Textbook of Mathematics

6

National Book Foundation
as
Federal Textbook Board, Islamabad

Textbook of

MATHEMATICS

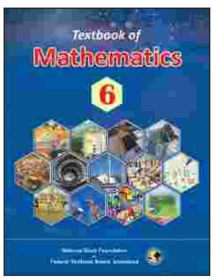
Grade 6

National Book Foundation
as
Federal Textbook Board
Islamabad

OUR MOTTO

- Standards
- Outcomes
- Access
- Style

Textbook of
Mathematics Grade - 6



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Preface

This textbook for Mathematics grade VI has been developed according to the Revised Curriculum 2017. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners. The main objective of this book is to develop higher order thinking processes by systematically building upon the foundation of learning from the previous grades. A key emphasis of the present textbook is on creating real life linkages of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they go up the learning curve and for them to fully grasp the conceptual basis that will be built upon in subsequent grades.

An amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its books. The present book features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement and the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this book.

May Allah guides and helps us (Ameen).

National Book Foundation

بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

شروع اللہ کے نام سے جو بڑا مہربان، نہایت رحم والا ہے

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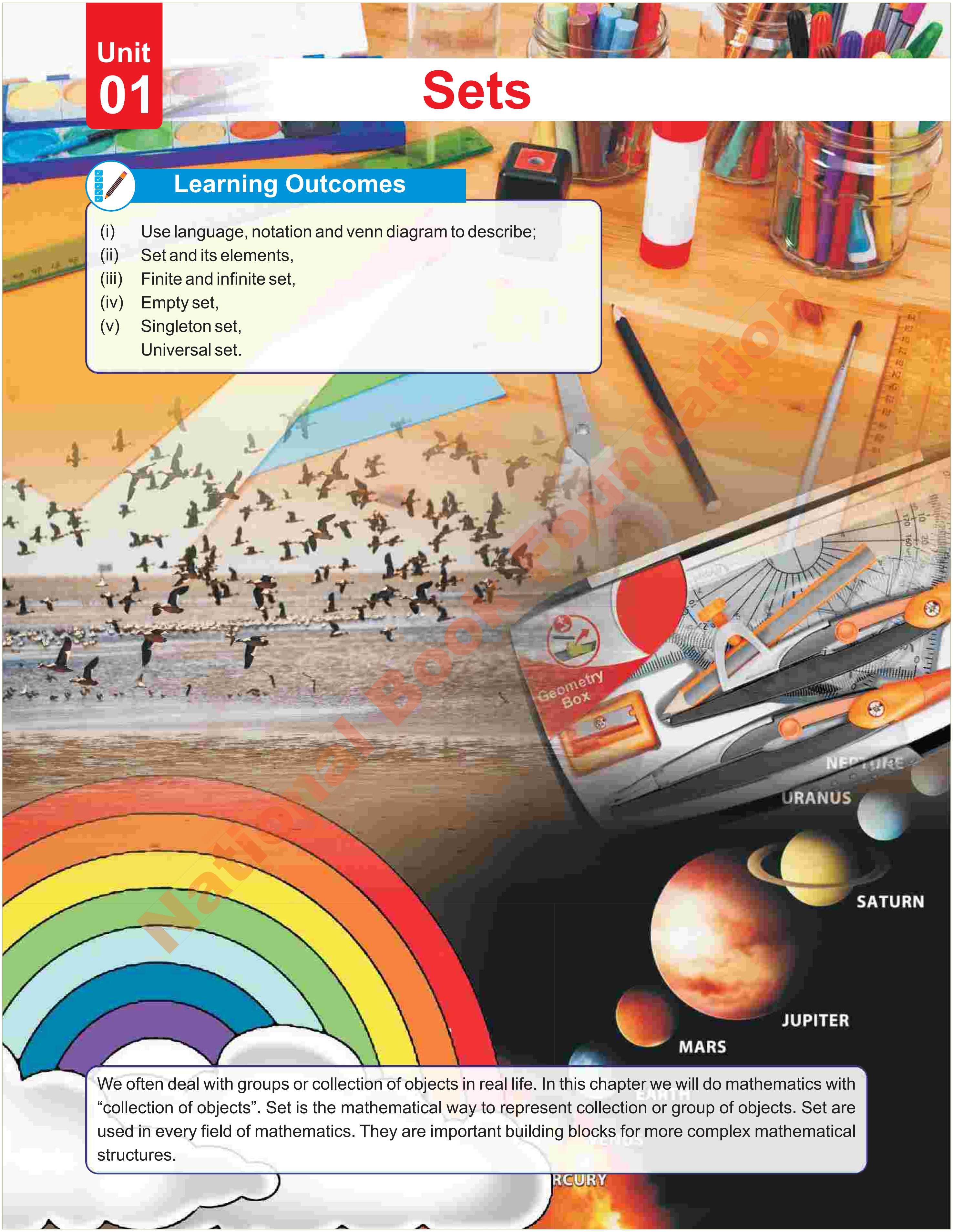
Unit 01

Sets

Learning Outcomes

(i) Use language, notation and venn diagram to describe;
(ii) Set and its elements,
(iii) Finite and infinite set,
(iv) Empty set,
(v) Singleton set,
Universal set.



We often deal with groups or collection of objects in real life. In this chapter we will do mathematics with "collection of objects". Set is the mathematical way to represent collection or group of objects. Set are used in every field of mathematics. They are important building blocks for more complex mathematical structures.

SETS

In this world around us, things exist in collections. There are many words to describe collection of objects, like a flock of birds, a pile of books, a group of people, a team of players, bouquet of flowers, etc.

In mathematics, the term “set” is used to describe “a collection of **“well-defined”** and **distinct** objects”.

Consider the following collection of days of week.

Days of week.

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday

Well-defined collection

Collection of **distinct** objects

This is a collection of **well-defined** and **distinct** objects because from the description "days of week" the reader is able to list all the elements of the collection. No two objects in this collection are the same. Hence it is a **set**.

Consider the collection of best players of cricket team.

Best players of cricket team

every one may not agree on which players are the best.

Not a well defined collection

Collection of distinct objects

The collection of best players of cricket team is not well defined and hence **not a set.**

Key fact

In a set
- The word "distinct" means that the objects of a set must all be different.
- Elements cannot be repeated.
- Order of elements does not matter.

Key fact

In general, a set is not any collection of objects. The objects in a set must be **well-defined** and **distinct**.

Each object in a set is called an **element** or member of the set. Elements of a set can be symbols, numbers or objects. We relate a set and its elements by using the symbol $\in$. In the following set

$A = \{1, 3, 5, ..., 19\}$

Since 3 is is an element of A we write $3 \in A$,

and 2 is not an element of A we write $2 \notin A$

Check Point

Let B be collection of all tall students in the class. Is B a set?

Check Point

A is the set of odd numbers less than 20.

- List all the elements of A in set notation.
- is $2 \in A$?

Key fact

The symbol

$\in$ denotes "is an element of "

$\notin$ denotes "is not an element of"

Check Point

State weather each of the following statement is true or false, if it is false explain why.

- $p \in \{p, e, n\}$
- $\{p\} \in \{p, e, n\}$
- $1 \in$ set of even number
- Train $\in \{t, r, a, i, n\}$

There are at least two common methods of specifying a set. One is to **describe** the elements of a set in words and other is to **list** elements of the set. Sets are usually represented by capital letters A, B, C,...

N = Natural numbers less than 10

$N = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

B = vowels in english alphabets

$B = \{a, e, i, o, u\}$

C = even numbers less than 100

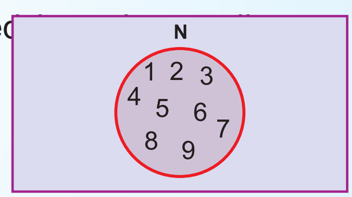
$C = \{0, 2, 4, ..., 98\}$

Key fact

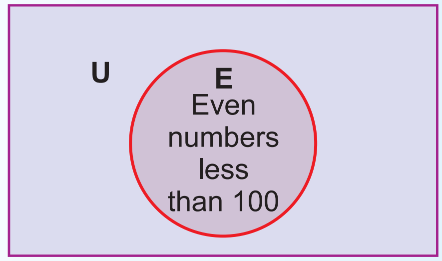
Some times, it may not be possible to list all the elements of a set, in such case we begin to list a few elements and then use ..., to show that pattern continues up to the last element.

We can represent a set using rectangle and circle. This geometrical representation of the set is called **venn diagram.**

(i) The set of natural numbers less than 10 are represented as follows.



(ii) U = Whole numbers less than 100
U = {0, 1, 2,..., 99}
(iii) E = Even numbers less than 100
E = {0, 2, 4,..., 98}



Finite and infinite set

If all the elements of a set are listed one after another, and the process eventually stops, such set is called **finite set.**

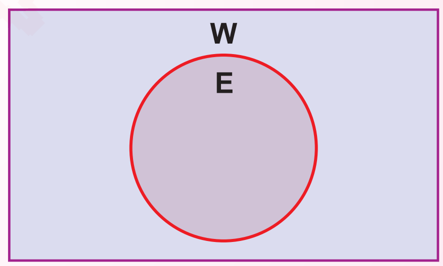
The number of elements in a finite set A is denoted by **n (A)**.

X = natural numbers less than 12
X = {1, 2, 3,..., 11} is finite set
n (X) = 11

Key fact
Infinite set never ends.

If all the elements of a set are listed one after another, and the process eventually does not stops, such set is called **infinite set.**

W = whole numbers
W = {0, 1, 2, 3,...,}
E = Even numbers
E = {0, 2, 4,...,}



are infinite sets

we write infinite set by listing a first few elements and then putting three dots "..." showing that the pattern continues and there are infinite more elements of the set to come.

Check Point

State number of elements in each of the following sets.
- Odd numbers divisible by 2.
- Dinosaur living on the Earth.
- Triangles having four vertices.
- Vowels in the word "PAKISTAN".
- Even numbers between 10 and 12.

Empty Set

If we consider the set B consisting of all human beings who visited mars, then B does not contain any element, that is B = { }

A set which contains no element is called **empty** or **null** set. Empty set is denoted by the symbol **"φ"** (pronounced as "phi").

The following sets are empty sets.

E = even numbers between 2 and 4 = φ = { }

B = human beings who visited Venus = φ = { }

Key fact

Set of natural numbers less than 1 is an empty set.

Singleton set

Consider the set C consisting of natural number between 5 and 7, then C = {6}, the set C contains exactly one element.

A set consisting of only one element is called **singleton set.**

Key fact

Set of whole numbers less than 1 is a singleton set.

Key fact

The set {φ} is not an empty set. It contains one element, the symbol φ.

Universal set

The universal set is the largest possible set of all elements under consideration for a specific situation.

There are 40 students in grade six in a school.

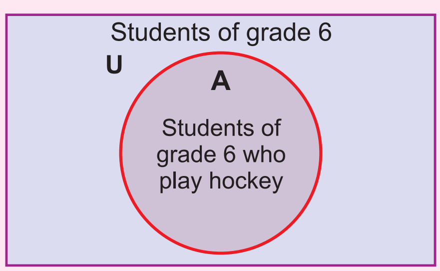
Let A be the set of students of grade 6 who play hockey.

In above set A, we are looking for the students of class six who play hockey, not the whole class six.

The students of grade six is the **universal** set for this particular situation.

A universal set is denoted by **U**.

The venn diagram representation of above set is as follows.



Check Point

State whether each of the following statements is true or false.

- W = Whole numbers = {0, 1, 2}
- {φ} = φ
- n (φ) = 0
- 4 ∉ even numbers
- i ∈ {a, e, i, o, u}

U = Whole numbers less than 30

U = {0, 1, 2,..., 29}

A = Even number less than 30.

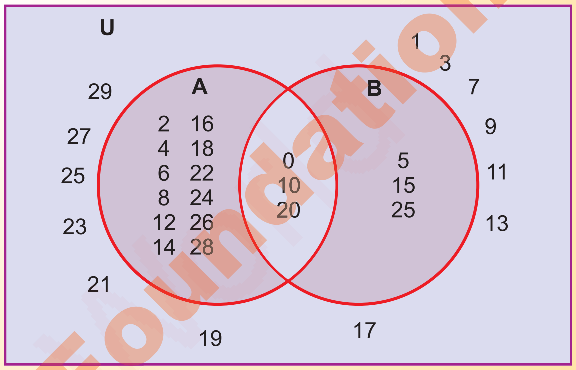
A = {0, 2, 4,..., 28}

B = Multiples of 5 less than 30.

B = {5, 10, 15, 20, 25}

are represented through venn diagram as follows.

The overlapping region of two circles represent common elements to both sets i.e. 0, 10, 20.



U = Whole numbers less than or equal to 30.

U = {0, 1, 2, 3, ..., 30}

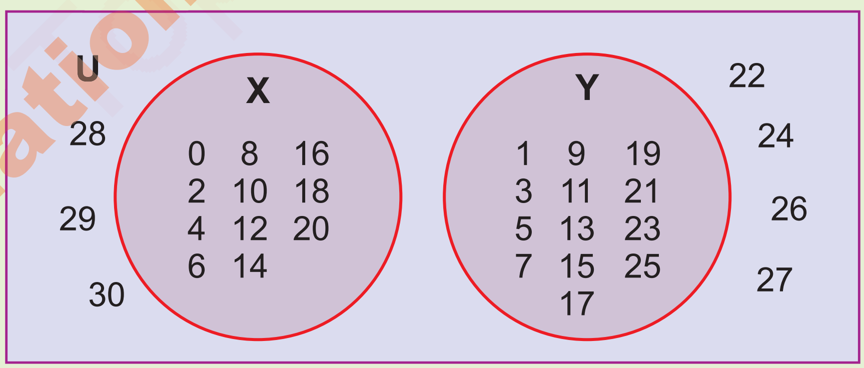
X = Even numbers less than or equal to 20.

X = {0, 2, 4,..., 20}

Y = Odd numbers less than or equal to 25.

Y = {1, 3, 5,..., 25}

are represented by venn diagram as follows.



Exercise 1.1

1. **Which of the following collections are sets.**
   (i) All colours in rainbow.
   (ii) Consonants of English alphabets.
   (iii) Students in your class.
   (iv) Very intelligent employees of a company.
   (v) Prime numbers less than 100.
   (vi) Very hard working students in your class.
   (vii) All letters in the word “Mathematics”.
   (viii) All outcomes of flipping a coin.
   (ix) All results of rolling a die.
   (x) Months of solar year.
   (xi) Islamic month of fasting.
2. **Which of the following statements are true? Explain your answer.**
   (i) $\{2, 4, 6, 8, ..., 100\} = \{2, 4, 6, 8, ...\}$
   (ii) $\{1, 3, 5, 7\} = \{7, 3, 5, 1\}$
3. **Describe the following sets.**
   (i) $A = \{0, 2, 4, 6, 8\}$
   (ii) $B = \{1, 3, 5, 7, 9\}$
   (iii) $X = \{2, 3, 5, 7, 11, 13, 17, 19\}$
   (iv) **Y = {mercury, venus, earth, mars, jupiter, saturn, uranus, neptune}**
   (v) **C = {potato, carrot, raddish, beet, sweet potato}**
4. **List the elements of the following sets.**
   (i) **N = natural numbers greater than or equal to 20.**
   (ii) **P = prime numbers between 10 and 30.**
   (iii) **M = multiples of 5 greater than 5.**
   (iv) **A = primary colours**
   (v) **B = persons having ages 200 years and above.**

5. **Which of the following represent the universal set for the given sets and represent them through venn diagram.**

A = All consonants of English alphabets,

B = All vowels of English alphabets, C = All English alphabets.

D = {a, b, c, d, e, f, g, h}

6. **List the elements of the following sets and represent them through venn diagram**
U = Natural numbers less than or equal to 20,
X = Prime numbers greater than 5 and less than 20,
Y = Odd numbers less than or equal to 20.

7. **Fill in the blanks with $\in$ or $\notin$.**
   - (i) 5 ________ {2, 4, 6,...}
   - (ii) 1 ________ {0, 4, 6, 8}
   - (iii) 2 ________ odd numbers less than 19.
   - (iv) 39 _______ sum of dots appearing when two dice are rolled.
   - (v) 100 ________ {2, 4, 6,...}
   - (vi) Blue ________ set of primary colours.
   - (vii) Grass ________ {carrot, potato, onion}
   - (viii) 6 ________ set of outcomes when a die is rolled.
   - (ix) A number divisible by 2 _________ set of whole numbers.
   - (x) The month of Ramzan __________ the set of solar months.

8. **Let U = colours of rainbow**
A = primary colours
Represent the sets through venn diagram

I have learnt

- Set is a collection of well defined and distinct objects.
- Venn diagram is a geometrical representation of sets.
- A set consisting of limited number of elements is called a finite set, otherwise it is called infinite set.
- A set having no element is called empty set.
- A set consisting of only one element is called singleton set.
- A universal set is the largest possible set of all elements under consideration for a specific situation.
- In venn diagram the overlapping region of two circles represents the elements which are common to both sets.

Words Board

- Set
- Well defined
- Distinct
- Venn diagram
- Finite set
- Infinite set
- Empty set
- Singleton set
- Universal set

Review Exercise 1

1. Encircle the correct option in the following statements.

(i) If A = Students in your class then A is a/an __________ set.

