0.1 Evaluation metrics

Let, $\mathbf{b}_g^G$ is the binary hash code of an arbitrary image $\mathbf{x}_g^G \in \mathbf{X}_G$ and $|\mathbf{X}_G| = U_2$. $d_H(\mathbf{b}_g^G, \mathbf{b}_q^Q)$ is the HD between hash pair $\mathbf{b}_g^G$ and $\mathbf{b}_q^Q$, where $\mathbf{b}_q^Q$ is the hash code of a query image $\mathbf{x}_q^Q \in \mathbf{X}_Q$. The relationship between cosine similarity and normalized Euclidean distance between two binary hash codes $\mathbf{b}_g^G$ and $\mathbf{b}_q^Q$ of length K can be expressed as follows:

$$\begin{aligned} d_H(\mathbf{b}_g^G, \mathbf{b}_q^Q) &= \frac{K}{4} \left\| \frac{\mathbf{b}_g^G}{\|\mathbf{b}_g^G\|_2} - \frac{\mathbf{b}_q^Q}{\|\mathbf{b}_q^Q\|_2} \right\|_2^2 \\ &= \frac{K}{2} \left(1 - cos(\mathbf{b}_g^G, \mathbf{b}_q^Q)\right) \end{aligned} \tag{1}$$

Retrieval performance is evaluated using two metrics: mAP and nDCG (**?**). The quality of retrieval is measured using mAP, while nDCG assesses the rank quality of the retrieved images across all query images. We discuss the computation process for both these metrics as below in the following subsections.

0.1.1 Mean average precision

$mAP@p$ represents the mAP for the top-p retrieved images from $\mathbf{X}_G$. It is calculated by finding the average precision ($AP_q@p$) for each $\mathbf{x}_q^Q \in \mathbf{X}_Q$ based on the top-p retrieved images from $\mathbf{X}_G$. Let, $\mathbf{x}_r^G$ is the r-th ranked image from top-p retrieve images. Then $AP_q@p$ is defined as,

$$AP_q@p = \frac{\sum_{r=1}^{p} P_q(r) R_q^{mAP}(r)}{\sum_{r=1}^{p} R_r^{mAP}(r)} \tag{2}$$

where, $P_q(r)$ denotes the precision for the top-r retrieval of query image $\mathbf{x}_q^Q$ and is defined by,

$$P_q(r) = \frac{\sum_{r=1}^{r} R_q^{mAP}(r)}{r} \tag{3}$$

$$R_q^{mAP}(r) = \begin{cases} 1, & \text{if } \mathbf{y}_r^G = \mathbf{y}_q^Q \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Finally,

$$mAP@p = \frac{1}{U_3} \sum_{\mathbf{x}_q^Q \in \mathbf{X}_Q} AP_q@p \tag{5}$$

0.1.2 Normalized discounted cumulative gain

In order to compute $nDCG@p$ for top-p retrieval, first we need to calculate $DCG@p$ for top-p retrieval. The mathematical formulation for $DCG_q@p$ of query image $\mathbf{x}_q^Q$ is given by,

$$DCG_q@p = \sum_{r=1}^{p} \frac{2^{R_q^{nDCG}(r)} - 1}{log_2(r+1)} \tag{6}$$

We normalize this by dividing it with the maximally achievable value or Ideal DCG (iDCG). Finally to obtain,

$$nDCG_q@p = \frac{DCG_q@p}{iDCG_q@p} \tag{7}$$

where $iDCG_q@p = DCG_q@p$ of ideal ranking or best possible ranking. The relevance score for image $\mathbf{x}_q^Q$ is defined by,

$$R_q^{nDCG}(r) = V \tag{8}$$

where $V \in \{0, 1, 2, 3\}$ represents the number of characteristics of $\mathbf{x}_q^Q$ and $\mathbf{x}_r^G$ are matched.

Finally,

$$nDCG@p = \frac{1}{U_3} \sum_{\mathbf{x}_q^Q \in \mathbf{X}_Q} nDCG_q@p \tag{9}$$