CONTENTS

EXAMPLE:

An experimental set consisting of 6 rubber belonging to 3 different rubber types was used to investigate whether the thicknesses of various table tennis rubber types are equal. In these experiments, it is statistically evaluated whether the thicknesses determined for all 3 rubber types are equal. The data obtained are included in Table 1.1. Analyze whether the thickness of these 3 rubber types is equal or not at the $\alpha = 0.05$ significance level.

Table 1.1.

Smooth Rubber	Pips Out Rubber	Anti-spin Rubber
2.3	1.5	2.3
2.2	1.6	2.00
2.15	1.1	1.5
2.1	1.8	2.1
1.9	1.7	1.8
2.0	1.4	1.5

Population: 18

Sample Size: 3 individuals in each rubber types

Independent Variable: Exercise Program (Smooth Rubber, Pips Out Rubber, Anti-spin Rubber).

Dependent Variable: Rubber thickness

Factors and Level of Factors: The factor is the rubber types with three types (Smooth Rubber, Pips Out Rubber, Anti-spin Rubber).

ARRANGEMENT OF THE EXPERIMENTAL

Determining the Number of Observation (Replication):

The number of the observation in the experiment is 3.

Smooth Rubber	Pips Out Rubber	Anti-spin Rubber
2.3	1.5	2.3
2.2	1.6	2.00
2.15	1.1	1.5
2.1	1.8	2.1
1.9	1.7	1.8
2.0	1.4	1.5

Determination of the Order of the Experiment (Randomization):

Random experiment.

Determination of the Order of the Experiment (Blocking):

Various types of rubber are used to measure rubber thicknesses. These measurement processes aim to achieve precise results by examining the characteristics of different rubber components.

We are writing the mathematical model:

$Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ (i = 1 2 3…18 , j = 1, 2, 3)

Y_{ij} : Response is the dependent variable

μ : General mean value

ε_{ij} : Error Term

1.2 The Assumption of Normality

We use Shapiro -Wilks Test for normality assumption.

S-W Test Formula: $\frac{\sum i=1nai\ X\ x_{(i)}}{\sum i=1n(x_i-\bar{x})^2}$

Step-1:
We establish the hypothesis for normality.

H_0: *The distribution is normal.*

H_1: *The distribution is not normal.*

Step-2:
Then we sort the observation values from small to large and complete the necessary operations for the S-W Test. These operations are included in Table 1.2.

Table 1.2.

i	$x_{(i)}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	a_i	$a_i \times x_i$
1	1.2	-0.636	0.4050	0.4886	0.2443
2	1.4	-0.436	0.1901	0.3253	0.1952
3	1.5	-0.336	0.1130	0.2552	0.2552
4	1.5	-0.336	0.1130	0.2027	0.2432
5	1.5	-0.336	0.1130	0.1587	0.2063
6	1.6	-0.236	0.0557	0.1197	0.1675
7	1.7	-0.136	0.0185	0.0837	0.1256
8	1.8	-0.036	0.0013	0.0496	0.0744
9	1.8	-0.036	0.0013	0.0116	0.0174
10	1.9	0.064	0.0041	-0.0164	−0.0295
11	2.0	0.164	0.0269	-0.0496	−0.0942
12	2.0	0.164	0.0269	-0.0837	−0.1674
13	2.1	0.264	0.0697	-0.1197	−0.2514
14	2.1	0.264	0.0697	-0.1587	−0.3327
15	2.15	0.314	0.0986	-0.2027	−0.4364
16	2.2	0.364	0.1325	-0.2552	−0.5614
17	2.3	0.464	0.2153	-0.3253	−0.7480
18	2.3	0.464	0.2153	-0.4886	−1.1232
	$\bar{x}=\frac{33.05}{18} \approx$ 1,8361		$\sum(x_i - \bar{x})^2$= 1.9084		$(\sum a_i \times x_i)^2$ =1,7982

Step-3:
We are calculate Shapiro-Wilk Test and we are looking at the table value from the Shapiro-Wilk Test table.

$$W_{s-w} = \frac{1.7982}{1.8361} = 0.979 \qquad W_{Table} = 0.89$$

We compare the two resulting values.

$$W_{S-W} > W_{Table}$$

H_0 is not rejected so we can say that "The distribution is normal" at the α = 0.05 significance level.

The output of the results calculated with SPSS:

Tests of Normality

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Type	Statistic	df	Sig.	Statistic	df	Sig.
Rubber	Smooth Rubber	,143	6	,200*	,989	6	,987
	Pips Out Rubber	,153	6	,200*	,957	6	,794
	Anti Spin Rubber	,203	6	,200*	,920	6	,505

Levene's Test for Homogeneity of the Variance

After testing that the observation values are normal, we are analyzed the homogeneity of the variance test analysis with the Levene's test.

