Usama Bilal

FA17-BCS-095

BCS-8A

Q No 1

(a)

Total $x_1 = 30$

Total $x_2 = 52$

Total $Y = 89$

Mean of $x_1 = 6$

Mean of $x_2 = 10.4$

Mean of $Y = 17.8$

Sum of squares of $x_1 = 58$

Sum of squares of $x_2 = 41.2$

Sum of products of $x_1 y = 85$

Sum of products of $x_2 y = 81.5$

Sum of products of $x_1 x_2 = 39$

Regression equation $= \hat{y} = b_1 x_1 + b_2 x_2 + a$

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$$b_1 = \frac{(SP x_1 y) \times (SS x_2) - (SP x_1 x_2) \times (SP x_2 y)}{(SS x_1) \times SS(x_2) - (SP x_1 x_2) \times SP(x_1 x_2)}$$

$$= \frac{327.4}{868.6} = 0.37693$$

$$b_2 = \frac{(SP x_2 y) \times (SS x_1) - (SP x_1 x_2) \times (SP x_1 y)}{(SS x_1) \times (SS x_2) - (SP x_1 x_2) \times (SP x_1 x_2)}$$

$$= \frac{1406.2}{868.6} = 1.6189$$

$$a = My - b_1 Mx_1 - b_2 Mx_2$$

$$= 17.8 - (0.98 \times 6) - (1.62 \times 10.4)$$

$$= -1.29841$$

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$$\hat{y} = 0.37693 x_1 + 1.61893 x_2 - 1.29841$$

$$\begin{bmatrix} 5 & 30 & 52 \\ 30 & 238 & 351 \\ 52 & 351 & 580 \end{bmatrix} \begin{bmatrix} a \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 89 \\ 613 \\ 1007 \end{bmatrix}$$

$$\begin{bmatrix} l_{11} \cdot 1 & l_{11} u_{12} & l_{11} u_{13} \\ l_{21} & l_{21} u_{12} + l_{22} & l_{21} u_{13} + l_{22} u_{23} \\ l_{31} & l_{31} u_{12} + l_{32} & l_{31} u_{13} + l_{32} u_{23} + l_{33} \end{bmatrix} = \begin{bmatrix} 5 & 30 & 52 \\ 30 & 238 & 351 \\ 52 & 351 & 580 \end{bmatrix}$$

$l_{11} = 5$, $l_{21} = 30$, $l_{31} = 52$

$u_{12} = \frac{1912}{l_{11}} = 6$, $u_{12} = 6$

$u_{13} = \frac{913}{a} = 10.4$

$l_{21} u_{12} + l_{22} = 0.22$

$\boxed{l_{22} = 58}$

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Q1 (b)

Curve $= ax^b$

taking log we get normal equations as

$$\Sigma Y = nA + b\Sigma X$$

$$\Sigma XY = A\Sigma x + b\Sigma X^2$$

Sum $x = \Sigma X = 21$

$\Sigma y = 32.85$

$X = \log_{10}(x)$

$\Sigma X = 0 + 0.301 + 0.4771 + 0.602 + 0.6998 + 0.7782$

$= 2.8573$

$Y = \log_{10}(y)$

$\Sigma X = 0.4742 + 0.6294 + 0.7168 + 0.7853 + 0.8325 + 0.8751$

$= 4.3134$

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$\Sigma x^2 = 0 + 0.0906 + 0.2276 + 0.3625 + 0.4886 + 0.6655$

$= 1.7748$

$x \cdot y = 0 + 0.1895 + 0.342 + 0.4728 + 0.5819 + 0.6809$

$= 2.2671$

Using these values:

$6A + 2.8573b = 4.3134$

$2.8573A + 1.77485 = 2.2671$

using these two equations we get

$A = 0.474 \quad b = 0.5143$

$a = \log^{-1}_{10}(A) = 2.9783$

$y = ax^b$

$y = 2.9783 \cdot x^{0.5143}$

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Date: M T W T F S

Q No 2

(a)

(i) Quadratic

Forward diff

$$f''(0.75) = \frac{f(1.25) - 2f(1) + f(0.75)}{(0.25)^2}$$

$$= \frac{0.8 - 2 + 1.33}{0.0625}$$

$$f''(0.75) = 2.1328$$

Backward diff

~~$f''(x_i)$~~

$$f(0.75) = \frac{f(0.75) - 2f(0.5) + f(0.25)}{(0.25)^2}$$

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$$f''(0.75) = \frac{1.333 - 2(2) + 4}{0.0625}$$

$$= 01.332$$

Central diff

$$f''(x_i) = \frac{f(1) - 2f(0.75) + f(0.5)}{(0.25)^2}$$

$$= \frac{1.2(1.333) + 2}{0.0625}$$

$$= 5.3344$$

ii) Cubic

Forward diff:-

$$f'''(0.75) = \frac{f(1.5) - 3f(1.25) + 3f(1) - f(0.75)}{(0.25)^3}$$

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$$f'''(0.75) = \frac{-0.0666}{0.015625}$$

$$= 4.2624$$

Backward :-

$$f'''(0.75) = f(0.75) - 3f(0.5) + 3(0.25) - f(0)$$

$$= 469.331$$

Central :-

$$f'''(0.75) = \frac{f(1.25) - 2f(1) + 2f(0.5) - f(0.25)}{(0.25)^3}$$

$$= \frac{0.8 - 2 + 4 - 4}{0.01562 5}$$

$$= 76.8$$

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$\Sigma X^2 = 0 + 0.0906 + 0.2276 + 0.3625 + 0.4886 + 0.6555$

$= 1.7748$

$X.Y = 0 + 0.1895 + 0.342 + 0.4728 + 0.5819 + 0.6809$

$= 2.2671$

Using these values:

$6A + 2.8573b = 4.3134$

$2.8573A + 1.7748b = 2.2671$

using these two equations we get

$A = 0.474 \quad b = 0.5143$

$a = \log^{-1}_{10}(A) = 2.9783$

$y = ax^b$

$y = 2.9783 \cdot x^{0.5143}$

Usama Bilal

Date

M T W T F S

Q NO 2 (b)

$$F(h) = \frac{y(x+h) - y(x-h)}{2h}$$

$$= \frac{-\frac{1}{0.05+0.0128} + \frac{1}{0.05-0.0128}}{2(0.0128)}$$

$$= 428.05289$$

$$F(h/2) = 406.66273$$

$$\left(\frac{h}{2}\right) = \frac{4F\left(\frac{h}{2}\right) - F\left(\frac{h}{2}\right)}{4-1}$$

$$= 399.5327$$

Step-size increase:-

$$F\left(\frac{h}{2^2}\right) = \frac{-\frac{1}{0.05+0.032} + \frac{1}{0.05-0.0032}}{2(0.0032)}$$

$$= 401.64515$$

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$$F_1\left(\frac{h}{2^2}\right) = \frac{4F\left(\frac{h}{2^2}\right) - F\left(\frac{h}{2}\right)}{4-1}$$

$$= 399.927$$

$$F_2\left(\frac{h}{2^2}\right) = \frac{4^2 F_1\left(\frac{h}{2^2}\right) - F_1\left(\frac{h}{2}\right)}{4^2-1}$$

$$= 400.00195$$

Exact value:

$$y'(0.05) = 400.009$$

$$y'(0.05) = \left(\frac{1}{x^2}\right)_{x=0.05}$$

$$= \frac{1}{0.0025} = 400$$