NUMERICAL COMPUTING

ASSIGNMENT 4

SYED AWAIS KAZMI
SP18-BCS-154

Question #1

t(sec)	1	2	3	4	5	6
x (m)	0.0201	0.0844	0.3444	1.01	2.366	4.7719

Find the velocity and acceleration of the slider at time t= 6 sec.

Solution

We know that

$$velocity = \frac{dx}{dt}$$

$$Acceleration = \frac{d^2x}{dt}$$

Using three point backward difference formula at t=6

$$velocity = f'(6) = \frac{f(x_i-2)-4f(x_i-1)+3f(x)}{2h}$$

$$h = \frac{6-1}{6} = 0.833$$

$$f'(6) = \frac{1.0100 - (4)(2.3600) + (4.7719)}{(2)(0.833)}$$

$$= 3.5444 \quad \text{units/sec}$$

Acceleration =

$$f''(6) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2}$$

$$= \frac{1.0100 - (2)(2.3600) + 4.7719}{(0.833)^2}$$

$$= 1.5395 \quad \text{units/sec}^2$$

Question #2

Find $\frac{dy}{dx}$ $\frac{d^2y}{dx^2}$ for $x = 0.2$

x	0	0.1	0.2	0.3	0.4	0.5
y	0	0.10017	0.20134	0.30452	0.41076	0.52115

Solution We know that

$$\text{velocity} = \frac{dy}{dx} \quad \text{and}$$

$$\text{acceleration} = \frac{d^2y}{dx^2}$$

At $x = 0.2$

$$f'(x) = \frac{dy}{dx} = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

$$\left\{ h = \frac{0.5-0}{6} = \frac{1}{12} \right\} \qquad = \frac{-0.10017 + 0.30452}{(1/6)}$$

$$= 1.767$$

$$\frac{d^2y}{dx^2} \text{ or } f''(x) = \frac{f(x_{i-1}) - 2f(x_i) + f(x_{i+1})}{h^2}$$

$$= \frac{0.10017 - 2(0.20134) - 0.3052}{(1/12)^2}$$

$$= 0.28944$$

Question # 3

Compute $f'(3.1)$ and $f'(3.2)$

x	1	2	3	4	5
$f(x)$	0	1.4	3.3	5.6	8.1

Solution

We have to first find $f(3.1)$ and $f(5.2)$ using newton forward differente formula

Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	0				
		1.4			
2	1.4		0.5		
		1.9		-0.1	
3	3.3		0.4		-0.1
		2.3		-0.2	
4	5.6		0.2		
		2.5			
5	8.1				

Here at $x = 3.1$

$$p = \frac{x - x_0}{h} = \frac{3.1 - 1}{1} = 2.1$$

$$f(3.1) = 0 + (2.1)(1.4) + \frac{(2.1)(2.1-1)(0.5)}{2!}$$

$$+ \frac{(2.1)(2.1-1)(2.1-2)}{3!}(-0.1)$$

$$+ \frac{(2.1)(2.1-1)(2.1-2)(2.1-3)}{4!}(-0.1)$$

$$f(3.1) = 2.94 + 0.5775 - 0.00385 + 0.00086$$
$$= 3.5145$$

At $x = 3.2$ $\quad p = \frac{3.2-1}{1} = 2.2$

$$f(3.2) = 0 + (2.2)(1.4) + \frac{(2.2)(2.2-1)}{2!}(0.5)$$

$$+ \frac{(2.2)(2.2-1)(2.2-2)}{3!}(-0.1) + \frac{(2.2)(2.2-1)(2.2-2)(2.2-3)(-0.1)}{4!}$$

$= 3.7329$

$$f(3.2) = 3.08 + 0.66 - 0.0088 + 0.00176$$

We have

$f(3) = 3.3 \qquad f(3.1) = 3.5145$

$f(3.2) = 3.7329$

Using backward finite difference formula we have

$$f'(3.1) = \frac{f(3.1) - f(3)}{3.1 - 3}$$

$$= \frac{3.5145 - 3.3}{0.1} = 2.145$$

Using backward finite difference formula we have

$$f'(3.2) = \frac{f(3.2) - f(3.1)}{3.2 - 3.1}$$

$$= \frac{3.7329 - 3.5145}{0.1} = 2.184$$

Therefore using numerical differentiation we have

$\boxed{f'(3.1) = 2.145}$ $\boxed{f'(3.2) = 2.184}$

Question # 4

$$\int_0^2 \frac{dx}{1+x^2}$$

i) Trapezoidal rule, $n=6$

Solution

$$h=\frac{b-a}{n} \Rightarrow h=\frac{12-0}{6}=2$$

x	0	2	4	8	10	12
y	1	1/5	1/17	1/64	1/101	1/145

$$\int_0^{12} y\,dx = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$=\frac{2}{2}\left[\left(1+\frac{1}{145}\right)+2\left(\frac{1}{5}+\frac{1}{17}+\frac{1}{64}+\frac{1}{101}\right)\right]$$