Levene's Test Formula:

$$\mathbf{W} = \frac{(N-k)\sum_{j=1}^{k} n_j(\bar{z}_j - \bar{z})^2}{(k-1)\sum_{j=1}^{n}\sum_{j=1}^{k}(z_{ij} - \bar{z}_j)^2}$$

Step-1:
We establish the hypothesis for normality.

H_0: *The variances are homogeneous.*

H_1: *The variances are not homogeneous.*

Step-2:
We are calculate $z_{.j}$, $\bar{z}_{.j}$ and z_{ij} . So we know $z_{ij} = |Y_{ij} - \bar{Y}_j|$. Then this value are included in Table 1.3.

Tablo 1.3.

S1	$Z_{.1}$	S2	$Z_{.2}$	S3	$Z_{.3}$
2.3	0.192	1.5	0.016	2.3	0.434
2.2	0.092	1.6	0.084	2.00	0.134
2.15	0.042	1.1	0.416	1.5	0.366
2.1	0.008	1.8	0.284	2.1	0.234
1.9	0.208	1.7	0.184	1.8	0.066
2.0	0.108	1.4	0.116	1.5	0.366
$T_{.1}$= 12.65	$Z_{.1}$= 0.65	$T_{.2}$= 9.1	$Z_{.2}$= 1.1	$T_{.3}$= 11.2	$Z_{.3}$= 1.59
$\bar{T}_1$= 2.108	$\bar{Z}_{.1}$= 0.1083	$\bar{T}_2$= 1.516	$\bar{Z}_{.2}$= 0.1833	$\bar{T}_3$= 1.866	$\bar{Z}_{.3}$= 0.265

$T_{..}$ = 32.95 $\bar{T}_{..}$= 1.83

$Z_{..}$ = 3.34 $\bar{Z}_{..}$= 0.1855

Step-3:
We are calculate Levene's Test and we are looking at the table value from the F-Table.

$$W = \frac{(18-3)[6(0.1083-0.1855)^2+6(0.1833-0.1855)^2+ 6(0.265-0.1855)^2]}{(3-1)[(0.192-0.1083)^2+ \ldots\ldots\ldots\ldots\ldots +(0.366-0.265)^2]} = 2.2538$$

W_C =2.2538 $F_{2;15;0.05}$ = 3.68

We compare the two resulting values.

$$W_C < F_{2;15;0.05}$$

H_0 is not rejected so we can say that "The variances are homogeneous." at the α = 0.05 significance level.

The output of the results calculated with SPSS:

Test of Homogeneity of Variances

Rubber

Levene Statistic	df1	df2	Sig.
2,304	2	15	,134

We are calculate based on mean.

ANOVA Table

Since normality and homogeneity assumptions were met, an ANOVA table was created to analyze whether the thickness of 3 different rubber types was the same.

F formula:

$$F = \frac{\frac{SS_{BG}}{df_1}}{\frac{SS_{WG}}{df_2}}$$

Step-1:
We establish the hypothesis for normality.

H_0: $\mu_{S1} = \mu_{S2} = \mu_{S3}$

H_1: $At\ least\ one\ mean\ value\ is\ different\ from\ the\ others.$

Step-2:
We are calculate SS_T , SS_{BG} and SS_ε .Than we create ANOVA Table. This table is Table 1.5.

$$SS_T = \sum_{i=1}^{n}\sum_{j=1}^{k} Y^2{}_{iJ} - \frac{T_{..}{}^2}{N} = (2.3)^2 + \ldots + (1.5)^2 - \frac{(32.95)^2}{18} = 2.0056$$

$$SS_{BW} = \sum_{j=1}^{k} \frac{T_{.j}^2}{n} - \frac{T_{..}^2}{n} = \frac{(12.65)^2+(9.1)^2+(11.2)^2}{6} - \frac{(32.95)^2}{18} = 1.0619$$

$$SS_\varepsilon = SS_T - SS_{BW} = 2.0056 - 1.0619 = 0.9437$$

Step-3:

We are calculate F value and we are looking at the table value from the F-Table.

$F_C = 0.5309/0.0629 = 8.4403$, $F_{2;15;0.05} = 3.68$

$$F_C > F_{2;15;0.05}$$

H_0 is rejected so we can say that "It has been analyzed that the thickness of the three rubber types is not equal."at the $\alpha = 0.05$ significance level.

Table 1.4.

