
APPENDIX A - FORMULAS FOR SPC CALCULATIONS

INTRODUCTION

This appendix outlines the formulas used by SPC calculations.

The Sections in the Appendix describe:

- **SHEWHART CALCULATIONS**
- **CUSUM CALCULATIONS**
- **EWMA CALCULATIONS**

SHEWHART CALCULATIONS

This Section describes the calculations used for:

- **Mean (XBar) Charts**
- **Individual Sample (X) Chart**
- **Median (Me) Chart**
- **Range (R) Chart**
- **Standard Deviation (S) Chart**
- **Moving Range (mR) Chart**
- **Mid-Range for Median (Mr) Chart**
- **Range For Median (R) Chart**
- **Capability Calculations**

Variables Used

The following variables are used in the calculations:

N	Number of Samples
n	Subgroup Size
k	Number of Subgroups in the Sample Set
CL	Center Line
LCL	Lower Control Limit
UCL	Upper Control Limit
\bar{X}_0 or \overline{Me}_0 or \overline{Mr}_0	Process Standard
σ_0	Process Tolerance

NOTE: Subgroups are independent, not overlapping (i.e., $N = nk$).

Mean (XBar) Charts

Trace The values plotted are the mean of each subgroup:

$$\bar{X} = \sum_{i=1}^n \frac{X_i}{n}$$

Limits (Based on Data) The center line is the mean of all subgroup means, and the control limits are based on the mean of all subgroup ranges:

$$CL = \bar{\bar{X}} = \sum_{i=1}^k \frac{\bar{X}_i}{k}$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

(See **Range (R) Chart** for \bar{R} and Table A-1 for A_2 .)

Limits (Based on Standards) The center line is the process standard entered, and the control limits are based on the process tolerance entered:

$$CL = \bar{X}_0$$

$$UCL = \bar{X}_0 + A\sigma_0$$

$$LCL = \bar{X}_0 - A\sigma_0$$

(See Table A-1 for A .)

Individual Sample (X) Chart

Trace The values plotted are the individual sample points:

$$X$$

Limits (Based on Data) The center line is the mean of all samples, and the control limits are based on the moving range:

$$CL = \bar{X} = \sum_{i=1}^N \frac{X_i}{N}$$

$$UCL = \bar{X} + E_2 \bar{mR}$$

$$LCL = \bar{X} - E_2 \bar{mR}$$

where: $E_2 = 2.66 = \frac{3}{d_2}$, $d_2 = 1.128$

(See Table A-1 for d_2 .)

(See **Moving Range (mR) Chart** for \overline{mR} .)

**Limits
(Based on Standards)**

The center line is the process standard entered, and the control limits are based on the process tolerance entered:

$$CL = \bar{X}_0$$

$$UCL = \bar{X}_0 + 3\sigma_0$$

$$LCL = \bar{X}_0 - 3\sigma_0$$

Median (Me) Chart

Trace

The values plotted are the median of each subgroup:

$$Me = \text{median}(X_1 \dots X_n)$$

**Limits
(Based on Data)**

The center line is the median value of all subgroup medians, and the control limits are based on the median of all subgroup ranges:

$$CL = \overline{Me} = \text{median}(Me_1 \dots Me_k)$$

$$UCL = \overline{Me} + A_4 \tilde{R}$$

$$LCL = \overline{Me} - A_4 \tilde{R}$$

(See **Range For Median (R) Chart** for \tilde{R} and Table A-2 for A_4 .)

**Limits
(Based on Standards)**

The center line \overline{Me}_0 is the process standard entered, and the control limits are based on the process tolerance σ_0 entered:

$$CL = \overline{Me}_0$$

$$UCL = \overline{Me}_0 + \tilde{A}\sigma_0$$

$$LCL = \overline{Me}_0 - \tilde{A}\sigma_0$$

(See Table A-2 for \tilde{A} .)

