Implementation of the Rhie-Chow Interpolation

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In the projection algorithm, the pressure correction is found by solving the Poisson equation

$$\delta t \sum_{f=1}^{N_{kf}} \left(\frac{\partial p'}{\partial x_i} \right)^f n_i^f |\partial \Omega_k^f| = \sum_{f=1}^{N_{kf}} \widetilde{\phi}_{fp}^{n+1}, \tag{1}$$

where $p' = p^{n+1} - p^n$, and $\widetilde{\phi}_{fp}^{n+1}$ is the mass flux based on the intermediate velocity and Rhie-Chow smoothing

$$\tilde{\phi}_{fp}^{n+1} = \tilde{\phi}_f^{n+1} + \left[\overline{\frac{\delta p^n}{\delta x_i}} - \left(\frac{\delta p^n}{\delta x_i} \right)^f \right] n_i^f |\partial \Omega_k^f| \delta t. \tag{2}$$

Subsequently, the velocity components and fluxes are updated separately,

$$u_i^{n+1} = \tilde{u}_i^{n+1} - \delta t \frac{\delta p'}{\delta x_i}, \tag{3}$$

$$\phi_f^{n+1} = \widetilde{\phi}_{fp}^{n+1} - \left(\frac{\delta p'}{\delta x_i} \right)^f n_i^f |\partial \Omega_k^f| \delta t. \tag{4}$$

To implement this algorithm, we first update the intermediate velocity field from the momentum solutions by

$$u_i^* = \widetilde{u}_i^{n+1} + \delta t \frac{\delta (p^n)}{\delta x_i}, \tag{5}$$

which gives the flux

$$\phi_f^* = \widetilde{\phi}_f^{n+1} + \delta t \overline{\frac{\delta p^n}{\delta x_i}} n_i^f |\partial \Omega_k^f|. \tag{6}$$

Substituting Eq. (6) to Eq. (2), we get

$$\tilde{\phi}_{fp}^{n+1} = \phi_f^* - \left(\frac{\delta p^n}{\delta x_i} \right)^f n_i^f |\partial \Omega_k^f| \delta t. \tag{7}$$

Introducing Eq. (7) to Eq. (1) and moving pressure gradient to left hand side,

$$\delta t \sum_{f=1}^{N_{kf}} \left(\frac{\partial p^{n+1}}{\partial x_i} \right)^f n_i^f |\partial \Omega_k^f| = \sum_{f=1}^{N_{kf}} \phi_f^*, \tag{8}$$

We solve this Poisson equation to obtain p^{n+1}. To make the velocity updates, correction steps in Eqs. (3) and (4) are reformulated using the auxiliary flux and velocity terms in Eqs. (5) and (6),

$$u_i^{n+1} = u_i^* - \delta t \frac{\delta p^{n+1}}{\delta x_i}, \tag{9}$$

$$\phi_f^{n+1} = \phi_f^* - \left(\frac{\delta p^{n+1}}{\delta x_i} \right)^f n_i^f |\partial \Omega_k^f| \delta t. \tag{10}$$