(a) empty (b) finite (c) infinite (d) singleton

(ii) B is the set of months of Eid then it is a/an _________ set.

(a) empty (b) finite (c) infinite (d) universal

(iii) C = First 10 multiples of 3 then n (C) = _________ .

(a) 3 (b) 6 (c) 9 (d) 10

(iv) D = Students of your class of age 5 years is a/an _____ set.

(a) empty (b) singleton (c) infinite (d) universal

(v) E = Multiples of 4 is a/an ________ set.

(a) empty (b) singleton (c) finite (d) infinite

(vi) **Set of colours of rainbow has _______ elements.**

(a) 6 (b) 5 (c) 7 (d) 9

(vii) **Set of triangles is a/an __________ set.**

(a) empty (b) infinite (c) finite (d) singleton

(viii) **Set of even number divisible by 3 is a/an ________ set.**

(a) empty (b) finite (c) infinite (d) singleton

(ix) **Set of natural numbers between 1 and 3 is a/an _________ set.**

(a) infinite (b) singleton (c) empty (d) universal

(x) **The universal set U for a set of even numbers is set of ________ numbers.**

(a) odd (b) natural (c) prime (d) whole numbers

2. **Represents the following sets through venn diagram.**

(i) **U = {1, 2, 3,..., 50},**
A = multiples of 3 less than 50,
B = multiples of 5 less than or equal to 50.

(ii) **U = {1, 2, 3,..., 40},**
A = {1, 2, 3,..., 20},
B = {21, 22, 23,..., 40}.

3. **Represent the following sets through venn diagram**
U = {0, 1, 2, ... , 10}
A = {0, 2, 4, 6, 8}
B = {1, 2, 3, 4, 6, 7, 9}

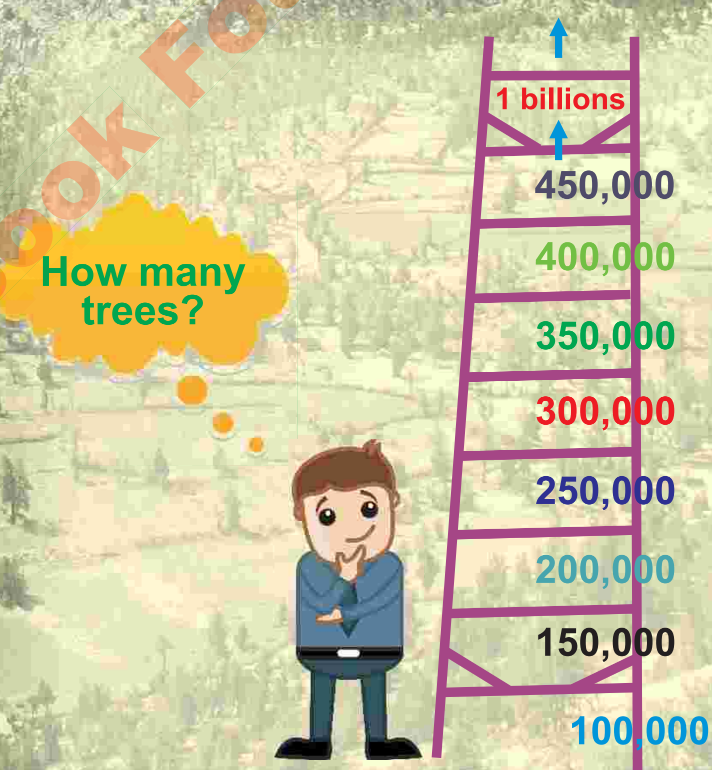
4. **Represent the following sets through venn diagram**
A = {0, 2, 4, 6, 8, ..., 48}, B = {2, 3, 5, 7, 11, 13, ..., 47}, U = {0, 1, 2, 3, ..., 50},
What do you guess the shade portion of circles represent?

Unit
02

Whole Numbers

Learning Outcomes

(i) Differentiate between natural and whole numbers. Represent a given list of whole numbers on a number line.
(ii) Whole number $<$ or $>$ a given whole number.
(iii) Whole number $\leq$ or $\geq$ a given whole number.
(iv) Add and subtract two given whole numbers using number line.
(v) Verify commutative and associative law (under addition) of whole numbers. Recognize '0' as additive identity.
(vi) Multiply and divide two given whole numbers. Verify commutative and associative laws {multiplication} of whole numbers.
(vii) Recognize '1' as multiplicative identity. Verify distributive law of multiplication over addition.
(viii) Verify distributive law of multiplication over subtraction (with positive difference)



The forest near Ziarat district in Balochistan, is second largest Juniper (Snober) forest in the world. Its trees are estimated up to 4000 to 5000 years old.
Pakistan's federal government plans to plant 100 million trees within 5 years and started a massive project to add 350,000 hectors of trees both by planting and natural regeneration
Whole numbers describe number of trees, population of world and are used for counting objects..
Think of all numbers we use every day for counting!

WHOLE NUMBERS

Natural and Whole Numbers

What would you do without numbers?

You couldn't not even count and keep track of important events in your life.

Once people began to count, how did they remember the numbers they had counted?

You read books like this with numbered pages. They needed a way to write down the numbers. So they invented **Numerals.**

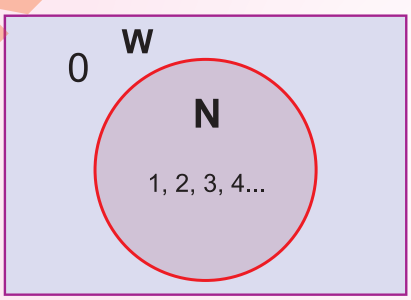
This World is full of numbers.

Key fact

Numerals are symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 used to construct numbers.

The numbers 1, 2, 3,... are called **Natural numbers,** denoted by **N.**
N = {1, 2, 3,...}
The numbers starting from zero that is 0, 1, 2, 3,... are called **whole numbers** denoted by **W**.
W = {0, 1, 2, 3,...}

Set of whole numbers and natural numbers are represented through venn diagram as follows



Key fact

The only whole number which is not natural is Zero **'0'**

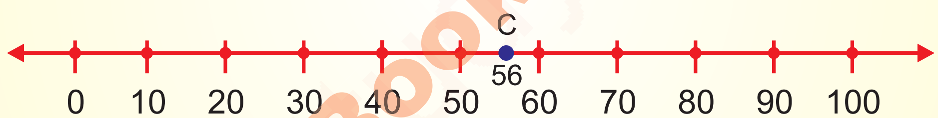
Representation of Whole numbers on number line

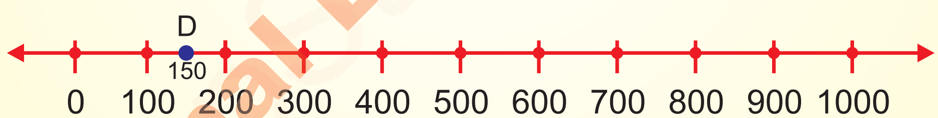
What is a number line?

A number line is the geometrical representation of numbers on a straight line.

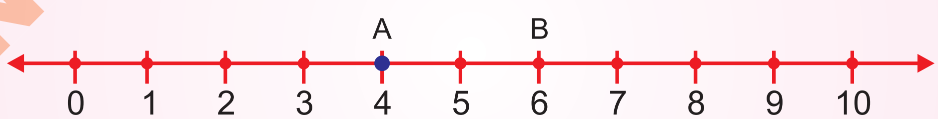
A straight horizontal line divided into number of parts with a unit distance between them is called a **number line.**

The points A, B, C, D represent the whole numbers 4, 6, 56, 150 respectively on number lines.





On a number line we can easily make the comparison between two numbers. On the number line greater number lies to the right of specified whole number, and the smaller number lies to the left of the number.



6 is greater than 5, it lies to the right of 5 and 4 being smaller than 5 lies to the left of 5. We can use following symbols for comparison of numbers.

(i) The symbol $<$ stands for "less than" like $5 < 8$ means '5 is less than 8.

(ii) The symbol $>$ stands for "greater than" like $11 > 9$ means 11 is greater than 9.

(iii) The symbol $\leq$ stands for "less than or equal to" like $x \leq 3$, means "x is less than or equal to 3".

(iv) The symbol $\geq$ stands for "greater than or equal to" as $x \geq 9$, means "x is greater than or equal to 9".

Key fact

Whole number less than 1 is 0

Check Point

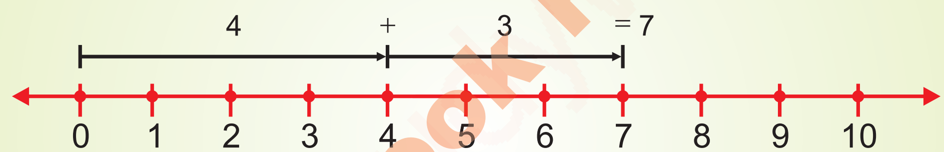
Represent the following whole number on a number line

- 200, 360, 450 and 500
- Whole numbers less than 8

Addition and subtraction of whole numbers on numer line

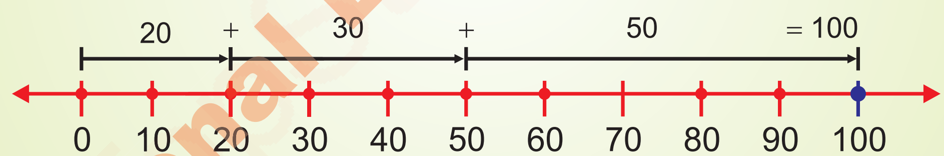
We can add whole numbers on a number line as follows

(i)



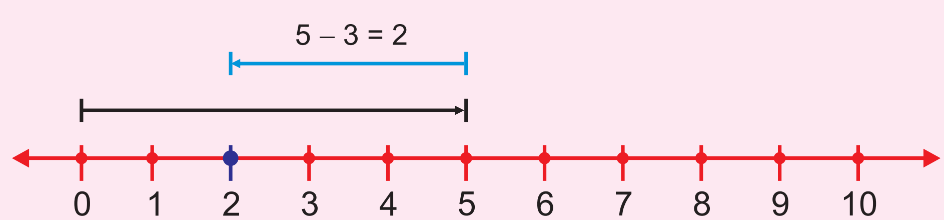
Represent 20 + 30 + 50 on a number line

(ii)



Subtracting a whole number from another is just like addition, but we move backward instead of forward, we move to the left.

Subtract 3 from 5 on number line



Properties of whole numbers

Commutative Law of Addition

Consider $5 + 4 = 9$

$4 + 5 = 9$

From above We see that

$5 + 4 = 4 + 5$

Similarly $1 + 11 = 12$

$11 + 1 = 12$

$1 + 11 = 11 + 1$

This shows that sum of two whole numbers is the same, no matter in which order they are added. This law is called commutative law of addition.

Key fact

Subtraction of whole numbers is not commutative.

Associative Law of Addition of Whole numbers

$2 + (4 + 3) = 2 + 7 = 9$

$(2 + 4) + 3 = 6 + 3 = 9$

from above

$(2 + 4) + 3 = 2 + (4 + 3)$

We see that the results of additions are the same even if we change the grouping of numbers.

While adding whole numbers, we can group them in any order. This is called associative law of addition of whole numbers.

Key fact

Subtraction of whole numbers is not associative.

Zero '0' As Additive Identity

Consider

$0 + 3 = 3$

$0 + 1 = 1$

$0 + 9 = 9.$

$0 + 0 = 0$

When we add **0** to any whole number, we get the same whole number again., **Zero** is called **identity** for addition, because addition of **zero** “**0**” to any whole number does not change the number.

Key fact

Subtraction is the inverse operation of addition.

Exercise 2.1

1. **Write the smallest natural and smallest whole number.**

2. **Is every natural number also a whole number?**

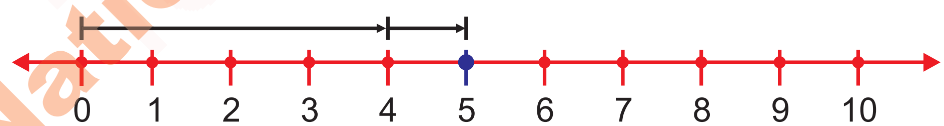
3. **Write the following numbers and represent them on number line.**

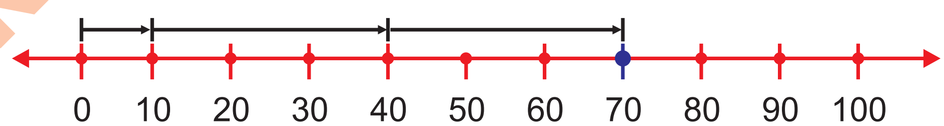
   (i) The greatest 1 digit number.
   (ii) All natural numbers greater than 9 and less than 19.
   (iii) Whole numbers greater than or equal to 8.
   (iv) Whole numbers less than 12 and greater than 5.
   (v) Natural numbers lying between 3 and 10.
   (vi) Whole number ≥ 6 and ≤ 11.
   (vii) Whole numbers ≤ 8 but > 6.
   (viii) Whole number ≤ 14 but > 9.
   (ix) Even numbers less than 10.
   (x) Odd numbers greater than 5 and less than 15.

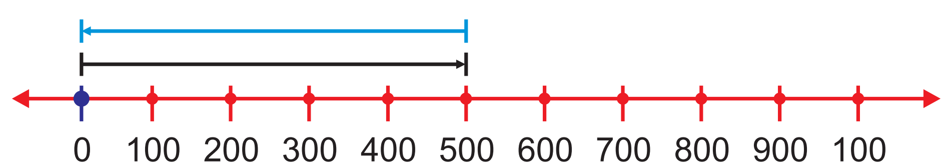
4. **Represent the following arithmetic operations on number line**

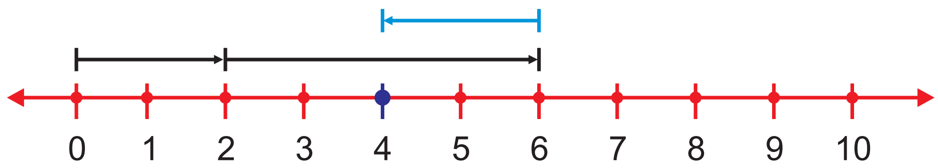
   (i) $2 + 5$
   (ii) $7 - 4$
   (iii) $2 + 3 - 4$
   (iv) $2 + 2 + 2$
   (v) $6 - 6$

5. **Find the arithmetic operation for the following number line.**

(i)


(ii)


(iii)


(iv) 

6. Complete the following

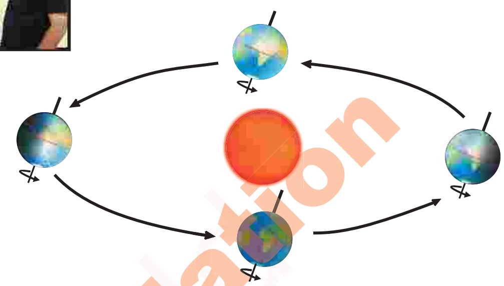
(i) □ + 3 = ▱ + 20
(ii) 1 + 0 = □ + ▱
(iii) 0 + □ = 0
(iv) 10 + (□ + 20) = (10 + 30) + ▱
(v) □ + (6 + 4) = (9 + 6) + ▱
(vi) 7 + □ = 5 + ▱
(vii) (□ + 53) + 15 = 19 + (▱ + 15)

7. Discuss and verify associative law of addition for the following.

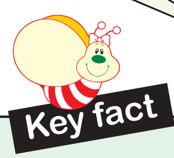
(i) (38 + □) + 17 = ▱ + (13 + 17)
(ii) □ + (29 + 48) = (19 + 29) + ▱
(iii) (12 + □) + 37 = ▱ + (51 + 37)
(iv) □ + (85 + 62) = (17 + 85) + ▱
(v) (25 + □) + 68 = 25 + (21 + ▱)
(vi) (10 + 35) + 24 = □ + (▱ + ⏢)
(vii) (□ + ▱) + 95 = 104 + (37 + ⏢)
(viii) □ + (85 + 90) = (34 + ▱) + ⏢
(ix) 10 + (□ + 20) = (▱ + 30) + ⏢
(x) (11 + 12) + 15 = □ + (▱ + ⏢)

Multiplication of Whole Numbers.

It takes the earth 365 days to complete its revolution around the sun.
One year has 365 days.
How many days are there in 2 years?



We can find the number of days in 2 years by multiplication
1 year = 365 days.
2 years = 2 x 365 days = 730 days.



Key fact

Multiplication of whole numbers is equivalent to repeated addition of whole numbers.

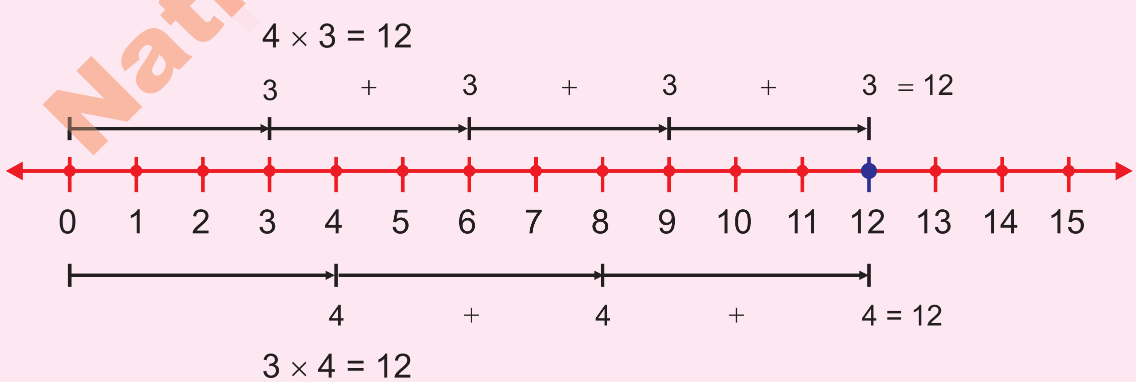
$4 \times 3 = 12$

$3 + 3 + 3 + 3 = 12$

$4 \times 3 = 3 + 3 + 3 + 3$

We can represent the product of two numbers on a number line.

