Question: Assignment (ii) Date - 17.09.2020 (CRS1908)

Possibility of implementing swap gate using single qubit gates only:-

⇒ claim:
We will prove we cannot implement a swap gate using two single input qubit gates.

~~claim~~ If possible let, ~~we can~~ we can.

Then the matrix of swap gate is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = A$$

Let the two single input qubit gates are

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ & } \begin{pmatrix} p & q \\ r & s \end{pmatrix}.$$

Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} p & q \\ r & s \end{pmatrix} = A$

$$\Rightarrow \begin{pmatrix} ap & aq & bp & bq \\ ar & as & br & bs \\ cp & cq & dp & dq \\ cr & cs & dr & ds \end{pmatrix} = A$$

$\Rightarrow ap = br = cq = ds = 1$

and, ~~all~~ other entries are zero.

Take one $aq = 0$

i.e either $a = 0$ or $q = 0$

But $ap = 1$ & $cq = 1$ implies $a \neq 0$, $q \neq 0$.

So contradiction

We are done

(*) Assignment-II

(*) Given a 2-input AND gate in classical computer, given a circuit diagram in terms of quantum gates.

⇒ We will do using Toffoli-Gate.

The truth table is

a	b	c	a_1	b_1	$c_1 = c \oplus ab$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

If we ~~put~~ choose $c = 0$, then we can get AND-gate

If we ~~put~~ choose, $c = 1$ (part) then we get

$c_1 = 1 \oplus ab = \bar{ab}$, which is NAND gate

~~For OR-gate we will do.~~

Similarly we can construct OR-gate & XOR gate