Sectoral Shocks and Inflation in Small Open Economies

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ECON 743

This presentation

1. Preamble: How it started
2. How I get here
3. What I am doing
4. What I have
5. Where I go

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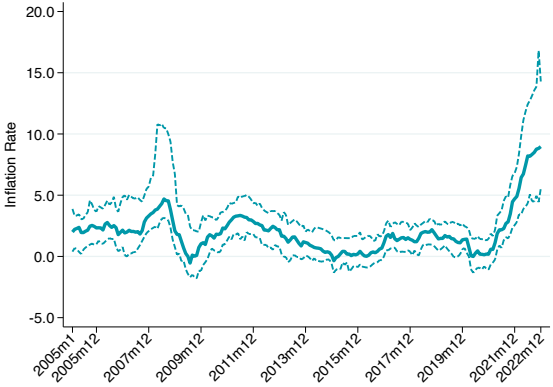
- Bottom-line
  - Do not be afraid of reaching out to people
  - Find something that you **really** like (easier said than done but true!)

How I get here

- Tons of attention on inflation' sources amid the COVID-19 pandemic due to disparate shocks: sectoral and aggregate shocks
  - Highlighted the important role of sectoral supply and demand shocks in affecting inflation.
- Most (if not all) work centered in advanced economies
- I did not understand why since inflation was rising everywhere

Inflation was rising everywhere...

(a) Advanced Economies



(b) Emerging Markets

with same response of monetary policy everywhere as well

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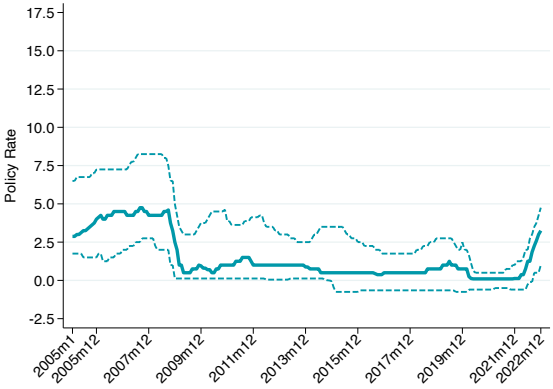

(b) Emerging Markets

To the question

- Got me thinking: What can we learn from these closed economy approaches when applied to SOEs?
  - Are sectoral shocks equally important for inflation in SOEs as in closed economies?
  - Need to think harder on: goods trade, financial trade, and nominal rigidities.

Related Literature

- Inflation in *closed economies* with production networks, and sectoral shocks

  Pasten, Schoenle, and Webber (2020), Guerrieri, Lorenzoni, Straub, and Werning (2021, 2022), Gourinchas, Kalemli-Özcan, Penciakova, and Sander (2021), Baqaee and Farhi (2022), La'O and Tahbaz-Salehi (2022), Rubbo (2022), Afrouzi and Bhattarai (2022), di Giovanni, Kalemli-Özcan, Silva, and Yıldırım (2022, 2023), Ferrante, Graves, and Iacovello (2022), Luo and Villar (2022),...

  $\Longrightarrow$ Provide a inflation decomposition (sufficient statistics) in a small open economy setup.

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- Inflation in *open economies* with sectoral shocks

  Gali and Monacelli (2005), Comin and Johnson (2020), Fornaro and Romei (2022), Ho, Sarte, and Schwartzmann (2022), di Giovanni, Kalemli-Özcan, Silva, and Yıldırım (2022)

  $\Longrightarrow$ Embed realistic production network to study inflation in small open economies.

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  - Think of inflation in the space rather than time dimension: same logic.
  - Find out a useful inflation decomposition

What I have: Static Model (Sketch)

- F inellasticaly supplied factors (not produced goods)
- M imported goods: can be part of final consumption or used as intermediate inputs
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- $\boldsymbol{P}_D$: vector of prices for goods N.
- $\boldsymbol{P}_M$: vector of prices for imports M.
- $\boldsymbol{W}$: vector of factor prices F.