Range (R) Chart

Trace

The values plotted are the range of each subgroup:

$$R = \max(X_1 \dots X_n) - \min(X_1 \dots X_n)$$

**Limits
(Based on Data)**

The center line is the mean of all subgroup ranges, and control limits are based on this mean:

$$CL = \bar{R} = \sum_{i=1}^k \frac{R_i}{k}$$

$$UCL = D_4 \bar{R}$$

$$LCL = D_3 \bar{R}$$

(See Table A-1 for D_3 and D_4 .)

**Limits
(Based on Standards)**

The center line and control limits are based on the process tolerance σ_0 entered:

$$CL = d_2 \sigma_0$$

$$UCL = D_2 \sigma_0$$

$$LCL = D_1 \sigma_0$$

(See Table A-1 for d_2 , D_1 , and D_2 .)

Standard Deviation (S) Chart

Trace The values plotted are the standard deviation of each subgroup:

$$S = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{n-1}$$

**Limits
(Based on Data)**

The center line is the mean of all subgroup standard S_i deviations, and the control limits are based on this mean:

$$CL = \bar{S} = \sum_{i=1}^k \frac{S_i}{k}$$

$$UCL = B_4 \bar{S}$$

$$LCL = B_3 \bar{S}$$

(See Table A-1 for B_3 and B_4 .)

**Limits
(Based on Standards)**

The center line and control limits are based on the process tolerance σ_0 entered:

$$CL = c_4 \sigma_0$$

$$UCL = B_6\sigma_0$$

$$LCL = B_5\sigma_0$$

(See Table A-1 for c_4 , B_5 and B_6 .)

Moving Range (mR) Chart

Trace The values plotted are the absolute difference between the current sample and the previous sample

$$mR = |X_i - X_{i-1}|$$

**Limits
(Based on Data)** The center line the mean of the moving ranges between all subgroups, and the control limits are based on this mean:

$$CL = \overline{mR} = \sum_{i=1}^{N-1} \frac{mR_i}{N-1}$$

$$UCL = D_4\overline{mR}$$

$$LCL = D_3\overline{mR}$$

Where: $D_4 = 3.267$, $D_3 = 0$

(See Table A-1 for D_4 and D_3 .)

**Limits
(Based on Standards)** The center line and control limits are based on the process tolerance σ_0 entered:

$$CL = d_2\sigma_0$$

$$UCL = D_2\sigma_0$$

$$LCL = D_1\sigma_0$$

Where: $d_2 = 1.128$, $D_1 = 0$, $D_2 = 3.686$

(See Table A-1 for d_2 , D_2 , and D_1 .)

Mid-Range for Median (Mr) Chart

Trace The values plotted are the mid-ranges of each subgroup:

$$Mr = \frac{R}{2} = \frac{\max(X_1 \dots X_n) - \min(X_1 \dots X_n)}{2}$$

**Limits
(Based on Data)** The center line is the median of all subgroup mid-ranges, and the control limits are based on the median of all subgroup ranges:

$$CL = \overline{Mr} = \text{median}(Mr_1 \dots Mr_k)$$

$$UCL = \overline{Mr} + A_5 \tilde{R}$$

$$LCL = \overline{Mr} - A_5 \tilde{R}$$

(See **Range For Median (R) Chart** for \tilde{R} and Table A-2 for A_5 .)

**Limits
(Based on Standards)**

The center line is the process standard entered, \overline{Mr}_0 , and the control limits are based on the process tolerance σ_0 entered:

$$CL = \overline{Mr}_0$$

$$UCL = \overline{Mr}_0 + \tilde{A}' \sigma_0$$

$$LCL = \overline{Mr}_0 - \tilde{A}' \sigma_0$$

(See Table A-2 for \tilde{A}' .)

Range For Median (R) Chart

Trace

The values plotted are the ranges of each subgroup:

$$R = \max(X_1 \dots X_n) - \min(X_1 \dots X_n)$$

**Limits
(Based on Data)**

The center line is the median of all subgroup ranges, and the control limits are based on this mean:

$$CL = \tilde{R} = \text{median}(R_1 \dots R_k)$$

$$UCL = D_6 \tilde{R}$$

$$LCL = D_5 \tilde{R}$$

(See Table A-2 for D_5 and D_6 .)