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*
Between Groups	1.0619	2	0.5309	8.4403
Within Groups	0.9437	15	0.0629	
Total	2.0056	17		

The output of the results calculated with SPSS:

ANOVA

Rubber

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1,062	2	,531	8,439	,004
Within Groups	,944	15	,063		
Total	2,006	17			

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Rubber

	(I) Type	(J) Type	Mean Difference (I-J)	Std. Error	Sig.
Tukey HSD	Smooth Rubber	Pips Out Rubber	,59167*	,14482	,003
		Anti Spin Rubber	,24167	,14482	,249
	Pips Out Rubber	Smooth Rubber	-,59167*	,14482	,003
		Anti Spin Rubber	-,35000	,14482	,070
	Anti Spin Rubber	Smooth Rubber	-,24167	,14482	,249
		Pips Out Rubber	,35000	,14482	,070
Scheffe	Smooth Rubber	Pips Out Rubber	,59167*	,14482	,004
		Anti Spin Rubber	,24167	,14482	,279
	Pips Out Rubber	Smooth Rubber	-,59167*	,14482	,004
		Anti Spin Rubber	-,35000	,14482	,085
	Anti Spin Rubber	Smooth Rubber	-,24167	,14482	,279
		Pips Out Rubber	,35000	,14482	,085
LSD	Smooth Rubber	Pips Out Rubber	,59167*	,14482	,001
		Anti Spin Rubber	,24167	,14482	,116
	Pips Out Rubber	Smooth Rubber	-,59167*	,14482	,001
		Anti Spin Rubber	-,35000*	,14482	,029
	Anti Spin Rubber	Smooth Rubber	-,24167	,14482	,116
		Pips Out Rubber	,35000*	,14482	,029

Multiple Comparisons

Dependent Variable: Rubber

			95% Confidence Interval	
	(I) Type	(J) Type	Lower Bound	Upper Bound
Tukey HSD	Smooth Rubber	Pips Out Rubber	,2155	,9678
		Anti Spin Rubber	-,1345	,6178
	Pips Out Rubber	Smooth Rubber	-,9678	-,2155
		Anti Spin Rubber	-,7262	,0262
	Anti Spin Rubber	Smooth Rubber	-,6178	,1345
		Pips Out Rubber	-,0262	,7262
Scheffe	Smooth Rubber	Pips Out Rubber	,1987	,9847
		Anti Spin Rubber	-,1513	,6347
	Pips Out Rubber	Smooth Rubber	-,9847	-,1987
		Anti Spin Rubber	-,7430	,0430
	Anti Spin Rubber	Smooth Rubber	-,6347	,1513
		Pips Out Rubber	-,0430	,7430
LSD	Smooth Rubber	Pips Out Rubber	,2830	,9003
		Anti Spin Rubber	-,0670	,5503
	Pips Out Rubber	Smooth Rubber	-,9003	-,2830
		Anti Spin Rubber	-,6587	-,0413
	Anti Spin Rubber	Smooth Rubber	-,5503	,0670
		Pips Out Rubber	,0413	,6587

*. The mean difference is significant at the 0.05 level.

Pips_out-Smooth: The difference between Pips_out and Smooth is statistically significant (p = 0.0026). On average, there is a difference of approximately 0.59 units between the Pips_out and Smooth groups. Looking at the confidence interval, it is seen that this difference is statistically significant (confidence interval between -0.97 and -0.22).

Anti_spin-Smooth: The difference between Anti_spin and Smooth is not statistically significant (p = 0.24). That is, there is no statistically significant difference between the Anti_spin and Smooth groups (confidence interval between -0.62 and 0.13).

Anti_spin-Pips_out: The difference between Anti_spin and Pips_out is not statistically significant, but since the p-value is 0.07, a trend can be observed at a certain level of significance for this comparison. That is, there may not be a significant difference between the Anti_spin and Pips_out groups, but in some cases they may differ (confidence interval between -0.03 and 0.73).

Based on these results, we can say that there is a statistically significant difference between Pips_out and Smooth.

However, there does not appear to be a clear difference between Anti_spin and other groups

Homogeneous Subsets

Rubber

	Type	N	Subset for alpha = 0.05	
			1	2
Tukey HSD[a]	Pips Out Rubber	6	1,5167	
	Anti Spin Rubber	6	1,8667	1,8667
	Smooth Rubber	6		2,1083
	Sig.		,070	,249
Duncan[a]	Pips Out Rubber	6	1,5167	
	Anti Spin Rubber	6		1,8667
	Smooth Rubber	6		2,1083
	Sig.		1,000	,116
Scheffe[a]	Pips Out Rubber	6	1,5167	
	Anti Spin Rubber	6	1,8667	1,8667
	Smooth Rubber	6		2,1083
	Sig.		,085	,279

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 6,000.

5. REFERENCES

1. https://tenismesa.es/gb/content/rubbers-thickness.html
2. https://www.megaspin.net/store/extra/rubber-guide.asp
3. https://pingsunday.com/effect-of-sponge-thickness-in-table-tennis/
4. https://www.masatenisi.com/lastikler

   (The websites where I obtained information to learn about the thicknesses of table tennis rubbers according to their types and to compile the data.)

5. Oral Erbas S, Olmus H. , Nazman E. Araştırmacılar İçin SPSS Uygulamalı İstatistiksel Deney Tasarımı