④ Proof of No-cloning theorem. Asim Manna (CMS1905)

In quantum mechanical device, it is impossible to create an independent and identical quantum state of an arbitrary unknown quantum state.

⇒ If possible let, there exists a operator U (unitary) which can be create an identical quantum state and ~~work for~~ and U i.e for unknown state $|\psi\rangle$,

$$U(|\psi\rangle|s\rangle) = |\psi\rangle|\psi\rangle$$

where $|s\rangle$ is pure state.

Let, $|\varphi_1\rangle$ and $|\varphi_2\rangle$ be two unknown quantum state,

then $U(|\varphi_1\rangle|s\rangle) = |\varphi_1\rangle|\varphi_1\rangle$ — ①

and $U(|\varphi_2\rangle|s\rangle) = |\varphi_2\rangle|\varphi_2\rangle$ — ②

Since, Now $\langle\varphi_1|\varphi_2\rangle^2 = \langle\varphi_1|\varphi_2\rangle\langle\varphi_1|\varphi_2\rangle$

$= \langle\varphi_1|\langle\varphi_1|\varphi_2\rangle|\varphi_2\rangle$,

$= (U(|\varphi_1\rangle|s\rangle))^{\dagger}\, U(|\varphi_2\rangle|s\rangle)$

$= \langle s|\langle\varphi_1|U^{\dagger}U|\varphi_2\rangle|s\rangle$

$= \langle s|\langle\varphi_1||\varphi_2\rangle|s\rangle$

$= \langle\varphi_1||\varphi_2\rangle\langle s||s\rangle$

$= \langle\varphi_1||\varphi_2\rangle\cdot 1$ since

$= \langle\varphi_1||\varphi_2\rangle$, This is real no.

So, if, $\langle\varphi_1||\varphi_2\rangle = 0$, then $|\varphi_1\rangle = |\varphi_2\rangle$

if $|\varphi_1|\varphi_2\rangle = 1$ then $|\varphi_1\rangle$ & $|\varphi_2\rangle$ are orthogonal to each other

It is contradiction,

So we are done