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Problem 1. (6 points) Find the equation of the plane normal to the vector **N** = [-3 6 2]^T and passing through the point $P_0 = (4, 2, 5)$

The plane is the set of all points $P(x, y, z)$ such as the vector $\overline{P_oP}$ from P_o to P is orthogonal to the vector $\vec{N}$:

$$\langle \overrightarrow{P_oP}, \vec{N} \rangle = 0 \qquad \overrightarrow{P_oP} = \begin{bmatrix} x - 4 \\ y - 2 \\ z - 5 \end{bmatrix}$$

$$\langle \overrightarrow{P_oP}, \vec{N} \rangle = \begin{bmatrix} x - 4 & y - 2 & z - 5 \end{bmatrix} \begin{bmatrix} -3 \\ 6 \\ 2 \end{bmatrix} = 0$$

$$= -3(x-4) + 6(y-2) + 2(z-5) = 0$$

$$= -3x + 6y + 2z - 10 = 0$$

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Problem 2. (6 points) Let

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 4 \end{pmatrix}$$

Find the base for the null space of A, N(A).

Transform A to reduced row echelon form:

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} x_1 + x_3 = 0 \\ x_2 + x_3 = 0 \end{matrix}$$

OR

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} x_1 + x_2 + 2x_3 = 0 \\ x_2 + x_3 = 0 \end{matrix}$$

Regardless of which set of equations we use,

$$x_2 = -x_3 \quad , x_1 = -x_3 \quad , x_1 = x_2$$

Let $x_3 = \alpha \rightarrow N(A) = \alpha \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

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Problem 3. (7 points) Determine a least squares solution to the following system:

$$\begin{aligned} -x_1 + x_2 &= 10 \\ 2x_1 + x_2 &= 5 \\ x_1 - 2x_2 &= 20 \end{aligned}$$

$$\begin{bmatrix} -1 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -25 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & -1 & 20 \\ -1 & 6 & -25 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -6 & 25 \\ 6 & -1 & 20 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -6 & 25 \\ 0 & -35 & 130 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -6 & 25 \\ 0 & -7 & 26 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -6 & 25 \\ 0 & 1 & -26/7 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 19/7 \\ 0 & 1 & -26/7 \end{array}\right] \to \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 19/7 \\ -26/7 \end{bmatrix}$$

OR

$$\left.\begin{aligned} 6x_1 - x_2 &= 20 \\ x_1 - 6x_2 &= 25 \end{aligned}\right\} \text{ solve simultaneously to get}$$

$$x_1 = \frac{19}{7}, \quad x_2 = -\frac{26}{7}$$

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Problem 4. (6 points) Let $\mathbf{x} = [1\ 1\ 1\ 1]^T$ and $\mathbf{y} = [8\ 2\ 2\ 0]^T$. Find the vector projection of **x** onto **y**.

$$\alpha = \frac{\langle \vec{x}, \vec{y} \rangle}{\|\vec{y}\|} = \frac{[1\ 1\ 1\ 1]\begin{bmatrix}8\\2\\2\\0\end{bmatrix}}{\left\{[8\ 2\ 2\ 0]\begin{bmatrix}8\\2\\2\\0\end{bmatrix}\right\}^{1/2}} = \frac{8+2+2+0}{[8^2+2^2+2^2+0^2]^{1/2}}$$

$$\alpha = \frac{12}{\sqrt{72}} = \frac{12}{6\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \quad \text{scalar projection}$$

$$\vec{P} = \alpha \frac{\vec{y}}{\|\vec{y}\|} = \sqrt{2}\,\frac{1}{6\sqrt{2}}\begin{bmatrix}8\\2\\2\\0\end{bmatrix} = \frac{1}{3}\begin{bmatrix}4\\1\\1\\0\end{bmatrix} \quad \text{vector projection}$$