$4 \times 3 = 3 + 3 + 3 + 3 = 12$



Commutative law of Multiplication of whole numbers

Consider

$5 \times 7 = 35$

$7 \times 5 = 35$

$5 \times 7 = 7 \times 5 = 35$

Above example shows that the product of two whole numbers is the same, no matter in which order they are multiplied. This law is called the commutative law for multiplication.

Key fact

Division of whole numbers is not commutative.

Associative law of Multiplication of whole numbers

Consider the product

$(6 \times 7) \times 4 = 42 \times 4 = 168$

$6 \times (7 \times 4) = 6 \times 28 = 168$

We see that the product in both cases is the same

$(6 \times 7) \times 4 = 6 \times (7 \times 4) = 168$

While multiplying whole numbers , we can group them in any order. This law is called associative law of multiplication of whole numbers.

Use appropriate law to find the product $5 \times 346 \times 2$

$5 \times 346 \times 2 = (5 \times 2) \times 346$

$= 10 \times 346 = 3460$

ONE '1' as multiplicative identity

Consider

$1 \times 14 = 14$

$34 \times 1 = 34$

$1 \times 0 = 0$

When we multiply **1** by any whole number, we get the same whole number again., **One** "**1**" is called **identity** for multiplication, because multiplication of **one** "**1**" by any whole number does not change the number.

Division of whole numbers

Earth takes 24 hours to complete its rotation about its axis. 1 day is equal to 24 hours how many days are in 72 hours?

We can find the number of days by division.
24 hours = 1 days.
72 hours = 72 ÷ 24 = 3 days

Key fact

The quotient of two numbers depends on the order of the numbers.

Quotient of 3 and 6 = $\frac{3}{6} = \frac{1}{2} \notin W$

Quotient of 6 and 3 $= \frac{6}{3} = 2 \in W$

$$\frac{3}{6} \neq \frac{6}{3}$$

Division is an inverse operation of multiplication which can be checked as follows.

$$\frac{80}{5} = 16$$

$$5 \times 16 = 80$$

Check Point

Division by zero is not possible why?

Distributive law of multiplication over addition/subtraction

Distributive property of multiplication over addition/subtraction relates the operation of multiplication with addition/subtraction.

(i) Consider

$35 \times (10 + 2) = 35 \times 12 = 420$

$(35 \times 10) + (35 \times 2) = 350 + 70 = 420$

We see that

$$\underbrace{\underbrace{35 \times (10 + 2)} = \underbrace{(35 \times 10) + (35 \times 2)}}_{\text{Distributive property}}$$

Whole numbers also obey distributive law of multiplication over subtraction.

use suitable law to find the following products

(i) $6 \times (20 - 8)$

$6 \times (20 - 8) = (6 \times 20) - (6 \times 8)$

$6 \times 12 \quad = 120 - 48$

(ii) 21×103

$21 \times 103 = 21 \times (100 + 3)$

$= (21 \times 100) + (21 \times 3)$

$= 2100 + 63$

$= 2163$

(iii) 21×97

$21 \times 97 = 21 \times (100 - 3)$

$= (21 \times 100) - (21 \times 3)$

$= 2100 - 63$

$= 2,037$

Exercise 2.2

1. Check the following by changing the division into multiplication.

(i) $\frac{24}{6}=4$ (ii) $\frac{12}{2}=6$

(iii) $\frac{7}{7}=1$ (iv) $\frac{92}{23}=4$

(v) $\frac{0}{9}=0$ (vi) $\frac{450}{10}=45$

2. Use distributive property over addition or subtraction to solve the following.

(i) 25×12 (ii) 105×106 (iii) 18×12

(iv) 14×102 (v) 35×19 (vi) 16×105

(vii) 346×99

3. Write and verify the distributive property of multiplication for the following products.

(i) $251 \times (30 + 1)$ (ii) 90×103

(iii) $47 \times (50 + 2)$ (iv) 17×39

(v) $51 \times (100 - 2)$ (vi) 63×37

4. Replace each □, ▱, ⏢ and ○ with an appropriate whole number.

(i) 3 × (□ + 8) = (3 × 28) + (3 × 8) = ▱

(ii) (16 × 12) + (6 × 12) = (16 + 6) × 12 = □

(iii) 5 x (4 – □) = (5 × 4) - (▱ × 1) = ⏢

(iv) (11 × 10) - (9 × 10) = (11 – 9) × □ = ▱

(v) (15 + 7) × 9 = (□ × 9) + (▱ × 9) = ⏢

(vi) 5 × (□ – 13) = (5 × 18) – (5 × 13) = ▱

(vii) (8 + 7) × 15 = (8 × □) + (▱ × 15) = ⏢

(viii) (□ × 8) – (▱ × 8) = (10 – 3) × 8 = ⏢

(ix) 16 × (13 + 7) = (□ × ▱) + (16 × 7) = ⏢

(x) (9 × 23) – (□ × ▱) = 9 × (⏢ – 3) = ○

I have learnt

Words Board

- Natural numbers
- Whole numbers
- Sum
- Difference
- Product
- Quotient
- Additive Identity
- Multiplicative Identity
- Closure
- Commutative
- Associative
- Distributive

- Numerals are symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 used to construct numbers.
- The set {1, 2, 3,...} is called set of natural numbers denoted by N and the set {0, 1, 2,...} is the set of whole numbers denoted by W.
- The only whole number which is not natural is zero.
- A number line is the geometrical representation of numbers on a straight line.
- On number line the greater number lies to the right of given whole number, the smaller number lies to the left of the number.
- The symbol $<$ is used for "less than".
- The symbol $>$ is used for "greater than".
- The symbol $\leq$ is for "less than or equal to".
- The symbol $\geq$ is used for "greater than or equal to".
- Set of whole numbers satisfies commutative and associative laws with respect to addition and multiplication.
- Zero "0" is the additive identity and one "1" is the multiplicative identity.
- Subtraction and division are inverse operations of addition and multiplication respectively.

Review Exercise 2

1. Encircle the correct option for the following statements.

(i) The only whole number which is not natural is ____________.

(a) 1 (b) 0 (c) 9 (d) 2

(ii) Numerals are the symbols used to construct ____________.

(a) numbers (b) sentences (c) phrases (d) none of these

(iii) ____________ is called the additive identity of whole numbers.

(a) 0 (b) 1 (c) 2 (d) 3

(iv) Division of whole numbers by ____________ is not possible.
(a) 1 (b) 0 (c) 2 (d) 3

(v) __________ is called the multiplicative identity of whole numbers.
(a) 0 (b) 1 (c) 2 (d) 3

(vi) $(2 + 3) + 5 = 2 + (3 + 5)$ is called __________ law of addition.
(a) associative (b) commutative (c) distributive (d) none of these

(vii) $0 + 1 = 1 + 0 = 1$ is called additive __________.
(a) associative (b) commutative (c) distributive (d) identity

(viii) $3 + 5 = 5 + 3$ is called __________ law of addition.
(a) associative (b) commutative (c) distributive (d) identity

(ix) $(3 + 2) \times 7 = (3 \times 7) + (2 \times 7)$ is called __________ law of multiplication over addition.
(a) associative (b) commutative (c) distributive (d) identity

(x) $1 \times 5 = 5 \times 1 = 5$ is called multiplicative __________.
(a) associative (b) commutative (c) distributive (d) identity

2. Fill in the following boxes

(i) $3 + 6 = \square + 3 = \square$
(ii) $0 + \square = \square + 4 = 4$
(iii) $27 \times \square = \square + 27 = 27$
(iv) $\square + 0 = \square + 47 = 47$
(v) $15 + (\square + 12) = (15+5) + \square$
(vi) $10 + (14 + \square) = (10 + \square) + 9$
(vii) $\square \times (10 + 2) = (21 \times \square) + (\square \times 2)$
(viii) $13 \times (\square - 2) = (13 \times 100) - (13 \times \square)$
(ix) $1 \times (\square + 13) = (\square \times 2) + (1 \times \square)$
(x) $251 \times (\square + 1) = (251 \times 10) + (\square \times 1)$

3. Write and verify the distributive property of multiplication.

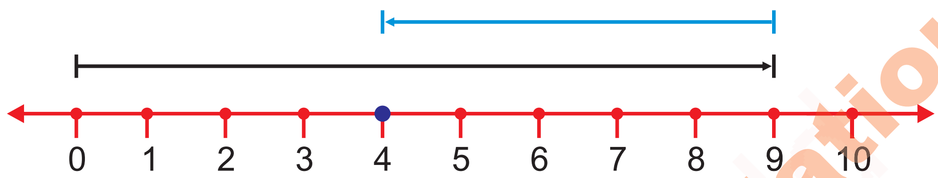
(i) 100×102 (ii) 49×51

(iii) 103×59 (iv) 19×19

(v) 350×401 (vi) 520×98

4. Write and arithematic operation for the following number lines.

(i)



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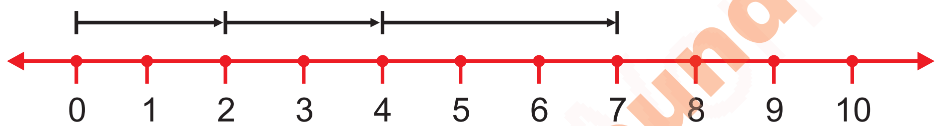

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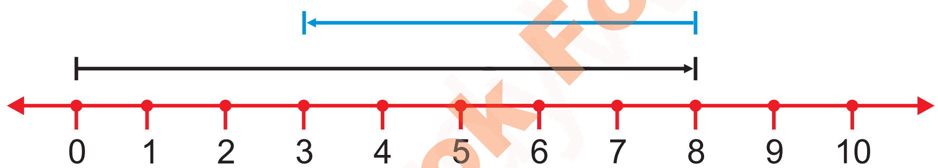
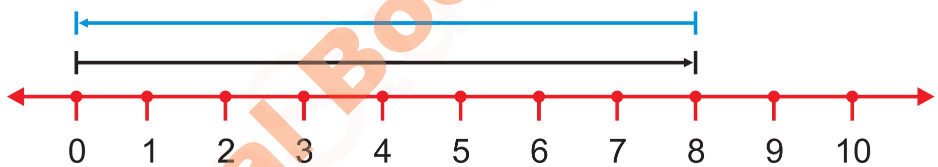


(iv)



Unit 03 Factors and Multiples

Learning Outcomes

(i) Recognize factors and multiples of numbers.
(ii) Test for divisibility.
(iii) Find prime factors of a given number and express its factors in the index notation.
(iv) Find HCF and LCM.
- Prime Factorization.
- Long Division Method.

(v) Apply problem solving strategies using HCF and LCM in real life situations.

Real life applications are essential to help students acquire logical thinking and to show them the relevance of mathematical concepts throughout their learning. The main purpose of this teaching is to improve the students understanding of HCF and LCM. The students could handle real life problems and to enhance higher order thinking skills.

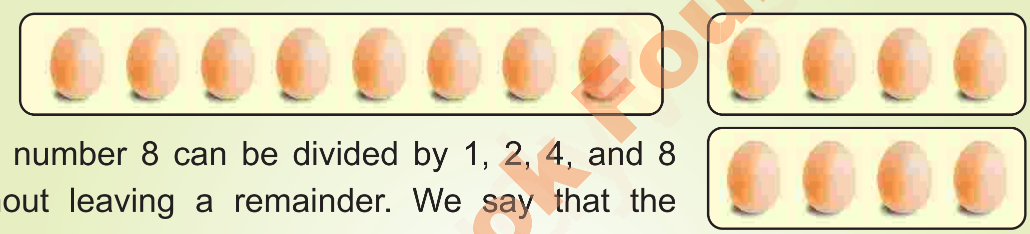
FACTORS

Let us put 40 chairs in the room. How can we arrange them?

We can put the chairs into 8 rows with 5 chairs in each row.

Can we think of other ways the chairs can be arranged? How many different ways can we arrange these chairs. If there are 48 chairs how will we arrange them?

We can arrange 8 eggs into equal rows in more than one way. Here are two ways.



The number 8 can be divided by 1, 2, 4, and 8 without leaving a remainder. We say that the factors of 8 are 1, 2, 4 and 8.

$1 \times 8 = 8$
$2 \times 4 = 8$

$8 \div 1 = 8$
$8 \div 8 = 1$
$8 \div 2 = 4$
$8 \div 4 = 2$

Are there other factors of 8? Is 3 a factor of 8?

No. 8 cannot be divided exactly by any other number

The factor of a number are the numbers that it can be divided exactly by. The number 8 can be divided exactly by 1, 2, 4 and 8. So, the factors of 8 are 1, 2, 4 and 8.

List the factor of 40

$40 = 1 \times 40$
$40 = 2 \times 20$
$40 = 4 \times 10$
$40 = 5 \times 8$

Stop listing when the numbers start to repeat. →

$40 = 8 \times 5$

The factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

Find the factors of 45.

$45 = 1 \times 45$
$45 = 3 \times 15$
$45 = 5 \times 9$

The factors of 45 are 1, 3, 5, 9, 15, 45

Multiple

What are some multiple of 3?

$1 \times 3 = 3$ $2 \times 3 = 6$ $3 \times 3 = 9$ $4 \times 3 = 12$

$1 \times 3 = \boxed{3}$
$2 \times 3 = \boxed{6}$
$3 \times 3 = \boxed{9}$
$4 \times 3 = \boxed{12}$
. .
. .
. .

← multiples of 3

3, 6, 9 and 12 are the first four multiples of 3

What is the 5th multiple of 3?

$5 \times 3 = 15$

The 5th multiple of 3 is 15.

Mubeen wants to find the multiples of 10.

$1 \times 10 = 10$	$6 \times 10 = 60$
$2 \times 10 = 20$	$7 \times 10 = 70$
$3 \times 10 = 30$	$8 \times 10 = 80$
$4 \times 10 = 40$	$9 \times 10 = 90$
$5 \times 10 = 50$	$10 \times 10 = 100$

The first six multiples of 10 are 10, 20, 30, 40, 50 and 60.
Can you say what the 9th multiple of 10 is?

Key fact

Multiple

- A multiple of a number is the product of that number and any number.
- A number is a factor of all its multiples.

Is 21 a multiple of 3?

$$\begin{array}{r} 7 \\ 3\overline{)\,21} \\ -21 \\ \hline 0 \\ \hline \end{array}$$

$7 \times 3 = 21$
Yes, 21 is the 7th multiple of 3.

21 can be divided exactly 3. So, 3 is a factor of 21 and 21 is a multiple of 3.

Is 41 a multiple of 5?

$$\begin{array}{r} 8 \\ 5\overline{)\,41} \\ -40 \\ \hline 1 \\ \hline \end{array}$$

41 can not be divided exactly by 5. So, 5 is not a factor of 41 and 41 is not a multiple of 5.

Find the first 5 multiple of 3, 8 and 13
The first 5 multiple of 3 are: 3, 6, 9, 12, 15
The first 5 multiple of 8 are: 8, 16, 24, 32, 40
The first 5 multiple of 13 are: 13, 26, 39, 52, 65

Is 6 a factor of 42?
$42 \div 6 = 7$, so 6 is a factor of 42
Is 8 a factor of 50?
$50 \div 8 = 6$ and remainder 2, so 8 is not a factor of 50.

$$\begin{array}{r} 6 \\ 8\overline{)\,50} \\ 48 \\ \hline 2 \\ \hline \end{array}$$

Find the factors of 12.

$12 = 1 \times 12$
$12 = 2 \times 6$
$12 = 3 \times 4$

The factors of 12 are 1, 2, 3, 4, 6, 12

What number is a factor of all numbers.

Exercise 3.1

1. **Write down the first 4 multiples of:**

(i) 5	(ii) 6	(iii) 7
(iv) 9	(v) 11	(vi) 14

2. **Write down the multiples of 5 between 4 and 31.**

3. **Identify the numbers that have 18 as a factor.** **54, 126, 198, 240, 320**

4. **Identity the numbers below which are factors of 144.**
   1, 2, 3, 4, 8, 9, 12, 14, 16, 32, 48, 144

5. **Identify the multiples of 8 from the following numbers.**
   14, 24, 32, 54, 56, 36, 72, 30, 64, 18, 40, 78, 96, 108, 120

6. **Find all factors of:**

(i) 15	(ii) 21	(iii) 32
(iv) 29	(v) 40	

Tests Of Divisibility

Is 639 divisible by 2 or 3?

We can find out whether 639 is divisible by 2 or 3 by actual division.

