question:

Possibility of implementing swap gate using single qubit gates only:-

→ claim:
We will prove, we cannot implement a swap gate using two single input qubit gates.

~~claim~~ If possible let, ~~we can~~ we can.

Then, the matrix of swap gate is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = A$$

let, The two single input qubit gates are

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \& \quad \begin{pmatrix} p & q \\ r & s \end{pmatrix}.$$

Then

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} p & q \\ r & s \end{pmatrix} = A$$

$$\Rightarrow \begin{pmatrix} ap & aq & bp & bq \\ ar & as & br & bs \\ cp & cq & dp & dq \\ cr & cs & dr & ds \end{pmatrix} = A$$

$$\Rightarrow \quad ap = br = cq = ds = 1$$

and, ~~other~~ other entries are zero.

Take one $aq = 0$

i.e. either $a = 0$ or $q = 0$

But $ap = 1 \ \& \ cq = 1$ implies $a \neq 0, q \neq 0$.

so, contradiction

we are done