{A}^T\mathrm{d}\bar{\boldsymbol{\lambda}}$$

Export share changes. Since export demand is exogenous, we can write

$$\bar{x}_i = \frac{P_iX_i}{\mathcal{M}} \Longrightarrow \mathrm{d}\bar{x}_i = \bar{x}_i(\widehat{P}_i + \widehat{X}_i - \widehat{\mathcal{M}}),$$

which staking into a vector form

$$\mathrm{d}\bar{\boldsymbol{x}} = diag(\bar{\boldsymbol{x}})(\widehat{\boldsymbol{P}}_D + \widehat{\boldsymbol{X}} - \mathbf{1}_N\widehat{\mathcal{M}}) \tag{24}$$

C.4 Putting all equations together

$$
\begin{aligned}
\mathrm{d}\bar{\boldsymbol{\Lambda}} &= \boldsymbol{\Phi}_D^F \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_F^F \widehat{\boldsymbol{W}} + \boldsymbol{\Phi}_M^F \widehat{\boldsymbol{P}}_M + \boldsymbol{A}^T \mathrm{d}\bar{\boldsymbol{\lambda}} \qquad (F \text{ equations}, F+N+F+N \text{ unknowns}) \\
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \boldsymbol{\Psi}^T \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}_D^C + \boldsymbol{\Phi}_D \right) \widehat{\boldsymbol{P}}_D + \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}_M^C + \boldsymbol{\Phi}_M \right) \widehat{\boldsymbol{P}}_M + \boldsymbol{\Psi}^T \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Psi}^T \boldsymbol{\Phi}_F \widehat{\boldsymbol{W}} \qquad (N \text{ equations}, N \text{ add. unknowns}) \\
\widehat{\boldsymbol{P}}_D &= -\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}\boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}\boldsymbol{\Gamma}\widehat{\boldsymbol{P}}_M \qquad (N \text{ equations, no new unknowns}) \\
\mathrm{d}\bar{\boldsymbol{\Lambda}} &= diag(\bar{\boldsymbol{\Lambda}}) \left(\widehat{\boldsymbol{W}} + \widehat{\boldsymbol{L}} - \boldsymbol{1}_F \widehat{\mathcal{M}} \right) \qquad (F \text{ equations, no new unknowns}) \\
\mathrm{d}\bar{\boldsymbol{x}} &= diag(\bar{\boldsymbol{x}})(\widehat{\boldsymbol{P}}_D + \widehat{\boldsymbol{X}} - \boldsymbol{1}_N \widehat{\mathcal{M}}) \qquad (N \text{ equations, no new unknowns})
\end{aligned}
$$

This is a system of $2F + 3N$ unknowns: $(\mathrm{d}\bar{\boldsymbol{\Lambda}}, \widehat{\boldsymbol{W}}, \mathrm{d}\bar{\boldsymbol{\lambda}}, \widehat{\boldsymbol{P}}_D, \mathrm{d}\bar{\boldsymbol{x}})$ on the same number of equations, and thus it pins down all necessary objects.

Note that the distribution of factor shares ($\mathrm{d}\bar{\boldsymbol{\Lambda}}$) and domar weights ($\mathrm{d}\bar{\boldsymbol{\lambda}}$) changes crucially depends on the substitution matrices ($\boldsymbol{\Phi' s}$ matrices). As a result, to the extent that substitution patterns are encapsulated in these matrices, they affect aggregate inflation in the small open economy via changing factor shares. It is in this sense that elasticities of substitution matters for inflation in this model, something that does not hold in the closed economy as these terms cancels out, to a first-order.