Consider

$$\begin{array}{r|l} & 319 \\ 2 & 639 \\ & \underline{6} \\ & 3 \\ & \underline{2} \\ & 19 \\ & \underline{18} \\ & 1 \end{array} \qquad \begin{array}{r|l} & 213 \\ 3 & 639 \\ & \underline{6} \\ & 3 \\ & \underline{3} \\ & 9 \\ & \underline{9} \\ & 0 \end{array}$$

So, 639 is divisible by 3 and not by 2. But this is a time consuming practice. There are some simple rules for determining as to whether or not a given number is divisible by any other number. These rules are known as "Tests of Divisibility".

TESTS OF DIVISIBILITY

It is often useful to know if a number is divisible by another number. Here are some simple divisibility tests to help you.

A number is divisible by 2 if it ends in 0, 2, 4, 6 or 8

39604
1000
82 788
1020
964

all are divisible by 2

A number is divisible by 3 if the sum of its digits is divisible by 3

79 is NOT divisible by 3 since 7 + 9 = 16 and3 does not go evenly in 16
72 is divisible by 3 since 7 + 2 = 9

75
36
7512

A number is divisible by 4 if its last two digits are divisible by 4

679 320
567 24

A number is divisible by 5 if it ends in 0 or 5

4805
5005
840

A number is divisible by 6 if it is divisible by both 2 and 3

48 { ends in 8
4+8 = 12

4506
ends in 6 | 4 + 5 + 6 = 15

A number is divisible by 8 if the last 3 digits are divisible by 8

13 592 is divisible by 8

$$\begin{array}{r} 74 \\ 8\overline{)\,592} \\ 592 \\ \hline \end{array}$$

38848

A number is divisible by 9 if the sum of its digits is divisibile by 9

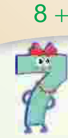
171
1 + 7 + 1 = 9
812 754
8 + 1 + 2 + 7 + 5 + 4 = 27

A number is divisible by 10 if it ends in 10

200
1120
400
590

A number is divisible by 11 if the difference between the sums of alternate digits is either 0 or a multiple of 11

4563 is divisible by 11 as (4+5) -(6+3) = 9-9=0.
918291 is divisible by 11 as (9+8+9) – (1+2+1) = 26–4 = 22.



Test of divisibility for 7 is complex and beyond the scope of students at this level.

Which of the numbers 2, 3, 4, 5, 8, 9, 10 and 11 exactly divide

(i) 8880 (ii) 3245

(i): 8880 is divisible by 2 being even.

8880 is divisible by 3 as 8+8+8+0 = 24 is divisible by 3.

8880 is divisible 4 as 80 is divisible by 4.

8880 is divisible by 5 and 10 as last digit is zero.

8880 is divisible by 8 as last 3 digits (880) are divisible by 8.

$\therefore$ 8880 is divisible by 2,3,4,5,8 and 10.

(ii) 3245 is divisible by 5 as last digit is 5.

3245 is divisible by 11 as (3+4) - (2+5) = 7 - 7 = 0

$\therefore$ 3245 is divisible by 5 and 11.

Exercise 3.2

1. State which of the following are divisible by 2, 3, 4 or 5 and give a reason for each.

(i) 26	(ii) 63	(iii) 110
(iv) 435	(v) 6123	(vi) 1036
(vii) 3000		

2. State wheather the following are divisible by 6, 8, 9, 10, 11 and give a reason for each.

(i) 7170	(ii) 4128	(iii) 9900
(iv) 43212	(v) 4455	(vi) 7172
(vii) 819291	(viii) 24120	

3. From the list, 5, 8, 81, 85, 60, 26, 54, 45. Write numbers divisible by 5.

4. Is 6 a factor of 432? Give reason.

Prime Factorization

Find the factors of 5, 11, 19.

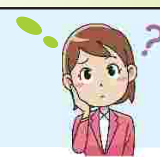
Factors of 5 are : 1 and 5

Factors of 11 are : 1 and 11

Factors of 19 are : 1 and 19

PRIME NUMBERS

A number which has only two different factors, 1 and the number itself, is a prime number. 5, 11 and 19 are prime numbers.



Find the factors: 8, 15, 26

Factor of 8 are : 1, 2, 4, 8

Factor of 15 are : 1, 3, 5 , 15

Factor of 26 are : 1, 2, 13, 26

COMPOSITE NUMBER

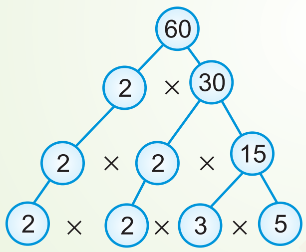
A number which is product of two or more prime factors is called composite number 8, 15 and 26 are composite numbers. similarly, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 … are composite numbers.

Prime Factorization:

Find the prime factors of 60

$60 = 2 \times 2 \times 3 \times 5$

2, 3, 5 are called prime factors of 60, because all are prime.

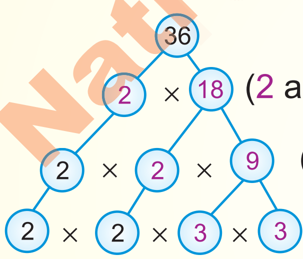


Factors Tree

A factor tree can be used to express a composite number as a product of its prime factors.

- A composite number can be expressed as the product of two or more prime numbers also called prime factors.
- The decomposition of a composite number into prime factors is known as prime factorization.

Find the prime factors of 36 using a factor tree.



(2 and18 are factors 2 is prime 18 is composite)

(2 and 9 are factors 2 is prime 9 is composite)

(3 is prime)

The prime factors of 36 are $2 \times 2 \times 3 \times 3$

$$36 = 2 \times 2 \times 3 \times 3$$
$$= 2^2 \times 3^2 \text{ (representation in index form)}$$

Index Notation:

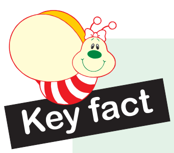
The prime factor of 36 are $2 \times 2 \times 3 \times 3$
Which can be expressed in index notation as $= 2^2 \times 3^2$

The following can be expressed by using index notation as:

(i) $11 \times 11 = 11^2$

(ii) $2 \times 2 \times 7 \times 7 \times 7 = 2^2 \times 7^3$

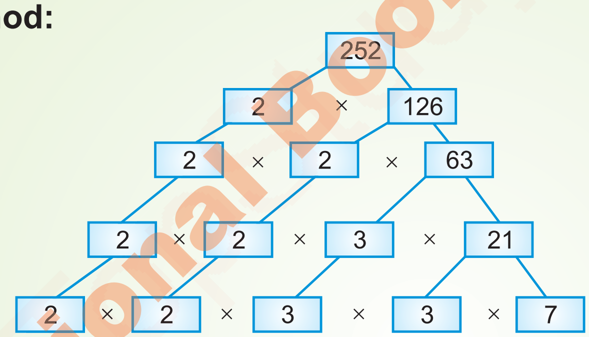
(iii) $5 \times 5 \times 5 \times 11 \times 11 \times 11 \times 19 = 5^3 \times 11^4 \times 19$



- Two numbers are called co-primes if their HCF is one (1) e.g. 10 & 17.
- Product of two prime number is never a prime number e.g. 3 x 5 = 15.
- Sum of two prime numbers may or may not be a prime number e.g.
  $2 + 3 = 5$, $5 + 7 = 12$

Express 252 in prime factors.

First Method:



$252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$ (in index notation)

Second Method:

2	252
2	126
3	63
3	21
7	7
	1

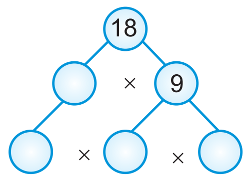
Start by smallest prime number and continue until we get 1.

$252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$

Exercise 3.3

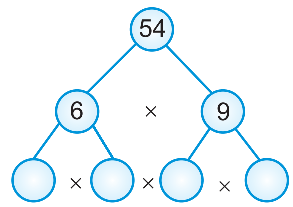
1. Complete each of the following factor tree:

(i)



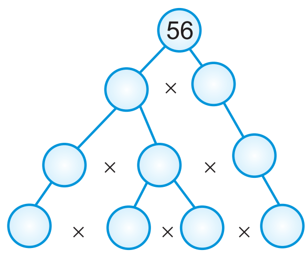
18 = ... × .... × ....

(ii)



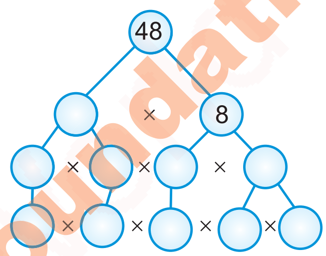
54 = ... × ..... × .... ×....

(iii)



56 = ... × .... × .... × ....

(iv)



48 = ... × .... × .... × .... × ....

2. Find prime factors of these numbers.

(i) 12 (ii) 20 (iii) 48 (iv) 76 (v) 88

(vi) 256 (vii) 576 (viii) 1296

3. Express the following in the index notation:

(i) $2 \times 2 \times 2$ (ii) $7 \times 7 \times 7 \times 7 \times 7$

(iii) $3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4$ (iv) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 5$

4. **Express the following as prime factors and write answer in index notation.**

(i) 72 (ii) 275 (iii) 300 (iv) 162

5. **Find the prime factors using the "factor tree" and write the answers in index notation.**

(i) 28 (ii) 136 (iii) 80 (iv) 130

Highest Common Factor (HCF)

There are 108, 180 and 216 students in three classes respectively. Buses are to be hired to take the students for school trip. Find the maximum number of students who can sit in a bus if each bus carries an equal number of students.

To find maximum number of students who can sit in a bus, we need to learn HCF.

List the factors of 100.

$100 = 1 \times 100$
$100 = 2 \times 50$
$100 = 4 \times 25$
$100 = 5 \times 20$
$100 = 10 \times 10$
$100 = 20 \times 5$

Stop listing when the numbers start to repeat.

The factors of 100 are 1, 2, 4, 5, 10, 20, 25 and 50

Find the factors of 6 and 10.

$6 = 1 \times 6$
$6 = 2 \times 3$
$6 = 3 \times 2$

Stop

$10 = 1 \times 10$
$10 = 2 \times 5$
$10 = 5 \times 2$

The factors of 6 are 1, 2, 3, and 6.   The factors of 10 are 1, 2, 5 and 10.

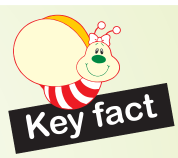
The common factors of 6 and 10 are 1 and 2.
So, we can say that the highest common factor of 6 and 10 is 2.

Find the highest common factor of 18 and 36.

$18 = 1 \times 18$
$18 = 2 \times 9$
$18 = 3 \times 6$

$36 = 1 \times 36$
$36 = 2 \times 18$
$36 = 3 \times 12$
$36 = 4 \times 9$

The common factors of 18 and 36 are 1, 2, 3, and 18.
So, the highest common factor is 18.



The highest common factor (HCF) is the largest factor which in common to both numbers.

Find HCF of 18 and 24.

Step 1: Write 18 and 24 as product
of prime factors.

Step 2: Pick the common prime factors.

Step 3: HCF of 18 and 24 is the product of the common prime factors.

Prime Factors of 18
$= 2 \times 3 \times 3$

2	18
3	9
3	3
	1

Prime Factors of 24
$= 2 \times 2 \times 3 \times 3$

2	24
2	12
2	6
3	3
	1

$18 = \boxed{2} \times \boxed{3} \times 3$
$24 = \boxed{2} \times \boxed{3} \times 2 \times 2$
HCF of 18 and 24 $= 2 \times 3$
$= 6$

Find the greatest number which exactly divides 950 and 300.

The greatest number which divides both 950 and 300 is their HCF.

There are two methods to find HCF.
- Prime factorization
- Division method

HCF by prime factorization

Prime factors of 950 and 300 are:

2	950
5	475
5	95
19	19
	1

2	300
2	150
3	75
5	25
5	5
	1

$950 = \boxed{2} \times 5 \times 5 \times 19$
$300 = \boxed{2} \times 2 \times 3 \times 5 \times 5$

HCF of 950 & 300 $= 2 \times 5 \times 5$
$= 50$

HCF by division method

$$\begin{array}{r|l} 300 & 950 \quad (3 \\ & 900 \\ \hline 50 & 300 \quad (6 \\ & 300 \\ \hline \end{array}$$

HCF = 50

What is the HCF of 19 and 23?

Find the HCF by prime factorization 84, 108, 132

$$\begin{array}{c|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{c|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{c|l} 2 & 132 \\ \hline 2 & 66 \\ \hline 3 & 33 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

Prime factors of 84 $= \boxed{2} \times \boxed{2} \times \boxed{3} \times 7$

Prime factors of 108 $= \boxed{2} \times \boxed{2} \times \boxed{3} \times 3 \times 3$

Prime factors of 132 $= \boxed{2} \times \boxed{2} \times \boxed{3} \times 11$

HCF $= 2 \times 2 \times 3$

$= 12$

HCF OF 84, 108, 132 is 12

Key fact

HCF by Long Division Method

In the long division method, the greater number is divided by the smaller number. The remainder is then taken as the divisor and the first divisor as dividend. The process of division is continued till the remainder is zero. The last divisor is the HCF of the given numbers.

Find the HCF of 546, 616 and 224 by division method.

$$\begin{array}{r} 546\,\overline{)\,616\,}\,(1 \\ -546 \\ \hline 70\,\overline{)\,546\,}\,(7 \\ -490 \\ \hline 56\,\overline{)\,70\,}\,(1 \\ -56 \\ \hline 14\,\overline{)\,56\,}\,(4 \\ -56 \\ \hline 0 \end{array} \qquad \begin{array}{r} 14\,\overline{)\,224\,}\,(16 \\ -14 \\ \hline 84 \\ -84 \\ \hline 0 \end{array}$$

HCF of 546, 616 and 224 is 14.

Exercise 3.4

1. **Find the HCF of the following numbers by using prime factors:**
   (i) 36, 84 (ii) 108, 315 (iii) 180, 250, 126
   (iv) 735, 1050, 455 (v) 288, 300, 216
2. **Find the HCF of the following numbers by the long division method:**
   (i) 63, 84 (ii) 105, 90 (iii) 360, 444
   (iv) 288, 600, 936 (v) 1078, 693, 847 (vi) 1320, 840, 500, 650
3. **Find the greatest common divisor of 36, 45 and 72.**
4. **Find the greatest number which exactly divides 595, 357 and 102 .**
5. **Find the highest common factor of 56 and 84 using both methods.**
6. **Three strings with length of 140cm, 168cm and 210cm are cut into pieces of same length. Find the greatest length of pieces if no string is left over.**
7. **Find the greatest sum of money that is exactly contained in Rs. 162, Rs. 252 and Rs. 342.**

Least common multiples (LCM)

Mr. Shahzad changes his car's oil every 3 months rotates the tires every 6 months and replaces the air filter once a year. If he completed all three tasks in April, what will be the next month he again completes all three tasks?

For this, we need to learn LCM.

Find the common multiples of 2 and 3.

$1 \times 2 = 2$	$6 \times 2 = 12$
$2 \times 2 = 4$	$7 \times 2 = 14$
$3 \times 2 = 6$	$8 \times 2 = 16$
$4 \times 2 = 8$	$9 \times 2 = 18$
$5 \times 2 = 10$	$10 \times 2 = 20$

$1 \times 3 = 3$	$6 \times 3 = 18$
$2 \times 3 = 6$	$7 \times 3 = 21$
$3 \times 3 = 9$	$8 \times 3 = 24$
$4 \times 3 = 12$	$9 \times 3 = 27$
$5 \times 3 = 15$	$10 \times 3 = 30$

So the least common multiple is 6.

6, 12 and 18 are some common multiples of 2 and 3.

Find the least common multiple of 4 and 5.

Multiple of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

Multiple of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...

So, the least common multiple of 4 and 5 is 20.

Key fact

The smallest number which is common multiple of two or more number is called least common multiple (LCM) of the numbers.

Find the LCM of 6 and 9.

Prime factors of 6 = 2×3

Prime factors of 9 = 3×3

Common factors of 6 and 9 = 3

Non-common factor of 6 and 9 = 2, 3

LCM = Product of common and non-common factors

= $2 \times 3 \times 3$

= 18

Step 1: Find prime factor of 6 and 9.

Step 2: Identitfy the common prime factor.

Step 3: LCM of the two number is the product of their common prime factors and non-common prime factors.

There are two methods to find LCM.

- Prime factorization
- Division method

The LCM of 30 and 36 by prime factorization and division method is as follows.

Prime factorization

$30 = \boxed{2} \times \boxed{3} \times 5$

$36 = \boxed{2} \times 2 \times \boxed{3} \times 3$

LCM of 30 and 36 = $\overbrace{2 \times 2 \times 3}^{\text{common factors}}$ ... $2 \times 2 \times 3 \times 3 \times 5$ (common factors: 2, 3; non-common factors: 2, 3, 5)

= 180

2	30
3	15
5	5
	1

2	36
2	48
3	9
3	3
	1

We can also find LCM by division method

2	30, 36
3	15, 18
	5, 6 → Stop dividing

LCM = $2 \times 3 \times 5 \times 6$

= 180

Step 1: Choose a lowest divisor which divides all or at least two of the numbers carry forward with are not divisiable.

Step 2: The process is continue until no two numbers have any common divisor.

Step 3: The product of divisors and remainders is the LCM.

The LCM of 36, 48 and 56 by prime factorization method.

