

# Implementation of the Rhee-Chow Interpolation

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In the projection algorithm, the pressure correction is found by solving the Poisson equation

$$\delta t \sum_{f=1}^{N_{kf}} \left( \frac{\partial p'}{\partial x_i} \right)^f n_i^f |\partial\Omega_k^f| = \sum_{f=1}^{N_{kf}} \tilde{\phi}_{fp}^{n+1}, \quad (1)$$

where  $p' = p^{n+1} - p^n$ , and  $\tilde{\phi}_{fp}^{n+1}$  is the mass flux based on the intermediate velocity and Rhee-Chow smoothing

$$\tilde{\phi}_{fp}^{n+1} = \tilde{\phi}_f^{n+1} + \left[ \overline{\frac{\delta p^n}{\delta x_i}} - \left( \frac{\delta p^n}{\delta x_i} \right)^f \right] n_i^f |\partial\Omega_k^f| \delta t. \quad (2)$$

Subsequently, the velocity components and fluxes are updated separately,

$$u_i^{n+1} = \tilde{u}_i^{n+1} - \delta t \frac{\delta p'}{\delta x_i}, \quad (3)$$

$$\phi_f^{n+1} = \tilde{\phi}_{fp}^{n+1} - \left( \frac{\delta p'}{\delta x_i} \right)^f n_i^f |\partial\Omega_k^f| \delta t. \quad (4)$$

To implement this algorithm, we first update the intermediate velocity field from the momentum solutions as

$$u_i^* = \tilde{u}_i^{n+1} + \delta t \frac{\delta(p^n)}{\delta x_i}, \quad (5)$$

which gives the flux

$$\phi_f^* = \tilde{\phi}_f^{n+1} + \delta t \overline{\frac{\delta p^n}{\delta x_i}} n_i^f |\partial\Omega_k^f|. \quad (6)$$

Substituting Eq. (6) to Eq. (2), we get

$$\tilde{\phi}_{fp}^{n+1} = \phi_f^* - \left( \frac{\delta p^n}{\delta x_i} \right)^f n_i^f |\partial\Omega_k^f| \delta t. \quad (7)$$

Introducing Eq. (7) to Eq. (1) and moving pressure gradient to the right hand side,

$$\delta t \sum_{f=1}^{N_{kf}} \left( \frac{\partial p^{n+1}}{\partial x_i} \right)^f n_i^f |\partial\Omega_k^f| = \sum_{f=1}^{N_{kf}} \phi_f^*, \quad (8)$$

We solve this Poisson equation to obtain  $p^{n+1}$ . To make the velocity updates, correction steps in Eqs. (3) and (4) are reformulated using the auxiliary flux and velocity terms in Eqs. (5) and (6),

$$u_i^{n+1} = u_i^* - \delta t \frac{\delta p^{n+1}}{\delta x_i}, \quad (9)$$

$$\phi_f^{n+1} = \phi_f^* - \left( \frac{\delta p^{n+1}}{\delta x_i} \right)^f n_i^f |\partial\Omega_k^f| \delta t. \quad (10)$$