**Limits
(Based on Standards)**

The center line and control limits are based on the process tolerance σ_0 entered:

$$CL = d_4 \sigma_0$$

$$UCL = D_6 d_4 \sigma_0$$

$$LCL = D_5 d_4 \sigma_0$$

(See Table A-2 for D_5 and D_6 .)

Capability Calculations

**Control Band
(CB)**

$$CB = UCL - LCL$$

**Tolerance Band
(TB)**

$$TB = 2\sigma_0 = (\bar{X}_0 + \sigma_0) - (\bar{X}_0 - \sigma_0) = UpperSpec - LowerSpec$$

n = subgroup size

**Process Capability
Index (Cp or Cpk)**

$$Cp = \frac{TB}{\sqrt{n}CB} = \frac{TB}{ProcessCapability}$$

$$Cpk = \min\left(\frac{UpperSpec - \bar{X}}{3S}, \frac{\bar{X} - LowerSpec}{3S}\right)$$

Table A-1. Factors for Computation of Shewhart Charts

n	d ₂	d ₃	A ¹	A ₂ ²	D ₁ ³	D ₂ ⁴	D ₃ ⁵	D ₄ ⁶	c ₄ ⁷	B ₃ ⁸	B ₄ ⁹	B ₅ ¹⁰	B ₆ ¹¹
2	1.128	0.853	2.121	1.880	0.000	3.686	0.000	3.267	0.7979	0.000	3.267	0.000	2.606
3	1.693	0.888	1.732	1.023	0.000	4.358	0.000	2.574	0.8862	0.000	2.568	0.000	2.276
4	2.509	0.880	1.500	0.729	0.000	4.698	0.000	2.282	0.9213	0.000	2.266	0.000	2.088
5	2.356	0.864	1.342	0.577	0.000	4.918	0.000	2.114	0.9400	0.000	2.089	0.000	1.964
6	2.534	0.848	1.225	0.483	0.000	5.078	0.000	2.004	0.9515	0.030	1.970	0.029	1.874
7	2.704	0.833	1.134	0.419	0.204	5.204	0.076	1.924	0.9594	0.118	1.882	0.113	1.806
8	2.847	0.820	1.061	0.373	0.388	5.306	0.136	1.864	0.9650	0.185	1.815	0.179	1.751
9	2.970	0.808	1.000	0.337	0.547	5.393	0.184	1.816	0.9693	0.239	1.761	0.232	1.707
10	3.078	0.797	0.949	0.308	0.687	5.469	0.223	1.777	0.9727	0.284	1.716	0.276	1.669
11	3.173	0.787	0.905	0.285	0.811	5.535	0.256	1.744	0.9754	0.321	1.679	0.313	1.637
12	3.258	0.778	0.866	0.266	0.922	5.594	0.283	1.717	0.9776	0.354	1.646	0.346	1.610
13	3.336	0.770	0.832	0.249	1.025	5.647	0.307	1.693	0.9794	0.382	1.618	0.374	1.585
14	3.407	0.763	0.802	0.235	1.118	5.696	0.328	1.672	0.9810	0.406	1.594	0.399	1.563
15	3.472	0.756	0.775	0.223	1.203	5.741	0.347	1.653	0.9823	0.428	1.572	0.421	1.544
16	3.532	0.750	0.705	0.212	1.282	5.782	0.363	1.637	0.9835	0.448	1.552	0.440	1.526
17	3.588	0.744	0.728	0.203	1.356	5.820	0.378	1.622	0.9845	0.466	1.534	0.458	1.511
18	3.640	0.739	0.707	0.194	1.424	5.856	0.391	1.608	0.9854	0.482	1.518	0.475	1.496
19	3.689	0.734	0.688	0.187	1.487	5.891	0.403	1.597	0.9862	0.497	1.503	0.490	1.483
20	3.735	0.729	0.671	0.180	1.549	5.921	0.415	1.585	0.9869	0.510	1.490	0.504	1.470
21	3.778	0.724	0.655	0.173	1.605	5.951	0.425	1.575	0.9876	0.523	1.477	0.516	1.459
22	3.819	0.720	0.640	0.167	1.659	5.979	0.434	1.566	0.9882	0.534	1.466	0.528	1.448
23	3.858	0.716	0.626	0.162	1.710	6.006	0.443	1.557	0.9887	0.545	1.455	0.539	1.439
24	3.895	0.712	0.612	0.157	1.759	6.031	0.451	1.548	0.9892	0.555	1.445	0.549	1.429
25	3.931	0.708	0.600	0.135	1.806	6.056	0.459	1.541	0.9896	0.565	1.435	0.559	1.420