2	36
2	18
3	9
3	3
	1

2	48
2	24
2	12
2	6
3	3
	1

2	56
2	28
2	14
7	7
	1

Prime factors of 36 = $2 \times 2 \times 3 \times 3$

Prime factors of 48 = $2 \times 2 \times 2 \times 2 \times 3$

Prime factors of 56 = $2 \times 2 \times 2 \times 7$

LCM = Product of common factors and non common factors.

$= 2^4 \times 3^2 \times 7$

$= 16 \times 9 \times 7$

$= 1008$

The LCM of 50, 75, 80 by division method.

Start with the smallest common prime factor →

5 is a common prime factor of 5 and 15, so we divide 5 and 15 by 5 →

2	50, 75, 80
5	25, 75, 40
5	5, 15, 8
	1, 3, 8

Stop dividing when there are no common prime factors between any two numbers.

LCM = Product of divisors x product of numbers left

$= 2 \times 5 \times 5 \times 3 \times 8$

$= 1200$

Find the least number of biscuits which can be equally divided among 9, 12 and 18 children.

3	9, 12, 18
3	3, 4, 6
2	1, 4, 2
	1, 2, 1

To find least number of biscuits, we will find LCM.

So the least number of biscuits = $3 \times 3 \times 2 \times 2$

$= 36$

In a school the duration of each period for primary classes is 30 minutes and the duration of each period for other classes is 35 minutes. If first period of school starts at 8:30 am, after how long will their periods start together? What would be the time at that moment?

$$\begin{array}{r|l} 5 & 30,\ 35 \\ \hline & 6,\ \ 7 \end{array}$$

LCM = $5 \times 6 \times 7$
= 210 minutes
= 3 hours and 30 minutes
Time = 8:30 + 3:30
= 12 pm

After 3 hours and 30 minutes (at 12 pm) the period will start together.

Exercise 3.5

1. Find the LCM of the following by taking multiples:

(i) 12, 14 (ii) 3, 6, 9

2. Find the LCM by prime factorization.

(i) 14, 21 (ii) 18, 24, 36 (iii) 84, 72, 112
(iv) 90, 45, 75 (v) 49, 70, 105, 84 (vi) 480, 560, 720

3. Find the LCM by division method.

(i) 25, 30, 35 (ii) 60, 96, 108 (iii) 70, 210, 126
(iv) 105, 140, 280 (v) 120, 256, 288

4. Find the least number which, when divided by 14, 21 and 18, gives no remainder.

5. Find the length of the shortest piece of string that can be cut into equal lengths of 16cm, 20cm, 24cm.

6. On a light house a red light flashes after 10 minutes, a green light after 12 minutes and a yellow light after 15 minutes. If they flash together at 9 pm, at what time will they flash together again?

7. **Find the shortest length of a pipe that can be measure exactly with 5m, 15m and 25m long measuring tapes respectively.**

8. **Three bells ring at interval of 8 minutes, 15 minutes and 24 minutes respectively. If they ring together at 3 pm, at what time will they next ring together again.**

Words Board

- Factors
- Multiples
- Prime Numbers
- Composite Number
- Index Notation
- HCF
- LCM
- Test of Divisibility

I have learnt

- A number which divides a given number completely, leaving no remainder, is called factor of given number.
- A **prime number** is a number which has only two different factors, 1 and the number itself. Prime numbers are 2, 3, 5, 7, 11, 13, 17 etc.
- A **composite number** is a number which has more then two different factors. Composite numbers are 4, 6, 12, 15, 24, 32 etc.
- A composite number can be expressed as the product of two or more prime numbers.
- The process of expressing a composite number as the product of prime factors is called **prime factorization**.
- The largest of the factors common to two or more numbers is called the **Highest Common Factor** (HCF) of the numbers.
- The smallest of the common multiples of two or more numbers is called **Least Common Multiple** (LCM) of the numbers.

Review Exercise 3

1. **Encircle the correct option?**

(i) A number which divides a given number completely is called a:

(a) factor (b) multiple (c) remainder (d) none of these

(ii) 93 is a multiple of:

(a) 3 (b) 6 (c) 7 (d) 13

(iii) Number of multiples of 7 between 21 and 50 is:

(a) 3 (b) 4 (c) 5 (d) 6

(iv) 7521 is divisible by:

(a) 2 (b) 3 (c) 4 (d) 7

(v) A number is divisible by 6 if it is divisible by:

(a) 2 (b) 3 (c) 2 and 3 (d) 12

(vi) 990 is divisible by:

(a) 2,3,5,11 (b) 2,3,11,13 (c) 3, 11, 17 (d) 5, 7, 9

(vii) 5^4 = ...

(a) 5+5+5+5 (b) 5+5+5+5+5 (c) 5×5×5×5 (d) 20

(viii) There are .... methods for finding HCF.

(a) 2 (b) 4 (c) 5 (d) 3

(ix) HCF of 16 and 48 is

(a) 16 (b) 8 (c) 12 (d) 48

(x) LCM of 4 and 8 is

(a) 4 (b) 8 (c) 16 (d) 32

2. Write the factors of 84 and 96.
What are the common factors of 84 and 96?

3. Here is a set of numbers:-
5, 9, 12, 14, 15, 18, 20, 21, 24, 27, 28, 30, 33, 36, 39, 41, 42, 44, 45, 49, 50
Which of the above numbers are multiples of

(I) 2 (ii) 3 (iii) 4 (iv) 5

(v) 6 (vi) 7 (vii) 8 (viii) 9

4. One of the numbers 2, 3, 4, 5, 6, 8, 9, 11 does not divide 6480. Find it.

5. Express the following as a product of prime factors. Write the answer in index notation.

(i) 350 (ii) 6912

6. Find the greatest mass that can be taken an exact number of times from 360g, 504g and 672g.
7. A rectangular field measures 308m by 228m. Fencing posts are placed along its sides at equal distances apart. If the posts are as far apart as possible, what is the distance between them?
8. Find the smallest mass that can be measured out in equal amount of 6kg, 9kg and 12 kg.
9. Find the least length of a rope which can be cut into pieces of lengths 30m, 36m and 54m.
10. Find the smallest sum of money that is an exact multiple of Rs. 72, Rs. 80 and Rs. 96.

Unit 04

Integers

Learning Outcomes

(i) Recognize and identify integers,
- Negative integers,
- Positive integers,
- Zero (0) as neutral integers.

(ii) Represent integers on a number line.

(iii) Identify integers on number line;
- As positive integers,
- As negative integers,
- As neutral integer.

(iv) Arrange a given list of integers in ascending and descending order.

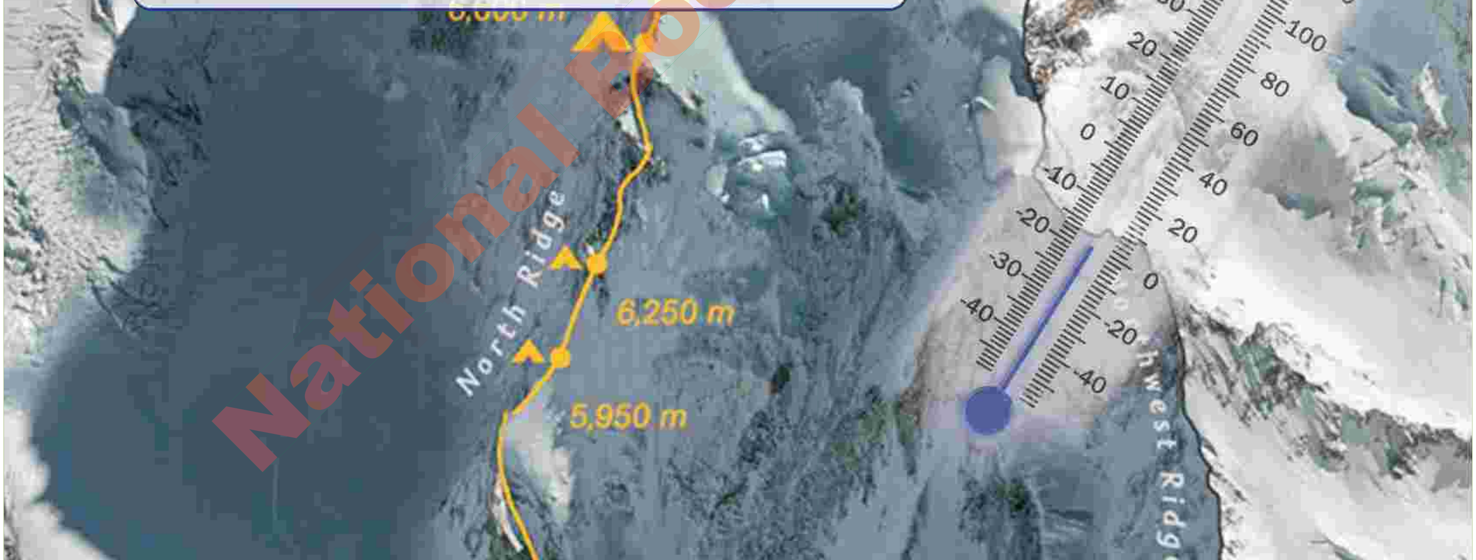
(v) Recognize absolute or numerical value of a number as its distance from zero on the number line.

(vi) Arrange the absolute or numerical values of the given integers in ascending and descending order.

(vii) Subtract two integers with same and opposite sings.

(viii) Multiply two integers with same and opposite sings.

(ix) Divide two integers with same and opposite sings.



There is one place in Pakistan where the temperature is so cold that even the thought of it freezes us. The place is non other than K2 base camp. K2 is the highest peak in Pakistan and 2nd highest peak in the world with height 28,251 feet above see level.

The temperature at K2, base camp (17,700 feet) remains below the freezing point for the entire year. In winter the temperature drops to 50° C below freezing point. To express many real life situations, such as temperature, height, depth's, gain or loss we use positive, negative numbers and zero called integers.

Integers

The extreme weather in Pakistan includes high and low temperature. In winter 2019, the temperature at K-2 dropped 47 degrees centigrade below the freezing point.

On the other hand, in May 2010, the temperature in Larkana rose to 53 degrees centigrade. It was not only the hottest temperature ever recorded in Pakistan, but also the hottest temperature recorded in the continent of Asia.

Ancient city of Mohenjo-daro

We can express temperature at K-2 below freezing point using negative numbers as $-47^\circ C$ and the hottest temperature at Larkana above freezing point using positive number $+53^\circ C$. These numbers are member of the set of integers.

An **Integer** is any number from the set $\{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$

Integers can be represented on a number line.

On number line, positive integers are represented as points to the right of zero, and negative integers as points to the left of zero.

On number line, any positive integer is to the right of a negative integer. Number will increase in value if we move from left to right on a number line.

Point A $=-4$, B $=-2$, C $=1$, D $=5$, E $=-225$, F $=-150$, G $=60$ and H $=175$ are represented on number line as follows.

Key fact

- Integers greater than zero are **positive integers.**
- Integers less than zero are **negative integers.**
- **Zero** is neither positive nor negative.

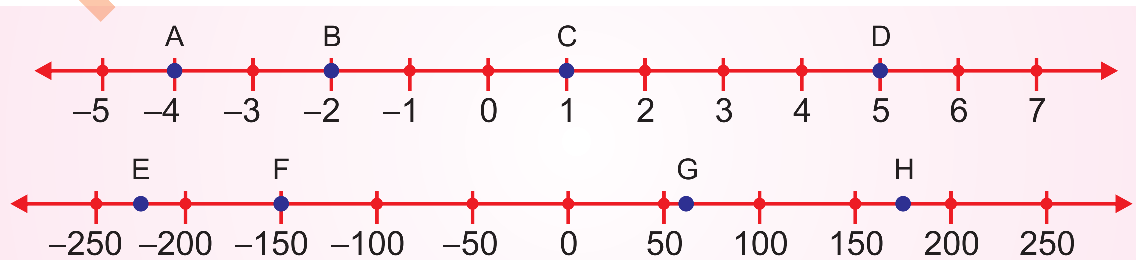
Key fact

- Comparing a positive and negative integer, the positive integer will always be greater.

$1 > -10,000$,

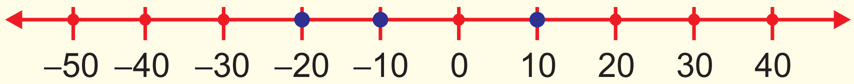
- Negative number is always less than zero and positive number is always greater than zero.

$-10,000 < 0$ and $1 > 0$



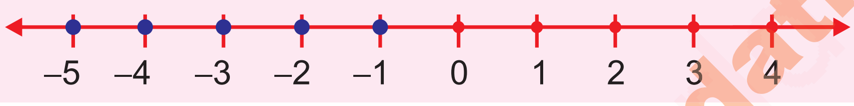
Represent the following integers on number line.
(i) –20 , 10, –10 (ii) integers less than zero (iii) integers greater than 50.

(i)

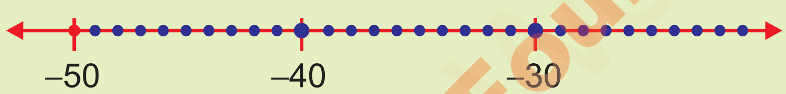


–20 is to the left of –10 on number line so –20 <–10, 10 is to the right of –10 on the number line 10> –10

(ii) Integers less than zero = {–1, –2, –3,...}



(iii) Integers greater than –50 = {–49, –48, –47,...}



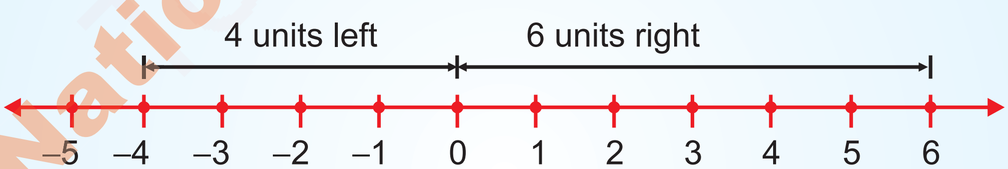
Key fact

- All negative integers are smaller than any positive integer.
- The symbols "<"(less than) and ">" (greater than) are used to compare integers.

Key fact

Positive integers are written without + sign.

The **absolute value** of an integer is its distance from zero on the number line. The symbol for absolute value is | |.



On the number line above, –4 is at a distance of 4 units from zero. The absolute value of –4 is 4
we write | –4 | = 4
and 6 is at 6 units from 0. The absolute value of 6 is 6
we write | 6 | = 6

Key fact

Absolute value of an integer is always positive.

Exercise 4.1

1. Write an integer for each situation and represent on number line.

(i) 10°F below 0.
(ii) A rise of 8°C in temperature.
(iii) 150 meters above see level.
(iv) A loss of 2000 feet in altitude.
(v) A deduction of Rs. 3650 from pay.
(vi) Three years ago from now.
(vii) Water level in a reservoir that is 5 meter above the normal range.
(viii) 25 points lost in a game.
(ix) 4 seconds before the rocket is being launched.
(x) The greatest negative integer.
(xi) The smallest positive integer.
(xii) Set of integers less than zero.
(xiii) Set of integers greater than zero.
(xiv) Set of integers between -1 and 1.
(xv) Set of integers greater than -2 and less than 4.
(xvi) Set of integers between -100 and 100.
(xvii) Set of integers greater than -1.
(xviii) Set of integers greater than -1000 and less than zero.
(xix) Set of integers greater than -6 and less than 6.
(xx) Set of integers greater than 100 and less than 1000.

2. Arrange the integers –100, 0, 5, –5, –1000, –650, 700, –1, –34, –47, 100, –350

(i) In ascending order
(ii) In descending order

3. Fill in the box with $<$ or $>$

(i) 0 $\square$ 4 (ii) -1 $\square$ 0
(iii) 0 $\square$ -4 (iv) -5 $\square$ -4
(v) -33 $\square$ 33 (vi) 18 $\square$ -18

(vii)	$-7 \square -6$	(viii)	$0 \square -1000$
(ix)	$-10,000 \square -1$	(x)	$6 \square 0$
(xi)	$-10 \square 0$	(xii)	$-6 \square 0$
(xiii)	$-6 \square 6$	(xiv)	$15 \square 12$

4. **The lowest temperature ever recorded in K-2 is $-50^\circ C$. The lowest temperature ever recorded in Quetta is $-11^\circ C$. Complete -50 ___ -11 with $<, =$ or $>$**

5. **Monthly average temperatures of a city are given below.**

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
–4	0	13	22	38	42	39	37	28	20	18	–2

List the temperature in order from least to greatest and tell which month is hottest and which one is coldest.

6. **Use a number line to represent each of the following.**

(i) –6, –4, 0, 1, 3, 10, 9

(ii) –120, 400, –360, 0, 320, 200

(iii) Set of integers greater than –60 and less than -10.

(iv) Set of integers between –100 and zero.

7. **Arrange the absolute values of the following integers in descending order**

–40, –30, 79, 0, 14, –36, 36, –29, 80, –65.

Addition of integers (same sign)

During the month of January a few changes in the temperature of a city were observed.

The temperature was 3°C and rose by 7°C.
The temperature was –5°C and fell by 3°C.
What is new temperature in both cases?

The news temperatures in both cases can be found by addition of integers.