$$=1.5754$$

ii) Trapezoidal rule, $n=8$

$$h=\frac{b-a}{n} \Rightarrow \frac{12-0}{8}=\frac{3}{2}$$

x	0	3/2	3	9/2	6	15/2	9	21/2	12
y	1	4/13	1/10	4/35	1/37	4/229	1/82	4/445	1/145

$$\int_0^{12} y\,dx = \frac{h}{2}\left[(y_0+y_8)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$=\frac{3}{4}\left[\left(1+\frac{1}{145}\right)+2\left(\frac{4}{13}+\frac{1}{10}+\frac{4}{35}+\frac{1}{37}+\frac{4}{229}+\frac{1}{82}+\frac{4}{445}+\frac{1}{145}\right)\right]$$

$$=\frac{3}{4}(2.0342719)=1.52570$$

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iii) Simpson's Rule 1/3 rule , n = 6

$$h = \frac{b-a}{n} \Rightarrow \frac{12-0}{6} = 2$$

x	0	2	4	6	8	10	12
y	1	1/5	1/17	1/37	1/65	1/101	1/145

$$\int_0^{12} y\,dx = \frac{h}{3}\Big[(y_0+y_5)+4(y_1+y_3+y_5)$$
$$+2(y_2+y_4)\Big]$$

$$= \frac{2}{3}\Big[\left(1+\frac{1}{145}\right)+4\left(\frac{2}{5}+\frac{1}{37}+\frac{1}{101}\right)+$$
$$\left(\frac{1}{17}+\frac{1}{65}\right)\Big]$$

$$= \frac{2}{3} \times (2.10302491) = 1.40201$$

iv) Simpson's 1/3 Rule n = 8

$$h = \frac{b-a}{n} \Rightarrow \frac{12-0}{8} = \frac{3}{2}$$

x	0	1.5	3	4.5	6	7.5	9	10.5	12
y	1	4/13	1/10	4/35	1/37	4/229	1/82	4/445	1/82

$$\int_0^{12} y\,dx = \frac{h}{3}\Big[(y_0+y_8)+4(y_1+y_3+y_5+y_7)$$
$$+2(y_2+y_6+y_8)\Big]$$

$$= \frac{h}{2}\Big[\left(1+\frac{1}{145}\right)+4\left(\frac{4}{13}+\frac{4}{35}+\frac{4}{229}+\frac{4}{445}\right)$$
$$+2\left(\frac{1}{10}+\frac{1}{37}+\frac{1}{82}\right)\Big]$$

$= \frac{1}{2}\left[\frac{14.5}{145} + 1.52483 + \frac{2112}{7585}\right]$

$= \frac{1}{2}\left[2.81017\right] = 1.40508$

v) vi) Simpson's $\frac{3}{8}$ Rule, $n = 6$

$h = \frac{12}{6} = 2$

$n = 6$ where $n + 1 = 7$ gives 7 co-ordinates

x	0	2	4	6	8	10	12
y	1	1/5	1/17	1/37	1/65	1/101	1/145

$$\int_0^{12} y\,dx = \frac{3h}{8}\left[[y_0 + y_8] + 3[y_1 + y_2 + y_4 + y_5] + 2(y_3)\right]$$

$$= \frac{3}{4}\left[\left(1 + 1/145\right) + (3)\left(1/5 + 1/17 + \frac{1}{65} + \frac{1}{101}\right) + (2)(1/37)\right]$$

$$= \frac{3}{4}\left(145/145 + 0.85233 + 2/37\right)$$

$$= 3/4\,(1.91327) = 1.435$$

vii) Boole's Rule using five functional evaluations

$$h = \frac{12}{4} = 3$$

x	0	3	6	9	12
y	1	$1/10$	$1/37$	$1/82$	$1/145$

$$\int_0^{12} y\,dx = \frac{2h}{45}\left(7y_0 + 32y_1 + 12y_2 + 32y_2 + 7y_3\right)$$

$$= \frac{(2)(3)}{45}\left[(7)(1) + (32)\left(\frac{1}{10}\right) + 12\left(\frac{1}{37}\right) + (32)\left(\frac{1}{82}\right) + (7)\left(\frac{1}{145}\right)\right]$$

$$= 1.46171$$

viii) Weddle's rule $n = 6$

$$h = \frac{a-b}{n} \Rightarrow \frac{12-0}{6} = 2$$

x	0	2	4	6	8	10	12
y	1	$1/5$	$1/17$	$1/37$	$1/65$	$1/101$	$1/143$

$$\int_0^{12} y\,dx = \frac{3h}{10}\left(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right)$$

$$= \frac{(3)(2)}{10}\left[1 + (5)\left(\frac{1}{5}\right) + \left(\frac{1}{17}\right) + (6)\left(\frac{1}{37}\right) + \left(\frac{1}{65}\right) + 5\left(\frac{1}{101}\right) + \frac{1}{145}\right]$$

$$= 1.37566$$