NOTE: These formulas represent "3 sigma control".
For "2 sigma control", replace "3" with "2".

1. $A = 3/\sqrt{n}$
2. $A_2 = 3/d_2\sqrt{n}$
3. $D_1 = d_2 - 3d_3$
4. $D_2 = d_2 + 3d_3$

5. $D_3 = 1 - 3d_3/d_2$
6. $D_4 = 1 + 3d_3/d_2$
7. $c_4 = 4(n-1)/(4n-3)$
8. $B_3 = 1 - 3/(c_4\sqrt{2(n-1)})$
9. $B_4 = 1 + 3/(c_4\sqrt{2(n-1)})$
10. $B_5 = c_4 - 3/(\sqrt{2(n-1)})$
11. $B_6 = c_4 + 3/(\sqrt{2(n-1)})$

Table A-2. Factors for Median Charts and Mid-Range Charts

n	\bar{A}	\bar{A}'	A_4	A_5	D_5	D_6	d_4
2	2.121	2.121	2.224	2.224	0.000	3.865	0.954
3	2.014	1.806	1.265	1.137	0.000	2.745	1.588
4	1.637	1.637	0.829	0.829	0.000	2.375	1.978
5	1.615	1.532	0.712	0.679	0.000	2.179	2.257
6	1.387	1.458	0.562	0.590	0.000	2.055	2.472
7	1.385	1.402	0.520	0.530	0.078	1.967	2.645
8	1.233	1.358	0.441	0.486	0.139	1.901	2.791
9	1.240	1.322	0.419	0.453	0.187	1.850	2.916
10	1.260	1.293	0.369	0.427	0.227	1.809	3.024

CUSUM CALCULATIONS

Variables Used The following variables are used in the calculations:

- N number of observations
- T target value
- D critical shift
- α risk factor

CUSUM Chart

Trace The cumulative sum of deviations from a target value is plotted:

$$s_t = \sum_{i=0}^t \left(X_i - \left(T \pm \frac{D}{2} \right) \right)$$

**Limit
(Critical Boundary)**

The critical boundary is based on the standard deviation of the samples and the critical shift and risk factor entered:

$$h = \left(\frac{S^2}{\pm D} \right) 1 n \left(\frac{1}{\alpha} \right)$$

$$\text{where } S = \sqrt{\sum_{i=1}^N \frac{(X_i - \bar{X})^2}{N-1}}$$

If you set the critical shift equal to the standard deviation, the formula simplifies to:

$$h = \pm D 1 n \left(\frac{1}{\alpha} \right)$$

EWMA CALCULATIONS

Variables Used

The following variables are used in the calculations:

N	number of samples
T	target value
λ	weighting factor
S	standard deviation
CL	center line
UCL	upper control limit
LCL	lower control limit

EWMA Chart

Trace

The predicted next value is plotted:

$$\hat{X}_{t+1} = \hat{X}_t + \lambda (X_t - \hat{X}_t)$$

Limits

The center line is the target value and the control limits are based on the standard deviation and weight factor:

$$CL = T$$

$$UCL = T + 3S \sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = T - 3S \sqrt{\frac{\lambda}{2-\lambda}}$$

$$\text{where: } S = \sqrt{\sum_{t=1}^N \frac{(X_t - \bar{X})^2}{N-1}}$$