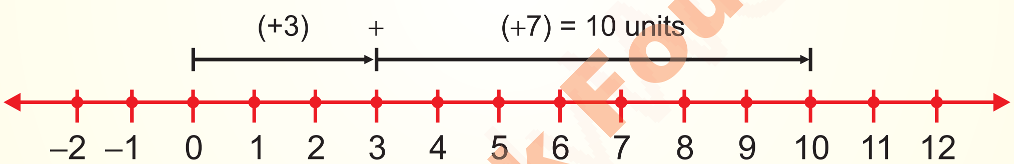
(i) Initial temperature $= +3$

Rise in temperature $= +7$

New temperature $= (+3) + (+7)$

$= +10 = 10\,^{\circ}C$

We can use the number line to find the sum as follows

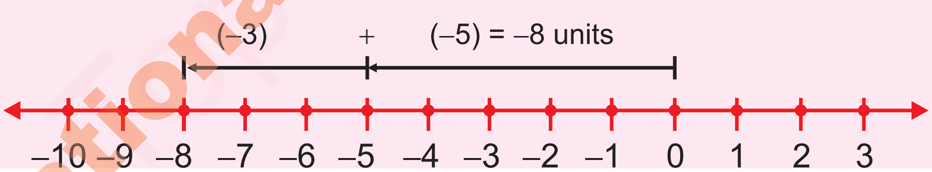


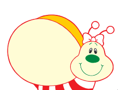
(ii) Initial temperature $= -5$

Fall in temperature $= -3$

New temperature $= (-5) + (-3) = -5 - 3 = -8$

$= -8\,^{\circ}C$





Key fact

To add two integers with the same signs, we add their absolute values. The sum is

- positive if both integers are positive.
- negative if both integers are negative.

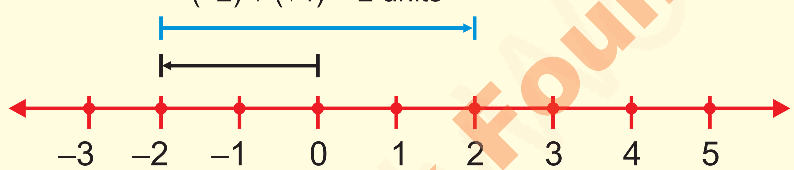
Addition of integers (opposite signs)

Can I find the new temperature when
- The temperature was $-2^{\circ}C$ and rose by 4 degrees.
- The temperature was $10^{\circ}C$ and dropped by 8 degrees.

The new temperatures in both cases can be found by addition of integers.

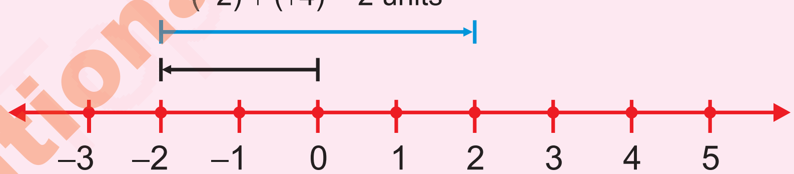
(i) Initial temperature $= -2$
Rise in temperature $= +4$
New temperature $= (-2) + (+4)$
$= -2 + 4 = 2\,^{\circ}C$

$(-2) + (+4) = 2$ units



(i) Initial temperature $= 10$
Fall in temperature $= -8$
New temperature $= (10) + (-8)$
$= 10 - 8 = 2\,^{\circ}C$

$(-2) + (+4) = 2$ units



Key fact

To add integers with different signs we **subtract** their absolute values
The sum is
- Positive if positive integer has the greatest absolute value.
- Negative if negative integer has the greatest absolute value.

Exercise 4.2

1. Calculate the following

(i) $(-4) + 8$ (ii) $(-1) + 8$

(iii) $(-15) + 2$ (iv) $9 + 9$

(v) $9 + (-9)$ (vi) $7 + (-3)$

(vii) $(-100) + (-30)$ (viii) $-430 + (-55)$

(ix) $(-105) + (-100)$ (x) $0 + (-10)$

2. Evaluate the following

(i) $-7 + (-11) + (-9)$ (ii) $-40 + (-50) + (60)$

(iii) $34 + 15 + (-13)$ (iv) $16 + (-15) + (-12)$

(v) $[-41 + (-13)] + [65 + (-31)]$ (vi) $[-17 + 12] + [-7 + 9]$

(vii) $[-36 + 40] + [41 + (-91)]$ (viii) $[4 + (-7)] + [-8 + (-12)]$

(ix) $[-16 + (-10)] + [-25 + (-12)]$ (x) $[16 + (-25)] + [-42 + (56)]$

3. Add the following on number line.

(i) $7, 3$ (ii) $-5, -4$

(iii) $-2, -2$ (iv) $6, -6$

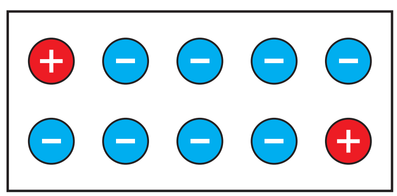
(v) $-6, 10$ (vi) $3, -8$

(vii) $-9, -3$ (viii) $5, 2$

(ix) $-8, 5$ (x) $-5, -3$

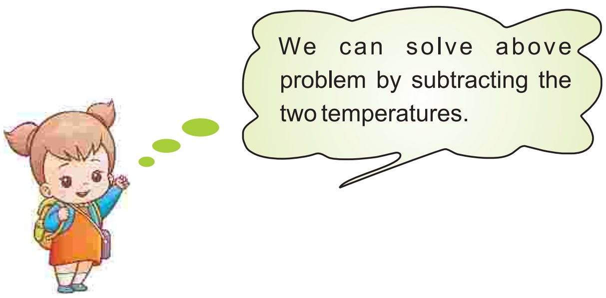
4. After attaining an altitude of 12,450 feet a rocket separated from its main engine. The engine fell into the ocean to a depth of 2,600 feet. How for did it fall?

5. Find the net charge of 2 positively charged particles and 8 negatively charged particles.



Subtraction of integers

At lunar noon, the temperature on the moon's surface was 120°C. By midnight the temperature was –160°C. What is the difference in temperature of lunar day and lunar night?



Temperature of lunar day = 120°C
Temperature of lunar night = –160°C
Difference in temperature of day and night = 120 – (–160) = 120 + 160
= 280°C

We can subtract first number from the second as follows.

(i) Subtract –9 from –3
–3 – (–9) = –3 + 9 = 6

(ii) Subtract 9 from –2
–2 – (9) = –2 – 9 = –11

(iii) Subtract 4 from 2
2 – (4) = 2 – 4 = –2

(iv) Subtract –98 from 0
0 – (98) = 0 + 98 = 98

(v) Subtract 55 from 0
0 – (55) = 0 – 55 = –55

(vi) Subtract –150 from –200
–200 – (–150) = –200 + 150 = –50

Key fact

To subtract integers
Change the sign of the integer being subtracted and add both integers according to the rule of addition.

Exercise 4.3

1. Simplify the following calculate without using number line.

(i)	$11-(3)$	(ii)	$14-(-6)$
(iii)	$-7-(7)$	(iv)	$-8-(-5)$
(v)	$-7-(-7)$	(vi)	$19-(4)$
(vii)	$0-(-12)$	(viii)	$8-(-5)$
(ix)	$30-(-15)$	(x)	$-8-(-7)$
(xi)	$-18+30$	(xii)	$13+(-8)$
(xii)	$-81+(-60)-(-10)$	(xiv)	$41-(-11)+(-7)$
(xv)	$-8-(-1)+19$	(xvi)	$[-4+(-14)]+[-39+29]$
(xvii)	$[29+(-48)]-[11-(-35)]$	(xviii)	$[-15-(-48)]-[-11-(-15)]$
(xix)	$-30+[(-12)-2]$	(xx)	$65-(-30)+[(-25)+(-10)]$

2. **In some hot places on earth temperature may reach 136°F, while in some cold places temperature may be as low as – 89°F. What is the difference between these two temperatures?**

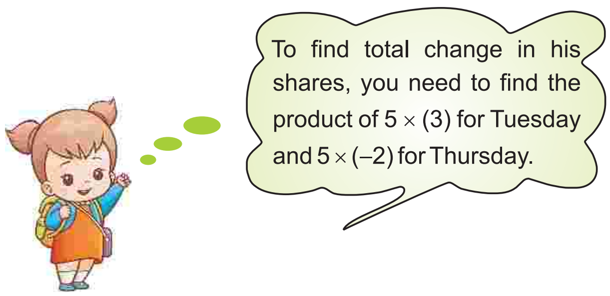
3. **In the first round of a game Ahmed lost 24 points. In the second round he gained 18 points. What was his score after two rounds?**

4. **The melting point of copper is 1,083°C. Its boiling point is 1,479°C greater than its melting point. What is the boiling point of copper?**

5. **A submarine has an altitude of –5500 feet and it dives down 1500 feet. What is its new altitude in feet?**

Multiplication of integers

The change in the price of stock in a corporation for Tuesday was increased by Rs. 3 per share and on Thursday it was decreased by Rs. 2 per share. If a person has 5 shares, which integers represent the total change in values of his shares on Tuesday and Thursday.



For Tuesday change in the values of shares can be found as follows
Increase in value per share $= 3$
Change in value of 5 shares $= (3) \times (5) = 15$
Similarly for Thursday change in values of shares is
Decrease per share $= -2$
Change in values of 5 shares $= (-2) \times (5) = -10$

We can multiply the following integers as

(i) $4 \times (-6) = -24$
(ii) $-1 \times 14 = -14$
(iii) $-100 \times 3 = -300$
(iv) $-11 \times (-7) = 77$
(v) $-7 \times (-9) \times (-6)$
$= -7 \times 54 = -378$
(vi) $-5 \times 8 \times (-12)$
$= -5 \times (-96) = 480$
(vii) $0 \times (-100) = 0$

Key fact

To multiply two integers, multiply their absolute values
- The product is possitive if both integers have same signs.
- The product is negative if both integers have opposite signs.

Division of integers

In month of March the temperature of a city dropped 24°F during 3 hours period. What integer represents the average temperature change per hour?

To find average temperature change you need to find $-24 \div 3$

Fall in temperature in 3 hours $= -24$

Average change in temperature per hour $= -24 \div 3$

$$= \frac{-24}{3} = -8\,^{\circ}F$$

We can divide the following integers as

(i) $15 \div (-5) = \frac{15}{-5} = -3$

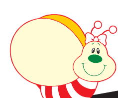
(ii) $-99 \div (-33) = \frac{-99}{-33} = 3$

(iii) $-72 \div 8 = \frac{-72}{8} = -9$

(iv) $250 \div (-50) = \frac{250}{-50} = -5$

(v) $108 \div 9 = \frac{108}{9} = +12$

(vi) $-100 \div -1 = \frac{-100}{-1} = 100$



Key fact

To divide two integers, divide their absolute value. The quotient will be

- positive when the two integers have the same signs.
- negative when the two integers have the opposite signs.

Exercise 4.4

1. Use an appropriate numeral in the box

(i) $5 \times \square = -10$ (ii) $-6 \times \square = -540$

(iii) $\square \times (-21) = -84$ (iv) $8 \times \square = -80$

(v) $-7 \times \square = 0$ (vi) $12 \times \square = -144$

(vii) $-100 \times \square = -200$ (viii) $-16 \times \square = 32$

(ix) $\square \times 12 = 48$ (x) $\square \times (-7) = -28$

(xi) $-314 \times \square = -3140$ (xii) $-125 \times \square = 0$

(xiii) $-12 \times \square = 60$ (xiv) $-15 \times \square = 120$

2. Fill in the box

(i) $63 \div (-7) = \square$ (ii) $210 \div 15 = \square$

(iii) $-90 \div (-5) = \square$ (iv) $-90 \div \square = -18$

(v) $-154 \div 77 = \square$ (vi) $136 \div \square = -4$

(vii) $-21 \div \square = 3$ (viii) $\square \div 14 = -7$

(ix) $\square \div (-13) = 3$ (x) $\square \div (-5) = -9$

3. Find the product of the following

(i) –36, –12 (ii) 345, –15

(iii) –18, 5 (iv) 11, –12

(v) –14, – 15 (vi) 12, 13

4. Complete the following by filling the box with appropriate sign.

(i) $5 \square (-2) = (-2) \square 5 = -10$

(ii) $(-9) \square 6 = 6 \square (-9) = -54$

(iii) $(-12) \square 12 = 12 \square (-12) = -1$

(iv) $7 \square (-13) = (-13) \square 7 = -91$

(v) $(-5) \square 5 = 5 \square (-5) = -1$

5. Fill in the blanks in the following

(i) $[(5) \times \square] \times (2) = (5) \times [(-2) \times \square]$

(ii) $(-4) \times [(2) \times \square] = [(-4) \times \square] \times (-6)$

(iii) $(-8) \times [(-6) \times \square] = [\square \times (-6)] \times (-3)$

(iv) $\square \times [0 \times (-100)] = [10 \times 0] \times \square$

(v) $[100 \times \square] \times 4 = 100 \times [31 \times \square]$

Words Board

- Integers
- Positive integers
- Negative integers
- Neutral
- Absolute value
- Sum
- Difference
- Product
- Quotient

I have learnt

- The set {..., –2, –1, 0, 1, 2,...} is called the set of integers.
- Zero is neither positive nor negative and hence is neutral integer.
- On a number line all positive integers lie to the right of zero and all negative integers lie to the left of zero.
- All negative integers are smaller than all positive integers and zero.
- The absolute value of an integer is its distance from zero on the number line.
- To add integers with the same sign we add their absolute values, the sum is positive if both integers are positive and the sum is negative if both integers are negative.
- To add integers with opposite sign, we subtract their absolute values and put the sign of the integer which has the greatest absolute value.
- To subtract integers we change the sign of the integers being subtracted and add both integers according to the rule of addition.
- The product / quotient of two integers is positive if both integers have same signs and product / quotient is negative if both integers have opposite signs.

Review Exercise 4

1. Encircle the correct option in the following statements:

i. The temperature above freezing point is represented by _______ integers.

(a) 0 (b) Negative (c) Positive (d) Both (b) and (c)

ii. **Depth of 100meter below sea level is represented by ______ m.**

(a) -100 (b) $+100$ (c) 0 (d) $+1$

iii. **If -30° represents rotation in clockwise direction then rotation in anticlockwise direction is represented by _________ .**

(a) -30° (b) $+30^\circ$ (c) $\pm30^\circ$ (d) 60°

iv. **On number line all positive integers lie to the ________ of zero.**

(a) left (b) right (c) above (d) below

v. **The absolute value of an integer is its distance from ________ .**

(a) negative integer (b) positive integer (c) 0 (d) 1

vi. **The absolute value of an integer can never be __________ .**

(a) positive (b) 1 (c) 0 (d) negative

vii. **$|-7|$ = __________ .**

(a) -7 (b) $+7$ (c) 0 (d) -1

viii. **-1000 __________ 0.**

(a) $\leq$ (b) $\leq$ (c) $>$ (d) $<$

ix. **-1 __________ -1000 .**

(a) $>$ (b) $<$ (c) $\leq$ (d) $\leq$

x. **On number line the value of integer _________ as we move to the right of the integer.**

(a) decreases (b) increases (c) stationery (d) none

xi. **Product of two integers with same signs is always __________ .**

(a) negative (b) positive (c) 0 (d) both (a) and (b)

xii. **The quotient of two integers with different signs is _________ .**

(a) negative (b) positive (c) 0 (d) both (a) and (b)

2. In the following table, temperatures in different conditions are given.

Temperature in Different conditions	
Condition	Temperature (in °Celcius)
Boiling point of water	100
Average oven temperature	180
Temperature of human body	37
Freezing point of water	0
Freezing point of mercury	–38.8
Temperature of dry ice	–80
Temperature of lunar day	120
Temperature of lunar night	–160

i. What is the coldest and hottest temperature given?

ii. What is the difference in temperature of the lunar day and lunar night?

iii. Arrange the temperatures in order from hottest to coldest.

3. If in first two rounds of a game Ali gained 18 and 20 points respectively. In third round he lost 15 points. What was his score after three rounds?

4. Mr. Umer owns 18 shares of stocks in a corporation. The change in the price of stock for Monday was reported Rs. 6 per share. Find total change in value of his shares of stock for Monday.

5. At noon the temperature was 35°F. Between noon and 6pm, the temperature drop down by 4°F and then rise up by 2°F. What was the temperature at 6pm?

6. Simplify the following.

(i) $4\times(-29)+8\times(-3)$ (ii) $-5\times(-3+4)$

(iii) $2\times(-1)+7\times(-2)$ (iv) $12\times(-21+9)+(-3)$

(v) $8+(-3)\times(-6+12)$ (vi) $-6\times3+4+(-9)$

(vii) $7+(-6)+3\times(-1)$ (viii) $-9\times(-2+0)$

(ix) $-5\times(-3\times4)$ (x) $-8\times(-8\times14)$

Unit
05

SIMPLIFICATION

Learning Outcomes

(i) Identify the following brackets:
- "——" vinculum,
- ( ) Curved brackets,
- { } Curly Brackets,
- [ ] Square brackets,

(ii) Recognize the order of preference for simplification involving brackets.

(iii) Simplify mathematical expression involving fractions and decimals using BODMAS rule.

Mathematics contributes directly to business, finance, health and every day life. Particularly, handling of mathematical problems involving fractions and decimals along with various arithmetic operations require a special technique, skill and practice. Conceptual understanding is required to overcome such kind of problems. By studying the concept of simplification, the students can apply and solve problems in real life situation and work place.