Notation (Only slide I promise)

$$\boldsymbol{\Omega} = \{\Omega_{ij}\} = \frac{P_j M_{ij}}{P_i Q_i}; \quad \boldsymbol{A} = \{a_{if}\} = \frac{W_f L_{if}}{P_i Q_i}; \quad \boldsymbol{\Gamma} = \{\Gamma_{im}\} = \frac{P_m M_{im}}{P_i Q_i}$$

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$$\boldsymbol{\lambda^M} = \frac{P_m(C_m + \sum_{i=1}^{N} M_{im})}{GDP}; \quad \boldsymbol{\chi} = \{\chi_m\} = \frac{P_m C_m}{GDP}; \quad \boldsymbol{b} = \{b_i\} = \frac{P_i C_i}{GDP}$$

Main Result so far...

- To a first-order, CPI inflation in a closed economy (Baqaee and Farhi, 2022) is

$$\mathrm{d}\log \boldsymbol{CPI} = -\boldsymbol{\lambda}^{T}\mathrm{d}\log \boldsymbol{Z} + \boldsymbol{\Lambda}^{T}\mathrm{d}\log \boldsymbol{W} \tag{1}$$

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  - Exporting is effectively **transferring** resources abroad: $\boldsymbol{x}^T \Psi_D$ and $\boldsymbol{x}^T \Psi_D \boldsymbol{A}$.
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- Takeaways
  - Opening up the economy **is** relevant: changes relevant shares.
  - Production networks do not add anything (relative to a closed economy) **unless** it is coupled with exporting/importing.

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  - Monetary policy?
    - Nothing in the current framework $\Longrightarrow$ more reasons to go fully dynamic.

Thank you!

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Key Equations

$$Q_i = C_i^D + X_i + \sum_{j=1}^{N} M_{ji}^D \quad \text{for each } i = 1, 2, ..., N \tag{2}$$

$$\bar{L}_f = \sum_{i=1}^{N} L_{if} \quad \text{for each } f = 1, 2, ..., F \tag{3}$$

$$\boldsymbol{\lambda} = \boldsymbol{\Psi}_D^T(\boldsymbol{b} + \boldsymbol{x}) \tag{4}$$

$$\boldsymbol{\lambda}^M = \boldsymbol{\chi} + \boldsymbol{\Gamma}^T\boldsymbol{\lambda} \tag{5}$$

$$\boldsymbol{\Lambda} = \boldsymbol{A}^T\boldsymbol{\lambda} \tag{6}$$

$$\mathrm{d}\log CPI = \tilde{\boldsymbol{b}}^T \mathrm{d}\log \boldsymbol{P}_D + \tilde{\boldsymbol{\chi}}^T \mathrm{d}\log \boldsymbol{P}_M \tag{7}$$

$$(\boldsymbol{b}^T + (\boldsymbol{x})^T)\mathbf{1}_N - (\boldsymbol{\lambda}^M)^T\mathbf{1}_M = 1 \tag{8}$$

$$\tilde{\boldsymbol{b}}^T\mathbf{1}_N + \tilde{\boldsymbol{\chi}}^T\mathbf{1}_M = 1 \tag{9}$$

$$\mathrm{d}\log \boldsymbol{P}_D = -\boldsymbol{\Psi}_D \mathrm{d}\log \boldsymbol{Z} + \boldsymbol{\Psi}_D\boldsymbol{A}\mathrm{d}\log \boldsymbol{W} + \boldsymbol{\Psi}_D\boldsymbol{\Gamma}\mathrm{d}\log \boldsymbol{P}_M \tag{10}$$

Another way to write CPI changes

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$$\mathrm{d}\log CPI = -\left(\lambda^T - \boldsymbol{x}^T\Psi_D\right)\frac{GDP}{E}\mathrm{d}\log \boldsymbol{Z} + \left(\Lambda^T - \boldsymbol{x}^T\Psi_D\boldsymbol{A}\right)\frac{GDP}{E}\mathrm{d}\log \boldsymbol{W} + \left((\lambda^M)^T - \boldsymbol{x}^T\Psi_D\Gamma\right)\frac{GDP}{E}\mathrm{d}\log \boldsymbol{P}_M$$

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