SIMPLIFICATION

Mr. Khalid has Rs. 600 and he gives one-half to his wife, one fourth to his daughter and one sixth to his son from the total amount. How much amount is left with Khalid?

$$=600-\left[\underbrace{\left(\frac{1}{2}\times 600\right)}_{\text{Wife's share}}+\underbrace{\left(\frac{1}{4}\times 600\right)}_{\text{Daughter's share}}+\underbrace{\left(\frac{1}{6}\times 600\right)}_{\text{Son's share}}\right]$$

Use BODMAS rule

$$=600-[300+150+100]$$

$$=600-[550]$$

$$=50 \text{ (Left with Khalid)}$$

$$=\text{Rs.}50$$

Sometimes, we need more than one operation to simplify the expression.

For solution of this type of problem involving numbers, mixed fractions and brackets, we use BODMAS rule

BODMAS rule

B - Brackets ⟶ First
O- Of (multiplication) ⟶ Second
D- Division ⟶ Third
M - Multiplication ⟶ Fourth
A- Addition ⟶ Fifth
S- Subtraction ⟶ Sixth

Brackets (Group Symbols According to Order)

First: " $\overline{\quad\quad}$ " Vinculum, bar bracket
Second: "( )" Small brackets, parentheses or curved brackets, round or simple brackets.
Third: "{ }" Braces, curly brackets, flower brackets or medium brackets.
Fourth: "[ ]" Square brackets or big brackets.

We simplify in the same order.

We can simplify $\frac{1}{3}+\left\{3\frac{1}{2}-\left(\frac{1}{2}+\overline{\frac{1}{3}-\frac{1}{4}}\right)\right\}$ as follows.

$$=\frac{1}{3}+\left\{3\frac{1}{2}-\left(\frac{1}{2}+\overline{\frac{1}{3}-\frac{1}{4}}\right)\right\}$$ Apply BODMAS rule

$$=\frac{1}{3}+\left\{\frac{7}{2}-\left(\frac{1}{2}+\overline{\frac{4-3}{12}}\right)\right\}$$ Convert mixed fraction into fraction and simplify under bar brackets

$$=\frac{1}{3}+\left\{\frac{7}{2}-\left(\frac{1}{2}+\frac{1}{12}\right)\right\}$$

BRACKETS

Brackets tell us which part of the sum is to be done first.

$$=\frac{1}{3}+\left\{\frac{7}{2}-\left(\frac{6+1}{12}\right)\right\}$$ Simplify expressions within curve brackets

$$=\frac{1}{3}+\left\{\frac{7}{2}-\frac{7}{12}\right\}$$

$$=\frac{1}{3}+\left\{\frac{42-7}{12}\right\}$$ Simplify within curly brackets

$$=\frac{1}{3}+\frac{35}{12}$$

$$=\frac{4+35}{12}$$ Simplify

$$=\frac{39}{12}$$

$$=3\frac{3}{12}$$ Convert into mixed fractions

Find the answer

$64 \div 4 \div 4 \div 4 + 2 \times 3 \div 3$

We simplify $2\frac{2}{3}\times\left[1\frac{1}{3}-\overline{\frac{1}{2}+\frac{1}{3}}\right]$ by the following rules.

$$=2\frac{2}{3}\times\left[1\frac{1}{3}-\overline{\frac{1}{2}+\frac{1}{3}}\right]$$ Apply BODMAS rule

$$=\frac{8}{3}\times\left[\frac{4}{3}-\overline{\frac{1}{2}+\frac{1}{3}}\right]$$ First convert mixed fractions into fraction

$$=\frac{8}{3}\times\left[\frac{4}{3}-\overline{\frac{3+2}{6}}\right]$$ Simplify under vinculum " $\overline{\quad}$ "

$$=\frac{8}{3}\times\left[\frac{4}{3}-\frac{5}{6}\right]$$

$$=\frac{8}{3}\times\left[\frac{4\times2-5}{6}\right]$$ Simplify the expression within the brackets

$$=\frac{8}{3}\times\left[\frac{8-5}{6}\right]$$

$= \frac{\cancel{8}^{4}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{6}_{3}}$ Simplify

$= \frac{4}{3}$

$= 1\frac{1}{3}$ Covert into mixed fraction

We simplify $\left[1\frac{7}{13} \times \left\{1\frac{2}{5} - \left(1\frac{5}{11} \div 7\frac{1}{2} \times \overline{1\frac{1}{4} + 1\frac{1}{2}}\right)\right\}\right]$ as follows.

$$\left[1\frac{7}{13} \times \left\{1\frac{2}{5} - \left(1\frac{5}{11} \div 7\frac{1}{2} \times \overline{1\frac{1}{4} + 1\frac{1}{2}}\right)\right\}\right]$$

$$= \left[\frac{20}{13} \times \left\{\frac{7}{5} - \left(\frac{16}{11} \div \frac{15}{2} \times \frac{5}{4}\ \frac{3}{2}\right)\right\}\right]$$

$$= \left[\frac{20}{13} \times \left\{\frac{7}{5} - \left(\frac{16}{11} \div \frac{15}{2} \times \frac{5+6}{4}\right)\right\}\right]$$

$$= \left[\frac{20}{13} \times \left\{\frac{7}{5} - \left(\frac{16}{11} \div \frac{15}{2} \times \frac{11}{4}\right)\right\}\right]$$

$$= \left[\frac{20}{13} \times \left\{\frac{7}{5} - \left(\frac{\cancel{16}}{\cancel{11}} \times \frac{2}{15} \times \frac{\cancel{11}}{\cancel{4}}\right)\right\}\right]$$

$$= \left[\frac{20}{13} \times \left\{\frac{7}{5} - \frac{8}{15}\right\}\right] = \left[\frac{20}{13} \times \left\{\frac{21-8}{15}\right\}\right]$$

$$= \left[\frac{\cancel{20}}{\cancel{13}} \times \frac{\cancel{13}}{\cancel{15}}\right] = \frac{4}{3} = 1\frac{1}{3}$$

Check Point

You lost Rs. 10 which was two fifth of your pocket money. What was your pocket money?

For simplication $1.8 + \left[2 + \left\{1.2 + \left(3.5 + \overline{0.6 - 0.1}\right)\right\}\right]$ apply the following procedure.

$$= 1.8 + \left[2 + \left\{1.2 + \left(3.5 + \overline{0.6 - 0.1}\right)\right\}\right]$$

$$= 1.8 + \left[2 + \left\{1.2 + \left(3.5 + 0.5\right)\right\}\right]$$

$$= 1.8 + \left[2 + \left\{1.2 + 4.0\right\}\right]$$

$$= 1.8 + \left[2 + 5.2\right]$$

$$= 1.8 + 7.2$$

$$= 9.0$$

Check Point

A book is 20mm thick. How many books would be required to make a packet 36cm high?

Exercise 5.1

Solve using BODMAS rule

1. $8\frac{2}{3}-5\frac{1}{3}\div2\frac{2}{3}+\frac{1}{3}$

2. $\frac{9}{20}-\left[\frac{1}{5}+\left\{\frac{1}{4}+\left(\frac{5}{6}-\overline{\frac{1}{3}+\frac{1}{2}}\right)\right\}\right]$

3. $\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]$

4. $\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{ of }4\frac{1}{3}\right)$

5. $2+\left\{2\frac{1}{4}-\left(\frac{5}{7}\div\overline{\frac{2}{3}-\frac{1}{7}}\right)\div\frac{8}{21}\right\}$

6. $\left(12\frac{3}{5}\div1\frac{4}{5}\right)\times\left(\frac{1}{8}\text{ of }1\frac{1}{7}\right)\div1\frac{5}{9}$

7. $1\frac{4}{5}\div\left[\frac{1}{25}\times\left\{1\frac{1}{4}+\left(3\frac{1}{3}\div2\frac{1}{2}\times1\frac{5}{16}\right)\right\}\right]\times\frac{1}{2}$

8. $\{(12.2\div2)\times3.6\}\div3$

9. $[\{0.4-(0.65\div5)\}\times(15-\overline{4.8-1.8})+8]-3$

10. $13.311\div[3.251+\{2.045-(1.9\times\overline{1.06-1.02})\}]$

Key Words

- DMAS rule
- BODMAS rule
- Of (Multiplication)
- Group Symbols
- $\overline{\quad}$ Vinculum,
- ( ) Curved brackets,
- { } Curly Brackets,
- [ ] Square brackets,

I have learnt

- While solving a problem, we first do the process of Division (D), then Multiplication (M), then Addition (A) and finally Subtraction (S). This rule is known as DMAS rule.
- While solving a problem involving brackets, we solve brackets first and then use DMAS rule.
- **BODMAS rule**
  - B - Brackets ⟶ First
  - O- Of (multiplication) ⟶ Second
  - D- Division ⟶ Third
  - M - Multiplication ⟶ Fourth
  - A- Addition ⟶ Fifth
  - S- Subtraction ⟶ Sixth
- **Brackets (Group Symbols According to Order)**
  - First: " $\overline{\quad\quad}$ " Vinculum, bar bracket
  - Second: "( )" Round or simple brackets
  - Third: "{ }" Braces, curly brackets
  - Fourth: "[ ]" Square brackets.

Review Exercise 5

1. Choose the Correct Option.

i. $\left(\frac{1}{2}\div\frac{4}{3}-\frac{1}{8}\right)\times\frac{8}{7}$ is

(a) $\frac{7}{2}$ (b) 1 (c) $\frac{9}{2}$ (d) $\frac{2}{7}$

ii. $72\div9$ of $\frac{1}{3}+\frac{2}{3}\times1\frac{1}{2}$

(a) 15 (b) 25 (c) 23 (d) 32

iii. $3.4\times1.8\div1.53+13.4$

(a) 17.8 (b) 16.8 (c) 17.4 (d) 16.4

iv. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of $875\div\frac{1}{5}$

(a) 1000 (b) 500 (c) 300 (d) 100

v. $1\frac{8}{13}\times60\frac{2}{3}\div2+17$

(a) 11 (b) 22 (c) 55 (d) 66

vi. $4\frac{4}{5} \div \frac{3}{5}$ of $5 + \frac{4}{5}$

(a) $1\frac{2}{5}$ (b) $2\frac{2}{5}$ (c) $2\frac{1}{5}$ (d) 10

vii. $\frac{7}{12} + \left(\frac{3}{8} \times \frac{4}{9} \div \frac{6}{18}\right) - \frac{5}{8}$

(a) $\frac{11}{12}$ (b) $\frac{9}{24}$ (c) $\frac{24}{11}$ (d) $\frac{11}{24}$

viii. 0.6 of 150

(a) 60 (b) 75 (c) 90 (d) 150

ix. Type of bracket “$\overline{\quad\quad}$” is called.

(a) Curve (b) Braces (c) Square (d) Vinculum

x. $5\frac{1}{4} - \left(\frac{1}{4} \text{ of } 5\frac{1}{4}\right)$

(a) $3\frac{15}{16}$ (b) $4\frac{15}{16}$ (c) $3\frac{16}{15}$ (d) $\frac{15}{16}$

Simplify.

2. $5\frac{1}{7} - \left\{3\frac{3}{10} \div \left(2\frac{4}{5} - \frac{7}{10}\right)\right\}$

3. $1 \div \frac{3}{7} \text{ of } (6 + 8 \times \overline{3-2}) + \left[\frac{1}{5} \div \frac{7}{25} - \left\{\frac{3}{7} + \frac{8}{14}\right\}\right]$

4. $9\frac{3}{4} \div \left[2\frac{1}{6} + \left\{4\frac{1}{3} - \left(1\frac{1}{2} + 1\frac{3}{4}\right)\right\}\right]$

5. $5\frac{1}{2} + \left[\frac{3}{2} \div \left\{\left(\frac{1}{4} - \frac{1}{8}\right) \times \frac{3}{4}\right\}\right]$

6. $2.04 + [1.56 \div \{2.4 - (1.8 \times \overline{0.3 + 0.6})\}]$

7. $[2.95 + \{3.02 \times (6.125 \div \overline{5.196 - 2.746})\}]$

8. $11.34 \times [3.42 + \{11.075 - (3.045 + 2.064 \div 1.032)\}]$

Unit 06

Ratio, Rate and Proportion

Learning Outcomes

(i) Recognize ratios.
(ii) Calculate the ratio of two numbers / quantities.
(iii) Reduce a given ratio into lowest form.
(iv) Recognize and use measures of rate.
(v) Apply problem solving strategies involving rates in real life situations.
(vi) Recognize proportion as a relation between two ratios.
(vii) Find direct and inverse proportion.
(viii) Apply problem solving strategies involving proportions in real life situations.

Fibonacci's Golden Ratio can be seen in the stunning spiral staircase in Italy designed by Giuseppe Memo in 1932. The golden ratio can be found through out the human body. Petals and leaves are often found in this distributions. Rule of golden ratio is mostly followed in the size of TV, LCD, papers etc.

Ratio

Areeba and Munazza went to bakery. Areeba bought 6 donuts and Munazza bought 3 donuts.


We can compare the number of items that and Areeba and Munazza bought using a ratio.

We say the **ratio** of the number of donuts bought by Areeba and Munazza is 6:3. In a ratio, ":" means "to". We read the ratio 6:3 as "6 to 3".

We can also say the ratio is 6:3.

A ratio is a comparison of like quantities.

In the ratio 6:3. the numbers 6 and 3 are known as **terms.**

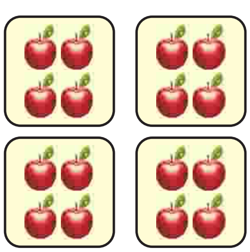
Aayan has a jug and a glass. The height of the jug is 29 cm. The height of the glass is 14 cm. What is the ratio of the height of the jug to the height of the glass.

We do not write units when we write a ratio
29 ~~cm~~ : 14 ~~cm~~

The ratio of the height of the jug to the height of the glass is 29 : 14.

Equivalent ratios

Umer has 16 apples and 12 mangoes. The ratio of the number of apples to the number of mangoes is 16:12. He then put all the items into groups of four. There are now 4 groups of apples and 3 groups of mangoes. We can say that the ratio of the number of apples to the number of mangoes is 4:3.



The ratio 16:12 and 4:3 are equivalent ratios.
4:3 is the ratio in its simplest form

$$\begin{array}{rcccl} \times 2 & 4 & : & 3 & \times 2 \\ \times 2 & 8 & : & 6 & \times 2 \\ & 16 & : & 12 & \end{array} \qquad \begin{array}{rcccl} \div 2 & 16 & : & 12 & \div 2 \\ \div 2 & 8 & : & 6 & \div 2 \\ & 4 & : & 3 & \end{array}$$

Key fact

- If we multiply or divide the terms of a ratio by the same non zero number. We get an equivalent ratio.
- The process will continue until we get the ratio in simplest form.
- We get the simplest form of a ratio when we cannot divide the terms further by any number except 1.

Here is a recipe for lemon juice that Mr. Aayan wants to use.

LEMON JUICE
(Makes 1litre)
- 1 cup of lemon squash
- 4 cups of water

Mr. Aayan uses 2 cups of lemon squash instead. How much water must he use to make the lemon juice?

Lemon squash : Water

$$\begin{array}{rcccl} \times 2 & 1 & : & 4 & \times 2 \\ = & 2 & : & 8 & \end{array}$$

He must use 8 cups of water to make lemon juice.

If he uses 3 cups of lemon squash instead, how much water must he use?

Lemon squash : water

$$\begin{array}{rcccl} \times 3 & 1 & : & 4 & \times 3 \\ = & 3 & : & 12 & \end{array}$$

Find the ratio of the age of father to the age of son.

Father
Age 36 years

Son
Age 12 years

Father's age : Son's age

÷ 2	36	:	12	÷ 2
÷ 2	18	:	6	÷ 2
÷ 3	9	:	3	÷ 3
	3	:	1	

We can also write
ratio of the age of father to the age of son is
$36 : 12 = \frac{36}{12}$
$= \frac{3}{1}$
$= 3 : 1$
36 : 12 is equivalent to 3 : 1

There are 18 boys and 24 girls in a class. Find the ratio of:
The number of boys to the number of girls.
The number of girls to the number of boys.

Ratio of the number of boys to the number of girls = 18 : 24
$= \frac{18}{24} = \frac{3}{4}$
$= 3 : 4$

Ratio of the number of girls to the number of boys = 24 : 18
$= \frac{24}{18} = \frac{4}{3}$
$= 4 : 3$

Key fact

The ratio of a to b, where a and b represent two quantities and b is not zero, is written as a : b or . $\frac{a}{b}$

The following ratios in the lowest form can be expressed as:

(i) 33 : 121 (ii) $\frac{5}{6} : \frac{7}{10}$

(iii) 0.1 : 0.5

(i) 33 : 121
$33 : 121 = \frac{33}{121}$
$= \frac{3}{11}$
33 : 121 = 3 : 11

(ii) $\frac{5}{6} : \frac{7}{10}$
$\frac{5 \times 5}{6 \times 5} = \frac{7 \times 3}{10 \times 3}$ (make the same denominator)
$\frac{25}{30} = \frac{21}{30}$
$= 25 : 21$

Key fact

- A ratio has no units.
- Comparison of ratio is possible when both the quantities have the same units.
- We do not write units 3kg : 4kg, we simply write 3 : 4

(iii) $0.1 : 0.5$

$0.1 : 0.5$

$= \frac{0.1}{0.5}$ (Ratio can be written as fraction)

$= \frac{1}{5}$

$= 1 : 5$

Key fact

In a:b, the first element a is called **antecedent** and second element b is called **consequent**.

Express each as a ratio of the first quantity to the second in its lowest form.

(i) 600 m, 1km (ii) 2kg, 1500g

(i) 600 m : 1 km

$= \frac{600\text{m}}{1\text{km}}$

$= \frac{600\text{m}}{1000\text{m}}$ (Convert to the same unit) 1km = 1000m

$= \frac{600}{1000}$

$= \frac{3}{5}$

$= 3 : 5$

(ii) 2 kg : 1500 g

$= \frac{2\text{kg}}{1500\text{g}}$

$= \frac{2000\text{g}}{1500\text{g}}$ (Convert to the same unit)

$= \frac{2000}{1500}$

$= \frac{4}{3}$

$= 4 : 3$

Check Point

A ratio is said to be in its simplest form if the HCF of its terms is?

Exercise 6.1

1. **Express each of the following in the lowest form:**

   (i) $12 : 72$ (ii) $0.4 : 20$ (iii) $\frac{3}{2} : \frac{1}{3}$

   (iv) $3 : 2\frac{1}{2}$ (v) $0.7 : 70$ (vi) $15 : 121$

2. **Express each as a ratio of the first quantity to the second in its lowest form:**

   (i) Rs 350, Rs 425 (ii) 210°, 360° (iii) 3 kg, 2000g

   (iv) 1200 m, 2 km (v) 1.5 hour, 30 min (vi) 6m, 80 cm

3. In a class of 25 students, there are 11 boys.

(i) How many girls are in the class?

(ii) What is the ratio of girls to boys?

(iii) What is the ratio of boys to girls?

4. Daily income of a labourer and carpenter is Rs 600 and Rs 800 respectively. Find the ratio between their income in the lowest form.

5. Ayesha earned Rs 1800 and spent Rs 1200 in a day. Find the ratio of her income to expenditure.

Rate

Abdullah bought one dozen eggs for Rs. 120. What is the cost of 1 egg?

Cost of 12 eggs = Rs. 120

Cost of 1 egg = Rs. $\frac{120}{12}$ (one dozen)

$= \text{Rs. } \frac{120}{12}$ ← Rupees / ← Eggs

$= \text{Rs. } \frac{120}{12}$

The rate of 1 egg = Rs 10

one dozen = 12

A car travels 660 km in 33 litres. If the car travels 100km, what will be the fuel consumption in litres.

660km travel by fuel = 33 litres

1 km travel by = $\frac{33}{660}$ ← Litres / ← Km

The rate = $\frac{1}{20}$ litre per km.

For 100km, the fuel consumed = $\frac{1}{20} \times 100$ = 5 liters.

Saira works 5 hours and earns Rs. 1500. How much will she get for working 15 hours.

For working 5 hours she earns = Rs. 1500

The pay for 1 hour = Rs. $\dfrac{1500}{5}$ ← Rupees / ← Hours

The rate for 1 hour = Rs. $\dfrac{1500}{5}$ = Rs. 300 per hour.

She will be paid for 15 hours = Rs. 300 $\times$ 15

= Rs. 4500 for 15 hours.

Key fact

- A rate is a comparison of two different quantities.
- Units are mentioned while writing rates.

Farhan can type 90 words in 2 minutes. How many words can he type in 5 minutes?

Farhan can type words in one minute = $\dfrac{90}{2}$

= 45 words per minutes

He can type in 5 minutes = 45 $\times$ 5

= 225 words in 5 minutes

Exercise 6.2

1. **Amjad earns Rs. 2500 in 5 days. What is his pay for 3 days?**
2. **A shopkeeper buys 70 packets of biscuits for Rs 2800. How much will he have to pay if he buys 150 such packets?**
3. **An amusement park has 350 visitors over the course of 7 hours. At this rate, how many visitors would they expect over 15 hours?**
4. **A car uses 20 liters of petrol to travel 170 km. How far can it travel if it has only 16 liters of petrol?**
5. **Uzair drives 232 km in 4 hours. At this rate how far can he drive in 7 hours?**
6. **The cost of 10 m pipe is Rs. 2200. What is the cost of 22 m of such a pipe?**
7. **Cost of 10 books is Rs. 1640. What is the cost of 4 books?**
8. **Rs. 500 is charged for 50 units of electricity. Find the cost of 20 units of electricity.**

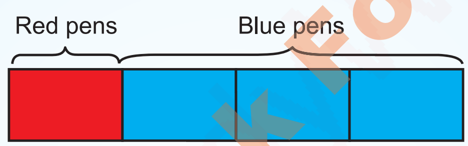
9. Moeed pays total of Rs. 60,000 rent for three months of a flat. Find his annual rent of flat.

10. Mubeen planted 600 trees in 30 days. How much trees will he plant in next 25 days.

Proportion

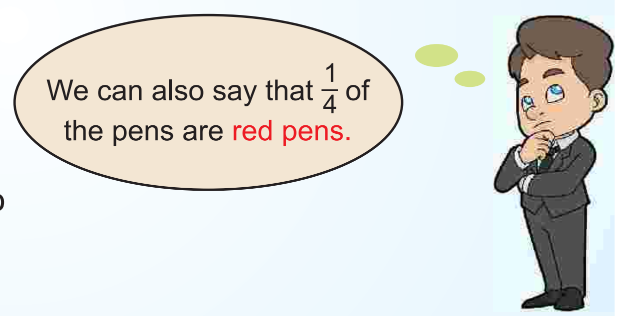
A departmental store sells pens in packs of four. There are 1 red and 3 blue pens in each pack.

We can compare the number of red pens to the total number of pens using ratio

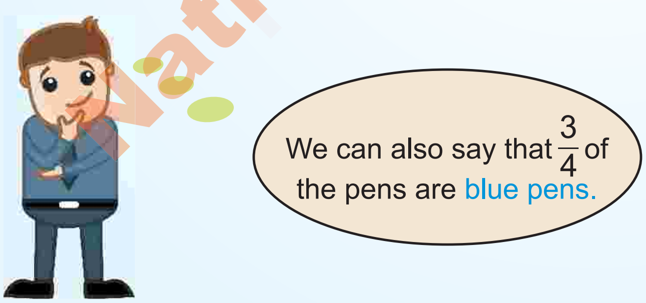


The ratio of the red pens to the total number of pens 1 : 4.

Since 1 in every 4 pens is a red pen, we say that the proportion of red pens is $\frac{1}{4}$ of the total number of pens.



The ratio of the number of blue pens to the total number of pens is 3 : 4

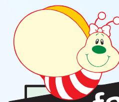


Since 3 in every 4 pens are blue pens, the proportion of blue pens is $\frac{3}{4}$ of the number of pens.

Look at the fractions

$$\frac{2}{3} = \frac{6}{9} \text{ (equivalent ratio)}$$

We can also write the above fractions as 2 : 3 = 6 : 9



Key fact

An equivalent ratio is also called a proportion.

The above expression can be written as 2 : 3 : : 6 : 9

The symbol ": :" is read as "is equivalent to" or "is the same as".

The number 2, 3, 6, 9 are called the terms of proportion.

In the proportion 2 : 3 : : 6 : 9

2 and 9 are called Extremes.

And 3 and 6 are called Means.

Product of Extremes = $2 \times 9 = 18$

Product of Means = $3 \times 6 = 18$

$\therefore$ Product of Extremes = Product of Means

Check whether 2, 5,10, 25 and 3, 7, 9, 28 are in proportion or not.

2 : 5 : : 10 : 25

$2 \times 25 = 5 \times 10$

$50 = 50$

Product of Means = $5 \times 10 = 50$

Product of Extremes = $25 \times 2 = 50$

Hence, are in proportion.

3 : 7 : : 9 : 28

$7 \times 9 = 3 \times 28$

$63 \neq 84$

Product of Means = $7 \times 9 = 63$

Product of Extremes = $28 \times 3 = 74$

Hence, are not in proportion.

TYPES OF PROPORTION

There are two types of proportion.
- Direct Proportion
- Inverse Proportion

Direct Proportion

The number of pens Zara bought and the total amount she paid for them are shown below.

No. of pens	1	2	3	4	5	6
Money Paid (Rs)	10	20	30	40	50	60

This table shows that the increase in the number of pens, the cost of pens also increase and keep the same ratio 10:1. These two quantities are in direct proportion.

$$\frac{10}{1}=\frac{20}{2}=\frac{30}{3}=\frac{40}{4}=\frac{50}{5}=\frac{60}{6}=10:1$$

If a, b, c, d are in direct proportion
then a : b : : c : d

$$\frac{a}{b}=\frac{c}{d}$$

$$a \times d = b \times c$$

Key fact

Direct Proportion

- It is a relation between two quantities such that when one quantity is increased, the other also increased in the same ratio:
- It is a relation between two quantities such that when one quantity is decreased , the other also decreased in the same ratio:

Let us consider any two ratio (from the above table).

2 : 20 and 5 : 50

Hence 2, 20 and 5, 50 are in direct proportion.

2 : 20 :: 5 : 50

$20 \times 5 = 50 \times 2$

$100 = 100$

Find the value of 'x' if x, 4, 8, 16 are in direct proportion.

$x : 4 :: 8 : 16$

$\frac{x}{4} = \frac{8}{16}$

$x = \frac{8}{16} \times 4$

$x = 2$

If a car uses 28 litres of petrol for 168 km, how far will it travel on 15 liters.

Let car travels x km in 15 litres

km : km = litres : litres

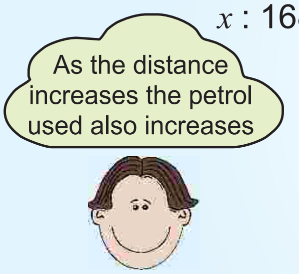
$x : 168 :: 15 : 28$

$\frac{x}{168} = \frac{15}{28}$

$x = \frac{15}{28} \times 168$

$x = 90$ km

The car will travel 90 km in 15 litres.



Distance (km)	Petrol (litres)
168 ↑	28 ↑
x	15

Inverse Proportion

The time taken by a bus moving between two towns at various speeds is shown in the following table.

Speed (km/h)	100	50	25	12.5
Time (hours)	1	2	4	8

As we see that speed of bus reduces, more time is required to reach the destination. The speed is inversely proportional to the time.

If a, b, c, d are in inverse proportion

then a : b : : d : c

$$\frac{a}{b} = \frac{d}{c}$$

$$a \times c = b \times d$$

Key fact

Inverse Proportion

- It is a relation between two quantities such that when one quantity is increased, the other decreased in the same ratio:
- It is a relation between two quantities such that when one quantity is decreased , the other increased in the same ratio:

5 men complete a task in 4 days. If there are 5 more men, how many days will they require to complete the same task?

Days	Men
4	5
x	10

Let the number of days be x

$$5 : 10 : x : 4$$

$$\frac{x}{4} = \frac{5}{10}$$

$$10\,x = 20$$

$$x = \frac{20}{10}$$

$$x = 2$$

It is an inverse proportion.
As the number of men increases the number of days decreases.

If 2 pipes fill a tank in 30 minutes, how many pipes of the same diameter can fill that tank in 5 minutes?

Let the number of pipes be x.

Number of pipes	Time (minutes)
2	30
x	5

It is an inverse proportion.

$$x : 2 :: 30 : 5$$

$$x \times 5 = 2 \times 30$$

$$x = \frac{2 \times 30}{5}$$

$$x = 12$$

Therefore the number of pipes = 12

Exercise 6.3

1. **Find the value of m in each of the following**

   (i) 7 : m : : 21 : 40

   (ii) 1: 11 : : m : 121

   (iii) 4 : 9 : : 16 : m

2. **A worker earns Rs. 560 in 4 days. How much will he earn in 14 days?**
3. **Kaleem cycled 36 km in 4 hours. How for can be go in 9 hours?**
4. **Mr. Iftikhar can type 120 words in 2 minutes. How many words can he type in 1 hour?**
5. **Sana paid Rs. 2400 to have a wall 2m by 3m painted. How much would she have to pay for a wall 4m by 4.5m?**
6. **A car travels 120 kilometers an hour. Find the distance traveled by the car in 90 minutes.**
7. **If 9 men can paint a school in 6 days, how long will 6 men take to paint that school?**
8. **Hareem takes 100 steps for walking a distance of 80 m. Find the distance covered by her in 175 steps.**
9. **Hassan takes 30 minutes to walk from his house to school at the speed of 4 kilometers per hour. How long will he take if he walks at the speed of 6 kilometers per hour?**
10. **Faiza used 75 GB data in 25 days. How much data will she use in 15 days.**
11. **100 men eat a certain quantity of food in one month. For how many days will that food be sufficient if 25 men join them?**
12. **A water tank can be emptied in 50 minutes by 5 pumps. How long will it take if 1 pump is out of order?**
13. **A project can be completed by 75 workers in 40 days. But project manager brought 15 more workers after 8 days. In how many days will the remaining work be finished?**

Words Board

- Ratio
- Rate
- Proportion
- Direct proportion
- Inverse proportion
- Equivalent Ratio
- Product of Means
- Product of Extremes

I have learnt

- A **ratio** is expressed as a fraction of the first quantity over the second. To find the ratio of two quantities, we must first express them in the same units. A ratio has no unit.
- A **rate** expresses a relationship involving two quantities of different kinds.
- A ratio is unchanged if both the terms of ratio are multiplied or divided by the same non-zero number.
- A proportion is a statement showing that two ratios are equivalent. If a, b, c, d are in proportion then $a : b :: c : d$.
- Direct proportion is a relation between two quantities such that when one quantity is increased (decreased), the other quantity is also increased (decreased) in the same ratio. If a, b, c, d are in direct proportion then $a : b :: c : d$ or $\frac{a}{b} = \frac{c}{d}$ or $ad = bc$
- Inverse proportion is the relation between two quantities such that when one quantity is increased (decreased), the other quantity is decreased (increased) in the same ratio. If a, b, c, d are in inverse proportion then $a : b :: d : c$ or $\frac{a}{b} = \frac{d}{c}$ or $ac = bd$

Review Exercise 6

1. Encircle the correct answer in the following questions:-

i. Ratio is the comparison of different quantities having the same...

(a) meaning (b) units (c) direction (d) length

ii. Which one cannot be the term of a ratio.

(a) -1 (b) 1 (c) 0 (d) 0.1

iii. a : b is equivalent to

(a) $a \div b$ (b) $b \div a$ (c) $a - b$ (d) $b - a$

iv. **Ratio between 12 m and 16 m is written as.**

(a) 12m : 16m (b) 12 : 16 (c) 3 : 4 (d) 3m : 4m

v. **The reduced form of the 16 : 20 is**

(a) 8 : 10 (b) 4 : 5
(c) 16 : 20 (d) 5 : 4

vi. **In the proportion c : d : : e : f the terms d and e are.**

(a) extremes (b) means
(c) equivalent (d) internal

vii. **A proportion is the statement such that two... are equivalent.**

(a) ratios (b) quantities (c) terms (d) differences

viii. **In the proportion 3 : 6 : : x : 12, the value of x is**

(a) 1 (b) 2 (c) 3 (d) 6

ix. **Which one is equivalent to the ratio 2 : 3**

(a) $\frac{18}{22}$ (b) $\frac{22}{18}$ (c) $\frac{27}{18}$ (d) $\frac{18}{27}$

x. **In 5 : 7, if we increase first element of the ratio up to 15, what will be the second element?**

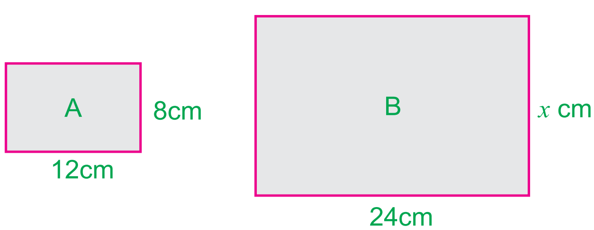
(a) 21 (b) 14 (c) 7 (d) 3.5

2. **The length and width of a rectangle are 6m and $\frac{9}{5}$ m respectively. Find the ratio of the width to the length.**

3. **Hanan is 18 years old. His elder brother Asim is 6 years older than Hanan. Find the ratio of Hanan's age to the Asim's age.**

4. **Ratio of mass of sugar to the mass of soap is 5 : 8. How many times soap is heavier than sugar ?**

5. **Find width of rectangle B if all quantities are in proportion.**



6. **Find the value of p if 3, p, 27, 36 are in**
   (i) Direct proportion (ii) Inverse proportion
7. **On a country map, the scale is written as 1 : 8000. Find the distance between two cities in kilometers if distance on the scale is 6cm.**
8. **A group of 80 people has food for 45 days. How long will food last, if 20 more people join them?**
9. **While traveling a certain distance, the wheel of a carriage of circumference 3.5m, makes 500 revolutions. How many revolutions will a wheel of circumference 4 m make while traveling the same distance?**

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Ministry of Federal Education & Professional Training
Vide letter No. F.1-1/2017-NCC Dated: 20th January 2020

قومی ترانہ

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تو نشانِ عزمِ عالی شان ارضِ پاکستان
مرکزِ یقین شاد باد!

پاک سر زمین کا نظام قُوّتِ اُخوّتِ عوام
قوم، مُلک، سلطنت پائندہ تابندہ باد!
شاد باد منزلِ مُراد!

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